The RTN and STN Models of Industrial Production Processes

The authors

Nomenclature

The RTN Model

δ	Length of the time slot.
E_i^{req}	Energy required to complete task i.
i	Index of the task.
$i+$	Index of the subsequent task of task i.
$N_{i,t}$	Binary variable modeling the start of task i.
P_i^L, P_i^H	Minimum and maximum processing powers of task i, respectively.
$P_{i,t}$	Power of task i at time slot t.
$\Pi_{EL,t}$	Energy consumption of the production process at time t.
Pr_t	Electricity price at time slot t.
r	Index of the resource.
r^{d}	Index of the product located at the transfer destination.
r^{f}	Index of the product located at the final stage.
r^{s}	Index of the product located at the transfer start point.
$R_{r,t}$	Value of resource r at time t.
$S_{i,t}$	Processing status of task i at time slot t.
t	Index of the discrete time slot.
τ_i^L, τ_i^H	Minimum and maximum processing times of task i, respectively.
$W_{r^{\mathrm{d}}}$	Maximum waiting time of resource r^{d}.
$w_{r^{\mathrm{d}}}$	Transfer time of resource r^{d}.
$\mu_{EL,i,\theta}$	Energy consumption of task i, θ time slots after the task starts.
$\mu_{r,i,\theta}$	Amount of resource r consumed/generated by task i, θ time slots after the start of task i.

The STN Model

$Cost$	Total energy cost across time horizon.
C_{ik}	Material consumption rate of task i at operating point k (kg/h).
Δt	Length of time interval.
Δt_{tik}	Time duration that task i operates at point k within interval t (h).
E_t	Energy consumption of factory at time t (kWh).
f	Index of the factory.
F	Set of factories.
G_{ik}	Material production rate of task i at operating point k (kg/h).
i	Index of the production task.
i^{end}	Total number of production tasks.
I^{P}	Set of production tasks.
I^{S}	Set of production states.
k	Index of the operating point.
K_i	Set of operating points of task i.
P_{ik}	Electricity consumption power of task i at operating point k (kW).
Pr_t	Electricity price at time t (\$/kWh).
S_{ti}	Amount of material i at the end of time interval t (kg).
S_i^0	Initial amount of material i (kg).
$S_i^{\max}$	Upper limit of material i (kg).
S_i^{tar}	Target amount of material i (kg).
t	Index of time interval.
T	Set of time intervals in the time horizon.
t^{end}	Final time interval.

I. The Resource Task Network (RTN) Model

A. Resource Task Network

We use the production process of steel manufacturing as an example to illustrate the modeling method of the RTN. The production process of a steel plant is shown in Fig. 1, which can be divided into four stages: 1. melting, 2. decarburization, 3. refining, and 4. casting. The four production stages, as well as the transfer tasks between them, are modeled as tasks, indexed by i. The schedule of tasks is based on discrete time slots indexed by t, with a length of δ (e.g., $\delta = 5$ minutes). The binary variable $N_{i,t}$ models the start of task i, with $N_{i,t} = 1$ representing that task i starts at time slot t.

The production equipment and products (intermediate products and final products) are modeled as resources, indexed by r. $R_{r,t}$ represents the value of resource r at time t. For instance, if r represents the EAF, $R_{r,t} = 1$ means that the EAF is idle at time slot t and can be used for melting. Since the product needs to be transported between different production links, to distinguish the product before and after transportation, it is modeled as different resources, represented by superscripts s and d, respectively; i.e., r^{s} (r^{d}) is the index of the product located at the transfer start point (destination).

The interaction between tasks and resources is modeled by the interaction matrix M, in which an entry $\mu_{r,i,\theta}$ represents the amount of resources r consumed/generated by task i, θ time slots after the start of task i. For example, the melting task (task i) generates molten steel (resource r) at 80 minutes (16 time slots with a time slot length of $\delta = 5$ minutes) after the starting time, so entry $\mu_{r,i,16}$ is set to 1.

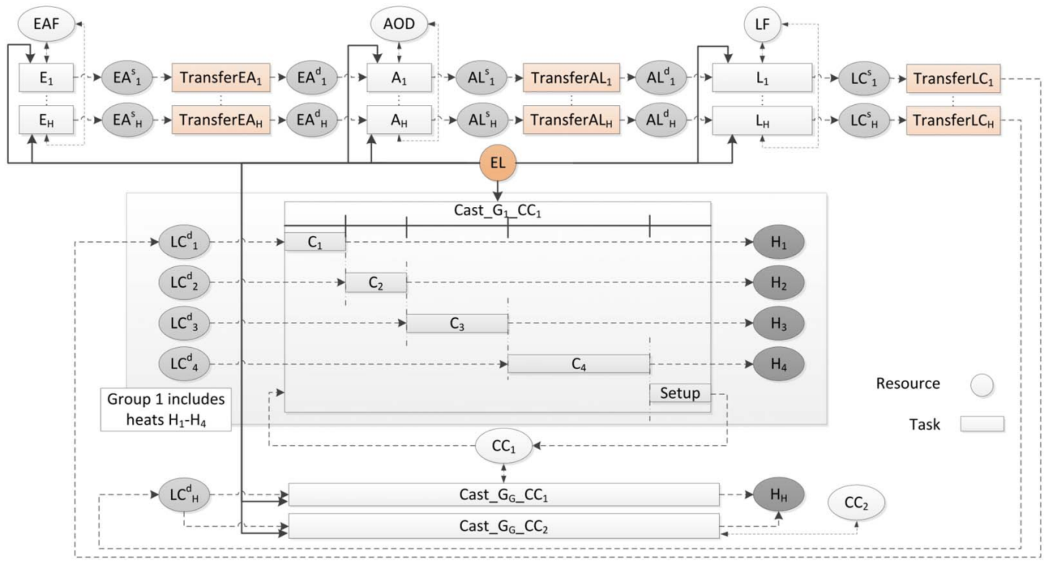

Fig. 1. Resource task network for basic scheduling of a steel plant.

Note that in the conventional RTN model, tasks are bound to batches; that is, the process and transfer of different batches are modeled as different tasks. Specific indexes, modeling methods, and examples can be found in Ref. [1]. This modeling method means that the scale of the RTN model of the same production line will increase with the production target (the number of batches), which will limit the scalability of the model.

B. Mathematical Formulation of the Basic RTN Model

The modeling approaches herein were selected from Ref. [1] and reduced for clarity and brevity.

a) Resource balance: The quantity of resources at the end of each time slot $R_{r,t}$ is determined by the initial quantity (i.e., the quantity at the end of the previous time slot) $R_{r,t-1}$ and the changes to the resources caused by the tasks within the time slot. This principle of resource balance can be represented as a constraint for both equipment and products in the following form:

$$R_{r,t} = R_{r,t-1} + \sum_i \sum_{\theta=0}^{\tau_i} \mu_{r,i,\theta} N_{i,t-\theta} \quad \forall r, t \tag{1}$$

Imposing this constraint on the equipment limits the use of a device to a single task within a given time period. However, this leads to the occurrence of rounding errors, which will be further analyzed in the case study. Similarly, the energy consumption of production process $\Pi_{EL,t}$ can be represented with a comparable constraint, although there is no direct coupling between the energy consumption across different time slots:

$$\Pi_{EL,t} = \sum_i \sum_{\theta=0}^{\tau_i} \mu_{EL,i,\theta} N_{i,t-\theta} \quad \forall t \tag{2}$$

where $\mu_{EL,i,\theta}$ is the energy consumption of task i, θ time slots after the task starts.

b) Task Execution: The RTN models each batch of products individually, and task modeling is also based on batches. Therefore, each batch needs to have each task applied to it only once:

$$\sum_t N_{i,t} = 1 \quad \forall i \tag{3}$$

For transfer tasks, the constraint below is added to ensure immediate execution (4). Constraint (4) requires that there be no waiting time for intermediate products at the start of transfer. The rationale behind this is twofold: first, this is a common requirement in IPPs such as steel manufacturing; second, it avoids a modeling approach where manufacturing is completed in one stage, followed by a waiting period, followed by transfer, followed by waiting again; avoiding this approach maintains the model's simplicity.

$$R_{r^{\mathrm{s}},t} = 0 \quad \forall r^{\mathrm{s}}, t \tag{4}$$

c) Waiting Time Limit: Intermediate products can wait for a period of time between two production stages rather than immediately proceeding to the next stage. This is one source of the energy flexibility of IPPs. However, this waiting time usually has an upper limit because materials such as molten steel need to be further processed before their temperature drops to a certain threshold. The duration limit of the waiting process (including the transfer time) is expressed as follows for resource r:

$$\delta \sum_t R_{r^{\mathrm{d}},t} + w_{r^{\mathrm{d}}} \leq W_{r^{\mathrm{d}}} \quad \forall i \tag{5}$$

where $R_{r^{\mathrm{d}},t} = 1$ indicates that the product has arrived at the transfer destination at time slot t and is waiting; therefore, $\sum_t R_{r^{\mathrm{d}},t}$ is the waiting time, $w_{r^{\mathrm{d}}}$ is the transfer time and $W_{r^{\mathrm{d}}}$ is the maximum waiting time of resource r^{d}.

d) Product Delivery: By the end of the final time slot, each batch needs to reach the final stage, i.e., complete all the manufacturing processes:

$$R_{r^{rmf},T} = 1, \forall r^{rmf} \tag{6}$$

where r^{rmf} is the index of the product located at the final stage.

e) Objective: The goal of factory production scheduling is not a constraint on its internal production process. For the completeness of the model, we use an objective function as an example, which aims to minimize the production energy cost:

$$\min. \sum_t Pr_t \Pi_{EL,t} \tag{7}$$

where Pr_t is the electricity price at time slot t.

f) Multiple operating modes: In actual operation, a device may operate in one of several states, not just in the "on" and "off" states. The basic RTN model mentioned above can model this feature by modeling the different operating states of the device as different tasks. For example, if a device can operate at 50% of its rated power, then the interaction matrix needs to model its processing time as twice the rated operating state (i.e., $\mu_{r,i,2\tau} = 1$, where τ is the nominal processing time), and the energy consumption per time slot is half the nominal value.

C. Flexible operating mode

If a device can adjust its operating point in each period or even continuously adjust within the operating boundaries, then its processing time can be adjusted within a feasible range. This constraint can be expressed in terms of the start time of the subsequent task (i.e., the end time of the task, as we assume the transfer tasks are executed immediately) as follows:

$$\sum_{t'=t+\tau_i^L}^{t+\tau_i^H} N_{i+,t'} \geq N_{i,t} \tag{8}$$

where τ_i^L and τ_i^H are the minimum and maximum processing times of task i, respectively, and i and $i+$ are the indices of the task that can be flexibly adjusted and its subsequent task, respectively.

Under the flexible operating mode, we also need to introduce the power variable of the device operation (a continuous variable), which is subject to the following constraints:

$$P_i^L \cdot S_{i,t} \leq P_{i,t} \leq P_i^H \cdot S_{i,t} \quad \forall i, t \tag{9}$$

where P_i^L and P_i^H are the minimum and maximum processing powers of task i, respectively; $P_{i,t}$ is the power of task i at time slot t; $S_{i,t}$ is the processing status of task i at time slot t; and $S_{i,t} = 1$ indicates that task i is being processed at time slot t, which means that the device is on and usually cannot be interrupted until the end of the task. Naturally, $S_{i,t}$ is determined by the start times of the task and the subsequent task:

$$S_{i,t} - S_{i,t-1} = N_{i,t} - N_{i+,t} \quad \forall t \tag{10}$$

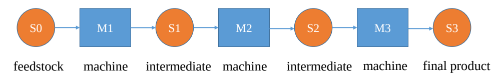

Fig. 2. A common state-task network model for industrial production process. M_i and S_i represent the ith machine ("task"') and its product ("state"), both included explicitly as network nodes. Here, the task can be an actual industrial device or several devices for the same production objective. The states can represent the feedstock, intermediate, or final products.

Finally, different control methods need to comply with the energy-material conversion conditions of processing itself, and a certain amount of energy needs to be invested to complete processing:

$$\sum_t \delta P_{i,t} \geq E_i^{\mathrm{req}} \quad \forall i \tag{11}$$

where E_i^{req} is the energy required to complete task i, which can be determined by the nominal power multiplied by the nominal processing time of the task if it is assumed that the same amount of energy is required for different processing modes.

II. The State-Task Network (STN) Model

The general-purpose STN (Fig. 2) can model a wide range of production processes arising in multiproduct/multipurpose industrial facilities [2]. Consider an arbitrary factory $f \in F$ (for brevity, the subscript f is omitted in this section, e.g., E_t for E_{ft}). Let i/i^{end} denote the index/total number of production tasks. Let $I^{\mathrm{P}}/I^{\mathrm{S}}$ denote the set of production tasks/states ($I^{\mathrm{P}} = \{1, 2, ..., i^{\mathrm{end}}\}$, $I^{\mathrm{S}} = \{0\} \cup I^{\mathrm{P}}$, $i = 0$ for the feedstock). Let t and T denote the index of time intervals and the whole time horizon, respectively ($T = \{0, 1, 2, ..., t^{\mathrm{end}}\}$, $t = 0$ for the initial or current time interval). Let k/K_i denote the index/set of the operating points of task i.

The conventional STN model assumes that industrial devices can only operate at one point within a time interval.

Factory f aims to minimize $Cost$, its total energy cost across T:

$$Cost = \sum_{t \in T} Pr_t E_t, \tag{12}$$

where Pr_t (\$/kWh) is the electricity price and E_{it} (kWh) is the energy consumption of factory f at t. E_t is decomposed to the energy consumption of the operating points of the tasks:

$$E_t = \sum_{i \in I^{\mathrm{P}}} \sum_{k \in K_i} P_{ik} \Delta t_{tik}, \ t \in T, \tag{13}$$

where P_{ik} (kW) is the electricity consumption power of task i at operating point k and Δt_{tik} (h) is the time duration (continuous) that task i is operating at point k within t. Naturally, the operation time of a task in all its operating points within a time interval, including the off state, is nonnegative (14) and sums up to the length of the interval Δt (15):

$$\Delta t_{tik} \in \{0, 1\} \times \Delta t, \ t \in T, i \in I^{\mathrm{P}}, k \in K_i. \tag{14}$$

$$\sum_{k \in K_i} \Delta t_{tik} = \Delta t, \ t \in T, i \in I^{\mathrm{P}}. \tag{15}$$

Note that we only focus on the duration or the length of time that the devices operate at their different operating points (e.g., "on"/"off") in the time periods, rather than the specific time to switch operating points. After determining the operating duration, the feasibility of the specific time to switch operating points is guaranteed by (14)-(15), and operations are left for the factory to implement.

Let S_{ti} (kg) denote the amount of material i at the end of time interval t. Let $S_i^{0/\max/\mathrm{tar}}$ denote the initial/upper limit/target amount of material i. The facility must meet its production target (16) including a buffer limit (17), and the initial states of the buffers are given by (18):

$$S_{ti} \geq S_i^0 + S_i^{\mathrm{tar}},\ t = t^{\mathrm{end}}, i \in I^{\mathrm{S}}. \tag{16}$$

$$0 \leq S_{ti} \leq S_i^{\max},\ t \in T, i \in I^{\mathrm{S}}. \tag{17}$$

$$S_{ti} = S_i^0,\ i \in I^{\mathrm{S}}, t = 0 \tag{18}$$

Let G_{ik}/C_{ik} (kg/h) denote the material production/consumption rate of task i at operating point k. The change in buffer states across time for the feedstock, intermediate, and final products is given by (19), (20), and (21), respectively:

$$S_{ti} = S_{(t-1)i} - \sum_{k \in K_{i+1}} C_{(i+1)k} \Delta t_{t(i+1)k},\ t \in T, i = 0. \tag{19}$$

$$S_{ti} = S_{(t-1)i} + \sum_{k \in K_i} G_{ik} \Delta t_{tik} - \sum_{k \in K_{i+1}} C_{(i+1)k} \Delta t_{t(i+1)k},$$
$$t \in T, i \in I^{\mathrm{P}} \setminus \{i^{\mathrm{end}}\}. \tag{20}$$

$$S_{ti} = S_{(t-1)i} + \sum_{k \in K_i} G_{ik} \Delta t_{tik},\ t \in T, i = i^{\mathrm{end}}. \tag{21}$$

Although STN is derived for devices with discrete operating points, it can also model the operation of nonadjustable devices as well as devices with inherently continuous operating points. We verify that the former can be modeled by only one operating point and the latter can be modeled by two operating points, corresponding to "on" (i.e., operating at rated power) and "off" states. For devices whose time to switch the operating point is not negligible, binary variables can be introduced in conventional ways. As an alternative, the continuous-time assumption of STN can still be used, which requires sacrificing model accuracy for computational efficiency. We will analyze the effect of this compromise in the case study.

References

[1] X. Zhang, G. Hug, and I. Harjunkoski, "Cost-Effective Scheduling of Steel Plants With Flexible EAFs," IEEE Trans. Smart Grid, vol. 8, no. 1, pp. 239–249, Jan. 2017.

[2] E. Kondili, C. Pantelides, and R. Sargent, "A general algorithm for short-term scheduling of batch operations—I. MILP formulation," Computers & Chemical Engineering, vol. 17, no. 2, pp. 211–227, Feb. 1993.