Actor-Critic Policy Gradient

Spring, 2021

Outline

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Policy Gradient

Different Schools of Reinforcement Learning

1. Value-based RL: solve RL through dynamic programming
   1. Classic RL and control theory
   2. Representative algorithms: Deep Q-learning and its variant
   3. Representative researchers: Richard Sutton (no more than 20 pages on PG out of the 500-page textbook), David Silver, from DeepMind
2. Policy-based RL: solve RL mainly through learning
   1. Machine learning and deep learning
   2. Representative algorithms: PG, and its variants TRPO, PPO, and others
   3. Representative researchers: Pieter Abbeel, Sergey Levine, John Schulman, from OpenAI, Berkelely

Policy Gradient

1. State-of-the-art RL methods are almost all policy-based
    1. **A2C and A3C**: Asynchronous Methods for Deep Reinforcement Learning, ICML'16. Representative high-performance actor-critic algorithm: `https://openai.com/blog/baselines-acktr-a2c/`
    2. **TRPO**: Schulman, L., Moritz, Jordan, Abbeel (2015). Trust region policy optimization: deep RL with natural policy gradient and adaptive step size
    3. **PPO**: Schulman, Wolski, Dhariwal, Radford, Klimov (2017). Proximal policy optimization algorithms: deep RL with importance sampled policy gradient

Policy Gradient

Review – Policy Gradient

$$\nabla \bar{R}_\theta \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t=1}^{T_n}\left(\sum_{t'=t}^{T_n}\gamma^{t'-t}r_{t'}^n - b\right)\nabla log p_\theta(a_t^n|s_t^n)$$

baseline

G_t^n : obtained via interaction

Policy Gradient

Review – Policy Gradient

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Very unstable

Policy Gradient

Review – Policy Gradient

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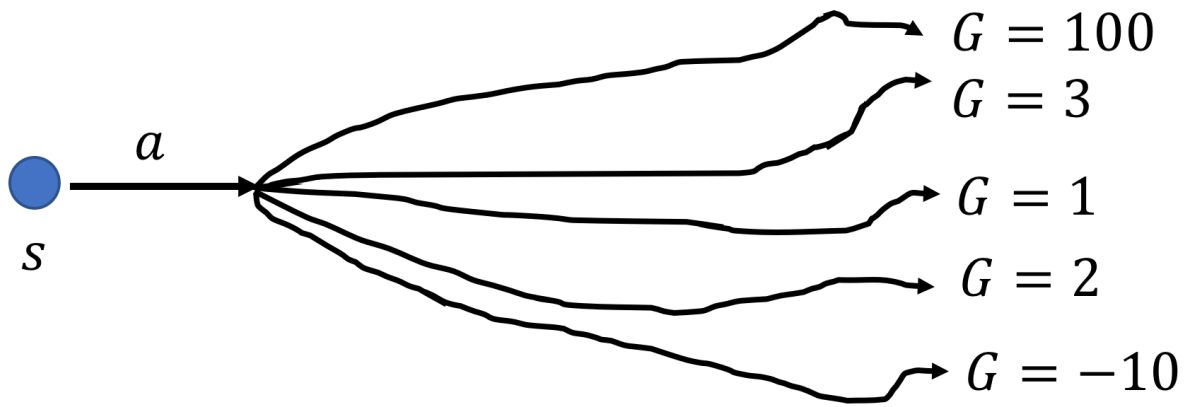
G_t^n : obtained via interaction

Very unstable



Policy Gradient

Review – Policy Gradient

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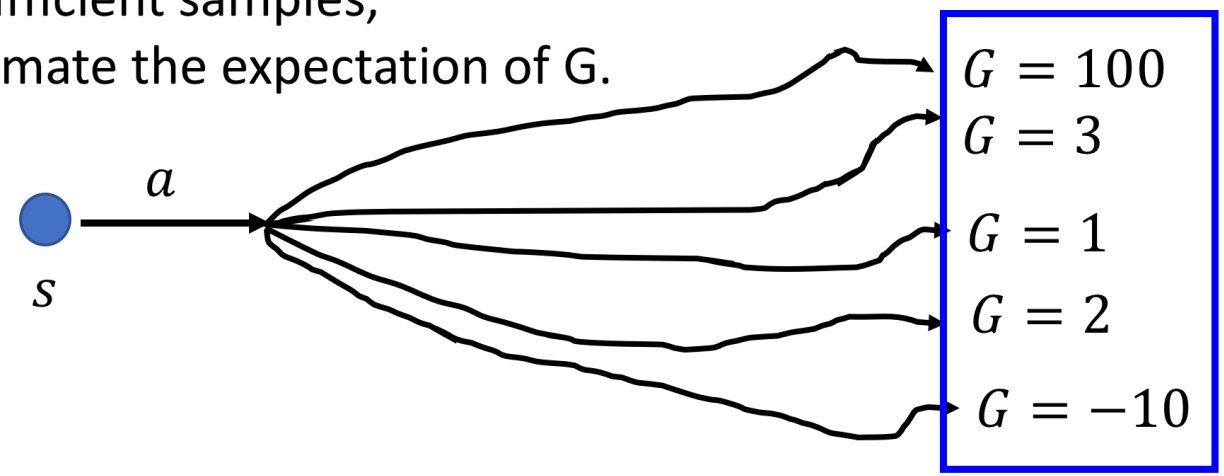
G_t^n : obtained via interaction

Very unstable

With sufficient samples,
approximate the expectation of G.



Policy Gradient

Review – Policy Gradient

baseline

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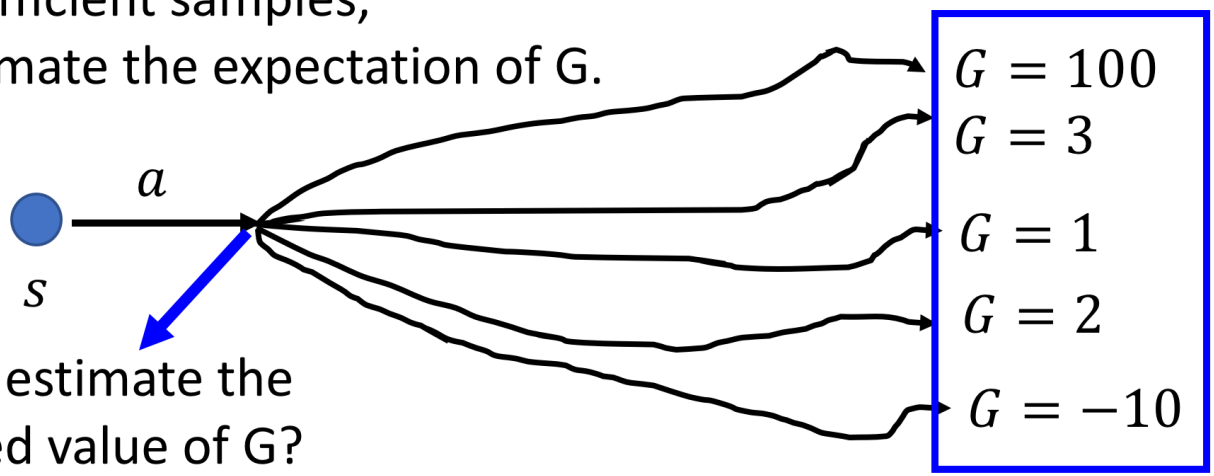
G_t^n : obtained via interaction

Very unstable

With sufficient samples, approximate the expectation of G.



Can we estimate the expected value of G?

Policy Gradient

Review – Q-Learning

- State value function $V^{\pi}(s)$
  - When using actor π, the *cumulated* reward expects to be obtained after visiting state s
- State-action value function $Q^{\pi}(s, a)$
  - When using actor π, the *cumulated* reward expects to be obtained after taking a at state s

Policy Gradient

Review – Q-Learning

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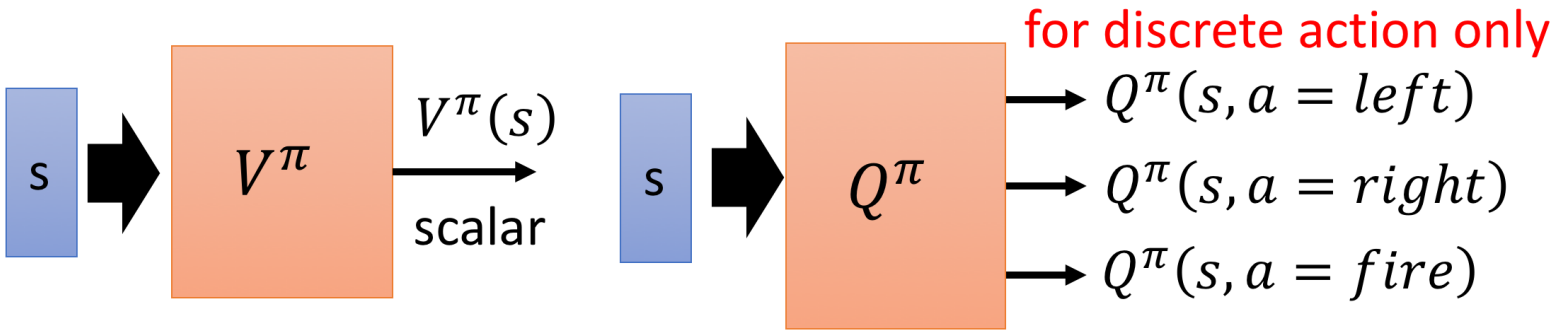

Estimated by TD or MC

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Actor-Critic

Actor-Critic

$$\nabla \bar{R}_\theta \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} \left(\sum_{t'=t}^{T_n} \gamma^{t'-t} r_{t'}^n - b \right) \nabla log p_\theta(a_t^n | s_t^n)$$

baseline

G_t^n : obtained via interaction

$$E[G_t^n] = Q^{\pi_\theta}(s_t^n, a_t^n)$$

Actor-Critic

Actor-Critic

$V^{\pi_\theta}(s_t^n)$

baseline

$$\nabla \bar{R}_\theta \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t=1}^{T_n}\left(\sum_{t'=t}^{T_n}\gamma^{t'-t}r_{t'}^n - b\right)\nabla log p_\theta(a_t^n|s_t^n)$$

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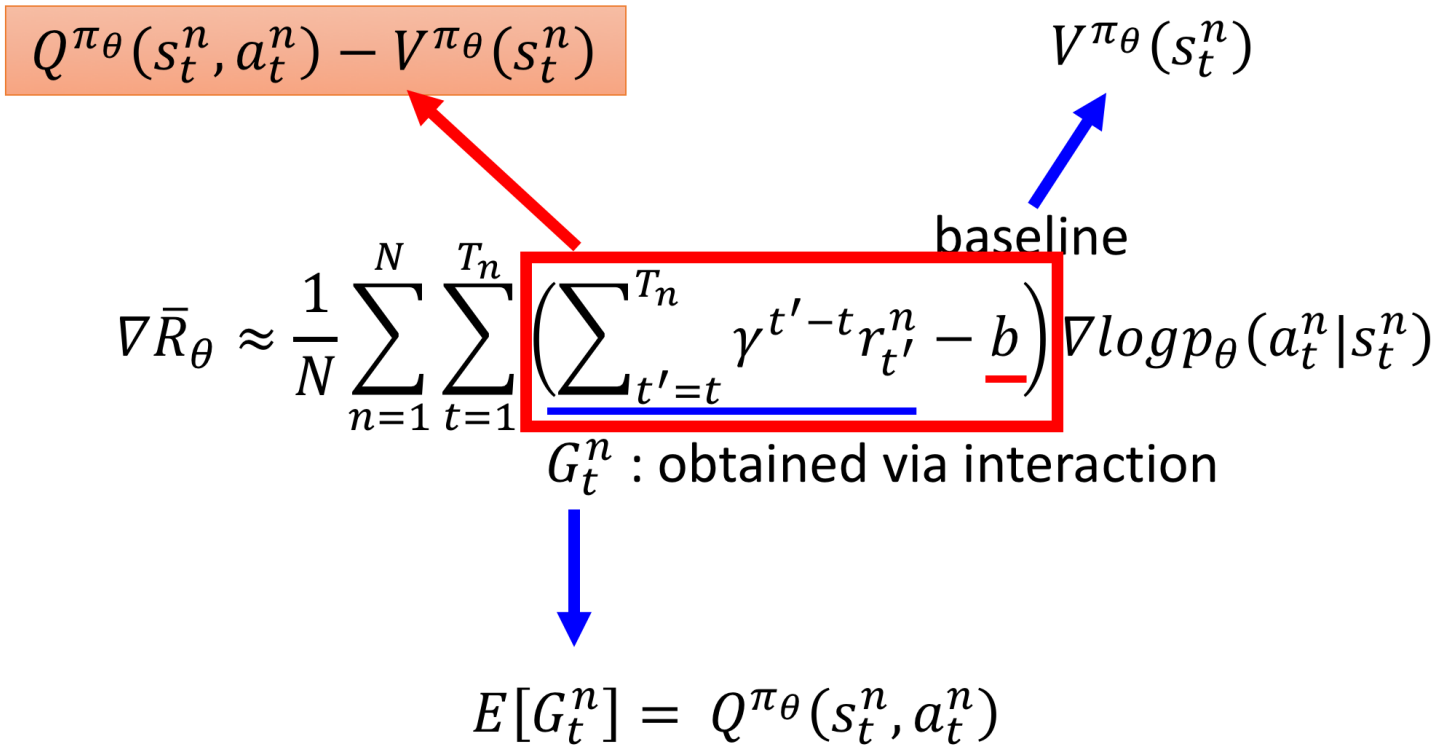
Actor-Critic

Actor-Critic



Actor-Critic

Advantage Actor-Critic (A2C)

$$Q^{\pi}(s_t^n, a_t^n) - V^{\pi}(s_t^n)$$

Estimate two networks? We can only estimate one.

Actor-Critic

Advantage Actor-Critic (A2C)

$Q^{\pi}(s_t^n, a_t^n) - V^{\pi}(s_t^n)$

Estimate two networks? We can only estimate one.

$$Q^{\pi}(s_t^n, a_t^n) = E[r_t^n + V^{\pi}(s_{t+1}^n)]$$

$$Q^{\pi}(s_t^n, a_t^n) = r_t^n + V^{\pi}(s_{t+1}^n)$$

Actor-Critic

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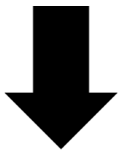

$$r_t^n + V^{\pi}(s_{t+1}^n) - V^{\pi}(s_t^n)$$

Estimate two networks? We can only estimate one.

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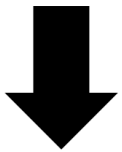

$$r_t^n + V^{\pi}(s_{t+1}^n) - V^{\pi}(s_t^n)$$

Estimate two networks? We can only estimate one.

Only estimate state value

A little bit variance

$$Q^{\pi}(s_t^n, a_t^n) = E[r_t^n + V^{\pi}(s_{t+1}^n)]$$

$$Q^{\pi}(s_t^n, a_t^n) = r_t^n + V^{\pi}(s_{t+1}^n)$$

Actor-Critic

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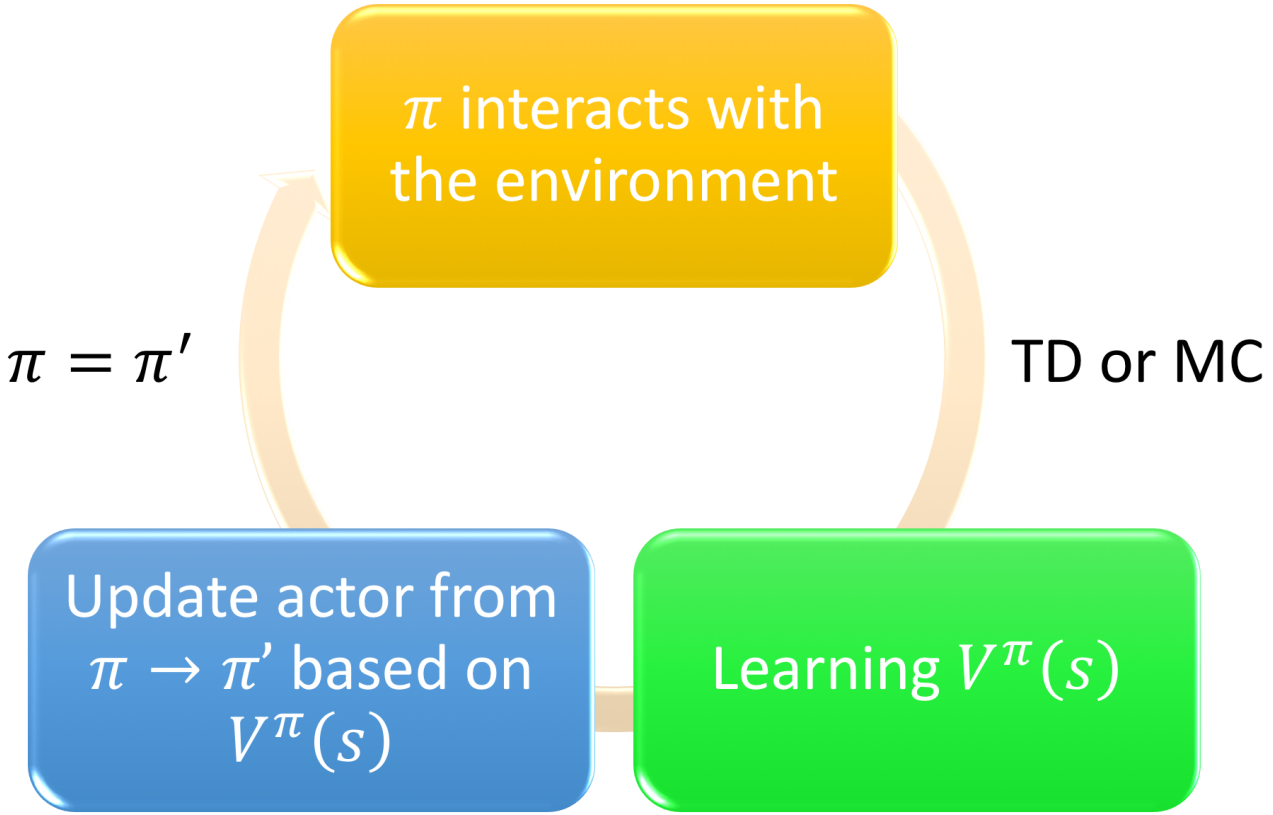
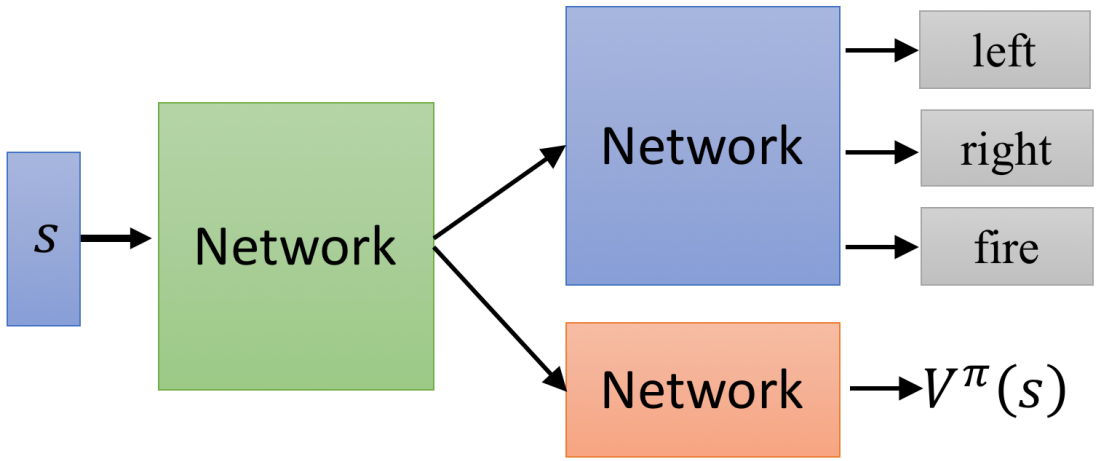


$$\nabla \bar{R}_{\theta} \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} \left(r_t^n + V^{\pi}(s_{t+1}^n) - V^{\pi}(s_t^n) \right) \nabla log p_{\theta}(a_t^n | s_t^n)$$

Actor-Critic

Advantage Actor-Critic (A2C)

- Tips
  - The parameters of actor $\pi(s)$ and critic $V^{\pi}(s)$ can be shared



- Use output entropy as regularization for $\pi(s)$
  - Larger entropy is preferred → exploration

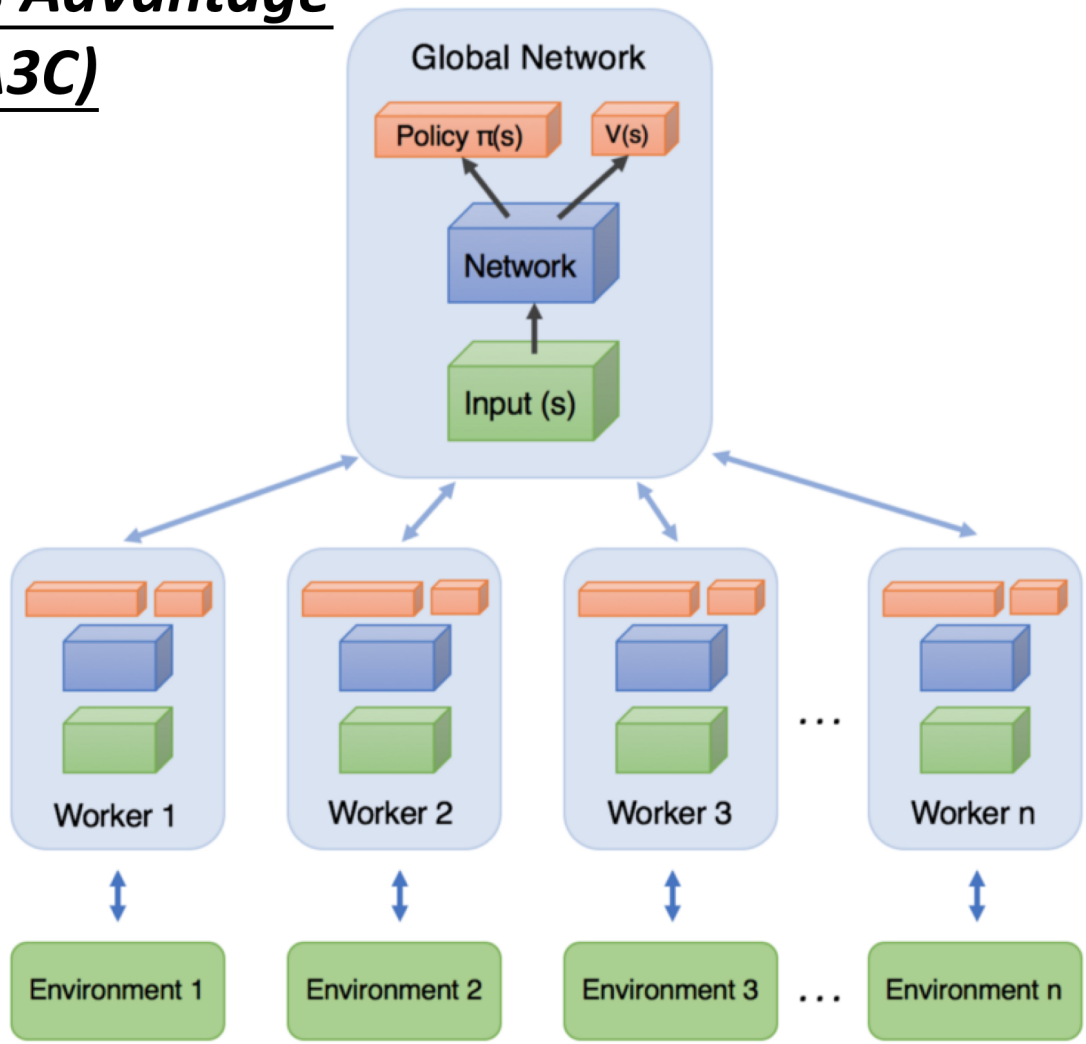
Actor-Critic

- Simple actor-critic algorithm based on action-value critic
- Using linear value function approximate $Q_w(s,a) = \phi(s,a)^T w$
  - Critic: Updates w by linear TD(0)
  - Actor: Updates θ by policy gradient

function QAC
  Initialise s, θ
  Sample $a \sim \pi_\theta$
  for each step **do**
    Sample reward $r = \mathcal{R}_s^a$; sample transition $s' \sim \mathcal{P}_{s,\cdot}^a$
    Sample action $a' \sim \pi_\theta(s', a')$
    $\delta = r + \gamma Q_w(s', a') - Q_w(s, a)$
    $\theta = \theta + \alpha \nabla_\theta \log \pi_\theta(s, a) Q_w(s, a)$
    $w \leftarrow w + \beta \delta \phi(s, a)$
    $a \leftarrow a', s \leftarrow s'$
  end for
end function

Actor-Critic

Asynchronous Advantage Actor-Critic (A3C)



Actor-Critic

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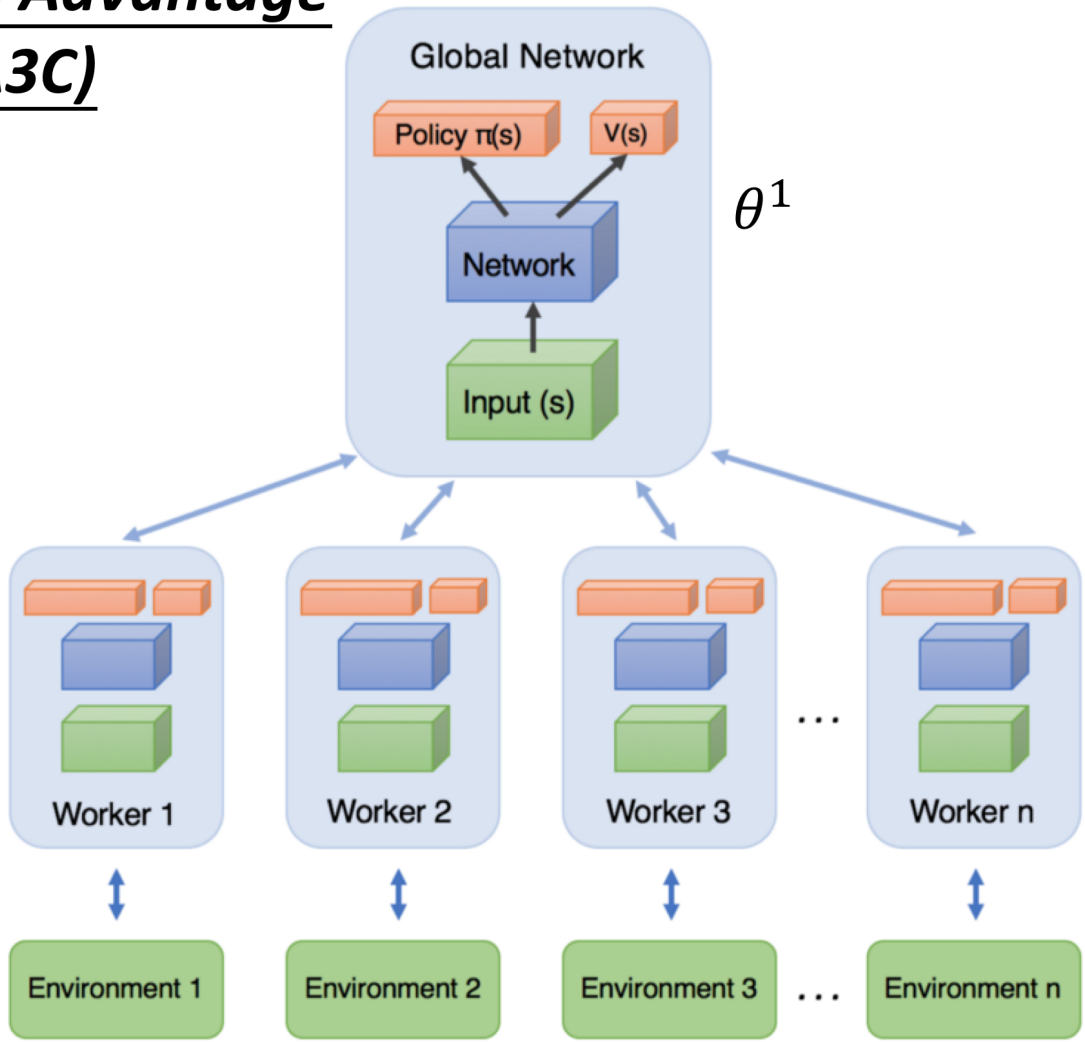
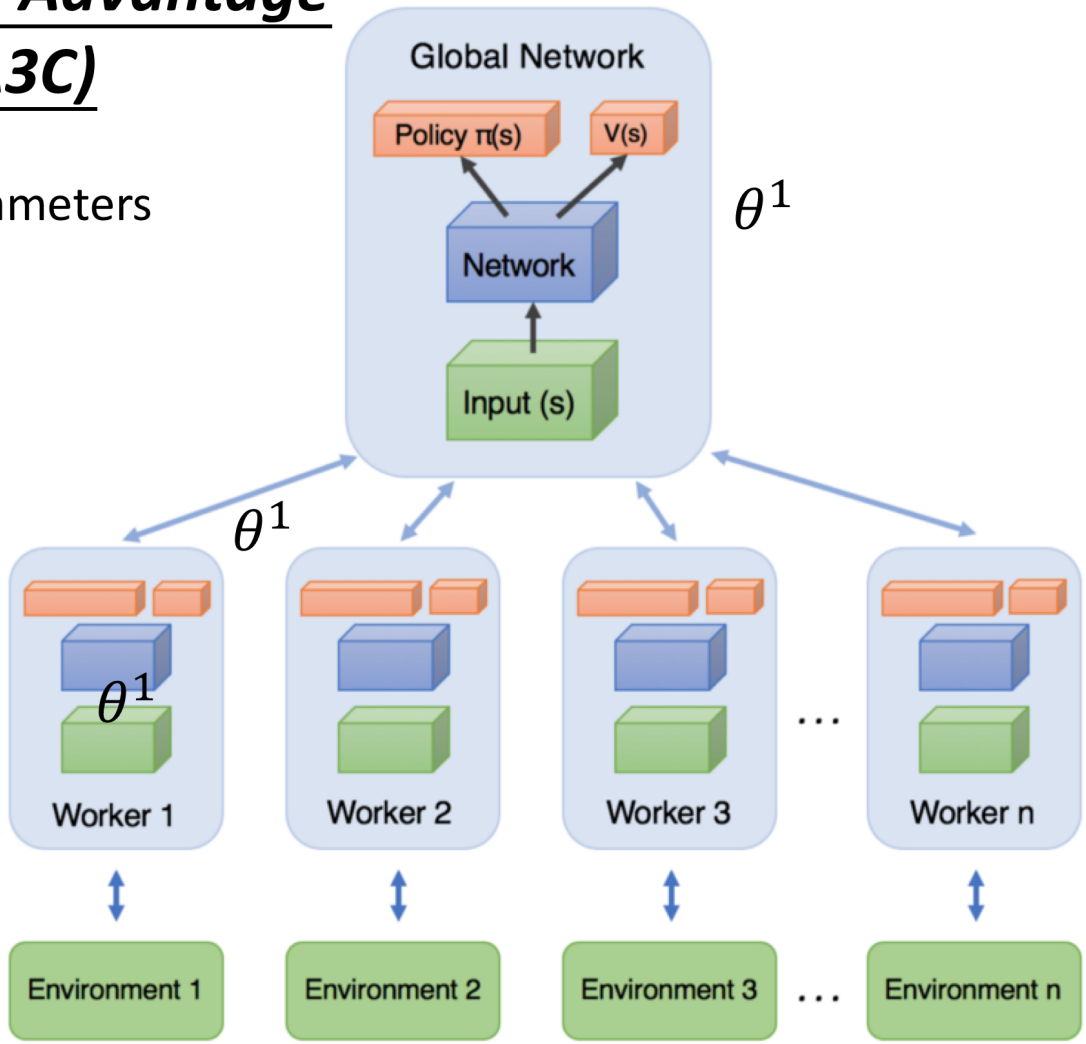


Actor-Critic

Asynchronous Advantage Actor-Critic (A3C)

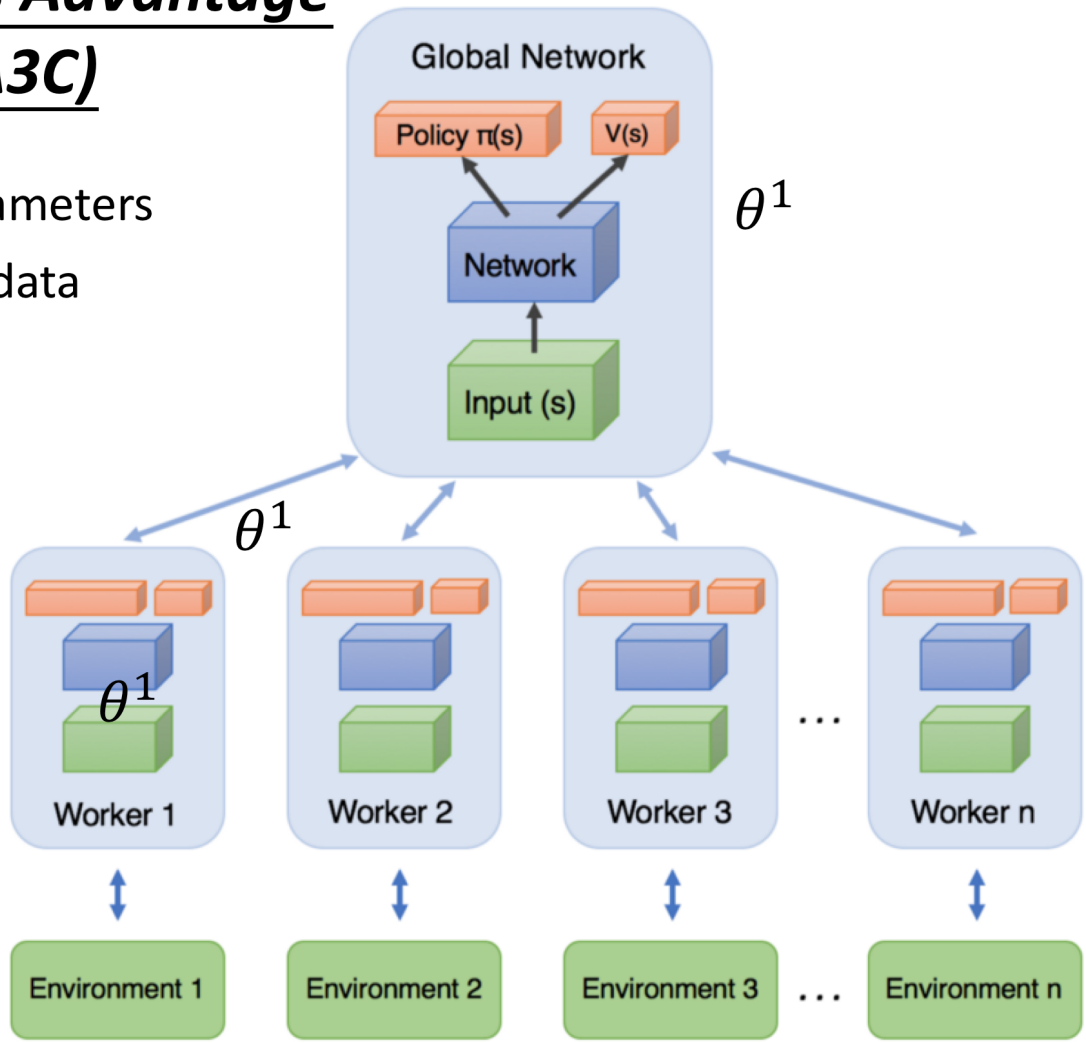
1. Copy global parameters



Actor-Critic

Asynchronous Advantage Actor-Critic (A3C)

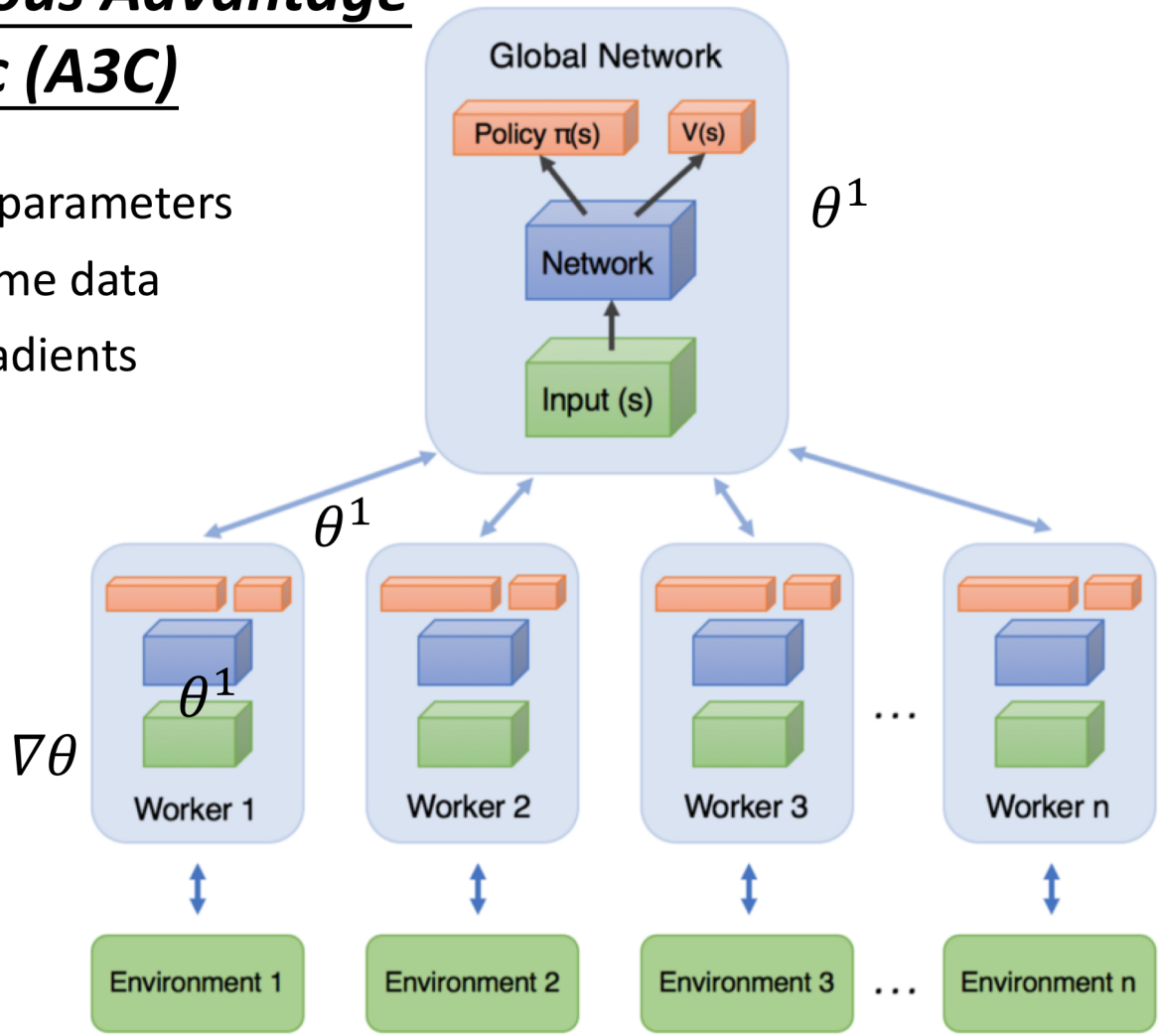
1. Copy global parameters
2. Sampling some data



Actor-Critic

Asynchronous Advantage Actor-Critic (A3C)

1. Copy global parameters
2. Sampling some data
3. Compute gradients



Actor-Critic

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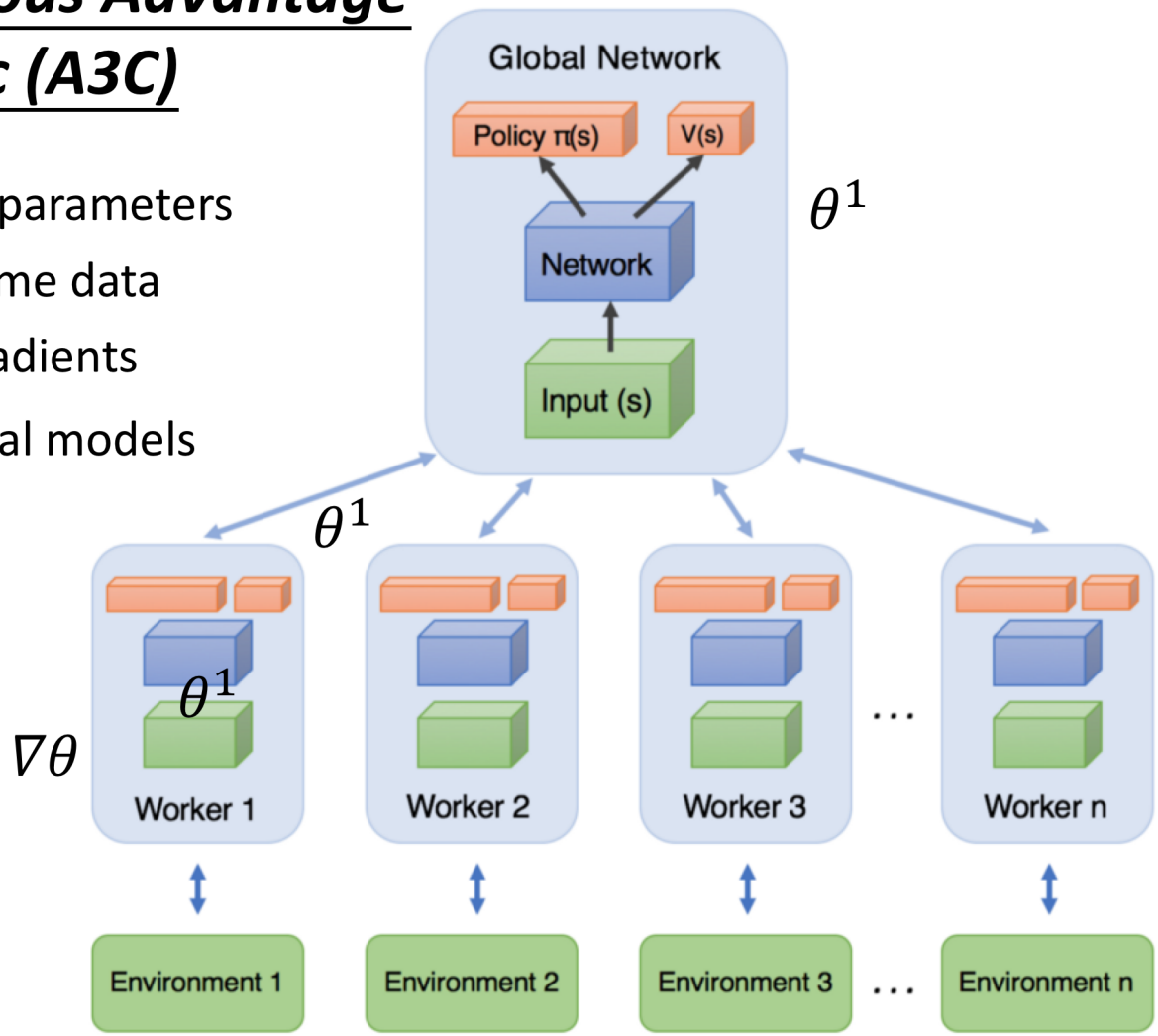

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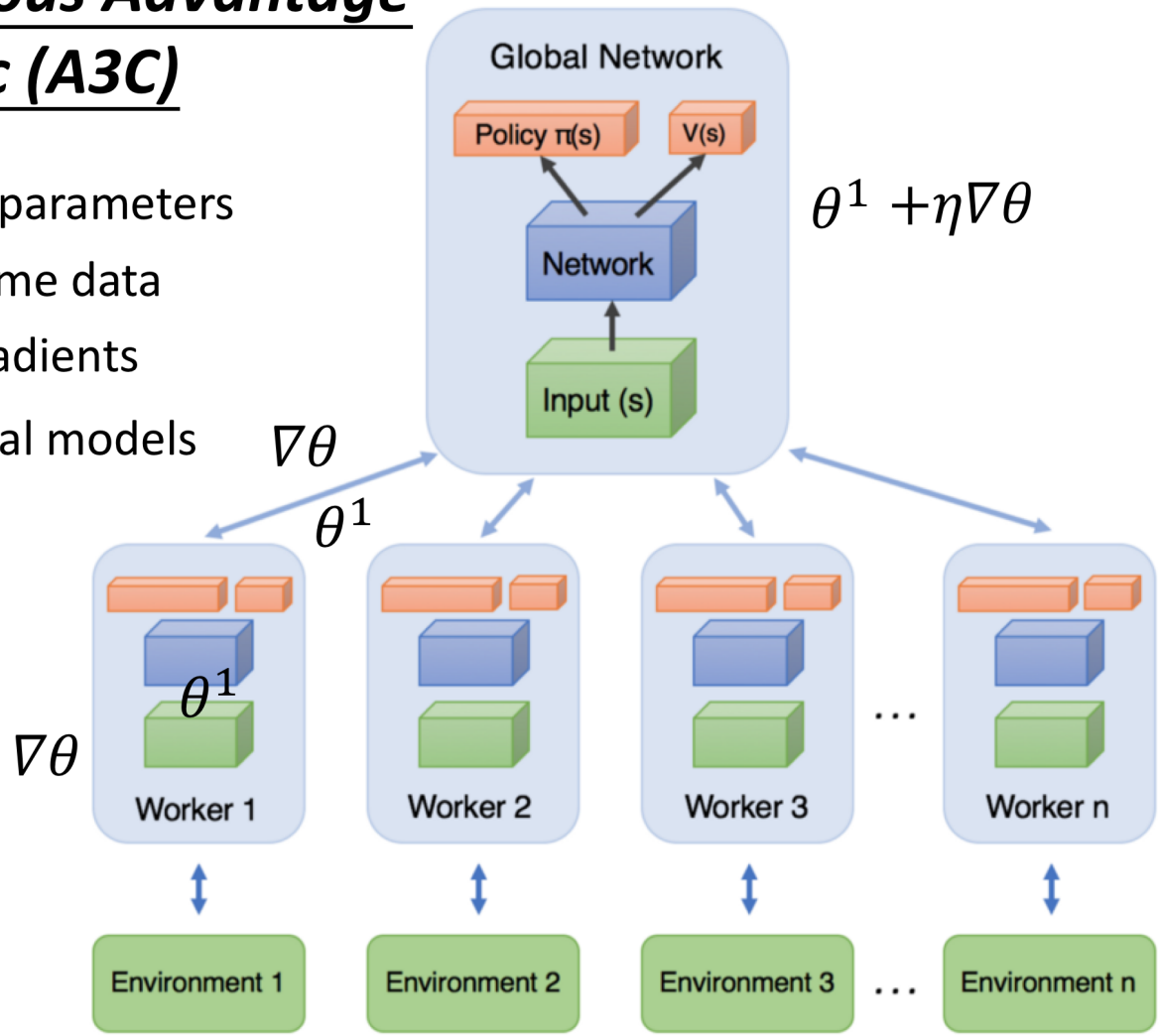

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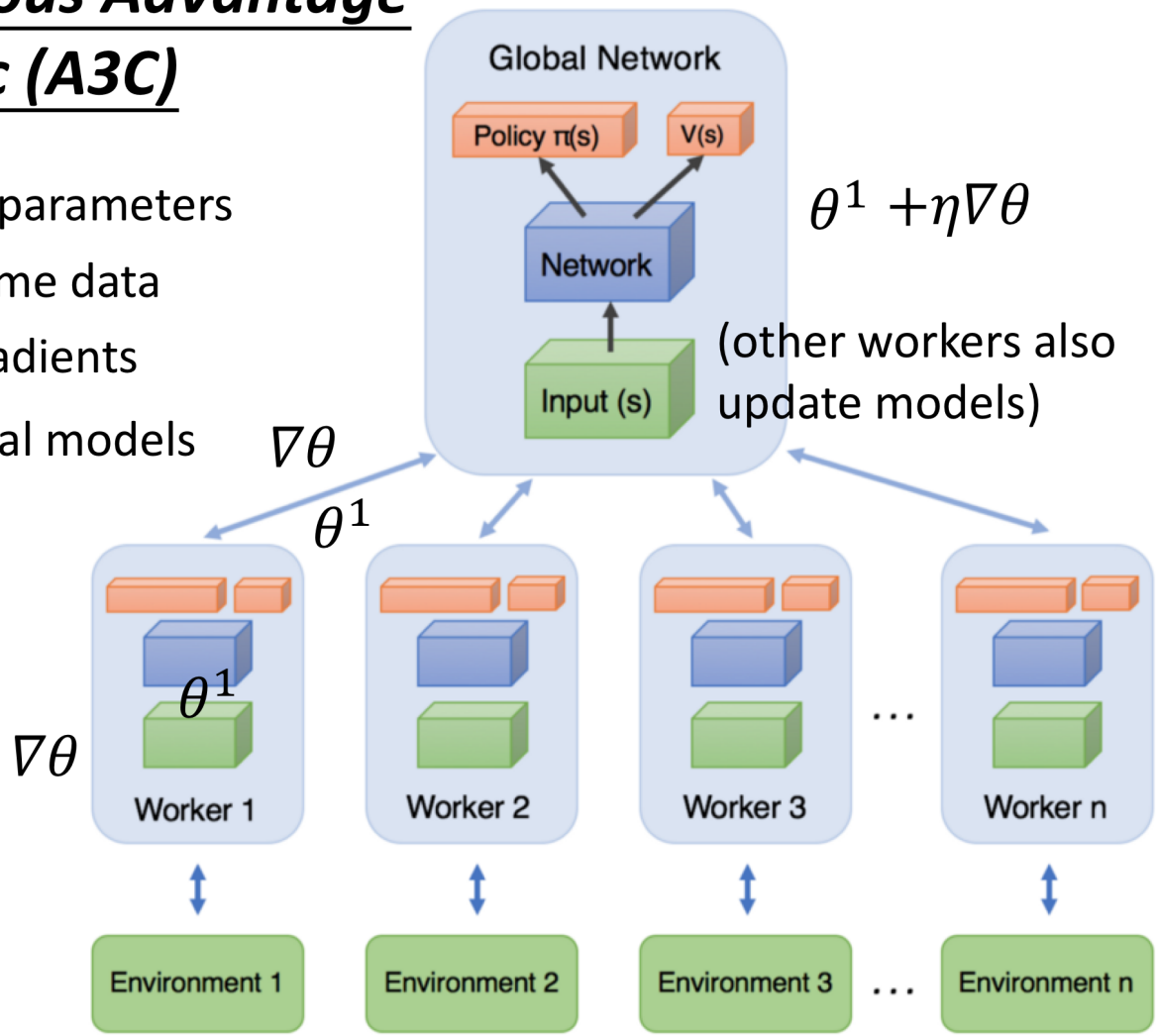

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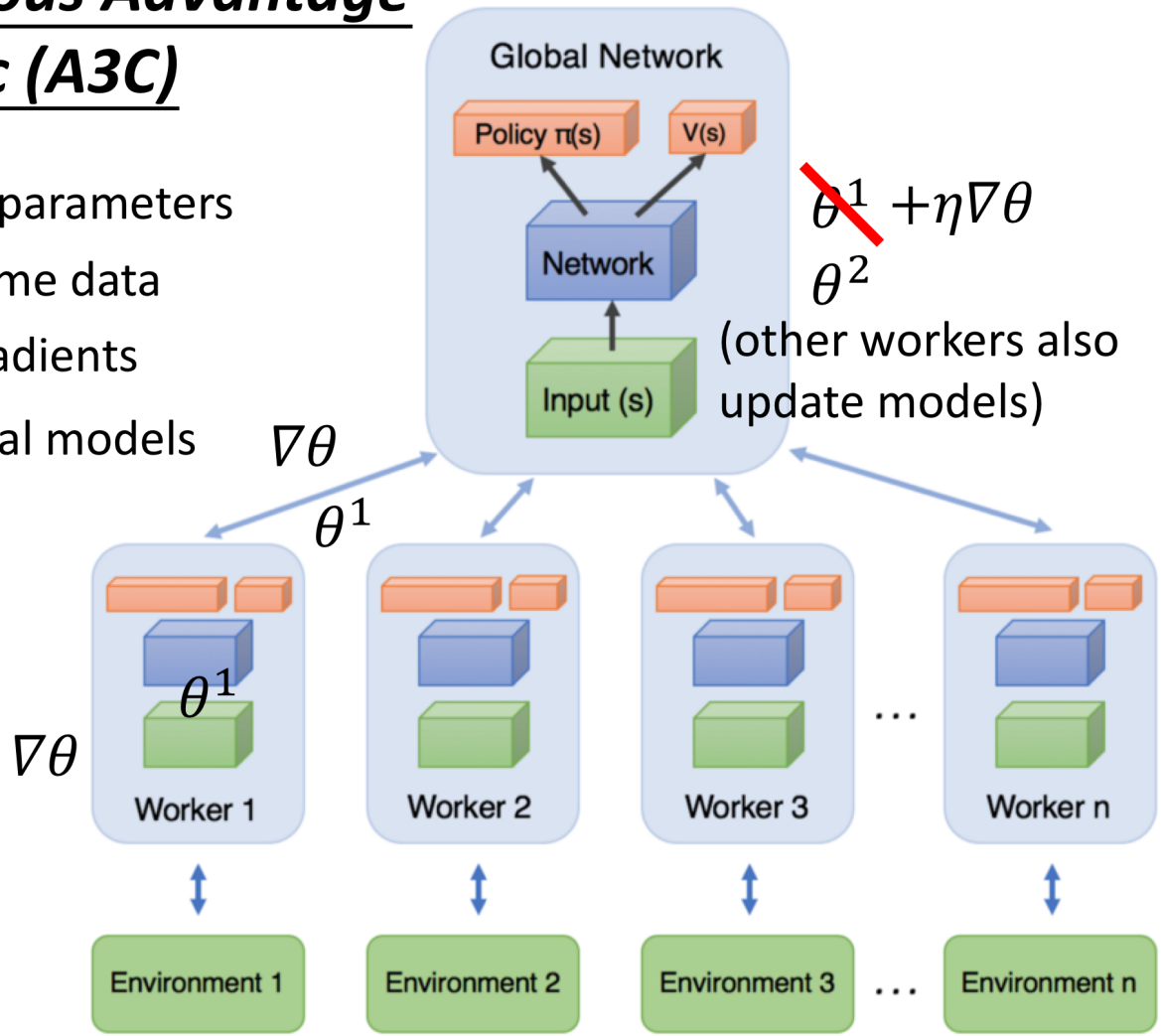


Asynchronous Advantage Actor-Critic (A3C)

- Multiple asynchronous actors-learners run in parallel
- Explore different parts of environment, i.e., different exploration policies in different learners
- Stabilize learning, reduce training time, and without large resource requirements
- Add the entropy of policy to objective function
- Use standard non-centered RMSProp update

$$g = \alpha g + (1 - \alpha)\Delta\theta^2 \text{ and } \theta \leftarrow \theta - \eta \frac{\Delta\theta}{\sqrt{g + \epsilon}}$$

Asynchronous Methods for Deep Reinforcement Learning, Proceedings of ICML, 2016

A3C Algorithm

Algorithm S3 Asynchronous advantage actor-critic - pseudocode for each actor-learner thread.

// Assume global shared parameter vectors θ *and* θ_v *and global shared counter* $T = 0$
// Assume thread-specific parameter vectors θ' *and* θ'_v
Initialize thread step counter $t \leftarrow 1$
repeat
  Reset gradients: $d\theta \leftarrow 0$ and $d\theta_v \leftarrow 0$.
  Synchronize thread-specific parameters $\theta' = \theta$ and $\theta'_v = \theta_v$
  $t_{start} = t$
  Get state s_t
  repeat
    Perform a_t according to policy $\pi(a_t|s_t;\theta')$
    Receive reward r_t and new state s_{t+1}
    $t \leftarrow t + 1$
    $T \leftarrow T + 1$
  until terminal s_t **or** $t - t_{start} == t_{max}$
  $R = \begin{cases} 0 & \text{for terminal } s_t \\ V(s_t, \theta'_v) & \text{for non-terminal } s_t \text{// Bootstrap from last state} \end{cases}$
  for $i \in \{t-1, \ldots, t_{start}\}$ **do**
    $R \leftarrow r_i + \gamma R$
    Accumulate gradients wrt θ': $d\theta \leftarrow d\theta + \nabla_{\theta'} \log \pi(a_i|s_i;\theta')(R - V(s_i;\theta'_v))$
    Accumulate gradients wrt θ'_v: $d\theta_v \leftarrow d\theta_v + \partial \left(R - V(s_i;\theta'_v)\right)^2 / \partial \theta'_v$
  end for
  Perform asynchronous update of θ using $d\theta$ and of θ_v using $d\theta_v$.
until $T > T_{max}$

Summary of Policy Gradient Algorithms

- The policy gradient has many equivalent forms

$$
\begin{aligned}
\nabla_\theta J(\theta) &= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s,a)\ v_t\right] && \text{REINFORCE} \\
&= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s,a)\ Q^w(s,a)\right] && \text{Q Actor-Critic} \\
&= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s,a)\ A^w(s,a)\right] && \text{Advantage Actor-Critic} \\
&= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s,a)\ \delta\right] && \text{TD Actor-Critic} \\
&= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(s,a)\ \delta e\right] && \text{TD}(\lambda)\text{ Actor-Critic} \\
G_\theta^{-1}\nabla_\theta J(\theta) &= w && \text{Natural Actor-Critic}
\end{aligned}
$$

- Each leads a stochastic gradient ascent algorithm
- Critic uses policy evaluation (e.g. MC or TD learning) to estimate $Q^\pi(s,a), A^\pi(s,a)$ or $V^\pi(s)$

Table of Contents

Q-learning-DDPG-TD3-SAC

1. Value based RL methods starting from Q-learning are also being developed over the years
2. **DDPG**: Deterministic Policy Gradient Algorithms, Silver et al. ICML 2014
3. **TD3**: Addressing Function Approximation Error in Actor-Critic Methods, Fujimoto et al. ICML 2018
4. **SAC**: Soft Actor-Critic: Off-Policy Maximum Entropy Deep Reinforcement Learning with a Stochastic Actor, Haarnoja et al. ICML 2018

Introduction

- It was previously believed that the deterministic policy gradient did not exist.
- Or could only be obtained when using a model.
- Until 2014, deterministic policy gradient was proposed, and showed that it has a simple model-free form.
- In 2016, an actor-critic algorithm based on deterministic policy gradient was proposed for continuous action spaces.

Deterministic Policy Gradient Algorithms, ICML 2014

Continuous Control with Deep Reinforcement Learning, ICLR 2016

Stochastic Policy

- The policy is stochastic and denoted by

$$\pi_\theta : \mathcal{S} \to \mathcal{P}(\mathcal{A}),$$

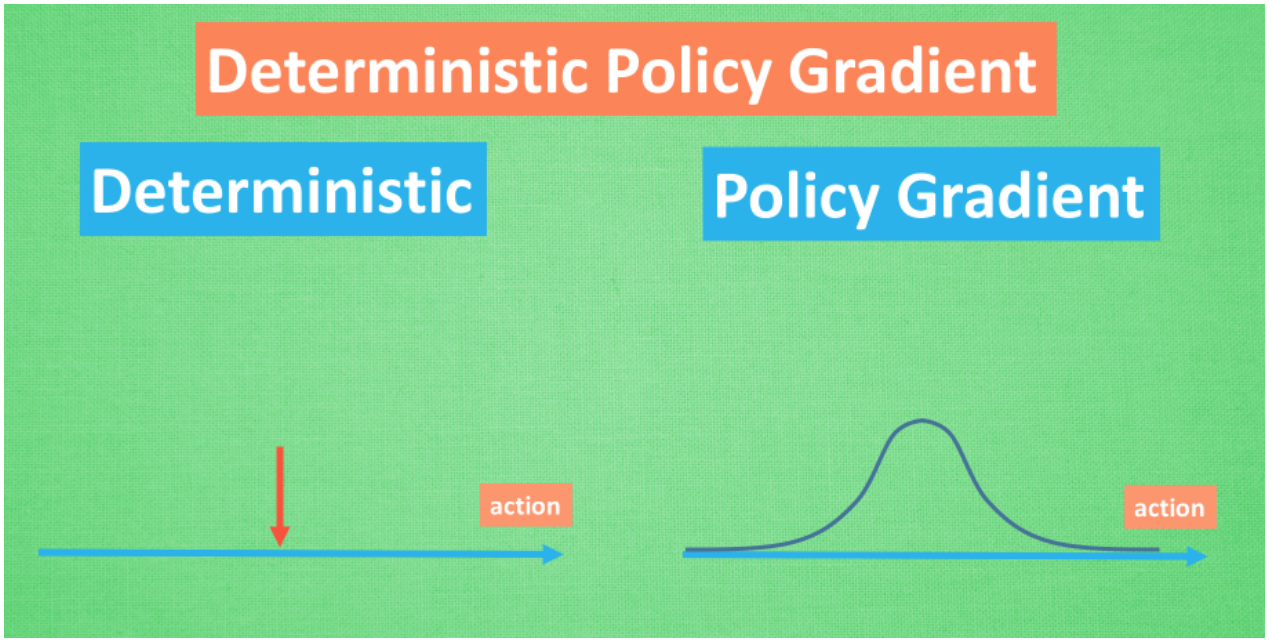
  where $\mathcal{P}(\mathcal{A})$ is the set of probability measures on $\mathcal{A}$ and $\theta \in \mathbb{R}^n$ is a vector of n parameters.
- $\pi_\theta(a_t|s_t)$ is the conditional probability density at a_t associated with the policy.
- For the same state s, the actions stochastically selected according to θ might be different.

Deterministic Policy

- Select a deterministic action:

$$a = \mu_\theta(s)$$

- action is *uniquely* determined at the state s w.r.t the parameter θ
- suitable for the continuous action space, especially if the action space has many dimensions



Deterministic Policy vs. Stochastic Policy

- For *stochastic* case: the policy gradient integrates over both state and action spaces

$$\begin{aligned}\nabla_\theta J(\pi_\theta) &= \int_{\mathcal{S}} \rho^\pi(s) \int_{\mathcal{A}} \nabla_\theta \pi_\theta(a|s) Q^\pi(s,a) \mathrm{d}a \mathrm{d}s \\ &= \mathbb{E}_{s\sim\rho^\pi, a\sim\pi_\theta} \left[\nabla_\theta \log \pi_\theta(a|s) Q^\pi(s,a)\right]\end{aligned}$$

- For *deterministic* case: the policy gradient only integrates over the state spaces since the action selections are deterministic

$$\begin{aligned}\nabla_\theta J(\mu_\theta) &= \int_{\mathcal{S}} \rho^\mu(s) \nabla_\theta \mu_\theta(s) \left.\nabla_a Q^\mu(s,a)\right|_{a=\mu_\theta(s)} \mathrm{d}s \\ &= \mathbb{E}_{s\sim\rho^\mu} \left[\nabla_\theta \mu_\theta(s) \left.\nabla_a Q^\mu(s,a)\right|_{a=\mu_\theta(s)}\right]\end{aligned}$$

Deterministic Policy vs. Stochastic Policy (2)

- Hence, computing the stochastic policy gradient may require more samples.
- Especially if the action space has many dimensions.
- It is shown that the deterministic policy gradient is the limiting case, as policy variance tends to zero, of the stochastic policy gradient.

Generalized Policy Iteration

The majority of model-free reinforcement learning algorithms are based on generalized policy iteration: interleaving policy evaluation with policy improvement.

- Policy evaluation methods estimate the action-value function

$$Q^{\mu}(s, a)$$

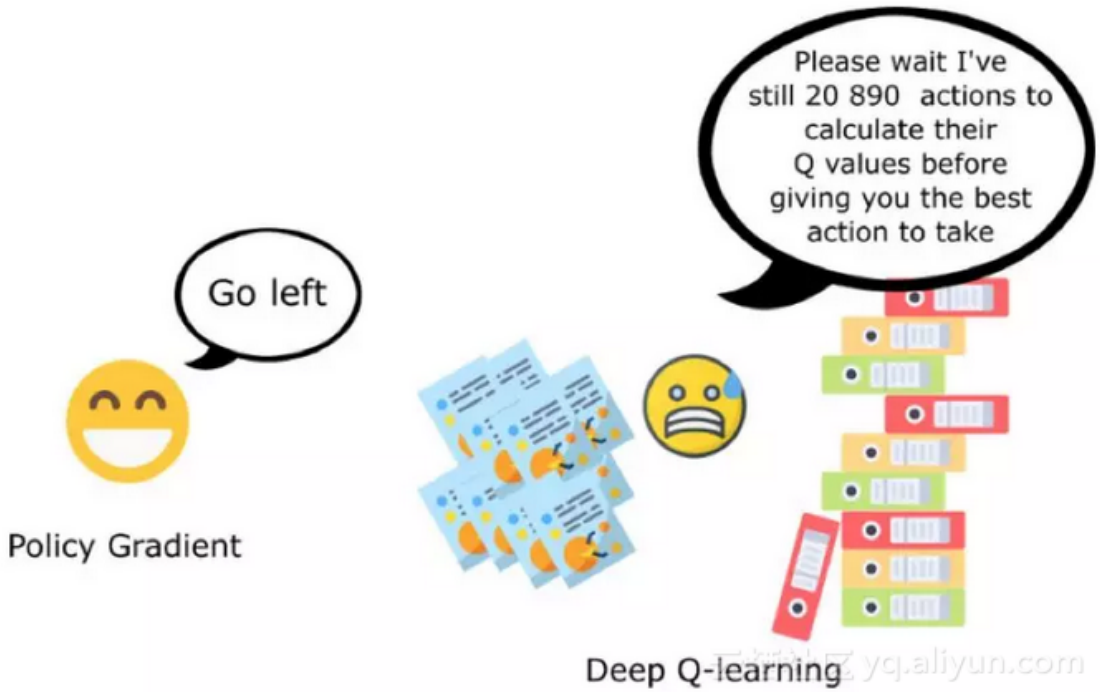
  - e.g. Monte-Carlo evaluation or TD learning
- Policy improvement methods update the policy w.r.t the $Q^{\mu}(s, a)$
  - the most common approach: greedy maximization

$$\mu^{k+1}(s) = \arg\max_{a} Q^{\mu^k}(s, a)$$

  - or soft maximization, etc.

Deterministic Policy Improvement

- In continuous action spaces, greedy policy improvement becomes problematic, requiring a global maximization at every step



- Instead, moving the policy in the direction of the gradient of Q, rather than globally maximizing Q

Deterministic Policy Inprovement (2)

- For each visited state s, the policy parameter θ^{k+1} are updated in proportion to the gradient $\nabla_\theta Q^{\mu^k}(s, \mu_\theta(s))$
- Each state suggests a different direction of policy improvement
- By taking an expectation w.r.t. the state distribution ρ^{μ^k}

$$\theta^{k+1} = \theta^k + \alpha \mathbb{E}_{s\sim\rho^{\mu^k}}[\nabla_\theta Q^{\mu^k}(s, \mu_\theta(s))]$$

- Policy improvement is decomposed into the gradient of action-value w.r.t. actions, and the gradient of policy w.r.t. policy parameters

$$\theta^{k+1} = \theta^k + \alpha \mathbb{E}_{s\sim\rho^{\mu^k}}\left[\nabla_\theta \mu_\theta(s) \nabla_a Q^{\mu^k}(s, a)\big|_{a=\mu_\theta(s)}\right]$$

  - By convention $\nabla_\theta \mu_\theta(s)$ is a *Jacobian matrix* such that each column is the gradient $\nabla_\theta[\mu_\theta(s)]_d$ of the d-th action dimension of the policy w.r.t. θ

Guarantee of Policy Improvement

- If change the policy, different states are visited and the state distribution ρ^{μ^k} will change

$$\theta^{k+1} = \theta^k + \alpha \mathbb{E}_{s \sim \rho^{\mu^k}} \left[\nabla_\theta \mu_\theta(s) \nabla_a Q^{\mu^k}(s, a) \big|_{a=\mu_\theta(s)} \right]$$

- This policy improvement method cannot guarantee improvement without taking account of the change to distribution
- recall the *(Stochastic) Policy Gradient Theorem*, there is no need to compute the gradient of the state distribution.

(Stochastic) Policy Gradient Theorem

For any differentiable policy $\pi_\theta(s, a)$, for any of the policy objective functions, the policy gradient is

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta} [\nabla_\theta \log \pi_\theta(s, a) Q^{\pi_\theta}(s, a)]$$

Deterministic Policy Objective Function

Analogously to the stochastic case $\rightarrow$

- Consider a deterministic policy $\mu_\theta : \mathcal{S} \rightarrow \mathcal{A}$ w.r.t. $\theta \in \mathbb{R}^n$
- Define a deterministic policy objective function

$$J(\mu_\theta) = \mathbb{E}[r_1^\gamma | \mu]$$

- Probability distribution $p(s \rightarrow s', t, \mu)$ and discounted state distribution $\rho^\mu(s)$

$$\begin{aligned} J(\mu_\theta) &= \int_{\mathcal{S}} \rho^\mu(s) r(s, \mu_\theta(s)) ds \\ &= \mathbb{E}_{s \sim \rho^\mu}[r(s, \mu_\theta(s))] \end{aligned}$$

Deterministic Policy Gradient Theorem

- Replace the immediate reward $r(s, \mu_\theta(s))$ with $Q^\mu(s, \mu_\theta(s))$

Deterministic Policy Gradient Theorem

Suppose that the MDP satisfies that $p(s'|s,a), \nabla_a p(s'|s,a), \mu_\theta(s), \nabla_\theta \mu_\theta(s), r(s,a), \nabla_a r(s,a), p_1(s)$ *are continuous in all parameters and variables* s, a, s'. *Then,*

$$\begin{aligned}\nabla_\theta J(\mu_\theta) &= \int_{\mathcal{S}} \rho^\mu(s) \nabla_\theta \mu_\theta(s) \nabla_a Q^{\mu^k}(s,a)|_{a=\mu_\theta(s)} ds \\ &= \mathbb{E}_{s\sim\rho^\mu}\left[\nabla_\theta \mu_\theta(s) \nabla_a Q^{\mu^k}(s,a)|_{a=\mu_\theta(s)}\right]\end{aligned}$$

Off-Policy Deterministic Actor-Critic

- Behaving according to a deterministic policy will not ensure adequate exploration, and may lead to sub-optimal solutions
- For adequate exploration, off-policy learning is employed
- A actor learns a deterministic target policy $\mu_\theta(s)$ from trajectories generated by an *arbitrary* stochastic behaviour policy $\beta(a|s)$
- A critic estimates the action-value function $Q^w(s,a) \approx Q^\mu(s,a)$, off-policy from trajectories generated by $\beta(a|s)$
  - use Q-learning to estimate the action-value function

Off-Policy Deterministic Actor

- First, we modify the policy objective function to be the value function of the *target policy* $\mu_\theta(s)$, averaged over the *state distribution* of the *behaviour policy* $\beta_\theta(a|s)$,

$$J_\beta(\mu_\theta) = \int_{\mathcal{S}} \rho^\beta(s) V^\mu(s) ds$$
$$= \int_{\mathcal{S}} \rho^\beta(s) Q^\mu(s, \mu_\theta(s)) ds$$

$$\nabla_\theta J_\beta(\mu_\theta) \approx \int_{\mathcal{S}} \rho^\beta(s) \nabla_\theta \mu_\theta(s) Q^\mu(s, a) ds$$
$$= \mathbb{E}_{s\sim\rho^\beta}\left[\nabla_\theta \mu_\theta(s) \nabla_a Q^\mu(s, a)\Big|_{a=\mu_\theta(s)}\right]$$

Deterministic Actor-Critic Algorithm

$$\begin{aligned}
\delta_t &= r_t + \gamma Q^w(s_{t+1}, \mu_\theta(s_{t+1})) - Q^w(s_t, a_t) \\
w_{t+1} &= w_t + \alpha_w \delta_t \nabla_w Q^w(s_t, a_t) \\
\theta_{t+1} &= \theta_t + \alpha_\theta \nabla_\theta \mu_\theta(s_t) \nabla_a Q^w(s_t, a_t)|_{a=\mu_\theta(s)}
\end{aligned}$$

Compatible Function Approximation

- Find a compatible function approximator $Q^w(s,a)$ such that the true gradient is preserved.

Theorem

A function approximator $Q^w(s,a)$ is compatible with a deterministic policy $\mu_\theta(s)$, $\nabla_\theta J_\beta(\theta) = \mathbb{E}\left[\nabla_\theta \mu_\theta(s) \nabla_a Q^w(s,a)|_{a=\mu_\theta(s)}\right]$, if

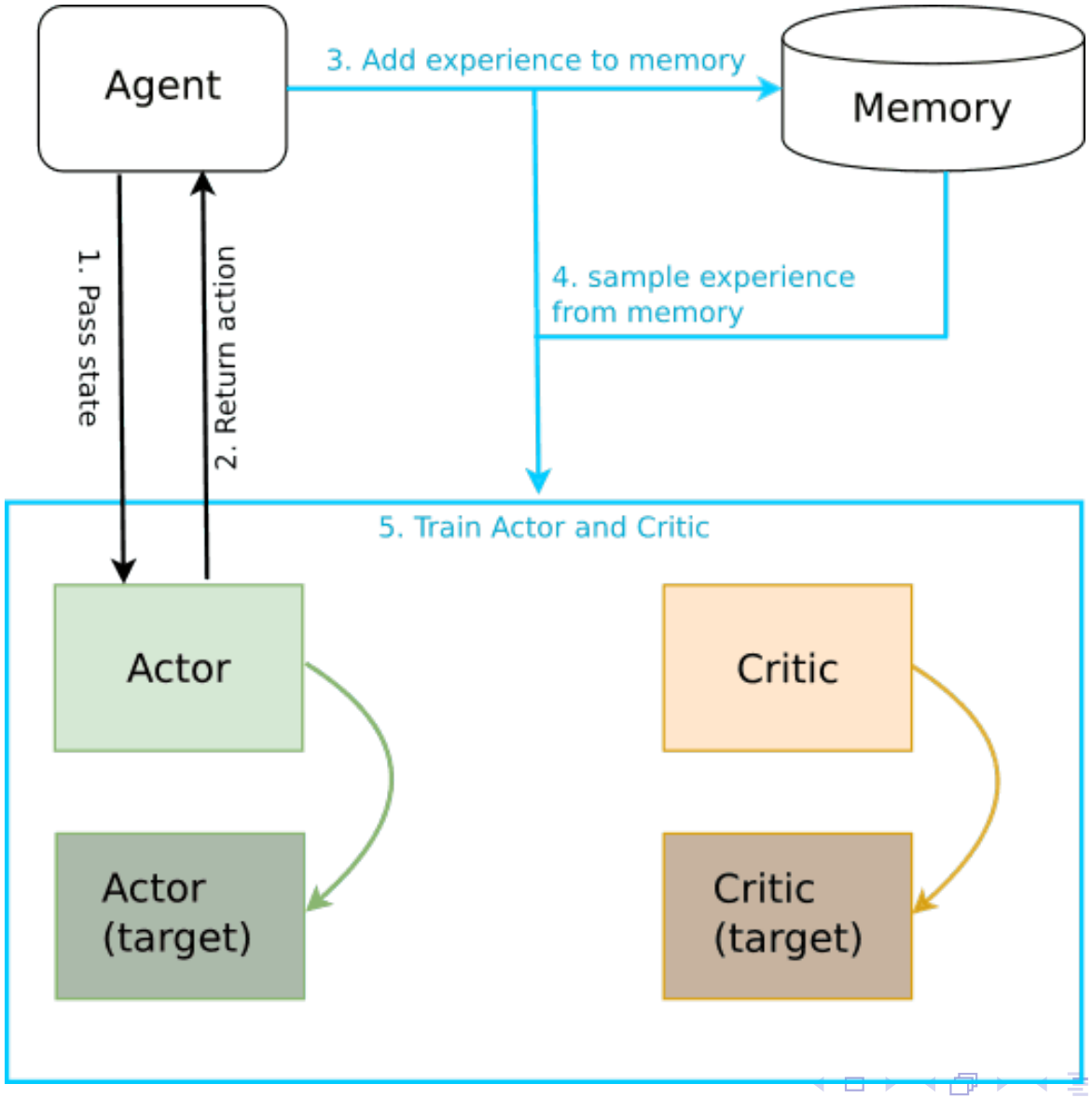
1. $\nabla_a Q^w(s,a)|_{a=\mu_\theta(s)} = \nabla_\theta \mu_\theta(s)^T w$ and
2. *w minimizes the mean-squared error, $MSE(\theta, w) = \mathbb{E}[\epsilon(s;\theta,w)^T \epsilon(s;\theta,w)]$ where $\epsilon(s;\theta,w) = \nabla_a Q^w(s,a)|_{a=\mu_\theta(s)} - \nabla_a Q^\mu(s,a)|_{a=\mu_\theta(s)}$*

Deep DPG (DDPG)

- DDPG is a kind of off-policy actor-critic algorithm using deep function approximators to learn policies in high-dimensional, continuous action spaces.
- The experience replay idea of DQN is introduced to learn across a set of uncorrelated transitions.
- Two sets of networks: critic network $Q(s, a|\theta^Q)$ and actor network $\mu(s|\theta^\mu)$, with weights θ^Q and θ^μ
  target network Q' and μ', with weights $\theta^{Q'}$ and $\theta^{\mu'}$ updated by:

$$\begin{aligned} \theta^{Q'} &\leftarrow \tau\theta^Q + (1-\tau)\theta^{Q'} \\ \theta^{\mu'} &\leftarrow \tau\theta^\mu + (1-\tau)\theta^{\mu'} \end{aligned} \quad \text{with } \tau \ll 1$$

DDPG Framework



DDPG Algorithm

Algorithm 1 DDPG algorithm

Randomly initialize critic network $Q(s, a|\theta^Q)$ and actor $\mu(s|\theta^\mu)$ with weights θ^Q and θ^μ.
Initialize target network Q' and μ' with weights $\theta^{Q'} \leftarrow \theta^Q$, $\theta^{\mu'} \leftarrow \theta^\mu$
Initialize replay buffer R
for episode = 1, M **do**
  Initialize a random process $\mathcal{N}$ for action exploration
  Receive initial observation state s_1
  for t = 1, T **do**
    Select action $a_t = \mu(s_t|\theta^\mu) + \mathcal{N}_t$ according to the current policy and exploration noise
    Execute action a_t and observe reward r_t and observe new state s_{t+1}
    Store transition (s_t, a_t, r_t, s_{t+1}) in R
    Sample a random minibatch of N transitions (s_i, a_i, r_i, s_{i+1}) from R
    Set $y_i = r_i + \gamma Q'(s_{i+1}, \mu'(s_{i+1}|\theta^{\mu'})|\theta^{Q'})$
    Update critic by minimizing the loss: $L = \frac{1}{N}\sum_i (y_i - Q(s_i, a_i|\theta^Q))^2$
    Update the actor policy using the sampled policy gradient:

$$\nabla_{\theta^\mu} J \approx \frac{1}{N} \sum_i \nabla_a Q(s, a|\theta^Q)|_{s=s_i, a=\mu(s_i)} \nabla_{\theta^\mu} \mu(s|\theta^\mu)|_{s_i}$$

    Update the target networks:

$$\theta^{Q'} \leftarrow \tau\theta^Q + (1-\tau)\theta^{Q'}$$
$$\theta^{\mu'} \leftarrow \tau\theta^\mu + (1-\tau)\theta^{\mu'}$$

  end for
end for

DDPG Algorithm (2)

Randomly initialize critic network $Q(s, a|\theta^Q)$ and actor $\mu(s|\theta^\mu)$ with weights θ^Q and θ^μ.
Initialize target network Q' and μ' with weights $\theta^{Q'} \leftarrow \theta^Q$, $\theta^{\mu'} \leftarrow \theta^\mu$
Initialize replay buffer R
for episode = 1, M **do**
  Initialize a random process $\mathcal{N}$ for action exploration
  Receive initial observation state s_1
  for t = 1, T **do**

行动策略为随机策略

  Select action $a_t = \mu(s_t|\theta^\mu) + \mathcal{N}_t$ according to the current policy and exploration noise
  Execute action a_t and observe reward r_t and observe new state s_{t+1}
  Store transition (s_t, a_t, r_t, s_{t+1}) in R
  Sample a random minibatch of N transitions (s_i, a_i, r_i, s_{i+1}) from R

经验回放

  Set $y_i = r_i + \gamma Q'(s_{i+1}, \mu'(s_{i+1}|\theta^{\mu'})|\theta^{Q'})$

目标网络

  Update critic by minimizing the loss: $L = \frac{1}{N}\sum_i (y_i - Q(s_i, a_i|\theta^Q)^2)$
  Update the actor policy using the sampled gradient:

$$\nabla_{\theta^\mu}\mu|_{s_i} \approx \frac{1}{N}\sum_i \nabla_a Q(s, a|\theta^Q)|_{s=s_t, a=\mu(s_t)} \nabla_{\theta^\mu}\mu(s|\theta^\mu)|_{s_t}$$

  Update the target networks:

$$\theta^{Q'} \leftarrow \tau\theta^Q + (1-\tau)\theta^{Q'}$$
$$\theta^{\mu'} \leftarrow \tau\theta^\mu + (1-\tau)\theta^{\mu'}$$

目标网络参数更新

  end for
end for

Example: TORCS

https://youtu.be/8CNck-hdys8