

Partial correlations

Today

- path diagrams
- partial and semi-partial correlations

Causal relationships

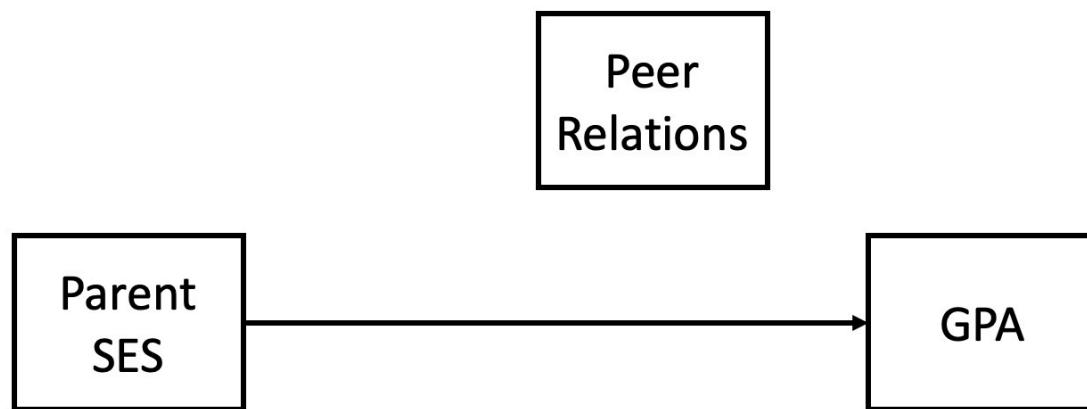
Does parent socioeconomic status *cause* better grades?

- $r_{GPA, SES} = .33, b = .41$

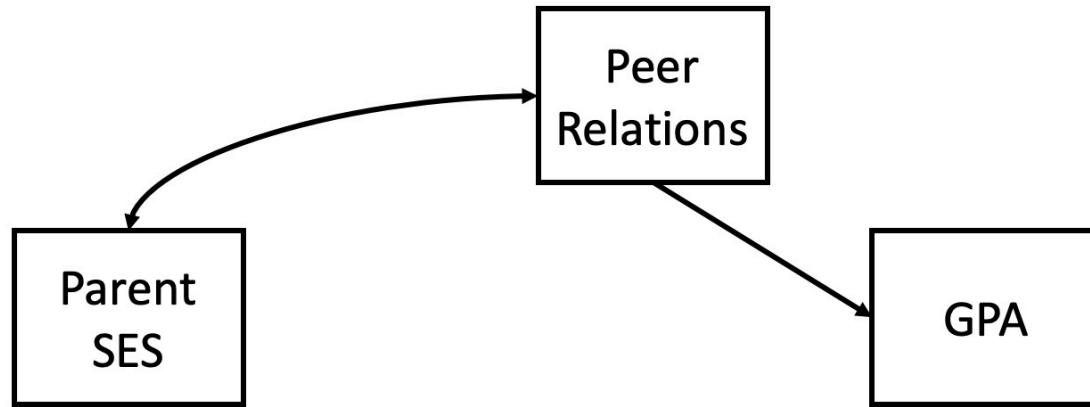
Potential confound: Peer relationships

- $r_{SES, peer} = .29$
- $r_{GPA, peer} = .37$

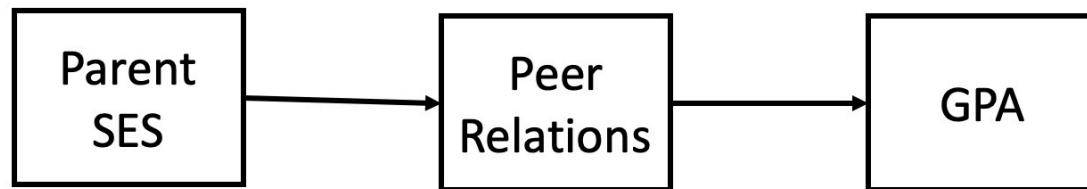
Does parent SES cause better grades?



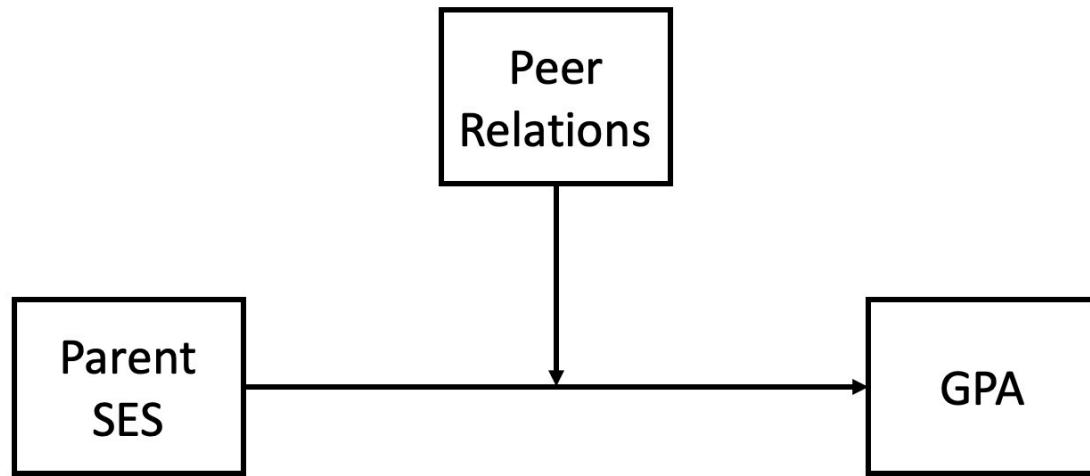
Spurious relationship



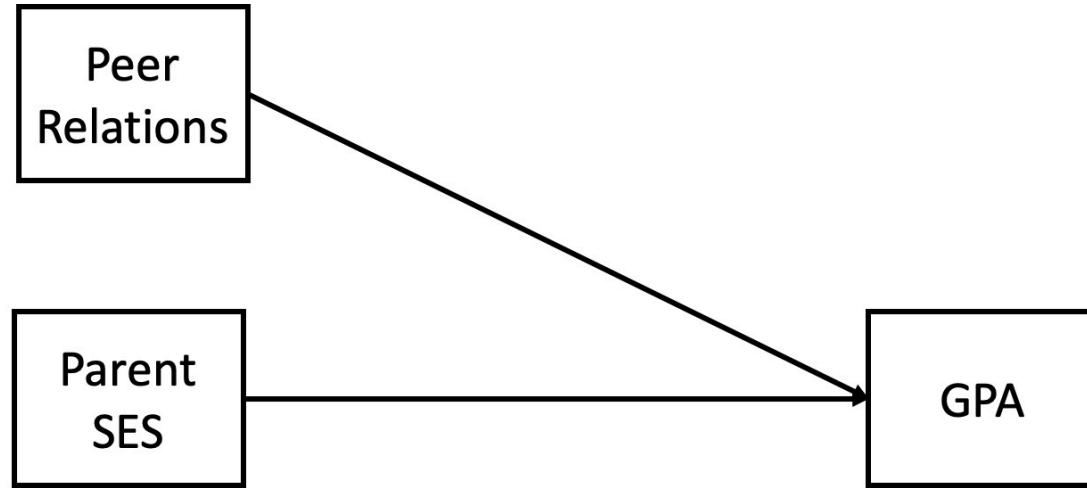
Indirect (mediation)



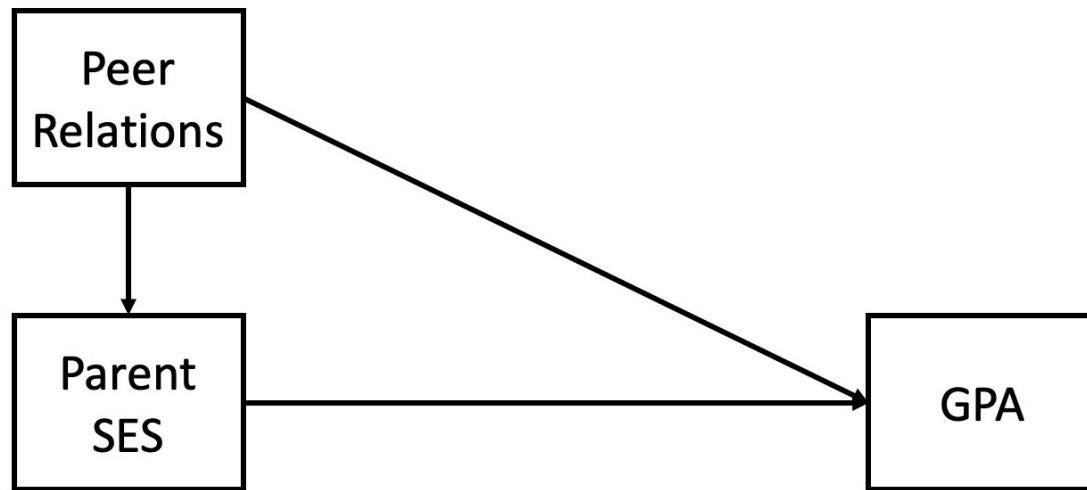
Interaction (moderation)



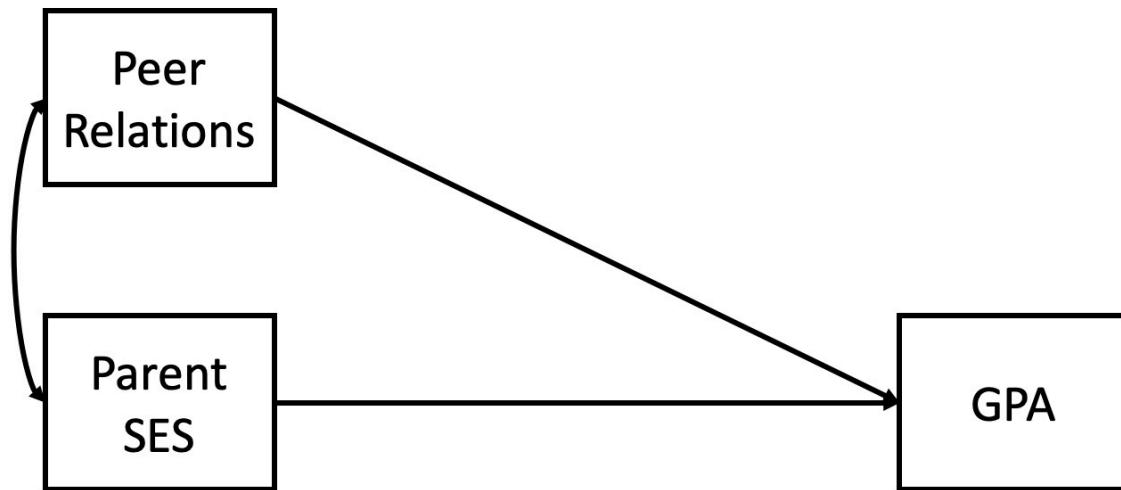
Multiple causes



Direct and indirect effects



Multiple regression



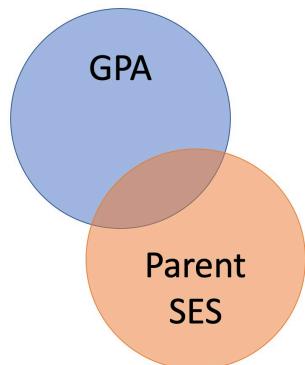
General regression model

$$\hat{Y} = b_0 + b_1 X_1 + b_2 X_2 + \cdots + b_k X_k$$

This is ultimately where we want to go.
Unfortunately, it's not as simply as multiplying
the correlation between Y and each X by the
ratio of their standard errors and stringing
them together.

Why?

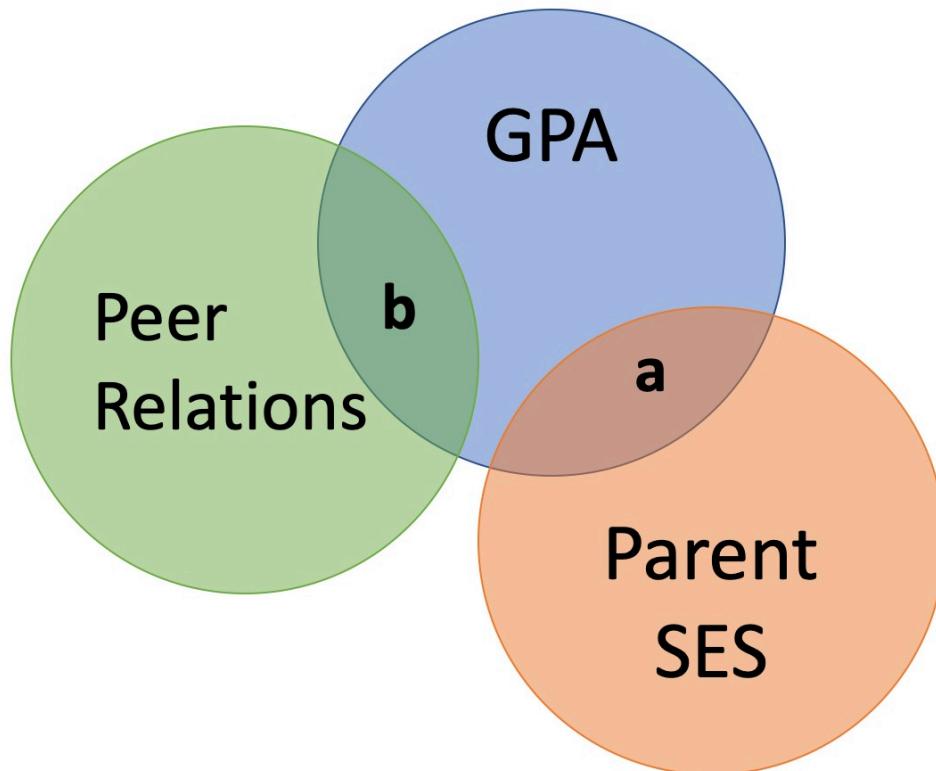
What is R^2 ?

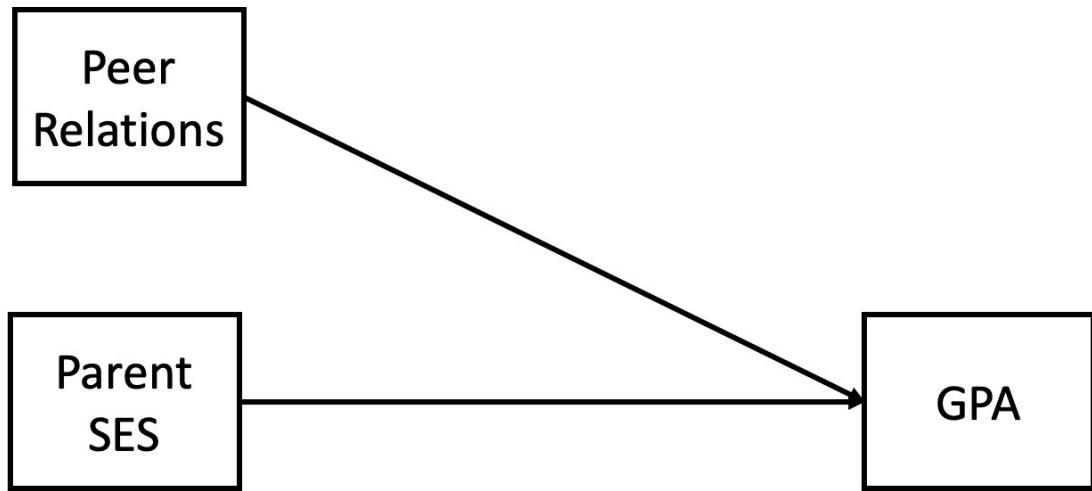


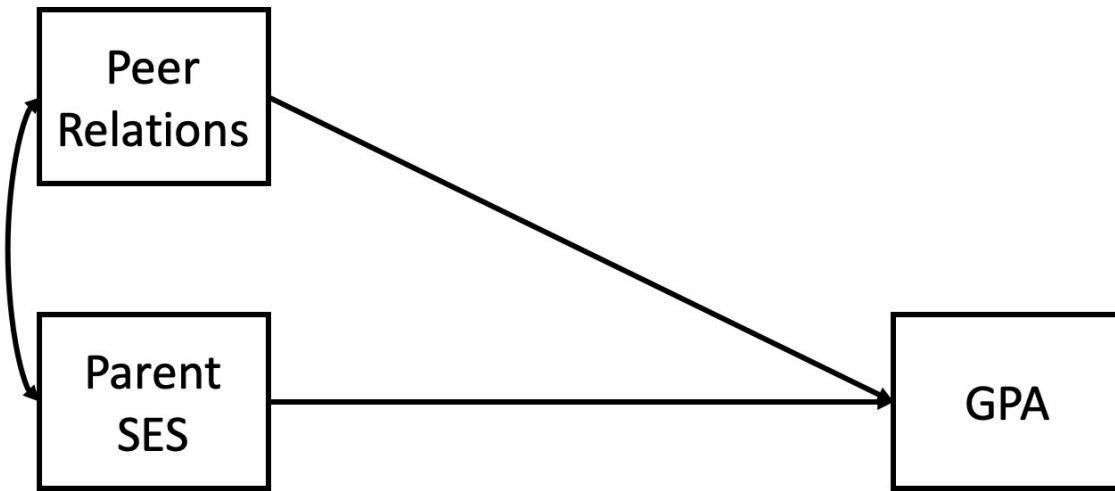
$$R^2 = \frac{s_{\hat{Y}}^2}{s_Y^2}$$

$$R^2 = \frac{SS_{\text{Regression}}}{SS_Y}$$

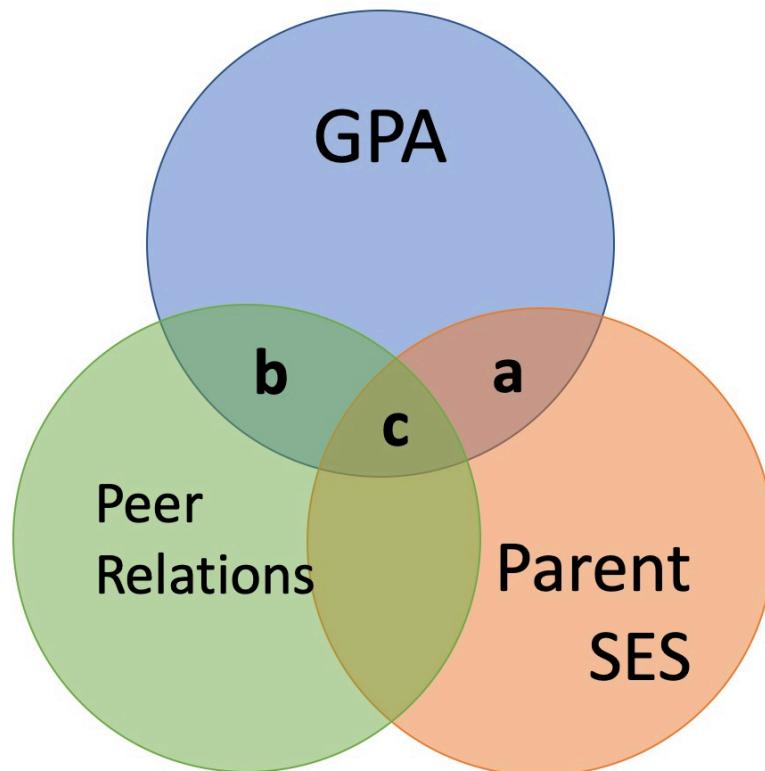
GPA = SES + Peer Relationships





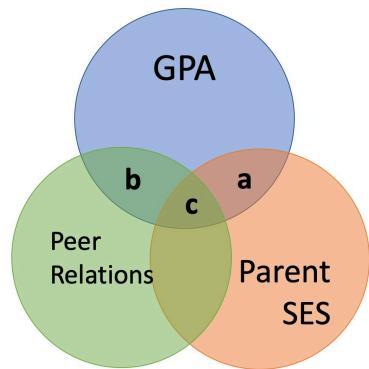


What is R^2 ?



What is R^2 ?

$$R^2_{Y.12} = a + b + c$$



How do we control for something?

Experimental Control

- Control variances (via equal groups)
- Randomly assign people to a group
- Conditions are the same except the IV

Statistical Control

- Control variance by removing unwanted variance from the other variables
- Control by "**partialling**"

Types of correlations

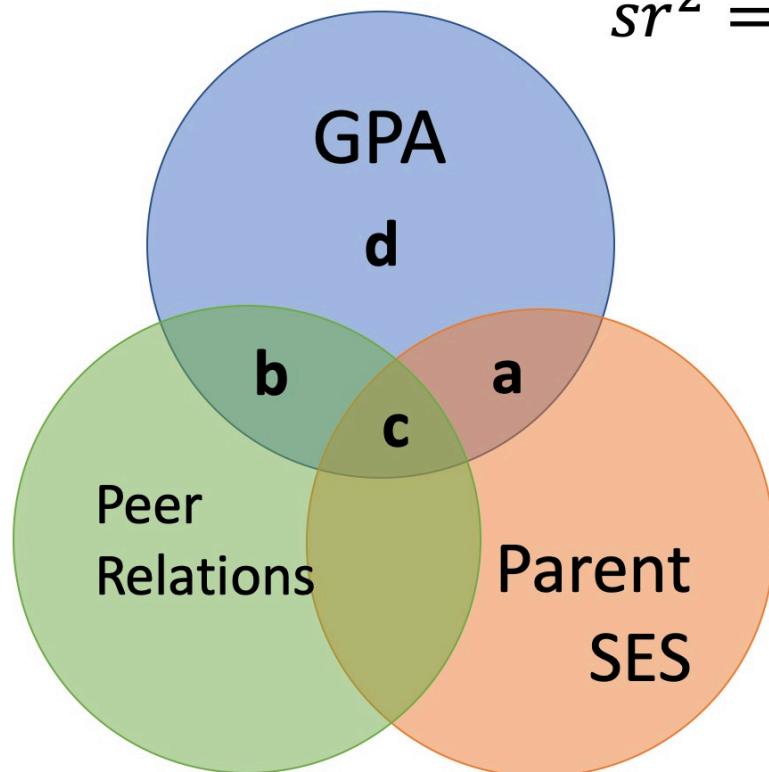
Pearson product moment correlation

- Standard correlation measure
- Ignores all outside variables
- aka bi-variate correlation
- Zero-order correlation
- Only two variables are X and Y

Semi-partial correlations

- This correlation assess the extent to which the part of X_1 *that is independent of* X_2 correlates with *all* of Y
- This is often the estimate that we refer to when we talk about **controlling for** another variable.
- Notation: sr or $r_{Y(1.2)}$; sr^2 is the semi-partial correlation, squared

$$sr^2 = \frac{a}{a + b + c + d}$$



Semi-partial correlations

$$sr_1 = r_{Y(1.2)} = \frac{r_{Y1} - r_{Y2}r_{12}}{\sqrt{1 - r_{12}^2}}$$

$$sr_1^2 = R_{Y.12}^2 - r_{Y2}^2$$

Notation:

- $R_{Y.12}^2$ is your standard R^2 -- variance explained in what is before the dot by what is after the dot. "*Variance explained in Y by X1 and X2*"
- $r_{Y(1.2)}$ is another way to write the semi-partial correlation.

Numbers to these correlations

$$r_{Y1} = .3 \quad r_{Y2} = .3 \quad r_{12} = .2$$

```
sr1 = (.3 - (.3*.2)) / sqrt(1 - (.2^2))
```

$$r_{Y1} = .3 \quad r_{Y2} = .3 \quad r_{12} = .5$$

```
sr2 = (.3 - (.3*.5)) / sqrt(1 - (.5^2))
sr1
```

```
## [1] 0.244949
```

```
sr2
```

```
## [1] 0.1732051
```

Types of correlations

Pearson product moment correlation

Semi-partial correlation

- This correlation assess the extent to which the part of X_1 *that is independent of* X_2 correlates with all of Y
- This is often the estimate that we refer to when we talk about **controlling for** another variable.
- Notation: sr or $r_{Y(1.2)}$; sr^2 is the semi-partial correlation, squared

Types of correlations

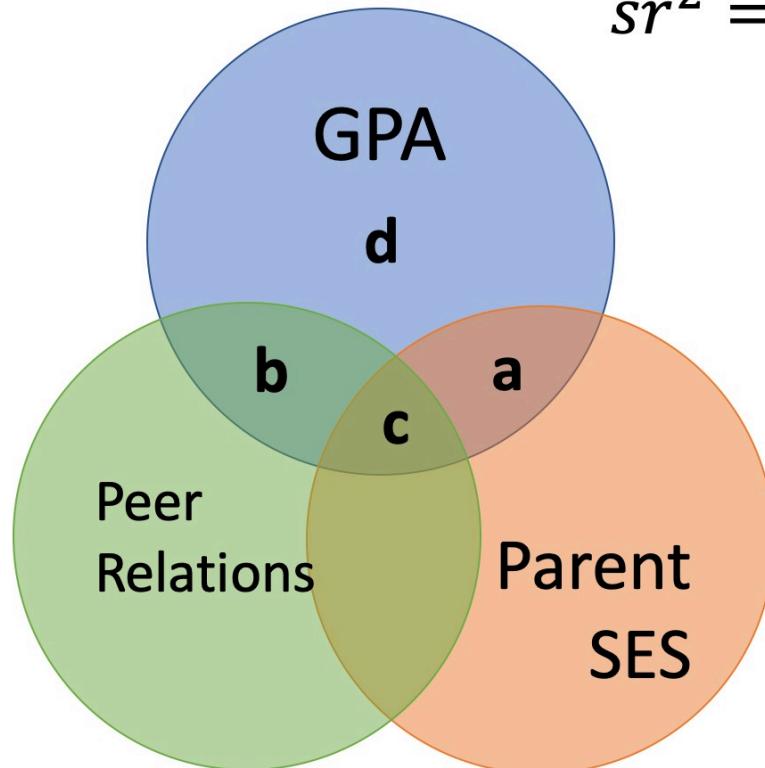
Pearson product moment correlation

Semi-partial correlation

Partial correlation

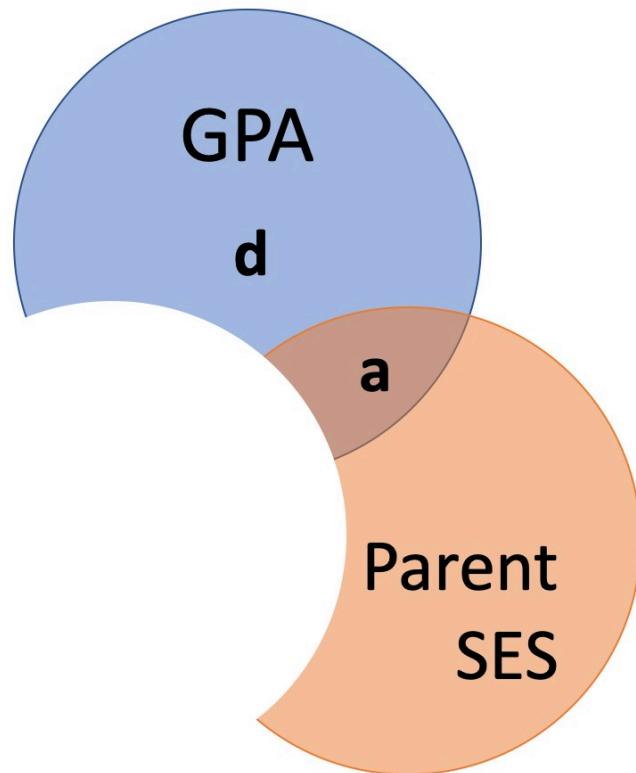
- The extent to which the part of X_1 *that is independent of X_2* is correlated with *the part of Y that is also independent of X_2* .
- Notation: pr or $r_{Y1.2}$; pr^2 is the partial correlation, squared

Partial Correlations



$$sr^2 = \frac{a}{a + b + c + d}$$

$$pr^2 = \frac{a}{a + d}$$



Partial correlations

$$\begin{aligned} pr_1 = r_{Y1.2} &= \frac{r_{Y1} - r_{Y2}r_{12}}{\sqrt{1 - r_{Y2}^2}\sqrt{1 - r_{12}^2}} \\ &= \frac{r_{Y(1.2)}}{\sqrt{1 - r_{Y2}^2}} \end{aligned}$$

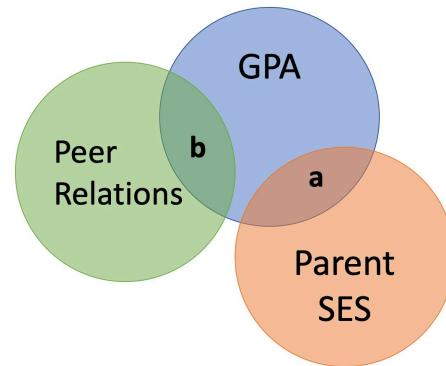
Partial correlation

$$pr^2 = \frac{R_{Y.12}^2 - r_{Y2}^2}{\sqrt{1 - r_{Y2}^2}}$$

$$sr^2 = R_{Y.12}^2 - r_{Y2}^2$$

What happens if X1 and X2 are uncorrelated?

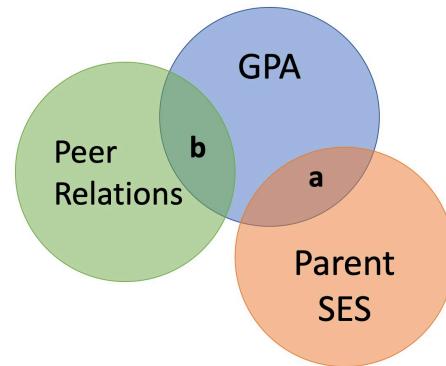
How does the semi-partial correlation compare to the zero-order correlation?



What happens if X1 and X2 are uncorrelated?

How does the semi-partial correlation compare to the zero-order correlation?

$$r_{Y(1.2)} = r_{Y1}$$

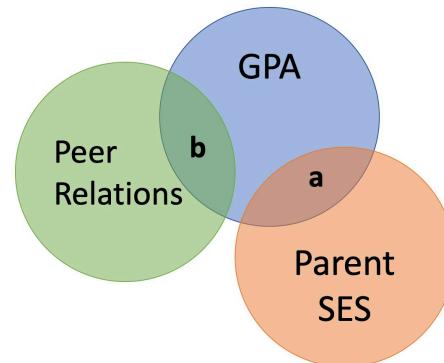


What happens if X1 and X2 are uncorrelated?

How does the semi-partial correlation compare to the zero-order correlation?

$$r_{Y(1.2)} = r_{Y1}$$

How does the partial correlation compare to the zero-order correlation?



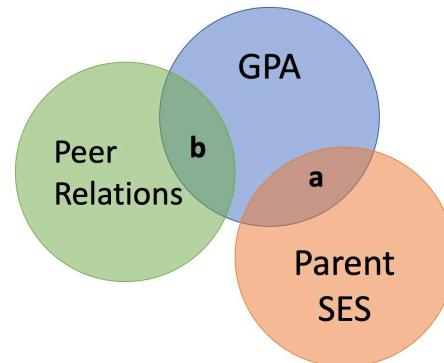
What happens if X1 and X2 are uncorrelated?

How does the semi-partial correlation compare to the zero-order correlation?

$$r_{Y(1.2)} = r_{Y1}$$

How does the partial correlation compare to the zero-order correlation?

$$r_{Y1.2} \neq r_{Y1}$$



When we use these?

Semi-partial: used when we want to show that some variable adds incremental variance in Y above and beyond another X variable. How much unique variance an IV adds to the TOTAL amount of variance.

- e.g., predicting Alzheimer's

Partial: used when some third variable, Z, is a plausible explanation of the correlation between X and Y. How much unique variance adds to the unaccounted variance in Y.sq

- e.g., predicting grades

Example with Flowers

Iris dataset from before. Sepal.Length,
Sepal.Width, Petal.Length

```
library(ppcor)
# zero-order correlation
round(cor(iris[,c("Sepal.Length", "Sepal.Width", "Petal.Length")]), 2)
```

| | Sepal.Length | Sepal.Width | Petal.Length |
|-----------------|--------------|-------------|--------------|
| ## Sepal.Length | 1.00 | -0.12 | 0.87 |
| ## Sepal.Width | -0.12 | 1.00 | -0.43 |
| ## Petal.Length | 0.87 | -0.43 | 1.00 |

Example with Flowers

Iris dataset from before. Sepal.Length,
Sepal.Width, Petal.Length

```
# semi-partial correlation  
round(spcor(iris[,c("Sepal.Length", "Sepal.Width", "Petal.Len
```

| | Sepal.Length | Sepal.Width | Petal.Length |
|-----------------|--------------|-------------|--------------|
| ## Sepal.Length | 1.00 | 0.28 | 0.91 |
| ## Sepal.Width | 0.52 | 1.00 | -0.67 |
| ## Petal.Length | 0.83 | -0.33 | 1.00 |

Example with Flowers

Iris dataset from before. Sepal.Length,
Sepal.Width, Petal.Length

```
# partial correlation  
# semi-partial correlation  
round(pcor(iris[,c("Sepal.Length", "Sepal.Width", "Petal.Length")]), 2)
```

| | Sepal.Length | Sepal.Width | Petal.Length |
|-----------------|--------------|-------------|--------------|
| ## Sepal.Length | 1.00 | 0.58 | 0.92 |
| ## Sepal.Width | 0.58 | 1.00 | -0.67 |
| ## Petal.Length | 0.92 | -0.67 | 1.00 |

e are part of Y that is independent of X

$$\hat{Y} = b_0 + b_1 X$$

$$e_i = Y_i - \hat{Y}_i$$

We can use this to construct a measure of X_1 that is independent of X_2 :

$$\hat{X}_{1.2} = b_0 + b_1 X_2$$

$$e_{X_1} = X_1 - \hat{X}_{1.2}$$

We run a regression where X_2 predicts X_1 . then we take the error from that, which is the part of X_1 that is independent of X_2 .

We can either correlate that value with Y, to calculate our semi-partial correlation:

$$r_{e,x_1,Y} = r_{y(1.2)}$$

Or we can calculate a measure of Y that is also independent of X_2 and correlate that with our X_1 residuals.

$$\hat{Y} = b_0 + b_1 X_2$$

$$e_y = Y - \hat{Y}$$

$$r_{e_{X_1}, e_y} = r_{y1.2}$$

Example

```
# create measure of Sepal.Length *independent* of Sepal.Width  
# model where X1 = SLength, X2 = SWidth  
# error is part of SLength that is independent of SWidth  
mod.sepals = lm(Sepal.Length ~ Sepal.Width, data = iris)  
indep.Sepals = broom::augment(mod.sepals)  
  
# correlate with Y = PLength to get semi-partial  
round(cor(indep.Sepals$.resid, iris$Petal.Length), digits = 2)  
  
## [1] 0.83
```

```
# Measure of Y = PLength that is also independent of X2 = SWi  
# create measure of Petal.Length independent of Sepal.Width s  
mod.petals = lm(Petal.Length ~ Sepal.Width, data = iris)  
indep.Petals = broom::augment(mod.petals)  
  
# Correlate those residuals with other residuals to get parti  
round(cor(indep.Sepals$.resid, indep.Petals$.resid), digits =  
  
## [1] 0.92
```

Who cares? Where are we going?

$$\hat{Y} = b_0 + b_1 X_1 + b_2 X_2 + \dots + b_p X_p$$

- Regression coefficients are "partial" regression coefficients
- Predicted change in \hat{Y} for a 1 unit change in X *holding all other predictors constant*
- Similar to semi-partial correlations -- represents part of each X

Next time...

Multiple regression!!