Interactions (V)

Last time...

Factorial ANOVA

- Estimated marginal means
- Estimated cell means
- Sums of squares

Power

- Omnibus test
- Coefficients

ANOVA vs Regression

Factorial ANOVA

Interaction tests whether there are differences in differences.

Simple main effect -- the effect of Factor A at a specific level of Factor B

Regression

Interaction tests whether slope changes.

Simple slopes -- the slope of Variable A at a specific level of Variable B

Power

The likelihood of finding an effect *if the effect actually exists.*

When calculating power for the omnibus test, use the expected multiple R^2 value to calculate an effect size: $f^2 = \frac{R^2}{1-R^2}$

To estimate power for a single coefficient, you need to consider (1) how much variance is accounted for by just the variable and (2) how much variance you'll account for in Y overall.
$f^2 = \frac{R^2_Y - R^2_{Y.X}}{1-R^2_Y}$

Effect sizes (interactions)

To start our discussion on powering interaction terms, we need to first consider the effect size of an interaction.

How big can we reasonably expect an interaction to be?

- Interactions are always partialled effects; that is, we examine the relationship between the product of variables X and Z only after we have controlled for X and controlled for Z. How does this affect the size of the relationship between XZ and Y?

Effect sizes (interactions)

The effect of XZ and Y will get **smaller** as X or Z (or both) is related to the product

The semi-partial correlation is always smaller than or equal to the zero-order correlation.

McClelland and Judd (1993)

Is it more difficult to find interaction effects in experimental studies or observational studies?

What factors make it relatively easier to find interactions in experimental work?

Influencing power in experimental studies

- No measurement error of IV
    - don't have to guess what condition a participant is in
    - measurement error is exacerbated when two variables measured with error are multiplied by each other
- Experimentalists are more likely to find cross-over interactions; observational studies may be restricted to fan interactions
    - cross-over interactions are easier to detect than fan interactions

Influencing power in experimental studies

- Experimentalists can concentrate scores on extreme ends on both X and Z
  - in observational studies, data tends to cluster around the mean
  - increases variability in both X and Z, and in XZ
- Experimentalists can also force orthognality in X and Z
- Experimentalists can study the full range of X in an experiment

McClelland and Judd's simulation

For the experiment simulations, we used 2 X 2 factorial designs, with values of X and Z equal to +1 and —1 and an equal number of observations at each of the four combinations of X and Z values.

```
X = rep(c(-1,1), each = 50)
Z = rep(c(-1,1), times = 50)
table(X,Z)
```

```
##       Z
## X      -1  1
##    -1 25 25
##    1  25 25
```

McClelland and Judd's simulation

For the field study simulations, they used values of X and Z that varied between the extreme values of +1 and —1. More specifically, in the field study simulations, values of X and Z were each sampled independently from a normal distribution with a mean of 0 and a standard deviation of 0.5. Values of X and Z were rounded to create equally spaced 9-point scales ranging from -1 to +1 because ranges in field studies are always finite and because ratings are often on scales with discrete intervals.

McClelland and Judd's simulation

For field studies

```
X = rnorm(n = 100, mean = 0, sd = .5)
Z = rnorm(n = 100, mean = 0, sd = .5)
X = round(X/.2)*.2
Z = round(Z/.2)*.2

psych::describe(data.frame(X,Z), fast = T)
```

```
##   vars   n  mean  sd median  min max range  skew kurtosis   se
## X    1 100 -0.05 0.5      0 -1.2 1.2   2.4  0.04    -0.41 0.05
## Z    2 100  0.03 0.5      0 -1.6 1.2   2.8 -0.44     0.63 0.05
```

For both: $\beta_0 = 0, \beta_X = \beta_Z = \beta_{XZ} = 1$.
$N = 100$, and randomly sampled normally distributed errors ($\mu = 0, \sigma = 4$)

```r
Y = 0 + 1*X + 1*Z + 1*X*Z + rnorm(n = 100, mean = 0, sd = 4)
summary(lm(Y ~ X*Z))
```

```
##
## Call:
## lm(formula = Y ~ X * Z)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -11.2002  -3.2569  -0.4427   3.1085  10.0917
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.7331     0.4710   1.557    0.123
## X             1.3502     0.9430   1.432    0.155
## Z             1.3674     0.9615   1.422    0.158
## X:Z          -2.8471     2.0902  -1.362    0.176
##
## Residual standard error: 4.679 on 96 degrees of freedom
```

```
set.seed(0305)
```

```
# for experimental studies
sim = 100
ebeta_xz = numeric(length = 1
et_xz = numeric(length = 100)
for(i in 1:sim){
  # simulate data
  X = rep(c(-1,1), each = 50)
  Z = rep(c(-1,1), times = 50


   Y = 0 + 1*X + 1*Z + 1*X*Z
    rnorm(n = 100, mean = 0,
  #run model
  model = lm(Y ~ X*Z)
  coef = coef(summary(model))
  #extract coefficients
  beta = coef["X:Z", "Estima
  t_val = coef["X:Z", "t valu
  #save to vectors
  ebeta_xz[i] = beta
  et_xz[i] = t_val
}
```

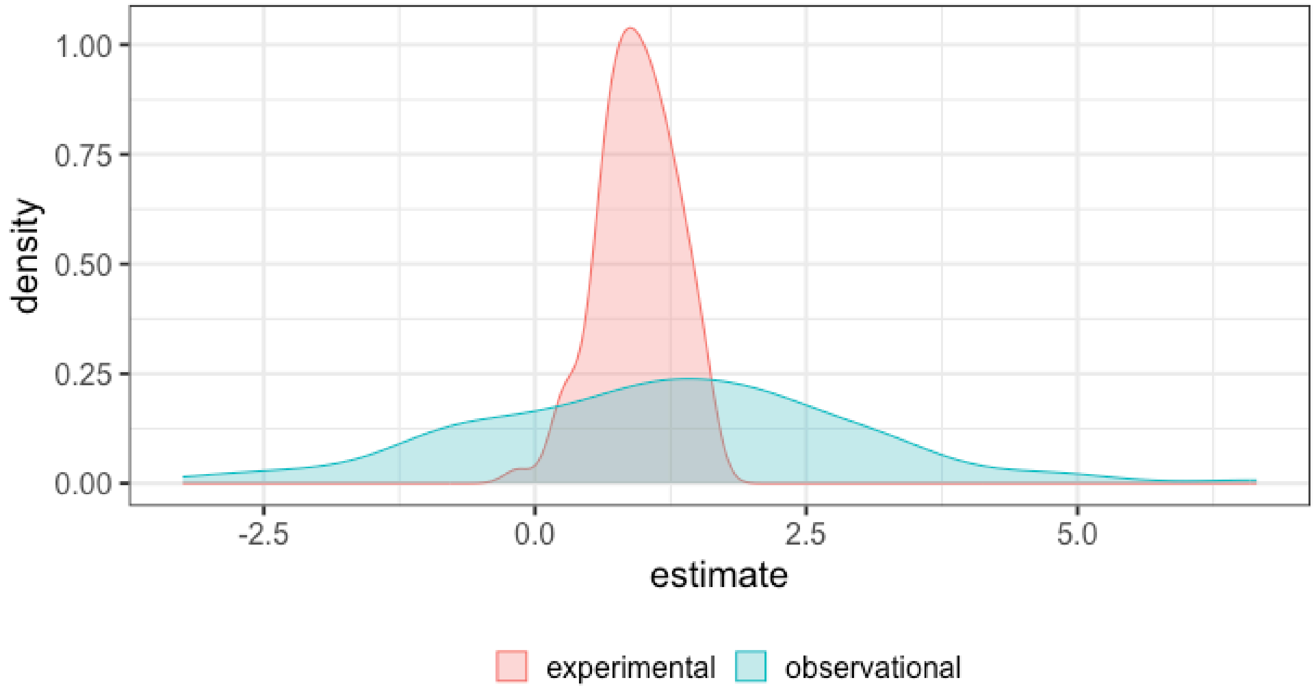
```
# for observational studies

obeta_xz = numeric(length = 1
ot_xz = numeric(length = 100)
for(i in 1:sim){
  # simulate data
  X = rnorm(n = 100, mean=0,
  Z = rnorm(n = 100, mean=0,
  X = round(X/.2)*.2
  Z = round(Z/.2)*.2
  Y = 0 + 1*X + 1*Z + 1*X*Z +
    rnorm(n = 100, mean = 0,
  #run model
  model = lm(Y ~ X*Z)
  coef = coef(summary(model))
  #extract coefficients
  beta = coef["X:Z", "Estima
  t_val = coef["X:Z", "t valu
  #save to vectors
  obeta_xz[i] = beta
  ot_xz[i] = t_val
}
```

```
mean(ebeta_xz)
```

```
## [1] 0.9440304
```

```
mean(obeta_xz)
```

```
## [1] 1.175444
```

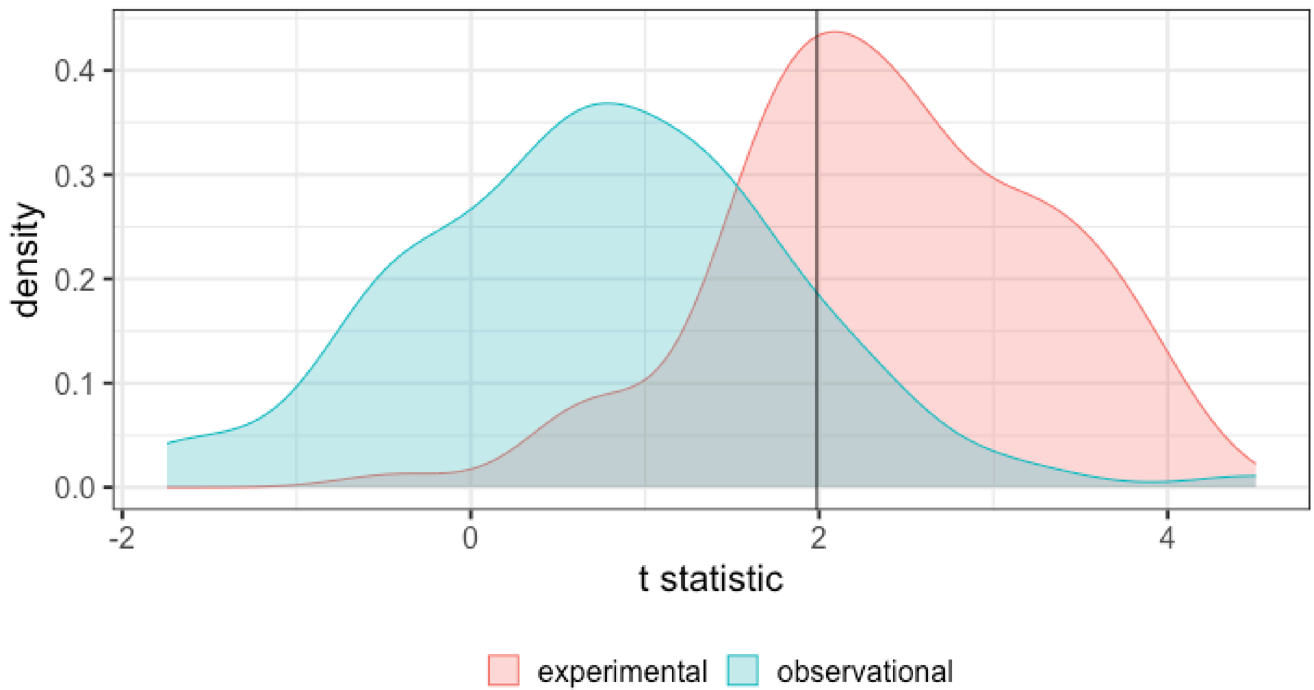


```r
mean(et_xz)
```

```
## [1] 2.383435
```

```r
mean(ot_xz)
```

```
## [1] 0.7411209
```



```r
cv = qt(p = .975, df = 100-3-1)
esig = et_xz > cv
sum(esig)
```

```
## [1] 66
```

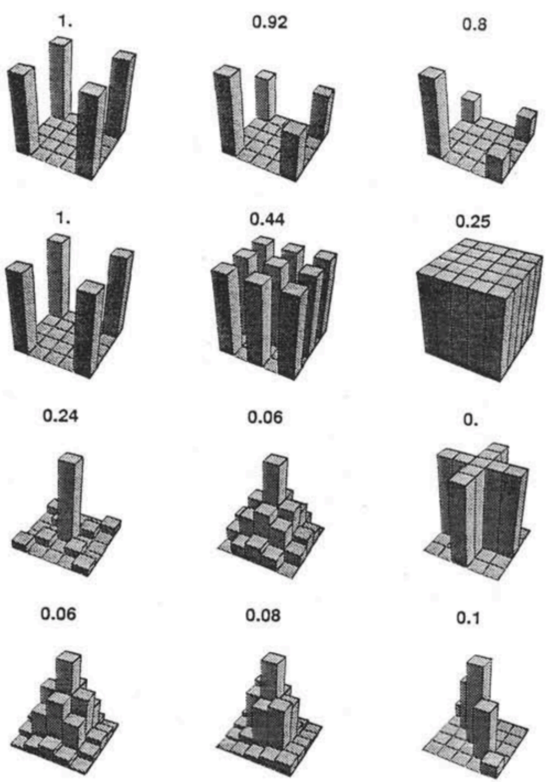
```r
osig = ot_xz > cv
sum(osig)
```

```
## [1] 12
```

In our simulation, 66% of experimental studies were statistically significant, whereas only 12% of observational studies were significant. Remember, we built our simulation based on data where there really is an interaction effect (i.e., the null is false).

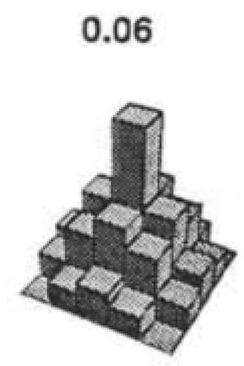
McClelland and Judd: 74% and 9%

Efficiency



Efficiency = the ratio of the variance of XZ (controlling for X and Z) of a design to the best possible design (upper left corner). High efficiency is better; best efficiency is 1.

Efficiency



If the optimal design has N observations, then to have the same power, any other design needs to have N*(1/efficiency).

So a design with .06 efficiency needs $\frac{1}{.06} = 16.67$ times the sample size to detect the effect.

Efficiency

This particular point has been "rediscovered" as recently as 2018:

- **you need *16* times the sample size to detect an interaction as you need for a main effect of the same size.**

- This generalizes to higher-order interactions as well. If you have a three-way interaction, you need 16*16 (256 times the number of people). More coming...

Observational studies: What NOT to do

Re-code X and Z into more extreme values (e.g., median splits)

- while this increases variance in X and Z, it also increases measurement error

Collect a random sample and then only perform analyses on the sub sample with extreme values

- reduces sample size and also generalizability

Observational studies: What NOT to do

What can be done?

M&J suggest oversampling extremes and using weighted and unweighted samples

Experimental studies: What NOT to do

Be mean to field researchers

Forget about lack of external validity and generalizability

Ignore power when comparing interaction between covariate and experimental predictors (ANCOVA or multiple regression with categorical and continuous predictors)

Polynomials

Non-linear relationships

Linear lines often make bad predictions -- very few processes that we study actually have linear relationships. For example, effort had diminishing returns (e.g., log functions), or small advantages early in life can have significant effects on mid-life outcones (e.g., exponentional functions). In cases where the direction of the effect is constant but changing in magnitude, the best way to handle the data is to transform a variable (usually the outcome) and run linear analyses.

```
log_y = log(y)
lm(log_y ~ x)
```

A small amount of anxiety is beneficial for performance on some tasks but too much is detrimental. When the **shape of the effect includes change(s) in direction**, then a **polynomial** term(s) may be more appropriate.

Polynomials are often a poor approximation for a non-linear effect. Correctly testing for non-linear effects usually requires (a) a lot of data and (b) making a number of assumptions about the data. Polynomial regression can be a useful tool for *exploratory* analysis and in cases when data are limited in terms of quantity and/or quality.

Polynomial regression

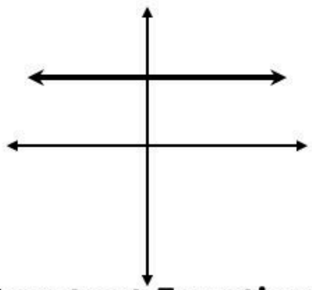
Polynomial regression is most often a form of hierarchical regression that systematically tests a series of higher order functions for a single variable.

$$\textbf{Linear: } \hat{Y} = b_0 + b_1 X$$

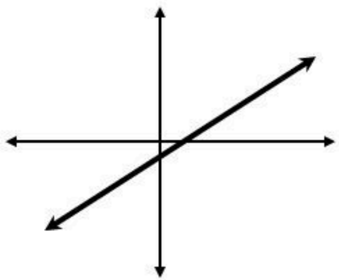
$$\textbf{Quadtratic: } \hat{Y} = b_0 + b_1 X + b_2 X^2$$

$$\textbf{Cubic: } \hat{Y} = b_0 + b_1 X + b_2 X^2 + b_3 X^3$$

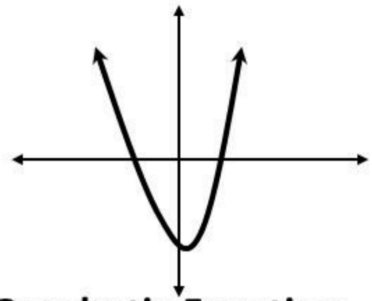
Graphs of Polynomial Functions:



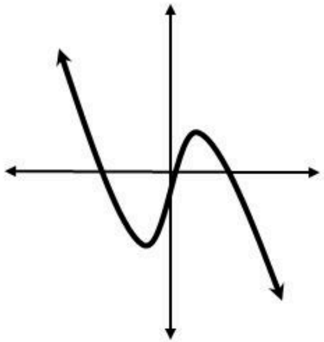
Constant Function
(degree = 0)



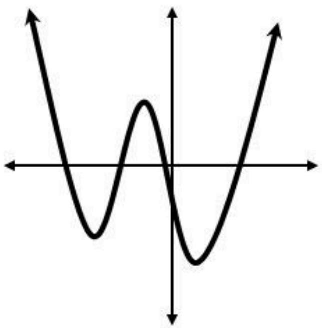
Linear Function
(degree = 1)



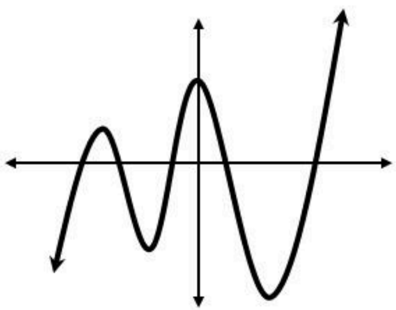
Quadratic Function
(degree = 2)



Cubic Function
(deg. = 3)

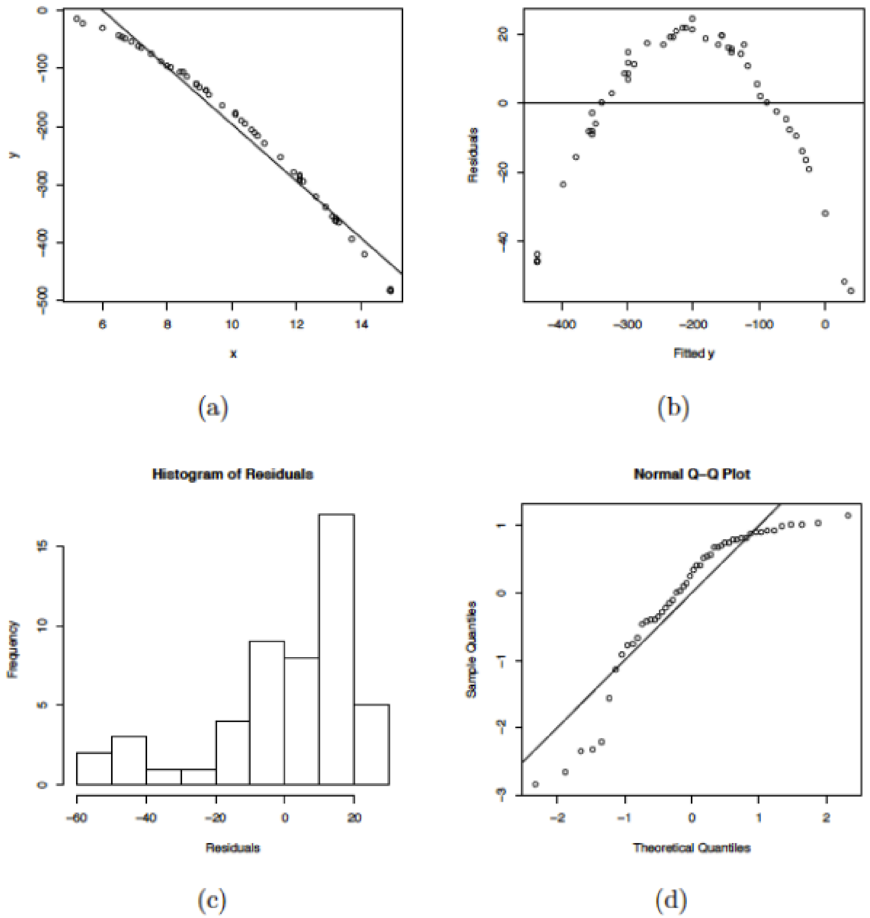


Quartic Function
(deg. = 4)



Quintic Function
(deg. = 5)

How do you know if somethir is up?



(a) Scatterplot of the quadratic data with the OLS line. (b) Residual plot for the OLS fit.

(c) Histogram of the residuals. (d) NPP for the Studentized residuals.

https://online.stat.psu.edu/stat501/book/export/html/962

Can a team have too much talent?

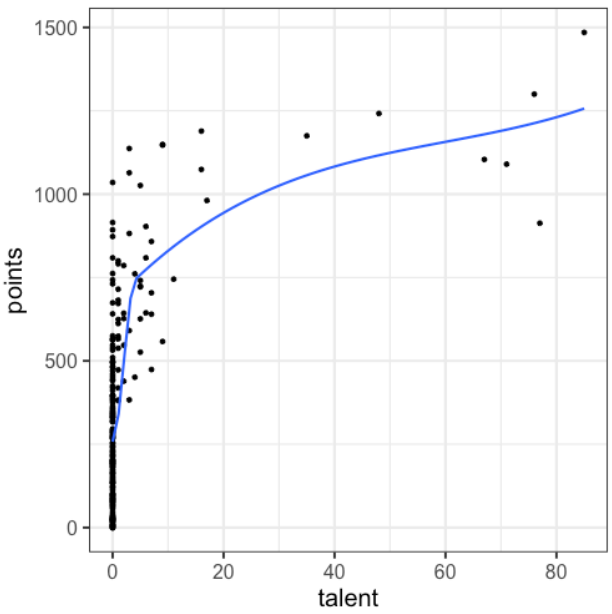
Do teams with too many talented players have poor intra-team coordination and perform worse than teams with a moderate amount of talent? They looked at 208 international football teams. Talent was the percentage of players during the 2010 and 2014 World Cup Qualifications phases who also had contracts with elite club teams. Performance was the number of points the team earned during these same qualification phases.

Swaab, R.I., Schaerer, M, Anicich, E.M., Ronay, R., and Galinsky, A.D. (2014). The too-much-talent effect: Team interdependence determines when more talent is too much or not enough. *Psychological Science 25*(8), 1581-1591.

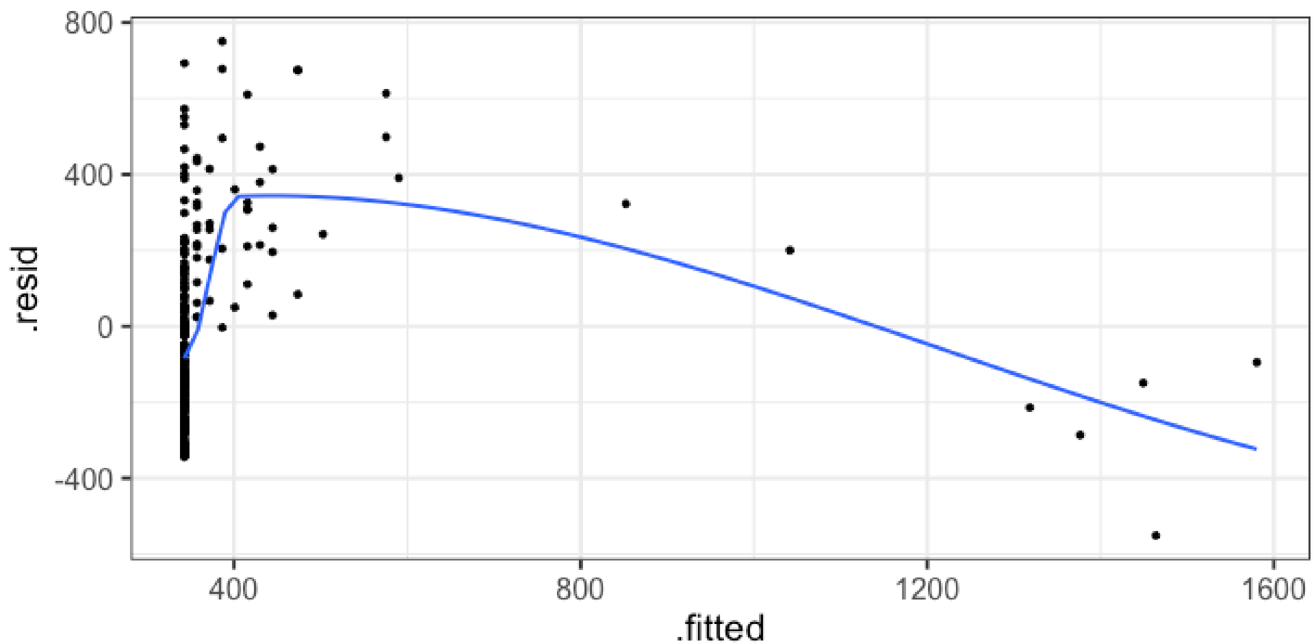
```r
head(football)
```

```
##       country points talent
## 1       Spain   1485     85
## 2     Germany   1300     76
## 3      Brazil   1242     48
## 4    Portugal   1189     16
## 5   Argentina   1175     35
## 6 Switzerland   1149      9
```

```r
ggplot(football, aes(x = tal
  geom_point() +
  geom_smooth(se = F) +
  theme_bw(base_size = 20)
```



```r
mod1 = lm(points ~ talent, data = football)
library(broom)
aug1 = augment(mod1)
ggplot(aug1, aes(x = .fitted, y = .resid)) +
  geom_point() +
  geom_smooth(se = F) +
  theme_bw(base_size = 20)
```



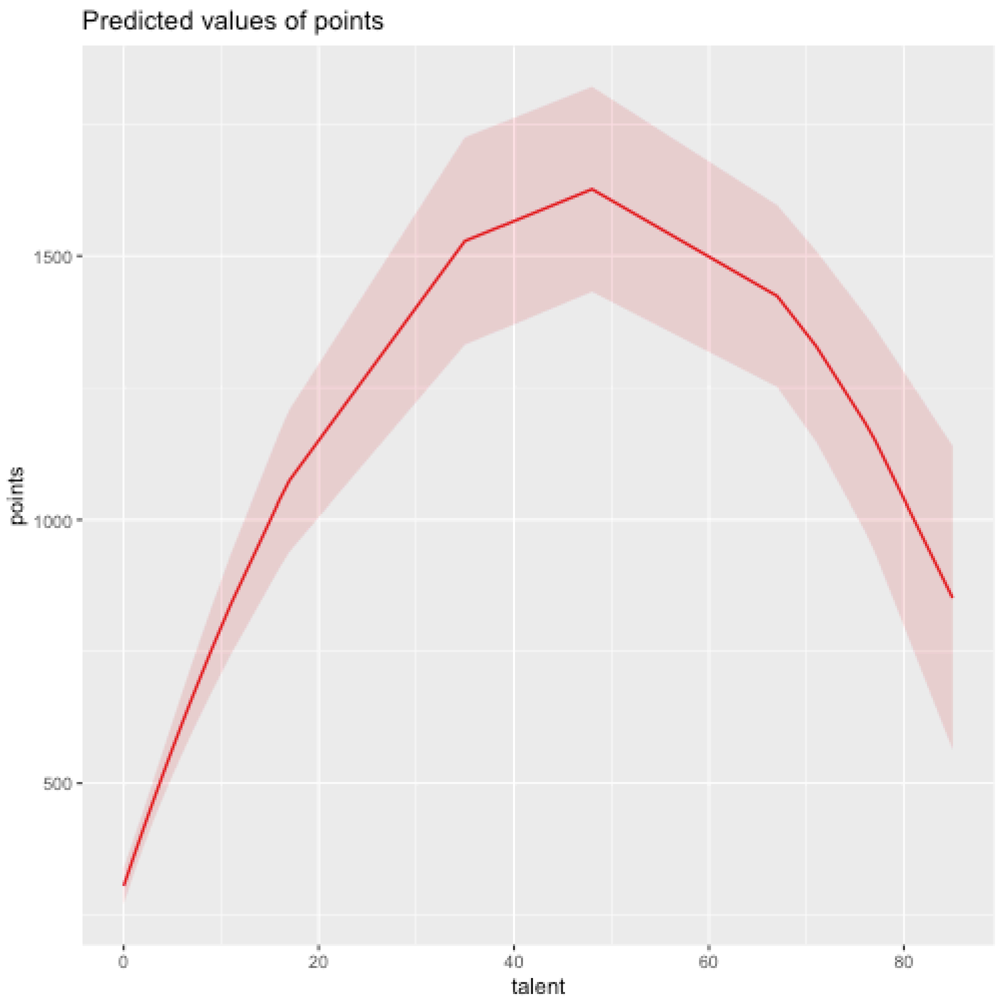
```
mod2 = lm(points ~ talent + I(talent^2), data = football)
```

The `I` stands for `identify`. It means that anything in `I()` should be treated according to it's arithmetic meaning. Sometimes R gets confused when you include `^` and `~` in the same formula. This `I()` prevents such issues.

```r
mod2 = lm(points ~ talent + I(talent^2), data = football)
summary(mod2)
```

```
##
## Call:
## lm(formula = points ~ talent + I(talent^2), data = football)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -384.66 -193.82  -35.34  152.11  729.66
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 305.34402   17.62668  17.323  < 2e-16 ***
## talent       54.89787    5.46864  10.039  < 2e-16 ***
## I(talent^2)  -0.57022    0.07499  -7.604 1.01e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 236.3 on 205 degrees of freedom
## Multiple R-squared:  0.4644,    Adjusted R-squared:  0.4592
## F-statistic: 88.87 on 2 and 205 DF,  p-value: < 2.2e-16
```

```r
library(sjPlot)
plot_model(mod2, type = "pred", terms = "talent")
```



Interpretation

The intercept is the predicted value of Y when $X = 0$

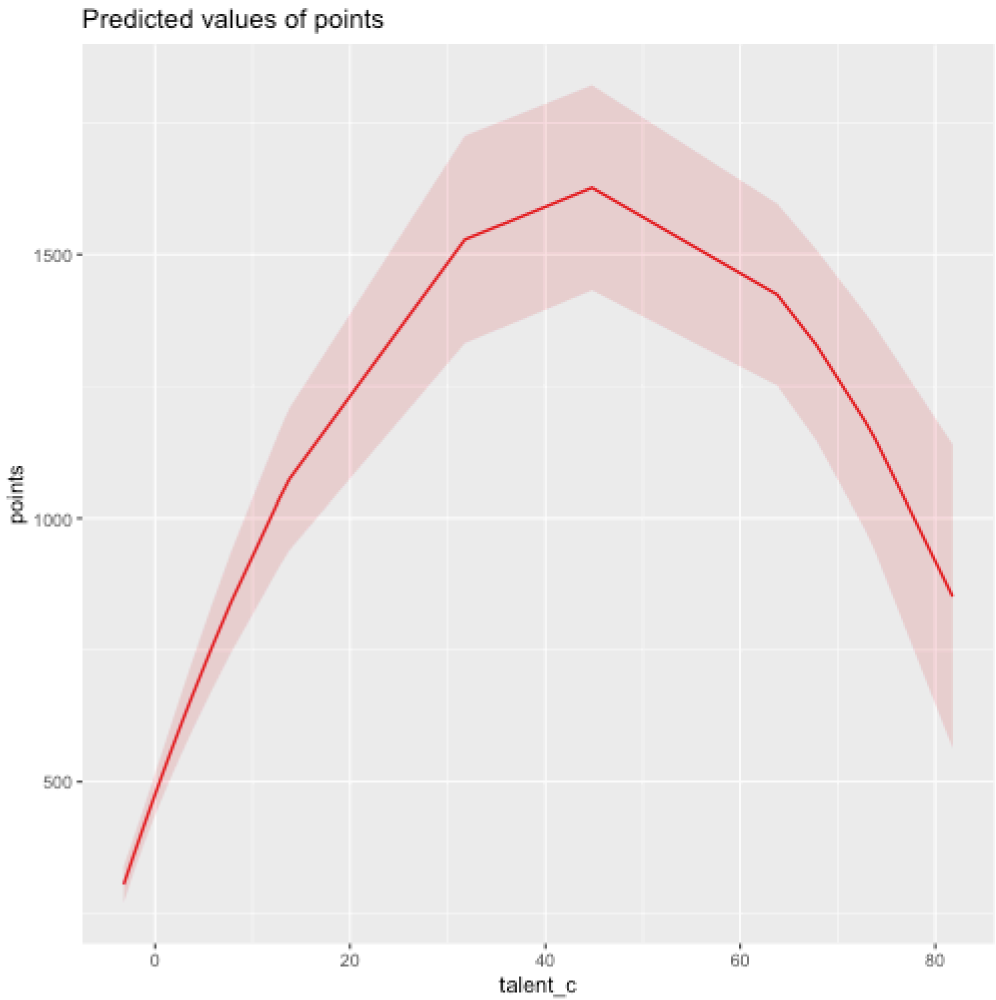
b_1 coefficient is the *tangent to the curve* when $X = 0$. In other words, this is the rate of change when X is equal to 0. If 0 is not a meaningful value on your X, you may want to center, as this will tell you the rate of change at the mean of X.

```
football$talent_c = football$talent - mean(football$talent)
mod2_c = lm(points ~ talent_c + I(talent_c^2), data = footbal
```

```r
summary(mod2_c)
```

```
##
## Call:
## lm(formula = points ~ talent_c + I(talent_c^2), data = football)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -384.66 -193.82  -35.34  152.11  729.66
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)   476.01572   19.94656  23.865  < 2e-16 ***
## talent_c       51.22982    5.00212  10.242  < 2e-16 ***
## I(talent_c^2)  -0.57022    0.07499  -7.604 1.01e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 236.3 on 205 degrees of freedom
## Multiple R-squared:  0.4644,	Adjusted R-squared:  0.4592
## F-statistic: 88.87 on 2 and 205 DF,  p-value: < 2.2e-16
```

```r
plot_model(mod2_c, type = "pred", terms = "talent_c")
```

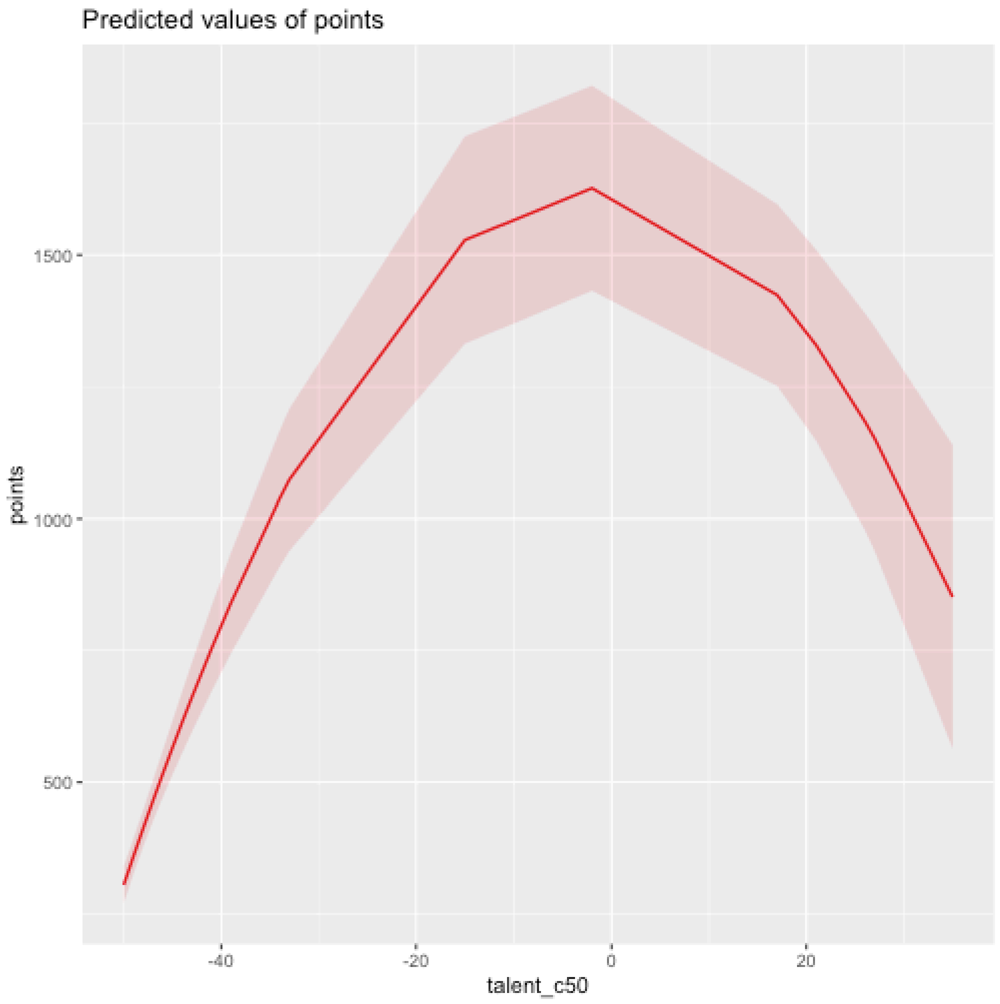


Or you can choose another value to center your predictor on, if there's a value that has a particular meaning or interpretation.

```
football$talent_c50 = football$talent - 50
mod2_50 = lm(points ~ talent_c50 + I(talent_c50^2), data = fo
```

```r
summary(mod2_50)
```

```
##
## Call:
## lm(formula = points ~ talent_c50 + I(talent_c50^2), data = football)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -384.66 -193.82  -35.34  152.11  729.66
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)     1624.68872   97.18998  16.717  < 2e-16 ***
## talent_c50        -2.12408    2.56568  -0.828    0.409
## I(talent_c50^2)   -0.57022    0.07499  -7.604 1.01e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 236.3 on 205 degrees of freedom
## Multiple R-squared:  0.4644,    Adjusted R-squared:  0.4592
## F-statistic: 88.87 on 2 and 205 DF,  p-value: < 2.2e-16
```


Predicted values of points
points
1500
1000
500
-40
-20
0
20
talent_c50

Interpretation

The b_2 coefficient indexes the acceleration, which is how much the slope is going to change. More specifically, $2 \times b_2$ is the acceleration:

> the rate of change in b_1 for a 1-unit change in X

You can use this to calculate the slope of the tangent line at any value of X you're interested in:

$$b_1 + (2 \times b_2 \times X)$$

```
tidy(mod2)
```

```
## # A tibble: 3 × 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)  305.      17.6         17.3  5.22e-42
## 2 talent        54.9      5.47        10.0  1.53e-19
## 3 I(talent^2)   -0.570    0.0750      -7.60 1.01e-12
```

At X = 10

```
54.9 + (2*-.570*10)
```

```
## [1] 43.5
```

At X = 70

```
54.9 + (2*-.570*70)
```

```
## [1] -24.9
```

Polynomials are interactions

A term for X^2 is a term for $X \times X$ or the multiplication of two independent variables holding the same values.

```
football$talent_2 = football$talent*football$talent
tidy(lm(points ~ talent + talent_2, data = football))
```

```
## # A tibble: 3 × 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)  305.      17.6         17.3  5.22e-42
## 2 talent        54.9      5.47        10.0  1.53e-19
## 3 talent_2      -0.570    0.0750      -7.60 1.01e-12
```

Polynomials are interactions

Put another way:

$$\hat{Y} = b_0 + b_1 X + b_2 X^2$$

$$\hat{Y} = b_0 + \frac{b_1}{2} X + \frac{b_1}{2} X + b_2 (X \times X)$$

The interaction term in another model would be interpreted as "how does the slope of X change as I move up in Z?" -- here, we ask "how does the slope of X change as we move up in X?"

When should you use polynomial terms?

You may choose to fit a polynomial term after looking at a scatterplot of the data or looking at residual plots. A U-shaped curve may be indicative that you need to fit a quadratic form -- although, as we discussed before, you may actually be measuring a different kind of non-linear relationship.

Polynomial terms should mostly be dictated by theory -- if you don't have a good reason for thinking there will be a change in sign, then a polynomial is not right for you.

Three-way interactions and beyond

Three-way interactions (regression)

Regression equation

$$\hat{Y} = b_0 + b_1X + b_2Z + b_3W + b_4XZ + b_5XW + b_6ZW + b_7XZW$$

The three-way interaction qualifies the three main effects (and any two-way interactions).

Like a two-way interaction, the three-way interaction is a conditional effect. And it is symmetrical, meaning there are several equally correct ways of interpreting it.

How do we describe a 3-way ANOVA?

A two-way (A x B) interaction means that the magnitude of one main effect (e.g., A main effect) depends on levels of the other variable (B). But, it is equally correct to say that the magnitude of the B main effect depends on levels of A. In regression, we refer to these as **conditional effects** and in ANOVA, they are called **simple main effects.**

A three-way interaction means that the *magnitude of one two-way interaction (e.g., A x B)* ***depends*** *on the levels of the remaining variable (C).*

A three-way interaction means that the *magnitude of one two-way interaction (e.g., A x B)* ***depends*** *on the levels of the remaining variable (C).*

It is equally correct to say that the magnitude of the A x C interaction depend on levels of B. Or, that the magnitude of the B x C interaction depends on levels of A. These are known as **simple interaction effects**.

```r
psych::describe(stress_data, fast = T)
```

```
##          vars   n  mean    sd median min  max range  skew kurtosis   se
## gender      1 150   NaN    NA     NA Inf -Inf  -Inf    NA       NA   NA
## bad_day     2 150  2.95  1.29      3   1    5     4  0.03    -1.11 0.11
## talk        3 150  2.57  1.20      2   1    5     4  0.32    -0.94 0.10
## stress      4 150 30.15 10.00     30   1   51    50 -0.03    -0.56 0.82
```

```r
table(stress_data$gender)
```

```
## 
## female   male 
##     67     83
```

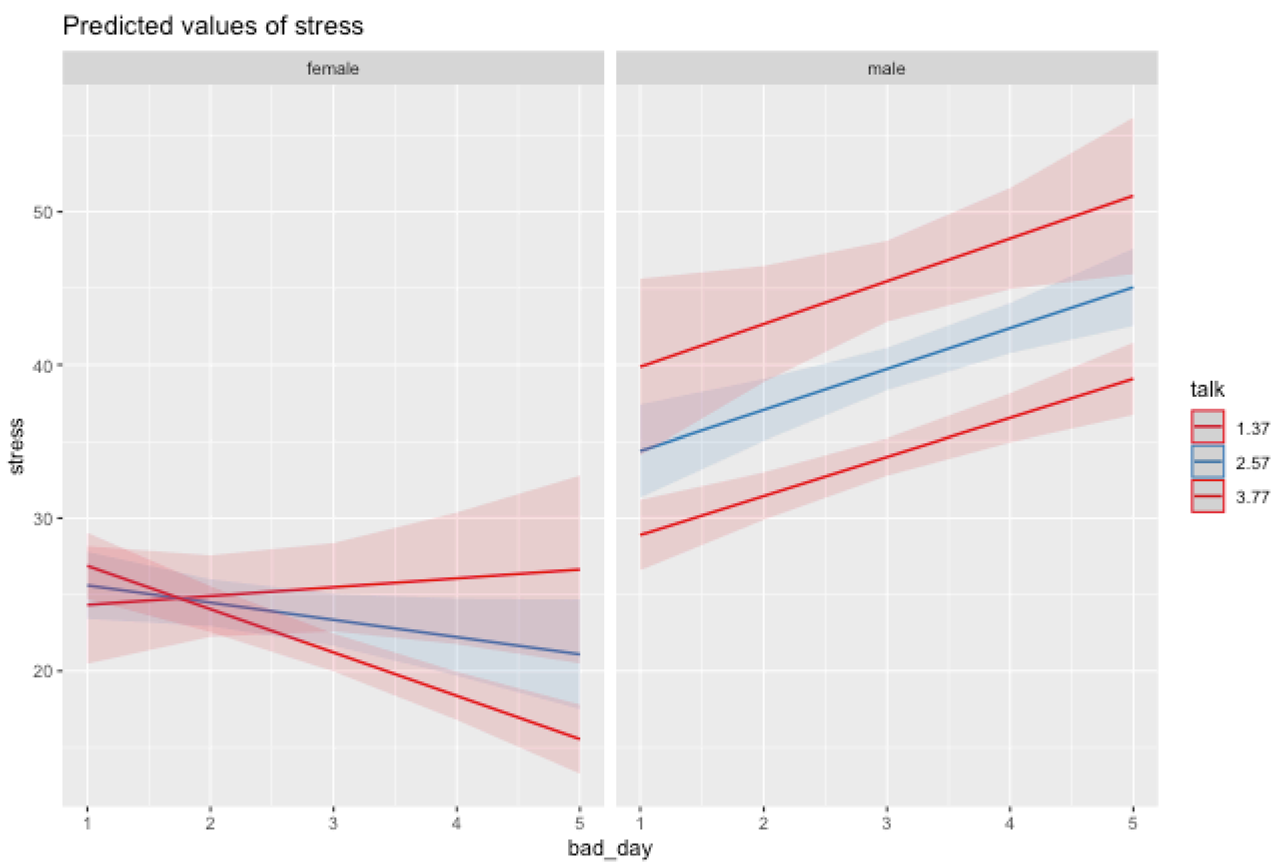
```r
mod_stress = lm(stress ~ bad_day*talk*gender, data = stress_d
summary(mod_stress)
```

```
##
## Call:
## lm(formula = stress ~ bad_day * talk * gender, data = stress_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -10.6126  -3.2974   0.0671   3.1129  10.7774
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)               20.3385     4.5181   4.502 1.39e-05 ***
## bad_day                    2.5273     1.6596   1.523  0.13003
## talk                       2.4870     1.3234   1.879  0.06227 .
## gendermale                -0.1035     5.7548  -0.018  0.98568
## bad_day:talk              -1.4220     0.4564  -3.116  0.00222 **
## bad_day:gendermale        -0.1244     2.0069  -0.062  0.95067
## talk:gendermale            1.9797     2.2823   0.867  0.38718
## bad_day:talk:gendermale    1.5260     0.7336   2.080  0.03931 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.733 on 142 degrees of freedom
```

```r
library(reghelper)
simple_slopes(mod_stress)
```

```
##     bad_day     talk gender Test Estimate Std. Error t value  df   Pr(>
## 1  1.661455 1.365903 sstest        5.8571     1.7437  3.3590 142 0.001
## 2  2.953333 1.365903 sstest        8.3892     1.5671  5.3535 142 3.386
## 3  4.245211 1.365903 sstest       10.9214     2.5606  4.2651 142 3.627
## 4  1.661455 2.566667 sstest       11.2788     1.4571  7.7406 142 1.687
## 5  2.953333 2.566667 sstest       16.1781     1.0985 14.7274 142 < 2.2
## 6  4.245211 2.566667 sstest       21.0775     1.6775 12.5647 142 < 2.2
## 7  1.661455 3.767431 sstest       16.7004     2.3888  6.9912 142 9.806
## 8  2.953333 3.767431 sstest       23.9670     1.4830 16.1608 142 < 2.2
## 9  4.245211 3.767431 sstest       31.2335     2.0578 15.1784 142 < 2.2
## 10 1.661455   sstest female        0.1245     0.7221  0.1724 142 0.863
## 11 2.953333   sstest female       -1.7125     0.5996 -2.8558 142 0.004
## 12 4.245211   sstest female       -3.5495     0.9450 -3.7561 142 0.000
## 13   sstest 1.365903 female        0.5850     1.0725  0.5455 142 0.586
## 14   sstest 2.566667 female       -1.1224     0.6181 -1.8160 142 0.071
## 15   sstest 3.767431 female       -2.8299     0.4630 -6.1115 142 9.006
## 16 1.661455   sstest   male        4.6396     1.0195  4.5510 142 1.137
## 17 2.953333   sstest   male        4.7741     0.6463  7.3862 142 1.177
## 18 4.245211   sstest   male        4.9085     0.9474  5.1813 142 7.418
## 19   sstest 1.365903   male        2.5451     0.5036  5.0533 142 1.315
## 20   sstest 2.566667   male        2.6700     0.6116  4.3655 142 2.427
## 21   sstest 3.767431   male        2.7950     1.2025  2.3244 142 0.021
```

```r
figures = plot_model(mod_stress, type = "int", mdrt.values =
figures[[4]]
```



As a reminder, centering will change all but the highest-order terms in a model.

```
stress_data = stress_data %>%
  mutate(bad_day_c = bad_day - mean(bad_day),
         talk_c = talk - mean(talk))
newmod = lm(stress ~ bad_day_c*talk_c*gender, data = stress_d
```

```r
tidy(mod_stress)
```

```
## # A tibble: 8 × 5
##   term                  estimate std.error statistic   p.value
##   <chr>                    <dbl>     <dbl>     <dbl>     <dbl>
## 1 (Intercept)             20.3       4.52     4.50   0.0000139
## 2 bad_day                  2.53      1.66     1.52   0.130
## 3 talk                     2.49      1.32     1.88   0.0623
## 4 gendermale              -0.104     5.75    -0.0180 0.986
## 5 bad_day:talk            -1.42      0.456   -3.12   0.00222
## 6 bad_day:gendermale      -0.124     2.01    -0.0620 0.951
## 7 talk:gendermale          1.98      2.28     0.867  0.387
## 8 bad_day:talk:gendermale  1.53      0.734    2.08   0.0393
```

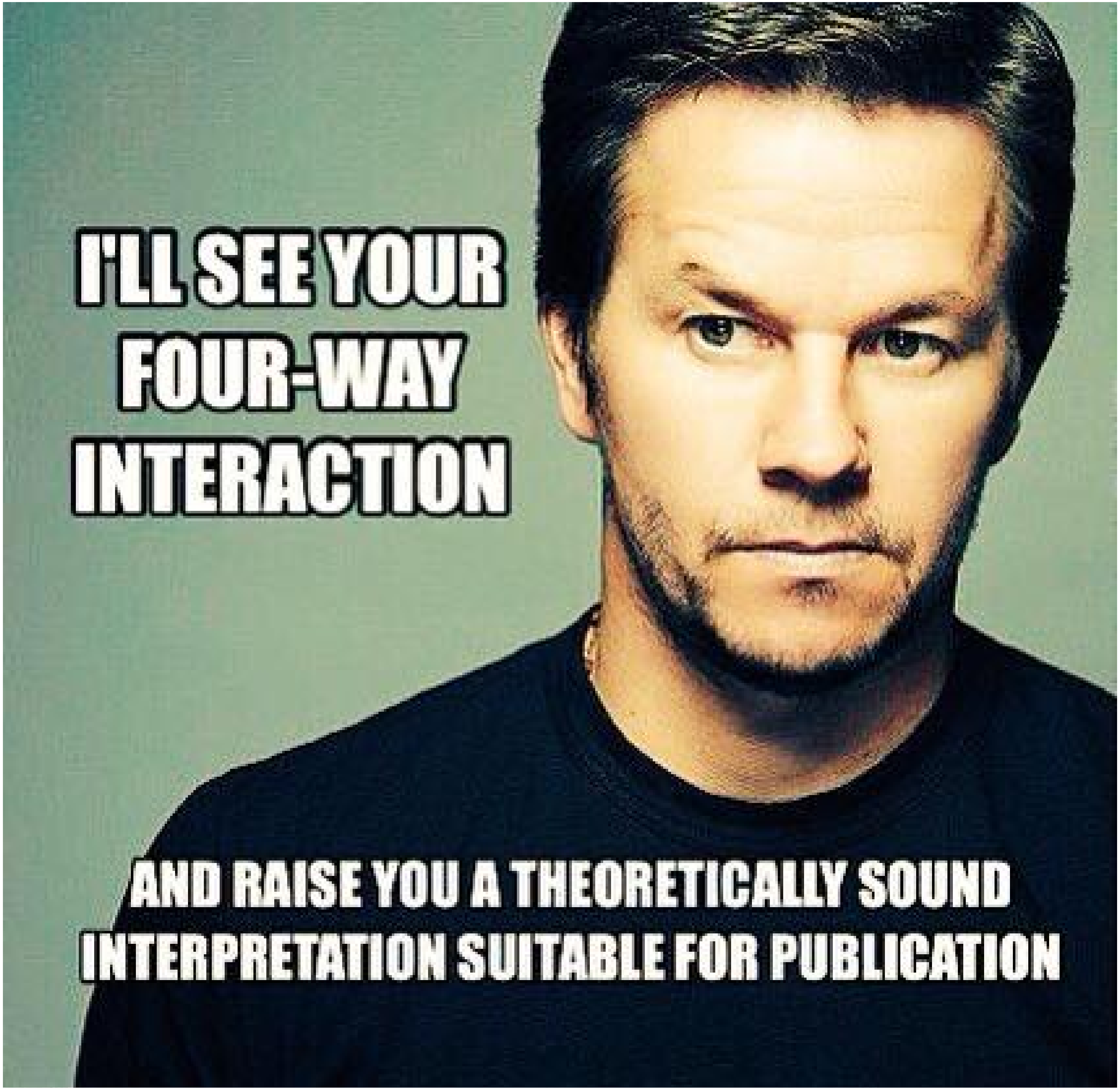
```r
tidy(newmod)
```

```
## # A tibble: 8 × 5
##   term                          estimate std.error statistic  p.value
##   <chr>                            <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)                      23.4      0.845     27.7  3.89e-59
## 2 bad_day_c                        -1.12     0.618     -1.82 7.15e- 2
## 3 talk_c                           -1.71     0.600     -2.86 4.94e- 3
## 4 gendermale                       16.2      1.10      14.7  2.20e-30
## 5 bad_day_c:talk_c                 -1.42     0.456     -3.12 2.22e- 3
## 6 bad_day_c:gendermale              3.79     0.870      4.36 2.47e- 5
## 7 talk_c:gendermale                 6.49     0.882      7.36 1.38e-11
## 8 bad_day_c:talk_c:gendermale       1.53     0.734      2.08 3.93e- 2
```

Four-way?

$$
\begin{aligned}
\hat{Y} = b_0 &+ b_1 X + b_2 Z + b_3 W + b_4 Q + b_5 XW \\
&+ b_6 ZW + b_7 XZ + b_8 QX + b_9 QZ + b_{10} QW \\
&+ b_{11} XZQ + b_{12} XZW + b_{13} XWQ + b_{14} ZWQ \\
&+ b_{15} XZWQ
\end{aligned}
$$

3-way (and higher) interactions are incredibly difficult to interpret, in part because they represent incredibly complicated processes. If you have a solid theoretical rationale for conducting a 3-day interaction, be sure you've collected enough subjects to power your test.


I'LL SEE YOUR FOUR-WAY INTERACTION
AND RAISE YOU A THEORETICALLY SOUND INTERPRETATION SUITABLE FOR PUBLICATION

Especially with small samples, three-way interactions may be the result of a few outliers skewing a regression line. If you have stumbled upon a three-way interaction during exploratory analyses, **be careful.** This is far more likely to be a result of over-fitting than uncovering a true underlying process.

Use at least one nominal moderator (ideally with only 2 levels), instead of all continuous moderators. This allows you to examine the 2-way interaction at each level of the nominal moderator, esp if one moderator is experimenter manipulated, which increases the likelihood of balanced conditions.

Next time...

Logistic Regression

Catch Up/Flex -- if nothing, then Ke will run the review session in class

March 27th = Exam 2!!!!!!!!!!

April 1st = discussion of Yarkoni & Westfall paper. You 100% need to read it. Not optional.

(everything after this slide is extra or code-based; feel free to play around, but you won't be tested on it)

```r
library(car)
fit = lm(Time~Speed*Noise, data = Data)
summary(aov(fit))
```

```
##               Df  Sum Sq Mean Sq F value   Pr(>F)    
## Speed          2 2805871 1402936 109.397  < 2e-16 ***
## Noise          2  341315  170658  13.307 4.25e-06 ***
## Speed:Noise    4  295720   73930   5.765 0.000224 ***
## Residuals    171 2192939   12824                     
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
Anova(fit, type = 2)
```

```
## Anova Table (Type II tests)
## 
## Response: Time
##              Sum Sq  Df  F value    Pr(>F)    
## Speed       2805871   2 109.3975 < 2.2e-16 ***
## Noise        341315   2  13.3075 4.252e-06 ***
## Speed:Noise  295720   4   5.7649 0.0002241 ***
## Residuals   2192939 171                       
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
fit = lm(Time~Speed*Noise, data = Data)
summary(fit)
```

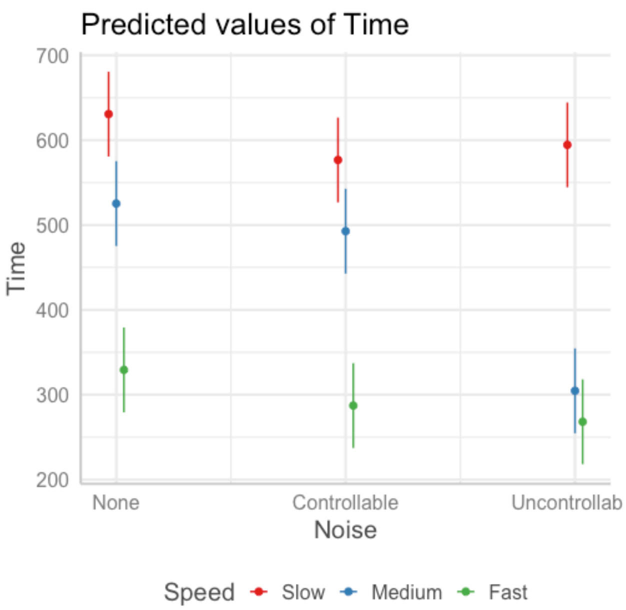
```
##
## Call:
## lm(formula = Time ~ Speed * Noise, data = Data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -316.23  -70.82    4.99   79.87  244.40
##
## Coefficients:
##                                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)                       630.72      25.32  24.908  < 2e-16 *
## SpeedMedium                      -105.44      35.81  -2.944  0.00369 *
## SpeedFast                        -301.45      35.81  -8.418 1.49e-14 *
## NoiseControllable                 -54.05      35.81  -1.509  0.13305
## NoiseUncontrollable               -36.28      35.81  -1.013  0.31243
## SpeedMedium:NoiseControllable      21.48      50.64   0.424  0.67201
## SpeedFast:NoiseControllable        12.01      50.64   0.237  0.81287
## SpeedMedium:NoiseUncontrollable  -184.39      50.64  -3.641  0.00036 *
## SpeedFast:NoiseUncontrollable     -24.84      50.64  -0.490  0.62448
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 113.2 on 171 degrees of freedom
```

```r
library(emmeans)
fit.grid = ref_grid(fit)
pairs(fit.grid, adjust = "holm")
```

```
##  contrast                                    estimate   SE  df t.ratio
##  Slow None - Medium None                        105.4 35.8 171   2.944
##  Slow None - Fast None                          301.4 35.8 171   8.418
##  Slow None - Slow Controllable                   54.1 35.8 171   1.509
##  Slow None - Medium Controllable                138.0 35.8 171   3.854
##  Slow None - Fast Controllable                  343.5 35.8 171   9.592
##  Slow None - Slow Uncontrollable                 36.3 35.8 171   1.013
##  Slow None - Medium Uncontrollable              326.1 35.8 171   9.106
##  Slow None - Fast Uncontrollable                362.6 35.8 171  10.124
##  Medium None - Fast None                        196.0 35.8 171   5.473
##  Medium None - Slow Controllable                -51.4 35.8 171  -1.435
##  Medium None - Medium Controllable               32.6 35.8 171   0.910
##  Medium None - Fast Controllable                238.1 35.8 171   6.648
##  Medium None - Slow Uncontrollable              -69.2 35.8 171  -1.931
##  Medium None - Medium Uncontrollable            220.7 35.8 171   6.162
##  Medium None - Fast Uncontrollable              257.1 35.8 171   7.180
##  Fast None - Slow Controllable                 -247.4 35.8 171  -6.908
##  Fast None - Medium Controllable               -163.4 35.8 171  -4.564
##  Fast None - Fast Controllable                   42.0 35.8 171   1.174
##  Fast None - Slow Uncontrollable               -265.2 35.8 171  -7.405
##  Fast None - Medium Uncontrollable               24.7 35.8 171   0.689
##  Fast None - Fast Uncontrollable                 61.1 35.8 171   1.707
```

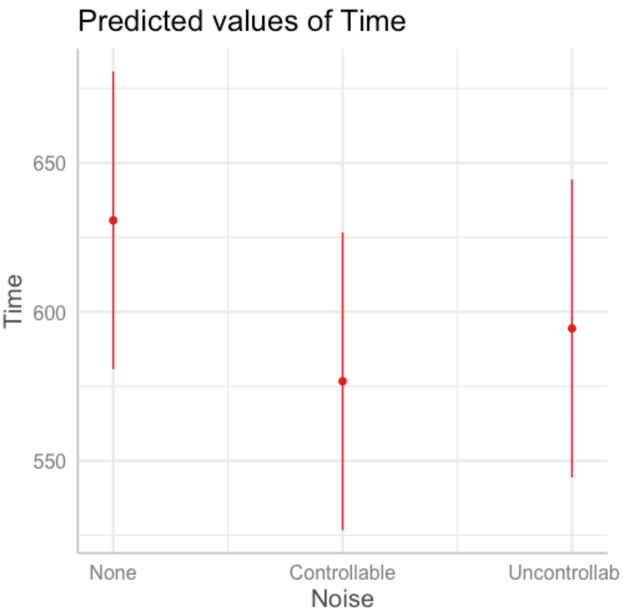
In `sjPlot()`, predicted values are the expected value of Y given all possible values of X, **at specific values of M**. If you don't give it all of M, it will choose every possible value.

```
library(sjPlot)
plot_model(fit, type = "pred
           terms = c("Noise"
  theme_sjplot(base_size = 2(
  theme(legend.position = "b(
```



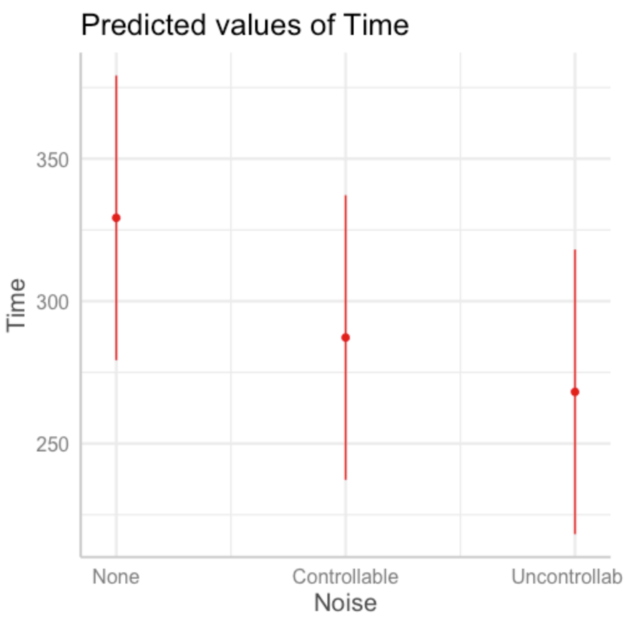
In `sjPlot()`, predicted values are the expected value of Y given all possible values of X, **at specific values of M**. If you don't specify levels of M, it will choose the lowest possible value.

```
library(sjPlot)
plot_model(fit, type = "pred
           terms = c("Noise"
  theme_sjplot(base_size = 2(
  theme(legend.position = "b(
```



In `sjPlot()`, predicted values are the expected value of Y given all possible values of X, **at specific values of M**. If you don't specify levels of M, it will choose the lowest possible value.

```
library(sjPlot)
plot_model(fit, type = "pred
            terms = c("Noise"
  theme_sjplot(base_size = 2(
  theme(legend.position = "b(
```



In `sjPlot()`, estimated marginal means are the expected value of Y given all possible values of X, **ignoring M**.

```
library(sjPlot)
plot_model(fit, type = "emm"
             terms = c("Noise")
  theme_sjplot(base_size = 2(
  theme(legend.position = "b(
```