Hypothesis Testing

Chung-I Li and Yu Shyr

Center for Quantitative Sciences
Vanderbilt University School of Medicine

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Learning Goals

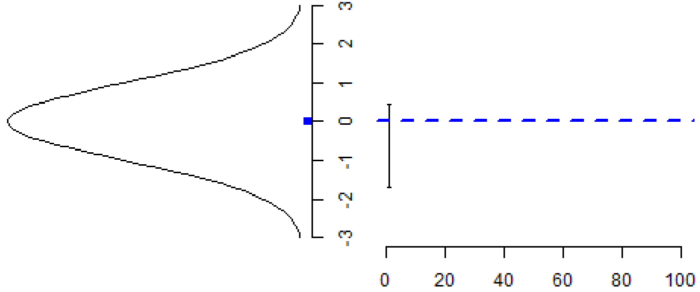
1. Learn the concepts of interval estimating and testing hypothesis via simulation
2. Learn how to conduct parametric and nonparametric tests
   - Single Population
     - testing one mean
     - testing one proportion
   - Two Populations
     - testing two means
     - testing two proportions
   - Paired Data
   - 3^+ groups

Concept of Interval Estimating

Let $X_1, \ldots, X_{30}$ be a random sample for normal distribution with mean $\mu = 0$ and standard deviation $\sigma = 1$. The 95% confidence interval estimator for μ is

$$[\bar{X} - 1.96\frac{1}{\sqrt{30}}, \bar{X} + 1.96\frac{1}{\sqrt{30}}]$$

where $\bar{X} = \sum_{i=1}^{30} X_i/30$.

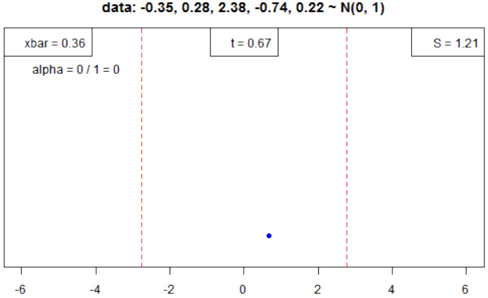


Concept of testing hypothesis

Let $X_1, \ldots, X_n$ be a random sample for a normal distribution $N(\mu, \sigma)$. Consider the testing

$$H_0 : \mu = 0 \text{ versus } H_1 : \mu = 2.$$

H_0 will be rejected if $|t = \frac{\bar{X}-0}{S/\sqrt{n}}| > 2.78$ where $\bar{X} = \sum_{i=1}^{n} X_i/n$ and $S^2 = \sum_{i=1}^{n}(X_i - \bar{X})^2/(n-1)$.



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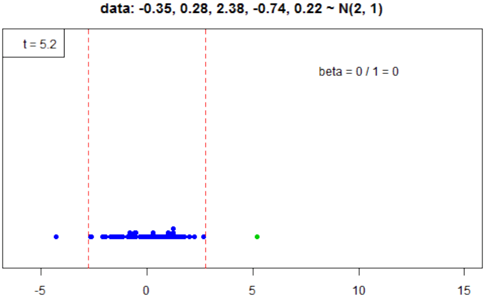


Learning Goals

parametric tests

- Single Population
  - testing one mean
  - testing one proportion
- Two Populations
  - testing two means
  - testing two proportions
- Paired Data
- 3^+ groups

Hypothesis Testing For Single Population Mean

```
> t.test(mydata$base_score, mu = 8)


One Sample t-test

data:  mydata$base_score
t = -1.748, df = 128, p-value = 0.0829
alternative hypothesis: true mean is not equal to 8
95 percent confidence interval:
 7.157 8.052
sample estimates:
mean of x
    7.605
```

Because the p-value 0.0829 is greater than 0.05, we cannot conclude that mean is not equal to 8.

Hypothesis Testing For Single Population Proportion

```
> prop.test(table(mydata$smoker), p = 0.6)


1-sample proportions test with continuity correction

data:  table(mydata$smoker), null probability 0.6
X-squared = 10.35, df = 1, p-value = 0.001295
alternative hypothesis: true p is not equal to 0.6
95 percent confidence interval:
 0.3702 0.5471
sample estimates:
     p
0.4574
```

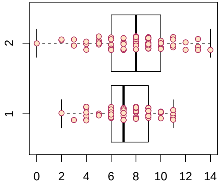
Because the p-value is less than 0.05, we conclude that the proportion is not equal to 0.6.

Hypothesis Testing For Two Population Means

```
> t.test(base_score ~ randomization, data = mydata)

Welch Two Sample t-test
data:  base_score by randomization
t = -1.46, df = 124.4, p-value = 0.147
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -1.5302  0.2313
sample estimates:
mean in group 1 mean in group 2
          7.262           7.912
```

Because p-value is greater than 0.05, we can't conclude that the mean difference bewteen two groups are different.



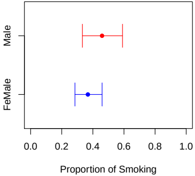
Hypothesis Testing For Two Population Proportions

Chi-Square Test

```
> prop.test(table(mydata$sex, mydata$smoker))

2-sample test for equality of proportions with continuity
correction
data:  table(mydata$sex, mydata$smoker)
X-squared = 0.6757, df = 1, p-value = 0.4111
alternative hypothesis: two.sided
95 percent confidence interval:
 -0.3049  0.1094
sample estimates:
prop 1 prop 2
0.4286 0.5263
```

Because the p-value is greater than 0.05, we can't conclude that proportions are different.



Hypothesis Testing For Two Population Proportions

Fisher's Exact Test

```
> fisher.test(table(mydata$sex, mydata$smoker))


Fisher's Exact Test for Count Data

data:  table(mydata$sex, mydata$smoker)
p-value = 0.3374
alternative hypothesis: true odds ratio is not equal to 1
95 percent confidence interval:
 0.2939 1.5480
sample estimates:
odds ratio
    0.6771
```

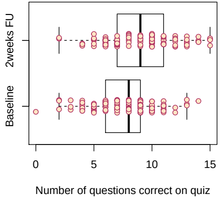
Because the p-value is greater than 0.05, we can't conclude that proportions are different.

Hypothesis Testing For Paired Data

```
> t.test(mydata$base_score, mydata$score1, paired = TRUE)

Paired t-test
data:  mydata$base_score and mydata$score1
t = -6.061, df = 128, p-value = 1.411e-08
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -1.6041 -0.8145
sample estimates:
mean of the differences
                 -1.209
```

Because the p-value is less than 0.05, we conclude that the mean difference is not equal to 0.



Hypothesis Testing For Paired Data

McNemar's Chi-squared Test

```
> mytable <- table(mydata$base_score >= 6, mydata$score1 >= 6, dnn =
c("Baseline",
+      "2weeks FU"))
> mytable

        2weeks FU
Baseline FALSE TRUE
   FALSE    10   15
   TRUE      3  101

> mcnemar.test(mytable)


McNemar's Chi-squared test with continuity correction

data:  mytable
McNemar's chi-squared = 6.722, df = 1, p-value = 0.009522
```

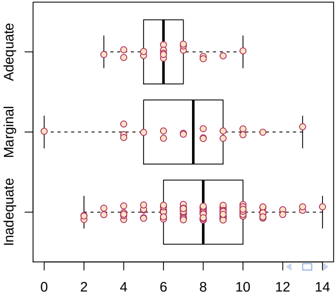
Because the p-value is less than 0.05, we conclude that proportions are not equal.

Hypothesis Testing For 3^+ Groups

```
> fit <- aov(base_score ~ pt_literacy, data = mydata)
> summary(fit)

             Df Sum Sq Mean Sq F value Pr(>F)
pt_literacy   1     42    42.3    6.69  0.011 *
Residuals   127    803     6.3
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Because the p-value is less than 0.05, we conclude that the means among three groups are not equal.



Quick Review

Parametric tests

- Single Population
  - testing one mean, t.test()
  - testing one proportion, prop.test()
- Two Populations
  - testing two means, t.test()
  - testing two proportions, prop.test()
- Paired Data
  - testing two means, t.test(,paired=TRUE)
  - testing two proportions, mcnemar.test()
- 3^+ groups, aov()

Exercise

Excerise

- Compare difference in chew_score (raw score on the 3 subjective health literacy questions) between male and female (sex).
- Compare difference in the number of questions correct on quiz between 2-week follow-up (score1) and 6-week follow-up (score2).
- Compare difference in chew_score among three levels of health literacy (pt_literacy).

Learning Goals

Nonparametric tests

- Two Populations means
- Paired data
- 3^+ groups

Hypothesis Testing For Two Population Mean

Wilcoxon rank sum test

```
> wilcox.test(base_score ~ randomization, data = mydata)


Wilcoxon rank sum test with continuity correction

data:  base_score by randomization
W = 1737, p-value = 0.1097
alternative hypothesis: true location shift is not equal to 0
```

Because the p-value is greater than 0.05, we can't conclude that the mean bewteen two groups are different.

Hypothesis Testing For Paired Data

```
> wilcox.test(mydata$base_score, mydata$score1, paired = TRUE)


Wilcoxon signed rank test with continuity correction

data:  mydata$base_score and mydata$score1
V = 1122, p-value = 1.472e-07
alternative hypothesis: true location shift is not equal to 0
```

Because the p-value is less than 0.05, we conclude that the mean difference bewteen two groups are different is not equal to 0.

Hypothesis Testing For 3^+ Groups

Kruskal-Wallis rank sum test

```
> kruskal.test(base_score ~ pt_literacy, data = mydata)


Kruskal-Wallis rank sum test

data:  base_score by pt_literacy
Kruskal-Wallis chi-squared = 7.877, df = 2, p-value = 0.01948
```

Because the p-value is less than 0.05, we conclude that the means among three groups are not equal.

Overview

Parametric and Nonparametric tests

- Single Population
  - testing one mean, t.test(), wilcox.test()
  - testing one proportion, prop.test()
- Two Populations
  - testing two means, t.test(), wilcox.test()
  - testing two proportions, prop.test()
- Paired Data,
  - two means,t.test(,paired=TRUE), wilcox.test(,paired=TRUE)
  - two proportions, mcnemar.test()
- 3^+ groups, aov(), kruskal.test()