

Fundamentals of Solid State Physics

Materials and Crystal Structures

Xing Sheng 盛 兴



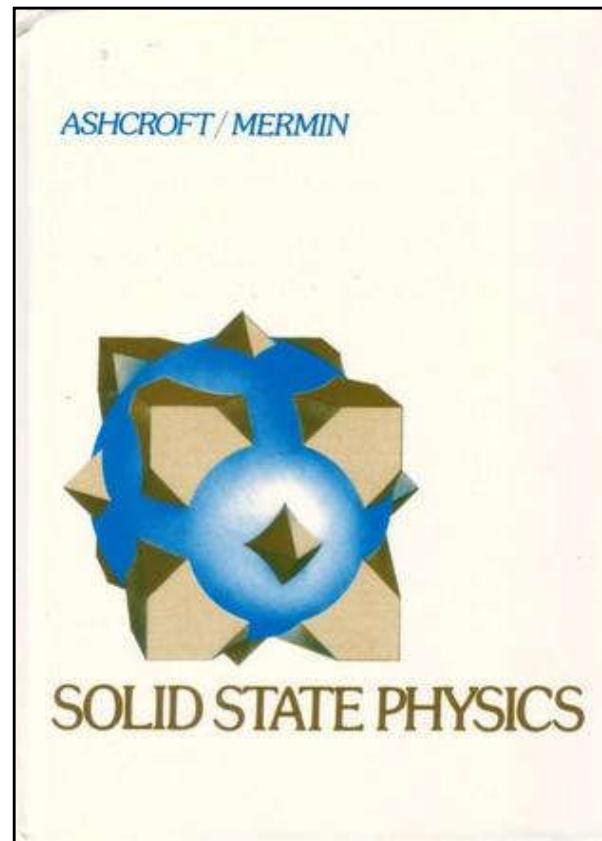
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This Class

- Introduction (Week 1)
- Materials and Crystal Structures (Week 2–3)
 - Crystal structures, lattices
 - Reciprocal space, Brillouin zones
 - Materials Characterization: Wave diffraction, the Bragg law
- Electronic Properties (Week 4–12)
- Thermal Properties (Week 13)
- Optical Properties (Week 14)
- Magnetic Properties (Week 15)

Further Reading

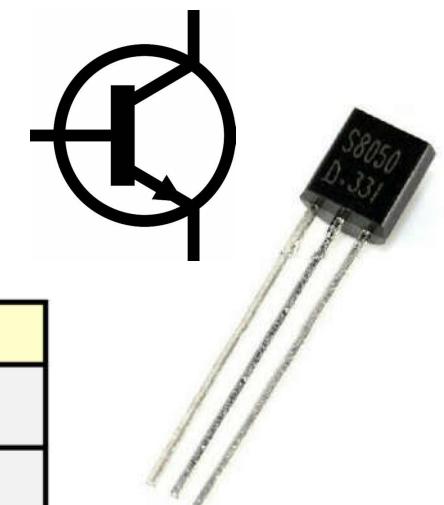
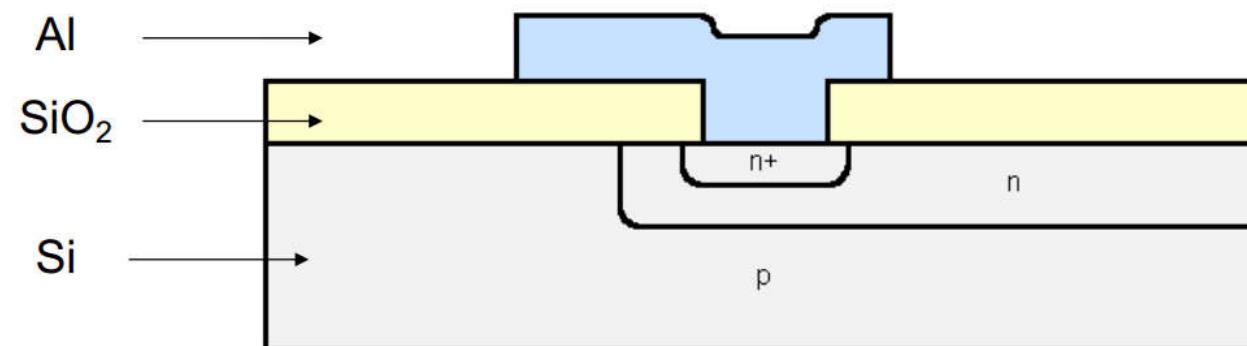
- **Ashcroft & Mermin, Chapter 4, 5, 6**



Importance of Materials

CMOS transistor

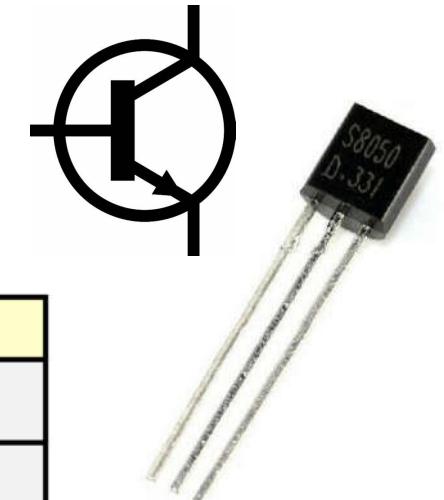
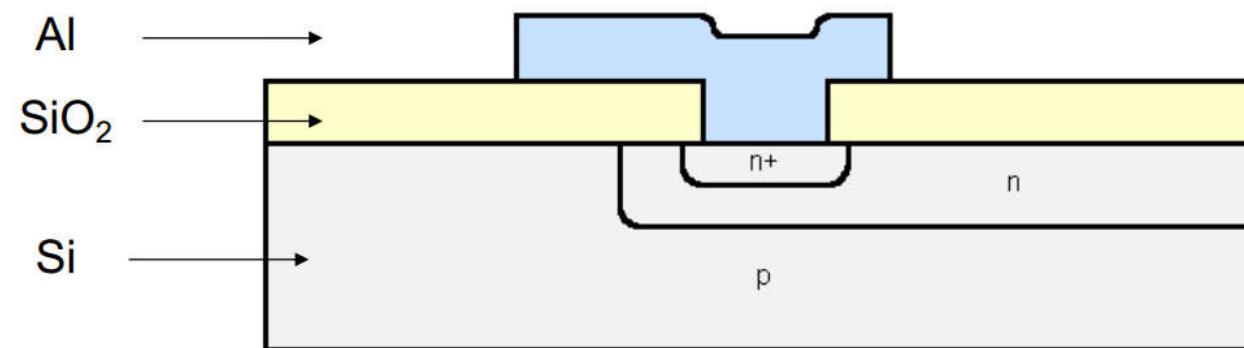
- Complementary Metal-Oxide-Semiconductor



Importance of Materials

CMOS transistor

- Complementary Metal-Oxide-Semiconductor



Metal



SiO₂



Silicon

Importance of Materials



Metal



SiO_2



Silicon

- Crystal Structures
 - polycrystalline, amorphous, single crystalline
- Electronics
 - conductor, insulator, semiconductor
- Optics (in the visible range)
 - reflective, transparent, absorbing

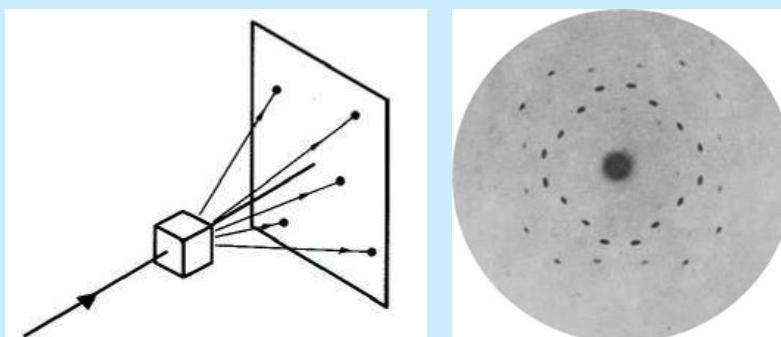
History of Crystal Structures

Discovery of X-ray



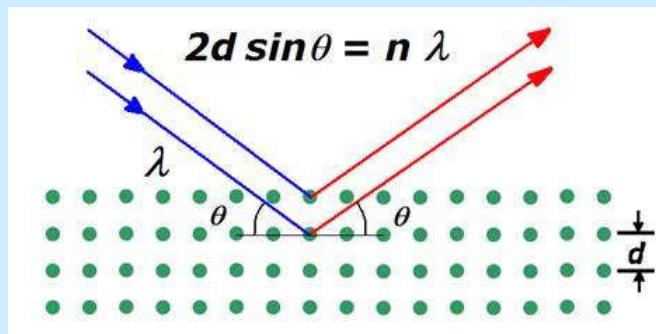
W. Rontgen (伦琴)
Nobel Prize in 1901

X-ray diffraction of crystals



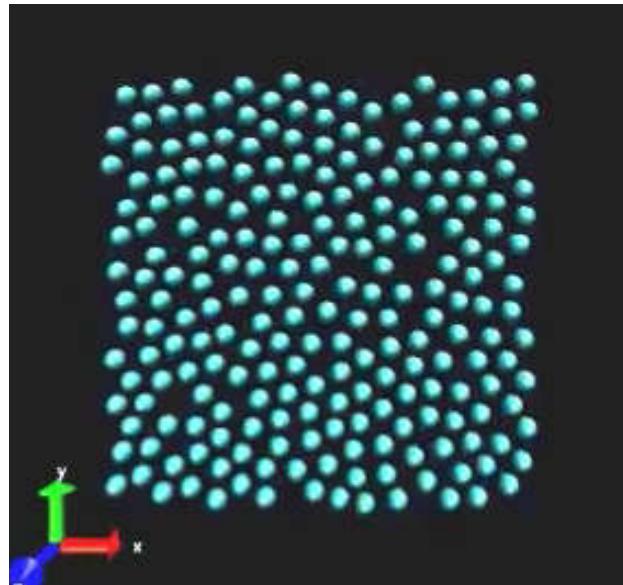
M. von Laue (劳厄)
Nobel Prize in 1914

Bragg's law

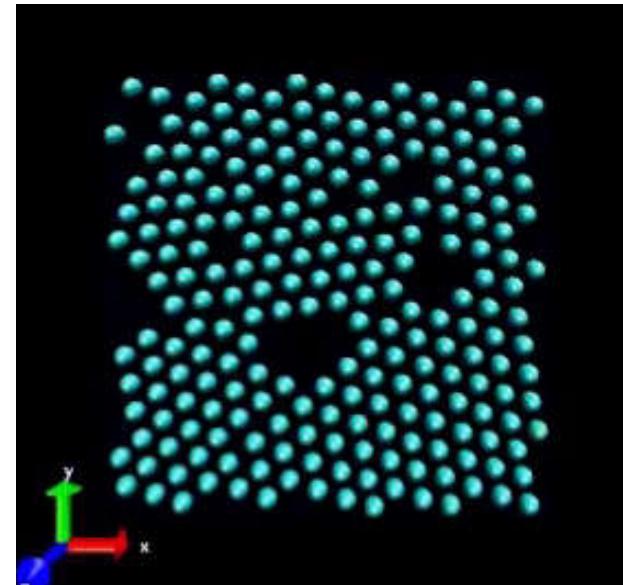


Bragg & Bragg
(布拉格父子)
Nobel Prize in 1915

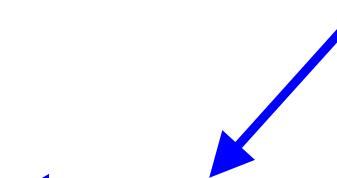
Crystal Structures



Video



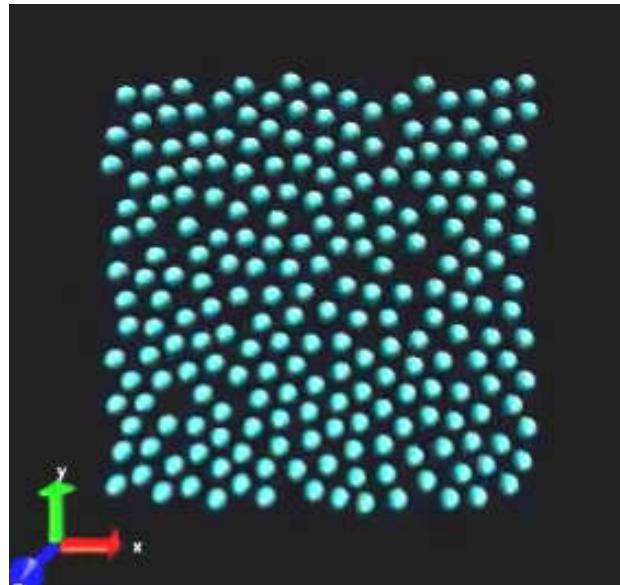
molecular dynamics simulation



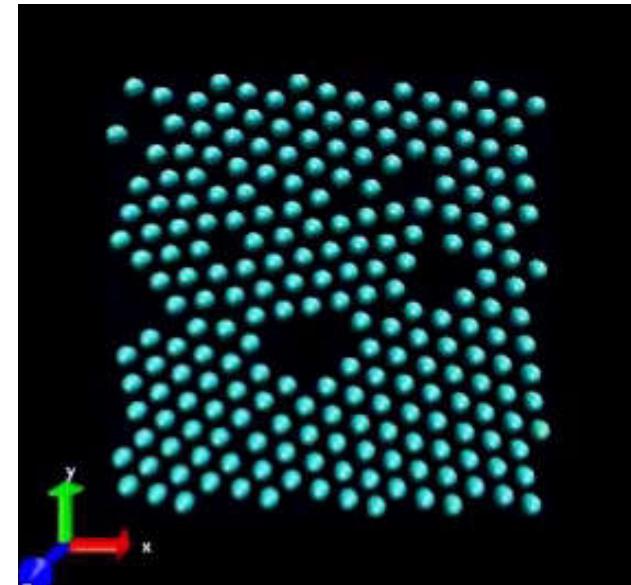
crystal structure:
*ordered, periodic arrays of atoms
(with translational symmetry)*

*Crystal is a microscopic (微观) concept,
not a macroscopic (宏观) concept.*

Crystal Structures



Video



our macroscopic world



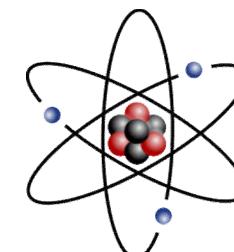
?

Chemical Bonding 化学键

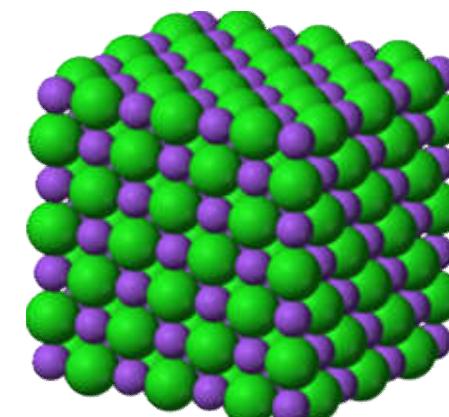
- Solids are formed by chemical bonding between atoms

- Metallic Bonding 金属键
- Ionic Bonding 离子键
- Covalent Bonding 共价键
- Van der Waals Bonding 范德华键
- Hydrogen Bonding 氢键
- ...

atom



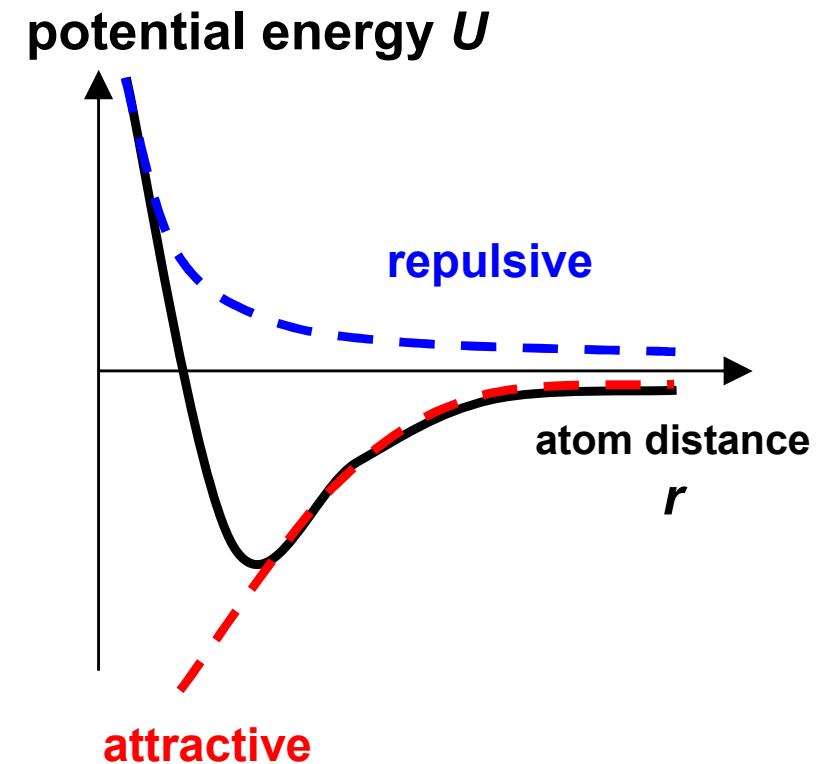
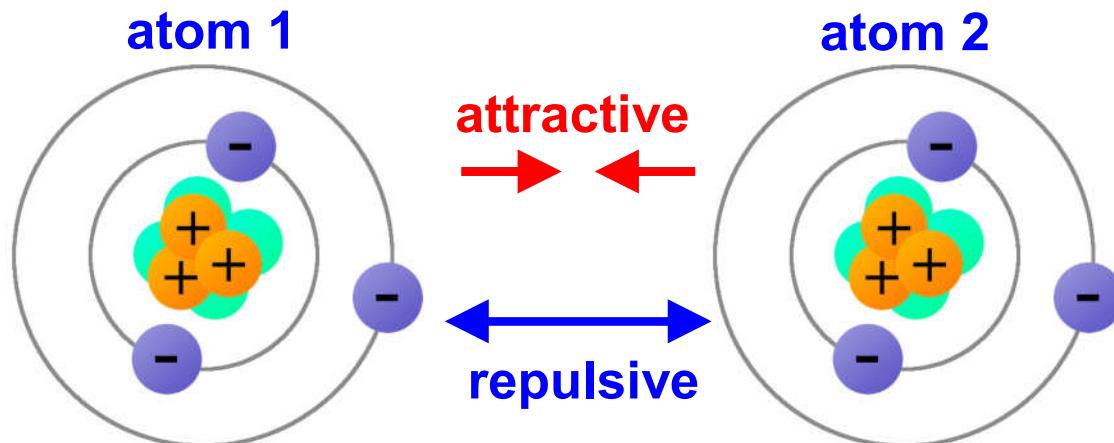
solid



- Valence electrons form bonds

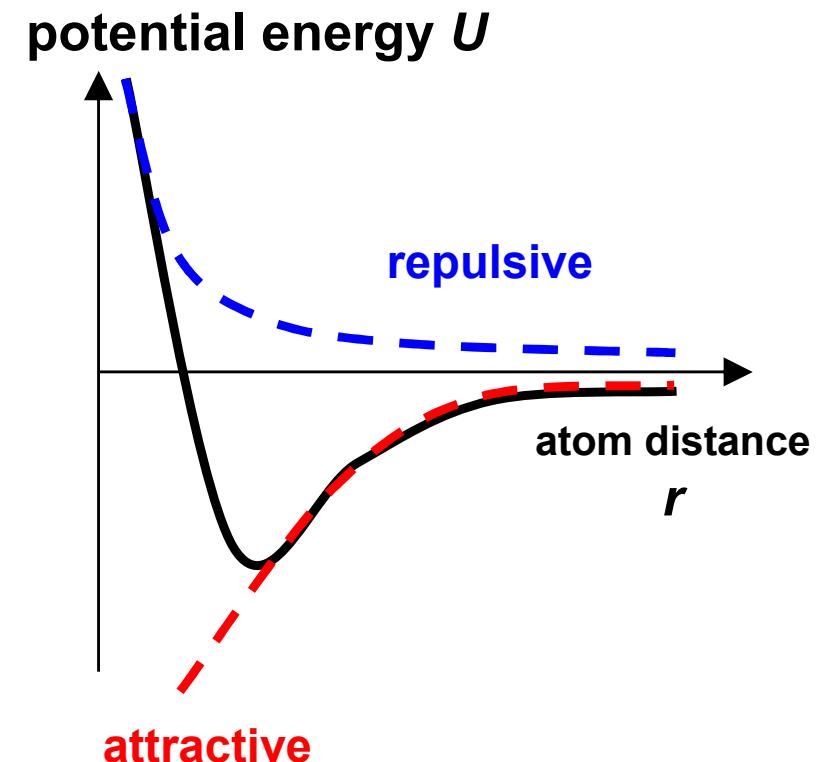
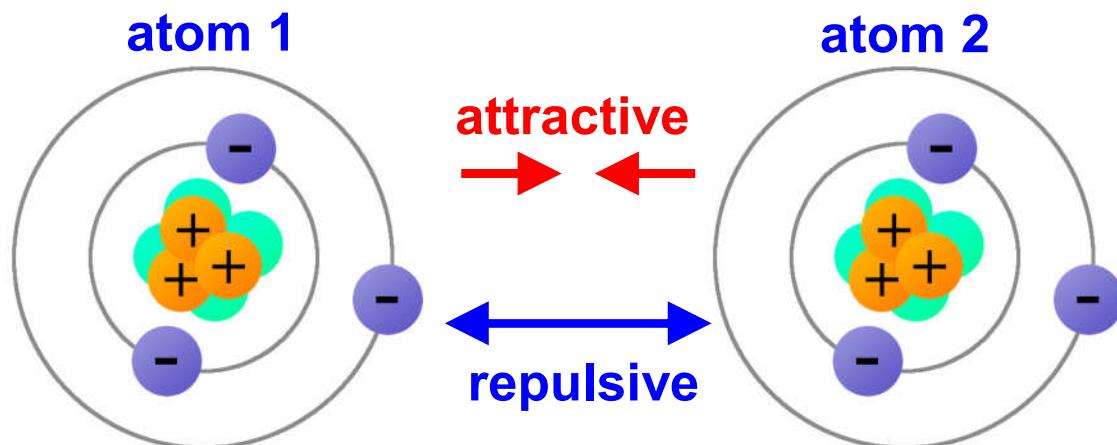
- Silicon (Si) $[1s^2 2s^2 2p^6] \underline{3s^2} 3p^2$

Atomic Interactions



- **Interatomic Potential U**
 - attraction: electrostatic (+ -)
 - repulsion: electrostatic (+ + / - -)
and Pauli exclusion principle

Atomic Interactions

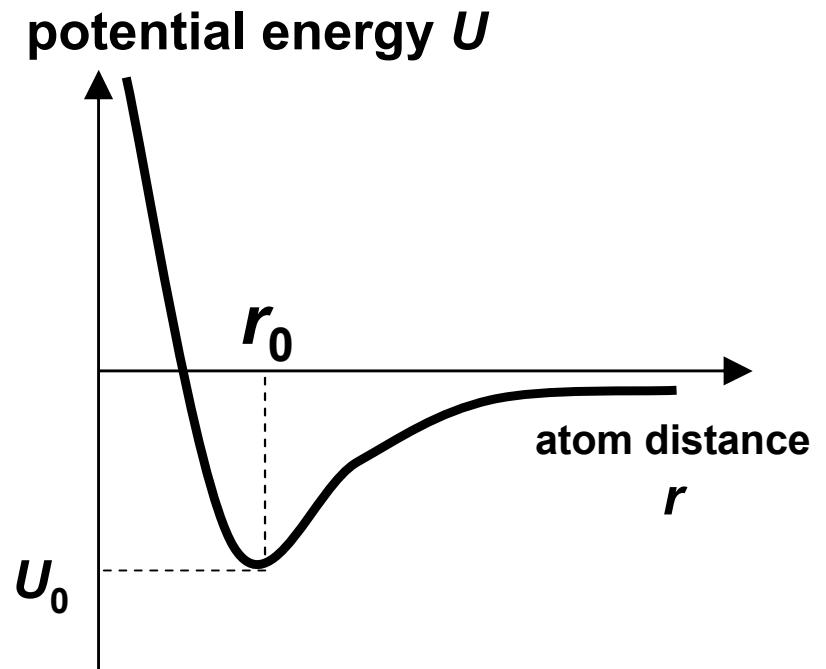
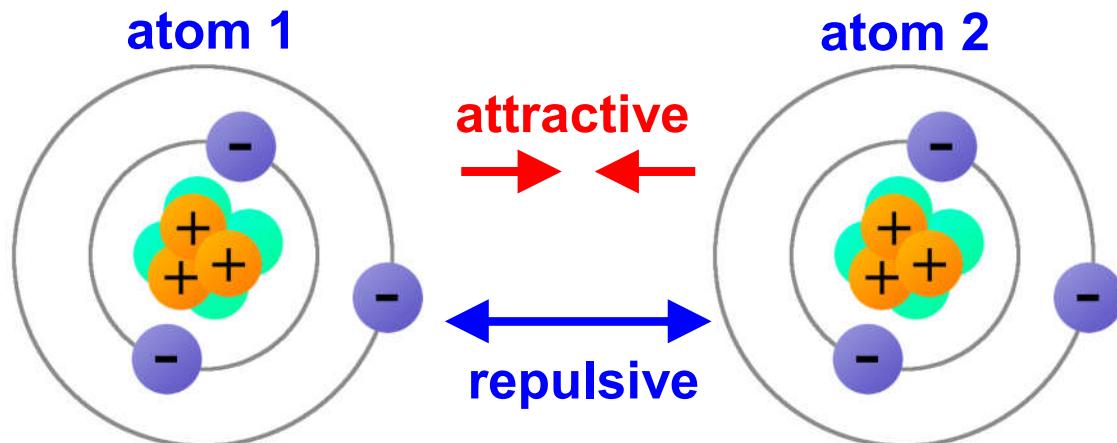


- Interatomic Potential U

$$U(r) = U_{\text{repulsion}}(r) - U_{\text{attraction}}(r)$$

U - potential energy (J, eV)
 r - atomic distance (nm, Å) 14

Atomic Interactions



- **Interatomic Potential U**

$-U_0$ - cohesive energy (结合能)
lowest energy state

$$U(r) = U_{\text{repulsion}}(r) - U_{\text{attraction}}(r)$$

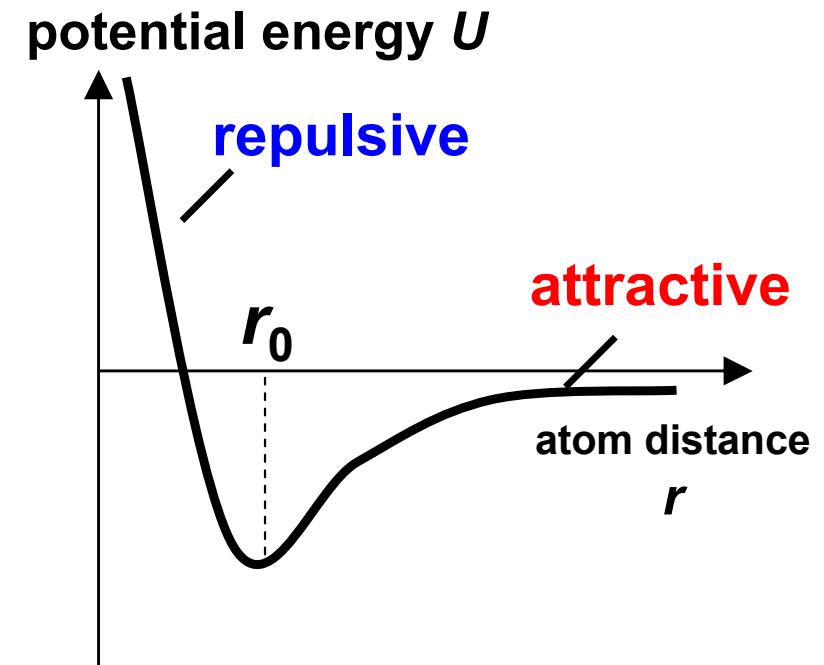
U - potential energy (J, eV)
 r - atomic distance (nm, Å) 15

Interatomic Potential: Examples

- Lennard-Jones (L-J)

$$U(r) = \frac{A}{r^{12}} - \frac{B}{r^6}$$

repulsive attractive



- Buckingham Potential

$$U(r) = A \exp\left(-\frac{r}{\rho}\right) - \frac{B}{r^6}$$

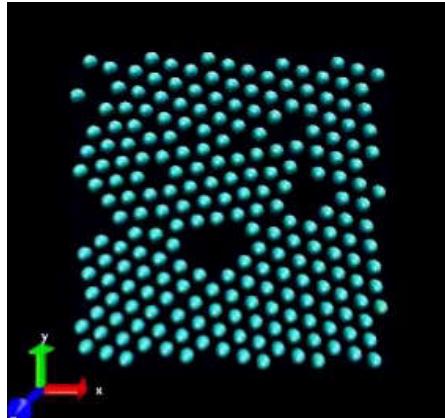
repulsive attractive

- Morse Potential

$$U(r) = D \left(e^{-2a(r-r_0)} - 2e^{-a(r-r_0)} \right)$$

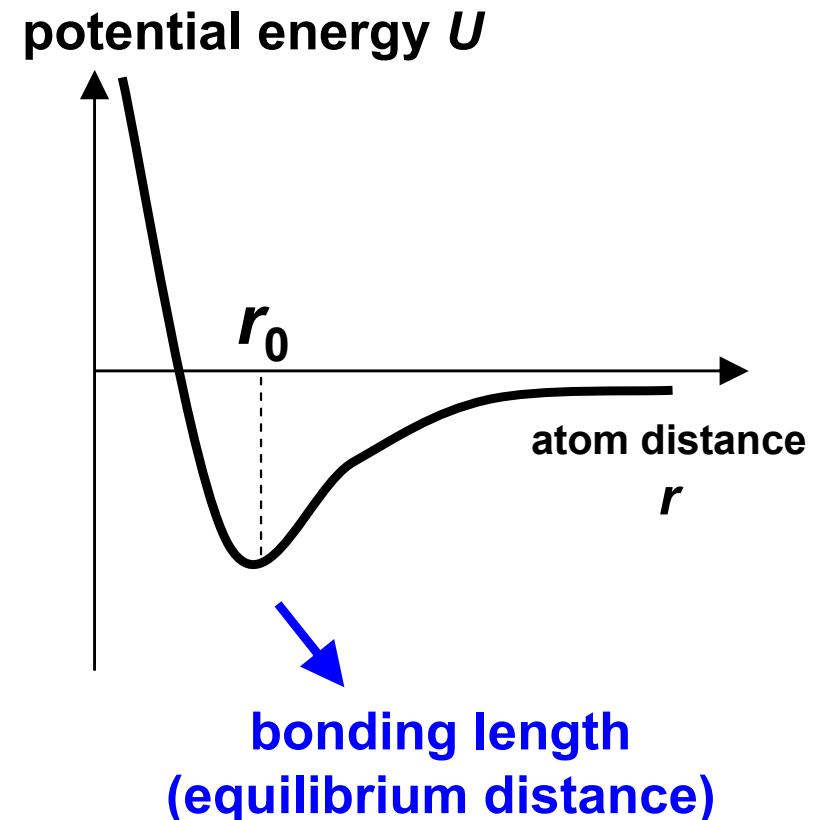
repulsive attractive

Atomic Interactions



optimal distance r_0 :
periodicity of crystals

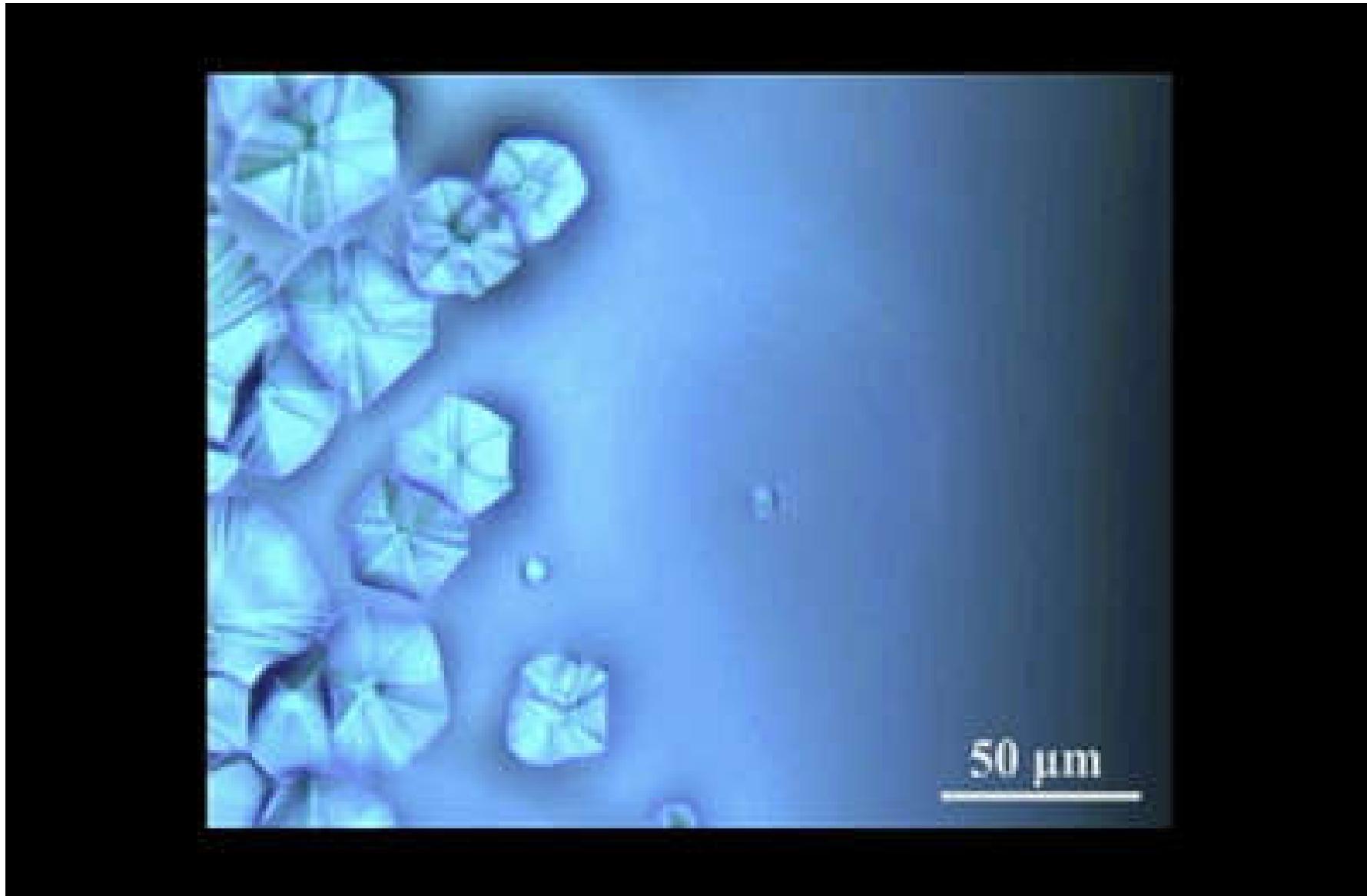
■ Interatomic Potential U



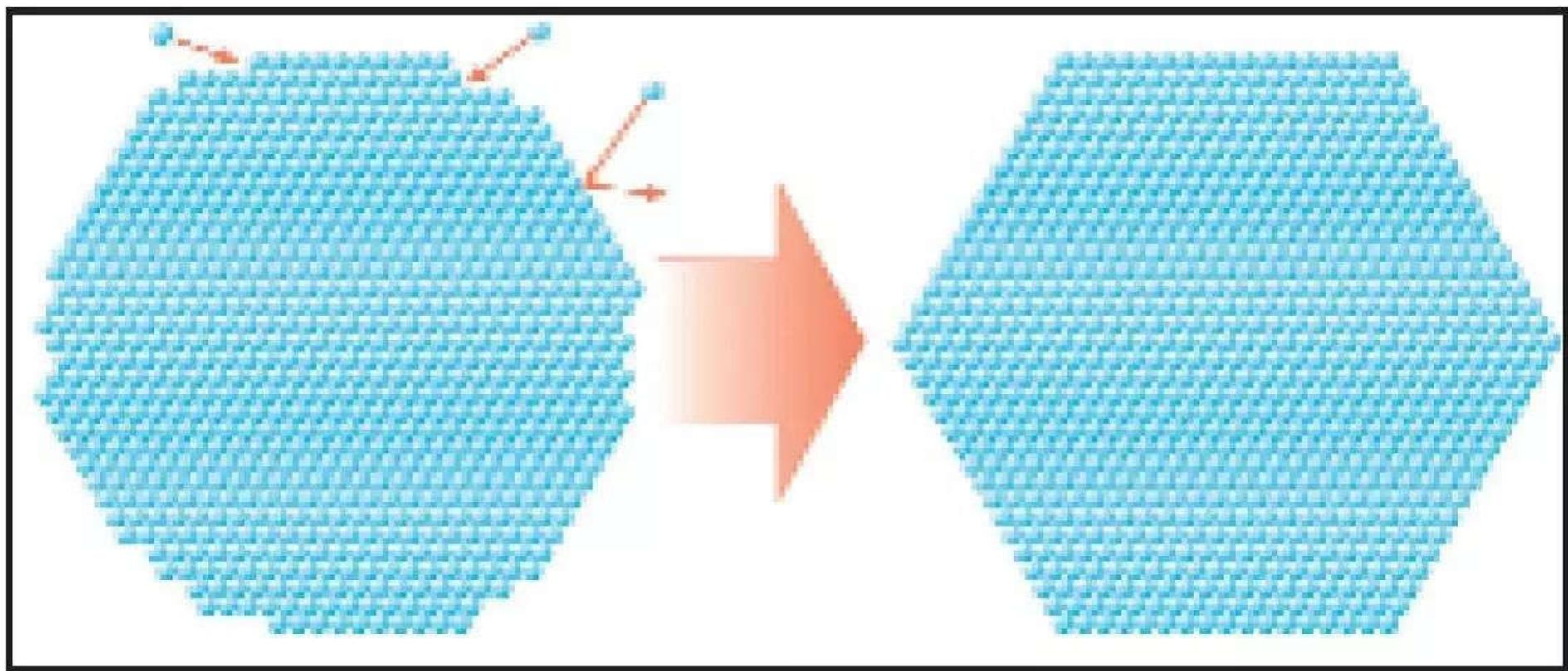
$$U(r) = U_{\text{repulsion}}(r) - U_{\text{attraction}}(r)$$

U - potential energy (J, eV)
 r - atomic distance (nm, Å) 17

Macroscopic Crystals



Macroscopic Crystals



Macroscopic Crystals



金刚石 diamond (C)



石英 quartz (SiO_2)



红宝石 ruby ($\text{Al}_2\text{O}_3:\text{Cr}$)



食盐 salt (NaCl)

Macroscopic Crystals



Silicon Crystal



石英 quartz (SiO_2)

Crystal Cave, Mexico

Crystal Structures

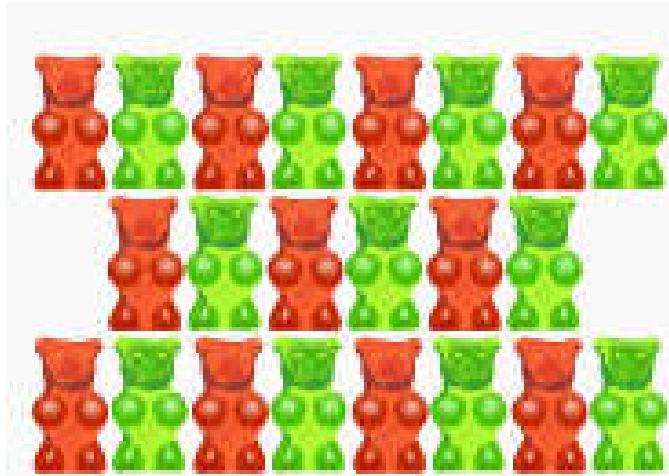
- **Vectors**
 - **Translational vectors**
 - **Primitive vectors**
- **Cells**
 - **Unit Cell, Conventional Cell**
 - **Primitive Cell, Wigner-Seitz Cell**
- **Lattice**
 - **Bravais Lattice**
 - **SC, BCC, FCC, HCP, ...**
- **Packing**
 - **Atomic Packing Factor**
- **Miller Index**
- **Coordination Number**

Crystal, Basis and Lattice

- Crystal 晶体
 - real material structures - physical concept, finite

- Basis 基元
 - single unit of an group of atoms

- Lattice 晶格/点阵
 - arrangement of points - mathematical concept, infinite
 - one point can represent *one or more* atoms

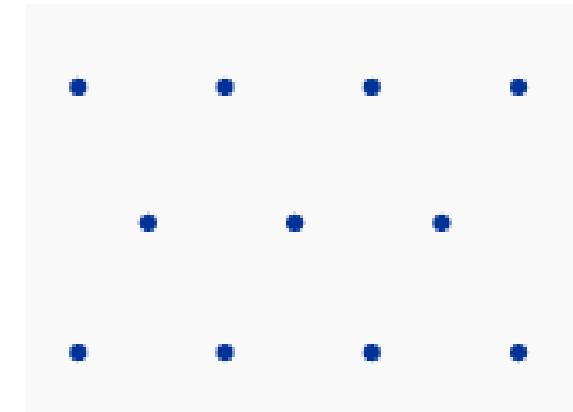


Crystal

=



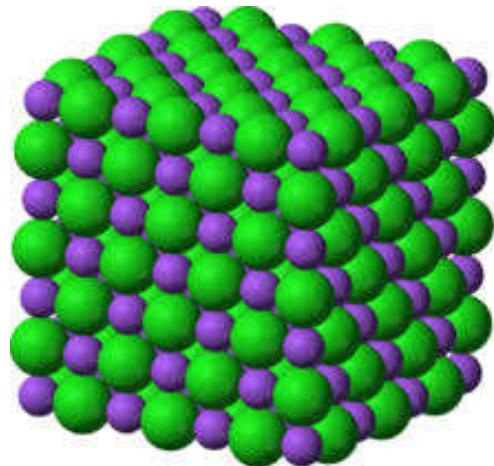
Basis



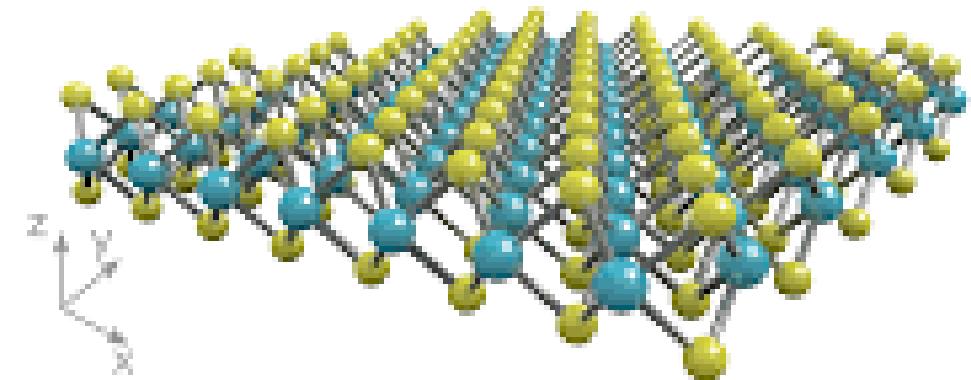
Lattice

Crystal, Basis and Lattice

Crystal

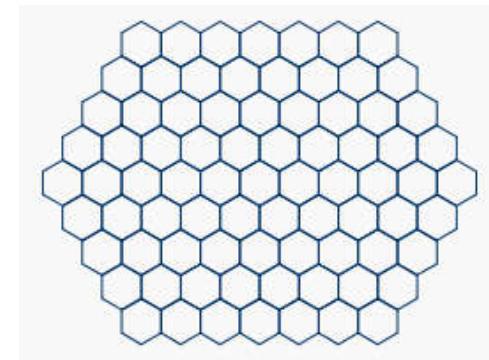
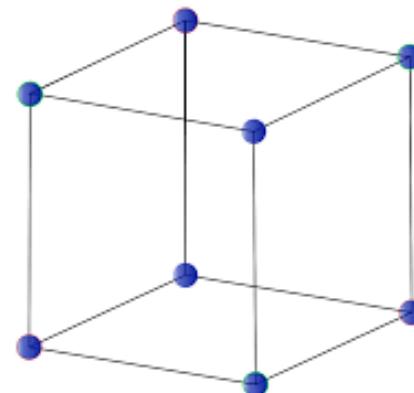


NaCl



monolayer MoS₂

Lattice



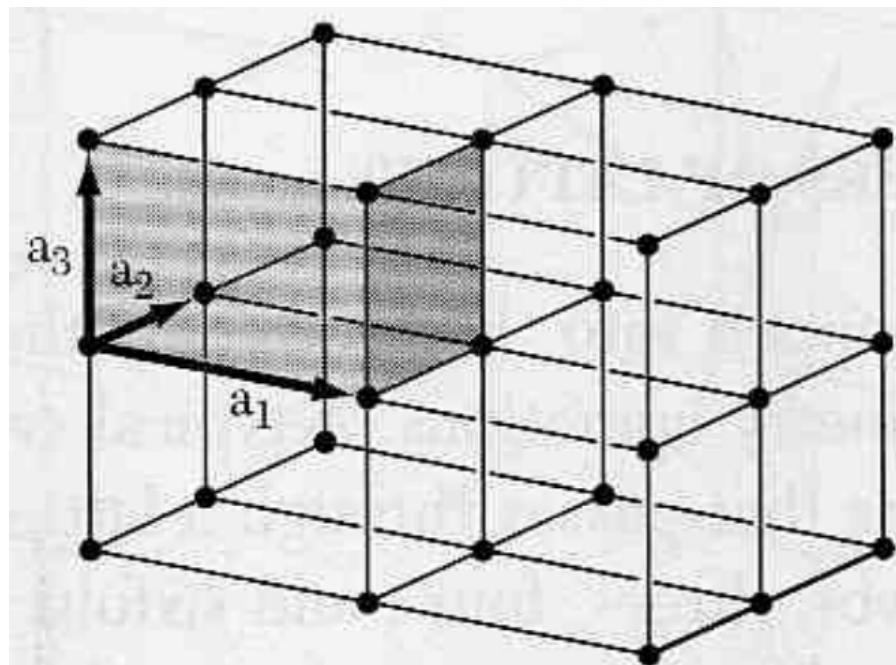
Bravais Lattice 布拉菲点阵

- Each point is ***exactly*** the same
- Position of each point

$$\mathbf{R} = n_1 \mathbf{a}_1 + n_2 \mathbf{a}_2 + n_3 \mathbf{a}_3$$

n_1, n_2, n_3 cover
all the integers

- $(\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3)$ primitive vectors 基矢量



translational symmetry
平移对称性

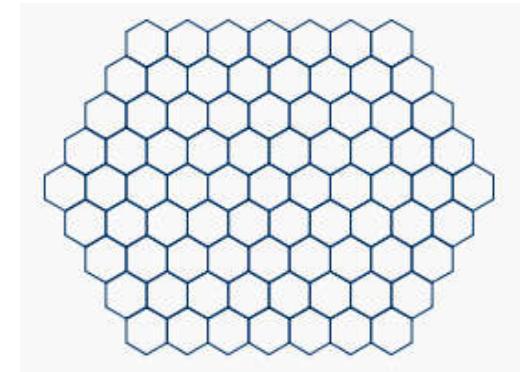
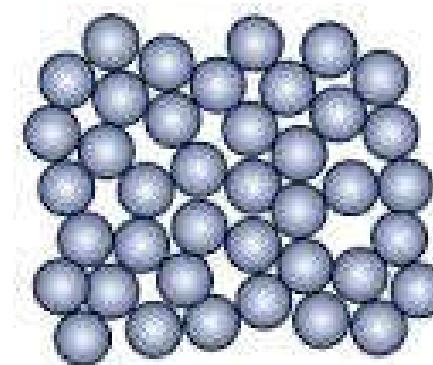
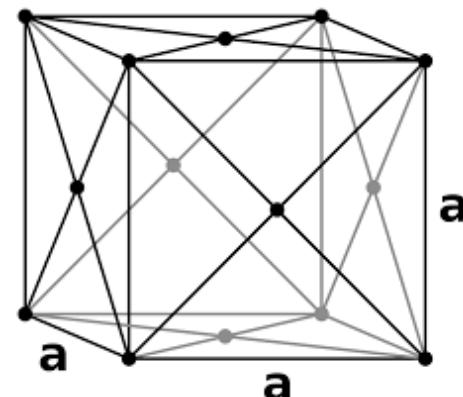
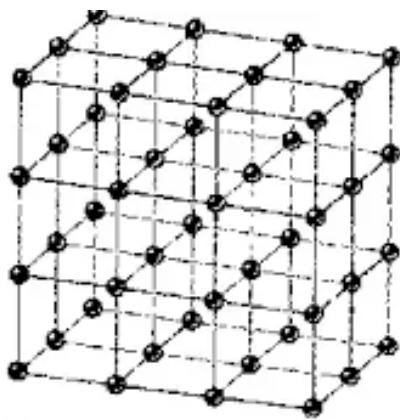
Bravais Lattice 布拉菲点阵

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$$\mathbf{R} = n_1 \mathbf{a}_1 + n_2 \mathbf{a}_2 + n_3 \mathbf{a}_3$$

n_1, n_2, n_3 cover
all the integers

Q: which is Bravais lattice, which is not?



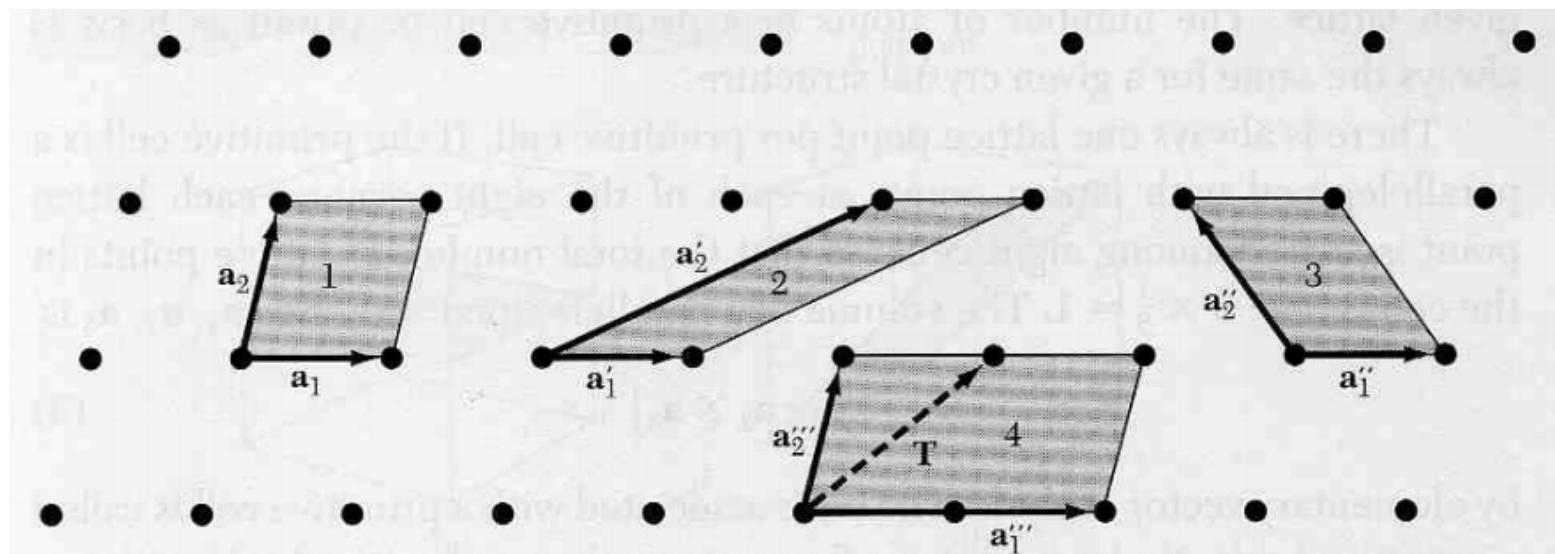
Bravais Lattice 布拉菲点阵

- Each point is *exactly* the same
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$$\mathbf{R} = n_1 \mathbf{a}_1 + n_2 \mathbf{a}_2 + n_3 \mathbf{a}_3$$

n_1, n_2, n_3 cover
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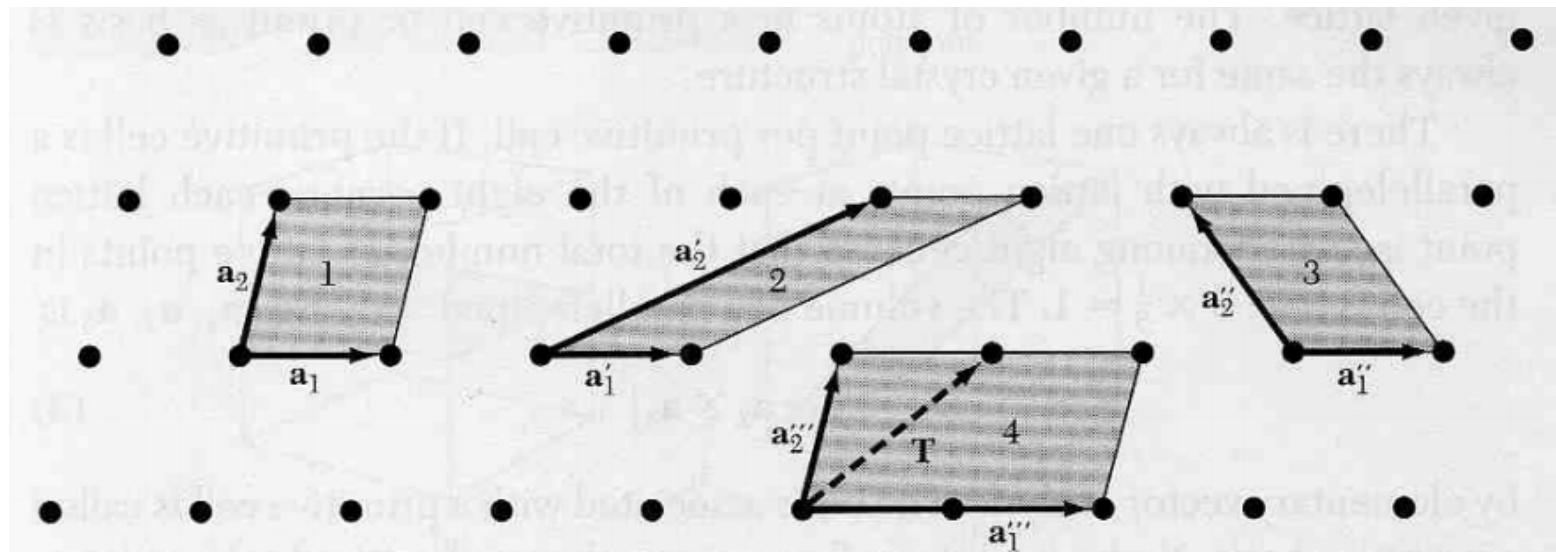
- $(\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3)$ primitive vectors 基矢量



Q: which are primitive vectors, which are not?

Lattice Cells 晶胞

- Repetitive Units to form the infinite lattice

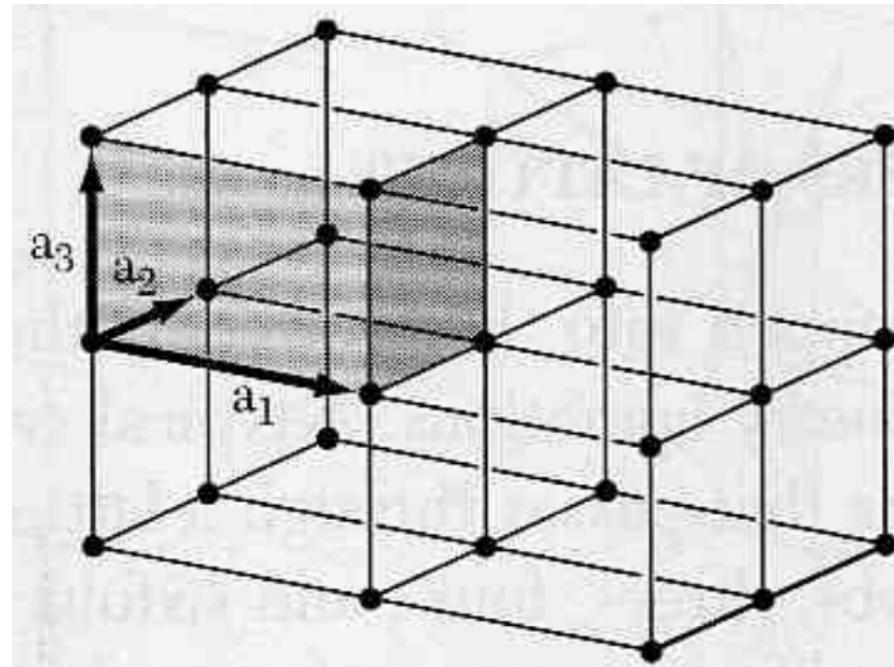


Lattice Cells 晶胞

- Primitive Cell 原胞/素胞

- A cell with the *smallest* volume
- A cell with *only one* lattice point

$$V_R = \mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{a}_3)$$

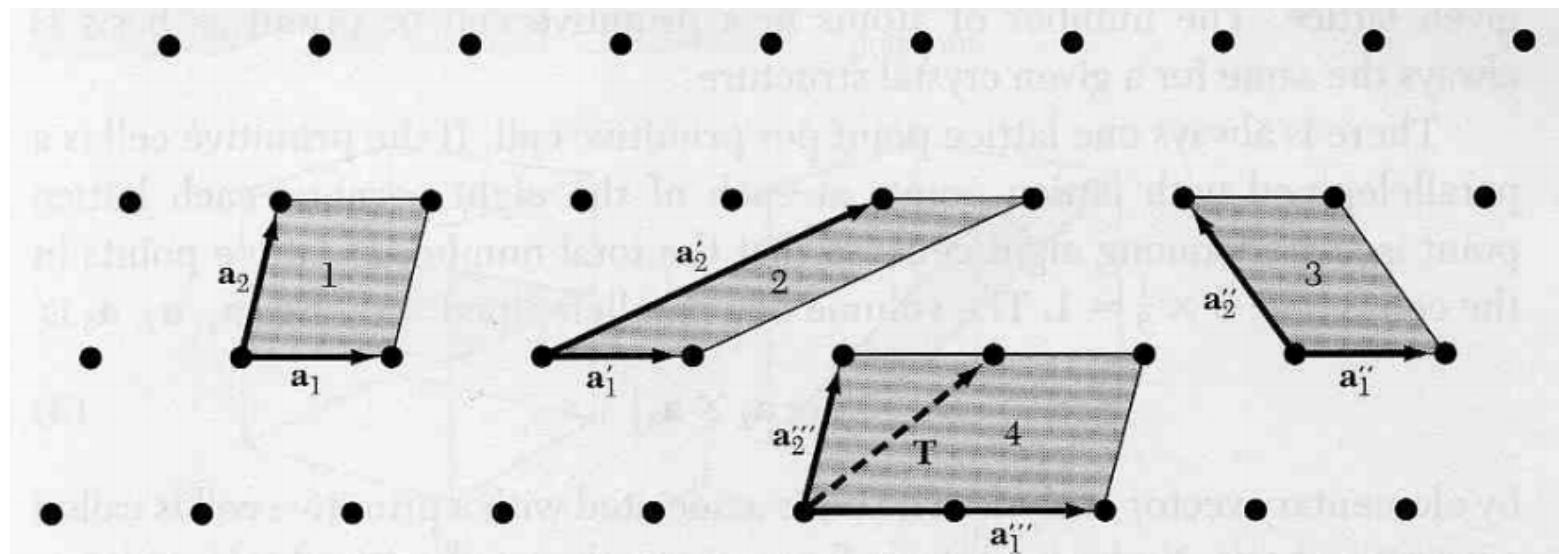


Q: how many atoms are in this primitive cell?

Lattice Cells 晶胞

■ Primitive Cell 原胞/素胞

- A cell with the *smallest* volume
- A cell with *only one* lattice point

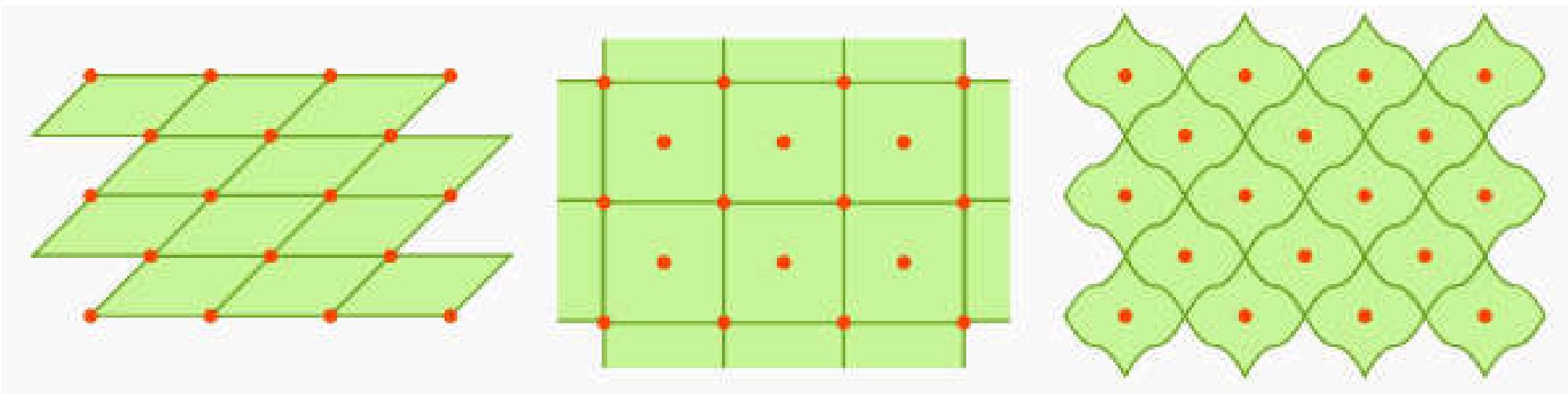


Q: which are primitive cells, which are not?

Lattice Cells 晶胞

- Primitive Cell 原胞/素胞

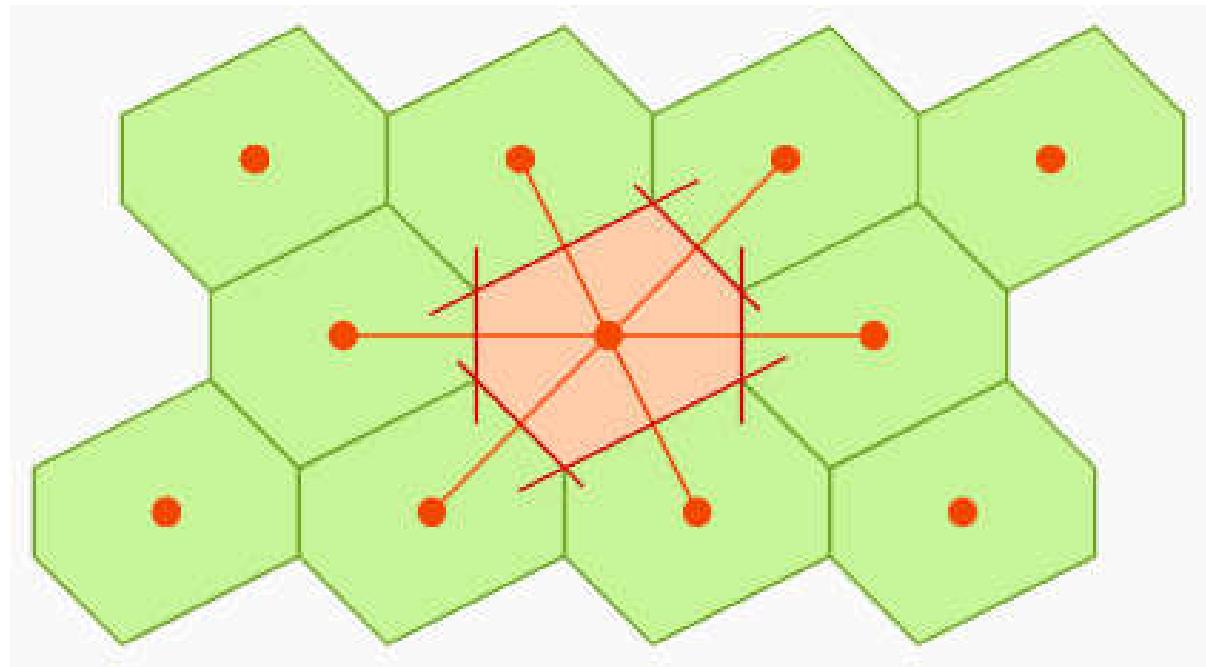
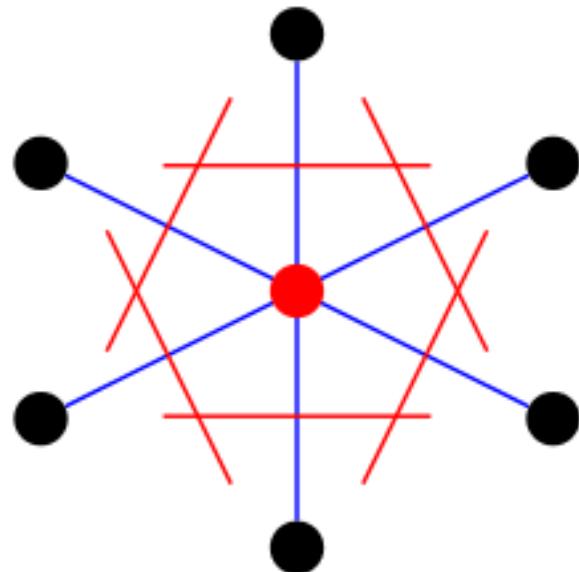
- A cell with the ***smallest*** volume
 - A cell with ***only one*** lattice point



Q: which are primitive cells, which are not?

Lattice Cells 晶胞

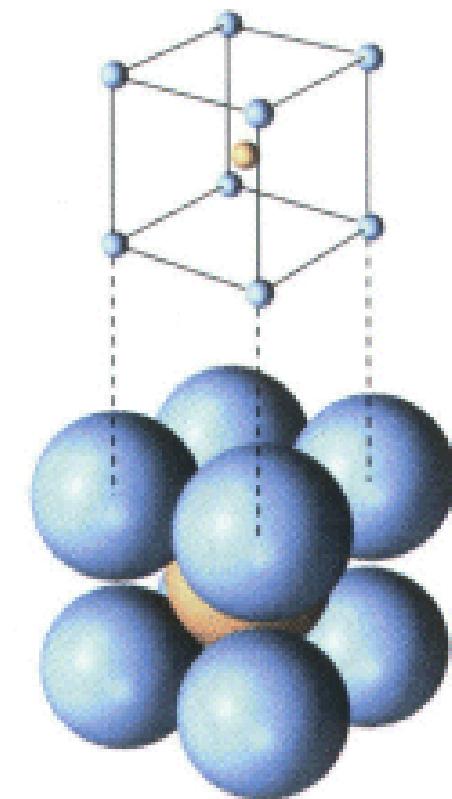
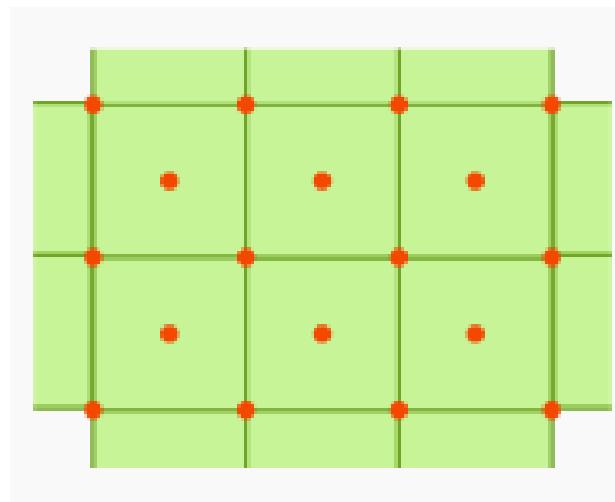
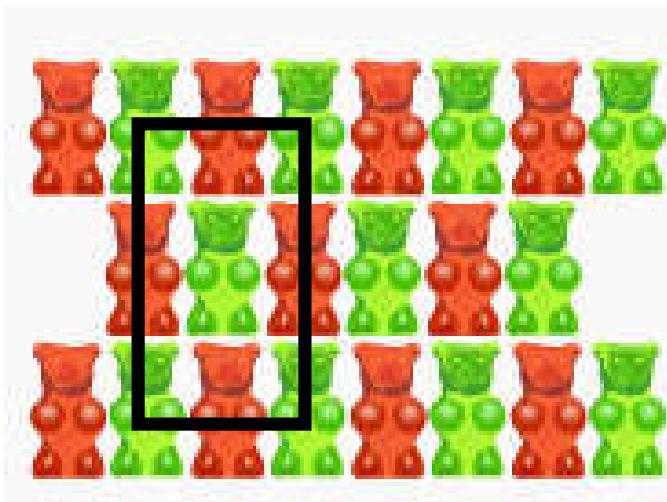
- Wigner-Seitz cell
 - A very special primitive cell



1. *draw lines to connect nearby points*
2. *at the midpoint and normal to these lines
draw new lines/planes*

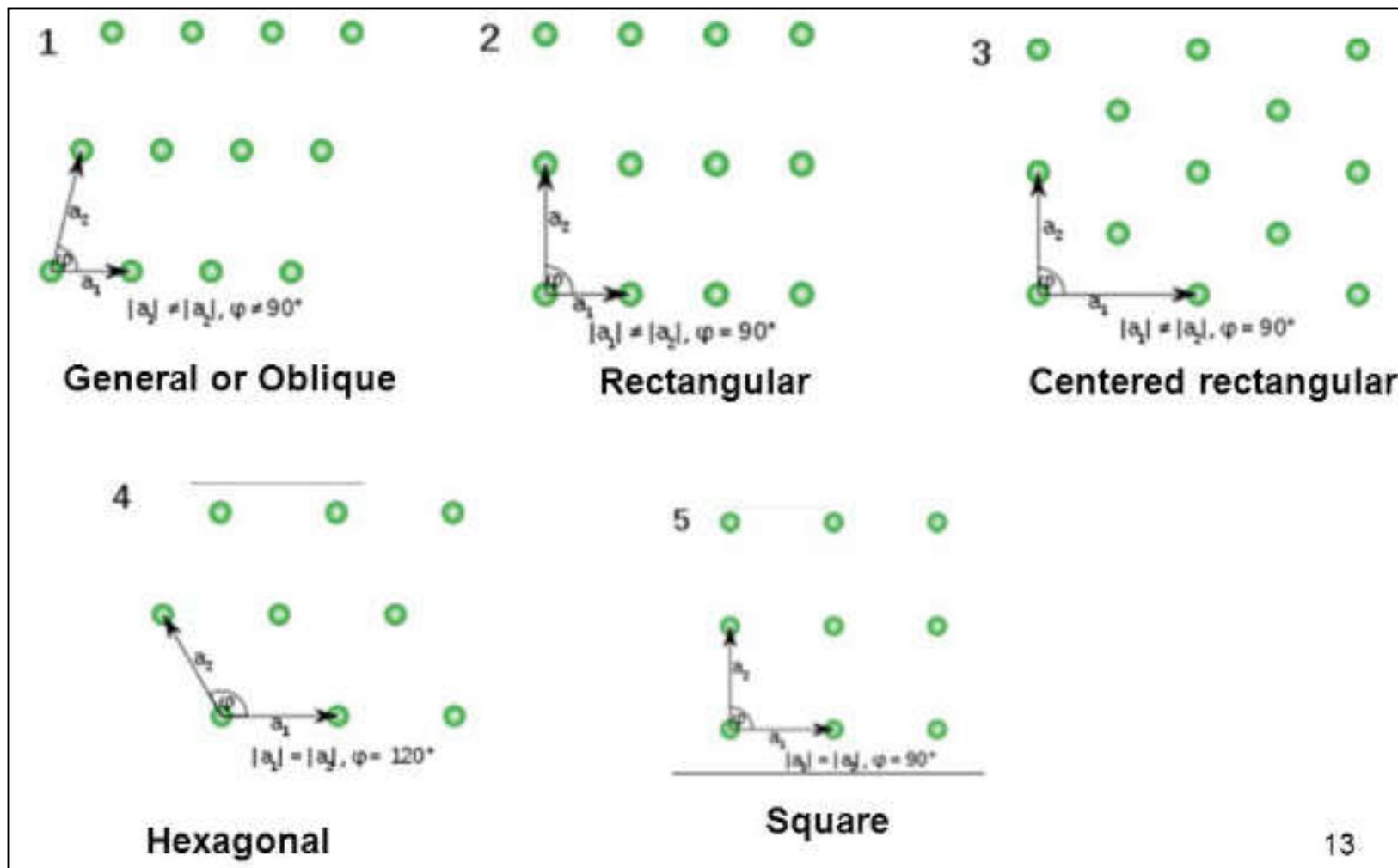
Lattice Cells 晶胞

- Compound Cell 复胞
 - A cell with *more than one* lattice point



2D Bravais Lattice

- There are 5 Bravais lattices in 2D



3D Bravais Lattice

- There are 14 Bravais lattices in 3D

7 crystal systems
(晶系)

P
Simple/Primitive

I
Body Centered

F
Face Centered

C
Base Centered

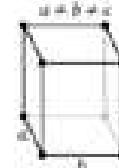
Triclinic



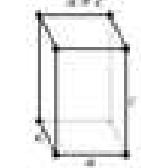
Monoclinic



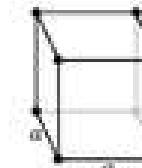
Orthorhombic



Tetragonal



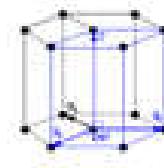
Cubic



Trigonal/rhombohedral

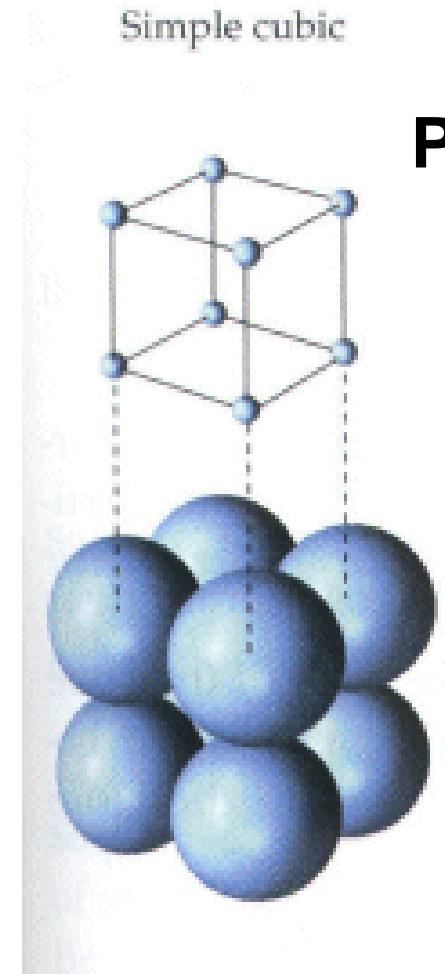


Hexagonal



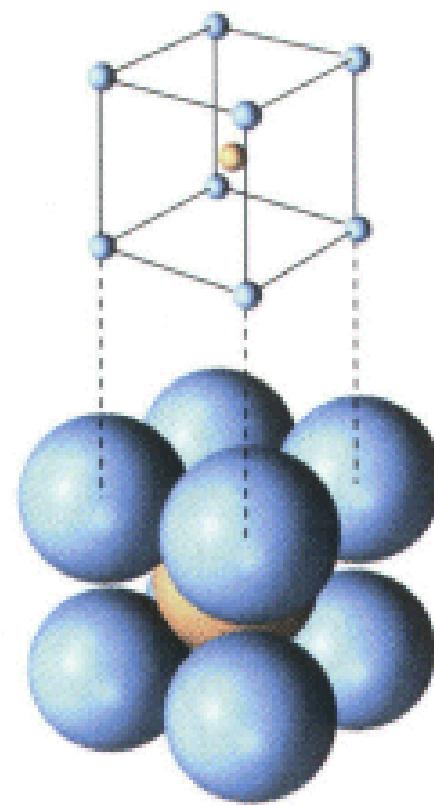
Bravais Lattice for Cubics 立方晶格

简单立方 SC



体心立方 BCC

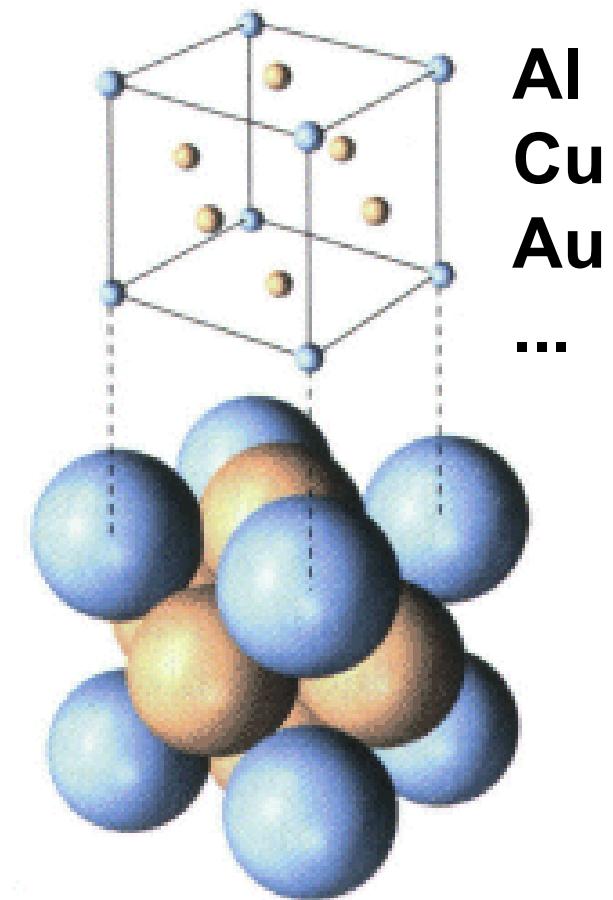
Body-centered cubic



面心立方 FCC

Face-centered cubic

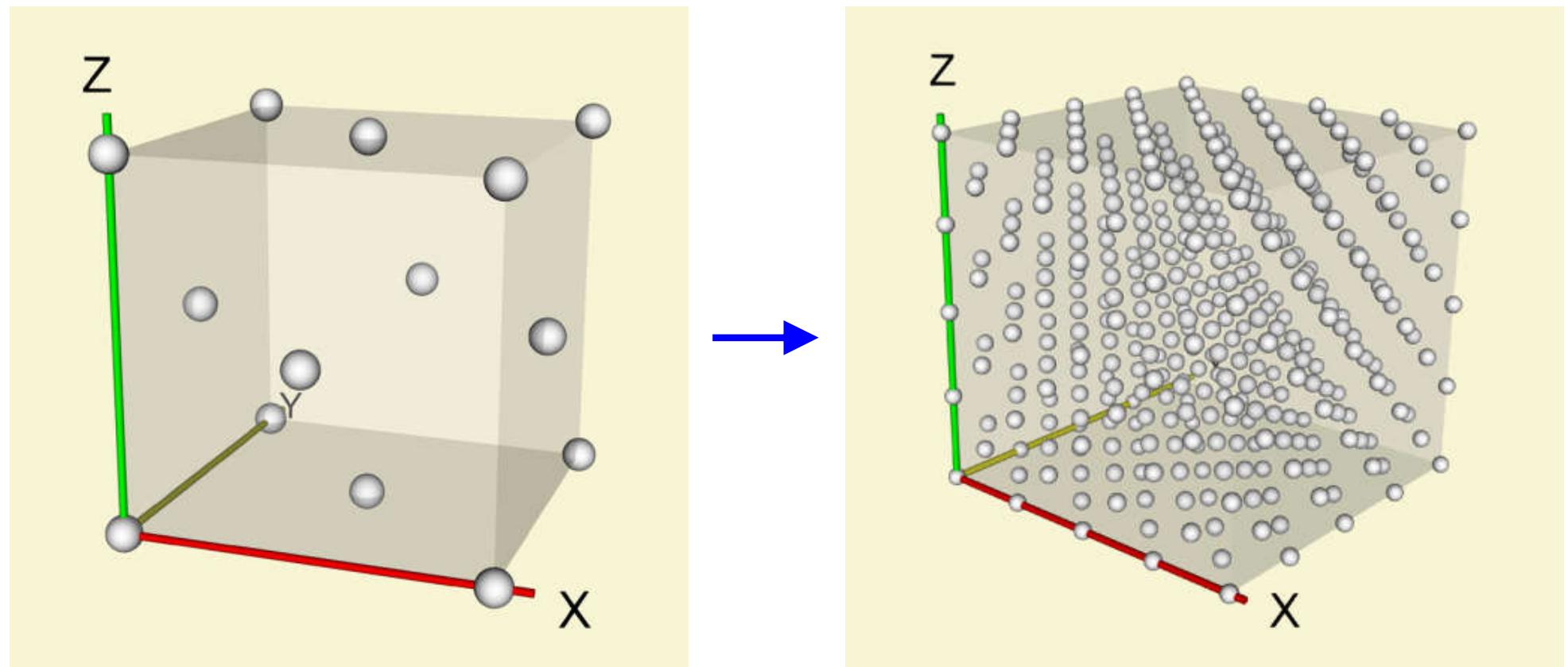
Li
Na
Cr
...



Q: Crystals with SC are not common, why?

FCC is a Bravais Lattice

Each point is *exactly* the same



2D Packing

- *Atomic Packing Factor (APF)* 填充因子
- $APF = (\text{area of circles}) / (\text{area of unit cell})$



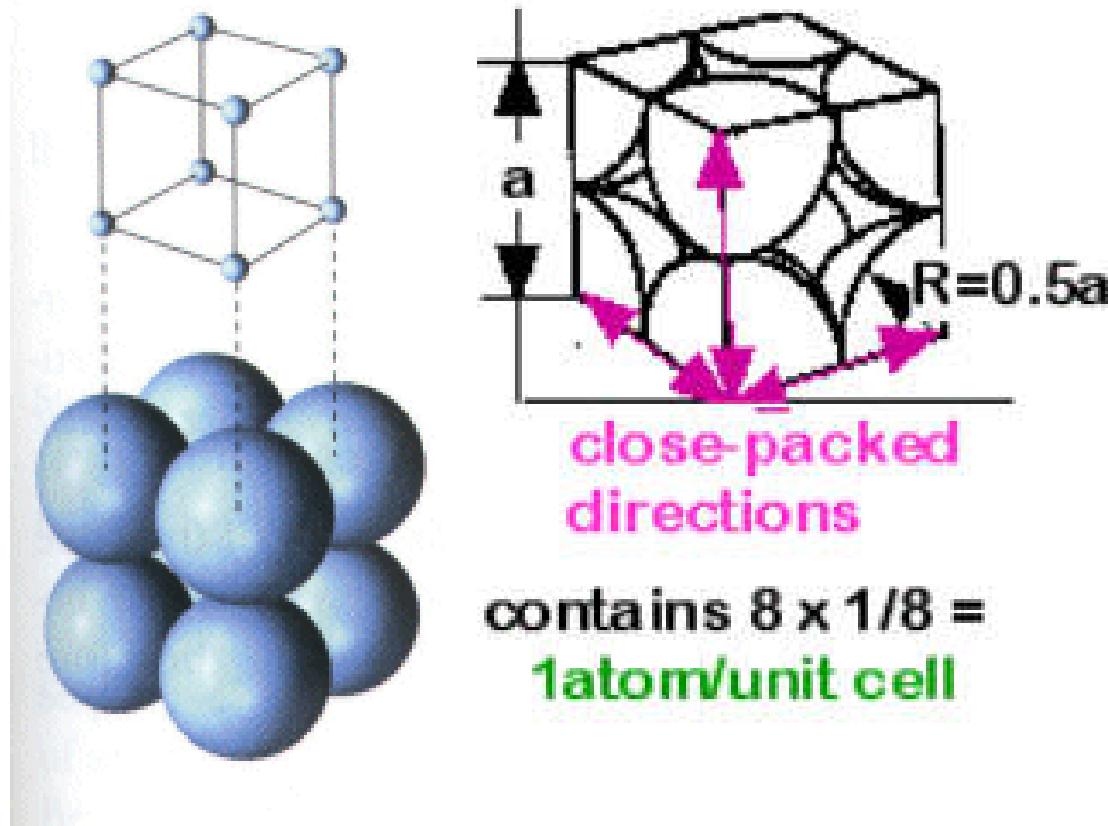
$$\begin{aligned} APF &= \frac{\pi(0.5a)^2}{a^2} = \frac{\pi}{4} \\ &= 0.785 \end{aligned}$$



$$\begin{aligned} APF &= \frac{\pi(0.5a)^2}{a * a * \sqrt{3} / 2} = \frac{\pi}{2\sqrt{3}} \\ &= 0.906 \end{aligned}$$

简单立方 SC

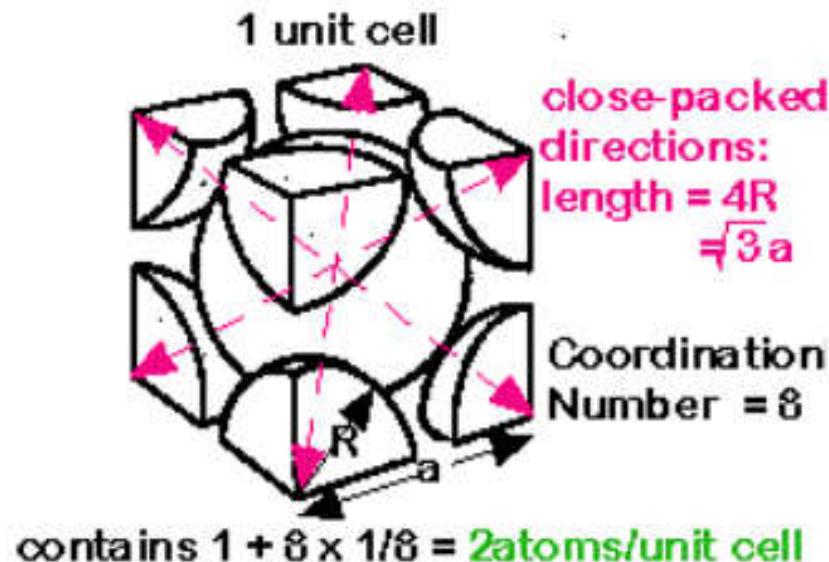
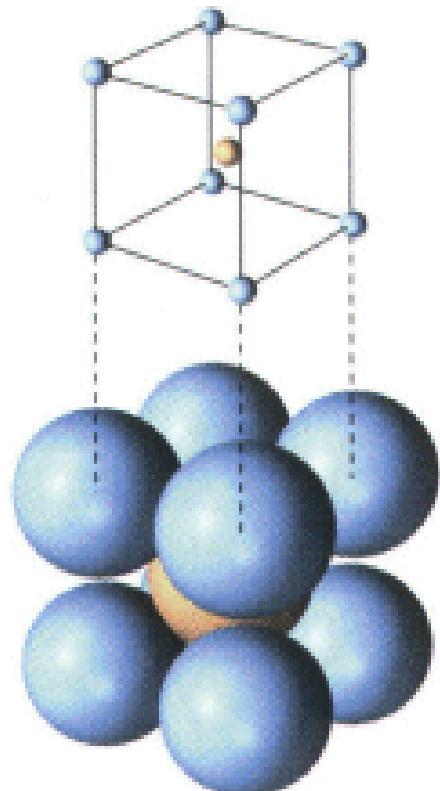
- APF = (volume of spheres) / (volume of unit cell)



$$APF = \frac{1 * \frac{4}{3} \pi (0.5a)^3}{a^3} = \frac{\pi}{6} = 0.523$$

体心立方 BCC

- APF = (volume of atoms) / (volume of unit cell)

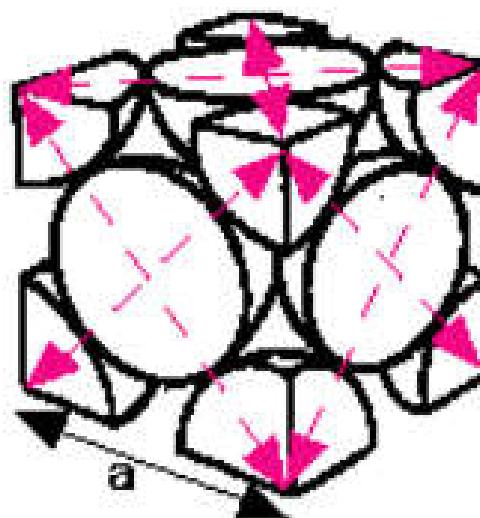
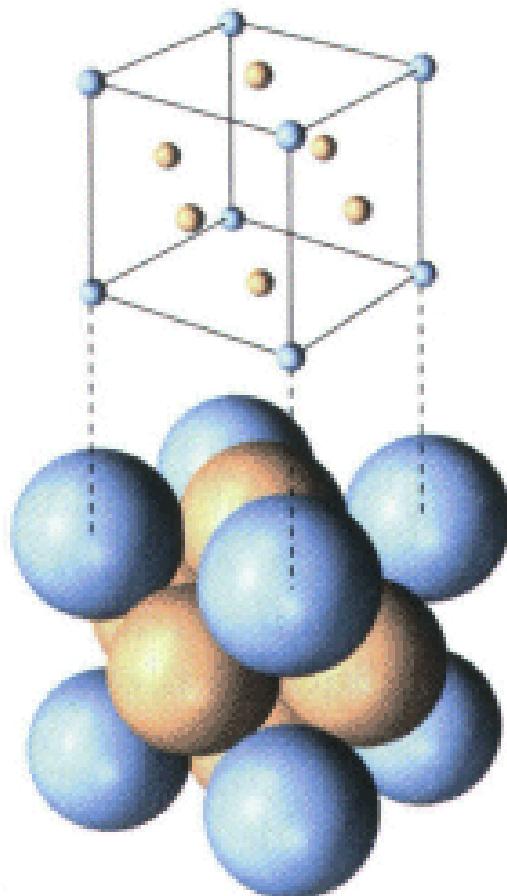


$$APF = \frac{2 * \frac{4}{3} \pi (\frac{\sqrt{3}}{4} a)^3}{a^3} = \frac{\sqrt{3}}{8} \pi = 0.681$$

体心立方 BCC

面心立方 FCC

- APF = (volume of atoms) / (volume of unit cell)



close-packed
directions:
length = $4R$
 $= \sqrt{2}a$

Coordination
Number = 12

contains $6 \times 1/2 + 8 \times 1/8 = 4$ atoms/unit cell

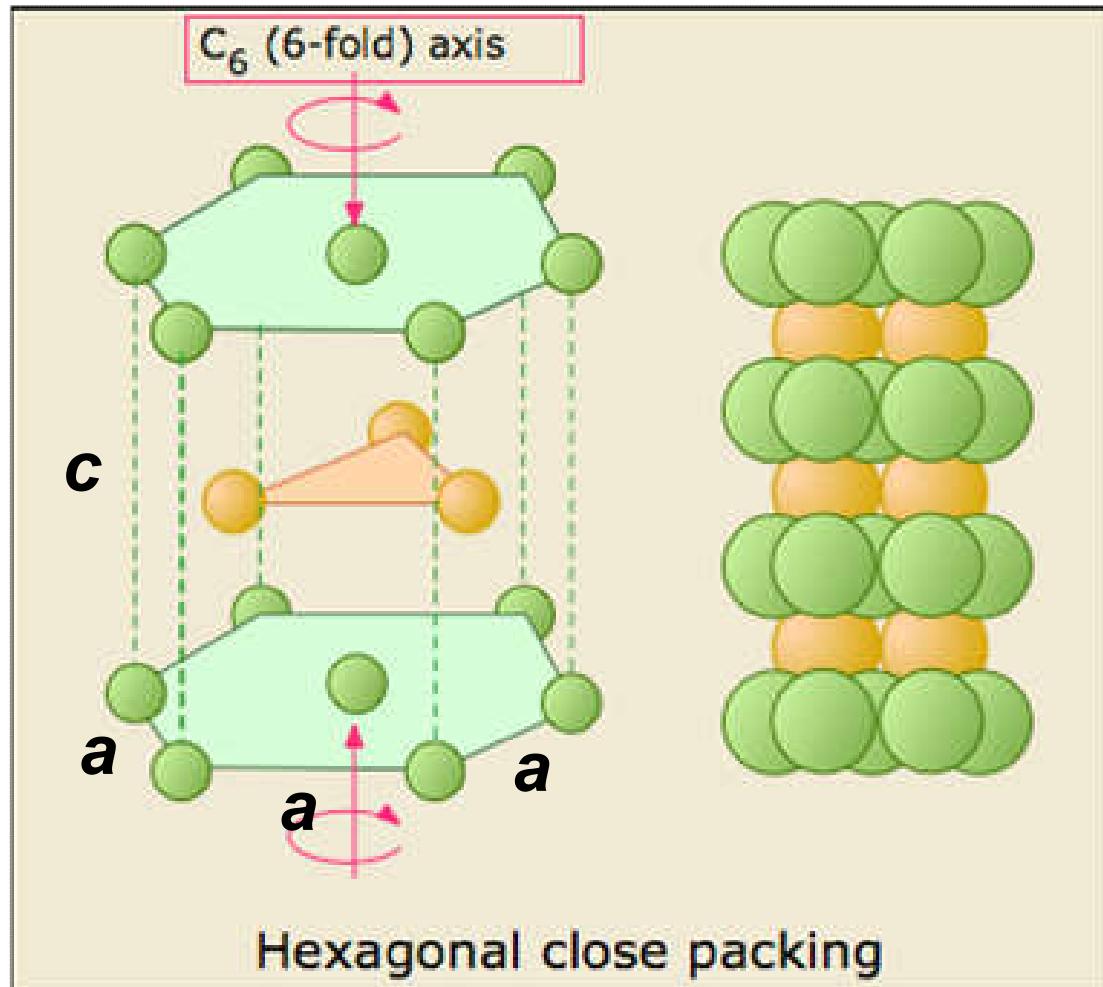
$$APF = \frac{4 * \frac{4}{3} \pi (\frac{\sqrt{2}}{4} a)^3}{a^3} = \frac{\sqrt{2}}{6} \pi = 0.740$$

Cubic Lattices

	SC	BCC	FCC
Volume, conventional cell	a^3	a^3	a^3
Lattice points per cell	1	2	4
Volume, primitive cell	a^3	$\frac{1}{2}a^3$	$\frac{1}{4}a^3$
Lattice points per unit volume	$1/a^3$	$2/a^3$	$4/a^3$
Number of nearest neighbors	6	8	12
Nearest-neighbor distance	a	$3^{1/2} a/2 = 0.866a$	$a/2^{1/2} = 0.707a$
Number of second neighbors	12	6	6
Second neighbor distance	$2^{1/2}a$	a	a
Packing fraction ^a	$\frac{1}{6}\pi$ $=0.524$	$\frac{1}{8}\pi\sqrt{3}$ $=0.680$	$\frac{1}{6}\pi\sqrt{2}$ $=0.740$

a: lattice constant / lattice parameter (晶格常数 / 晶格参数)

Hexagonal Close Packing (HCP) 六角密排



Mg, Zn, Ti,...

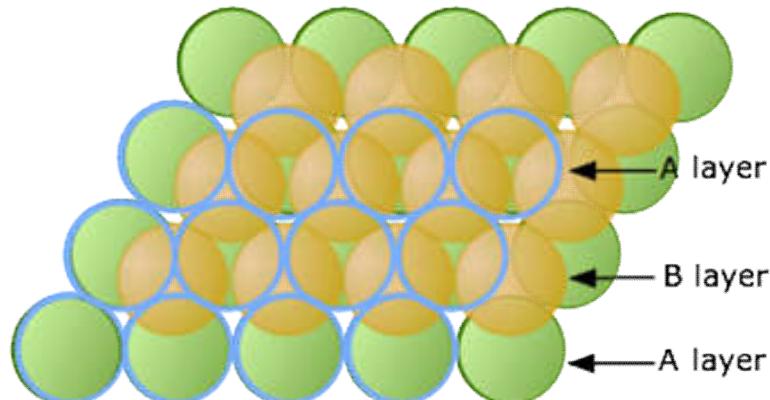
APF for HCP

$$APF = 0.740$$

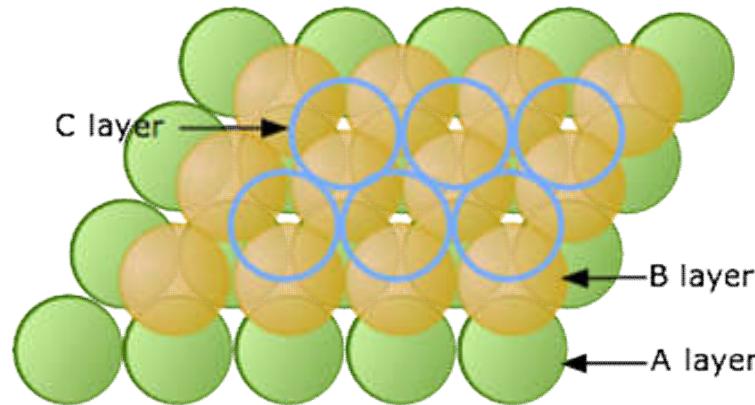
Q: Is HCP a Bravais lattice?
 $c/a = ???$

same as FCC

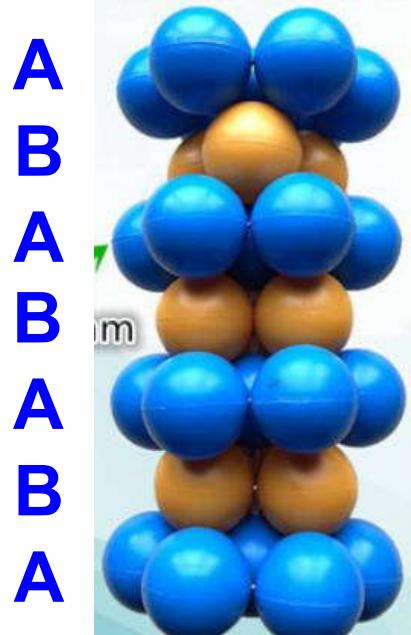
HCP and FCC



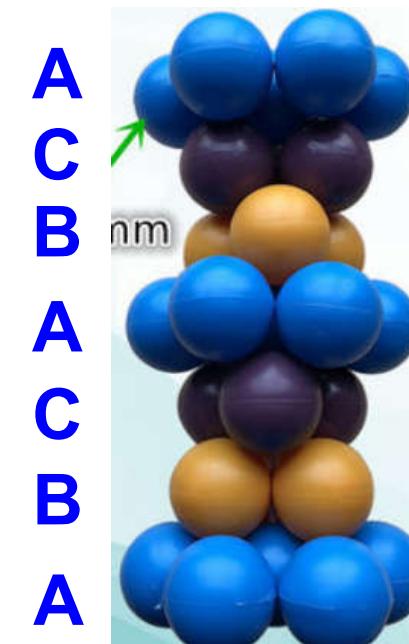
ABA hexagonal close packed



ABC face-centered cubic



HCP



FCC

HCP and FCC - A Little Story



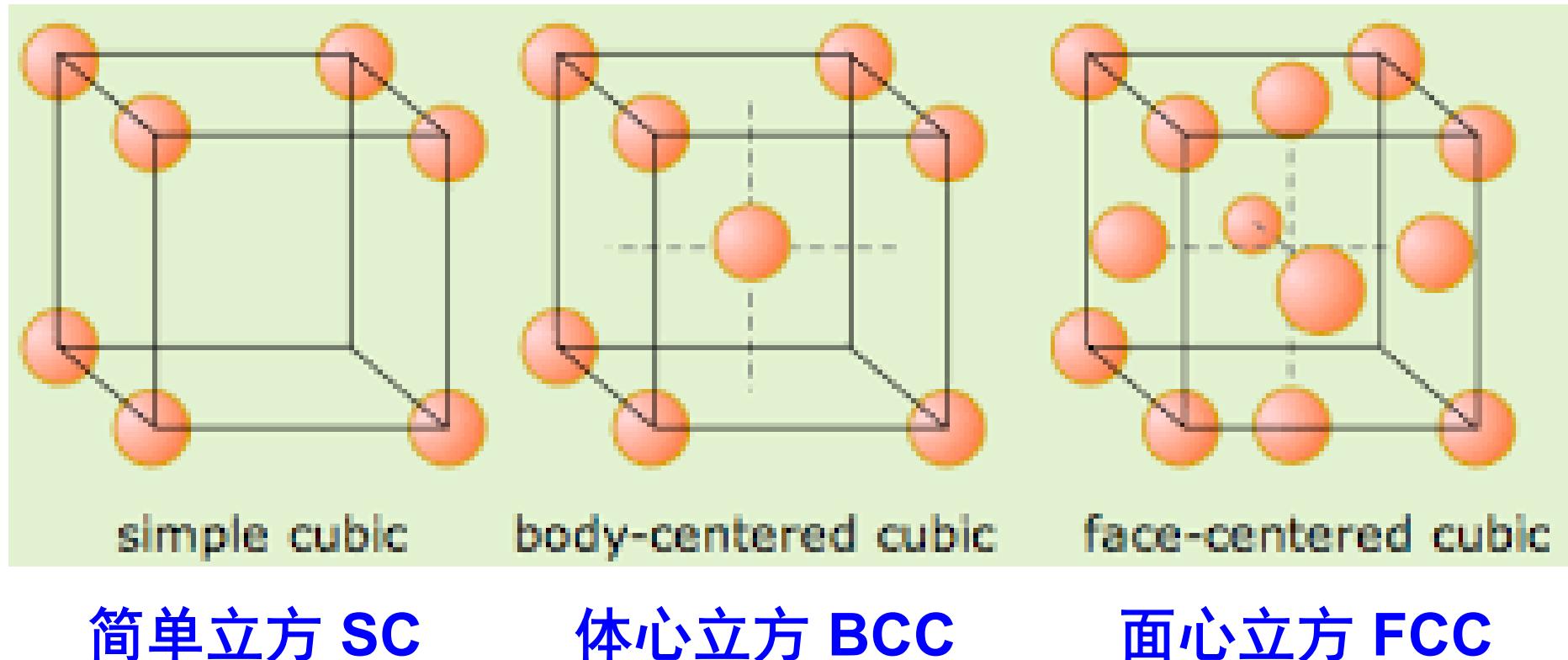
cannonball
pack

Q: FCC, or HCP?

- HCP and FCC are the densest way to pack equal-sized spheres in 3D space
 - J. Kepler's conjecture (开普勒猜想) in 1611
 - Everyone believes that it should be true, but no proof until ...
- T. Hales in Univ. Pittsburgh proved it in 1998

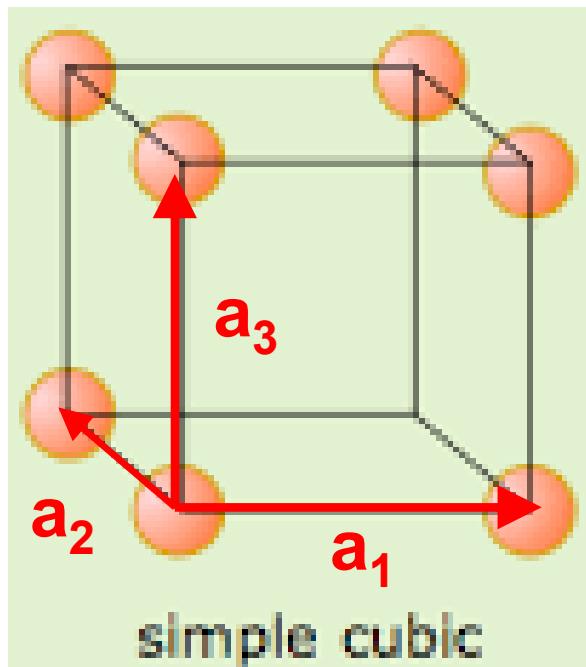
Cubic Lattices

- These are *conventional cells* 惯用晶胞
- They are *not primitive cells* for BCC and FCC. *Why?*



Q: How to draw primitive cells for BCC and FCC?

A Primitive Cell for SC



$$\mathbf{a}_1 = a\hat{\mathbf{x}}$$

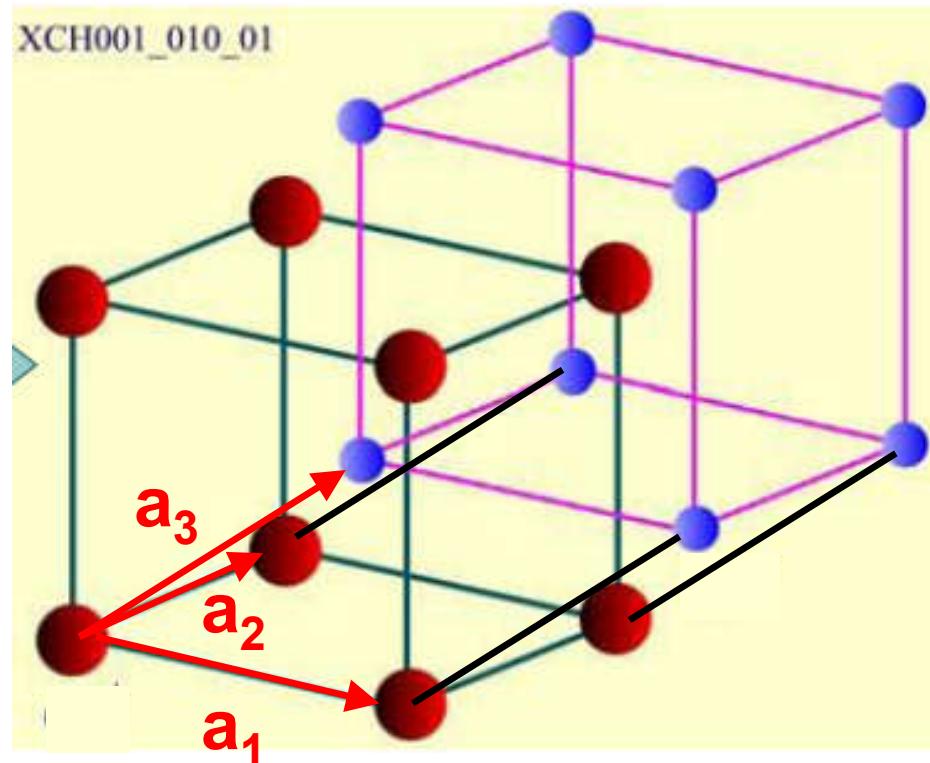
$$\mathbf{a}_2 = a\hat{\mathbf{y}}$$

$$\mathbf{a}_3 = a\hat{\mathbf{z}}$$

$(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$ are Cartesian coordinates

The volume of this primitive cell = a^3

A Primitive Cell for BCC

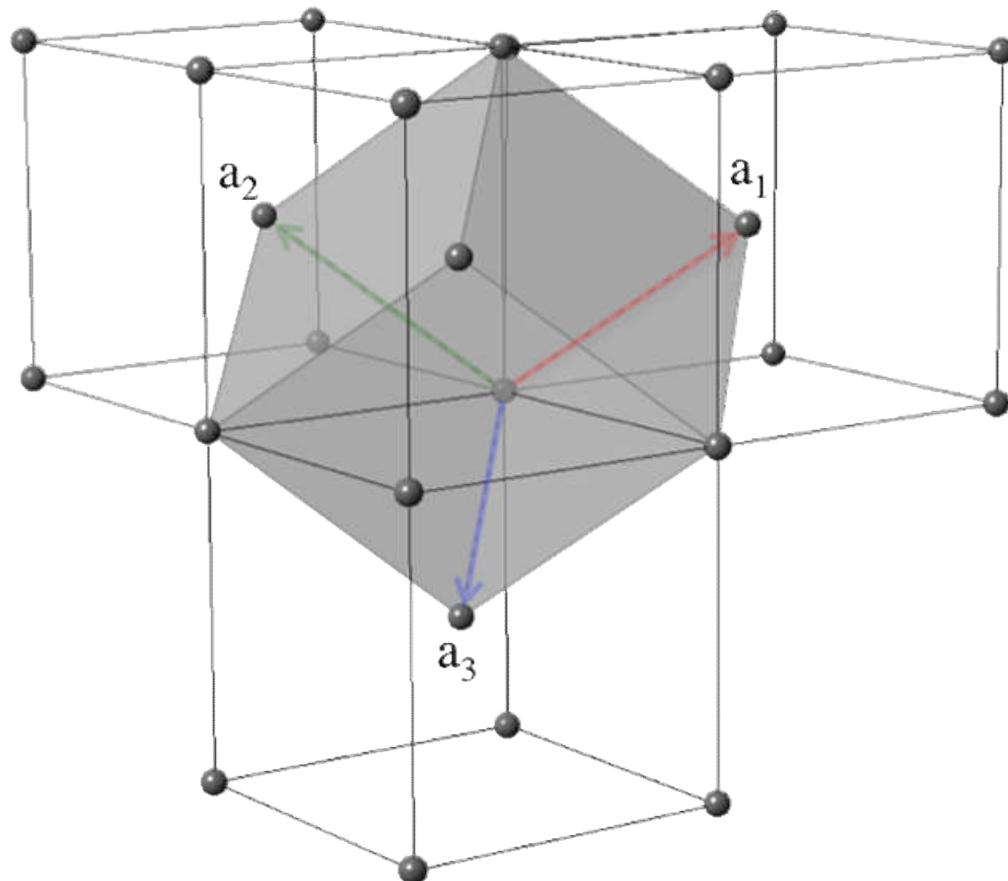


$$\mathbf{a}_1 = a\hat{\mathbf{x}}$$
$$\mathbf{a}_2 = a\hat{\mathbf{y}}$$
$$\mathbf{a}_3 = \frac{a}{2}(\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}})$$

Q: What is the volume of the primitive cell?

Another Primitive Cell for BCC

a more symmetric one



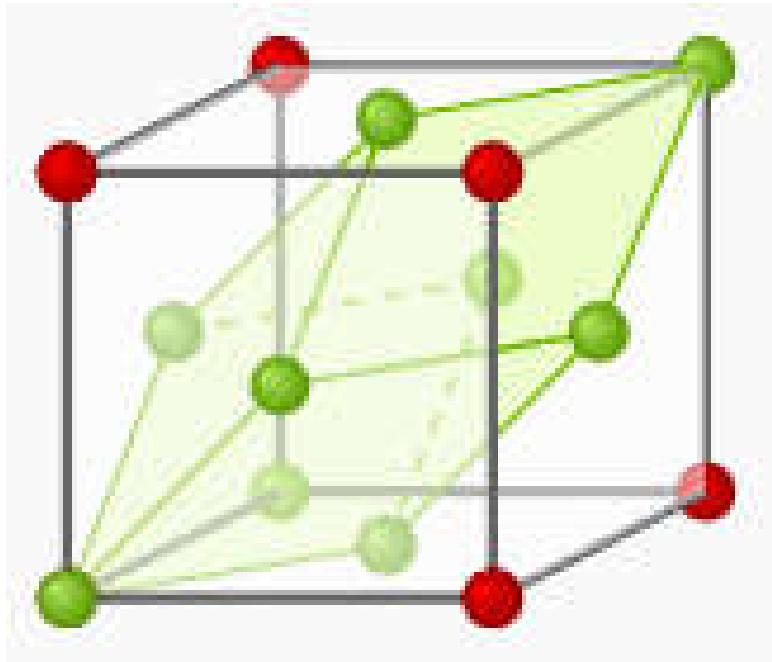
$$\mathbf{a}_1 = \frac{a}{2}(-\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}})$$

$$\mathbf{a}_2 = \frac{a}{2}(\hat{\mathbf{x}} - \hat{\mathbf{y}} + \hat{\mathbf{z}})$$

$$\mathbf{a}_3 = \frac{a}{2}(\hat{\mathbf{x}} + \hat{\mathbf{y}} - \hat{\mathbf{z}})$$

Q: What is the volume of the primitive cell?

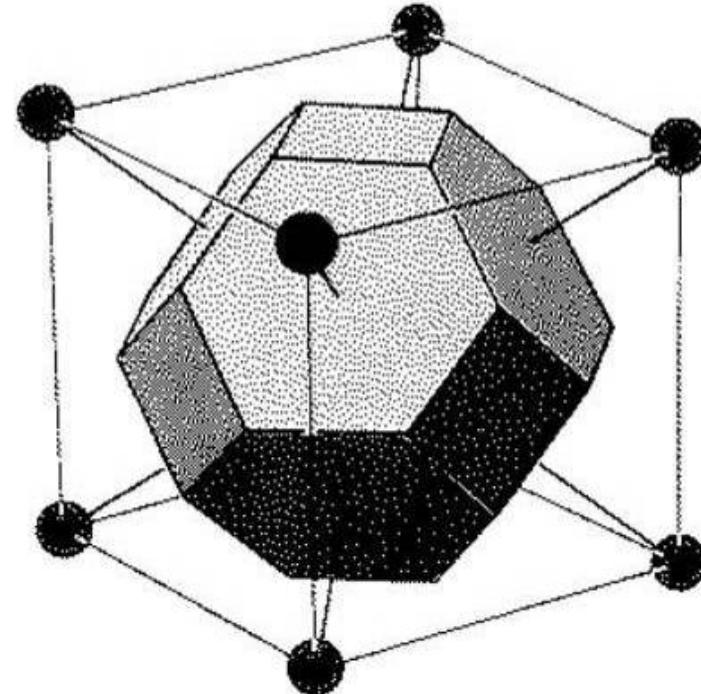
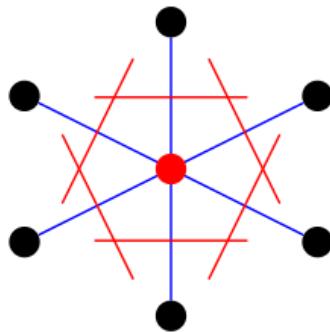
A Primitive Cell for FCC



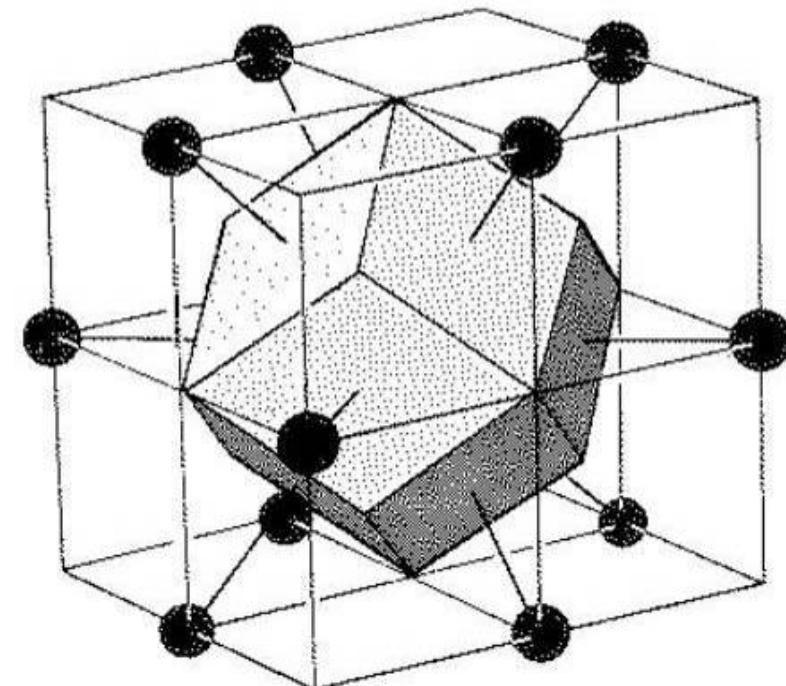
$$\mathbf{a}_1 = \frac{a}{2}(\hat{\mathbf{y}} + \hat{\mathbf{z}})$$
$$\mathbf{a}_2 = \frac{a}{2}(\hat{\mathbf{z}} + \hat{\mathbf{x}})$$
$$\mathbf{a}_3 = \frac{a}{2}(\hat{\mathbf{x}} + \hat{\mathbf{y}})$$

Q: What is the volume of the primitive cell?

Wigner-Seitz cells for BCC and FCC



体心立方 BCC

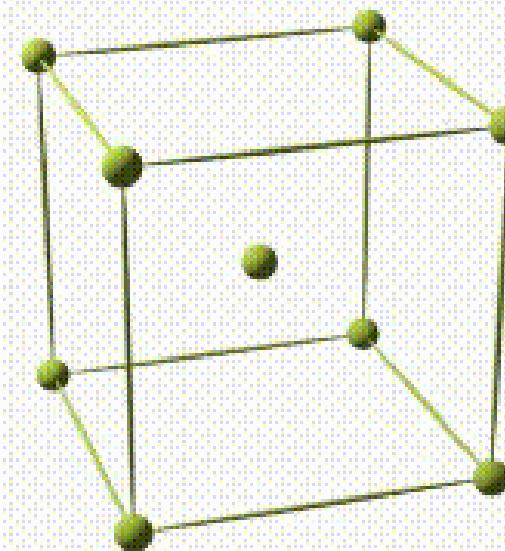


面心立方 FCC

Q: What is the volume of the Wigner-Seitz cell?

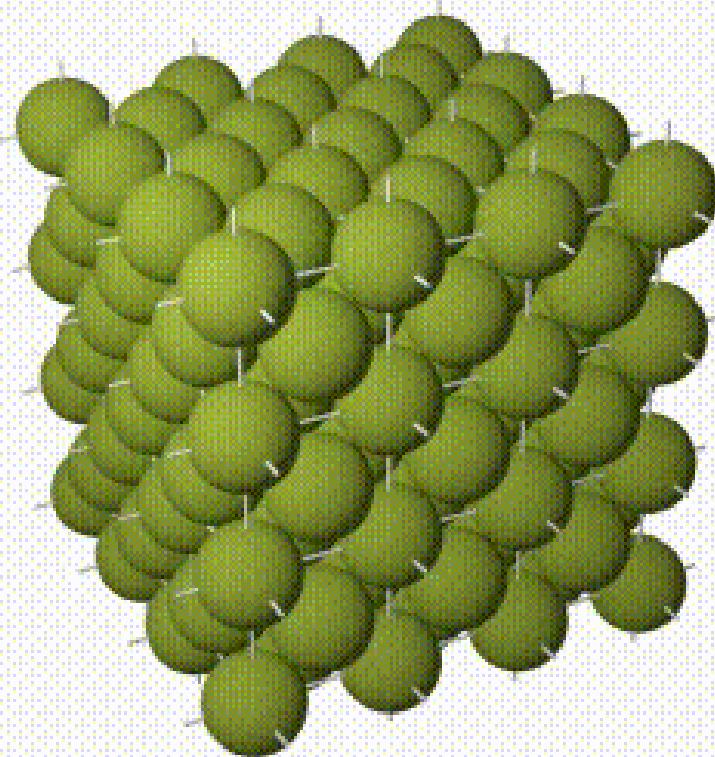
Review: BCC and FCC

The bcc structure



Review: BCC and FCC

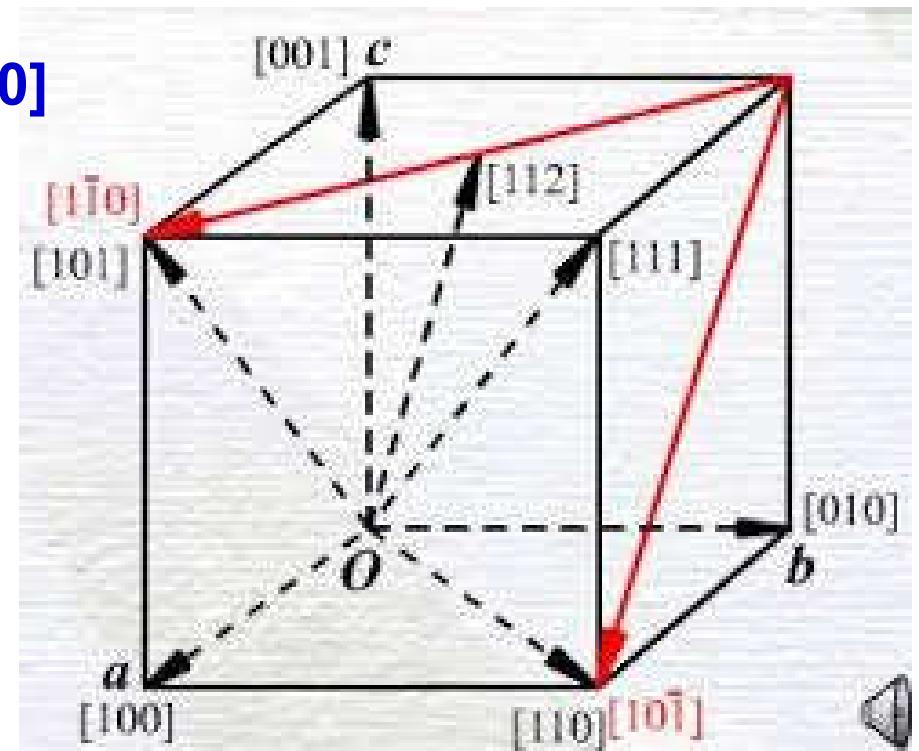
The fcc structure



Miller Indices - Direction 晶向

- crystal direction $[hkl]$
 - in cubic lattices, $[hkl]$ direction $\perp (hkl)$ plane
- $\langle hkl \rangle$
 - a group of similar directions
 - $\langle 001 \rangle$ includes $[001], [010], [100]$

h, k, l are integers with no common factors

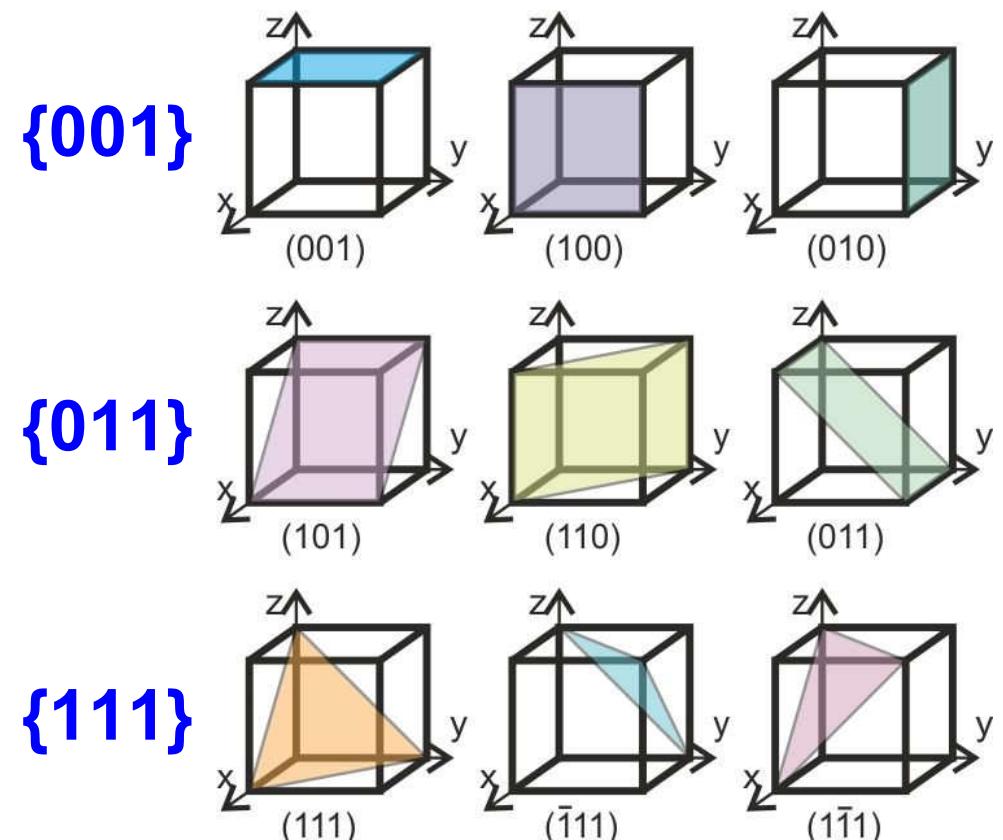


Miller Indices - Plane 晶面

- crystal plane (hkl)
 - intercepts at $(a_1/h, a_2/k, a_3/l)$
- $\{hkl\}$
 - a group of similar planes

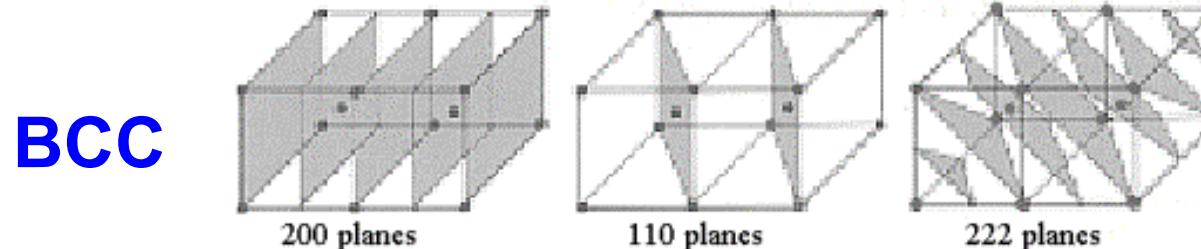
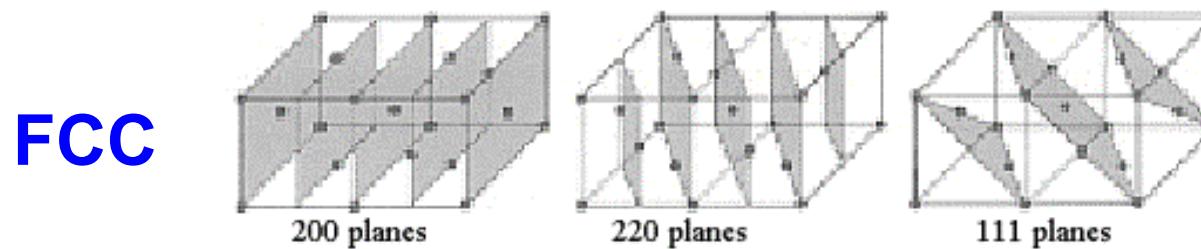
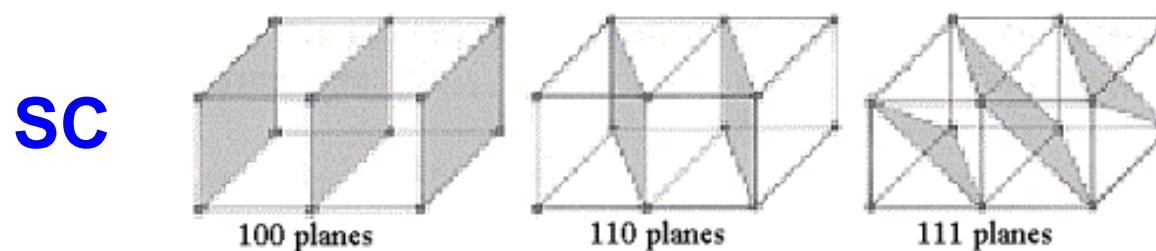
h, k, l are integers with no common factors

Q: Draw planes:
 $(120), (112), (131)$



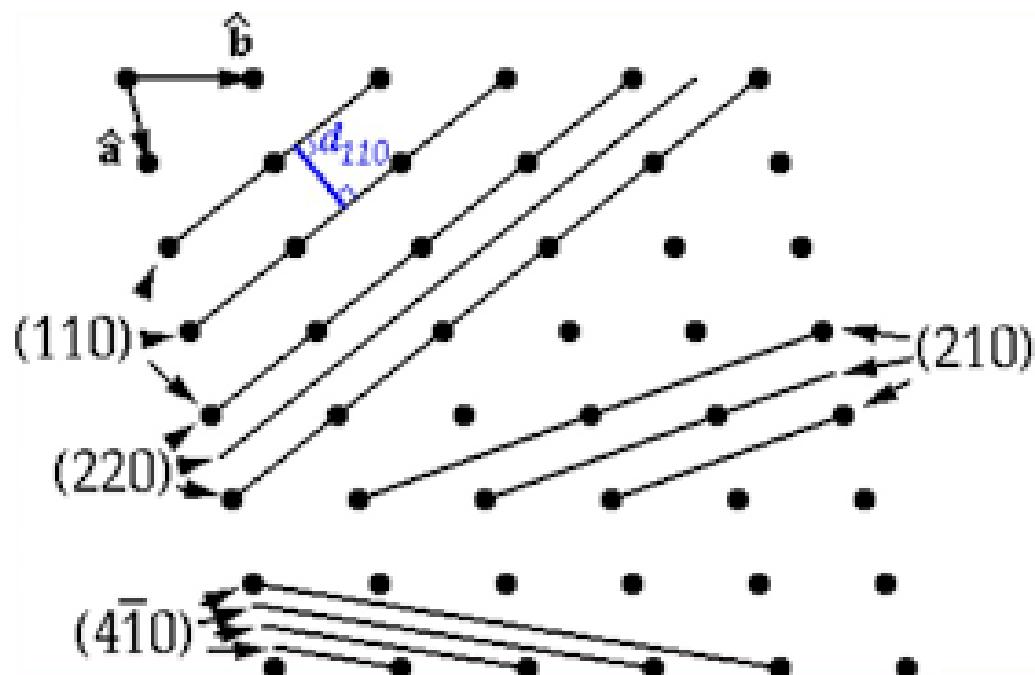
Lattice Plane 晶面

- any plane with at least 3 noncollinear Bravais lattice points
 - not always planes with Miller index, e.g., (200), (220), (222)



Interplanar Spacing 晶面间距

- Distance between adjacent lattice planes



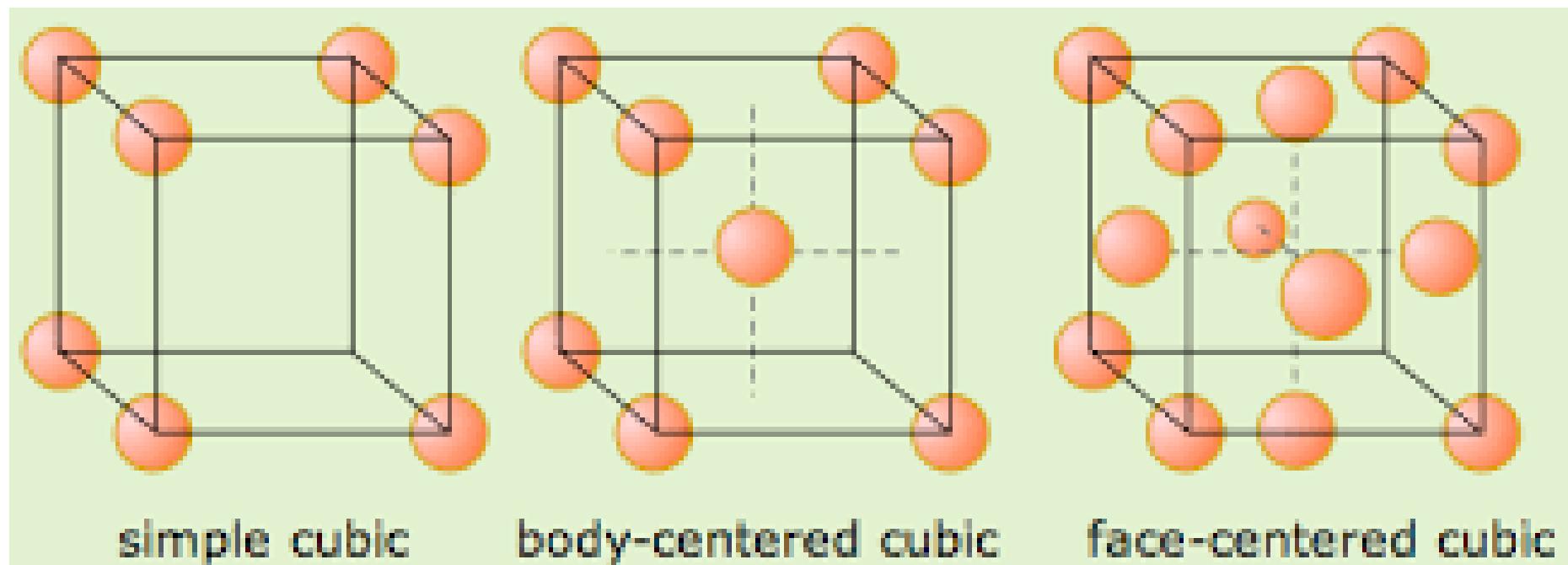
For cubic systems
(SC, BCC, FCC)

$$d_{(hkl)} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

Q: *Interplanar Spacing for
(120), (112), (131)*

Coordination Number 配位数

- The number of the nearest neighbors
 - 'kissing' number



简单立方 SC

6

体心立方 BCC

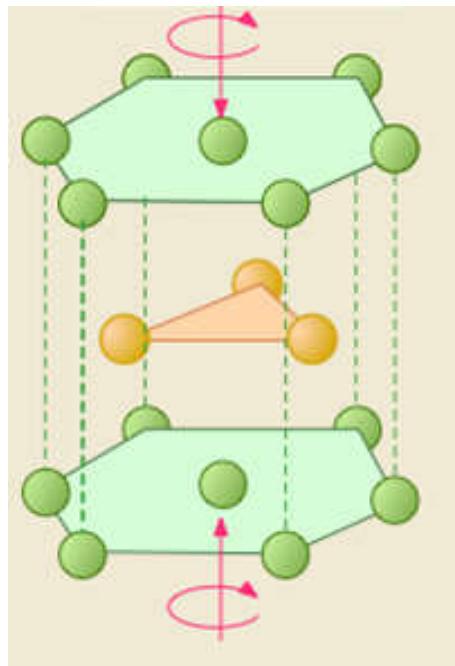
8

面心立方 FCC

12

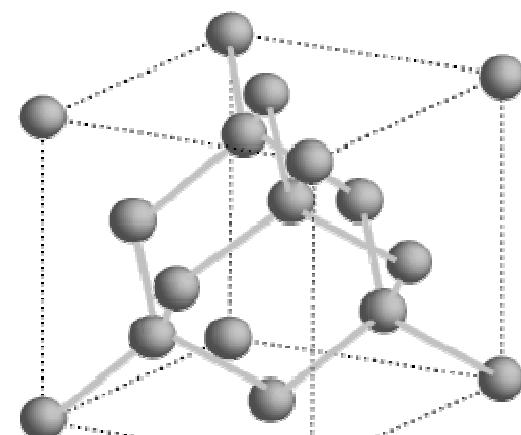
Coordination Number 配位数

- The number of the nearest neighbors
 - 'kissing' number



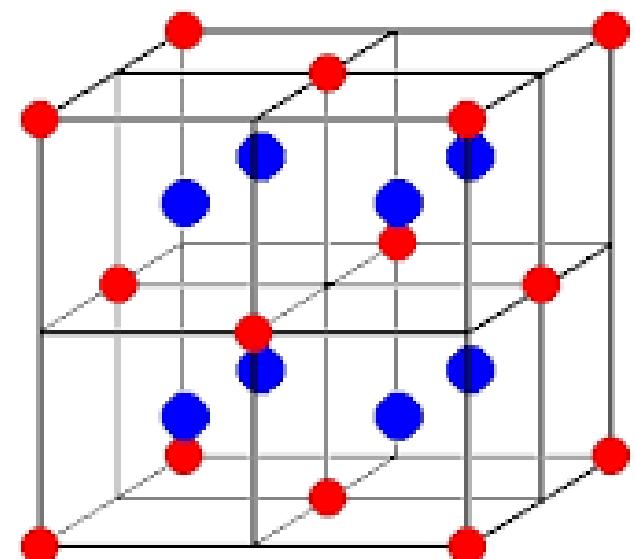
HCP

?



diamond

?

CaF₂

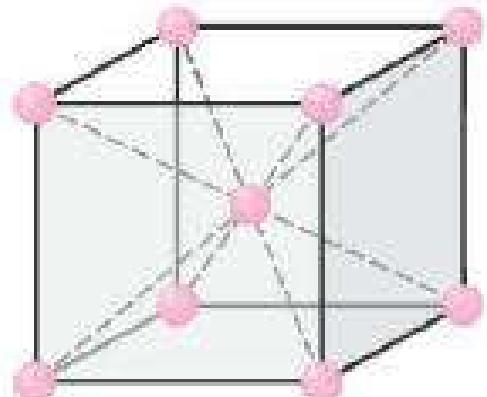
? and ?

Common Crystal Structures

- BCC, FCC, HCP
- Diamond 金刚石
 - C, Si, Ge
- Zinc Blende 闪锌矿
 - GaAs, InP, ZnS
- Halite / Rock Salt 岩盐
 - NaCl, KCl
- CsCl
- Fluorite 萤石
 - CaF₂
- Wurtzite 纤锌矿
 - GaN, ZnO
- Perovskite 钙钛矿
 - CaTiO₃, CsPbBr₃
- 1D, 2D structures
- ...

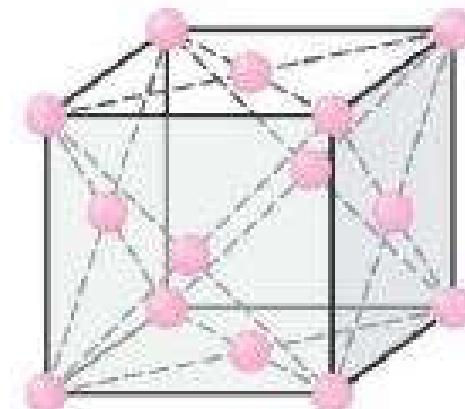
BCC, FCC, HCP

Li, Na, Cr, ...



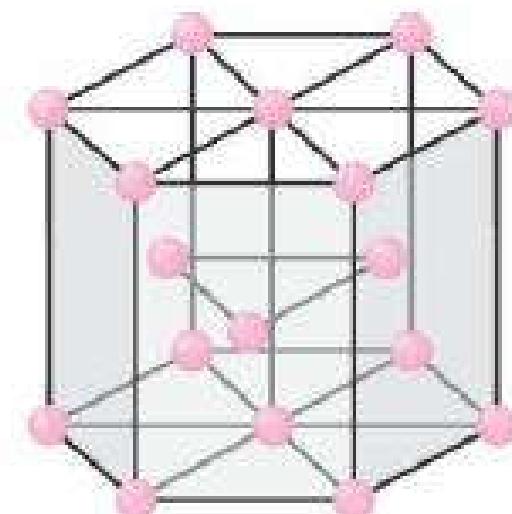
BCC

Al, Cu, Au, ...



FCC

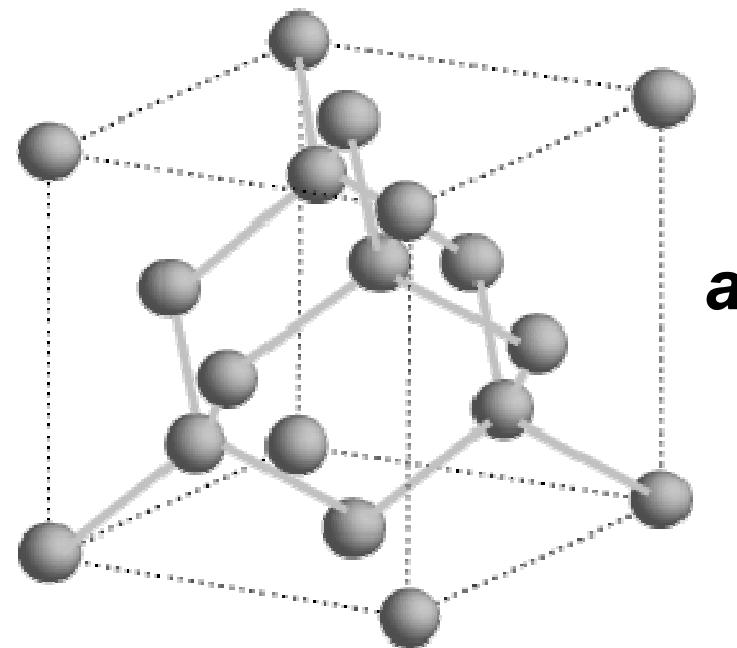
Mg, Zn, Ti, ...



HCP

Diamond Structure

- ## ■ C, Si, Ge, ...



lattice parameters for diamond structures

	a (Å)
C (diamond)	3.57
Si	5.43
Ge	5.66
α -Sn	6.49

Q:

What is the Bravais lattice?

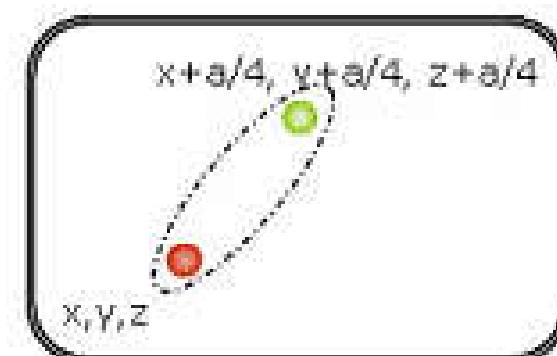
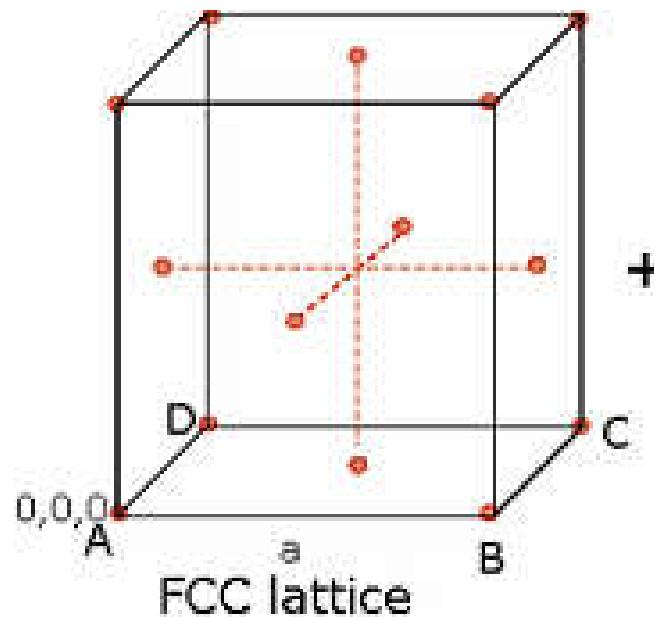
What is the atomic distance?

How many atoms in the cubic cell?

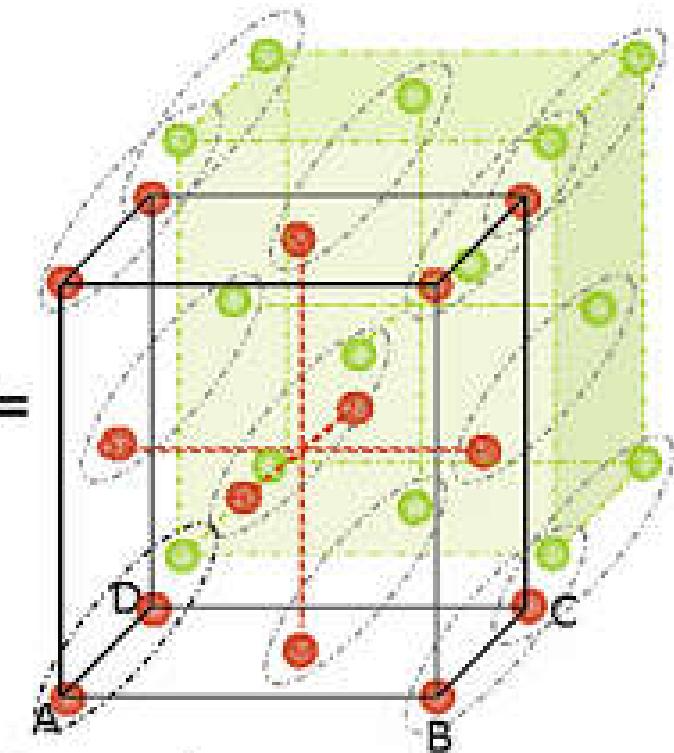
What is the APF?

Diamond Structure

- C, Si, Ge, ...



2 atom basis to be attached to each lattice point



Diamond structure
= 2 interpenetrating FCC's

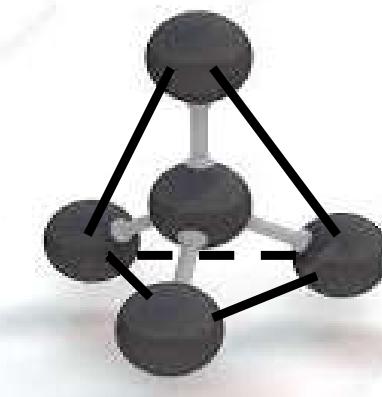
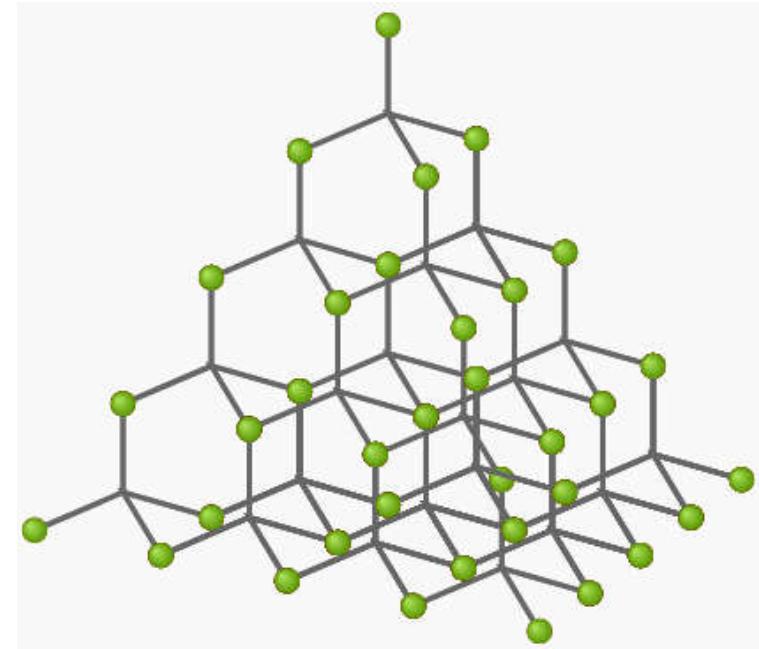
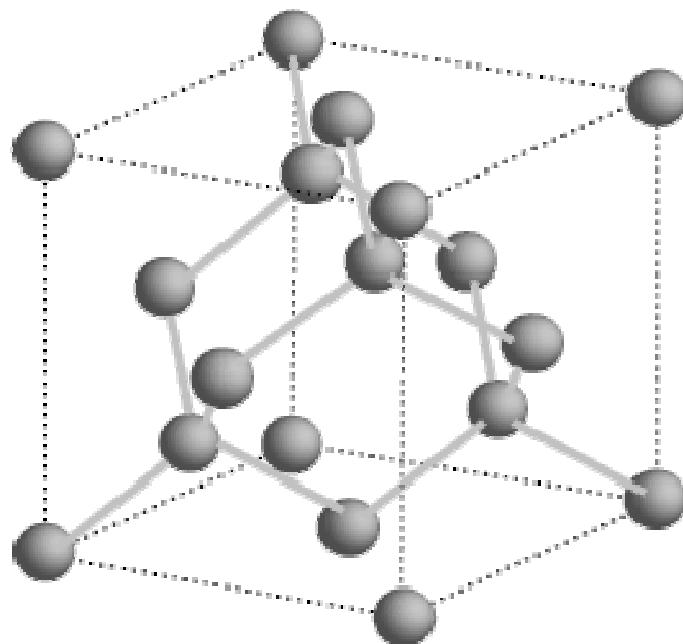
Lattice

Basis

Crystal

Diamond Structure

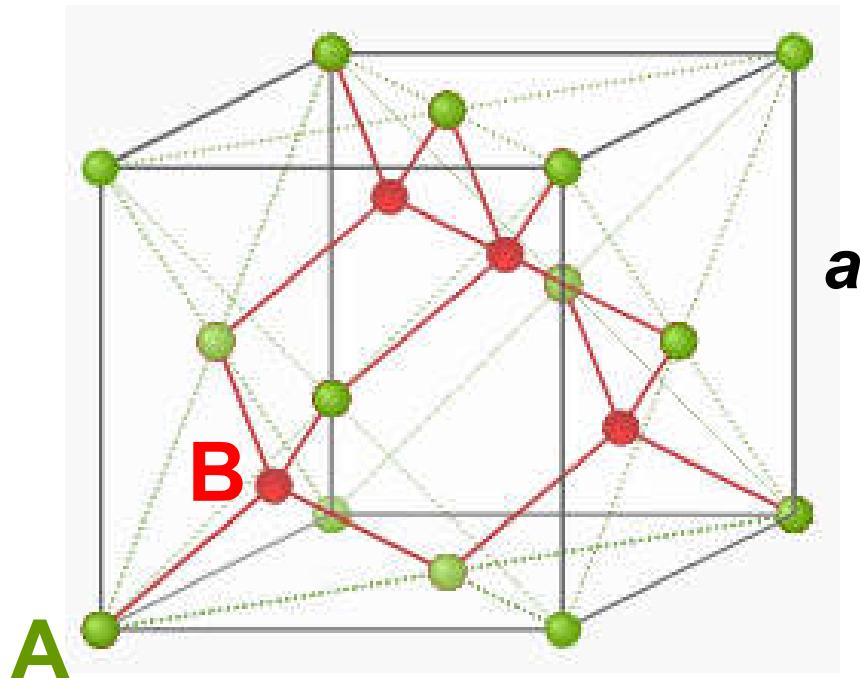
- C, Si, Ge, ...



tetrahedron
(正四面体)

Zinc Blende 闪锌矿

- GaAs, InP, ZnS, ...



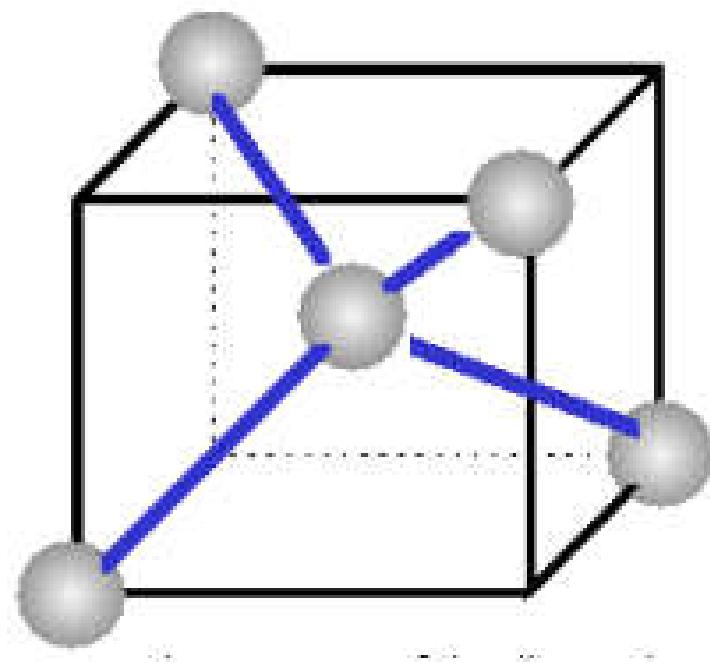
5	C	6	7	8	9	10			
B		N	O	F		Ne			
13	14	15	16	17	18				
Al	Si	P	S	Cl	Ar				
31	32	33	34	35	36				
Ga	Ge	As	Se	Br	Kr				
49	50	51	52	53	54				
In	Sn	Sb	Te	I	Xe				
81	82	83	84	85	86				
Tl	Pb	Bi	Po	At	Rn				

Q:

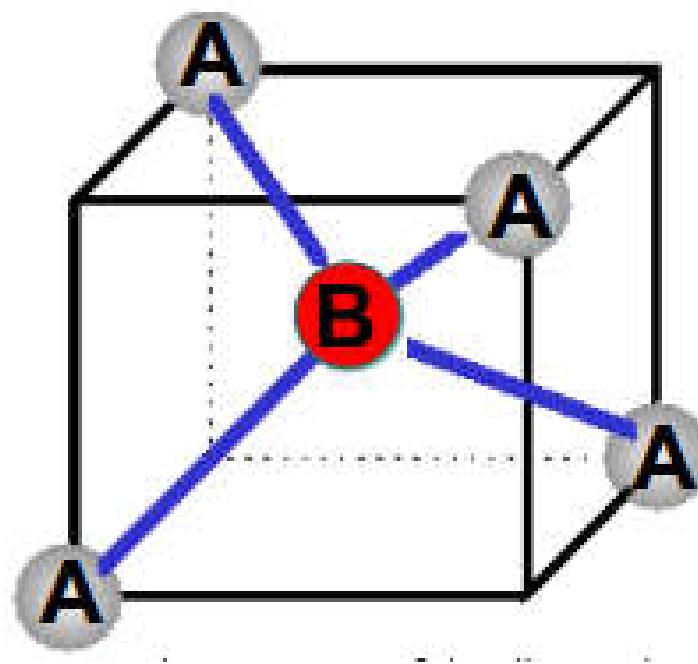
How many A and B atoms in the cubic cell?

Diamond vs. Zinc Blende

- FCC lattice



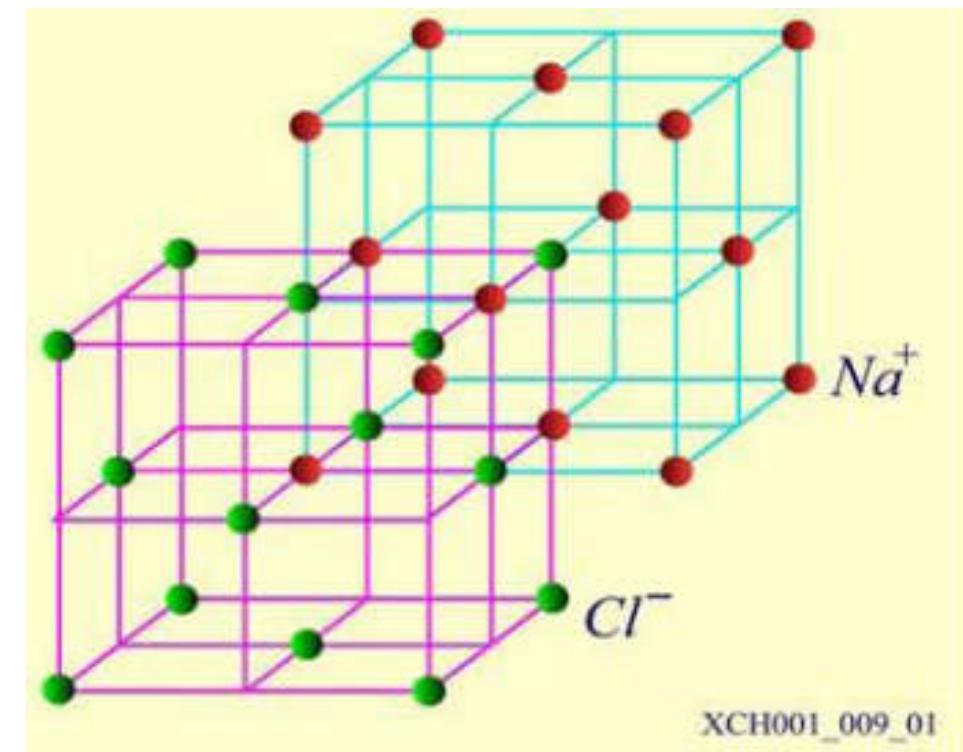
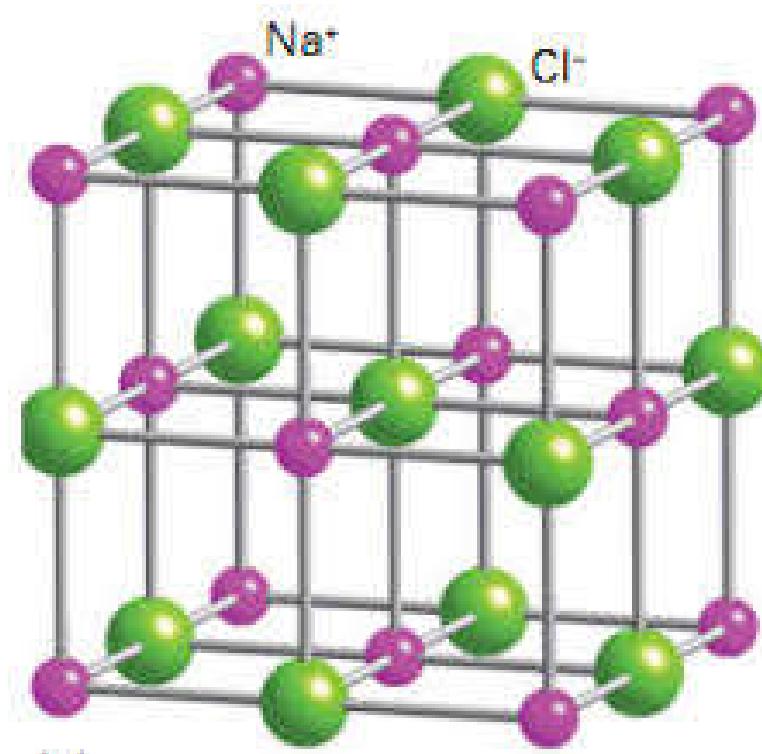
Si



GaAs

Halite / Rock Salt 岩盐

- NaCl, ...



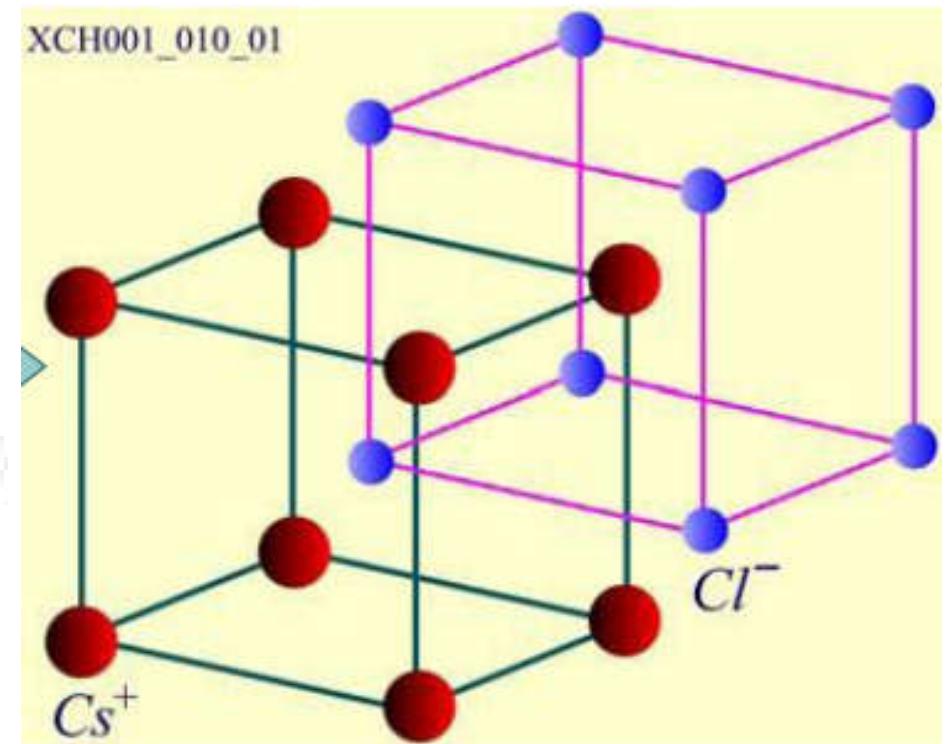
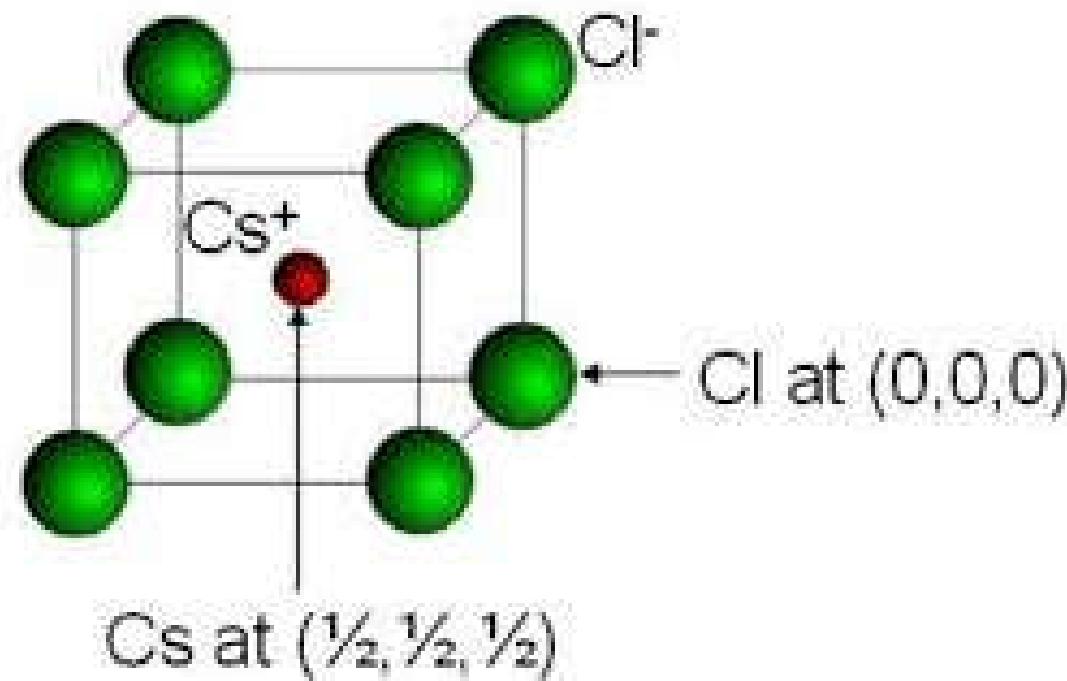
Q:

What is the Bravais lattice?

How many A and B atoms in the cubic cell?

CsCl

- CsCl, ...



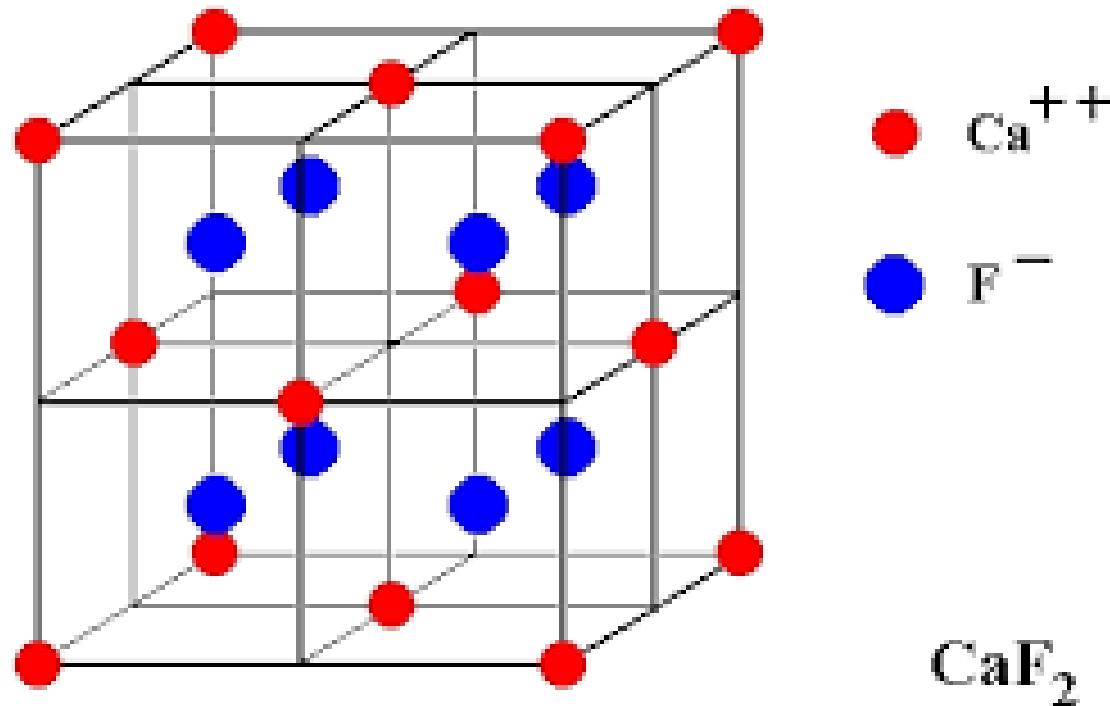
Q:

What is the Bravais lattice?

How many A and B atoms in the cubic cell?

Fluorite 萤石

- CaF_2 , ...



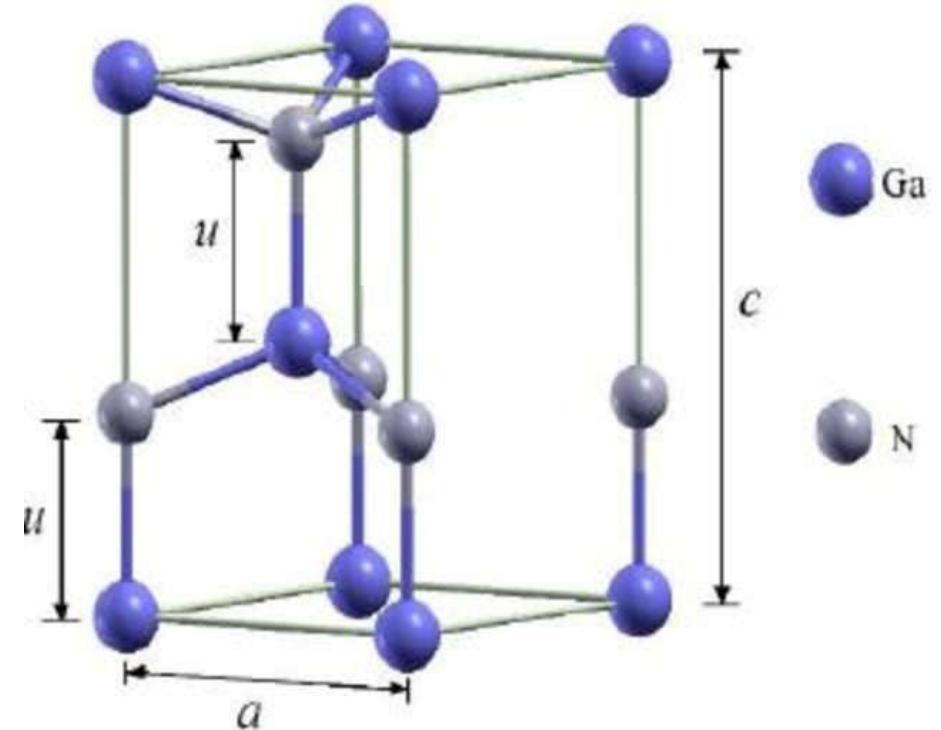
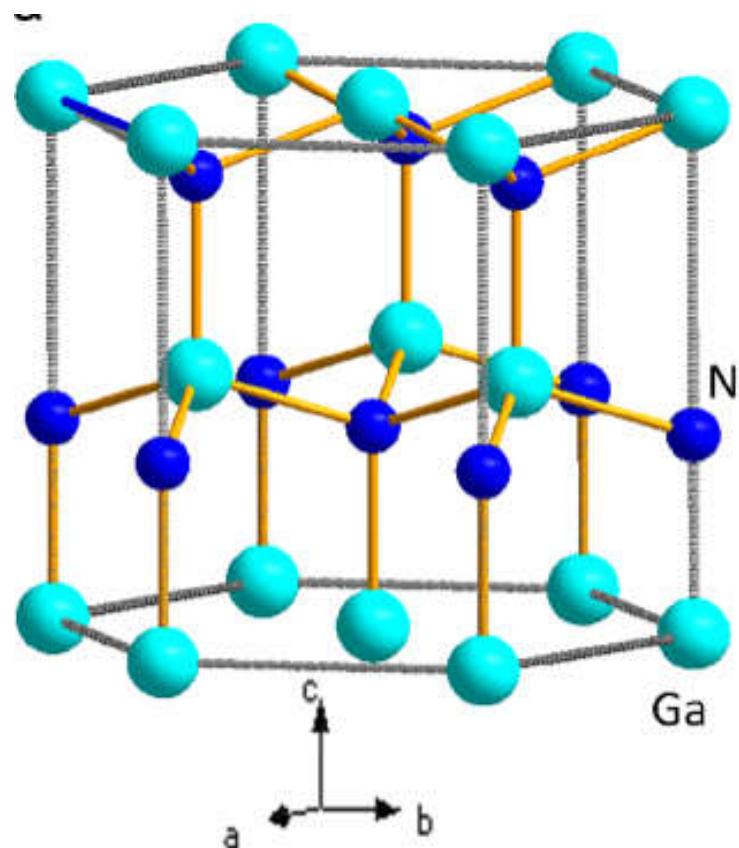
Q:

What is the Bravais lattice?

How many Ca and F atoms in the cubic cell?

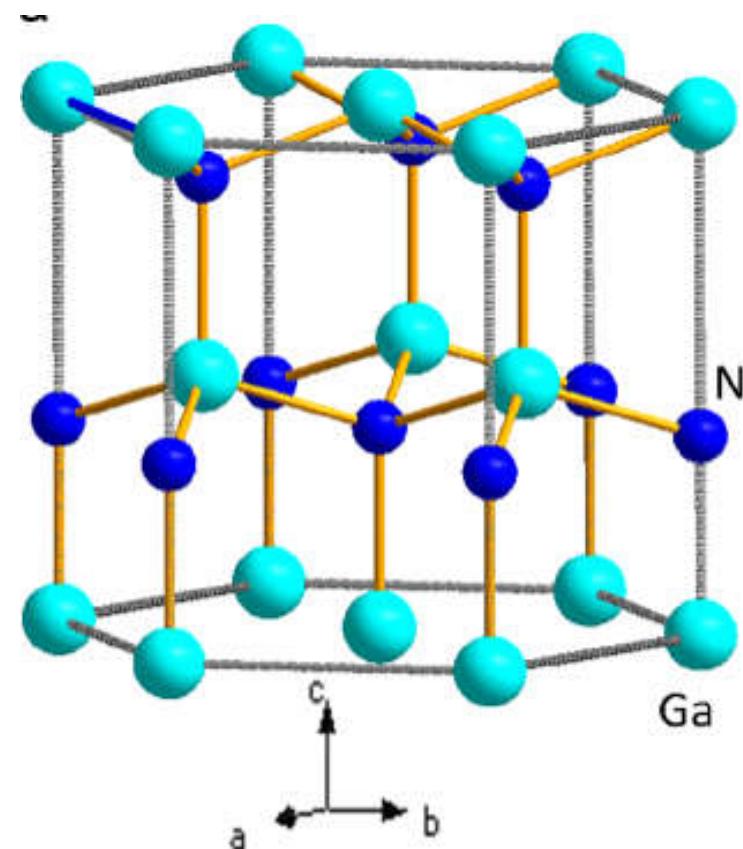
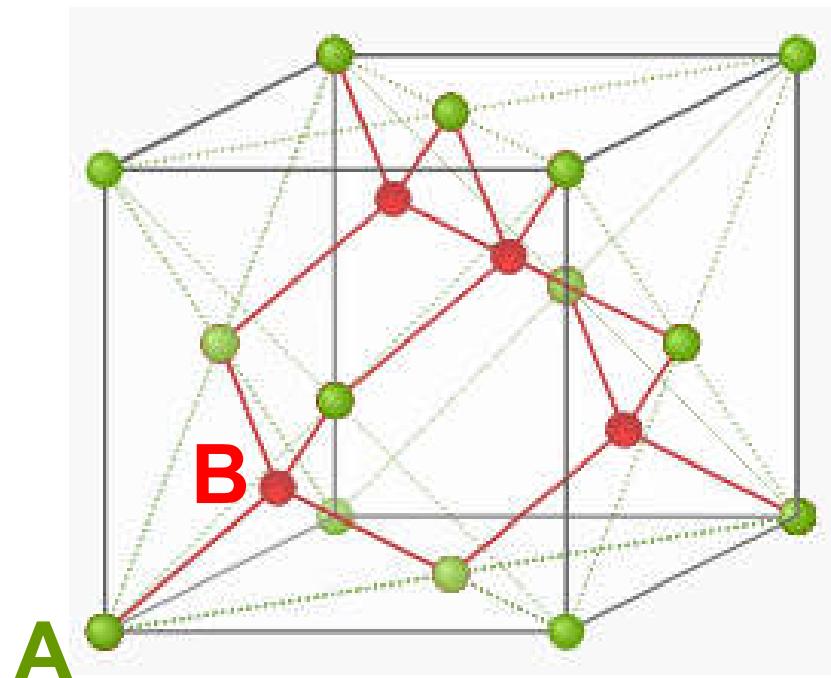
Wurtzite 纤锌矿

- GaN, ZnO, ...



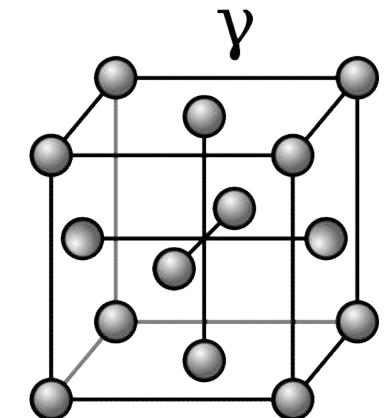
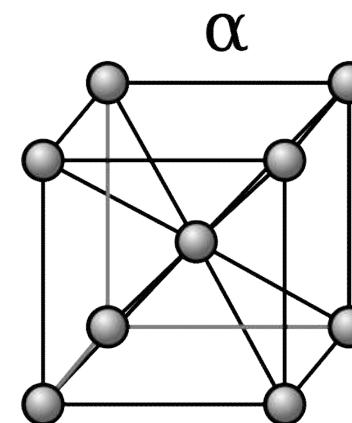
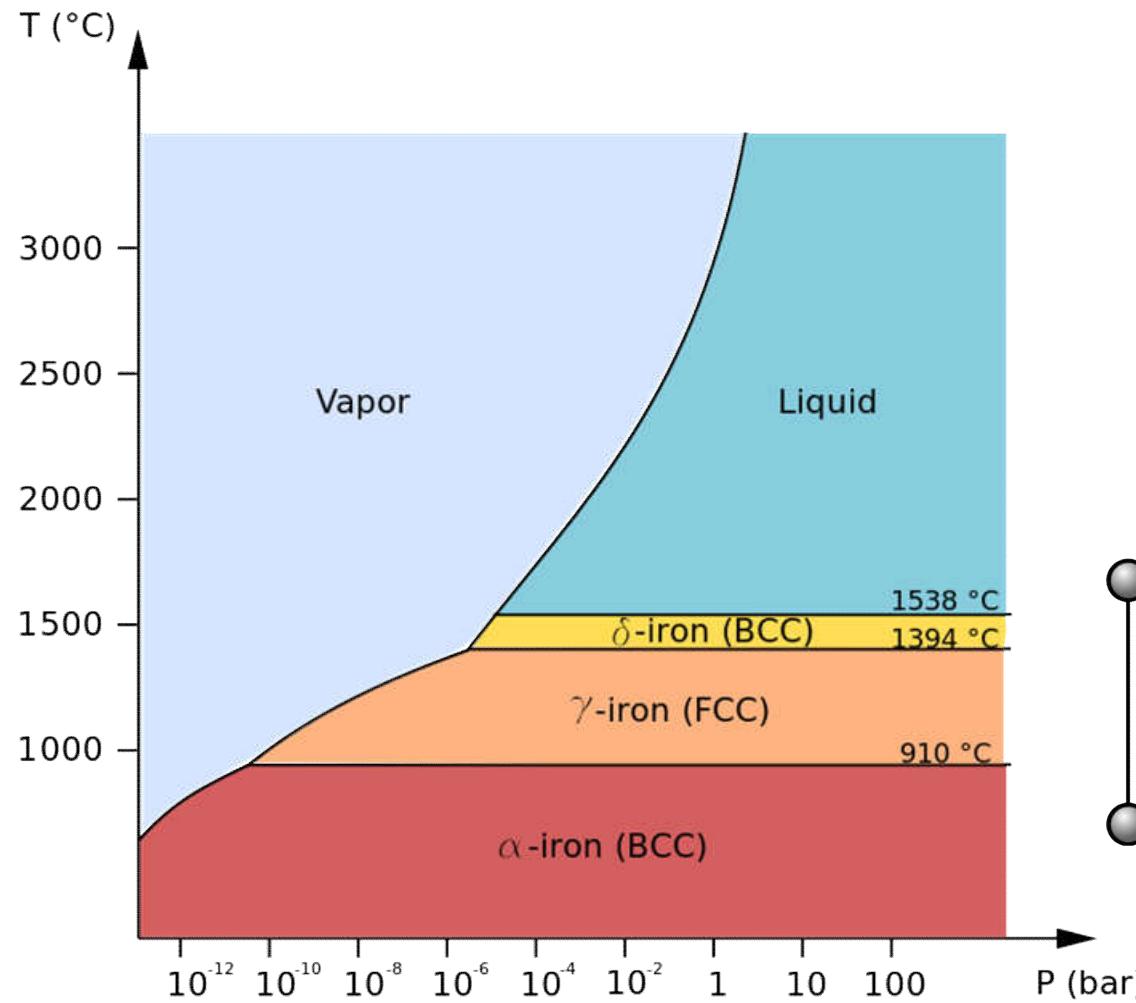
Allotrope 同素异构体

- ZnO can have Zinc Blende or Wurtzite structures

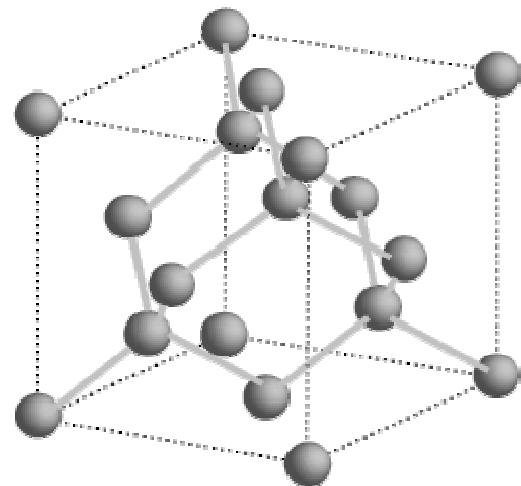


Allotrope 同素异构体

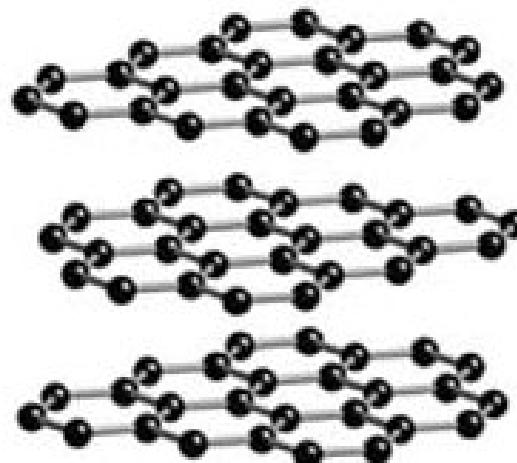
- Fe can have BCC or FCC structures



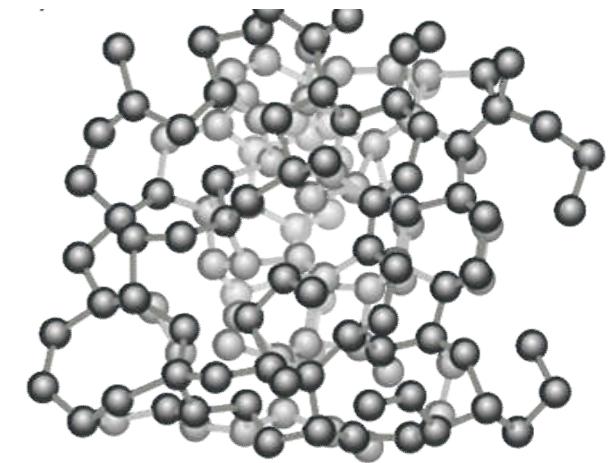
Allotropes for Carbon



diamond



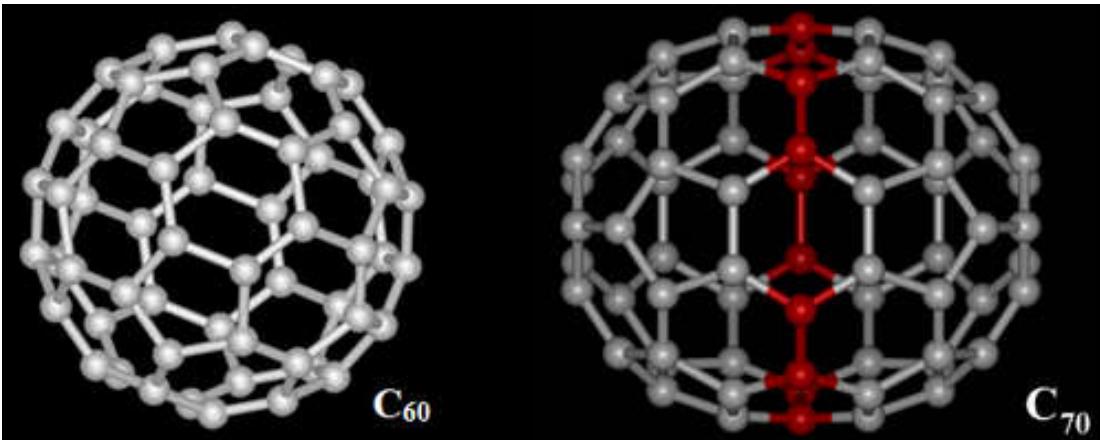
graphite



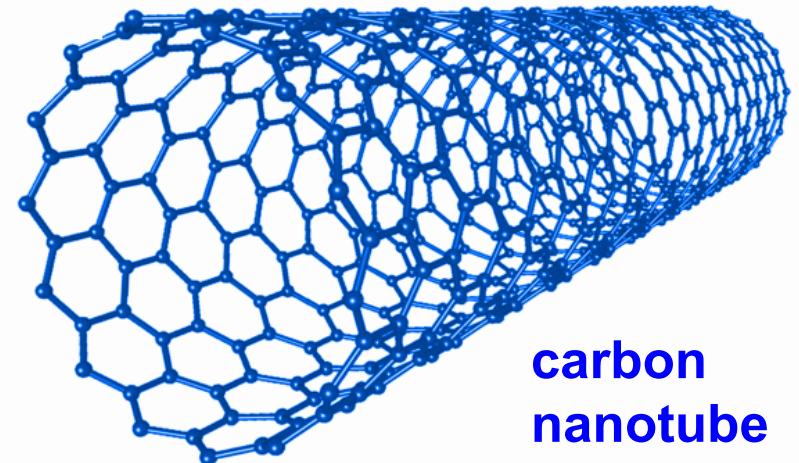
amorphous
carbon

Q: which one is electrically conductive, diamond or graphite?

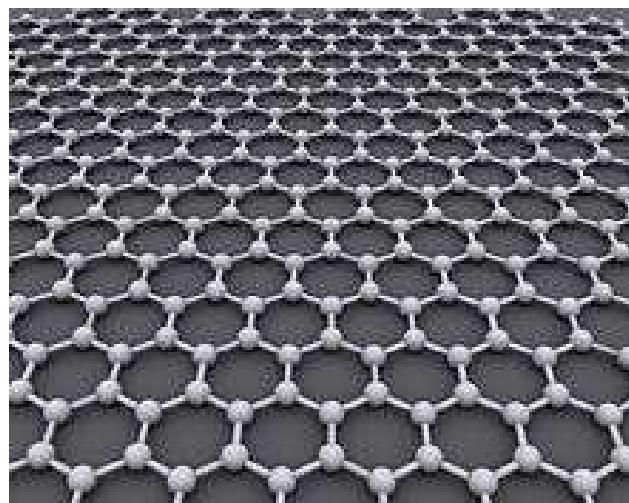
Allotropes for Carbon



H. Kroto, R. Curl, R. Smalley
1996 Nobel Prize in Chemistry



S. Iijima, *Nature* 354, 56 (1991)

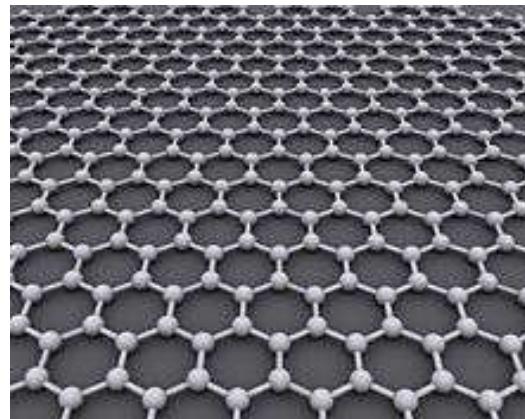


graphene

A. Geim, K. Novoselov
2010 Nobel Prize in Physics

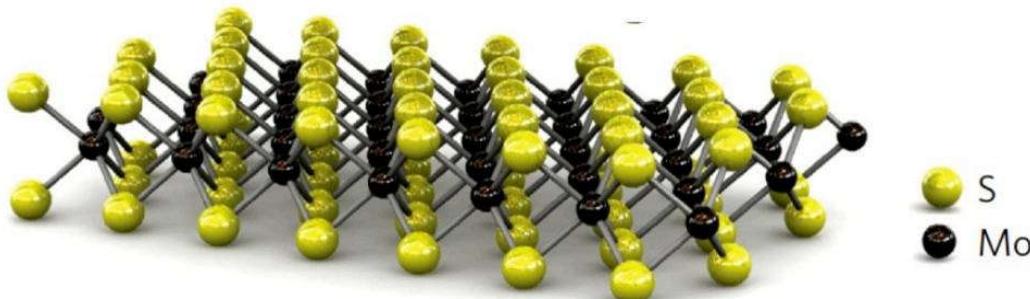
2D Materials

- Single atomic layer crystal

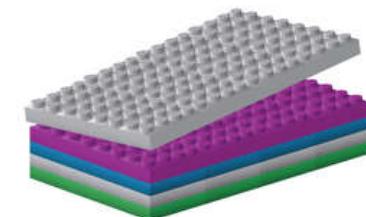
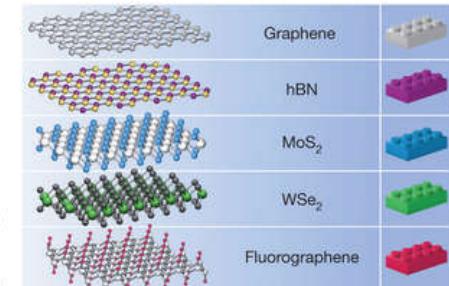
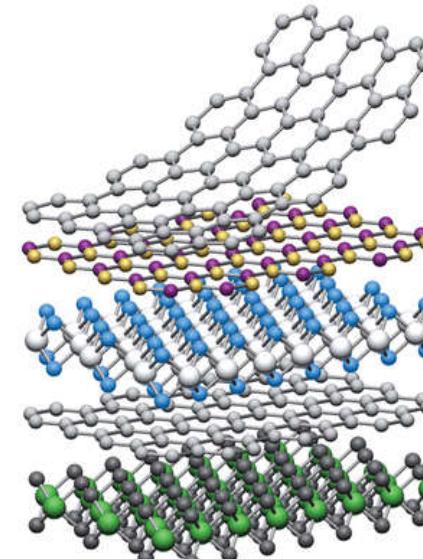


graphene

A. Geim, K. Novoselov
2010 Nobel Prize in Physics



Transition metal dichalcogenide (TMDC)
 MoS_2 , WSe_2 , ...



Materials are Imperfect

"Crystals are like people, it is the defects in them which tend to make them interesting."

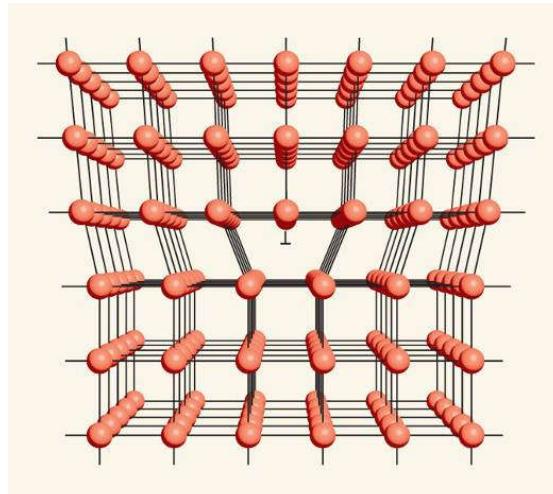
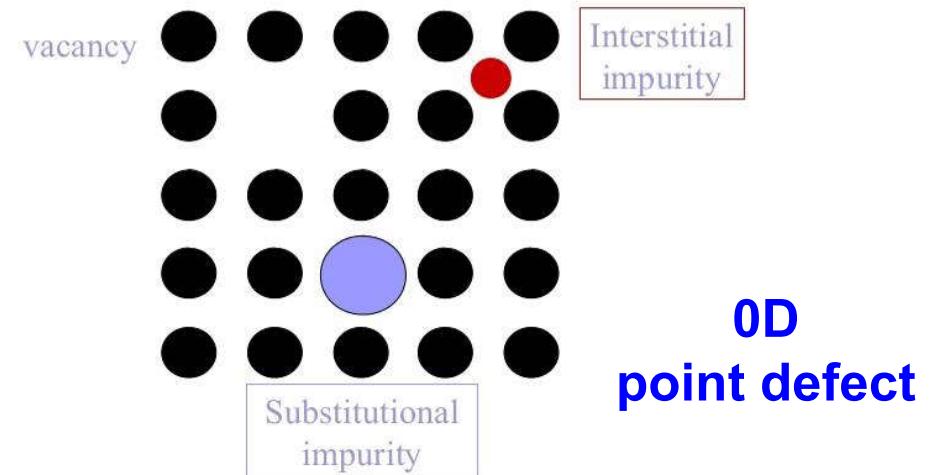
---- Colin Humphreys



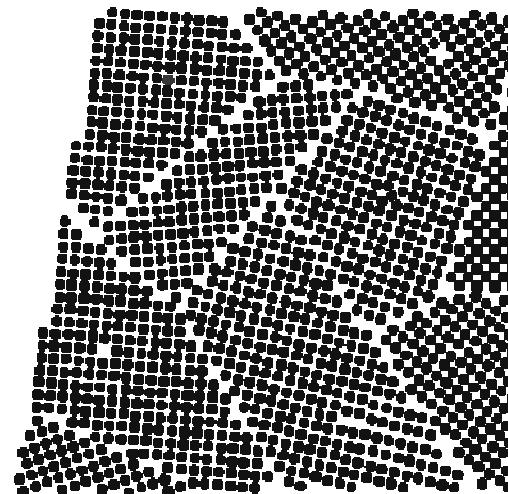
- Defects
 - 0D, 1D, 2D, 3D
- Crystallinity
 - Single Crystal, Polycrystal, Amorphous
 - Quasi-Crystal
 - Liquid Crystal

Defects in Crystals 缺陷

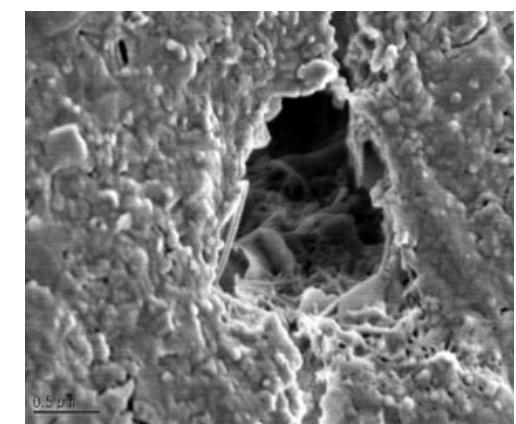
- **0D point defect 点缺陷**
- **1D line defect 线缺陷**
- **2D plane defect 面缺陷**
- **3D volume defect 体缺陷**



**1D
dislocation**

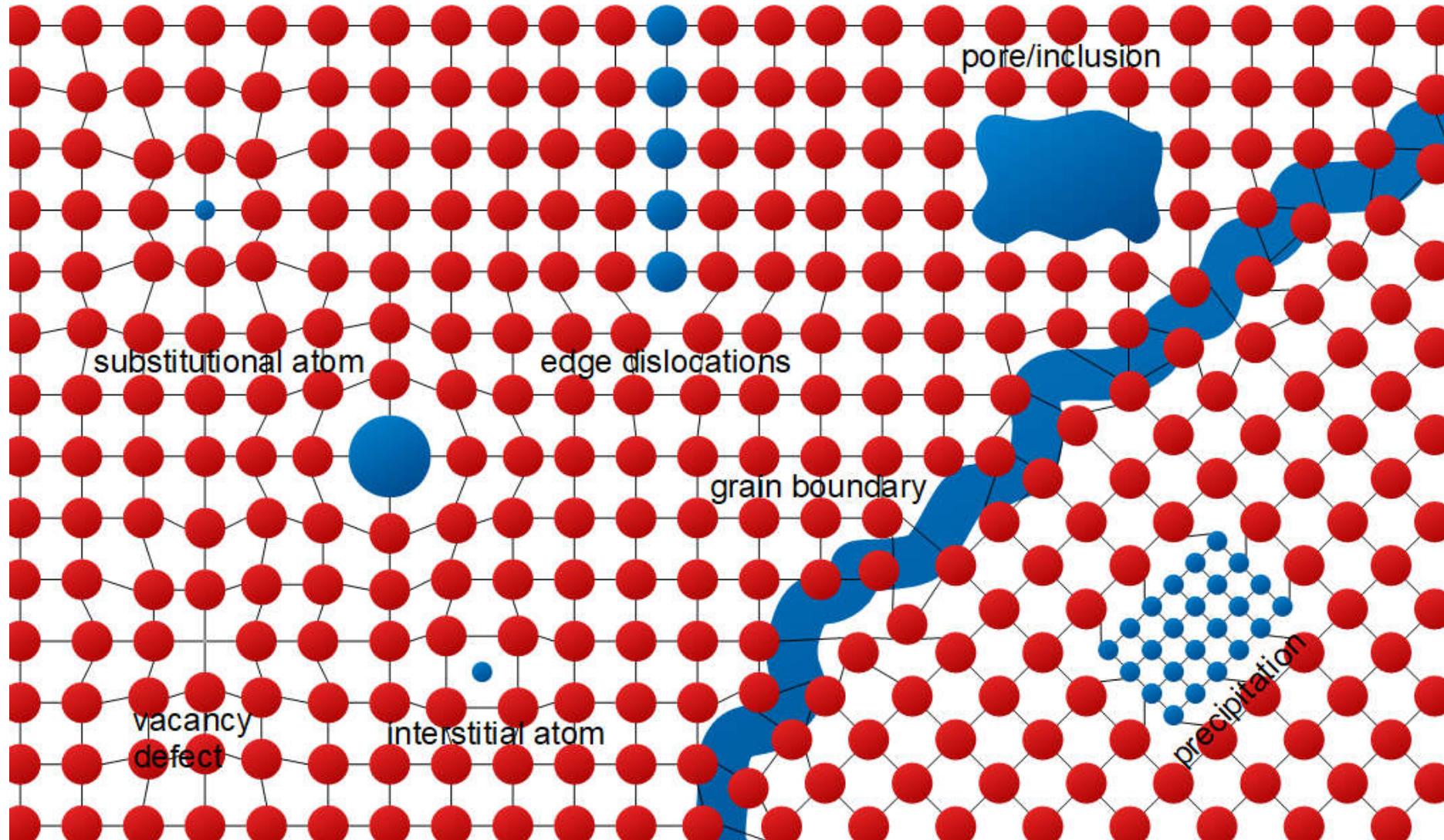


**2D
grain boundary**

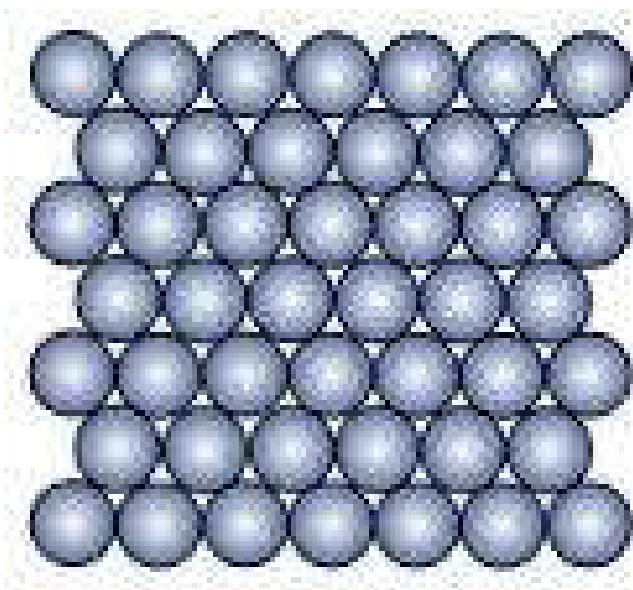


**3D
void / precipitate**

Defects in Crystals 缺陷

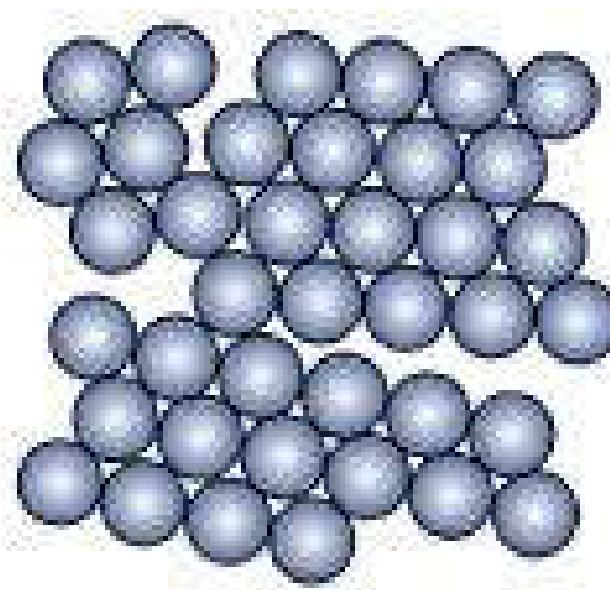


Crystallinity



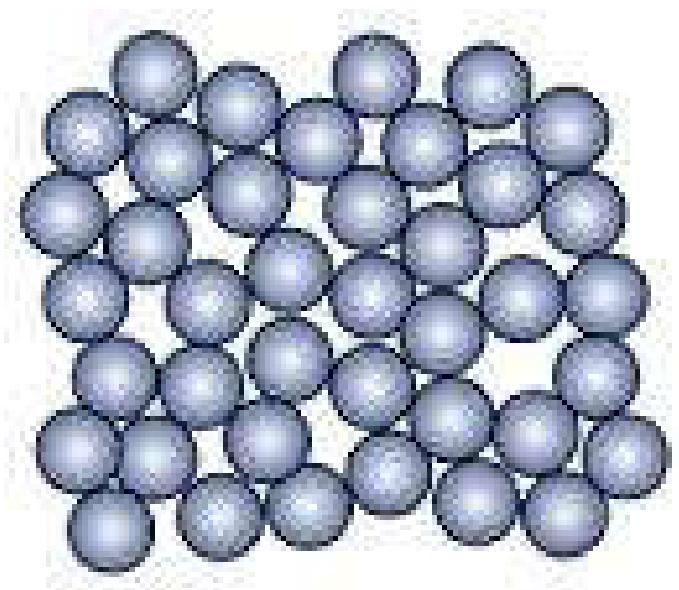
**single crystalline
monocrystalline**

单晶



**polycrystalline
multicrystalline**

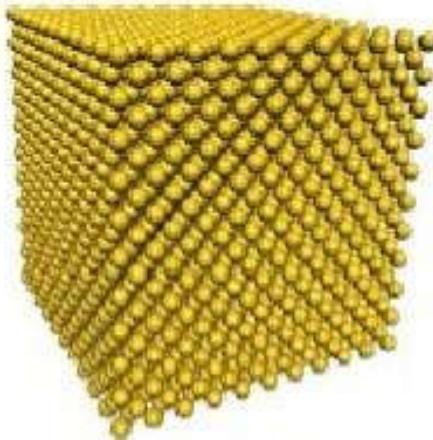
多晶



amorphous

非晶

Single Crystal / Mono Crystal



Quartz

Sugar



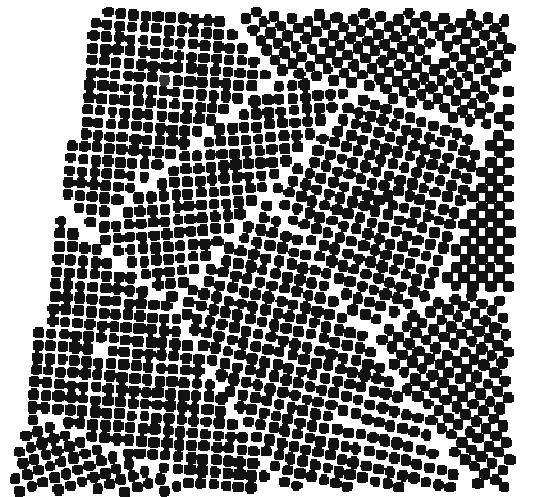
Silicon wafers,
GaAs, GaN, sapphire, ...



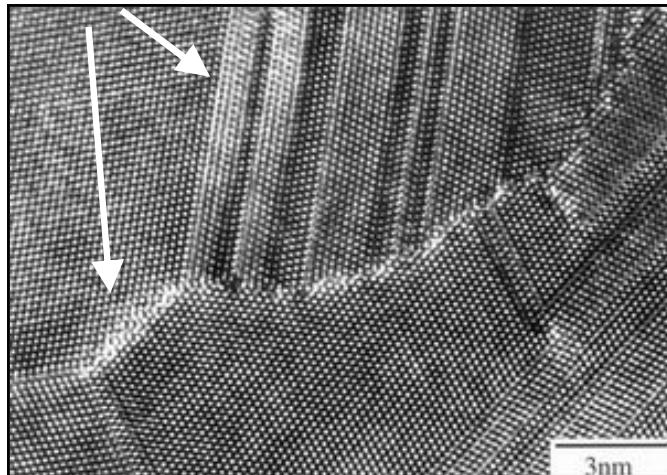
turbine blade



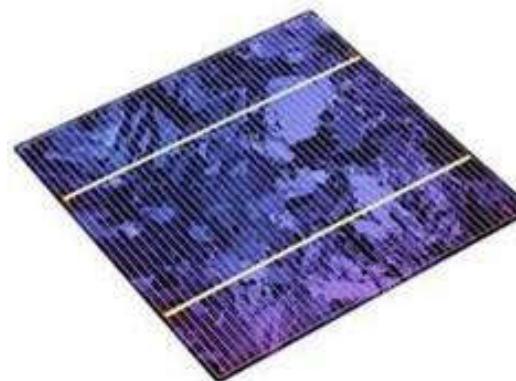
Polycrystal / Multicrystal



grain boundary



polycrystalline silicon



Poly-Crystalline
Solar Cell



Mono-Crystalline
Solar Cell



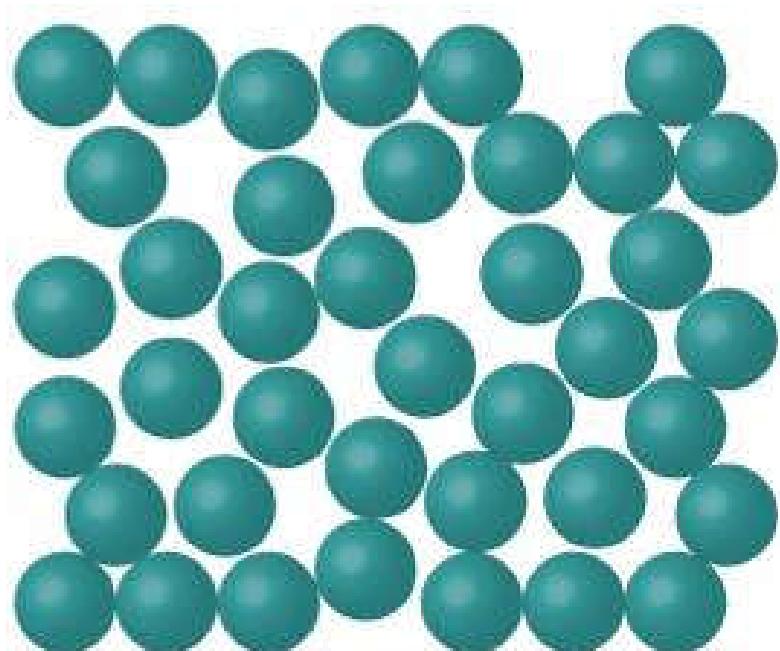
metals



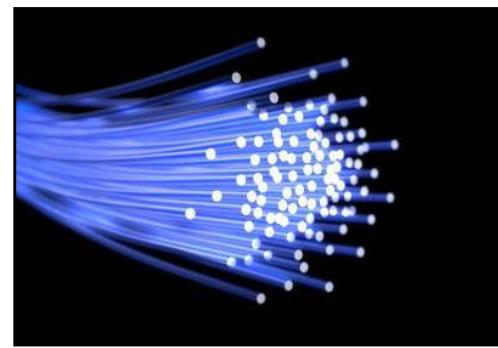
ceramics

Amorphous Materials

- Defects are everywhere ...



Amorphous



silica fiber



glass

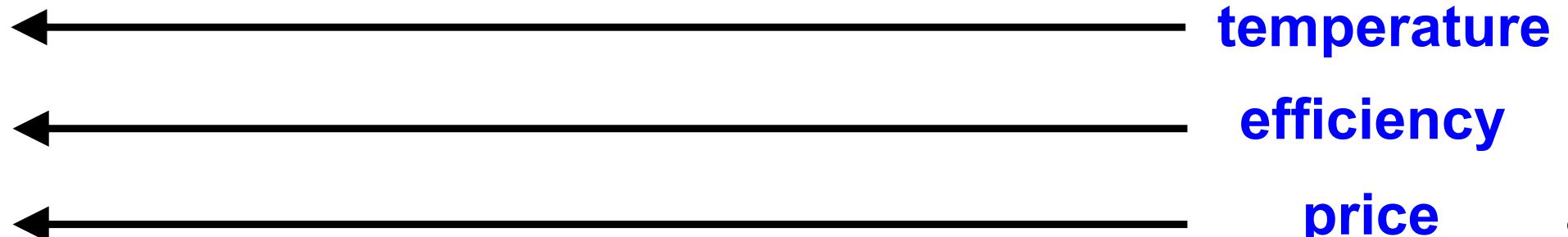
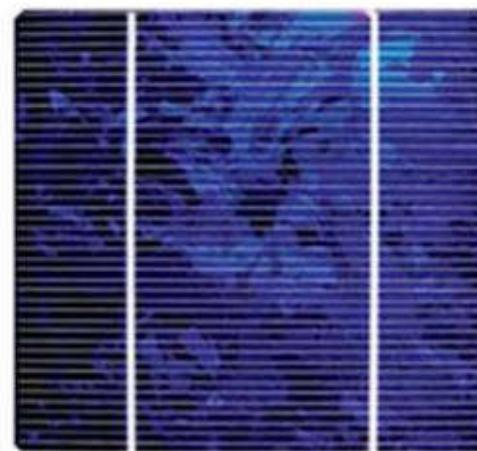


plastics

Crystallinity

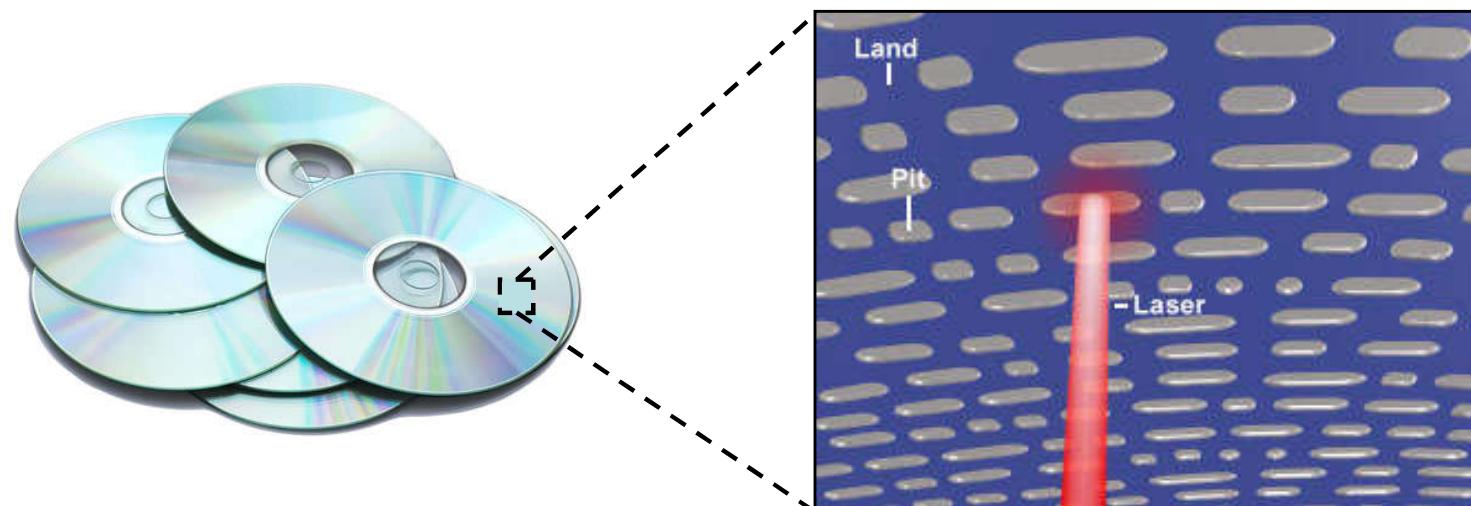
Silicon Solar Cells

Monocrystalline Polycrystalline Amorphous

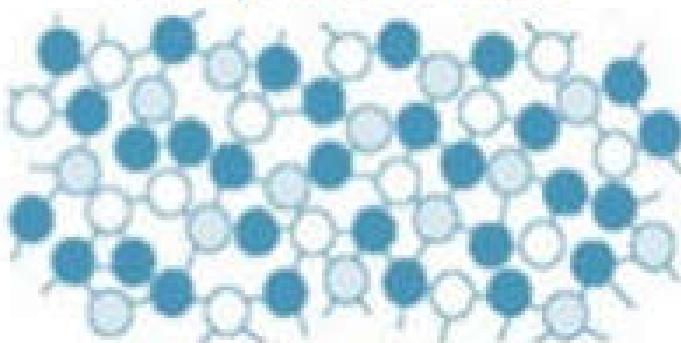


Optical Disc

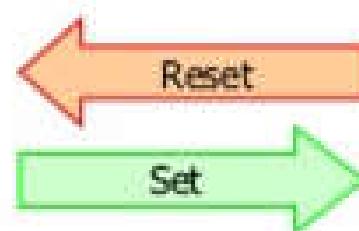
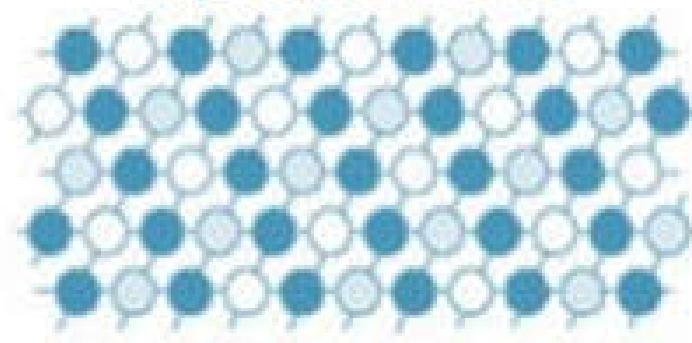
- Phase Change Memory



Amorphous Phase

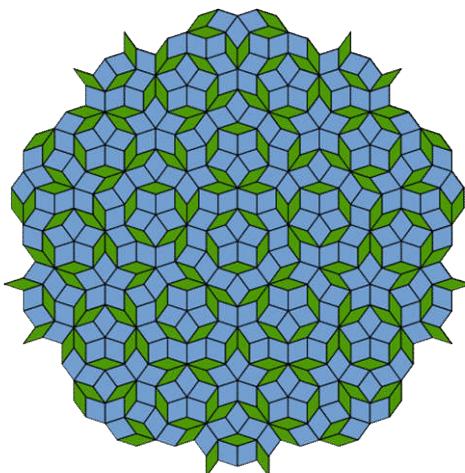


Crystalline Phase

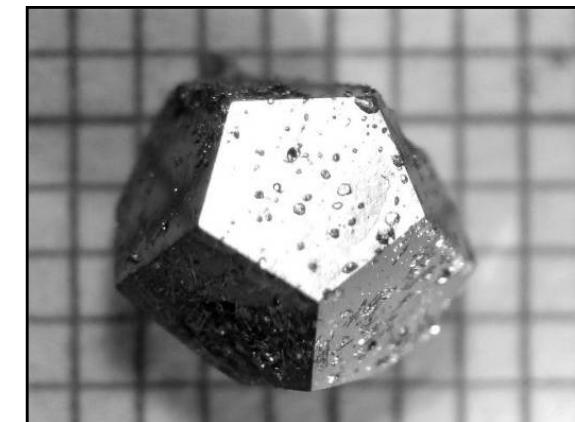
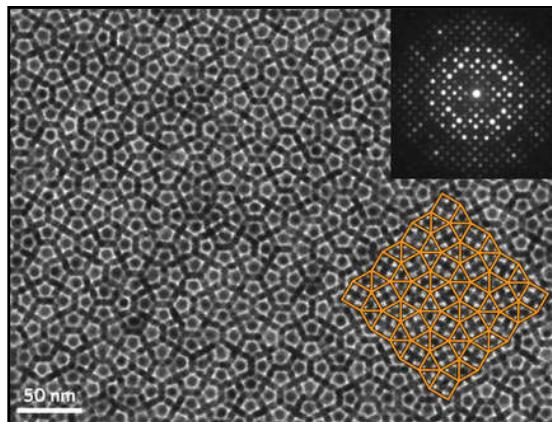


Quasi-Crystal 准晶

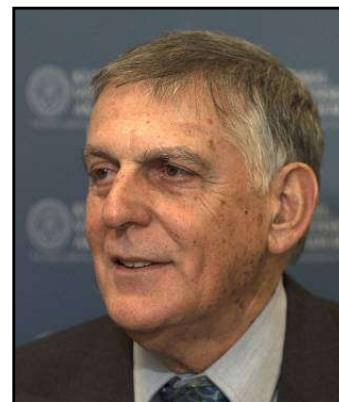
- Neither crystalline nor amorphous
 - 5, 8, 10, or 12-fold rotational symmetry
 - no translational symmetry



Penrose tiling



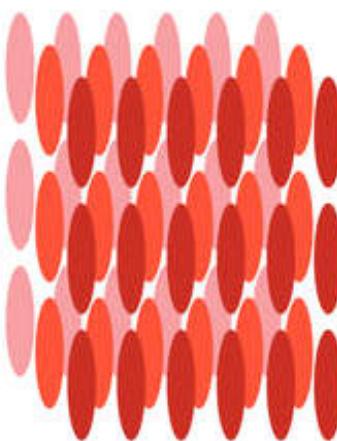
A Ho-Mg-Zn quasicrystal



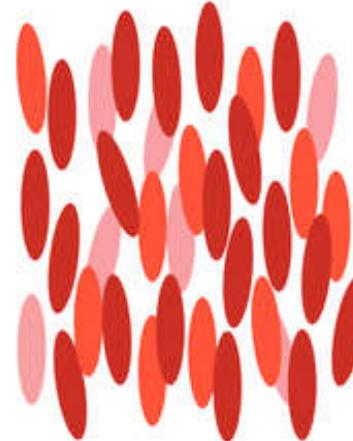
D. Shechtman
2011 Nobel Prize in Chemistry 85

Liquid Crystals 液晶

Crystalline Solid



Liquid Crystal



Isotropic Liquid



Liquid crystal display (LCD)

P. de Gennes
1991 Nobel Prize in Physics

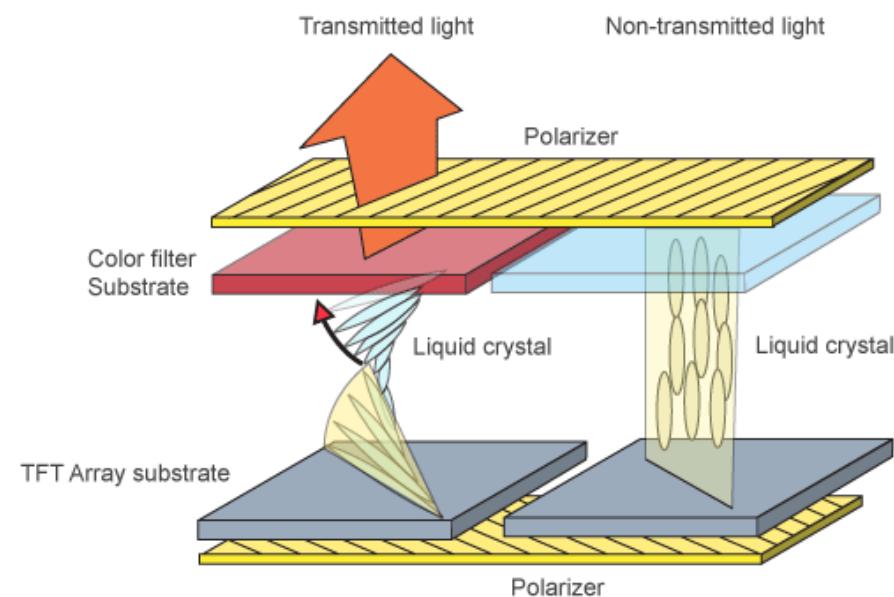


Diagram 2: The Fundamental Photonics of Liquid Crystal (Twisted Nematics)

Thank you for your attention