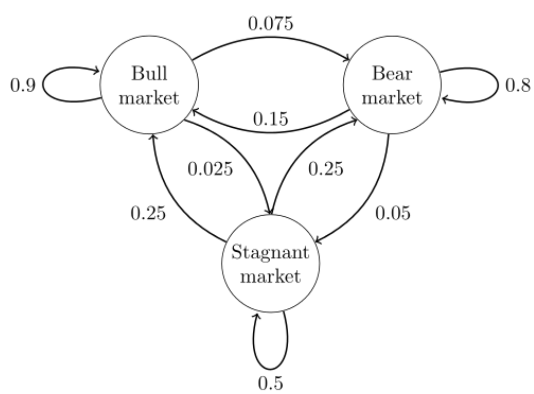
Revision Questions for CS4200/CS5200, On-line Machine Learning, Class 1

Disclaimer: This list of questions has been produced to help students in revision. There is no guarantee that the actual exam questions are in this list or that they will be in any way similar!

1. Give a definition of a depth one Markov chain.

2. This question is based on Wikipedia article "Markov chain"

   A week on a stock exchange can be characterised as bull market (with prices going up), bear market (with prices going down), or stagnant (with prices staying at about the same level). Let us describe the market by a Markov chain with the following transition probabilities:

   Source: Wikipedia, author: Gareth Jones

   (a) Write down the transition matrix M for the chain (assume the order bull, bear, stagnant).

   (b) Assuming the initial probabilities of bear market as 0.4, bull market as 0.4 and stagnant market as 0.2, calculate the probability to get the sequence of weeks bull, bull, stagnant, bear, bull. (Answer: $3.38 \cdot 10^{-4}$.)

   (c) Work out the 2- and 3-step transition probability matrices for the chain.

   Answer:

$$M^2 = \begin{pmatrix} 0.83 & 0.13 & 0.04 \\ 0.27 & 0.66 & 0.07 \\ 0.39 & 0.34 & 0.27 \end{pmatrix}$$

$$M^3 = \begin{pmatrix} 0.77 & 0.18 & 0.05 \\ 0.36 & 0.57 & 0.07 \\ 0.47 & 0.37 & 0.16 \end{pmatrix}$$

(d) Assuming the initial probabilities as above, calculate the probability of the 3rd week being bull market. (Answer: 0.55) Hint: if you are getting 0.39, you are probably multiplying matrices on the wrong side.

(e) Calculate the stationary distribution of the Markov chain. (Answer: $0.63, 0.31, 0.06$). Hint: to solve the system more comfortably, first multiply each equation by 10. Secondly, take any convenient value for one variable, e.g., x_3, and work out x_1 and x_2. Then normalise the vector for the components to sum up to 1. If you keep getting $(0.33, 0.33, 0.33)$, you are multiplying matrices on the wrong side.

3. The following sequence of weeks has been observed: bull, bull, bear, bull, stagnant, bear, bull, bull, bear, stagnant, stagnant, bull, bull, bear. Assuming that this sequence was generated by a first-order Markov chain, find the maximum likelihood estimate of the transition probabilities.

   Answer:

$$\begin{pmatrix} 0.43 & 0.43 & 0.14 \\ 0.67 & 0 & 0.33 \\ 0.33 & 0.33 & 0.33 \end{pmatrix}$$

4. Consider a set of three pages. Page 1 links 2 and 3; page 2 links 3; page 3 links 1. Calculate their page rank using the damping factor $d = 0.85$.

   Answer: $0.39, 0.21, 0.4$

   Hint: the transition matrix is

$$\begin{pmatrix} 0.05 & 0.475 & 0.475 \\ 0.05 & 0.05 & 0.9 \\ 0.9 & 0.05 & 0.05 \end{pmatrix}$$

5. What is the role of the damping factor? Explain its meaning in the browsing model and its theoretical significance.

6. Why have link-based ranking methods become less popular recently?

7. Describe the method of sampling from a univariate distribution based on the inverted distribution function F^{-1}. What are its drawbacks?

8. Formulate the detailed balance condition for a distribution p and a Markov chain with the transition matrix M. Prove that the condition is sufficient for p to be the stationary distribution.

9. Is it possible for a Markov chain to have a stationary distribution and not to converge to it? (*Answer:* yes. *Hint:* consider $M = I$.)

10. Describe the Metropolis-Hastings algorithm.

11. Suppose that the Metropolis-Hastings algorithm is used to sample from a univariate distribution with $p(x) \propto e^{-x^4}$. The Gaussian proposal distribution

$$q(v \mid u) = \frac{1}{\sqrt{2\pi}} e^{(v-u)^2/2}$$

is used. Let $h_t = 0.5$. Suppose that $x = 1$ has been sampled from the proposal distribution. Calculate the acceptance probability. (*Answer:* 0.39.)

12. Answer the previous question if the proposal distribution has variance σ^2 rather than 1. (*Hint:* does it matter?)

13. Can we get independent samples with MCMC?