

t_a, t_b

$$\bar{t}_a = (t_a \setminus \{a\}) \cup \{b\}, \quad \bar{t}_b = (t_b \setminus \{b\}) \cup \{a\}$$

Claim: $t_a = \bar{t}_b \iff t_b = \bar{t}_a$

Proof: $t_a = \bar{t}_b \iff t_a = (t_b \setminus \{b\}) \cup \{a\}$

$$\iff t_a \setminus \{a\} = t_b \setminus \{b\}$$

$$\iff (t_a \setminus \{a\}) \cup \{b\} = t_b$$

$$\iff \bar{t}_a = t_b \quad \square$$