SEMESTER I SESSION 2025/2026
SECI1013 DISCRETE STRUCTURE

ASSIGNMENT 2

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SECI1013: DISCRETE STRUCTURES

SESSION 2025/2026 – SEMESTER 1

ASSIGNMENT 2 (CHAPTER 2 – RELATION, FUNCTION & RECURRENCE

INSTRUCTIONS:

a. This assignment must be conducted in a group. Please clearly write the group members' names & matric numbers on the front page of the submission.
b. Solutions for each question must be readable and neatly written on plain A4 paper. Every step or calculation should be properly shown. Failure to do so will result in the rejection of the submission of the assignment.
c. This assignment consist of 7 questions (60 marks), contributing 5% of overall course marks.

Question 1 **[9 marks]**

Let D = {1,3,5}. Define R on D where $x, y \in D, xRy$ if 3x + y is a multiple of 6.

i) Find the element of R.
ii) Determine the domain and range of R.
iii) Draw the digraph of the relation
iv) Determine whether the relation R is assymetric?

Question 2 **[8 marks]**

Suppose R is an equivalence relation on the set A={x,y,z}. (x,y) $\in$ R and (y,z) $\in$ R. List all possible member of R and justify your answer.

Question 3 **[15 marks]**

Let B = {u, v, w, y} and R={(u,u), (u,w), (v,v), (v,w), (w,w), (w,y), (y,u), (y,v), (y,y)}

i) Construct the matrix of relation, M_R for the relation R on B
ii) List in-degrees and out-degrees of all vertices.
iii) Determine whether the relation R on the set B is a partial order relation. Check all variance Justify for answer.

Question 1

$D = \{1, 3, 5\}$

xRy, $3x + y =$ multiple of 6

$x = 1$, $y = 3$, $3x + y = 6$

$x = 3$, $y = 3$, $3x + y = 12$

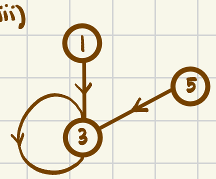
$x = 5$, $y = 3$, $3x + y = 18$

(i) $R = \{(1,3), (3,3), (5,3)\}$

(ii) domain $= \{1, 3, 5\}$

range $= \{3\}$

(iii)



(iv) no, relation R is irreflexive as the pair (3,3) exists

Question 2

- $R = \{(x,x), (y,y), (z,z), (x,y), (y,x), (x,z), (z,x), (y,z), (z,y)\}$
- R is reflexive because for all elements in A, it loops itself where $(x,x), (y,y), (z,z) \in R$.
- R is symmetric because every pair such as $(x,y), (y,z), (x,z)$ in R, the reverse pairs $(y,x), (z,y)$ and (z,x) are also in R.
- R is transitive because from $(x,y) \in R$ and $(y,z) \in R$, it follows that $(x,z) \in R$.
- R is reflexive, symmetric and transitive.
- So R is an equivalence relation on the set A.

$$M_R = \begin{array}{cc} & \begin{matrix} x & y & z \end{matrix} \\ \begin{matrix} x \\ y \\ z \end{matrix} & \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

All the main diagonals matrix elements are 1 and the matrix is reflexive.

$$M_R = \begin{array}{cc} & \begin{matrix} x & y & z \end{matrix} \\ \begin{matrix} x \\ y \\ z \end{matrix} & \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array} \quad M_R^T = \begin{array}{cc} & \begin{matrix} x & y & z \end{matrix} \\ \begin{matrix} x \\ y \\ z \end{matrix} & \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

$M_R = M_R^T$. The transpose matrix M_R. M_R^T is equal to M_R, so R is symmetric.

$$M_R \otimes M_R = \begin{array}{cc} & \begin{matrix} x & y & z \end{matrix} \\ \begin{matrix} x \\ y \\ z \end{matrix} & \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array} \otimes \begin{array}{cc} & \begin{matrix} x & y & z \end{matrix} \\ \begin{matrix} x \\ y \\ z \end{matrix} & \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array} = \begin{array}{cc} & \begin{matrix} x & y & z \end{matrix} \\ \begin{matrix} x \\ y \\ z \end{matrix} & \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

$M_R \otimes M_R = M_R$

$(x,y) \in R$ and $(y,z) \in R$, then $(x,z) \in R$.

The product of Boolean show that the matrix is transitive.

- R is reflexive, symmetric and transitive.
- So, R is an equivalence relation.

Question 3

(i)

$$M_R = \begin{array}{c} \\ u \\ v \\ w \\ y \end{array} \begin{array}{c} \begin{array}{cccc} u & v & w & y \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

(ii)

	u	v	w	y
in-degrees	2	2	3	2
out-degrees	2	2	2	3

(iii)

partial order - reflexive, antisymmetric, transitive

- $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$ reflexive, main diagonal all are 1.

- Relation R is antisymmetric, since $x \neq y$ either $(x,y) \notin R$ or $(y,x) \notin R$

- $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

  $(n_{14} = 1) \wedge (m_{14} = 0)$

  not transitive

$\therefore$ Relation R is not partial order.

Question 4

Let $f(x_1) = f(x_2)$, $f(x) = (x-1)^2$

$$(x_1 - 1)^2 = (x_2 - 1)^2 \qquad (\sqrt{})$$
$$x_1 - 1 = x_2 - 1 \qquad (+1)$$
$$x_1 = x_2$$

This shows that function f is one-to-one.

$f(x) = (x-1)^2$

$y = (x-1)^2$

$x = 1 + \sqrt{y}$

$f^{-1}(y) = 1 + \sqrt{y}$, $y \in [0, \infty)$

$x = f^{-1}(y) = 1 + \sqrt{y}$, $x \in [1, \infty)$

$$f(x) = f(f^{-1}(y)) = ((1+\sqrt{y}) - 1)^2 = (\sqrt{y})^2 = y$$

This shows that function f is onto.

$\therefore$ Since the function f is both one-to-one and onto, it is bijective.

Question 4 **[6 marks]**

Let

$$f: [1, \infty) \to [0, \infty), f(x) = (x-1)^2.$$

Determine whether the function f is **one–one**, **onto**, or **bijective**.
Show full working and justify your answer.

Question 5 **[9 marks]**

Let f and g be functions from the positive integers to the positive integers defined by

$$f(x) = 9x + 4, g(x) = \frac{3}{2}x - 1.$$

a) Find the inverse of $g(x)$.

b) Find the composition $(g \circ f)(x)$.

c) Find the composition $(f \circ g)(x)$.

d) Find the composition $(f \circ g \circ g)(x)$.

Question 6 **[6 marks]**

In a reactor, two intermediates mix to form product P. The initial temperatures are $P_0 = 4.0^{\circ}F$ and $P_1 = 5.0^{\circ}F$. Engineers observe that, for $t \geq 2$minutes, the update rule is:

"The new temperature is the previous temperature plus one-quarter of the temperature two minutes ago."

a) Write the recurrence relation that models this.

b) Using your recurrence, list $P_0, P_1, \ldots, P_5$ (exact values preferred).

Question 7 **[7 marks]**

Given the recurrence relation below,

$$s_1 = 2, s_n = s_{n-1}^2 - 1 \text{ for } n \geq 2.$$

a) Write a recursive algorithm to calculate the n^{th} term of the sequence

b) Trace the recursive steps to compute s_4. Show your working in a diagram.

Question 5

a) $g(x) = \frac{3}{2}x - 1$

$y = \frac{3}{2}x - 1$

$y + 1 = \frac{3}{2}x$

$x = \frac{2}{3}(y+1)$

$g^{-1}(y) = \frac{2}{3}(y+1)$

b) $(g \circ f)(x) = g(f(x))$

$= g(9x+4)$

$= \frac{3}{2}(9x+4) - 1$

$= \frac{27}{2}x + 6 - 1$

$= \frac{27}{2}x + 5$

c) $(f \circ g)(x) = f(g(x))$

$= f(\frac{3}{2}x - 1)$

$= 9(\frac{3}{2}x - 1) + 4$

$= \frac{27}{2}x - 9 + 4$

$= \frac{27}{2}x - 5$

d) $g(g(x)) = g(\frac{3}{2}x - 1)$

$= \frac{3}{2}(\frac{3}{2}x - 1) - 1$

$= \frac{9}{4}x - \frac{3}{2} - 1$

$= \frac{9}{4}x - \frac{5}{2}$

$(f \circ g \circ g)(x) = f(g(g(x)))$

$= 9(\frac{9}{4}x - \frac{5}{2}) + 4$

$= \frac{81}{4}x - \frac{45}{2} + 4$

$= \frac{81}{4}x - \frac{37}{2}$

Question 6

a) $P_t = P_{t-1} + \frac{1}{4}P_{t-2}$, $t \geq 2$

b) $P_0 = 4.0$

$P_1 = 5.0$

$P_2 = P_1 + \frac{1}{4}P_0 = 5.0 + \frac{1}{4}(4.0) = 6.0$

$P_3 = P_2 + \frac{1}{4}P_1 = 6.0 + \frac{1}{4}(5.0) = 7.25$

$P_4 = P_3 + \frac{1}{4}P_2 = 7.25 + \frac{1}{4}(6.0) = 8.75$

$P_5 = P_4 + \frac{1}{4}P_3 = 8.75 + \frac{1}{4}(7.25) = 10.5625$

Question 7

a) Input : n
output : S(n)

```
S(n) {
    if(n=1)
        return 2
    else return S(n-1)^2 -1
}
```

b) $S_n = S_{n-1}^2 - 1$

$S_1 = 2$
$S_2 = S_1^2 - 1 = 2^2 - 1 = 3$
$S_3 = S_2^2 - 1 = 3^2 - 1 = 8$
$S_4 = S_3^2 - 1 = 8^2 - 1 = 63$