Pendulum LQR Control

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State variables:

$x_1 = \theta$

$x_2 = \dot{\theta}$

Control input:

$u = \tau$

$\boxed{\tau : \text{torque}}$

System dynamics:

$\dot{x_1} = x_2$

$\dot{x_2} = -\frac{g}{l} sin(x_1) - \frac{k}{m} x_2 + \frac{1}{ml^2} u$

$\boxed{g : \text{gravitational acceleration}}$

$\boxed{l : \text{rod length}}$

$\boxed{k : \text{air drag coefficient}}$

$\boxed{m : \text{bob mass}}$

Linearized model:

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$A = \begin{pmatrix} \frac{\partial \dot{x_1}}{\partial x_1} & \frac{\partial \dot{x_1}}{\partial x_2} \\ \frac{\partial \dot{x_2}}{\partial x_1} & \frac{\partial \dot{x_2}}{\partial x_2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{g}{l} cos(x_1) & -\frac{k}{m} \end{pmatrix}$$

$$B = \begin{pmatrix} \frac{\partial \dot{x_1}}{\partial u} \\ \frac{\partial \dot{x_2}}{\partial u} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{ml^2} \end{pmatrix}$$

$\dot{x} = Ax + Bu$

Linear Quadratic Regulator:

$minimize\ J(x, u) = \int_0^\infty (\tilde{x}^T Q \tilde{x} + u^T R u)\ dt$

$\tilde{x} = x - x_d$

$u = -K\tilde{x}$

$K = R^{-1} B^T X$

$\boxed{K : \text{Optimal feedback gain}}$

CARE (Continuous Algebraic Riccati Equation):

$A^T X + XA + XBR^{-1}B^T X + C^T QC$

$\boxed{\text{Solve for } X}$