Constructing cointegrated cryptocurrency portfolios for statistical arbitrage

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Abstract

In this paper, we analyze the process of constructing cointegrated portfolios of cryptocurrencies. Our procedure involves a series of statistical tests, including the Johansen cointegration test and Engle-Granger two-step approach. Among our results, we construct cointegrated portfolios involving four cryptocurrencies: Bitcoin (BTC), Ethereum (ETH), Bitcoin Cash (BCH), and Litecoin (LTC). We develop a number of trading strategies under different entry/exit thresholds and risk constraints, and examine their performance in details through backtesting and comparison analysis. Our methodology can be applied more generally to create new cointegrated portfolio using other cryptocurrencies.

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1 Introduction

In 2009, Bitcoin (BTC) was introduced to become the first decentralized digital currency.[1] Since then, Bitcoin has been accepted by thousands of merchants and vendors, and the cryptocurrency market has seen enormous growth (Hileman and Rauchs, 2017). Its market capitalization measured in USD has grown from about $10bn in 2014 to over $200bn in 2018, with a peak around $828bn in Jan 2018.[2] Today, there are more than a thousand of cryptocurrencies besides Bitcoin, such as Ethereum (ETH), Litecoin (LTC), Bitcoin Cash (BCH), and more that are being traded on major cryptocurrency exchanges internationally.[3] Many alternative coins, also known as *altcoins*, were invented to be substitutes of Bitcoin for various reasons. For instance, Ethereum, a decentralized platform that run smart contracts,[4] was created to address the lack of a scripting language of Bitcoin for application development.[5] As another example, Bitcoin Cash was invented to address the scalability issue of Bitcoin due to its block size limit which put a cap on the amount of transactions Bitcoin network can process on a given time frame, etc. Such innovations have led to the rapid growths of many altcoins together with the rise in market size of cryptocurrency. However, being known as the earliest decentralized currency, Bitcoin have consistently been ranked on top in terms of traded volume, price and market capitalizations (see Table 1).

The rise of Bitcoin and many altcoins has created trading opportunities for investors and speculators. In fact, a recent study by Chuen et al. (2017) finds very low correlations between the CRyptocurrency Index (CRIX), a portfolio of cryptocurrencies, and traditional assets. Also, their empirical results show that including CRIX in an initial portfolio consisting only traditional assets can expand the efficient frontier and give additional utility to investors. Recall that S&P 500 returned around 21% in 2017, but Bitcoin gained an astounding 1,318% in one year. In an earlier study, Cheah and Fry (2015) consider the Bitcoin phenomenon as a speculative bubble with zero fundamental value. In mid December 2017, Bitcoin price reached all-time-high at over $19 thousand, with 24h volume approximately $23 billion. However, as price plummeted to a low $6000 in early February, Bitcoin has been considered one of the greatest bubbles in finance history.

The price surges of Bitcoin and Ethereum in early 2017 generated much attention to the cryptocurrency market and even triggered a massive series of Initial Coin Offerings (ICOs), which are similar to Initial Public Offerings (IPOs), but are for cryptocurrencies or tokens and less regulated. Many technology startups launched their own cryptocurrencies as a means of crowdfunding, though some ICOs were completely frauds or had been heavily targeted by hackers (Labbe, 2017). To purchase tokens, investors must have one of the mainstream coins which typically were Bitcoin and Ethereum. Such events created an increased demand that pushed the prices of these two coins even higher. One of the most popular ways for investors to gain access to the cryptocurrency market was through Coinbase[6] where investors could purchase coins directly from their bank accounts and then transfer the coins to different digital wallets and exchanges for trading purposes. Some exchanges even allow short selling of cryptocurrencies.

Unlike traditional equity market, most cryptocurrencies are highly correlated. For exam-

[1] See the bitcoin white paper, *Bitcoin: A Peer-to-Peer Electronic Cash System*, by Satoshi Nakamoto. Available at `https://bitcoin.org/bitcoin.pdf`

[2] Source: `https://coinmarketcap.com/charts/`

[3] Source: `https://coinmarketcap.com/rankings/exchanges/`

[4] Smart contracts are applications that allows exchanges of money, property, or anything of value to take place without the middleman once certain conditions are met.

[5] See the white paper, *A Next-Generation Smart Contract and Decentralized Application Platform*, by Vitalik Buterin, the founder of Ethereum. Available at `https://github.com/ethereum/wiki/wiki/White-Paper#decentralized-autonomous-organizations`

[6] Founded in 2012, Coinbase was one of the earliest online platform in the U.S. that allowed users to store, trade, and transfer digital currencies.

Rank	Name	Symbol	Price ($)	MarketCap ($)
1	Bitcoin	BTC	8239.57	141534282604
2	Ethereum	ETH	467.785	47238602070
3	XRP	XRP	0.454269	17859896217
4	Bitcoin Cash	BCH	828.355	14299695631
5	EOS	EOS	8.3154	7451841487
6	Stellar	XLM	0.314209	5896895909
7	Litecoin	LTC	84.5161	4869209762
8	Cardano	ADA	0.163161	4230286756
9	IOTA	MIOTA	1.02566	2850853030
10	Tether	USDT	0.998571	2503557642
11	TRON	TRX	0.036809	2420122242
12	Monero	XMR	140.502	2285241135
13	NEO	NEO	34.0835	2215427500
14	Dash	DASH	241.932	1988855554
15	Ethereum Classic	ETC	16.9501	1752226368
16	NEM	XEM	0.177986	1601874000
17	VeChain	VEN	2.6677	1479361013
18	Binance Coin	BNB	14.2613	1362132746
19	Tezos	XTZ	2.07289	1259257958
20	Zcash	ZEC	220.618	983003486

Table 1: Summary of top 20 cryptocurrencies ranked by market capitalization as of August 16, 2018. Source: CoinMarketCap.

ple, correlations on daily returns between the 4 major digital currencies, BTC, ETH, LTC, and BCH, were over 75% as shown in Figure 1. The high volatility of cryptocurrencies calls for the development of market-neutral trading strategies, and the high correlations among cryptocurrencies motivates us to investigate cointegration or mean-reversion strategies.

Mean-reversion trading strategies have been extensively researched and studied by academic researchers and practitioners who seek to understand the long-term co-movements of different asset prices and arbitrage from the mean-reverting property of the price spread. Strategies have been developed for many areas of traditional finance, including fixed income Wagner (2005), commodities and futures (Leung and Ward, 2015), as well as equities and ETFs (Kang and Leung, 2017; Leung and Li, 2015). However, very little research has been done for trading pairs or baskets of cryptocurrencies. In fact, most research on trading cryptocurrencies are based on technical analysis (Ha and Moon, 2018; Detzel et al., 2018), or arbitraging from price differences across exchanges (Makarov and Schoar, 2018).

In this paper, we analyze the process of constructing a cointegrated set of cryptocurrencies. The process involves a series of statistical tests, including the Johansen cointegration test (Johansen, 1988), and the classical 2-step approach developed by Engle and Granger (1987). Among our results, we construct cointegrated portfolios involving four cryptocurrencies: Bitcoin (BTC), Ethereum (ETH), Bitcoin Cash (BCH), and Litecoin (LTC), and the corresponding trading strategies are shown to be profitable under different entry/exit thresholds and risk constraints. To our best knowledge, the proposed cointegrated portfolio of 4 cryptocurrencies is new, and our methodology can be applied more generally to create new cointegrated portfolio using other cryptocurrencies.

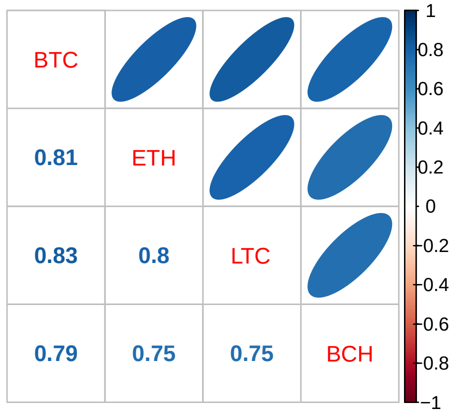


Figure 1: Correlation in daily returns of BTC, ETH, LTC, and BCH

2 Data

Before designing the trading systems, we first make general observations on how prices (in USD) of Bitcoin (BTC), Ethereum (ETH), Bitcoin Cash (BCH), and Litecoin (LTC) have co-moved since BCH was first introduced. To identify a cointegrating relationship among these crypto-assets, we employ both Johansen and Engle-Granger approaches to construct a mean-reverting portfolio to be tested for stationarity and tradability. Lastly, we design and back test trading systems for the mean-reverting portfolio under five different entry/exit threshold levels and incorporate a stop-loss exit and trailing stop exit.

A majority of cryptocurrency exchanges provides traders with user-friendly trading platforms with the intention to attract more users by reducing the complexity of buying and selling coins. For instance, Coinbase, in addition to the web application, offers mobile app that allows traders to store coins and make trades directly from their phones without having to know much about different types of order (i.e. limit, market order, etc.). For larger exchanges, such as Binance, Bitfinex and Bittrex, historical and live data are also accessible through RESTful APIs that allow traders to send price requests for analysis and manage their trading accounts. In this paper, we use the Open High Low Close Volume (OHLCV) data accessed through an API provided by CryptoCompare, a database in UK that provides raw and preprocessed cryptocurrency market data from multiple exchanges around the world. Historical data aggregated from all listed exchanges are also available in their inventory. However, due to price differences across exchanges, daily prices in U.S. dollars of Bitcoin (BTC), Ethereum (ETH), Litecoin (LTC) and Bitcoin Cash (BCH) were gathered only from Coinbase from December 20, 2017[7] to June 20, 2018 to construct our trading models.

[7]Bitcoin Cash only became available on Coinbase on December 20, 2017

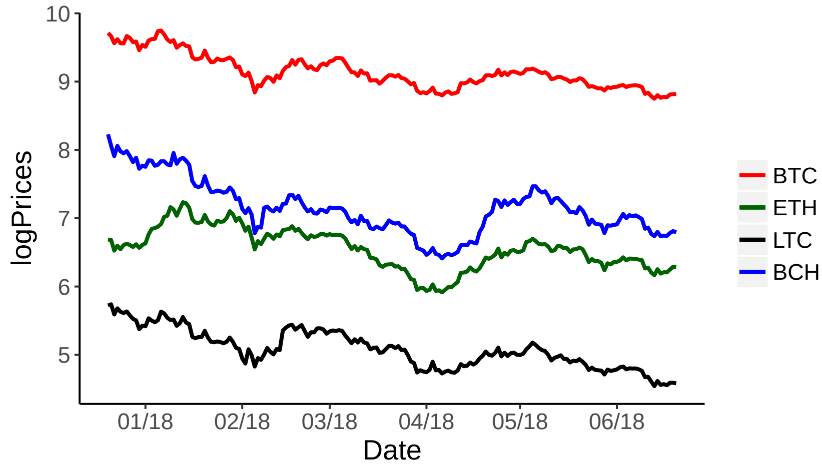


Figure 2: Log-prices of 4 main cryptocurrencies in Coinbase

As shown in Figure 2, price movements of all 4 cryptocurrencies during the testing period are nearly identical. This could imply a potential linear relationship between these 4 coins. Furthermore, if the time series of all 4 cryptocurrencies are stationary after 1-differencing, i.e. their prices are $I(1)$ processes, and there exists a linear combination whose residuals are stationary (or mean reverting), then these crypto-assets are said to be cointegrated. The linear combination (or number of shares to purchase for each crypto-asset[8]) that forms a stationary portfolio from non-stationary asset prices is often referred to as cointegrating vector. From a trading perspective, an investor can profit from purchasing this stationary portfolio, or so called the spread, when price is low and realize a profit when its price return to the mean price or cross above certain threshold. Similarly, an investor can short sell the spread when its price is high and realize a profit when price reverts to the mean price or cross below certain level. This characteristic of mean-reverting in price is extensively used by hedge fund managers and proprietary trading firms to construct pair-trading strategies, or so-called statistical arbitrage. One can think of the spread as a mutual fund or a basket of crypto-assets where the number of shares on each asset (can be positive or negative) would be predetermined by the cointegrating vector. And to purchase or sell the spread, an investor must be able to long and short on certain assets by the amount of shares suggested their cointegrating relationships.

3 Mean-Reverting Portfolio

Cointegration reflects how asset prices are tied together in long-term by a common stochastic trend even though they might drift apart in the short run. Therefore, cointegrated asset prices may appear to be more useful in developing quantitative trading models. Theoretical framework for constructing cointegrating time series in this paper include the Johansen cointegration test (Johansen, 1988), and the classical 2-step approach developed by Engle and Granger (1987). Note that, although the two approaches are somewhat equivalent in terms of establishing cointegrating relationships between given crypto-assets, the Johansen test tends to be more appropriate when it comes to modeling multivariate time series data. However, there are good reasons why the Engle-Granger approach is chosen for most financial applications, in particular, risk management. While the Johansen criterion seeks for

[8]Often referred to as weighted sum vector; however, since we perform regression directly on price series, this linear combination implies the number of shares to go long/short on each crypto-asset

the linear combination that is most stationary, Engle-Granger approach, based on ordinary least squares regression (OLS), seeks for the linear combination with minimum variance. As a result, Engle-Granger approach is generally a preferred method for most risk managing purposes, especially when involving risk measures. Also, one might be able to find more than one linear combination to form a mean-reverting portfolio; but to determine the appropriate model, we will need to examine both approaches and choose the cointegrating vector that would give the spread a faster rate of mean-reversion. In other words, we construct the spread in such way that yields highest profit when trading the mean-reverting crypto-portfolio.

3.1 Johansen Test

The main advantage of the Johansen test over the Engle-Granger approach is the ability to perform hypothesis testings directly on $I(1)$ price series themselves and derive cointegrating vectors without having to constantly check for stationarity. The order of cointegration, denoted rank($\mathbf{r}$), from Johansen criterion indicates the number of independent portfolios that can be formed by various linear combinations of the price time series, whereas using Engle-Granger approach we can only infer at most one linear combination if there exists one.

From Table 2, the Johansen test estimates of the order of integration from given price series with 3 confidence levels, i.e. 10%, 5%, and 1%. We reject the null hypothesis for $r = 0$ since the test value is greater than confidence level's value at 1%. Similarly, we accept the null hypothesis for $r = 1$ at 95% confidence. We infer that BTC, ETH, LTC and BCH are of 1^{st} order cointegrated, i.e rank($\mathbf{r}$) = 1. Reading off the first column of Table 3, we find a set of linear combinations for constructing a mean-reverting portfolio:

$$\text{SPREAD}_t = \text{BTC}_t - 2.22\text{ETH}_t - 29.43\text{LTC}_t - 0.19\text{BCH}_t \tag{1}$$

This portfolio is essentially a spread between BTC and the other three cryptocurrencies (ETH, LTC, BCH). Somewhat surprisingly, the names of Bitcoin (BTC) and BCH (Bitcoin Cash) would suggest a comoving relationship, but in this portfolio BCH carries the least weight compared to the other two cryptocurrencies, ETH and LTC, both in terms of units and cash amount. During this 6-month period from December 20, 2017 to June 20, 2018, prices of BTC were relatively high in comparison to ETH, LTC and BCH, as shown in Figure 2. Thus, it is sensible to have a relatively high (short) stake on LTC as suggested from equation (1). More importantly, we have established with 95% confidence that there exists 1 linear combination of these 4 crypto-assets whose prices are stationary in the long-term. This result is extremely important when constructing a mean-reverting portfolio using Engle-Granger approach since we already knew beforehand the existence of cointegration between crypto-assets' prices. In the next section, we apply the Engle-Granger approach to construct a cointegrating portfolio using again closing prices of BTC, ETH, LTC, and BCH.

	test	10%	5%	1%
r ≤ 3	1.97	6.50	8.18	11.65
r ≤ 2	11.25	12.91	14.90	19.19
r ≤ 1	22.25	18.90	21.07	25.75
r = 0	43.30	24.78	27.14	32.14

Table 2: Johansen estimates for testing cointegration

	BTC	ETH	LTC	BCH
BTC	1.00	1.00	1.00	1.00
ETH	-2.22	0.75	-25.79	69.87
LTC	-29.43	-28.66	85.76	1298.44
BCH	-0.19	-2.79	-6.16	-94.57

Table 3: Coefficients from the Johansen test for cointegration

3.2 Engle-Granger Two-Step Method

The classical approach for cointegration testing developed by Engle and Granger (1987) has the ability to capture long-term co-movements. We first perform a linear regression on some given $I(1)$ time series data, then test for stationarity of the residuals. It is crucial to ensure that residuals from the OLS model are stationary; otherwise, we may run into a potential issue, known as spurious regression. To assure stationarity, we then perform a series of unit-root tests on the OLS's residuals including the Augmented Dickey Fuller (ADF) test (Dickey and Fuller, 1979), the Phillips-Peron (PP) test (Phillips and Perron, 1988) and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test (Kwiatkowski et al., 1992).

As the goal is to perform stationarity check on price series, first order price-differences and residuals from the OLS model, testing hypotheses are designed as follows:

(i) Augmented Dickey Fuller test and Phillips-Peron test:

H_o: Presence of unit root in observable price series

H_a: Unit root does not exist in observable price series

(ii) Kwiatkowski-Phillips-Schmidt-Shin test:

H_o: Unit root does not exist in observable price series

H_a: Presence of unit root in observable price series

The Engle-Granger method involves a linear regression on given time series data which, in this case, is BTC; but it is not important which cryptocurrency is taken as the dependent variable. We propose the following OLS model:

$$\text{BTC}_t = c + \beta_1 \text{ETH}_t + \beta_2 \text{LTC}_t + \beta_3 \text{BCH}_t + \epsilon_t$$

Once a cointegrating relationship is established, we perform 3 stationarity tests on the residuals. Again, prices of these crypto-assets are said to be cointegrated only if prices are all $I(1)$ processes and the residuals ϵ_t from the above OLS model are stationary.

In many cases, financial data follow a random walk. BTC, ETH, LTC, and BCH in our study are not exempt from this assumption. According to the results from Table 4, we accept the null hypothesis (there exits unit-root in time series) from both ADF and PP tests which indicate that daily closing prices of all 4 cryptocurrencies are non-stationary and reject the null hypothesis from KPSS test which again suggests that prices are non-stationary. However, they all become stationary after the first differencing. Note that, one could theoretically reject the null hypothesis from KPSS test performed on ΔBCH series since its p-value is right on the edge of rejecting and accepting the null. Both ADF and PP tests, otherwise, suggest the non-existence of unit root from the time series. Therefore, we conclude BTC, ETH, LTC, and BCH are $I(1)$ processes.

	BTC_t	ETH_t	LTC_t	BCH_t	ΔBTC_t	ΔETH_t	ΔLTC_t	ΔBCH_t
ADF-test	0.49	0.36	0.24	0.41	0.01	0.01	0.01	0.01
PP-test	0.33	0.58	0.16	0.35	0.01	0.01	0.01	0.01
KPSS-test	0.01	0.01	0.01	0.01	0.10	0.10	0.10	0.05

Table 4: Summary of p-values from stationarity tests on cryptocurrencies' prices and their first differences

An OLS model is introduced to establish linear relationships between BTC with ETH, LTC and BCH, see Table 5. Notably, all the regressing coefficients appear to be statistically significant with p-values less than 1%. Also, the adjusted R-squared of 94.5% is considered remarkably high, especially in finance.

	Dependent variable:
	BTC
ETH	0.795^{***}
	(0.264)
LTC	27.020^{***}
	(1.770)
BCH	1.838^{***}
	(0.140)
Constant	$1{,}959.585^{***}$
	(176.725)
Observations	183
R^2	0.950
Adjusted R^2	0.949
Residual Std. Error	576.961 (df = 179)
F Statistic	$1{,}138.254^{***}$ (df = 3; 179)
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 5: Ordinary least square regression model for BTC

From Table 5, we obtain the following linear model:

$$BTC_t = 1,959.59 + 0.795ETH_t + 27.02LTC_t + 1.838BCH_t + \epsilon_t$$

Next, we conduct stationarity tests, i.e. ADF, PP and KPSS tests, on the residuals ϵ_t from this model. The results are as follows:

ADF test: rejecting the null-hypothesis $\Rightarrow$ no unit-root

PP test: rejecting the null-hypothesis $\Rightarrow$ no unit-root

KPSS test: accepting the null-hypothesis $\Rightarrow$ residuals are stationary

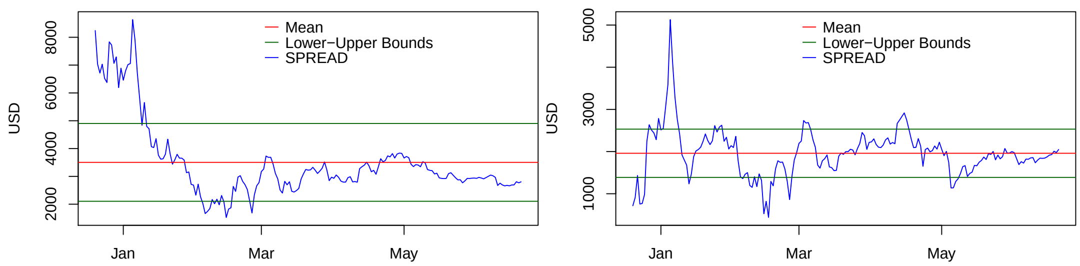
All 3 stationarity tests confirm that the residuals from linear regression model are stationary time series. Hence, prices of BTC, ETH, LTC and BCH are indeed cointegrated by our procedure. With this confirmation, the spread can be constructed using the suggested linear coefficients:

$$SPREAD_t = BTC_t - 0.795ETH_t - 27.02LTC_t - 1.838BCH_t \tag{2}$$

This is the portfolio we will use to trade.

3.3 Portfolio Time Series

The main purpose of our cointegration testing procedure is to construct a tradable mean-reverting portfolio. Recall that there are two linear models given in (1) and (2) in which the portfolio is a spread between BTC and three other cryptocurrencies. The fact that the linear combinations in the two models are highly similar further supports that BTC, ETH, LTC and BCH are cointegrated. Observing from Figures 3, it is hard to see which spread process appears to be *more* stationary or mean-reverting. However, from a trading perspective, periodic movements or fluctuations that lead to frequent crossing of the equilibrium levels from both directions are desired, especially given that entry and exit rules are based on deviations from the mean price (Chan, 2013). For this reason, Figure 3(b) suggests that model (2) from the Engle-Granger approach is more practical to trade, and is thus chosen for back-testing our trading system.

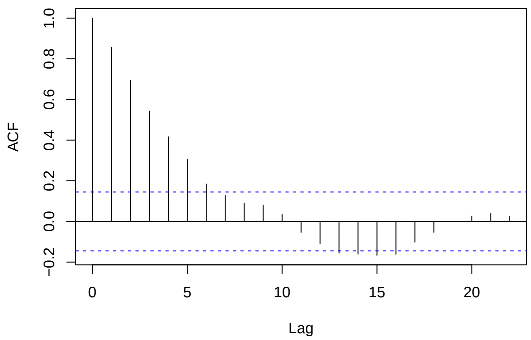


(a) (b)

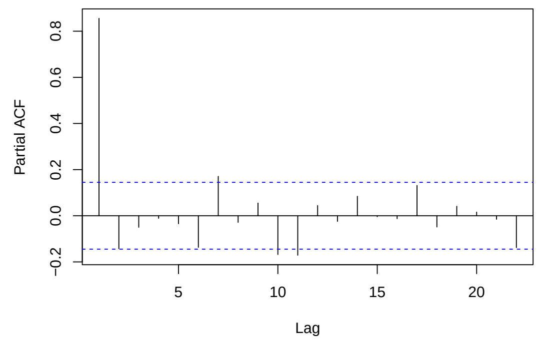
Figure 3: Spread constructed from two approaches: (a) Johansen methodology (b) Engle-Granger methodology

Next, we construct a time series model to capture dynamic of the spread in order to design the trading rules. Generally, we expect the spread to be highly mean-reverting since that is the main criterion we used to choose the 4 crypto-assets in the first place. In discrete time, the spread is often assumed to be stationary Autoregressive Moving Average (ARMA) processes since they are also mean-reverting by design. This is the basic requirement we set forth to designing our trading signals. From previous section, we derived from stationarity tests that the spread is reverting around the \$1,959.058 equilibrium. This helps us determine the amount of inventory that are willing to hold on the spread, but more importantly, it satisfies the stationarity requirement before we propose an appropriate ARMA model.

To estimate the orders and parameters of an ARMA process, we rely on sample autocorrelation function (ACF) and partial autocorrelation function (PACF) plots with test bounds, (see Box et al. (2015)). As suggested by ACF and PACF plots respectively in Figures 4(a) and 4(b), the residuals can be modeled by ARMA$(1, 0)$ or AR(1). We summarize the test in Table 6.



(a)



(b)

Figure 4: (a) ACF plot of the residuals from OLS model; (b) PACF plot of residuals from OLS model

	Dependent variable:
	residuals
AR(1) Coeff.	0.874***
	(0.036)
Intercept	1,919.026***
	(160.731)
Observations	183
Log Likelihood	−1,294.312
σ^2	80,800.910
Akaike Inf. Crit.	2,594.625
Note:	*p<0.1; **p<0.05; ***p<0.01

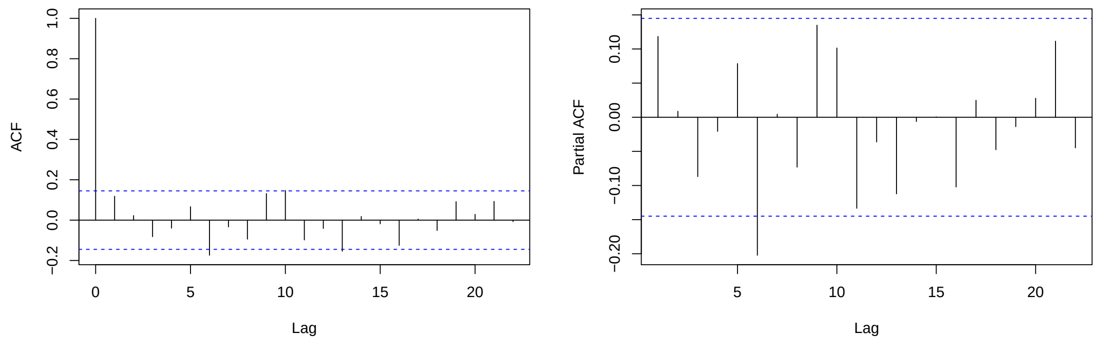
Table 6: Fitting the spread to the ARIMA$(1, 0, 0)$ model

To ensure that AR(1) is an appropriate model, Ljung-Box test is performed on residuals from the ARMA model to check for the overall randomness and no autocorrelation in the residual time series (see Box et al. (2015)). Ljung-Box test hypothesis:

H_o: Price series are independent, or no serial correlations

H_a: Price series are not independent, or they exhibit serial correlations

As shown in Figures 5(a) and 5(b), even with some lags outside the 95% confidence levels, the p-value of 0.1094 for the Ljung-Box test on residuals implies the failure to reject the null hypothesis that there is no autocorrelation in the residuals. We conclude that prices of the spread are AR(1) processes.



(a)

(b)

Figure 5: (a) Autocorrelation graph of the residuals from OLS model; (b) Partial autocorrelation graph of residuals from OLS model

4 Statistical Arbitrage Strategies

In trading practices, finding the right times for entry and exit is key to a profitable trading system (see Leung and Li (2016); Vidyamurthy (2004)). As previously mentioned, the default entry and exit thresholds in back-testing stage are typically set around 1 deviation, denoted by σ, above and below the mean spread level, assuming that traders can go both long and short on the spread. It is crucial to assure that these entry and exit levels are set to maximize profits in terms of the trading costs and transaction frequency. Thus, by back-testing a trading system with multiple entry/exit levels, we can get a sense of how profitable the system is by looking at different performance criteria such as profit and loss (P&L), Sharpe ratio, and more.

Define S_t as the spread at time t. A mean reversion strategy for trading the spread is described by the following linear combination:

$$S_t = \text{BTC}_t - \beta_1 \text{ETH}_t - \beta_2 \text{LTC}_t - \beta_3 \text{BCH}_t \tag{3}$$

where $(1, \beta_1, \beta_2, \beta_3) \coloneqq (1, -0.795, -27.02, -1.838)$ are the coefficients from the linear model (2).

To purchase 1 "unit" of the spread S, we buy 1 unit of BTC, -0.795 unit of ETH, -27.02 unit of LTC, and -1.838 unit of BCH. In other words, in addition to buying 1 BTC, we need to short sell 0.795 shares of ETH, 27.02 shares of LTC and 1.838 shares of BCH.[9] Different from traditional equity market, traders can purchase fractional shares up to 8 decimal places in the cryptocurrency market. It boils down to how the inventor of Bitcoin, Satoshi Nakamoto, decided to name Bitcoin in such way. When developing the Bitcoin, the creator Satoshi had decided its base unit to be 10^8 and the smallest fraction is called Satoshi. Many recent altcoins have adapted the same philosophy, though some have extended their smallest units up to 16 decimal places. For simplicity, we only go up to 3 decimal places on each cryptocurrency.

In addition, we only expose to 1 unit of the spread at any given time. That is to say when if we already held 1 unit of the spread and observe a deviation of the spread from the mean price, we do not add to our inventory any additional unit of S. As competitions in the cryptocurrency market continues to rise, many recent exchanges start charging very minimal

[9]Some exchanges, such as Bitfinex, allow for margin trading and short selling.

fee per transaction (typically 0.1% or less); thus, we assume no transaction cost associated with trading the spread.

Let us now state our trading rule:

(i) Long the spread/Exit short position when

$$S_t < \mu - c \cdot \sigma$$

(ii) Short the spread/Exit long position when

$$S_t > \mu + c \cdot \sigma$$

where μ is the mean value of the spread, σ is the standard deviation of the spread, and c is a multiple we choose to set our entry/exit thresholds. At any given time t where S_t falls under/over the lower/upper bound, the system will buy/sell 1 share of S then wait until the price revert to realize a profit from that particular trade. Here, we backtest and compare 5 different thresholds for entry/exit including $\pm 0.5\sigma$, $\pm 0.75\sigma$, $\pm 1\sigma$, $\pm 1.25\sigma$, and $\pm 1.5\sigma$ (deviations from the mean).

4.1 Trading the spread

Testing for multiple entry/exit levels allows us to analyze the advantages and disadvantages of the trading rules via multiple performance criteria. We summarize some trade statistics in Table 7 (see Appendix for definitions). When setting lower/upper bound at $\pm 1.5\sigma$, our system seems to outperform the rest in terms of annual Sharpe ratio, maximum drawdown, max/min/end equities, and gross profits.

Threshold level	$\pm 0.5\sigma$	$\pm 0.75\sigma$	$\pm\sigma$	$\pm 1.25\sigma$	$\pm 1.5\sigma$
Num.Txns	17	13	13	9	7
Num.Trades	8	6	6	4	3
Net.Trading.PL	7601.87	8112.87	8307.29	6313.43	8422.57
Avg.Trade.PL	950.06	1200.60	1233.00	1351.04	2518.30
Largest.Winner	1599.35	1599.35	1599.35	1605.06	2773.72
Largest.Loser	0.00	0.00	0.00	0.00	0.00
Gross.Profits	7600.50	7203.60	7398.03	5404.16	7554.91
Gross.Losses	0.00	0.00	0.00	0.00	0.00
Percent.Positive	100.00	100.00	100.00	100.00	100.00
Percent.Negative	0.00	0.00	0.00	0.00	0.00
Avg.Win.Trade	950.06	1200.60	1233.00	1351.04	2518.30
Avg.Losing.Trade	0.00	0.00	0.00	0.00	0.00
Avg.Daily.PL	950.06	1200.60	1233.00	1351.04	2518.30
Ann.Sharpe	37.22	56.98	71.88	81.09	109.00
Max.Drawdown	-2846.51	-2846.51	-2846.51	-2885.34	-1536.31
Profit.To.Max.Draw	2.67	2.85	2.92	2.19	5.48
Max.Equity	7996.42	8136.47	8330.90	6337.03	9334.01
Min.Equity	-1020.73	-1020.73	-1020.73	-1009.32	-152.05
End.Equity	7601.87	8112.87	8307.29	6313.43	8422.58

Table 7: Statistics for trading strategies with different entry and exit thresholds

Figure 6: Trading system setting entry/exit level at $\pm\sigma$

The mean-reverting behavior of the spread, as we can see in Figure 6, means that trades from all threshold levels yield positive profits, i.e. the percentages of profitable trades are all 100%. When S_t moved away from its mean in early January 2018, we see the advantage of setting a wide trading band ($\pm 1.5\sigma$). In fact, the yield from that particular trade had resulted in the highest realized profit, yet lower drawdowns in comparison to cases with narrower trading bands. As the band widens, the number of trades and transactions decreases as expected; but more importantly, net trading profit & loss (P&L) seems to be optimal when we set the bandwidth at $\pm 1.5\sigma$ which is consistent with all trading statistics from the table. Average P&L and average daily P&L also go up together with the largest gain when we increase the band. From asset management perspective, trading system with largest threshold level yield astonishing Sharpe ratio of 109 with lowest maximum drawdown. As a result, we observe that the trading system is optimal with the entry/exit level set at $\pm 1.5\sigma$.

4.2 Trading the spread with stop-loss

In the cryptocurrency market, we often observe larger price swings not only on daily basis, but also potentially anytime of the day. This is the reason why setting a larger entry/exit threshold could potentially boost our trading profits. At the same time, we cannot set it too large that the trading system would result in no-trades. On the other hand, when the entry/exit threshold levels get too narrow, traders would suffer larger drawdowns due to extremely high volatility or short term deviation in crypto-assets' prices. Therefore, in addition to testing multiple entry/exit levels, we propose adding stop-loss order on top of the existing 5 threshold levels trading systems, mainly to mitigate the effects of higher drawdown. Here, we set an arbitrary stop-loss order at 5%, meaning our system will automatically execute the stop-loss order a day after if today's price falls below 5% of the buy-in price.

According to Table 8, stop-loss orders prevent the trading system from unrealized gains that we could have earned when not implementing stop-loss orders. However, maximum drawdown from all 5 threshold levels systems are reduced. Noticeably, numbers of trades and transactions go up by a significant amount with lower net trading P&L, gross profits, and most importantly, the Sharpe ratio. Without zero-transaction cost assumption, making trades frequently could ultimately hurt the net trading P&L and gross profits of our system. Notably, when imposing a 5% stop-loss, most trades got executed at a lower price and thus, we do not fully capture the mean reverting benefit from the spread. In contrast to not having stop-loss, more than 70% of trades from all 5 systems resulted in losses. Both Sharpe ratios and end equities are significantly reduced.

Threshold level	$\pm 0.5\sigma$	$\pm 0.75\sigma$	$\pm\sigma$	$\pm 1.25\sigma$	$\pm 1.5\sigma$
Num.Txns	31	19	23	15	9
Num.Trades	15	9	11	7	4
Net.Trading.PL	2233.63	2285.58	3400.36	555.88	1672.08
Avg.Trade.PL	88.29	152.92	226.46	-50.48	201.10
Largest.Winner	954.18	1816.26	1816.26	1266.85	1506.03
Largest.Loser	-145.28	-533.75	-533.75	-625.11	-380.16
Gross.Profits	2539.48	2615.69	3978.36	1266.85	1506.03
Gross.Losses	-1215.11	-1239.38	-1487.26	-1620.24	-701.62
Percent.Positive	26.67	22.22	27.27	14.29	25.00
Percent.Negative	73.33	77.78	72.73	85.71	75.00
Avg.Win.Trade	634.87	1307.85	1326.12	1266.85	1506.03
Avg.Losing.Trade	-110.46	-177.05	-185.91	-270.04	-233.87
Avg.Daily.PL	88.29	152.92	226.46	-50.48	201.10
Ann.Sharpe	3.64	3.39	4.84	-1.29	3.64
Max.Drawdown	-791.28	-1325.03	-1325.03	-2010.37	-1225.63
Profit.To.Max.Draw	2.82	1.72	2.57	0.28	1.36
Max.Equity	2257.24	2640.13	3754.91	910.43	2583.51
Min.Equity	-267.28	-801.02	-801.02	-1486.36	-701.62
End.Equity	2233.63	2285.58	3400.36	555.88	1672.08

Table 8: Statistics for the trading strategy with 5% stop-loss

Figure 7: Trading system with 5% stop-loss, setting entry/exit level at $\pm\sigma$

4.3 Trading the spread with trailing stop-loss

While a stop-loss order limits the traders' net loss, a trailing stop is triggered by a large drawdown and is adjusted upward as the portfolio value increases. Hence, it has the advantage of continuously moving with the portfolio value and allowing the trader to potentially end with a gain even when the trailing stop is triggered by a drawdown.

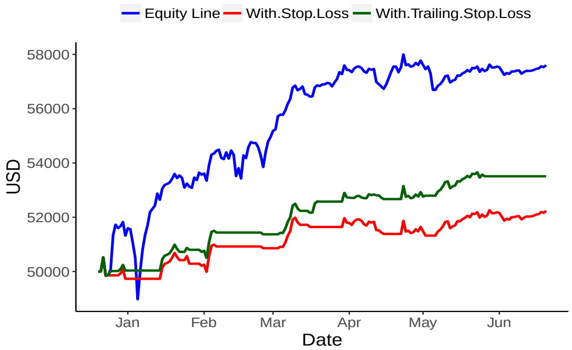
When implementing a strategy with trailing stop, there is also a possibility that trades get exited too early before a bigger upward price movement. This is the opportunity cost of having trailing stop rather than just implementing stop-loss alone. In our case, since trading signals are only triggered when prices of the spread fall above or drop below the entry/exit level, it could potentially prevent the trading system from executing additional trades as price fall within one deviation threshold. Also, if we know with certain level of confidence that the spread is a stationary process, it may not be ideal to implement either stop-loss or trailing stop . Nonetheless, we design additional trading systems setting a trailing stop at the same level as we did for stop-loss, which is 5%, to compare 3 trading systems with 5 threshold levels and evaluate the benefit of having one over the other using multiple trading performance criteria.

From Table 9, the number of trades and transactions increase significantly from the original trading system which could potentially be an expense when there are transaction costs associated with each trade. However, net trading P&L, average trade P&L, gross profits are higher than from the system with stop-loss. Here, the trailing stop has led to multiple exits with small profits as the spread reverts to the mean. As a result, percentages of profitable trades have gone up two times from the stop-loss trading systems and average daily P&L are also improved. Notably, maximum drawdown are drastically reduced while Sharpe ratios and end equities significantly increase.

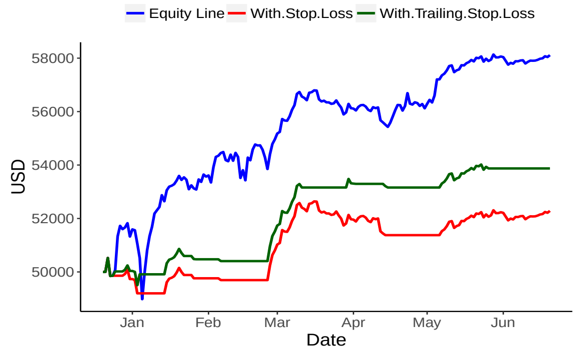
Threshold level	$\pm 0.5\sigma$	$\pm 0.75\sigma$	$\pm\sigma$	$\pm 1.25\sigma$	$\pm 1.5\sigma$
Num.Txns	36	24	26	20	10
Num.Trades	18	12	13	10	5
Net.Trading.PL	3509.31	3872.88	5080.65	4682.81	2114.03
Avg.Trade.PL	194.96	322.74	390.82	468.28	422.81
Largest.Winner	954.18	1816.26	1816.26	1266.85	1303.67
Largest.Loser	-137.35	-137.35	-137.35	-125.72	-179.62
Gross.Profits	3991.64	4440.42	5367.59	4849.46	2334.59
Gross.Losses	-482.33	-567.55	-286.94	-166.66	-220.56
Percent.Positive	66.67	58.33	76.92	80.00	60.00
Percent.Negative	33.33	41.67	23.08	20.00	40.00
Avg.Win.Trade	332.64	634.35	536.76	606.18	778.20
Avg.Losing.Trade	-80.39	-113.51	-95.65	-83.33	-110.28
Avg.Daily.PL	194.96	322.74	390.82	468.28	422.81
Ann.Sharpe	8.85	8.33	11.03	14.44	9.82
Max.Drawdown	-676.06	-1021.65	-676.06	-1040.13	-1066.16
Profit.To.Max.Draw	5.19	3.79	7.52	4.50	1.98
Max.Equity	3657.30	4020.86	5228.64	4830.80	2365.69
Min.Equity	-152.05	-497.65	-152.05	-516.12	-542.16
End.Equity	3509.31	3872.88	5080.65	4682.81	2114.03

Table 9: Statistics for the trading strategy with 5% trailing stop-loss

Starting with an initial amount of $50,000 and restricting the exposure of our trading systems to only 1 unit of the spread at a time, Figure 8 shows how the three systems perform with 5 proposed entry/exit threshold levels. Overall, the trading system with neither stop-loss nor trailing stop produced maximum profits with highest Sharpe ratios. Although traders may not prefer drawdowns from the initial trade, the spread reverted to the mean price and crossed above the exit level, yielding a total net profit of over $1,599 which is equivalent to 175% gain on the period from December 28, 2017 to January 14, 2018. By setting larger bands, we were able to capture full profit from the initial trade with little to no drawdown, see Figure 8(e). In addition, by not incorporating a stop-loss nor trailing stop, setting threshold at $\pm 1.5\sigma$ has yielded highest profit and Sharpe ratio out of all the proposed trading systems. Although when implementing trailing stop and stop-loss, we are able to reduce drawdowns from the initial system, trading performances also decline. In terms of profit maximization, the trading system without stop-loss nor trailing should be preferred since we have established that a portfolio comprised of BTC, ETH, LTC, and BCH is mean-reverting.



(a)



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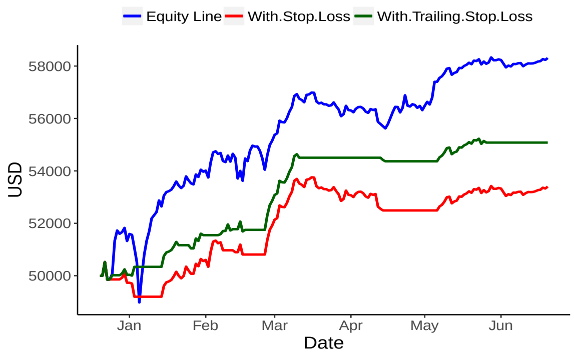

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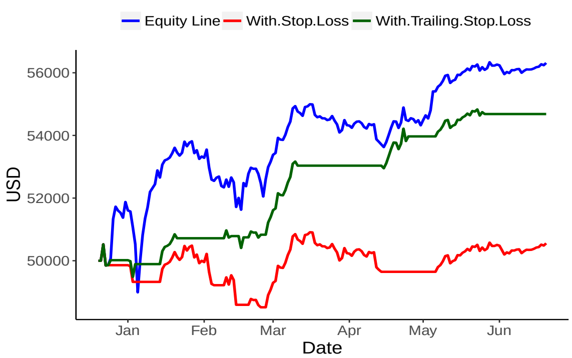

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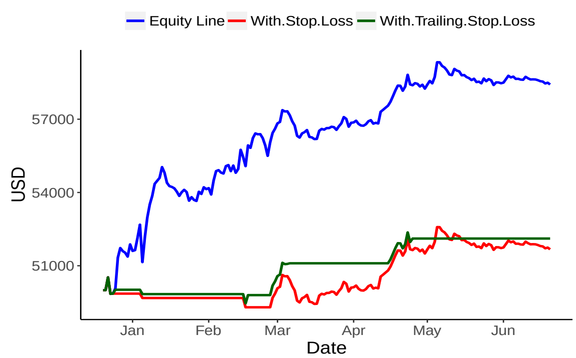

(e)

Figure 8:

Equity lines: (a) Entry/exit threshold $= 0.5\sigma$; (b) Entry/exit threshold $= 0.75\sigma$; (c) Entry/exit threshold $= \sigma$, (d) Entry/exit threshold $= 1.25\sigma$. (e) Entry/exit threshold $= 1.5\sigma$.

5 Conclusion

In this study, we have identified price cointegration among the prices of four cryptocurrencies, BTC, ETH, LTC, and BCH, using both the Johansen test and Engle-Granger procedure. Although the Johansen test implies a stationary portfolio, it is not a preferred approach for trading application because the resulting slow mean-reverting portfolio would result in very few to no trades. The Engle-Granger approach gives a mean-reverting and tradable portfolio since the fast mean-reverting spread would cross the entry and exit levels frequently. We find that setting greater entry and exit levels leads to larger profits. In comparison to trading systems with a stop-loss exit or trailing stop, the one without yields the highest profit but also largest drawdowns.

As the cryptocurrency market continues to grow with new coins and new exchanges, it is very important for individual investors, crypto-fund managers, as well as regulators to understand the price dependency among all cryptocurrencies, along with their derivatives. There are many directions for future research. For example, do the prices and volatilities of cryptocurrencies interact with the equity, bond, or commodity markets (see Narayan et al. (2017))? Can they be used as an inflation hedge like gold (see (Ghosh et al., 2004))? In this paper, we have considered several cryptocurrencies with the largest market capitalizations, but one can also develop a machine learning approach that automatically selects a small portfolio from a vast universe of cryptocurrencies (see Zhang et al. (2018)). Moreover, one can analyze and compare the price impacts of trading different cryptocurrencies. Results on this front will shed light on the process of selecting a cointegrated portfolio and the execution of trades.

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A Appendix: statistics for trading strategies

Num.Txns	Total number of transactions
Num.Trades	Total number of trades
Net.Trading.PL	Total profit & loss made due to all trades
Avg.Trade.PL	Average profit & loss made due to all trades
Largest.Winner	The maximum profit of trades
Largest.Loser	The maximum loss of trades
Gross.Profits	Total profits
Gross.Losses	Total losses
Percent.Positive	Percentage of profitable trades
Percent.Negative	Percentage of loss trades
Avg.Win.Trade	The average of profits of trades
Avg.Losing.Trade	The average of losses of trades
Avg.Daily.PL	Average profit & loss on daily basis
Ann.Sharpe	Sharpe ratio with 0 risk-free rate
Profit.To.Max.Draw	The ratio of profit to maximum drawdown
Avg.WinLoss.Ratio	The ratio of average profitable trades and average loss trades
Max.Equity	Maximum equity
Min.Equity	Mimum equity
End.Equity	Ending equity

Table 10: Trade statistics explanations