$|D| \times |I|$

- Cononical representation of dataset $D$ is matrix $M_D \in \{0,1\}^{|D|\times|I|}$ with $|D|$ rows and $|I|$ columns s.t.

    1) Sum of each column is the support of each item in $D$

    2) rows are sorted in decreasing order by $\| \|_1$, ties broken arbitrarily

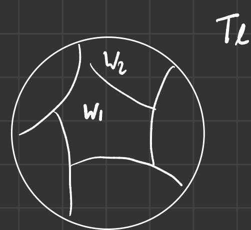
- $L_D$ = set of distinct lengths of transactions in $D$
- For each $\ell \in L_D$, let $s_\ell$ be the number of transactions of length $\ell$ in $D$

    let $T_\ell$ be the bag of transactions of length $\ell$ in $D$,

    $|T_\ell| = s_\ell$

- For each $\tau \in T_\ell$, let $W_\tau = \{ t \in T_\ell : t = \tau \}$
- $W_1, \ldots, W_{z_\ell}$ partion $T_\ell$

$$\sum_{i=1}^{z_\ell} |W_i| = s_\ell$$




- # of different ways of arranging transactions in $T_\ell$:

$$q_\ell = \binom{s_\ell}{|W_1|, \ldots, |W_{z_\ell}|} = \frac{s_\ell!}{|W_1|! \cdots |W_{z_\ell}|!}$$

// eq. for multiset permutation

- Total number of equivalent representations: $a_D = \prod_{\ell \in L_D} q_\ell$

# Calculating $a_{D'}$

- Suppose we swapped edges $(i,j), (k,\ell) \in E(G)$
- $M_D(u)$ denotes the transaction in row $u$ of the matrix $M_D$
- $W_{M_D(u)} = \{ t \in T_{r_u} : t = M_D(u) \}$, where $r_u = \| M_D(u) \|_1$
- $M_{D'}(u)$ denotes the transaction in row $u$ of the matrix $M_{D'}$ after the swap
- $W'_{M_D(u)}$ denotes the updated $W_{M_D(u)}$ after the swap
- $W'_{M_{D'}(u)}$ " " " $W_{M_{D'}(u)}$ " " "

get $a_{D'}()$:

for $u \in \{i, k\}$

$|W'_{M_D(u)}| \leftarrow |W_{M_D(u)}| - 1$

$|W'_{M_{D'}(u)}| \leftarrow |W_{M_{D'}(u)}| + 1$

$|W'_{M_D(u)}|! \leftarrow |W_{M_D(u)}|! / |W_{M_D(u)}|$ // $= (|W_{M_D(u)}| - 1)!$

$|W'_{M_{D'}(u)}|! \leftarrow (|W_{M_{D'}(u)}| + 1) \cdot |W_{M_{D'}(u)}|!$ // $= (|W_{M_{D'}(u)}| + 1)!$

$q'_{r_i} \leftarrow q_{r_i}$

$q'_{r_k} \leftarrow q_{r_k}$

for $u \in \{i, k\}$

$q'_{r_u} \leftarrow q'_{r_u} \cdot \dfrac{|W_{M_D(u)}|! \cdot |W_{M_{D'}(u)}|!}{|W'_{M_D(u)}|! \cdot |W'_{M_{D'}(u)}|!}$ // updates $q'_{r_i}$ twice if $r_i = r_k$

if $r_i \neq r_k$

$a_{D'} \leftarrow a_D \cdot \dfrac{q'_{r_i} q'_{r_k}}{q_{r_i} q_{r_k}}$

else

$a_{D'} \leftarrow a_D \cdot \dfrac{q'_{r_i}}{q_{r_i}}$

return $a_{D'}$

# Calculating $\log(a_D)$

- $\log(a_D) = \log\left(\prod_{\ell \in L_D} q_\ell\right) = \sum_{\ell \in L_D} \log(q_\ell)$

- $\log(q_\ell) = \log\left(\frac{s_\ell!}{|W_1|! \cdots |W_{z_\ell}|!}\right)$

$$= \log(s_\ell!) - \log(|W_1|! \cdots |W_{z_\ell}|!)$$
$$= \log(s_\ell!) - \sum_{i=1}^{z_\ell} \log(|W_i|!)$$

- $\log(s_\ell!) = \log(s_\ell) + \log(s_\ell - 1) + \cdots \log(1)$

$$= \sum_{j=1}^{s_\ell} \log(s_\ell - j + 1)$$

- $\log(|W_i|!) = \log(|W_i|) + \log(|W_i| - 1) + \cdots + \log(1)$

$$= \sum_{k=1}^{|W_i|} \log(|W_i| - k + 1)$$

- $\log(a_D) = \sum_{\ell \in L_D} \left[ \left( \sum_{j=1}^{s_\ell} \log(s_\ell - j + 1) \right) - \left( \sum_{i=1}^{z_\ell} \sum_{k=1}^{|W_i|} \log(|W_i| - k + 1) \right) \right]$

- $a_D / a_{D'} = 2^{\log(a_D / a_{D'})} = 2^{\log(a_D) - \log(a_{D'})}$ // what we need to calculate $P(G, G')$

## Calculating $\log(a_{D'})$

getLog$a_{D'}$():

for $u \in \{i, k\}$

$|W'_{M_D(u)}| \leftarrow |W_{M_D(u)}| - 1$

$|W'_{M_{D'}(u)}| \leftarrow |W_{M_{D'}(u)}| + 1$

$\log(|W'_{M_D(u)}|!) \leftarrow \log(|W_{M_D(u)}|!) - \log(|W_{M_D(u)}|)$

$\log(|W'_{M_{D'}(u)}|!) \leftarrow \log(|W_{M_{D'}(u)}| + 1) + \log(|W_{M_{D'}(u)}|!)$

$\log(q'_{r_i}) \leftarrow \log(q_{r_i})$

$\log(q'_{r_k}) \leftarrow \log(q_{r_k})$

for $u \in \{i, k\}$

$$\log(q'_{r_u}) \leftarrow \log(q'_{r_u}) + \log(|W_{M_D(u)}|!) + \log(|W_{M_{D'}(u)}|!) - \log(|W'_{M_D(u)}|!) - \log(|W'_{M_{D'}(u)}|!)$$

if $r_i \neq r_k$

$$\log(a_{D'}) \leftarrow \log(a_D) + \log(q'_{r_i}) + \log(q'_{r_k}) - \log(q_{r_i}) - \log(q_{r_k})$$

else

$$\log(a_{D'}) \leftarrow \log(a_D) + \log(q'_{r_i}) - \log(q_{r_i})$$

return $\log(a_{D'})$