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Estimation of Unit Values in Cross Sections without Quantity Information and Implications for Demand and Welfare Analysis

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Abstract: Household surveys frequently record only expenditure information. The lack of information about quantities purchased precludes the possibility of deriving household specific unit values. The aggregate price indexes derived from sources exogenous to the household survey are often not sufficient to identify all parameters and to provide plausible estimates. We use a theoretical result developed by Lewbel (1989) to construct "*pseudo*" unit values by 1) reproducing the variability of cross-sectional price variation using the variability of the budget shares, and 2) adding the variability to the aggregate price indexes published by the national statistical institute. The study estimates a complete quadratic demand system using a time series of cross-sections of Italian household budgets including, in turn, aggregate price indexes and "*pseudo*" unit values. The results show that the matrix of compensated price elasticities is negative semidefinite only if "*pseudo*" unit values are used. In order to have a counterfactual experiment, we then consider a household survey with actual unit values and compare them with "*pseudo*" unit values. The experiment shows that in most cases "*pseudo*" values maintain the relevant characteristics of the distribution of actual unit values. Overall, we conclude that "*pseudo*" unit values are better than aggregate price indexes for sound demand and welfare analysis.

Keywords: Unit values, Cross-section prices, demand analysis, Slutsky matrix properties

1. - Introduction

Empirical work on demand analysis generally relies on the assumption of price invariance across households, supported by the hypothesis that in cross-sectional data there are neither time nor spatial variations in prices. According to this assumption each family pays the same prices for homogeneous goods. Micro-data with this characteristic allow researchers to estimate Engel curves, describing the relationship between budget shares and the logarithm of household income, without accounting for price effects that are crucial for both behavioural and welfare applications. Slesnick (1998:2150) has remarked that “the absence of price information in the surveys creates special problems for the measurement of social welfare, inequality and poverty. ... Most empirical work links micro data with national price series on different types of goods so cross sectional variation is ignored. Access to more disaggregate information on prices will enhance our ability to measure social welfare, although it remains to be seen whether fundamental conclusions concerning distributional issues will be affected.”

In empirical works such limitation is usually by-passed by analysing time-series of cross-sections where price information comes from aggregate time series data. Plausible estimates of price effects require a series of cross-sections that is long enough and, if possible, aggregate price indexes that vary by month and location, usually by region or by province.

In general, household budget surveys of both developed and developing countries can be classified into two broad categories in increasing order of frequency of occurrence: 1) surveys of expenditure and quantities purchased expressed in a common unit, and 2) surveys of expenditure data only.

In the first case, where quantities and expenditure are both observed, cross-sectional prices are obtained as implicit prices, dividing expenditure by quantities, and are more properly referred to as “unit values.” When dealing with these surveys it is important to remember that a proper use of unit values in econometric analyses must take into account problems arising from the fact that unit values provide useful information about prices, but differ from market prices in many respects. The ratio between expenditure and quantities bought embed information about the choice of quality (Deaton 1987, 1988a, 1988b, 1990, 1998, Perali 2003). The level of the unit value of a composite good depends on the relative share of high-quality items and the composition of the aggregate good. Unit values can be highly variable also for supposedly homogeneous goods because the market offers many different grades and types.

On the other hand, when dealing with surveys that report only expenditure information, aggregate national price indexes are usually merged with household expenditure to obtain estimates of price elasticities. Unfortunately, this approach requires a long time series of cross sectional data to estimate a demand system with sufficient price variation and rely on very restrictive assumptions (Frisch 1959) which generally turn out to be rejected in empirical applications. Aggregate price indexes are in general highly correlated and the estimated elasticities are often not coherent with the theory. This problem has been object of the interest of other researchers (Coondoo et al. 2001, Dagsvik and Brubakk 1998, Lahatte et al. 1998).

For these reasons, surveys gathering exclusively expenditure data, such as the Italian household budget survey conducted by the National Statistical Institute - ISTAT and the majority of existing household budget surveys, have limited applicability in modern demand and welfare analysis. It is then important to devise an appropriate procedure to compute "*pseudo*" unit values using all those information available in the surveys, such as budget shares and demographic characteristics, which help in reproducing the distribution of the unit value variability as closely as possible. The theoretical background for this undertaking is provided in a study by Lewbel (1989a).

The main goal of this paper is to implement this method in a time series (1985-1996) of cross sections of Italian household budget surveys produced by ISTAT, and to check if *pseudo* unit values allow the estimation of well-behaved demand systems, suitable to perform welfare analysis.

The remainder of the paper is organized as follows. Section 2 presents the method used to derive national and regional consumer price indexes and unit values when quantity information is missing. We define a set of five different prices, ordered in terms of increased variability: 1) national price index P_N, 2) regional price index P_R, 3) *pseudo* unit values

$\hat{P}$, 4) regional *pseudo* unit values $\hat{P}_R$ and 5) regional *pseudo* unit values expressed in levels $\hat{P}_{RL}$. Section 3 describes the data sets used in the empirical analysis. Section 4, divided in three parts, presents the empirical results. In the first sub-section we discuss the properties of our price indexes based on a non-parametric analysis. The second subsection presents the estimates of different compensated price elasticities, obtained employing a quadratic AIDS model run on the set of five prices indices presented above. By comparing these estimated price elasticities we find that only *pseudo* unit values provide price effects with correct signs and meaningful economic interpretation. The third subsection presents the counterfactual experiment, based on a data set where actual unit values are available, investigating how closely the estimated *pseudo* unit values replicate the variability of the actual unit values. Finally, the conclusions summarize the experiment and the main results.

2. - The Theoretical Framework to Derive Unit Values when Quantity Information Is Missing

In this section, we present the method adopted to derive the set of prices $\wp = \{P_N, P_R, \hat{P}, \hat{P}_R, \hat{P}_{RL}\}$ that we use to implement the empirical analysis. We first define the national Italian price index P_N and the regional price index P_R. We then present Lewbel's (1989a) method to derive what we define *pseudo* unit values $\hat{P}$. This index $\hat{P}$ is then combined with the regional price index P_R to obtain the regional *pseudo* unit values $\hat{P}_R$. Noting that actual unit value P_U, defined as the ratio between expenditure and quantities, are in levels, as a last step we transform the index $\hat{P}_R$ into a corresponding price in level, $\hat{P}_{RL}$. It is relevant to remark that by not performing this last step (express *pseudo* unit values in levels) may have important consequences for the estimated price substitution and complementarity effects. In fact, these estimates would otherwise be the expression of the relative speed of variation through time of the indexes, that in the base month are the same across goods, and would not account for the relative importance of the aggregate goods in the basket.

A crucial aspect in understanding the procedure is to realize that the *pseudo* unit value $\hat{P}$ is defined over a range comparable to the range of "actual" unit values P_U, as if the latter had been normalized with respect to the unit value of a specific household chosen as a numeraire. On the other hand, $\hat{P}_{RL}$ is the *pseudo* unit value which more closely resembles actual unit values P_U, because it is in levels just as the actual unit values P_U are.

Summarizing, our general objective is to implement demand analysis on Italian household budget data. First, we need to construct a time series of cross-sections because with a single cross-section information on $\wp = \{P_N, P_R\}$ alone it would be very difficult to have an invertible data matrix. Having collected the national and regional price indexes from published sources, we reproduce as best we can the price variation of actual unit values which we then combine to the national and regional price indexes. In the empirical analysis of Section 4, we evaluate how the estimated Slutsky matrix changes as the price information used in the analysis moves from a low to a high level of variability.

2.1 - National and Regional Consumer Price Indexes

The Italian Statistical Institute - ISTAT publishes, on a monthly basis, consumer price indexes for more than 100 goods and services (*Indice Nazionale dei Prezzi al Consumo per l'Intera Collettività* - NIC in ISTAT terminology).

Definition 1 (National Elementary Price Index) *Let the indices j=1,...,J, t=1,...,T and m=1,...,12 be, respectively, the indices for the number of goods included in the basket, the years and the months. The price index for each elementary good j at national level is derived as:*

$$P_N^j = \frac{P_{t_{m+1}}^j}{p_{t_m}^j}.$$

This price index, published only at national level, is computed using information collected by ISTAT at provincial level for 930 products, and then aggregated at national level using appropriate weights.

The National Statistical Institute also publishes price indexes for aggregate goods for each province referring to the consumption habits, of the population of households whose heads are workers or white collars in non-agricultural sectors (*Indice dei Prezzi al Consumo per le Famiglie di Operai e Impiegati* - FOI in ISTAT terminology). This index is used by the Italian government for administrative purposes.

Definition 2 (Regional Elementary Price Index) *Define a set K with k elements, indexed by k=1,...,K, representing each group of goods* $j \in k$. *Then, the regional elementary price index* P_R^j *is obtained as:*

$$P_R^j = P_N + \left(P_N - P_R^k\right), \quad \text{for } j \in k$$

where the regional aggregate price index P_R^k *for group k is computed as the average of all provincial capitals by region. The index* P_R^k *refers to the consumption of households of workers and white collars in non-agricultural sectors.*

According to ISTAT, the *k* elements of the set *K* are: food, alcoholic beverages, clothing, housing and energy, furniture, health, transportation, communication, recreation, education, hotels and public services, and other goods and services.[1]

This procedure adds the spatial variability of the regional aggregate price index to the time dimension of the national elementary price index. Note that we aggregate the provincial price indexes in regional price indexes because in the Italian household budget survey we cannot identify households below the regional level. Note further, that the indexes

[1] The categories were five until 1989 and, as shown by the notation in the text, eleven from 1990 until 1996. The five categories adopted before 1990 are the following: Food, Clothing, Electricity and Fuel, Housing, Other goods and services.

P_R^j and P_R^k, at the provincial capital level, are as published by ISTAT, while the regional elementary price index defined above are computed for our experiment.

The next task is to match the monthly regional elementary price index P_R^j with all households interviewed at month m and living in region r=1,...,R where R=20, the number of Italian regions. We then aggregate each P_N^j into i=1,...,I groups corresponding to the goods selected for the empirical demand analysis. The aggregation uses Laspeyres indexes:

$$P_N^i = \sum_{j=1}^{n_i} \left[\frac{p_{t+1}^j}{p_t^j} \frac{p_t^j q_t^{ij}}{\sum_{j=1}^{n_i} p_t^j q_t^{ij}} \right] = \sum_{j=1}^{n_i} \left[\frac{p_{t+1}^j}{p_t^j} w_{ijt} \right] = \frac{\sum_{j=1}^{n_i} p_t^j q_t^{ij}}{\sum_{j=1}^{n_i} p_t^j q_t^{ij}},$$

where q^{ij} is the quantity of the *jth* good in group i=1,...,n, for j=1,...,n_i and n_i is the number of goods within group i, and the weights $w_{ijt} = p_t^j q_t^{ij} / \sum_{j=1}^{n_i} p_t^j q_t^{ij}$ are the household budget shares. Our time series collection of cross-sections of Italian household budget surveys runs from t=1985,...,1996. The base at which the indexes for all goods are equal to 100 is t_m=1985_1. Next, we pursue the objective to reconstruct the cross-sectional variability of prices.

2.2 - Cross-Sectional Price Variability from Demographic Information

Lewbel (1989a) proposes a method to estimate the cross-sectional variability of actual unit values by exploiting the demographic information included in generalized "within-group" equivalence scales or, more generally, demographic functions. These are defined here as the ratio of the group sub-utility function to the corresponding sub-utility function of a reference household, estimated without price variation, in place of "between-group" price variation. The method relies on the assumption that the original function is homothetically separable and "within-group" sub-utility functions are Cobb-Douglas.

Consider a separable utility function $U(u_1(q_1,d),...,u_n(q_n,d))$, where $U(u_1,...,u_n)$ is the "between-group" utility function and $u_i(q_i,d)$ is the "within-group" sub-utility function. The index i=1,...,n_i denotes the aggregate commodity groups, while n_i is the total number of goods q comprising group i. The vector of demographic characteristics, d, affects $U(.)$ through the direct effects on the within-group sub-utility function. We define the group equivalence scale $M_i(q,d)$ as

$$M_i(q,d)=\frac{u_i(q,d)}{u_i(q,d^h)},$$

where d^h describes the demographic profile of a reference household. Define a quantity index for group i as $Q_i(u_i,d^h)$ and rewrite the between-group utility function as:

$$U(u_i,...,u_n)=U\left(\frac{Q_1}{M_1},...,\frac{Q_n}{M_n}\right),$$

which is formally analogous to Barten's (1964) technique to introduce demographic factors into the utility function. Define further the price index for group i as $P_i=Y_i^h/Q_i$ where Y_i^h is expenditure on group i by the reference household. Barten's utility structure implies the following share demands for each household:

$$W_i=H_i(P_1M_1,...,P_nM_n,Y),$$

taking the form of $W_i^h=H_i(P_1,...,P_n,Y^h)$ for the reference household with scales M_i=1. The further assumption of homothetic separability admits two-stage budgeting and implies the existence of indirect sub-utility functions V_i such that $P_i=V_i(p_i,d^h)$. By analogy with the definition of group equivalence scales in utility space, it follows that:

$$M_i = \frac{V_i(p_i, d)}{V_i(p_i, d^h)},$$

and $V_i = M_i P_i$. Therefore, when demands are homothetically separable, each group scale depends only on relative prices within group i and on d, as expected given that homothetic separability implies strong separability.

Maximization of $u_i(q_i, d)$ subject to the expenditure $p_i q_i = y_i$ in group i gives the budget share for an individual good $w_{ij} = h_{ij}(p_i, d, y_i)$. For homothetically separable demands, then, the budget shares do not depend on expenditure $w_{ij} = h_{ij}(p_i, y_i)$. and integrate back in a simple fashion to $V_i = M_i P_i$. This information can be used at the between-group level in place of price data to estimate $W_i = H_i(V_1, \ldots, V_n, Y)$.

Under the assumption that the sub-group utility functions can be represented in a Cobb-Douglas form, with parameters specified as "shifting" functions of demographic variables alone as follows:

$$F_i(q_i, d) = k_i \prod_{j=1}^{n_i} q_{ij}^{m_{ij}(d)},$$

then the shares correspond to the demographic functions

$$w_{ij} = h_{ij}(p_i, d) = m_{ij}(d), \tag{1}$$

with

$$\sum_{j=1}^{n} w_{ij}(d) = \sum_{j=1}^{n} m_{ij}(d) = 1. \tag{2}$$

The implied indirect utility function is:

$$V_i(p_i, d) = M_i P_i = \frac{1}{k_i} \prod_{j=1}^{n_i} \left(\frac{p_{ij}}{m_{ij}} \right)^{m_{ij}(d)},$$

with

$$k_i(d) = \prod_{j=1}^{n_i} m_{ij}\left(d^h\right)^{-m_{ij}(d^h)},$$

where $k_i(d)$ is a scaling function depending only on the choice of the reference demographic levels.

These results support a simple procedure to estimate price variation in survey data without quantity information. Jointly estimate the m_{ij} equations and the fitted shares using the stochastic specification $\hat{w}_{ij} = \hat{h}_{ij} = m_{ij}(d) + \varepsilon_{ij}$, where ε is a spherical error term for the within-group budget shares. Then, further assuming with no loss of information, that $p_{ij}=P_i=1$ for all i and j, price information can be deduced from demographic information alone by using (1) and (2):

$$M_i P_i = M_i = \frac{1}{\hat{k}_i} \prod_{j=1}^{n_i} \left(\frac{1}{\hat{m}_{ij}}\right)^{\hat{m}_{ij}(d)} = \frac{1}{\hat{k}_i} \prod_{j=1}^{n_i} m_{ij}^{-m_{ij}}, \tag{3}$$

and

$$\hat{k}_i(d) = \prod_{j=1}^{n_i} \hat{m}_{ij}\left(d^h\right)^{\hat{m}_{ij}(d^h)},$$

by treating M_i as price data. It is important to note that the Cobb-Douglas assumption places restrictions only at the within-group level, while leaving the between-group demand equations free to be arbitrarily flexible. An approximation to equation (3) can be obtained by using the observed within-group budget shares. Interestingly, from an empirical point of view, a more flexible functional form such as a Translog may improve the fit of *pseudo* unit values with respect to actual unit values.

Given this setup, we can now formally define the *pseudo* unit values as:

Definition 3 (Pseudo Unit Values)

$$\hat{P}_i = M_i P_i = M_i = \frac{1}{k_i} \prod_{j=1}^{n_i} w_{ij}^{\;-w_{ij}}, \tag{4}$$

where k_i is the average of the subgroup expenditure for the ith group budget share.

The *pseudo* unit value is an index that can be compared to actual unit values after normalization of the actual unit values, choosing the value of a specific household as a numeraire. The index $\hat{P}^i$ summarizes the cross-section variabilities of prices that can be added to spatially varying price indexes to resemble unit values expressed in index form. Therefore, we can define the following regional *pseudo* unit value as:

Definition 4 (Regional Pseudo Unit Values)

$$\hat{P}_R^i = \hat{P}^i P_R^i,$$

where $P_R^i = \sum_{i=1}^{n_i} w_{ij} P_R^i.$

For *pseudo* unit values to look like actual unit values, the *pseudo* price index has to be transformed into levels.

2.3 - Nominal Prices and Substitution Effects

The transformation in nominal terms is fundamental to properly capture complementary and substitution effects. Cross-effects would otherwise be the expression of the differential speed of change of the good-specific price indexes through time.

Definition 5 (Regional Pseudo Unit Values in Levels)

$$\hat{P}_{RL}^i = \hat{P}_R^i y^i,$$

where y^i is the average expenditure of group i evaluated at the base year, t_m=1985[1].

Regional *pseudo* unit values in levels are then expressed in the same unit of measurement as actual unit values. Note that the average group expenditure y^i here acts as an aggregate price of the composite good *i*. The information comes from the household budget survey of the base month.

Early experiments with *pseudo* unit values with Italian household budget data (Perali 1999 and 2000) have provided comforting indications about the possibility of estimating regular preferences. In the present paper, we are interested in describing the effects of the use of the elements of the price set $\wp$ on the estimated matrix of cross-price elasticities.

3. - Data

Our datasets come from two different sources. The first one is represented by household budget data where only expenditure produced by the Italian National Statistical Institute (ISTAT) are recorded. The second source is represented by data on prices, quantity and expenditure obtained from budget data of rural households collected by the Italian Institute for the Analysis of Agricultural Markets (ISMEA). Here below we provide a brief description of both datasets.

3.1 - Italian Household Budget Data (ISTAT): Expenditure

Expenditure data comes from a series of repeated cross-sectional national household budget surveys conducted by the Italian Statistical Institute over a time interval ranging from 1985 to 1996. These surveys contain detailed information on monthly expenditure, covering private consumers' expenditure with a high level of detail concerning single items purchased. The survey central unit is the household, defined as a set of persons living together and characterized by the common use of their incomes. Within each cross-section, households are interviewed at different times during the year, on a monthly basis. Further, we know the geographic location of these households by region only. As we will see, this represents useful information when we match demographic and expenditure information with price data.

Table 1. Variable Labels and Definition

Label	**Definition**
Year	Year of sampling
Hage	Age of breadwinner
Nch05	No. of children 0-5 years old
Nch614	No. of children 6-14 years old
Nch1518	No. of children 15-18 years old
Nmaj	No. of adults
Fsize	Household size
Sex-m	Dummy=1 if male head of household, = 0 otherwise
Sex-f	Dummy=1 if female head of household, = 0 otherwise
Tj	Dummy=1 if wife works, = 0 otherwise
Ts	Dummy=1 if husband works, = 0 otherwise
L-dip	Dummy=1 if head hh is an employee, = 0 otherwise
L-ind	Dummy=1 if head hh is a self-employed, = 0 otherwise
Rural	Dummy=1 if hh lives in the countryside, = 0 otherwise
Food	Food share
Cloth	Clothing share
House	Housing share
Tracom	Transport and communication share
Educat	Education and leisure share
Other	Other goods share
Lnx	Logarithm of total expenditure

The samples of household budgets used in this paper are composed of more than 32,000 households per year, with the exception of 1996 where only 22,740 households were interviewed, for a total of more than 370,000 observations. From these surveys we have selected households in which the age of the head of household is between 19 and 75. In order to reduce the estimation burden, we have drawn a random sample of 2,134 households.

Household expenditure have been aggregated into six broad categories and transformed in budget shares: Food, Clothing, Housing, Transport and Communication, Education and Other goods and services. These shares are the dependent variables in our demand system. Price data have also been obtained from the Monthly Bulletin of Statistics published by ISTAT. The Italian Institute of Statistics produces different data on consumer price indexes. As explained in Section 2, ISTAT collects information on a consumer price index based on the consumption habits of the whole population and of the blue and white collar class. Both price indexes are available on a monthly base. However, while the national price index is published using a high level of disaggregation among goods and services at the national level, the latter is provided with a much lower level of disaggregation (only 5 categories from 1985 until 1996), and at the level of the 106 provincial capitals across 20 regions.

These price indexes have been used as a basis to obtain the set of price indexes discussed in Section 2. We have chosen January 1985 as the base for all the price indexes used in the empirical analysis. Price indexes have been matched to expenditure taking into account the period of the year in which the household was interviewed. This means that households interviewed in March have been matched with prices collected in the same month. At the same time, whenever regional variation was introduced, the matching between expenditure and prices has also taken into account the different regional residency of each household.

Table 1 reports names and definitions of all variables used in the estimation of the demand system, while means and standard deviations of the demographics and expenditure variables are shown in Table 2. Table 3 reports the descriptive statistics for the five set of prices and for the six broad categories of goods and services listed above.

Table 2. Sample Statistics for the Pooled Italian Household Surveys 1985-1996, No. of Households 2,134

Variable	Mean	Std. Dev.	Min.	Max.
Year	90.342	3.3416	85	96
Hage	49.982	13.4931	19	75
Nch05	0.181	0.4532	0	3
Nch614	0.336	0.6490	0	4
Nch1518	0.213	0.4832	0	3
Nmaj	1.987	1.0929	0	7
Fsize	2.985	1.3453	1	9
Sex-m	0.830		0	1
Sex-f	0.170		0	1
Tj	0.649		0	1
Ts	0.301		0	1
L-dip	0.488		0	1
L-ind	0.191		0	1
N-W	0.255		0	1
N-E	0.196		0	1
Centre	0.210		0	1
South	0.246		0	1
Islands	0.092		0	1
Rural	0.064		0	1
Food	0.290	0.1271	0.003	0.753
Cloth	0.085	0.0702	0.000	0.544
House	0.285	0.1291	0.007	0.948
Tracom	0.134	0.1208	0.001	0.900
Educat	0.064	0.0628	0.001	0.644
Other	0.142	0.1112	0.001	0.729
Lnx	14.581	0.6804	12.079	17.531

Table 3. Sample Statistics for Price Indexes

Variable	Mean	Std. Dev.	Min	Max
		Food Prices		
P_N	1.375	0.2231	1	1.791
P_R	1.388	0.2253	1	1.804
$\hat{P}$	1.046	0.1436	0.530	1.707
$\hat{P}_R$	1.454	0.3256	0.605	2.952
$\hat{P}_{RL}$	7148.8	2918.3	1690.0	18876.4
		Clothing Prices		
P_N	1.397	0.1985	1	1.756
P_R	1.452	0.2438	1	1.876
$\hat{P}$	0.839	0.1322	0.461	1.082
$\hat{P}_R$	1.214	0.2742	0.513	1.912
$\hat{P}_{RL}$	1802.5	1498.2	153.4	6742.2
		Housing Prices		
P_N	1.377	0.2456	1	1.826
P_R	1.388	0.2716	1	1.888
$\hat{P}$	0.821	0.2200	0.000	1.241
$\hat{P}_R$	0.386	0.3418	0.000	2.169
$\hat{P}_{RL}$	1572.9	1865.1	0.000	18089.1

Table 3. (Continued)Sample Statistics for Price Indexes

Variable	Mean	Std. Dev.	Min.	Max.
		Transport and Communication Prices		
P_N	1.326	0.2175	1	1.730
P_R	1.340	0.2258	1	1.804
$\hat{P}$	1.085	0.2220	0.292	1.717
$\hat{P}_R$	1.499	0.3841	0.303	2.617
$\hat{P}_{RL}$	3679.7	3522.2	204.1	17719.5
		Education and Leisure Prices		
P_N	1.460	0.2573	1	1.869
P_R	1.493	0.2638	1	1.912
$\hat{P}$	1.152	0.1933	0.361	1.558
$\hat{P}_R$	1.680	0.3946	0.483	2.650
$\hat{P}_{RL}$	1825.4	1706.4	79.9	7670.3
		Other Goods Prices		
P_N	1.460	0.2754	1	1.921
P_R	1.523	0.2987	1	2.041
$\hat{P}$	1.941	0.4196	0.461	2.952
$\hat{P}_R$	2.590	0.6600	0.480	4.742
$\hat{P}_{RL}$	5946.8	4646.1	573.2	25262.8

3.2 - Italian Rural Household Budget Data (ISMEA): Quantities, Expenditure, and Unit Values.

The empirical analysis of this work is based on a sub-sample of the 1995 ISMEA Survey on Socio-Economic Characteristics of Italian Rural Households. This is a nationwide farm household survey of 1,777 farm-households. The sampling is based on the last Agricultural Census conducted in 1992 by the Italian National Statistical Institute (ISTAT). The dataset from ISMEA has the appealing feature (limited to the food category) of recording the quantity of items bought by each household together with the price at which the item was bought.

The survey combines information about household and farm characteristics, time use, farm profits, off-farm money income, governmental and intra-household transfers, consumption, and information about the degree of autonomy in decision making by household members. The availability of this information is the basis for the estimation of both global and full income. The ISMEA data base merges four survey types (farm accounting survey, stylised time use survey, expenditure survey, and income survey) into one. The interview is more time consuming and costly, but it compels the interviewee to double check the price information revealed.

In this paper we limit our analysis to the food category as an aggregate. In Table 4 we report the composition of the food group with the relative descriptive statistics. The use of unit values in empirical analysis needs some words of qualification. The consumers' response to a price increase is either to buy less of the same composition of the aggregate good or to buy more lower-quality items. If we define a price increase of a commodity group as a proportionate increase in the prices of all the different qualities, unit values may change less than proportionately because households respond to a price increase by choosing less expensive qualities. Thus, it is likely that the estimate of parameters will be biased when computing demand system analysis with unit values.

In general, better-off households pay more for each unit of a commodity even if narrowly defined and, thus, presumably highly homogenous. Households with children trade off quantity and quality differently with respect to households without children. Poor households tend to have more children than rich households.

Table 4. Sample Statistics for the ISMEA Dataset Share Groups, No. of Households 1,777

Shares	Mean	Std. Dev.	Min.	Max.
Food	0.269	0.1094	0.020	0.817
Bread and cereal	0.209	0.1151	0	1
Meat	0.371	0.1648	0	0.835
Oil and Fat	0.057	0.0473	0	0.441
Milk	0.108	0.0732	0	0.715
Fruit and vegetable	0.082	0.0646	0	0.409
Sugar	0.060	0.0450	0	0.432
Beverage	0.113	0.1018	0	0.659
Bread and Cereal Sub-Group				
Bread	0.512	0.1947	0	1
Biscuits	0.146	0.1413	0	1
Flour	0.042	0.0702	0	0.794
Pasta	0.236	0.1522	0	1
Rice	0.064	0.0785	0	1
Meat Sub-Group				
Beef	0.373	0.2884	0	1
Pork	0.124	0.2065	0	1
Lamb	0.061	0.1886	0	1
Poultry	0.084	0.1952	0	1
White meat	0.052	0.1834	0	1
Cold meats	0.166	0.2330	0	1
Other meat	0.047	0.1844	0	1
Fresh fish	0.275	0.2982	0	1
Frozen fish	0.073	0.2079	0	1
Sugar Sub-Group				
Sugar	0.266	0.2602	0	1
Marmalade	0.133	0.2339	0	1
Coffee	0.608	0.2748	0	1
Other	0.097	0.2093	0	1

Therefore, unit values may be positively related to total outlays. If interpreted as choice variables, unit values may be simultaneously determined with the expenditure decision. As a consequence, if unit values are correlated with expenditure, then simultaneity should be accounted for.

Another important source of contamination of unit values is the occurrence of measurement errors in both recorded quantities and expenditure that will be transmitted to the measurement of unit values. Further, some households may not purchase every detailed good. Thus, neither expenditure nor unit values can be obtained from the observations that do not report expenditure. This feature of the data produces a sample selectivity bias. In the paper, we do not deal directly with these problems. Our objective is to learn something about the distribution of *pseudo* unit values compared to that of actual unit values and to test the regularity of the estimated price effects.

4. - Empirical Analysis

4.1 - Non-Parametric Description of Price Information

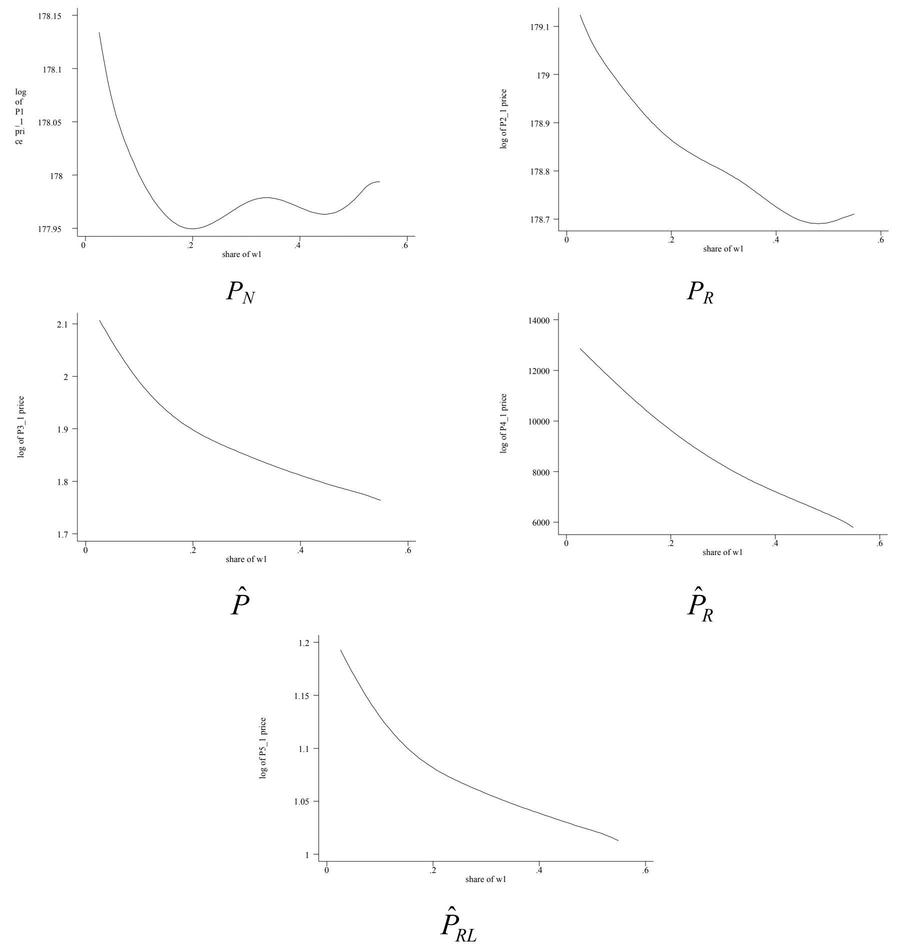
This section describes how the set of prices $\wp = \left\{P_N, P_R, \hat{P}, \hat{P}_R, \hat{P}_{RL}\right\}$ that we have constructed using the method developed in Section 2 differ in terms of cross-sectional variability. For reasons of space, we describe only the behaviour of food price indexes. The evidence gathered for this category can be extended to all other categories we have identified in our analysis.[2]

As a descriptive tool, we adopt the locally-weighted non-parametric regressions (Fan 1992). Compared to Kernel regressions, this technique has the advantage of minimizing the bias associated with the use of unequally spaced x's.[3]

[2] Empirical evidence is available upon request from the authors.

[3] Ordinary least squares (OLS) do not present the same problems as kernel regressions. When the regression function is linear, OLS will be unbiased and consistent. However, the problem with an OLS technique is that it cannot adapt to the shape of a non-linear regression function, independently of the sample size. In other words, as long as the relationship between the variables under investigation is not linear, the OLS will not allow unbiased and consistent estimates. In principle this is the main problem, given

Figure 1. Locally Weighted Regressions for Food Share



In order to implement such a technique Fan (1992) suggests estimating a series of local regressions. Instead of averaging the *w'*s

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that we do not know ex-ante what the exact relationship existing among our variables will be.

around x_i, as in kernel regression, and instead of running a regression using all the data points as in OLS, he adopts the best of both procedures and runs a regression using only the points "close" to x_i. As with kernel regression, we use a band-width to define "close," but instead of averaging, we run a weighted or GLS regression at x_i, where the weights are nonzero only within the band, and are larger the closer the observations are to x_i. By repeating this procedure for all points at which we want to estimate the regression function we get our locally weighted regression.

Figure 1 shows the relationship existing between the food share and the different definitions of food price indexes. Apart from the graph reported in the top left-hand side panel of Figure 1, all others graphs show a negative relationship between price and food share. In case of the national elementary price index $\left(P_N^i\right)$ the budget share for food is negative for low values of the food share, while it increases for medium and high values of the share. This is in part a direct consequence of the highly reduced variability we have to face when using the national price index across households in the sample. By moving across panels in Figure 1, we see that the relationship between food share and price changes. In particular if we look at the bottom right-hand side panel, we can see a clear negative relationship between food share and price index: the higher the price paid by a household, the lower its food share. This result stems from the inclusion in $\hat{P}_R^i$ and $\hat{P}_{RL}^i$ of all possible variability we could gather from the household budget survey information.

4.2 - Economic Robustness of Price Information: the Slutsky Property

The demand analysis has been performed using a system with six budget shares, that include: food, clothing, housing, transport and communication, education and leisure, and other goods. Estimation has been carried out for the set of prices listed in Section 2 and reported in Table 3. Compensated own and cross-price elasticities for each of the five set of prices are reported[4] in Tables 5, 6, 7, 8 and 9.

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[4] The own and cross-price elasticities are computed at the average of each price.

Table 5. Compensated Own- and Cross-Price Elasticities using National Elementary Price Indexes

	Food	Cloth	House	Tracom	Educ	Other
Food	−1.325	0.134	0.355	0.474	−0.130	0.487
	0.108	*0.109*	*0.266*	*0.181*	*0.142*	*0.314*
Cloth	0.504	3.379	1.853	−2.624	−1.545	−1.563
	0.409	*0.822*	*0.731*	*0.466*	*0.501*	*0.890*
House	1.140	1.612	1.072	−0.240	0.338	−3.935
	0.866	*0.641*	*2.204*	*1.013*	*0.679*	*1.228*
Tracom	0.619	−0.926	−0.094	−0.199	0.267	0.339
	0.239	*0.153*	*0.413*	*0.326*	*0.249*	*0.344*
Educ	−0.560	−1.823	0.460	0.889	3.310	−2.273
	0.621	*0.506*	*0.929*	*0.845*	*0.937*	*1.202*
Other	1.040	−0.896	−2.596	0.549	−1.104	3.015
	0.714	*0.432*	*0.666*	*0.574*	*0.515*	*1.058*

Note: Standard errors are in italics.

Comparing price elasticities for the five sets of prices we notice that own-price elasticities computed using national $\left(P_N^i\right)$ and regional $\left(P_R^i\right)$ elementary price indexes are positive for four and three budget shares, respectively. This result is definitely in contrast with economic theory. Moreover the magnitude of the elasticities does not seem to provide any economic meaning.

For instance, looking at the own-price elasticity for the clothing share in Table 5, we see that it is positive and with a value of 3.379. This result becomes even worse when using the regional price indexes (Table 6). Similar comments apply to the elasticities of the other budget shares, as shown in Tables 5 and 6.

Table 6. Compensated Own- and Cross-Price Elasticities using Regional Elementary Price Indexes

	Food	Cloth	House	Tracom	Educ	Other
Food	–0.807	–1.111	0.146	0.303	0.328	1.137
	0.422	*0.333*	*0.219*	*0.187*	*0.158*	*0.436*
Cloth	–7.848	32.219	2.408	3.751	–5.840	–24.682
	3.081	*7.929*	*1.163*	*2.211*	*1.699*	*7.240*
House	0.469	1.104	–0.928	1.370	0.414	–2.440
	0.732	*0.568*	*0.666*	*0.542*	*0.386*	*0.791*
Tracom	0.445	0.780	0.624	–0.931	–0.398	–0.515
	0.274	*0.361*	*0.225*	*0.265*	*0.227*	*0.404*
Educ	1.415	–3.543	0.548	–1.163	0.724	2.022
	0.693	*0.997*	*0.511*	*0.617*	*1.042*	*1.238*
Other	2.121	–6.487	–1.398	–0.652	0.877	5.545
	0.928	*0.710*	*0.424*	*0.483*	*0.605*	*0.862*

Note: Standard errors are in italics.

Table 7. Compensated Own- and Cross-Price Elasticities using *Pseudo* Unit Values

	Food	Cloth	House	Tracom	Educ	Other
Food	–1.197	0.172	0.122	0.303	0.158	0.448
	0.035	*0.018*	*0.006*	*0.022*	*0.019*	*0.026*
Cloth	0.400	–0.806	0.105	0.094	0.052	0.166
	0.038	*0.053*	*0.007*	*0.037*	*0.036*	*0.048*
House	0.277	0.112	–0.830	0.136	0.091	0.211
	0.010	*0.007*	*0.004*	*0.012*	*0.007*	*0.011*
Tracom	0.335	0.044	0.064	–0.830	0.055	0.332
	0.024	*0.018*	*0.007*	*0.023*	*0.020*	*0.029*
Educ	0.561	0.080	0.130	0.177	–1.243	0.305
	0.064	*0.054*	*0.009*	*0.063*	*0.074*	*0.066*
Other	0.771	0.125	0.148	0.515	0.150	–1.694
	0.046	*0.035*	*0.007*	*0.047*	*0.031*	*0.063*

Note: Standard errors are in italics.

Moving through Tables 7, 8, and 9 we observe that the elasticities show a remarkable improvement, both in terms of sign and magnitude. Own-price elasticities for all budget shares are negative and statistically significant. However, differences exists between these elasticities. In particular, own and cross-price elasticities for $\hat{P}^i$ and $\hat{P}^i_R$ are similar both in sign and magnitude, but the magnitude of the own-price elasticity for the food share seems to be extremely high. However, the own price elasticity for $\hat{P}^i_{RL}$ is lower and equal to −0.360. This result seems to be more in line with the theory which predicts that the demand for food is less elastic than that for the other goods.

Table 8. Compensated Own- and Cross-Price Elasticities using Regional *Pseudo* Unit Values

	Food	**Cloth**	**House**	**Tracom**	**Educ**	**Other**
Food	–1.105	0.158	0.115	0.282	0.144	0.407
	0.032	*0.018*	*0.006*	*0.021*	*0.016*	*0.023*
Cloth	0.450	–0.795	0.082	0.079	0.035	0.154
	0.050	*0.065*	*0.008*	*0.045*	*0.037*	*0.048*
House	0.309	0.079	–0.829	0.215	0.062	0.156
	0.009	*0.008*	*0.032*	*0.015*	*0.007*	*0.010*
Tracom	0.362	0.036	0.104	–0.851	0.051	0.303
	0.026	*0.021*	*0.010*	*0.023*	*0.019*	*0.027*
Educ	0.654	0.055	0.102	0.177	–1.291	0.308
	0.069	*0.058*	*0.009*	*0.066*	*0.077*	*0.075*
Other	0.883	0.119	0.123	0.508	0.149	–1.772
	0.051	*0.036*	*0.008*	*0.048*	*0.035*	*0.069*

Note: Standard errors are in italics.

Another interesting piece of evidence about $\hat{P}^i_{RL}$ is the sign of the cross elasticities between food and education and between food and leisure. Both are statistically significant, with the signs of these elasticities establishing a negative substitution effect between the two goods, which should not surprise given the composition of the education and leisure budget share.

These results clearly show that own and cross-price elasticities present a large variability in terms of both sign and magnitude depending on the set of prices used in the estimation of the demand system. In general, the higher the degree of heterogeneity in the prices, the better the economic and statistical results.

Table 9. Compensated Own- and Cross-Price Elasticities using Regional *Pseudo* Unit Values in Level

	Food	**Cloth**	**House**	**Tracom**	**Educ**	**Other**
Food	–0.360	0.011	0.103	0.159	–0.038	0.125
	0.018	*0.012*	*0.007*	*0.016*	*0.011*	*0.017*
Cloth	0.035	–0.739	0.095	0.134	0.128	0.347
	0.041	*0.071*	*0.014*	*0.053*	*0.046*	*0.058*
House	0.258	0.071	–0.840	0.367	0.040	0.103
	0.015	*0.010*	*0.040*	*0.017*	*0.009*	*0.013*
Tracom	0.210	0.053	0.195	–0.935	0.057	0.420
	0.021	*0.022*	*0.010*	*0.034*	*0.020*	*0.028*
Educ	–0.204	0.209	0.087	0.232	–1.028	0.706
	0.062	*0.076*	*0.018*	*0.078*	*0.084*	*0.085*
Other	0.317	0.265	0.104	0.803	0.329	–1.818
	0.042	*0.044*	*0.013*	*0.052*	*0.038*	*0.079*

Note: Standard errors are in italics.

4.3 - A Counter-Factual Experiment Comparing Actual and Estimated Unit Values

The second part of our experiment was performed on micro data collected by ISMEA in 1995, with the appealing feature of recording the quantity of items bought by each household and their market prices. This set of information allowed a direct comparison between actual and *pseudo* unit values obtained using the method discussed in Section 2, with the final aim of understanding the informative content that *pseudo* unit values can provide.

The comparison was performed by means of nonparametric densities, carried out over four commodity shares for both actual and *pseudo* unit values. To make the magnitude of the two unit values comparable we have normalized the actual unit values around their own average.

Before turning to the results, we have to deal with a major technical problem, namely the zero expenditure, which becomes even more severe when we deal with subgroups of food expenditure. In fact, for those families who do not consume certain items the survey does not record information on either the expenditure or the market price. This means that we cannot compute the actual unit values and, at the same time, the lack of expenditure information affects the procedure for computing the *pseudo* unit values using the Lewbel method. To overcome the first problem we imputed the average of the specific market price to all those families with missing information on specific food item consumption. Similarly, in order to estimate the *pseudo* unit value, we substituted the missing information with the number 1[5].

As for the elasticities, in this section we comment only on the results for the food category as an aggregate. However, it is worth mentioning that our method has produced results for food which are quite different from and much better then those for the other sub-categories. These differences might be due to the own consumption within households or the zero expenditure.

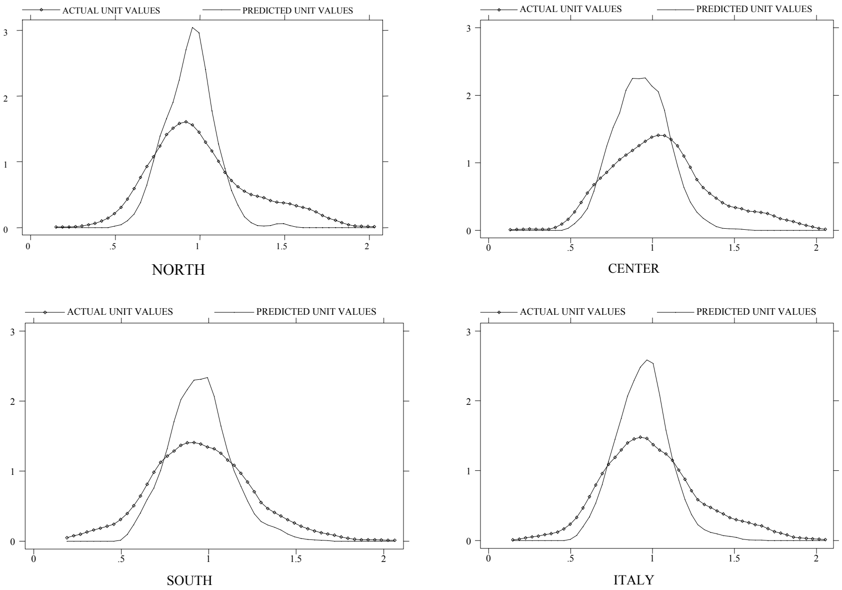
According to the density functions reported in Figure 2, the nonparametric densities for the actual and *pseudo* unit values look alike. Furthermore, even if the tails of the actual unit values are thicker than the tails of *pseudo* unit values, both are basically centered on 1. It is interesting to note that the shape of the actual and *pseudo* unit values differs across the three macroregions of Italy. This strengthens the effort to reproduce regional prices, as described in Section 2.

These evidence seems to support our method for recovering unit values from expenditure data and demographic characteristics. However,

[5] - Looking at equation 4, we can see that using this procedure we do not alter the estimation of the *pseudo* unit values.

problems related to the presence of zero expenditure in the data and/or to the use of a very specific functional form (Cobb-Douglas) for preferences in the Lewbel method should induce us to consider that this analysis is just a starting point for future in-depth research.

Figure 2. Nonparametric Densities for Actual and *Pseudo* Unit Values for Food Share



5 - Conclusions

The main objective of this work was to render household budget surveys that collect only information about expenditure, such as the Italian household survey conducted by ISTAT, suitable also for demand and welfare analysis. The lack of information about quantities bought precludes the possibility of deriving household specific prices (unit values) and of estimating complete demand systems on the basis of welfare analysis. As shown by the empirical demand analysis, the price information coming from aggregate price indexes derived from sources exogenous to the household survey may not be sufficient to provide plausible estimates.

We use a theoretical result developed by Lewbel (1989a) to construct *pseudo* unit values by reproducing the variability of cross-sectional price variation using the variability of the budget shares, and then adding the estimated variability to the aggregate price indexes published by the national statistical institute. We first describe the main features of the distribution of the constructed price set $\wp = \left\{P_N, P_R, \hat{P}, \hat{P}_R, \hat{P}_{RL}\right\}$ to make the changes in variability evident when adopting a specific choice for prices. The study then estimates a complete quadratic AI demand system using a time series of cross-sections of Italian household budgets including, in turn, aggregate price indexes and *pseudo* unit values, with the aim of showing the changes in the estimated price elasticities associated with the different prices. The results show that the matrix of compensated elasticities is negative definite only if *pseudo* unit values are used. Nominal *pseudo* unit values, which more closely reproduce actual unit values, give a set of own and cross-price effects that is more plausible. Lastly, we consider a household survey with actual unit values in order to conduct a counterfactual experiment aiming at comparing actual with *pseudo* unit values. The experiment shows that in most cases *pseudo* values maintain the relevant characteristics of the distribution of actual unit values. Overall, we conclude that *pseudo* unit values are better than aggregate price indexes for theoretically sound demand and welfare analysis.

Certainly, the adoption of *pseudo* unit values does no harm because the Lewbel method simply consists in adding cross-sectional price variability to aggregate price data.

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Appendix: Specification of the Translated Quadratic Demand System and Associated Price Elasticities

We choose to represent consumers' preferences using the Quadratic Almost Ideal Demand System demographically modified using a translating modifying term. The demographically translated cost function *C*(*u,p,d*), where *u* denotes the utility level, p prices and d demographic characteristics, is specified as:

$$\ln C(u,p,d) = \left[\ln A(p) + \frac{\varphi(u)B(p)}{1-\varphi(u)\lambda(p)}\right] + \ln\left[P^T(p,d)\right] =$$

$$= \left[\ln A(p) + \frac{B(p)}{\varphi^*(u)-\lambda(p)}\right] + \ln\left[P^T(p,d)\right] =$$

$$= \ln G(u,p) + \ln\left[P^T(p,d)\right],$$

where the term ln*A*(*p*) is specified as a Translog function

$$\ln A(p) = \alpha_0 + \sum_i \alpha_i \ln p_i + 0.5\sum_i\sum_j \gamma_{ij}^* \ln p_i \ln p_j,$$

and the price aggregator *B*(*p*) is defined as a Cobb-Douglas

$$B(p) = \beta_0 \prod_{i=1}^{n} p_i^{\beta_i}.$$

The term $\varphi^*(u) = 1/\varphi(u)$ is an index decreasing in utility $\varphi(u)$ for some monotonic function $\varphi(.)$ and the term λ(p) is a differentiable, homogeneous function of degree zero of prices *p*. The function

$P^T(p,d) = \prod_{i=1} p_i^{t_i(d)}$ is the translating term where demographic factors interact with prices. The demographic function includes a variable t indexing time to control for the year effect of the time-series of cross-sections and demographic attributes specified linearly:

$$\tau_i(d) = \alpha_{it} t + \sum_{k=1}^{K} \tau_{ik} d_k .$$

Note that the modified cost function is separable in the original preference structure $G(u,p)$ without demographic characteristics and the translating fixed cost term $P^T(p,d)$ grouping all demographic information .Welfare comparisons are therefore independent of the base level of utility, or income, chosen as the basis of the comparisons and are exact by construction (Lewbel 1989b and Blackorby and Donaldson 1991, Lewbel 1997). The inversion of the expenditure function gives the modified indirect utility function

$$\ln V(y,p,d) = \left[\left(\frac{\ln y^* - \ln A(p)}{B(p)} \right)^{-1} + \lambda(p) \right]^{-1},$$

where $\ln y^* = \ln y - \ln P^T$ is the PIGLOG indirect utility function. Roy's identity yields the following modified ordinary share equations

$$w_i = \alpha_i + \tau_i(d) + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \beta_i \left[\ln y^* - \ln A(p) \right] +$$
$$+ \frac{\lambda_i}{B(p)} \left[\ln y^* - \ln A(p) \right]^2 .$$

From this model, we obtain the uncompensated price elasticity ε^u_{ij} as:

$$\varepsilon^{u}{}_{ij} = \frac{\partial \ln q_i}{\partial \ln p_j} = \frac{\partial \ln w_i}{\partial w_i}\frac{\partial w_i}{\partial \ln p_i} =$$

$$= \frac{1}{w_i}\left\{\gamma_{ij} - \beta_j\left[\alpha_j + \sum_r \delta_{ir}\ln d_r + \sum_k \gamma_{ij}\ln p_k\right]\right\} -$$

$$\frac{1}{w_i}\left\{\frac{2\lambda_i}{B(p)}\ln\left(\frac{y^*}{A(p)}\right)\left(\alpha_j + \sum_r \delta_{ir}\ln d_r + \sum_k \gamma_{ij}\ln p_k\right)\right\} -$$

$$\frac{1}{w_i}\left\{\frac{\lambda_i\beta_j}{B(p)}\ln\left(\frac{y^*}{A(p)}\right)^2\right\} - \Lambda_{ij},$$

where Λ_{ij} is the Kronecker operator. The compensated price elasticities ε_{ij}, are derived as:

$$\varepsilon_{ij} = \varepsilon_{ij}^{u} + \eta_i w_j.$$