

An Integer Linear Programming Approach

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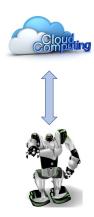
## **Outline**

- 1. Background of Cloud Robotics
- 2. Mapping Problem and Algorithm
- 3. Experimental Results
- 4. Future Work

**Background of Cloud Robotics** 

## **Background**

Take advantage of the Internet as a resource for massively parallel computation and resources.



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- Cloud provides a shared knowledge database
- Robots can download new skills instantly
- Or offload compute intensive task to the cloud
  - Image processing
  - Real-time Tracking

## **Background**

Take advantage of the Internet as a resource for massively parallel computation and resources.





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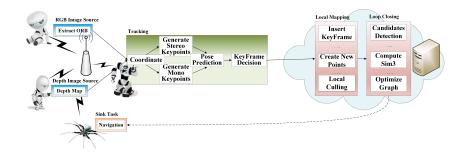
#### **Advantages**

- Cheaper, lighter hardware
- Longer battery life
- Less need for software pushes/updates

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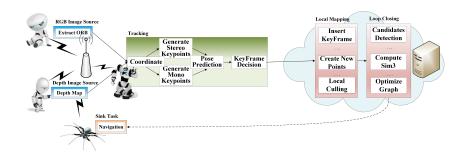
## Offloading in Cloud Robotics

## Simultaneous Localization And Mapping (SLAM)



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#### Simultaneous Localization And Mapping (SLAM)



#### Challenges

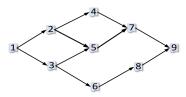
- Overhead of big data delivery, i.e., continuously collecting and processing sensing data
- Transfer rate of the cloud robotic network
- The existence of heterogeneity of robots

Mapping Problem and Algorithm

## Task Graph Model

Use a Directed Acyclic Graph (DAG) to represent the application model:

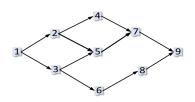
- Vertices: task steps
- Edges: data dependency
- Weight of Vertices: computational complexity
- Weight of Edges: intermediate data size



### Task Graph Model

Use a Directed Acyclic Graph (DAG) to represent the application model:

- Vertices: task steps
- Edges: data dependency
- Weight of Vertices: computational complexity
- Weight of Edges: intermediate data size



#### **Problem**

Map the task graph to the compute nodes, minimize the overall

#### makespan.

- assignment
- scheduling

#### **Formulation**

#### TABLE I: Notations

	Constants
$S_{ij}$	The size of intermediate data from task i to task
$R_{ik}$	The running time of task $i$ on node $k$
$B_{kl}$	The transmission rate from node k to node l
$\tau$	The set of time slots $T = \{1, 2,, t\}$
$pred_i$	The predecessors of task i
$succ_i$	The sucessors of task i
	Variables
	4.42 2.44 2.45 2.4 4.4 4.4 2.4

- $x_{ik}$  A binary variable indicating whether task i assigned to node k
- $x_{ik}^l$  A binary variable indicating whether task i is scheduled to run on node k starting at time slot
- yij A binary variable indicating whether task i is scheduled to run after i
- t<sup>n</sup> The start time of task i
- The finish time of task i

- min : te
- $s.t.: \sum_{k \in V_n} x_{ik} = 1, \forall i \in V_t.$  (

$$y_{jj} + y_{jj} = 1, \forall i, j \in V_t.$$
 (2)

$$x_{ik} \leq Z_{ik}, \forall i \in V_t, k \in V_n.$$
 (3)

$$t_i^s \ge \max_{j \in pred_i} \{t_j^e + \sum_{(k,l) \in E_n} \frac{x_{ik} x_{jl} S_{ji}}{B_{kl}}\}, \forall i \in V_t, \quad (4)$$

$$t_j^s \ge x_{ik}x_{jk}y_{ij}t_i^e, \forall i \in V_t, \forall j \in V_t, \forall k \in V_n$$
 (5)

$$t_i^e = t_i^s + \sum_{k \in V_D} x_{ik} R_{ik}, \forall i \in V_t,$$
(6)

- Const. (1) ensures that each task is assigned to one compute node
- Const. (2) specifies the global scheduling sequence
- Const. (4) specifies the task and data dependency
- Const. (5) ensures the non-overlapped assumption
- Const. (6) expresses the relationship of start time and finish time

## Reformulate to ILP (1/2)

#### **Nonlinear Constraint**

$$t_i^s \ge \max_{j \in pred_i} \{ t_j^e + \sum_{(k,l) \in E_n} \frac{x_{ik} x_{jl} S_{ji}}{B_{kl}} \}, \forall i \in V_t,$$

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$$\tag{4}$$

• Since  $x_{ik}$  is a binary variable, define a new variable  $f_{ijkl}$  as:

$$f_{ijkl} = x_{ik}x_{jl}, \quad \forall (i,j) \in E_t, \forall (k,l) \in E_n,$$
 (7)

• which can be equivalently replaced by:

$$0 \le f_{ijkl} \le x_{ik}, \quad \forall (i,j) \in E_t, \forall (k,l) \in E_n, \tag{8}$$

$$x_{ik} + x_{jl} - 1 \le f_{ijkl} \le x_{jl}, \quad \forall (i,j) \in E_t, \forall (k,l) \in E_n.$$
 (9)

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 (9)

#### Conversion

Constraint (3) can be rewritten in a linear form as:

$$t_i^s \geq \max_{j \in \textit{pred}_i} \{t_j^e + \sum_{(k,l) \in \mathcal{E}_n} \frac{f_{ijkl} S_{ji}}{B_{kl}}\}, \forall i \in V_t,$$

6

## Reformulate to ILP (2/2)

#### **Nonlinear Constraint**

$$t_{j}^{s} \ge x_{ik}x_{jk}y_{ij}t_{i}^{e}, \forall i \in V_{t}, \forall j \in V_{t}, \forall k \in V_{n}$$
(5)

## Reformulate to ILP (2/2)

#### **Nonlinear Constraint**

$$t_j^s \ge x_{ik} x_{jk} y_{ij} t_i^e, \forall i \in V_t, \forall j \in V_t, \forall k \in V_n$$
 (5)

- Discretize continuous time structure to a sequence of time slots
- Define another variable  $x_{jk}^I$  to indicate whether task i is scheduled run on node k starting at slot I
- Replace Constraint (5) by:

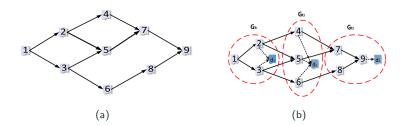
$$\sum_{k \in V_n} \sum_{l \in \mathcal{D}} x_{ik}^l = 1, \forall i \in V_t, \tag{10}$$

$$\sum_{l \in \mathcal{D}} x_{ik}^l = x_{ik}, \forall i \in V_t.$$
 (11)

$$t_i^s = \sum_{k \in V_n} \sum_{I \in \mathcal{D}} x_{ik}^I T_I, \forall i \in V_t.$$
 (12)

$$\sum_{i \in V_{t,V} = \max(1, l-P_{t+1})} \sum_{i=1}^{l} x_{ik}^{V} \leq 1, \forall i \in V_{t}, \tag{13}$$

## **Scaling Approach**

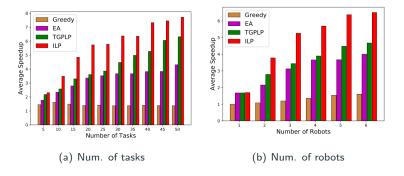


Task Graph Partitioning

- Partition task graph to smaller subgraphs
- Solve subproblems in which some parts of the problem have been fixed

# **Experimental Results**

## **Experimental Results**

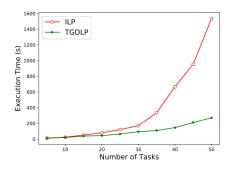


Performance Comparison

#### Observation

- ILP approach finds optimal solutions but TGPLP doesn't
- As the number of tasks/robots increases, TGPLP shows its advantage

## **Experimental Results**



Scalability Comparison

#### Observation

- It is quite time-consuming to get an optimal solution using ILP
- TGPLP approach, provides 'good-enough' results but within acceptable time

# Future Work

#### **Future Work**

- The method can be further improved by the introduction of heuristic based hot-start solutions;
- We intend to take into consideration some real-world applications under cloud robotic systems;
- And also develop online algorithms to adapt the scenarios with high dynamicity of moving robots.