European University of Bangladesh

2/4 Gabtoli, Mirpur, Dhaka 1216.

Admit Card

Name of Exam : **Final Exam Summer 2021**
Semester : **Summer 2021**
Student's Name : **Shemol Chandra Roy**
Student's ID : **210122009**
Batch : **18th Batch**
Program : **BSc in Computer Science & Engineering (Diploma)**

Courses in which to appear at:

SL	Course Title	Course Code	Credit
1	Discrete Mathematics [A]	CSE-123	3
2	Introduction to Electrical Engineering [A]	EEE-101	3
3	Physics [A]	PHY-101	3
4	Introduction to Electrical Engineering Sessional [A]	EEE-102	1.5
5	Mathematics-II (Ordinary and Partial Differential Equations) [A]	MTH-103	3
6	Physics Sessional [A]	PHY-102	1.5

S/he is allowed to sit for the above mentioned exam.

[Digitally Signed]

Controller of Examinations (EUB)

Instructions for Examinees:

1. Examinee should come to the examination hall with the Admit Card.
2. No examinee will be allowed to sit in the examination hall outside the seat plan.
3. No bag or book will be allowed in the examination hall.
4. Cell Phone must be kept switched off in the examination hall.
5. No examinee will be allowed to enter the examination hall after expiry of half an hour.
6. No examinee will be allowed to leave the exam hall within the first half an hour after the examination begins.
7. Any examinee adopting unfair means will be brought under disciplinary action including expulsion.
8. Any kind of misbehavior will be considered as a serious offence under the rules of the University.

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European University of Bangladesh
2/4 Gabtoli, Mirpur. Dhaka-1216

Final Exam Summer-2021

Name : Shemol chandra Roy
ID : 210122009
Program : BSC in computer science and Engineering (Eve)
Course Title : Mathematices-II
course Code : MTH-103
Section : A
Semester & year : 2nd year 1st Semester
Date : 12/08/2021
Total page no : 09

Ans to the question no : 1(a)

Given,

$$z = Ce^{mt} \sin mx \quad \text{--- (i)}$$

Differentiating (i) partially with respect to x and t.

$$\frac{\partial z}{\partial x} = Cm e^{mt} \cos mx \quad \text{--- (ii)}$$

$$\frac{\partial z}{\partial t} = Cm e^{mt} \sin mx \quad \text{--- (iii)}$$

Again Differentiating (ii) partially with respect to x and t respectively

$$\frac{\partial^2 z}{\partial x^2} = -Cm^2 e^{mt} \sin mx \quad \text{--- (iv)}$$

$$\frac{\partial^2 z}{\partial t^2} = Cm^2 e^{mt} \sin mx \quad \text{--- (v)}$$

Adding ~~(iv) & (v)~~

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial t^2} = 0$$

This is the required partial Differential

Ans to the question no : 1 (b)

Given,

$$xdx - ydx - xdy + zdz = 0$$

$$\therefore (x-y)dx - xdy + zdz = 0 \quad \text{--- (1)}$$

Here, $P = (x-y)$; $Q = -x$, $R = z$

And,

$$\frac{\partial P}{\partial y} = -1, \quad \frac{\partial Q}{\partial x} = -1; \quad \frac{\partial R}{\partial x} = 0; \quad \frac{\partial P}{\partial z} = 0; \quad \frac{\partial Q}{\partial z} = 0; \quad \frac{\partial R}{\partial y} = 0$$

$$P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} + \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right)$$

$$= (x-y)(0-0) + (-x)(0-0) + z(-1+1)$$

$$= 0$$

Hence the differential equation is Integrable

Also, $\frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y}$; $\frac{\partial R}{\partial x} = \frac{\partial P}{\partial x}$ & $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

The differential equation is exat.

Regrouping (1) we get;

$$xdx - ydx - xdy + zdz = 0$$

$$xdx - (ydx + xdy) + zdz = 0$$

$$x\,dx - d(xy) + z\,dz = 0$$

Integrating we get;

$$\frac{x^2}{2} - xy + \frac{z^2}{2} = c$$

Ans.

Ans to the question no: 2(a)

Lagrange's Equation:

A linear PDE of the form $Pp + Qq = R$, Q, R are functions of x, y & z is called Largrange's Equations.

Ans to the question no: 2(b)—1

$x^2p + y^2q - z^2 = 0$

$\therefore x^2p + y^2q = z^2$ ——— (1)

Here,

$P = x^2, \; Q = y^2, \; R = z^2$

Lagrange's auxiliary equation is

$$\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{z^2} \quad \text{——— (2)}$$

Taking 1st & 2nd term $\int \frac{dx}{x^2} = \int \frac{dy}{y^2}$

$\Rightarrow -\frac{1}{x} = -\frac{1}{y} + c$

$\Rightarrow \frac{1}{x} - \frac{1}{y} = -c$

$\Rightarrow \frac{1}{x} - \frac{1}{y} = c_1$

Taking 2nd & 3rd term $\int \frac{dy}{y^2} = \int \frac{dz}{z^2}$

$\Rightarrow -\frac{1}{y} = -\frac{1}{z} + c$

$\Rightarrow \frac{1}{y} - \frac{1}{z} = -c$

$\Rightarrow \frac{1}{y} - \frac{1}{z} = c$

Hence $\phi\left(\frac{1}{x} - \frac{1}{y}, \frac{1}{y} - \frac{1}{z}\right) = 0$ is the required solution

Ans to the question no : 2(b)—ii

Given,

$$2p - 3q = 3x^2 \sin(2y + 3x) \quad \text{--- (i)}$$

Here,

$$P = 2,\ Q = -3,\ R = 3x^2 \sin(2y + 3x)$$

Lagrange's auxilliary equation is

$$\frac{dx}{2} = \frac{dy}{-3} = \frac{dz}{3x^2 \sin(2y+3x)} \quad \text{--- (ii)}$$

Taking 1st and 2nd term

$$\frac{dx}{2} = \frac{dy}{-3}$$

$$\Rightarrow \int 3dx + \int 2dy$$

$$\Rightarrow \cancel{2y + 3x}$$

$$\Rightarrow 2y + 3x = c_1$$

$$u = 2y + 3x = c_1$$

Taking 1st & 3rd term

$$\frac{dx}{2} = \frac{dz}{3x^2 \sin(2y+3x)}$$

$$\Rightarrow \int 2dz = \int 3x^2 \sin c_1 \, dx$$

$$\Rightarrow 2z + c_2 = x^3 \sin c_1$$

$$\Rightarrow x^3 \sin c_1 - 2z = c_2$$

$$\Rightarrow \cancel{x^3 \sin(2y+3x)}$$

$$\Rightarrow x^3 \sin(2y+3x) - 2z = c_2$$

Hence $\varphi(2y+3x,\ x^3 \sin(2y+3x)) = 0$ is the required solution.

Ans to the question no: 3(i)

Given that,

$$(D^3_x - 7D_xD_y^2 - 6D^3_y)z = 0$$

let,

$z = \phi(y+mx)$ be the solution of (1)

putting $D_x = m$, $D_y = 1$ in equation(1)

Auxiliary equation

$$m^3 - 7m - 6 = 0$$

$$\Rightarrow m^3 - m - 6m - 6 = 0$$

$$\Rightarrow m^3 + m^2 - m^2 - m - 6m - 6 = 0$$

$$\Rightarrow m^2(m+1) - m(m+1) - 6(m+1) = 0$$

$$\Rightarrow (m+1)(m^2 - m - 6) = 0$$

$$\Rightarrow (m+1)(m^2 - 3m + 2m - 6) = 0$$

$$\Rightarrow (m+1)\{m(m-3) + 2(m-3)\} = 0$$

$$\Rightarrow (m+1)(m-3)(m+2) = 0$$

$\therefore m = -1, -2, 3$

C.F, $z_c = \phi_1(y-x) + \phi_2(y-2x) + \phi_3(y+3x)$ Ans

Ans to the question no : 3 (ii)

Given that,

$$(D_x^3 - 4D_x^2 D_y + 4D_x D_y^2)z = \cos(2x+y) \longrightarrow (1)$$

Let,

$z = \phi(y+mx)$ be the solution of (1)

putting $D_x = m$, $D_y = 1$ in equation (1)

Auxiliary equation

$$m^3 - 4m^2 + 4m = 0$$

$$\Rightarrow m(m^2 - 4m + 4) = 0$$

$$\Rightarrow m(m^2 - 2m - 2m + 4) = 0$$

$$\Rightarrow m\{m(m-2) - 2(m-2)\} = 0$$

$$\Rightarrow m(m-2)(m-2) = 0$$

$\therefore m = 0, 2, 2$

C.F, $z_c = \phi_1(y) + \phi_2(y+2x) + \phi_3(y+2x)$

$$P.I,\ z_p = \frac{1}{(D_x^3 - 4D_x^2 D_y + 4D_x D_y^2)} \cos(2x+y)$$

$$= \frac{1}{D_x (D_x - 2D_y)^2} \cos(2x+y)$$

$$= \frac{1}{D_x} \cdot \frac{x^2}{2!} \cos(2x+y)$$

$$= \frac{x^2}{2\times 2} \sin(2x+y)$$

$$= \frac{x^2}{4} \sin(2x+y)$$

~~G.S = $\phi_1(y+2x) + \phi$~~

$$\therefore G.S = \phi(y) + \phi_2(y+2x) + \phi_3(y+2x) + \frac{x^2}{4}\sin(2x+y)$$

Ans