Fundamentals of Solid State Physics

Semiconductors
- Intrinsic and Extrinsic

Xing Sheng 盛 兴

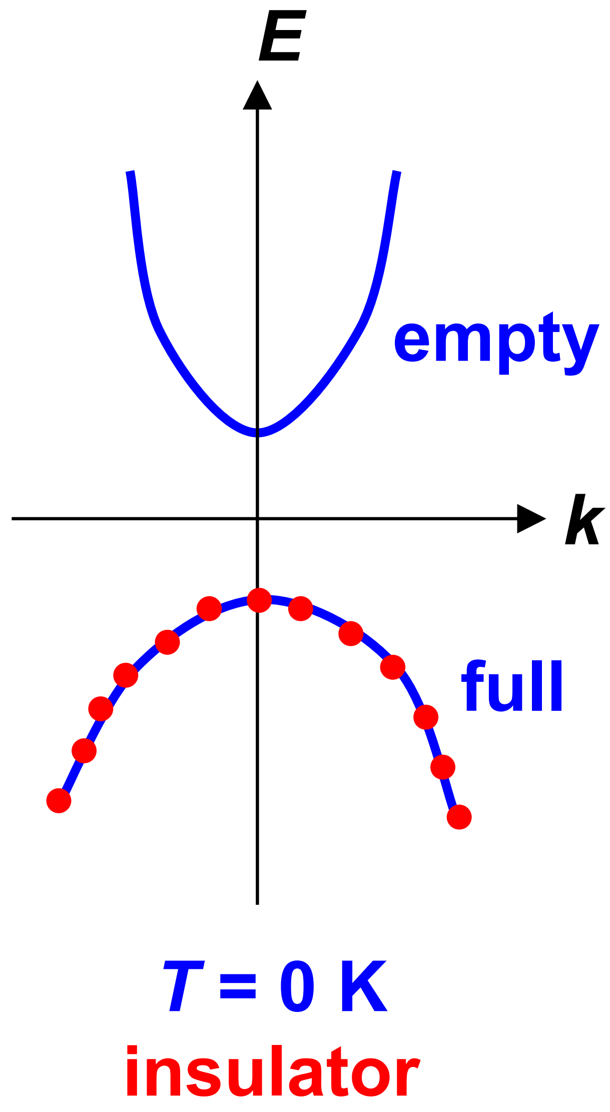
Department of Electronic Engineering
Tsinghua University
xingsheng@tsinghua.edu.cn

Density of Carriers

when $T = 0$ K

$$n_c = N_c(T)e^{-(E_c-\mu)/k_BT} = 0$$

$$p_v = P_v(T)e^{-(\mu-E_v)/k_BT} = 0$$



Density of Carriers

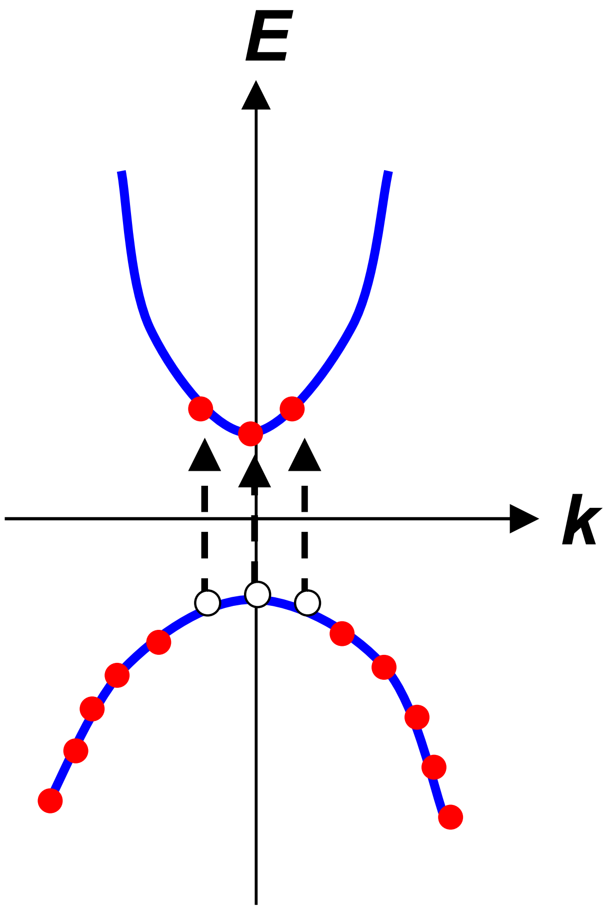
when T > 0 K

$$n_c = N_c(T)e^{-(E_c-\mu)/k_BT} > 0$$

$$p_v = P_v(T)e^{-(\mu-E_v)/k_BT} > 0$$

conductivity

$$\sigma = n_c e\mu_e + p_v e\mu_h$$



T > 0 K
thermalization 热激发
CB and VB are partly filled
conductor

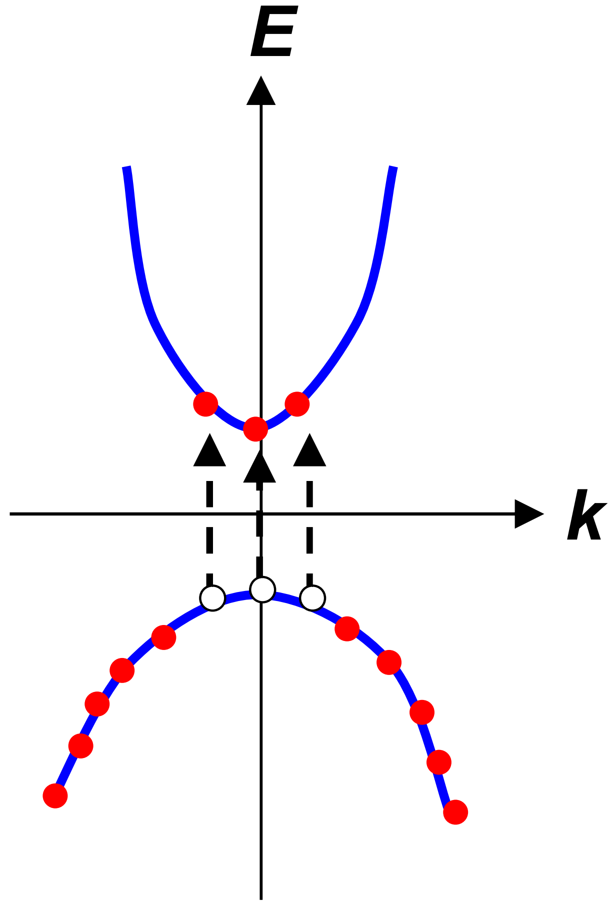
Density of Carriers

when T > 0 K

$$n_c = N_c(T)e^{-(E_c-\mu)/k_BT} > 0$$

$$p_v = P_v(T)e^{-(\mu-E_v)/k_BT} > 0$$

$$\boxed{\begin{aligned} n_c p_v &= N_v(T)P_v(T)e^{-(E_c-E_v)/k_BT} \\ &= N_v(T)P_v(T)e^{-E_g/k_BT} \end{aligned}}$$



mass action law

at equilibrium, $n_c p_v$ is a constant

Intrinsic Semiconductor 本征半导体

pure, no impurity, charge balance

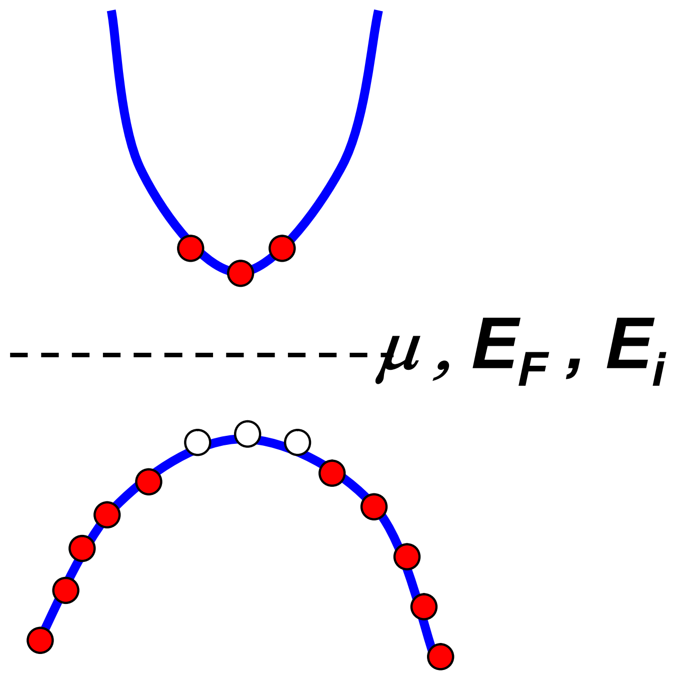
$$\boxed{n_c = p_v = n_i}$$

$$n_i = \sqrt{N_v(T)P_v(T)} \cdot e^{-E_g/2k_BT}$$

$$\mu = E_F = E_i$$

$$= E_c - k_BT\ln\left(\frac{N_c(T)}{n_i}\right)$$

$$= E_v + k_BT\ln\left(\frac{P_v(T)}{n_i}\right)$$



Intrinsic Semiconductor 本征半导体

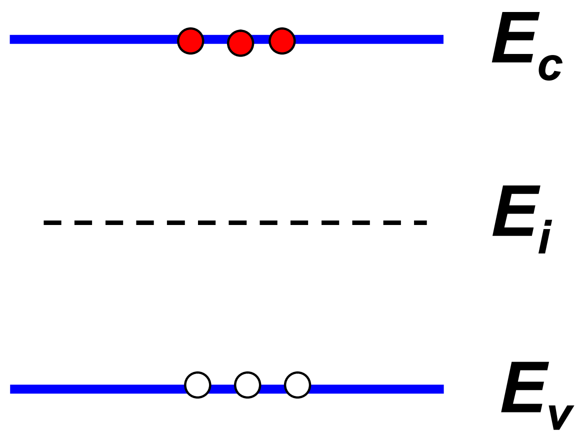
pure, no impurity, charge balance

$$\boxed{n_c = p_v = n_i}$$

$$\boxed{n_i = \sqrt{N_v(T) P_v(T)} \cdot e^{-E_g/2k_B T}}$$

$$\mu = E_F = E_i$$

$$= E_v + \frac{1}{2} E_g + \frac{3}{4} k_B T \ln\left(\frac{m_h^*}{m_e^*}\right)$$



The chemical potential / Fermi level / Intrinsic level is closed to the middle gap

Intrinsic Semiconductor 本征半导体

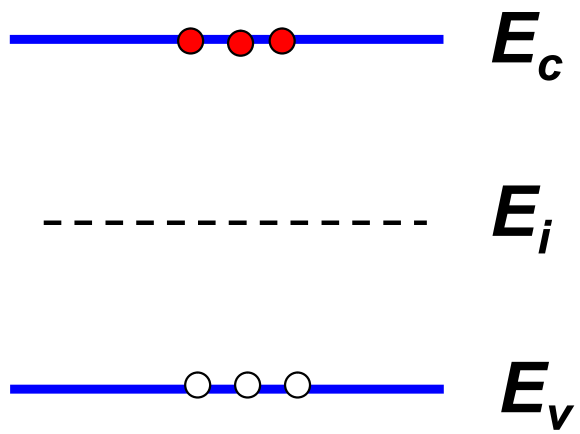
Example: intrinsic Si at 300 K

$$n_c = p_v = n_i \approx 10^{10}\,\mathrm{cm}^{-3}$$

$$\mu_e = 1500\ \mathrm{cm}^2/\mathrm{V/s}$$

$$\mu_h = 450\ \mathrm{cm}^2/\mathrm{V/s}$$

$$\sigma = n_c e \mu_e + p_v e \mu_h \approx 10^{-6}\ \mathrm{S/cm}$$

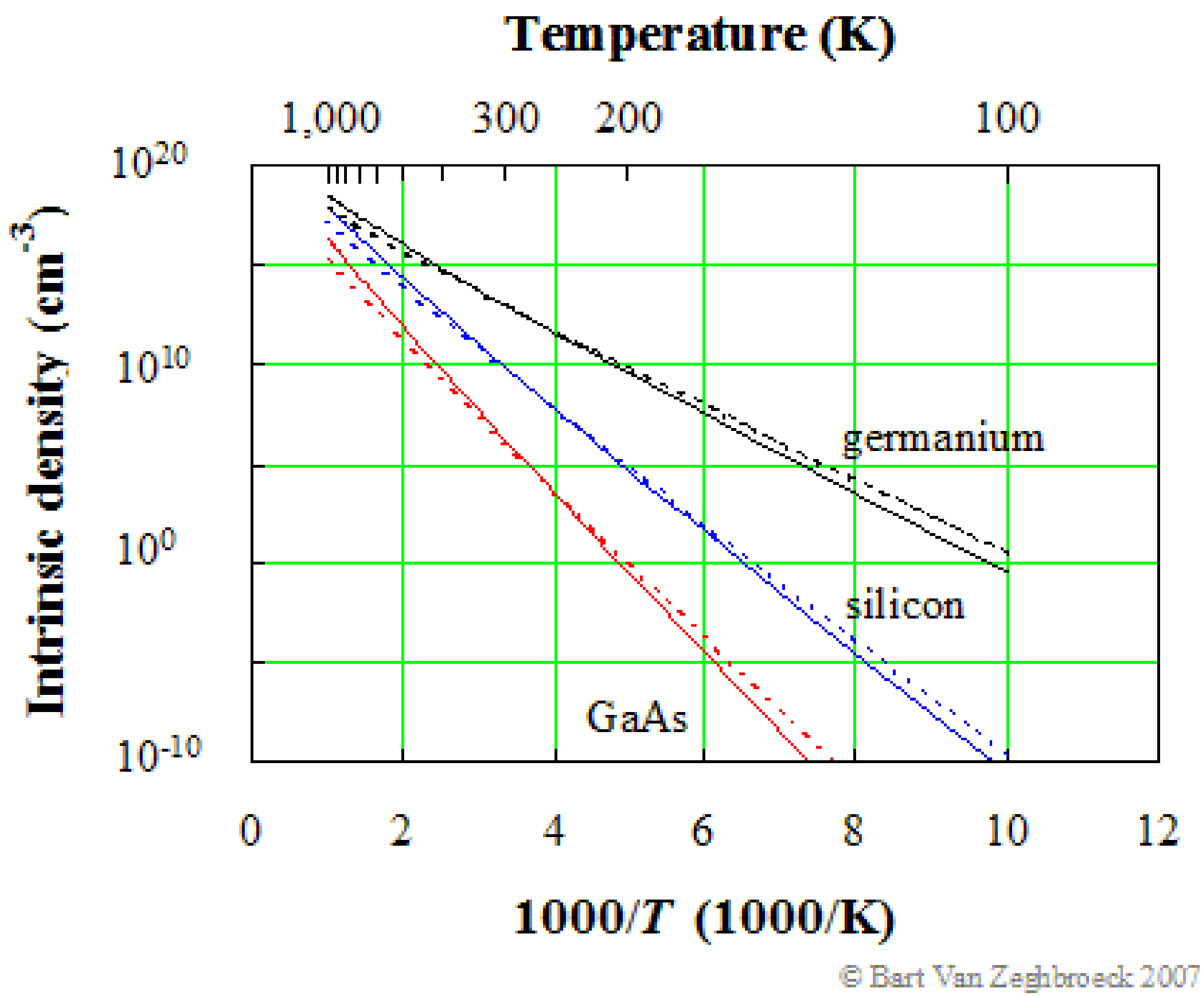


Intrinsic Semiconductor 本征半导体

temperature dependence of carrier concentration

$$n_i \propto T^{3/2} \cdot e^{-E_g/2k_BT}$$

$$\ln n_i \sim -\frac{E_g}{2k_BT}$$



Intrinsic Semiconductor 本征半导体

temperature dependence of carrier mobility μ

$$\mu = e\frac{\tau}{m^*}$$

at low *T*

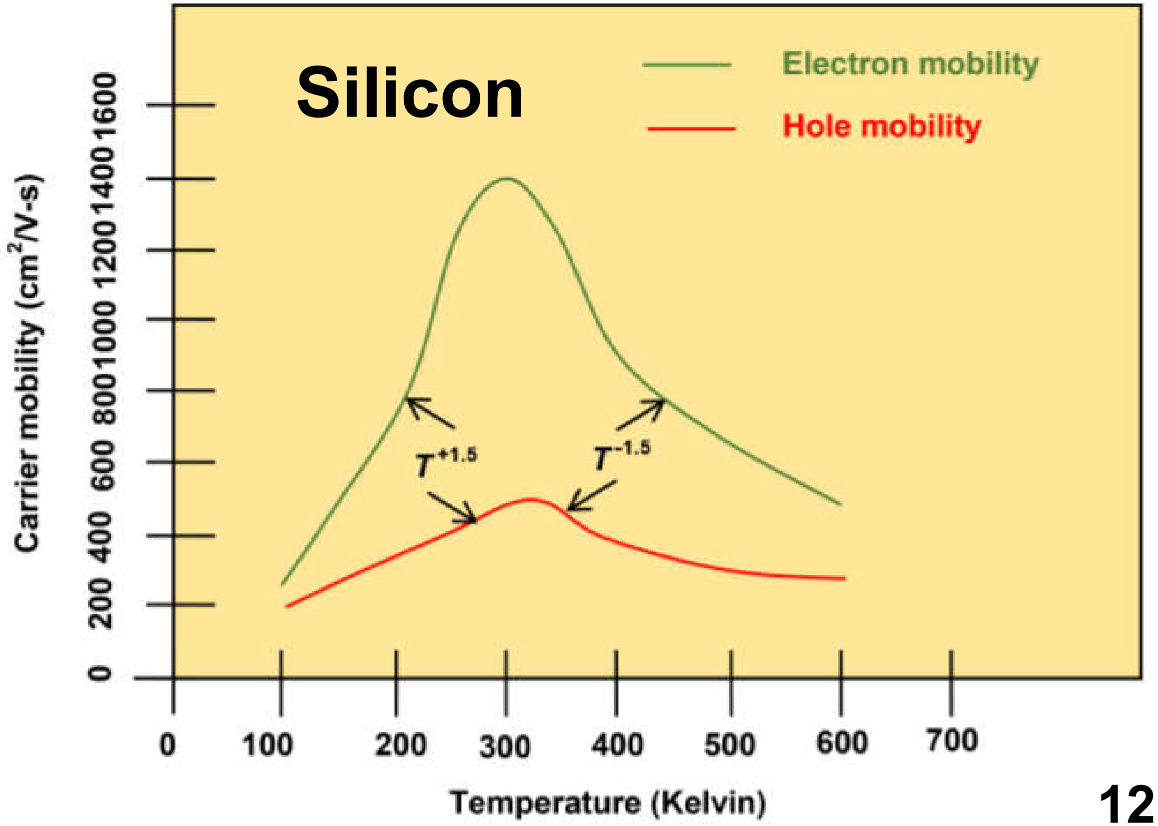
$$\mu \sim T^{3/2}$$

impurity scattering

at high *T*

$$\mu \sim T^{-3/2}$$

lattice scattering

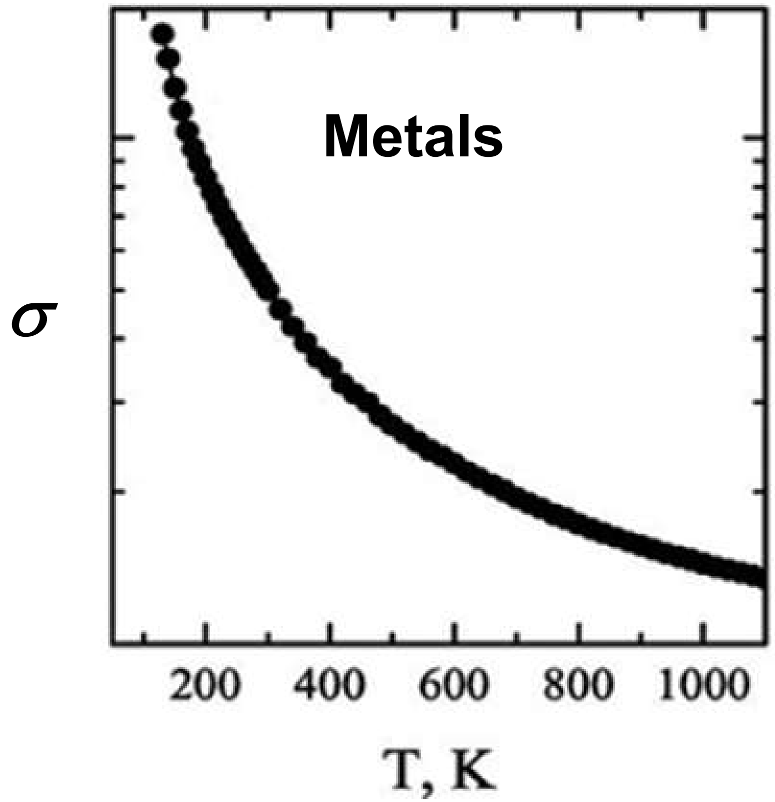


Temperature Dependence of σ

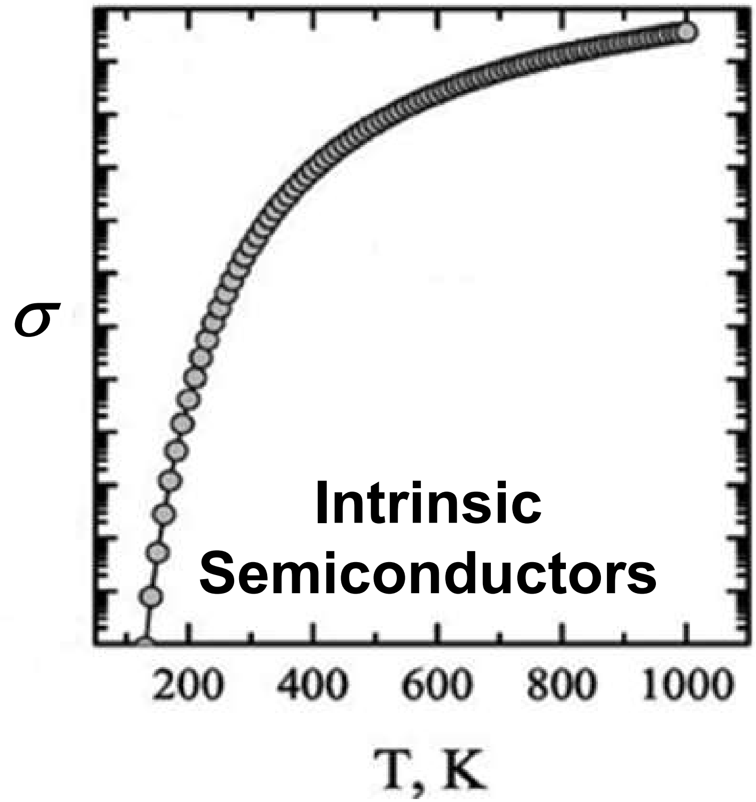
Metals and semiconductors have different temperature dependences of σ

$$\sigma = ne\mu$$

$$\mu = e\frac{\tau}{m^*}$$



μ dominates



***n* dominates**

Conductivity of Semiconductor

metals

	conductivity σ (S/m)
Ag	$6.3*10^7$
Al	$3.5*10^7$

insulators

wood	$10^{-14} \sim 10^{-16}$
glass	$10^{-11} \sim 10^{-15}$

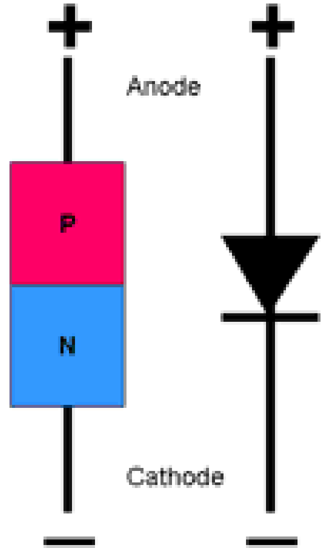
silicon with doping

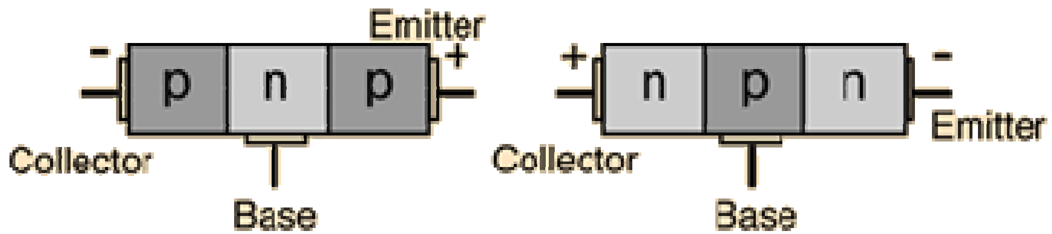
	conductivity σ (S/m)
0	10^{-6}
$1 / 10^9$	10^{-1}
$1 / 10^6$	10^2
$1 / 10^3$	10^5

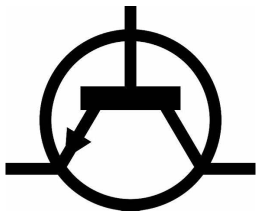
at T = 300 K

Doping Makes Functional Devices

ingots and wafers

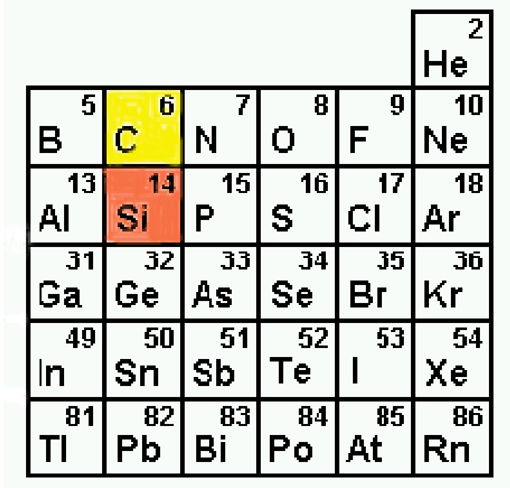


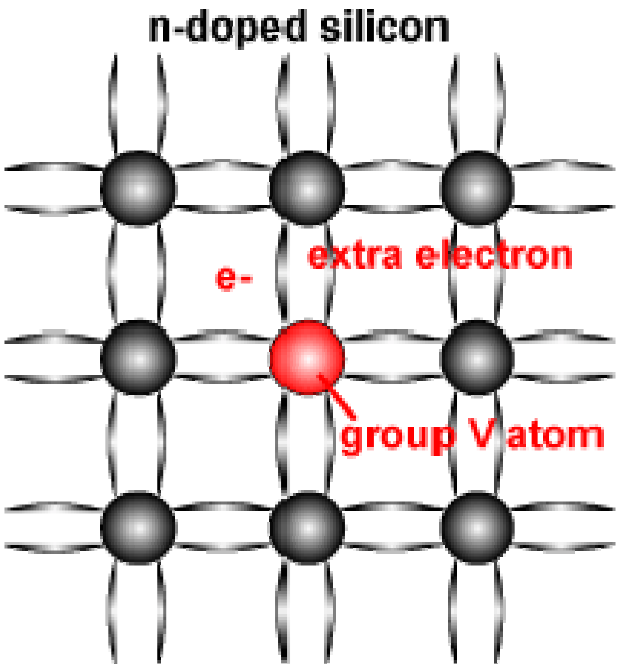


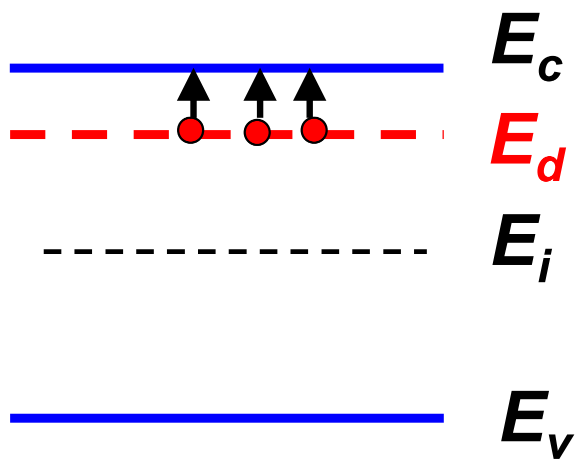


Extrinsic Semiconductor 掺杂半导体

For Si and Ge (group IV)
add Group V dopants: P, As, Sb, ...
create level E_d close to E_c with extra electrons
these electrons can be excited at low temperature
making Si more conductive
donor 施主 ----> n-doping



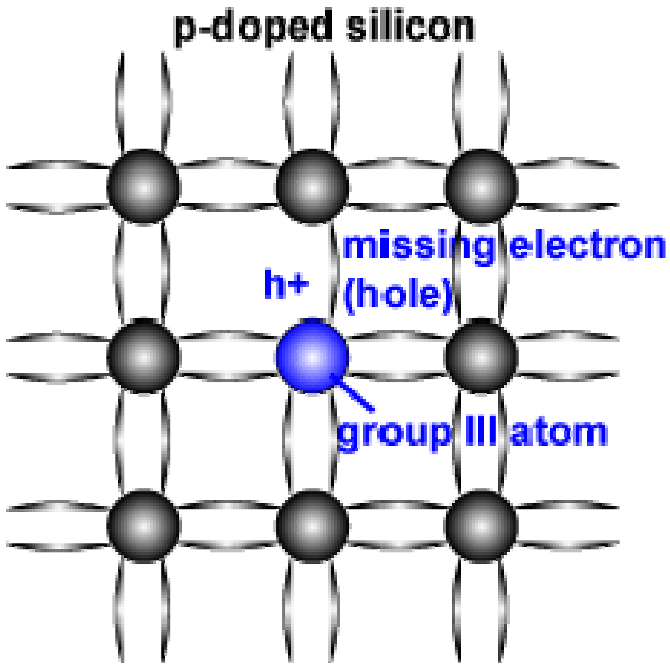


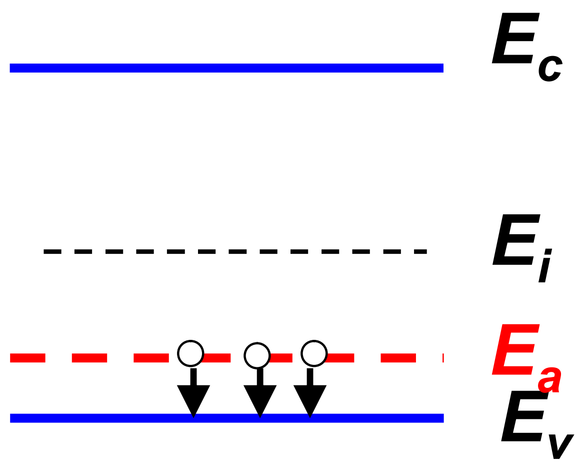


Extrinsic Semiconductor 掺杂半导体

For Si and Ge (group IV)
add Group III dopants: B, Al, Ga, ...
create level E_a close to E_v with extra holes
these holes can be excited at low temperature
making Si more conductive
acceptor 受主 ----> p-doping

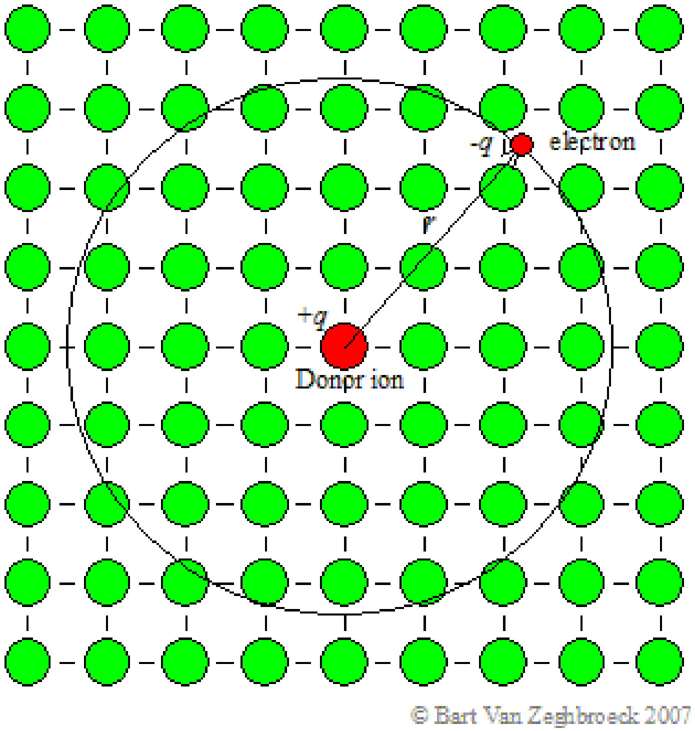
					2 He
5 B	6 C	7 N	8 O	9 F	10 Ne
13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn





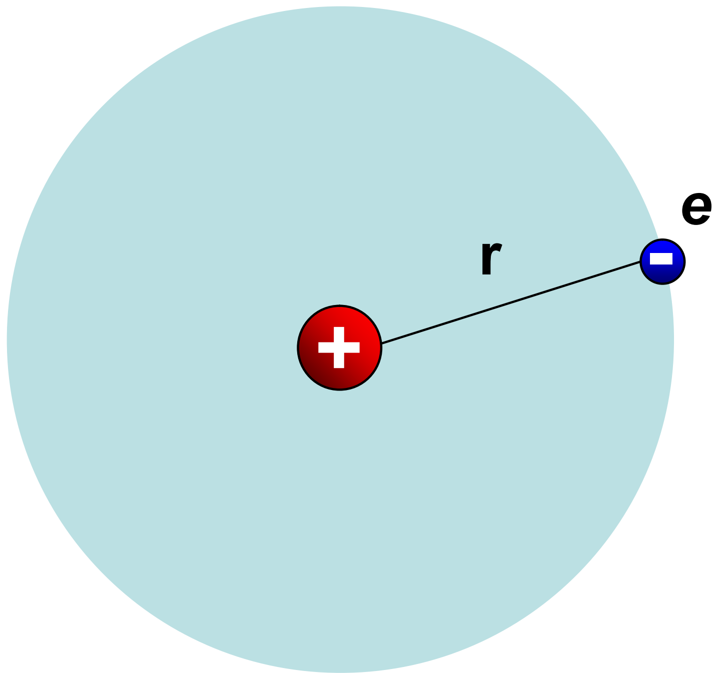
Ionization Energy of Dopants 电离能

Hydrogen-like Model



$$\Delta E = 13.6 \frac{m^*}{m_0} \frac{1}{\varepsilon_r^2} \text{eV}$$

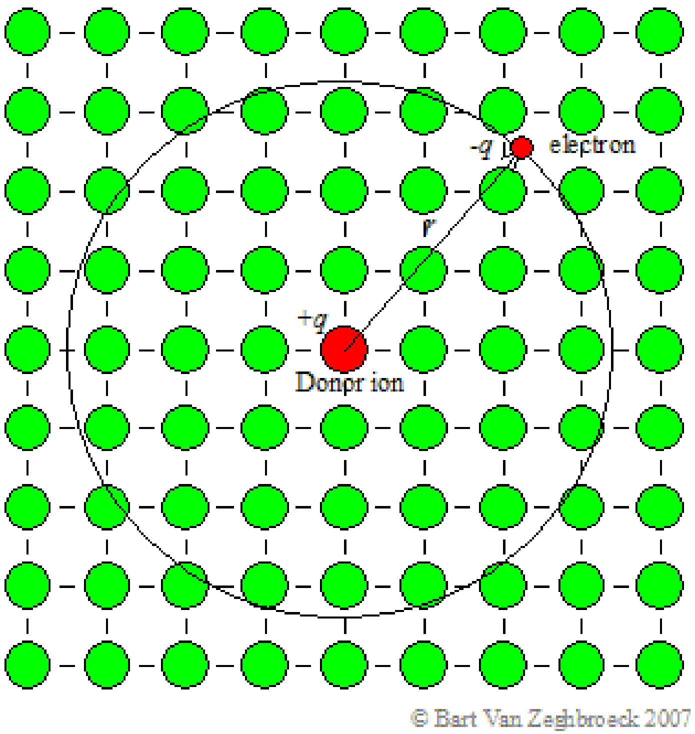
Hydrogen Atom



$$E_1 = -\frac{m_0 e^4}{8\varepsilon_0^2 h^2} = -13.6\,\text{eV}$$

Ionization Energy of Dopants 电离能

Hydrogen-like Model

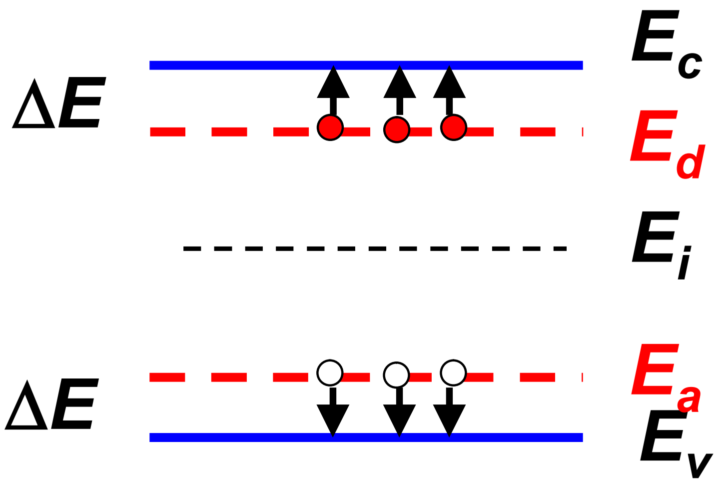


$$\Delta E = 13.6 \frac{m^*}{m_0} \frac{1}{\varepsilon_r^2} \text{eV}$$

m* **- effective mass ~ 0.1*m*$_0$**

ε_r - relative dielectric constant ~ 10

→ **ΔE ~ 0.01 eV**



Ionization Energy of Dopants 电离能

Table 28.2
LEVELS OF GROUP V (DONORS) AND GROUP III (ACCEPTORS) IMPURITIES IN SILICON AND GERMANIUM

GROUP III ACCEPTORS (TABLE ENTRY IS $\mathcal{E}_a - \mathcal{E}_v$)

	B	Al	Ga	In	Tl
Si	0.046 eV	0.057	0.065	0.16	0.26
Ge	0.0104	0.0102	0.0108	0.0112	0.01

GROUP V DONORS (TABLE ENTRY IS $\mathcal{E}_c - \mathcal{E}_d$)

	P	As	Sb	Bi
Si	0.044 eV	0.049	0.039	0.069
Ge	0.0120	0.0127	0.0096	—

ROOM TEMPERATURE ENERGY GAPS ($E_g = \mathcal{E}_c - \mathcal{E}_v$)

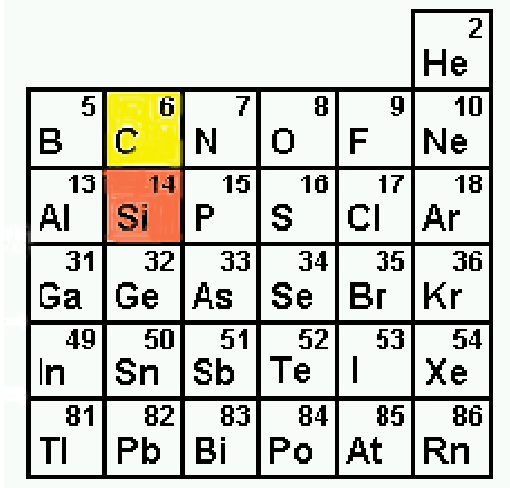
Si	1.12 eV
Ge	0.67 eV

Source: P. Aigrain and M. Balkanski, *Selected Constants Relative to Semiconductors*, Pergamon, New York, 1961.

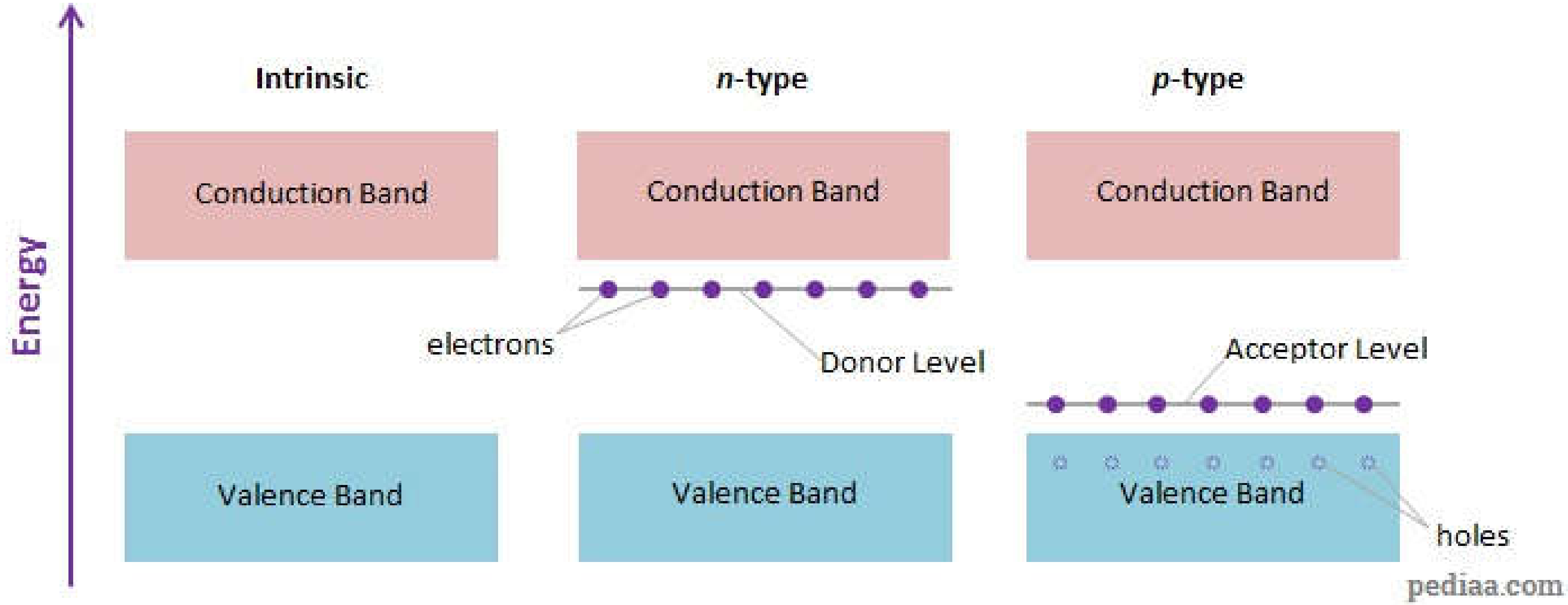
Ashcroft & Mermin p.580

Doping in Silicon

For Si (and Ge):
p dopant: **B, Al, Ga, ...**
n dopant: **P, As, Sb, ...**

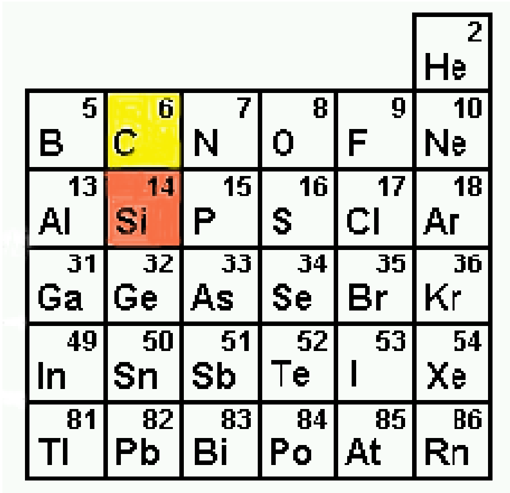


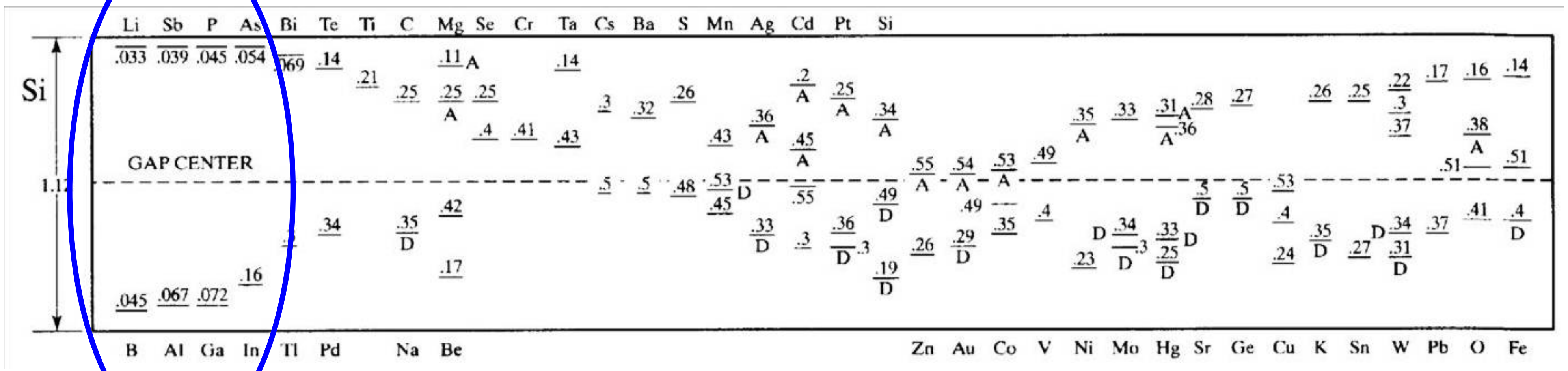
These dopants are shallow level defects, which can be excited to generated carriers closed to E_c or E_v (~ 0.01 eV), room temperature k_BT ~0.03 eV



Other Defects in Silicon

Many other elements are deep level defects, which cannot be excited, making traps for carriers and Si less conductive.





dopants

deep level defects
Na, K, Au, Cu, Fe, O, ...

Doping in GaAs

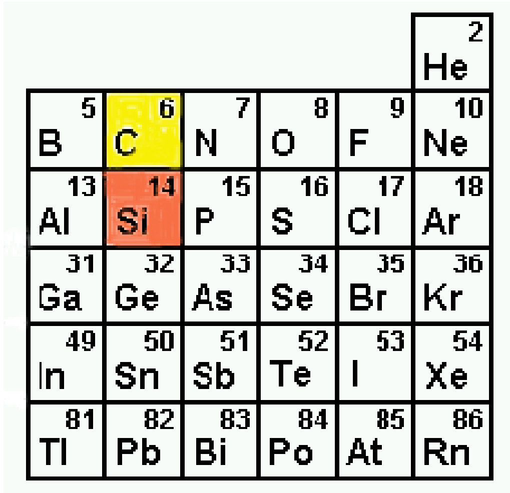
For GaAs:

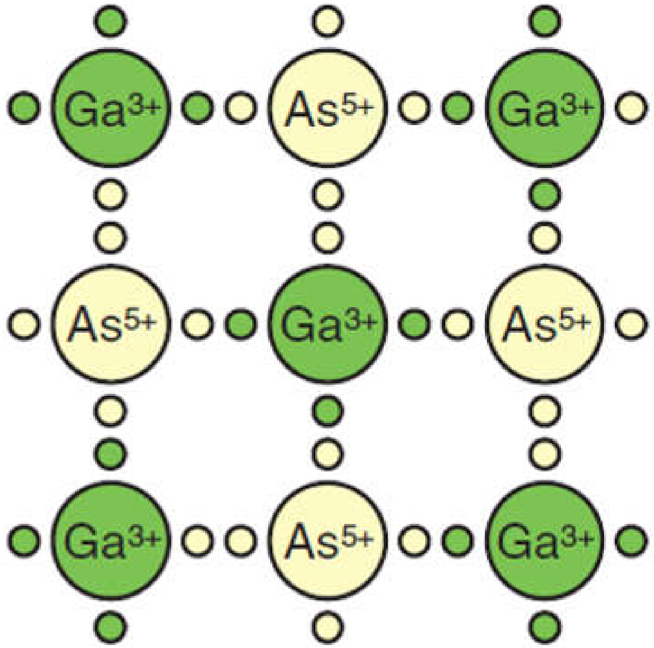
p dopant:

- replace Ga: Mg, Zn, Be
- replace As: C
- ...

n dopant:

- replace As: Se
- replace Ga: Si, Ge
- ...



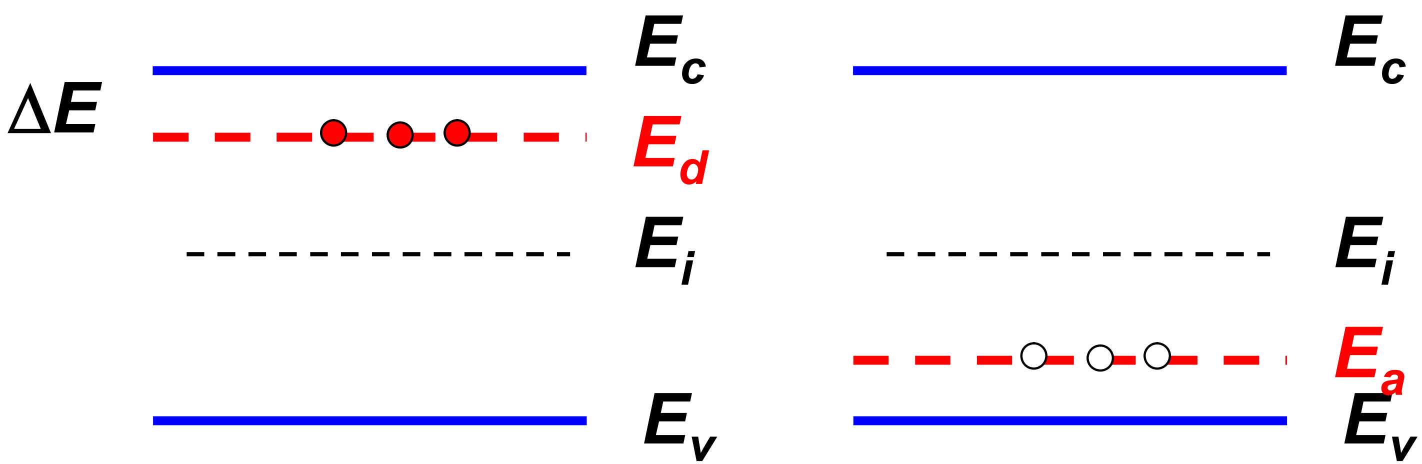


Extrinsic Semiconductor 掺杂半导体

when T = 0 K, carriers cannot be excited

$n_c \to 0$ $p_v \to 0$ **insulator**

electrons and holes are frozen



Extrinsic Semiconductor 掺杂半导体

when T > 0 K, carriers can be excited (ionization 电离)

shallow level dopants are easily activated at room temperature T = 300 K.

mass action law $$\boxed{n_c p_v = N_v(T)P_v(T)e^{-E_g/k_BT} = n_i^2}$$

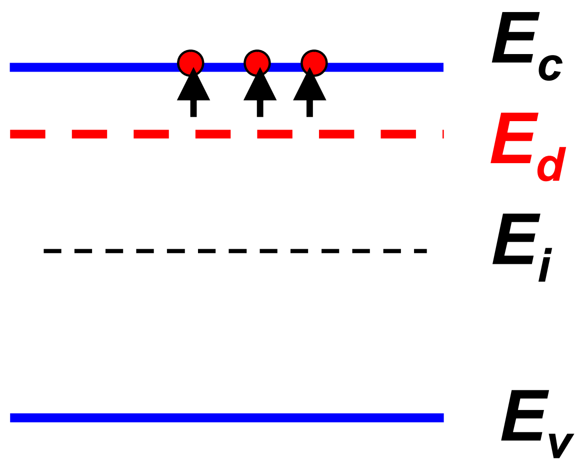
at equilibrium, $n_c p_v$ is a constant

If $N_D >> n_i$

For n-doping

$$\boxed{n_c = N_D}$$

$$\boxed{p_v = n_i^2 / n_c}$$



N_D - concentration of donor (cm^{-3})

Extrinsic Semiconductor 掺杂半导体

when T > 0 K, carriers can be excited (ionization)

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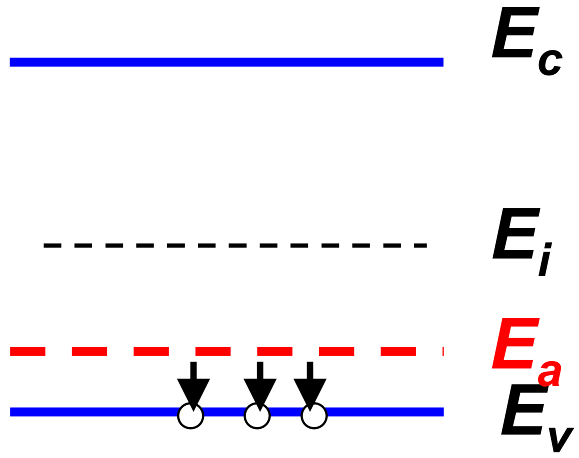
at equilibrium, $n_c p_v$ is a constant

If $N_A >> n_i$

For n-doping

$$\boxed{p_v = N_A}$$

$$\boxed{n_c = n_i^2 / p_v}$$



N_A - concentration of acceptor (cm^{-3})

Example - Silicon

For intrinsic Si at room temperature (T = 300 K)

$$\sigma = n_c e\mu_e + p_v e\mu_h$$

$$n_i \sim 10^{10}\,\mathrm{cm}^{-3}$$

$$\sigma \sim 10^{-6}\,\mathrm{S/cm}$$

atom density of Si

$$N_{Si} \sim 10^{22}\,\mathrm{cm}^{-3}$$

If we put 1 ppm (10^{-6}) P in Si

$$N_D \sim 10^{16}\,\mathrm{cm}^{-3} \gg n_i$$

$$n_c = N_D \sim 10^{16}\,\mathrm{cm}^{-3}$$

$$p_v = n_i^2 / n_c \sim 10^{4}\,\mathrm{cm}^{-3}$$

$$\sigma \sim 10^{0}\,\mathrm{S/cm}$$

the conductivity is increased by 10^6

the conductivity is related to the doping and weakly dependent on T

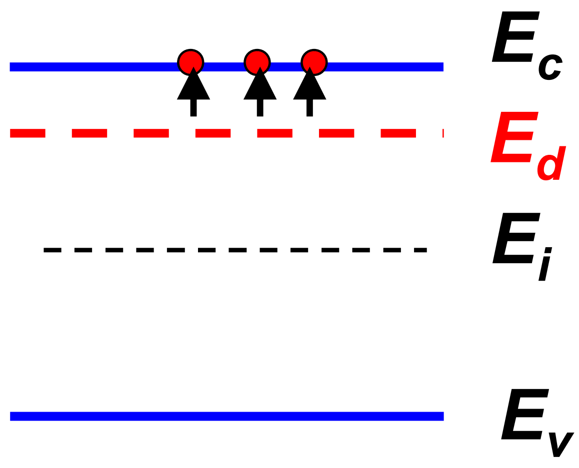
Chemical Potential / Fermi Level

For n-doping

$$n_c = N_D$$

$$p_v = n_i^2 / n_c$$

$$n_c \gg p_v$$

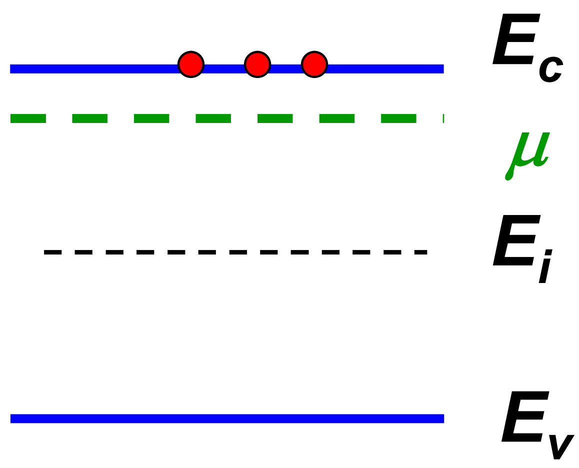


N_D - concentration of donor (cm^{-3})

$$n_c = N_c(T)e^{-(E_c-\mu)/k_BT}$$

$$p_v = P_v(T)e^{-(\mu-E_v)/k_BT}$$

$$\rightarrow \quad E_c - \mu \ll \mu - E_v$$



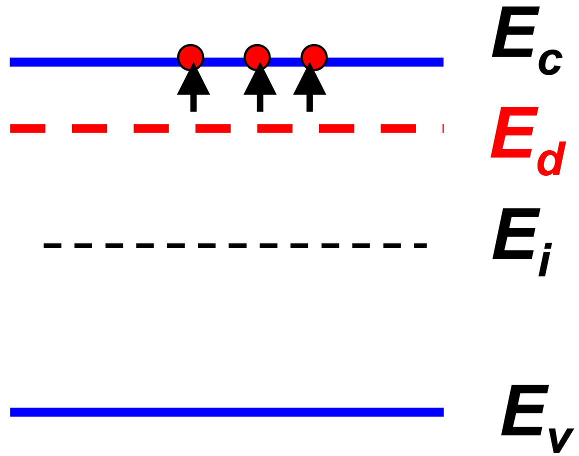
chemical potential / Fermi level moves closer to E_c

Chemical Potential / Fermi Level

For n-doping

$$n_c = N_D$$

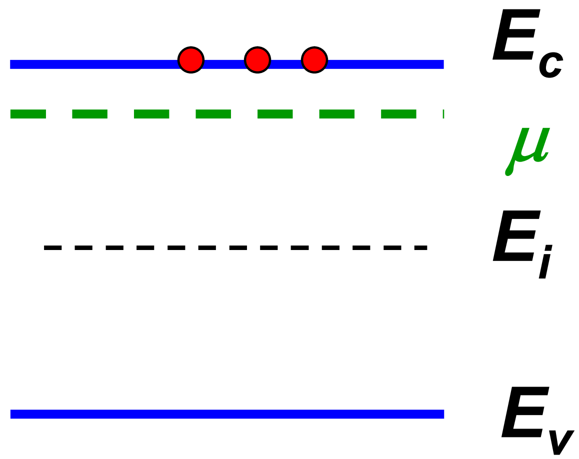
$$p_v = n_i^2 / n_c$$



$$\mu = E_c - k_B T \ln\left(\frac{N_c(T)}{n_c}\right)$$

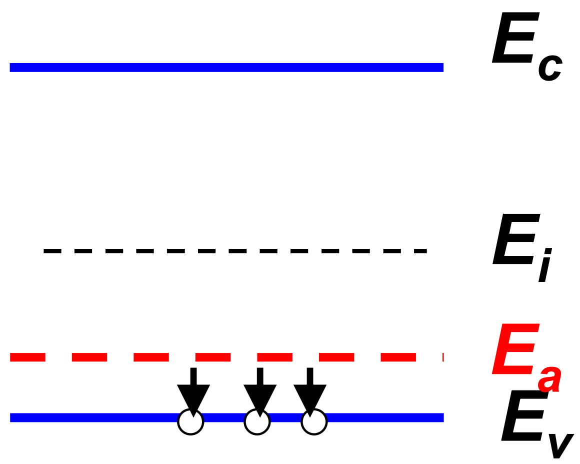
$$= E_i + k_B T \ln\left(\frac{N_D}{n_i}\right)$$

$$\approx E_v + \frac{1}{2}E_g + k_B T \ln\left(\frac{N_D}{n_i}\right)$$



Chemical Potential / Fermi Level

For p-doping



$$p_v = N_A$$

$$n_c = n_i^2 / p_v$$

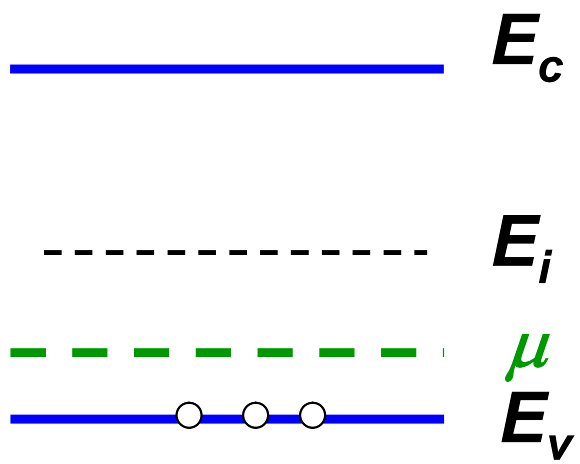
$$p_v \gg n_c$$

N_A - concentration of acceptor (cm^{-3})

$$n_c = N_c(T) e^{-(E_c - \mu)/k_B T}$$

$$p_v = P_v(T) e^{-(\mu - E_v)/k_B T}$$

$$\rightarrow \quad E_c - \mu \gg \mu - E_v$$



chemical potential / Fermi level moves closer to E_v

Chemical Potential / Fermi Level

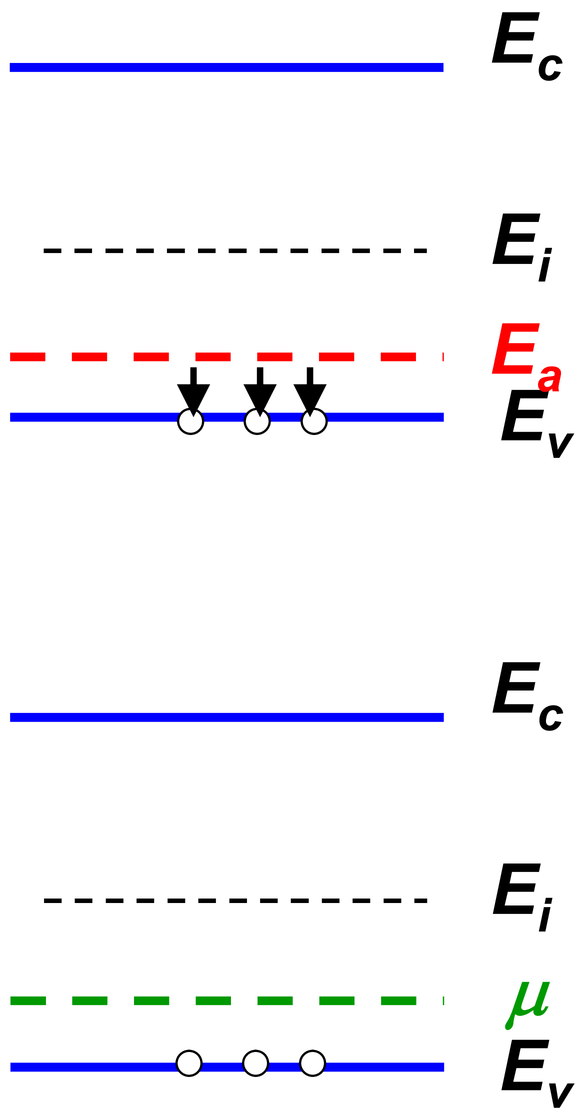
For p-doping

$$p_v = N_A \qquad n_c = n_i^2 / p_v$$

$$\mu = E_v + k_B T \ln\left(\frac{P_v(T)}{p_v}\right)$$

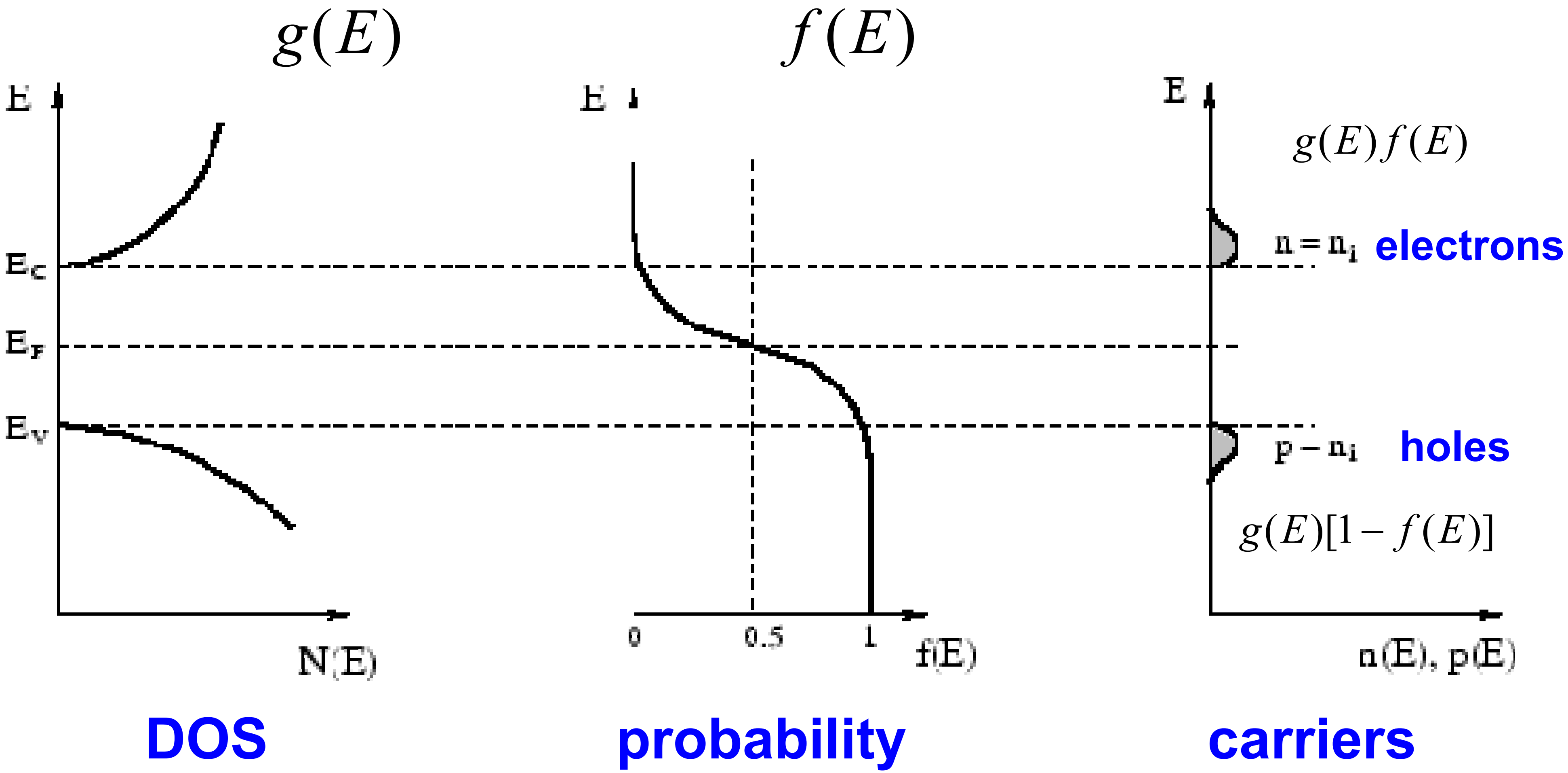
$$= E_i - k_B T \ln\left(\frac{N_A}{n_i}\right)$$

$$\approx E_v + \frac{1}{2}E_g - k_B T \ln\left(\frac{N_A}{n_i}\right)$$



Intrinsic vs. Extrinsic

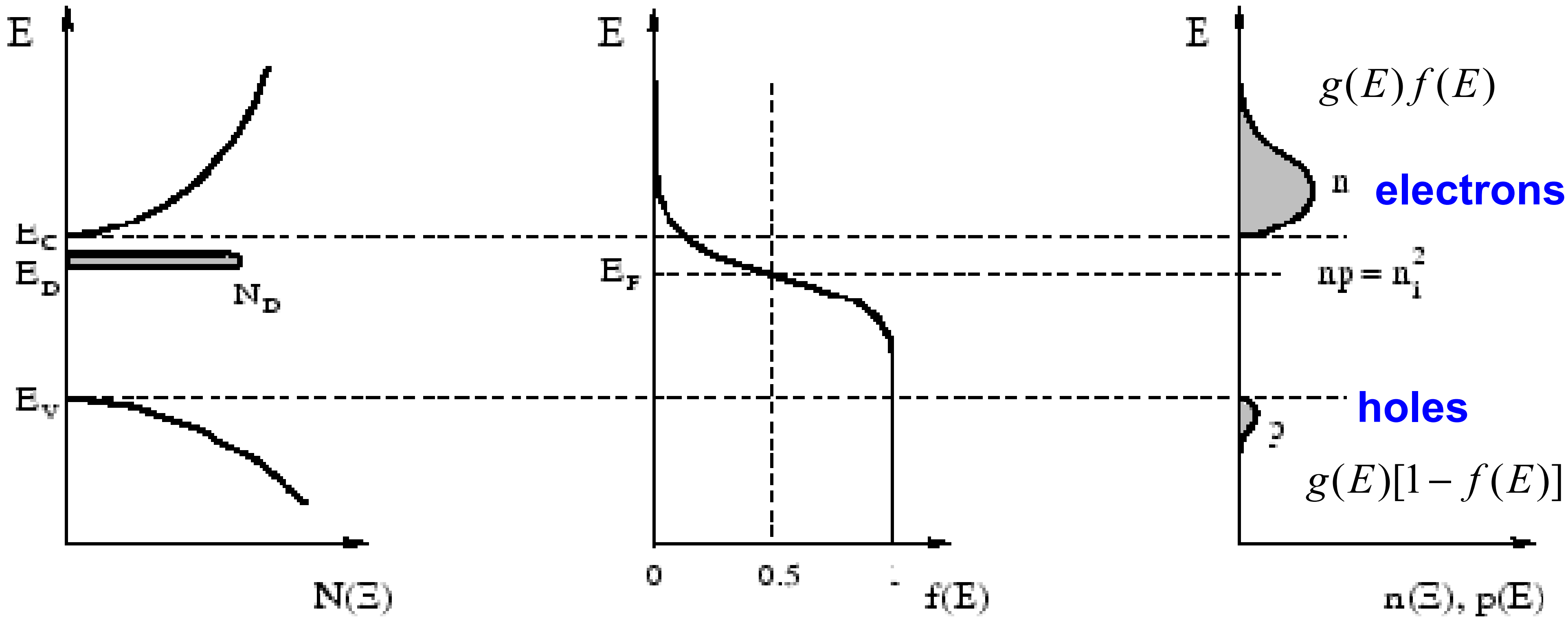
Intrinsic



Intrinsic vs. Extrinsic

Extrinsic (n-doping)

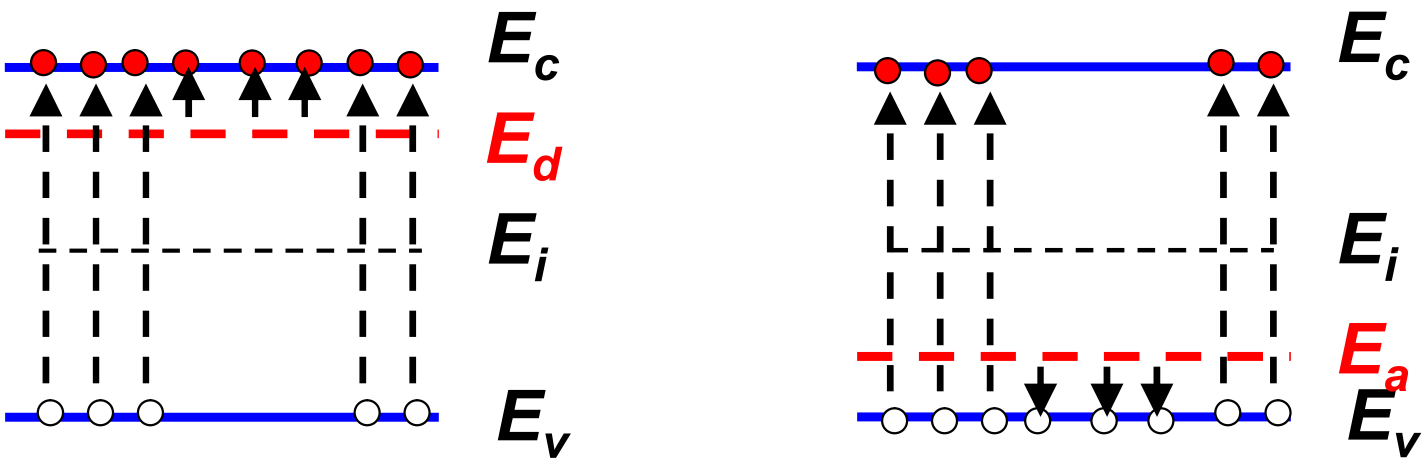
$g(E)$ $f(E)$



DOS probability carriers

At Very High Temperature

when *T* is very high, more carriers can be excited



$$n_c \approx p_v \approx n_i \gg \text{doping concentration}$$

similar to an intrinsic semiconductor

Extrinsic Semiconductor 掺杂半导体

temperature dependence of carrier concentration

low *T*, freeze-out

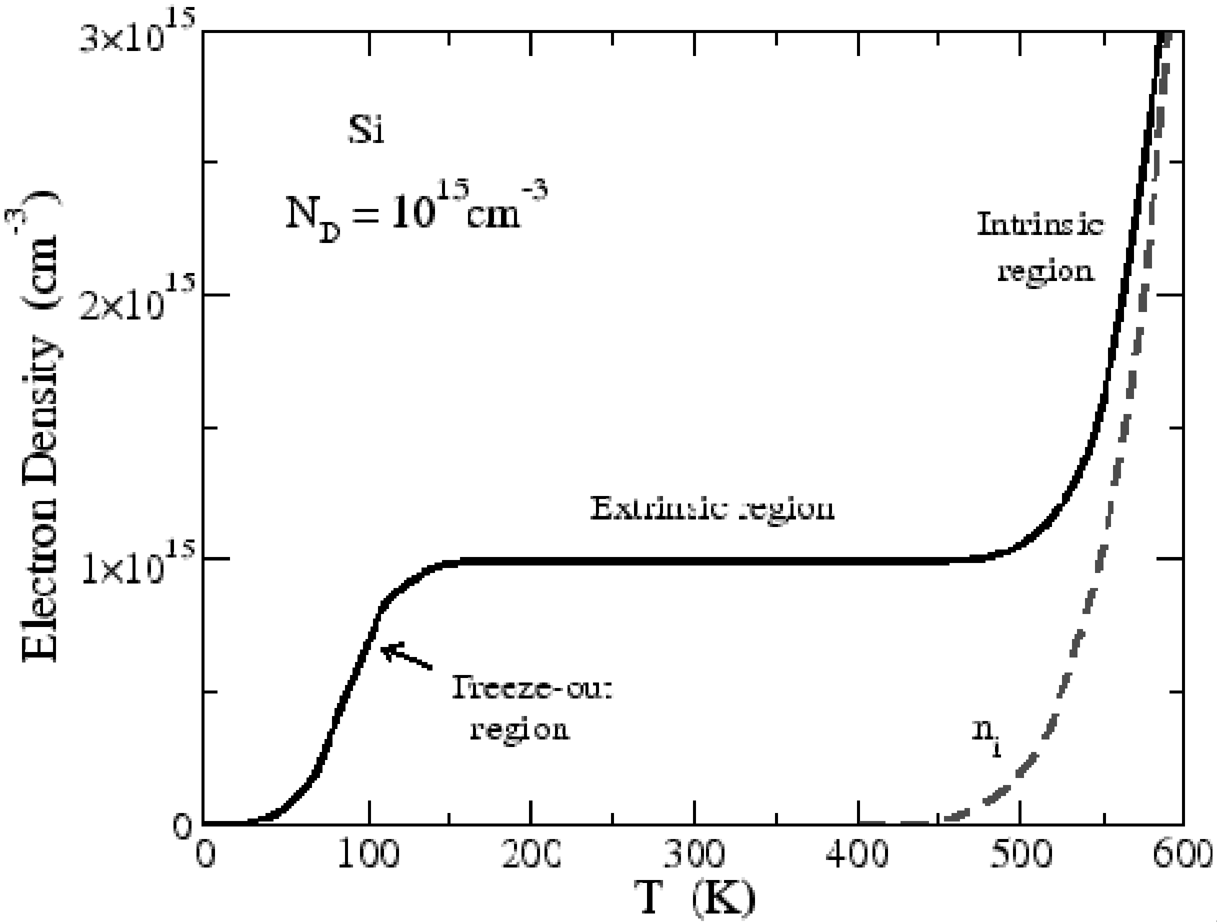
$$\boxed{n \sim e^{-(E_c - E_D)/k_B T}}$$

median *T*, extrinsic

$$\boxed{n = N_D}$$

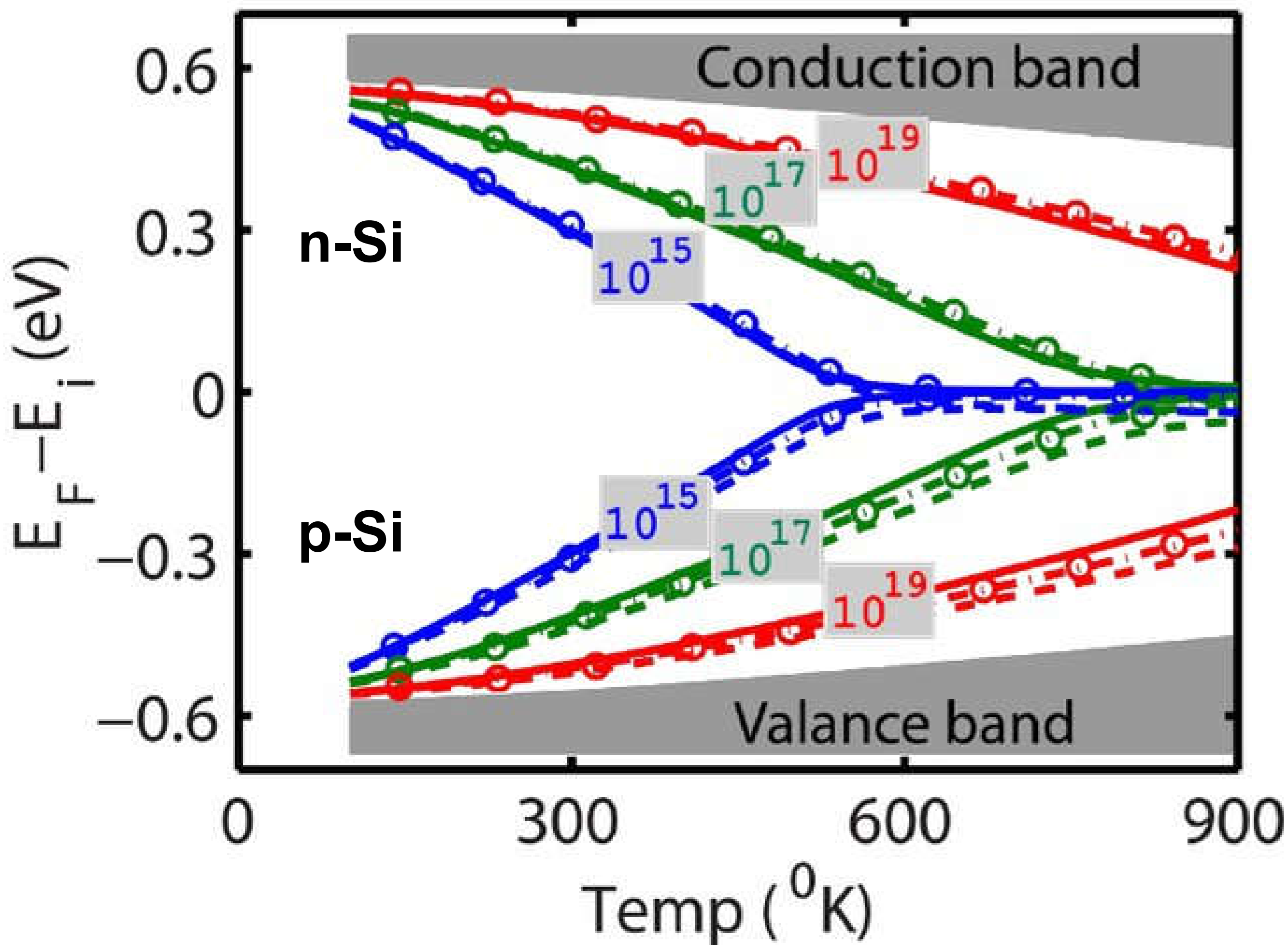
high *T*, intrinsic

$$\boxed{n \sim e^{-E_g/2k_B T}}$$



Extrinsic Semiconductor 掺杂半导体

temperature dependence of chemical potential



https://arxiv.org/abs/1110.2405

Extrinsic Semiconductor 掺杂半导体

temperature dependence of carrier concentration

low T, freeze-out

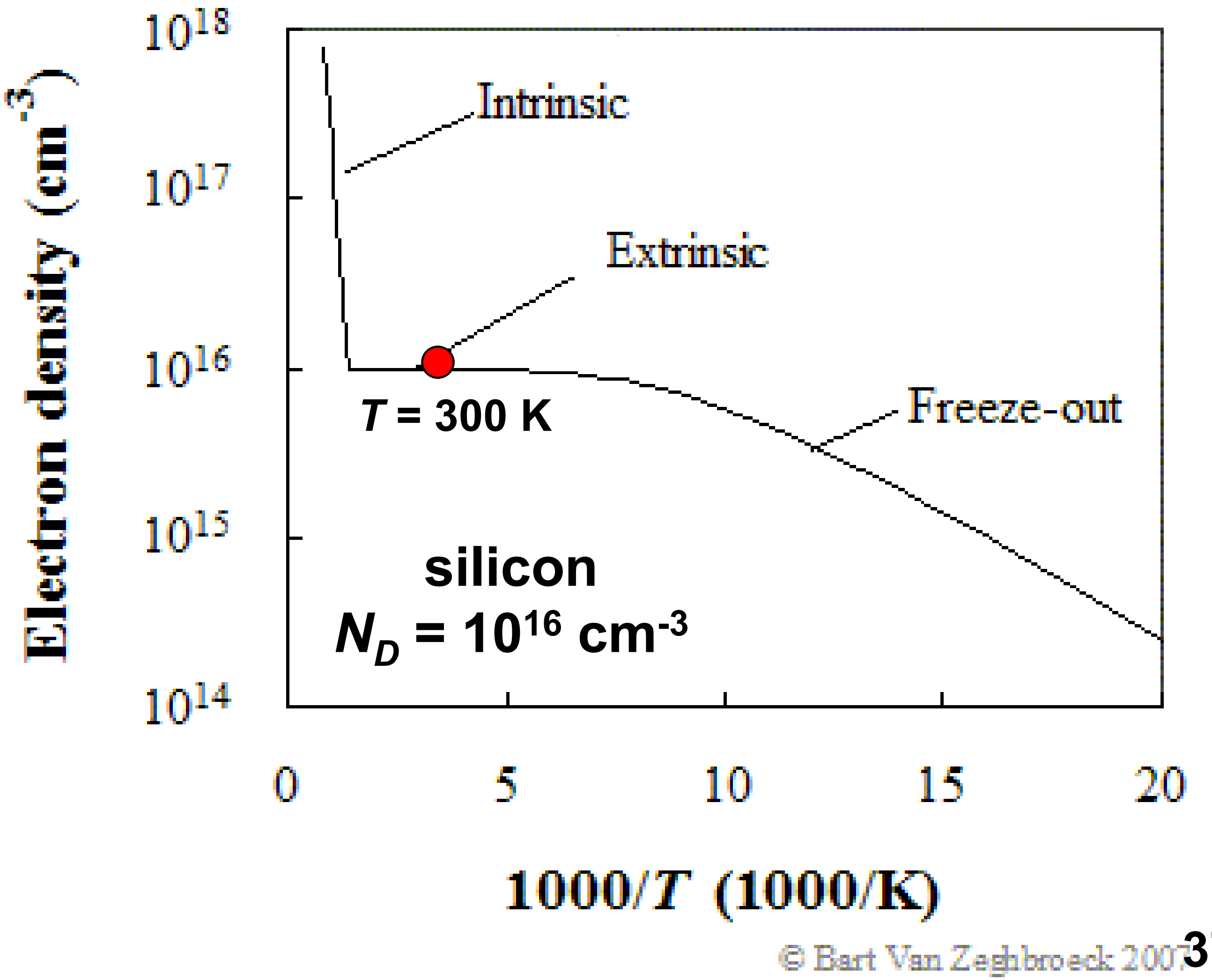
$$n \sim e^{-(E_c - E_D)/k_B T}$$

median T, extrinsic

$$n = N_D$$

high T, intrinsic

$$n \sim e^{-E_g/2k_B T}$$



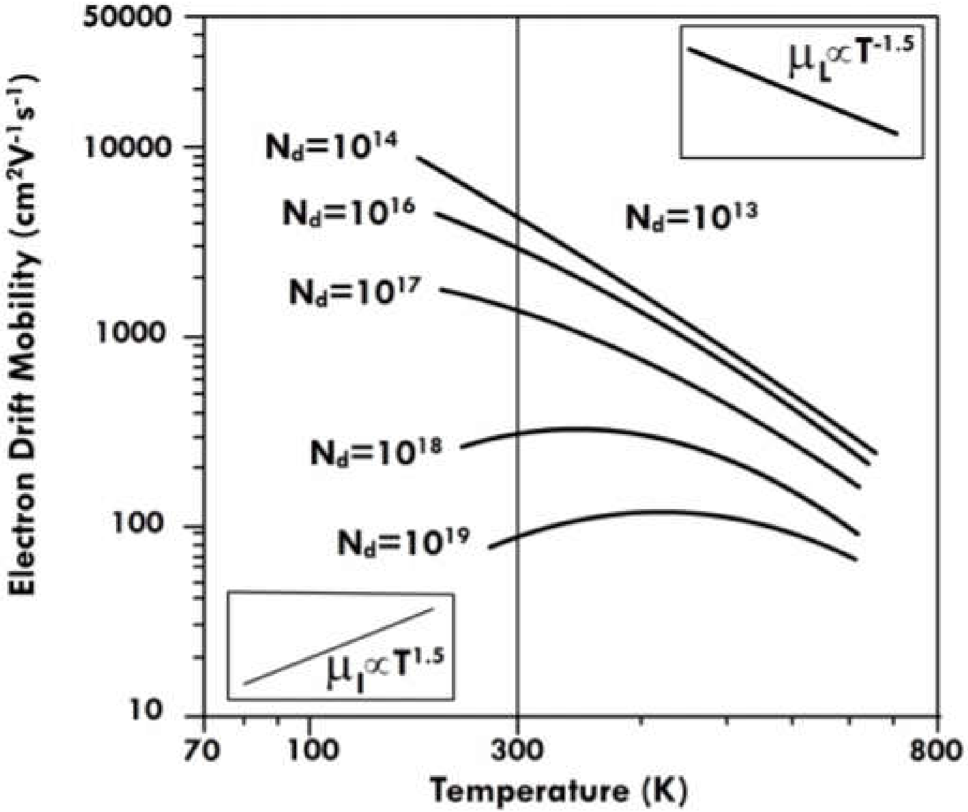
Extrinsic Semiconductor 掺杂半导体

temperature dependence of mobility μ

at low *T* $\boxed{\mu \sim T^{3/2}}$ at high *T* $\boxed{\mu \sim T^{-3/2}}$

impurity scattering lattice scattering

when doping increases, mobility decreases, due to more impurity scattering

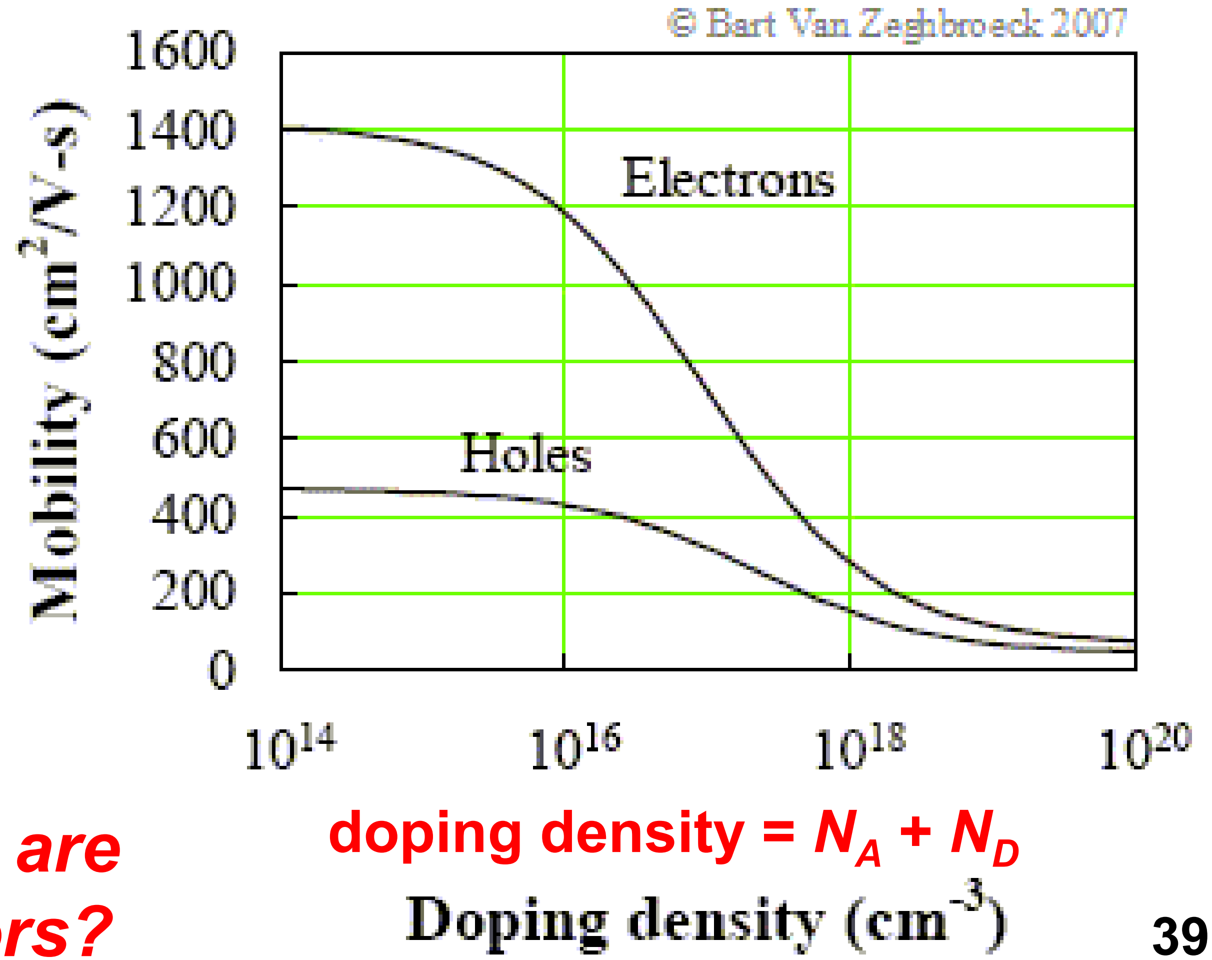


Extrinsic Semiconductor 掺杂半导体

doping dependence of mobility μ

silicon, T = 300 K

when doping increases, mobility decreases, due to more impurity scattering



doping density = $N_A + N_D$

Q: What happens if there are both donors and acceptors?

Extrinsic Semiconductor 掺杂半导体

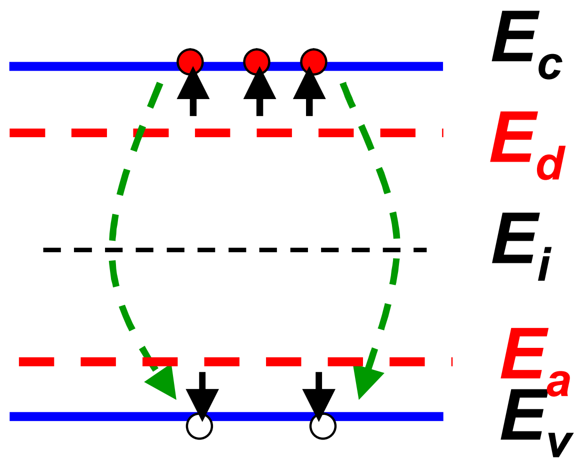
compensated semiconductor (补偿半导体)

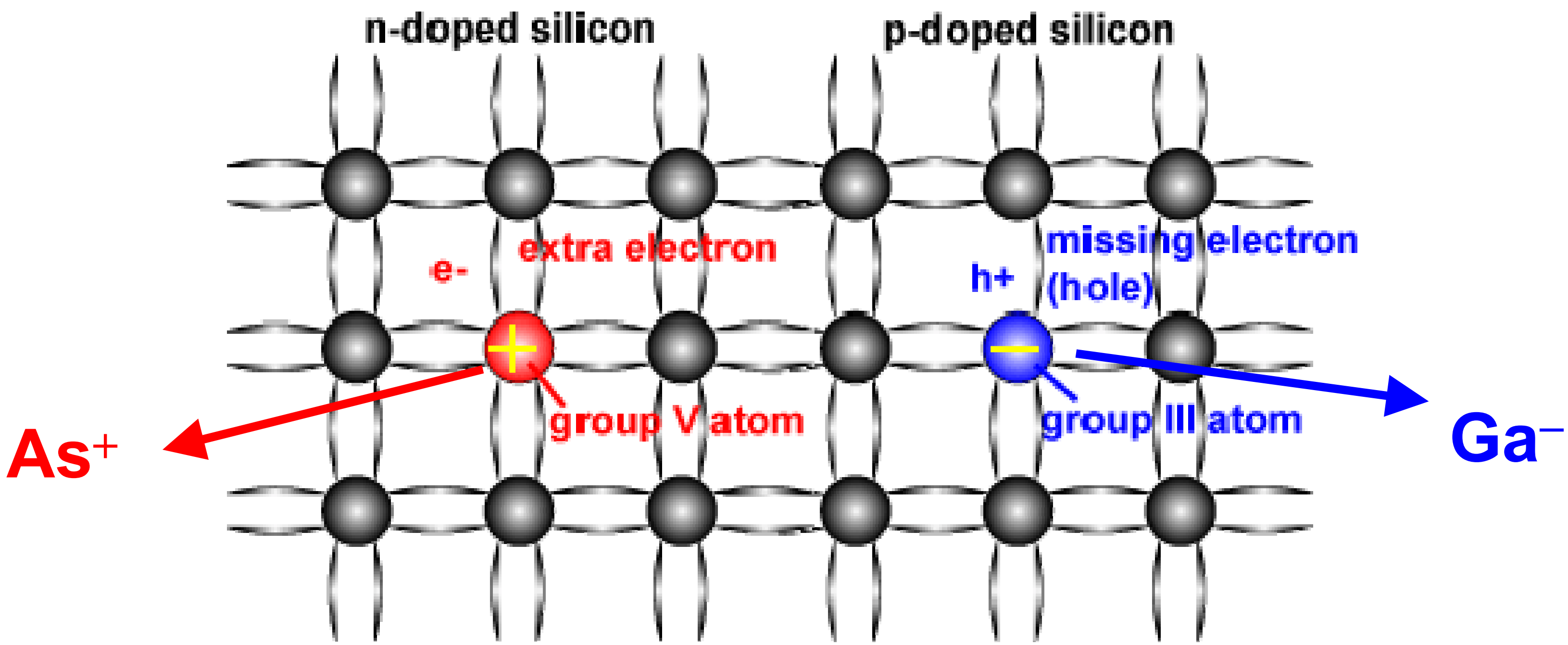
contains both donor and acceptor

$$\begin{cases} n_c + N_A = p_v + N_D \\ n_c p_v = n_i^2 \end{cases}$$

charge balance

mass action law





Extrinsic Semiconductor 掺杂半导体

compensated semiconductor (补偿半导体)

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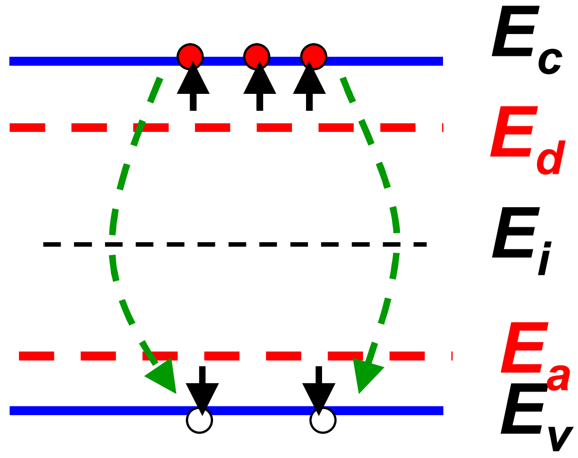


$$\begin{cases} n_c + N_A = p_v + N_D \\ n_c p_v = n_i^2 \end{cases}$$

charge balance

mass action law

if $N_A > N_D$ → **p-doping**

→ $$p_v = \frac{N_A - N_D}{2} + \sqrt{\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2}$$

if $N_A - N_D \gg n_i$ → $p_v = N_A - N_D$ $\quad n_c = n_i^2 / p_v$

Extrinsic Semiconductor 掺杂半导体

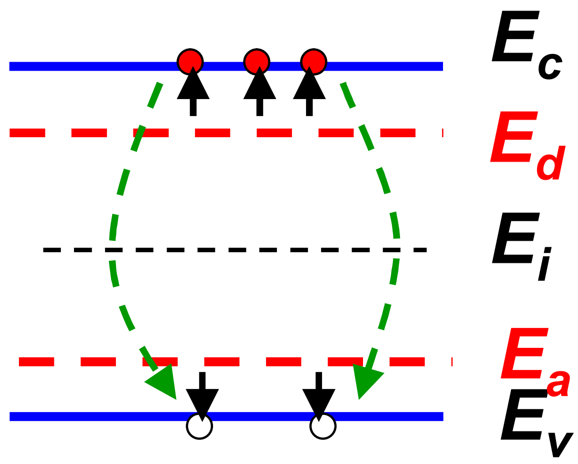
compensated semiconductor (补偿半导体)

contains both donor and acceptor

$$\begin{cases} n_c + N_A = p_v + N_D \\ n_c p_v = n_i^2 \end{cases}$$

charge balance

mass action law



if $N_D > N_A$ → **n-doping**

$$\rightarrow \quad n_c = \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2}$$

if $N_D - N_A \gg n_i$ → $n_c = N_D - N_A$ $\quad p_v = n_i^2 / n_c$

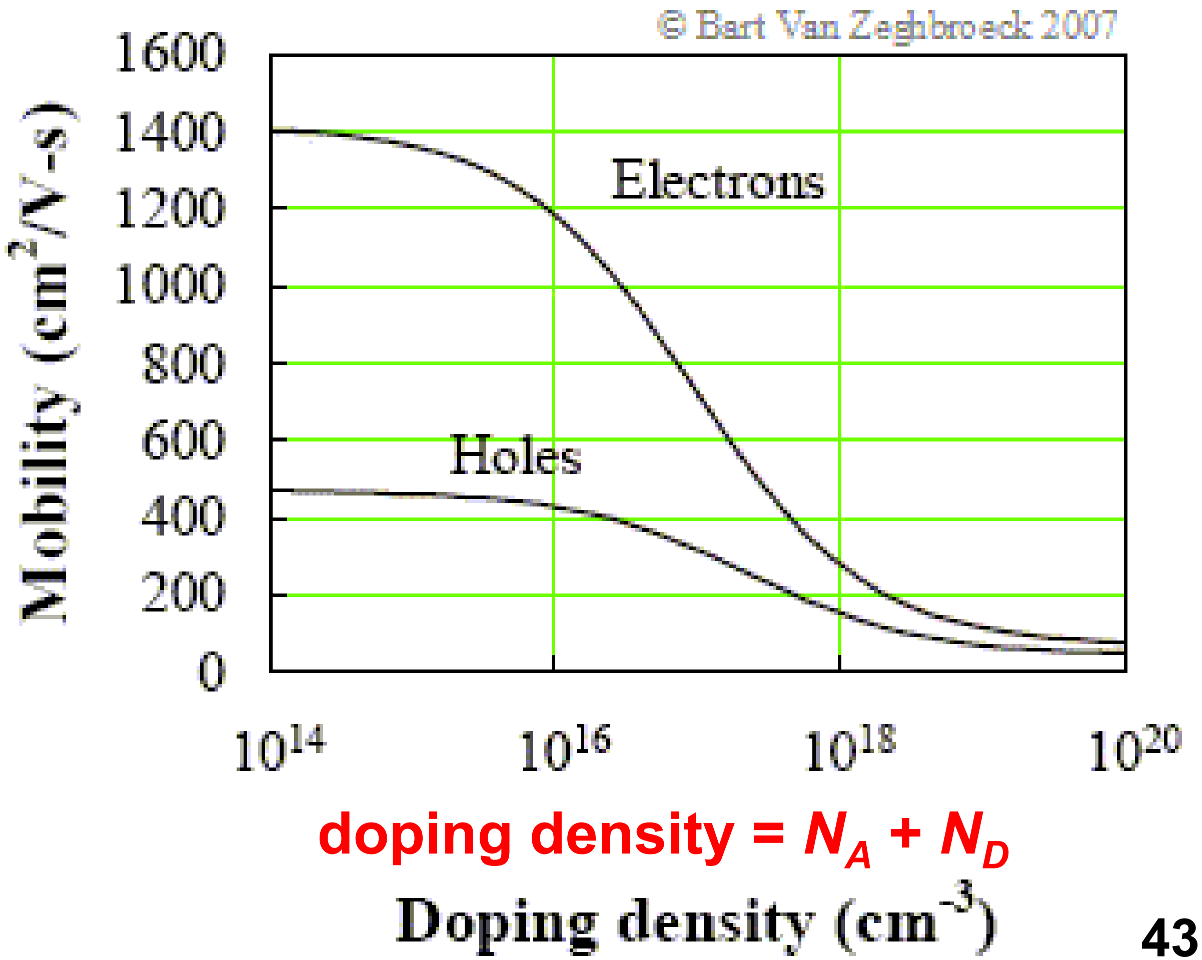
Extrinsic Semiconductor 掺杂半导体

in compensated semiconductor (补偿半导体) contains both donor and acceptor

the mobility μ depends on $N_A + N_D$

when doping increases, mobility decreases, due to more impurity scattering

silicon, T = 300 K



doping density = $N_A + N_D$

Mass Action Law - A Little Notion

- **The product of electron and hole concentrations is a constant, at a fixed temperature**

$$\boxed{n_c p_v = n_i^2 = N_v(T)P_v(T)e^{-E_g/k_B T}}$$

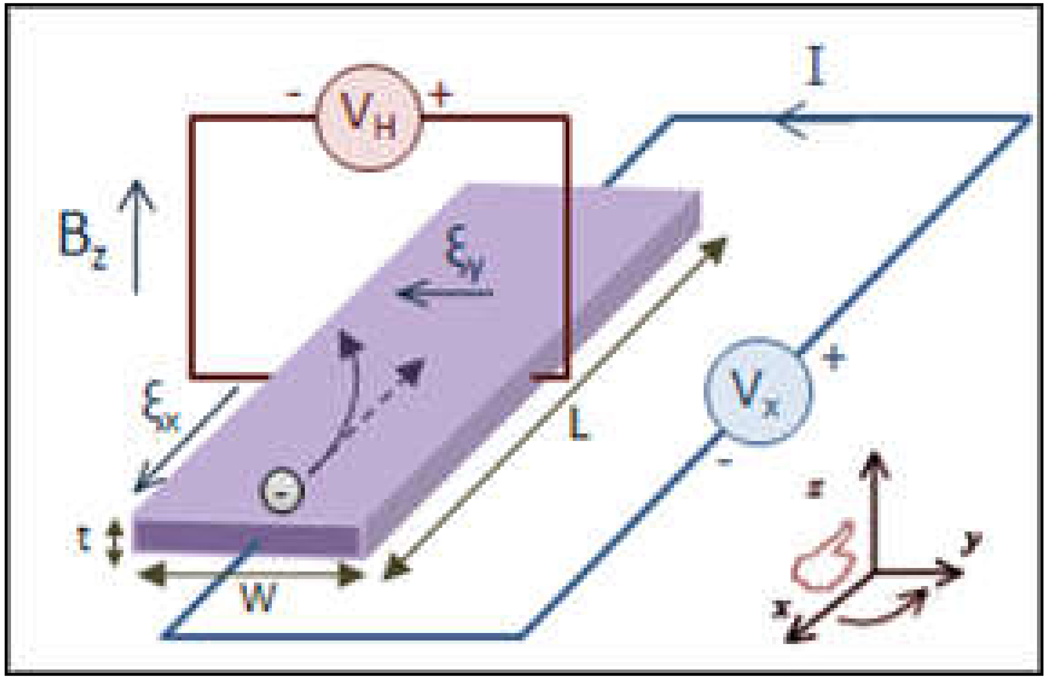
- **In water, the product of H^+ and OH^- concentrations is also a constant**

$$\boxed{[H^+][OH^-] = K_w = 10^{-14}\,(\text{mol/L})^2 \;\; (\text{at } 25\ ^\circ\text{C})}$$

- **Both are originated from classical statistics (non-degenerate, Maxwell-Boltzmann distribution), *not* related to quantum mechanics**

Measurement of Doping

- **Hall Effect 霍尔效应**
  - **A current flows through a conductor**
  - **V_H is generated when applying B_z**



Lorentz Force at equilibrium

$$eE_y = ev_x B_z$$

$$j_x = nev_x$$

$$E_y = \frac{1}{ne} j_x B_z = R_H j_x B_z$$

R_H - Hall coefficient

https://www.tf.uni-kiel.de/matwis/amat/mw2_ge/kap_2/backbone/r2_1_3.html

Measurement of Doping

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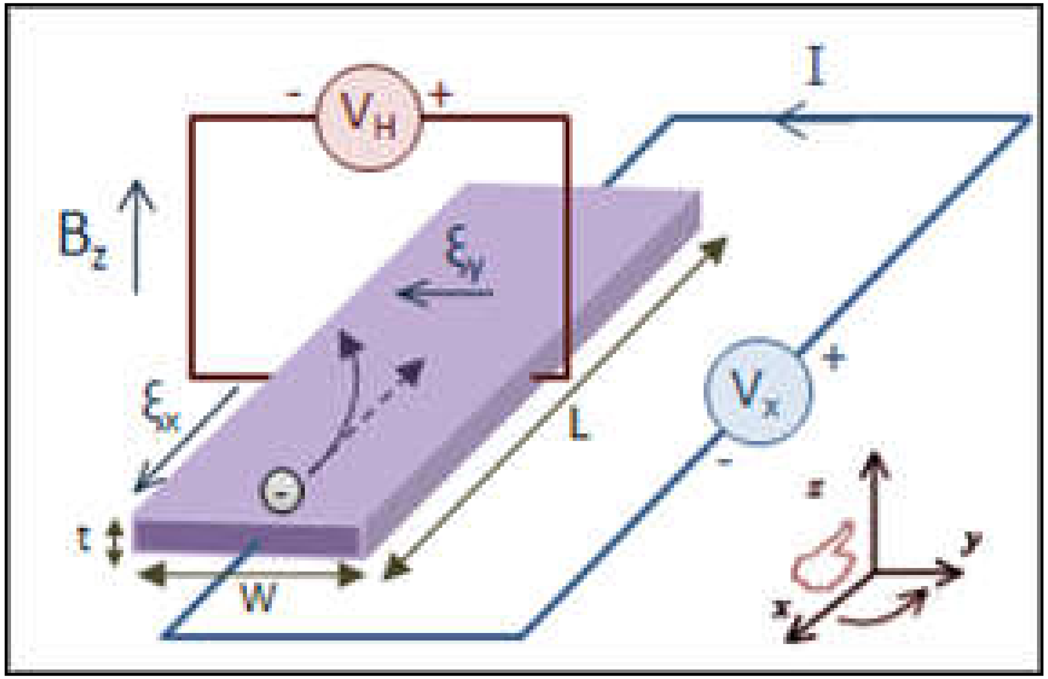


p-doping

$$R_H = \frac{1}{p_v e}$$

positive

n-doping

$$R_H = -\frac{1}{n_c e}$$

negative

https://www.tf.uni-kiel.de/matwis/amat/mw2_ge/kap_2/backbone/r2_1_3.html

Carrier Behaviors in Semiconductors

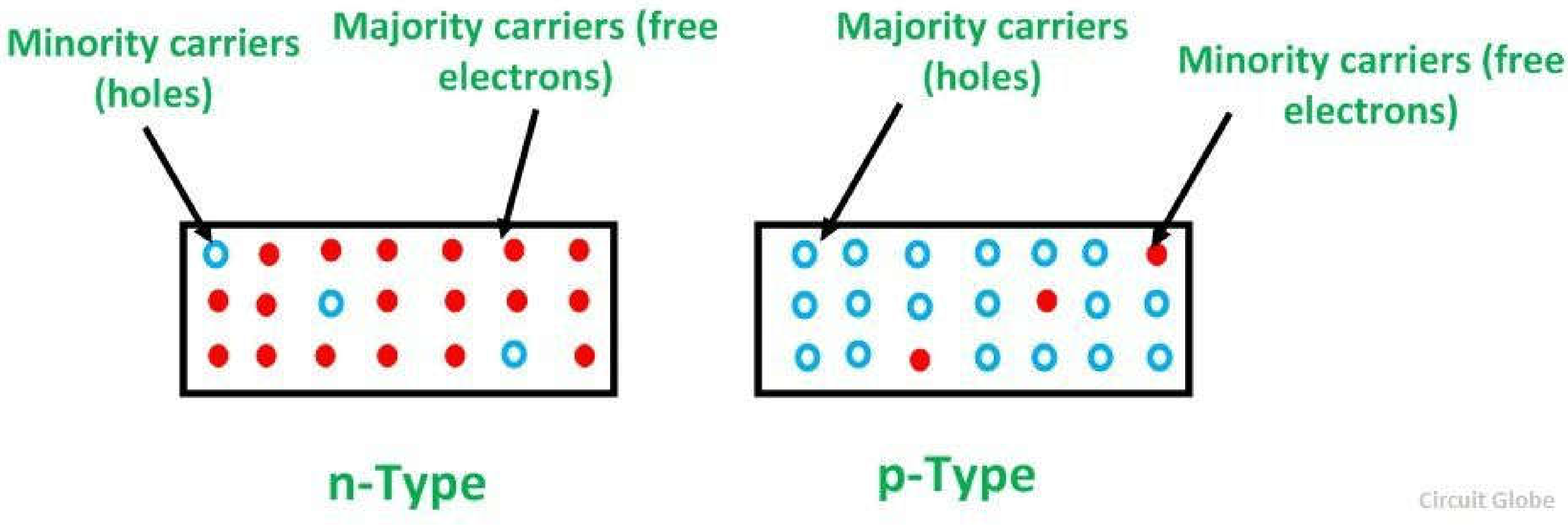
- Equilibrium carriers (平衡态载流子)
- Non-equilibrium carriers (非平衡态载流子)

- Current Flow
  - diffusion current
  - drift current

- Generation 产生
- Recombination 复合

Carrier Behaviors in Semiconductors

- **Majority carriers (多数载流子)**
- **Minority carriers (少数载流子)**
  - **minority carriers are very important, because their amount can be easily changed by current injection, optical absorption, etc.**



Equilibrium carriers (平衡态载流子)

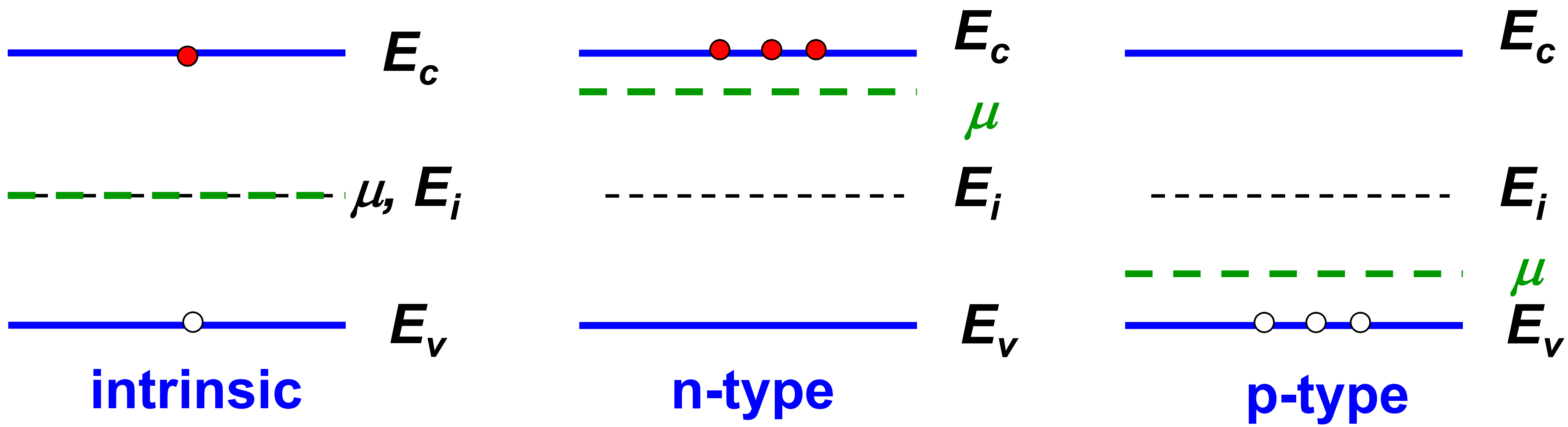
only thermal activation

$$n_c = N_c(T)e^{-(E_c-\mu)/k_BT}$$

$$p_v = P_v(T)e^{-(\mu-E_v)/k_BT}$$

$$\boxed{n_c p_v = N_v(T)P_v(T)e^{-E_g/k_BT} = n_i^2}$$

mass action law



Non-equilibrium carriers (非平衡态载流子)

activation by other energy sources
e.g., photon absorption, current injection, ...

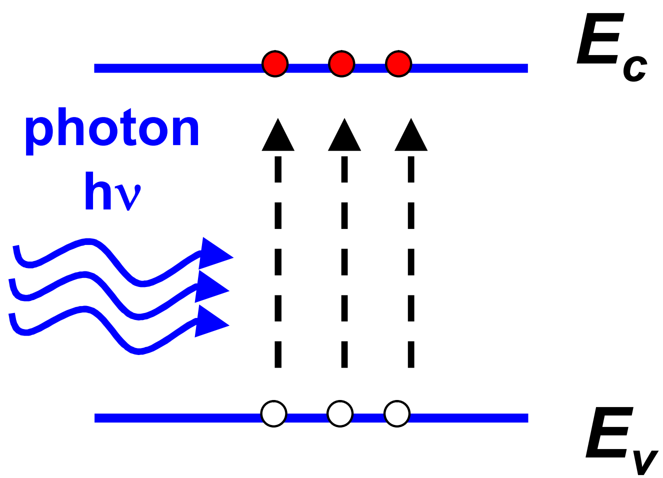
extra more carriers n_c and p_v

$$p_v \neq P_v(T)e^{-(\mu-E_v)/k_BT}$$

$$n_c \neq N_c(T)e^{-(E_c-\mu)/k_BT}$$

$$\boxed{n_c p_v \neq N_v(T)P_v(T)e^{-E_g/k_BT} = n_i^2}$$

mass action law is not valid



Non-equilibrium carriers (非平衡态载流子)

activation by other energy sources
e.g., photon absorption, current injection, ...

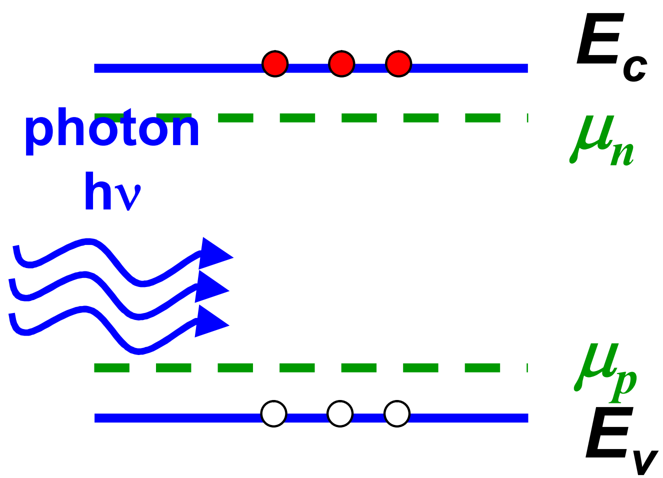
extra more carriers n_c and p_v

rewrite:

$$p_v = P_v(T)e^{-(\mu_p - E_v)/k_B T}$$

$$n_c = N_c(T)e^{-(E_c - \mu_n)/k_B T}$$

$$\boxed{\mu_n \neq \mu_p}$$



quasi-Fermi levels (准费米能级)

Current Flow

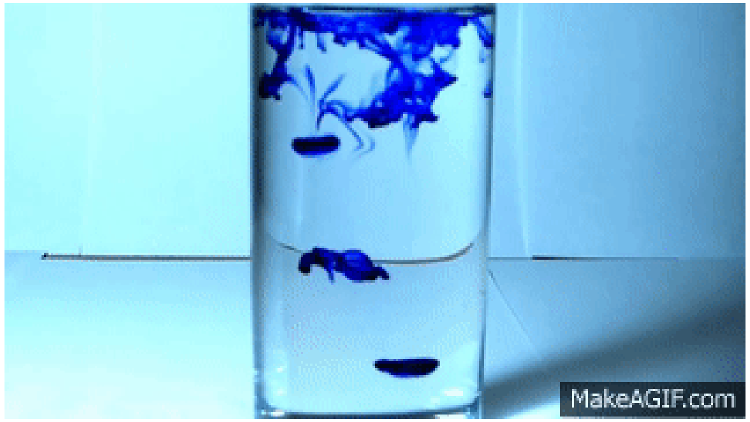
- **Diffusion current 扩散电流**
  - **caused by concentration gradient**

- **Drift current 迁移电流**
  - **caused by electric field**

Current Flow

- **Diffusion current 扩散电流**
  - caused by concentration gradient

ink in water



1D

$$j = -qD\frac{\partial n}{\partial x}$$

3D

$$\mathbf{j} = -q\mathbf{D}\nabla\mathbf{n}$$

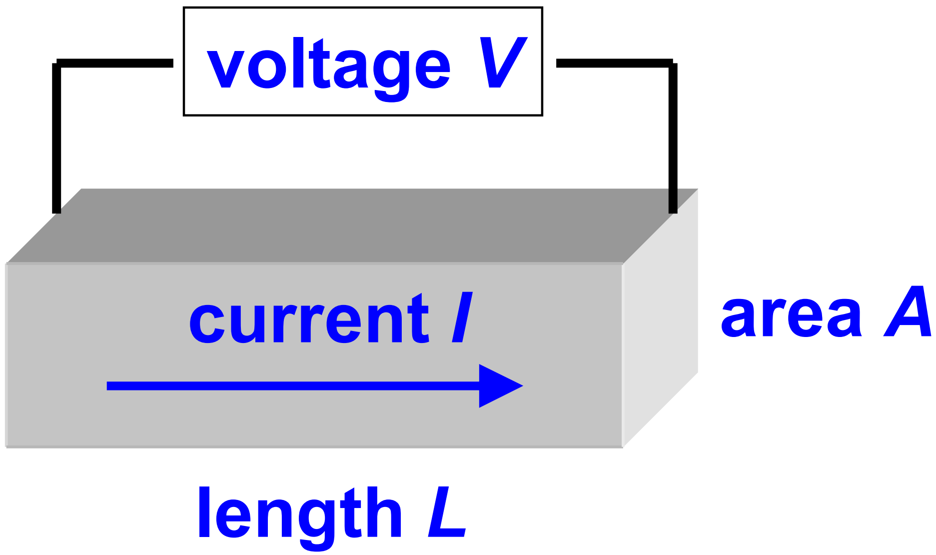
diffusivity 扩散系数

$$D = \mu\frac{kT}{q}$$

n - carrier concentration (#/m^3)
D - diffusivity (m^2/s) 扩散率
μ - mobility (m^2/V/s)

Current Flow

- **Drift current 迁移电流**
  - **caused by electric field**



Ohm's Law

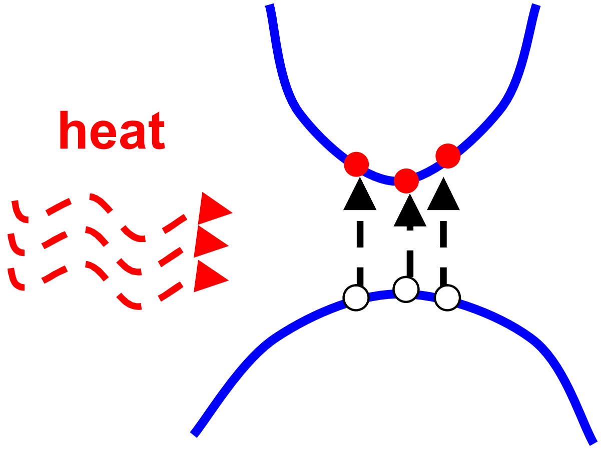
$$j = \sigma E = nq\mu E = nqv$$

n **- carrier concentration (#/m^3)**
D **- diffusivity (m^2/s) 扩散率**
μ **- mobility (m^2/V/s)**

Carrier Generation

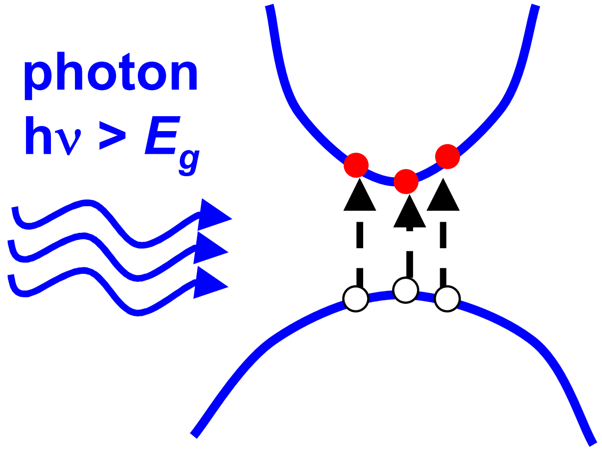
- **Excited by thermal energy**

$$\boxed{n_c p_v = N_v(T) P_v(T) e^{-E_g/k_B T} = n_i^2}$$



- **Excited by photons**
  - **photodetector**
  - **solar cells**
  - **...**

$$\boxed{h\nu = \mathbf{e} + \mathbf{h}}$$

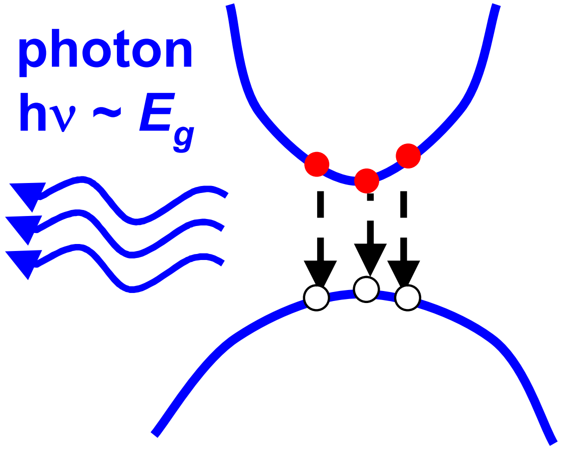


- **Other sources**
- **...**

Carrier Recombination 复合

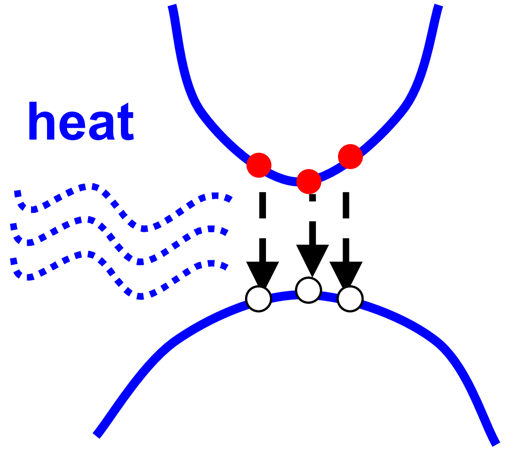
- Radiative 辐射
  - LED
  - laser
  - ...

$$\mathbf{e} + \mathbf{h} = \text{photon}$$



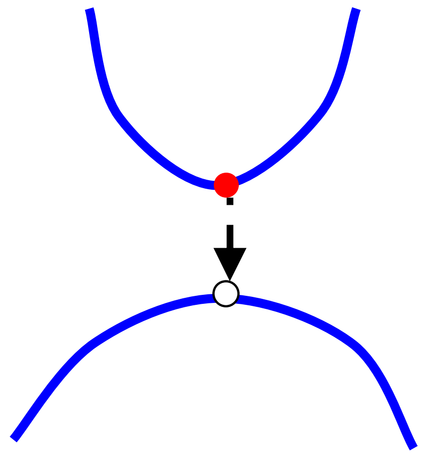
- Non-radiative recombination
  - defects, impurities, surfaces
  - lattice (phonons)
  - Auger process
  - ...

$$\mathbf{e} + \mathbf{h} = \text{phonons}$$



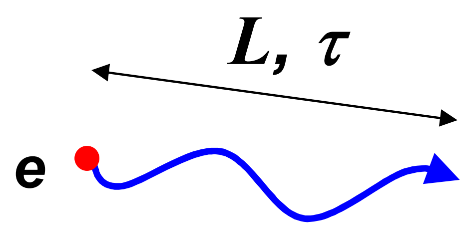
Carrier Recombination 复合

- 载流子寿命 Carrier lifetime τ_n and τ_p
  - the averaged time before the non-equilibrium carriers recombine
- 扩散率 Diffusivity D_n and D_p



- 扩散长度 Diffusion length L_n and L_p
  - the averaged distance carriers move before recombination

$$\boxed{L = \sqrt{D\tau}}$$

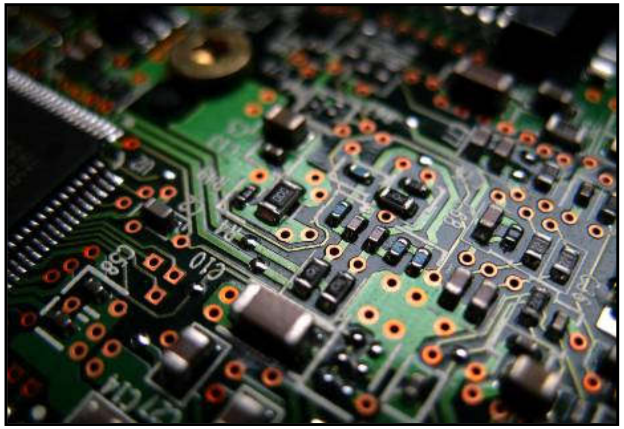


  - At non-equilibrium, minority carrier diffusion and lifetime are important for device performance

D - diffusivity (m^2/s) 扩散率

Semiconductors - Applications

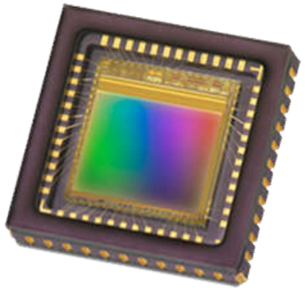
different carrier behaviors offer different applications



integrated circuits

LEDs

lasers



detectors

solar cells

Thank you for your attention