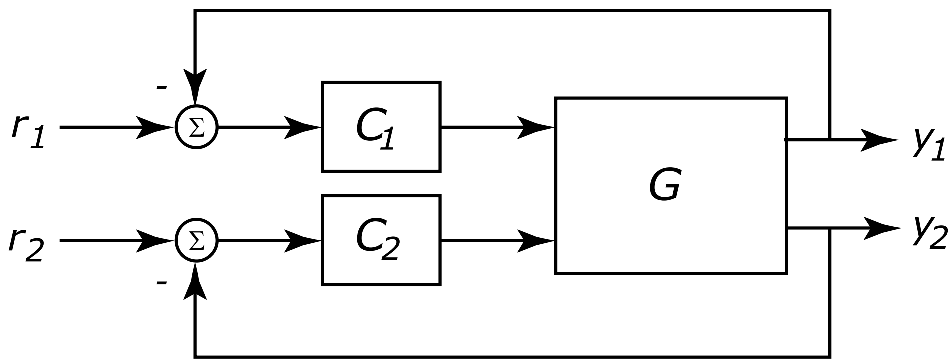
EL2520
Control Theory and Practice

Decentralized Control and Decoupling

Elling W. Jacobsen
School of Electrical Engineering and Computer Science
KTH, Stockholm, Sweden

Decentralized Control and the RGA

- *Decentralized control:* control each output with one control input, corresponds to a diagonal controller $F_y(s)$
- E.g., recall decentralized control of 2x2 system in Lecture 1:



- Questions
  – Will decentralized control give acceptable stability and performance?
  – What pairing of inputs and outputs should we use?

The Relative Gain Array - RGA

- Main problem with decentralized control: interactions between loops
- Question: how will closing other loops in a decentralized system affect the transfer-function from u_j to y_i ?

Consider two cases (assume G(s) square and invertible)

1. **All loops open**:

$$y_i = G_{ij}(s)u_j$$

2. **All other loops closed**: assume perfect control of all other outputs, i.e., $y_{k,k\neq i} = 0$. We then get, since $u = G^{-1}y$,

$$y_i = \frac{1}{(G^{-1}(s))_{ji}} u_j$$

- The ratio is called *the relative gain*

$$\lambda_{ij} = \frac{G_{ij}(s)}{1/(G^{-1}(s))_{ji}} = G_{ij}(s)(G^{-1}(s))_{ji}$$

The Relative Gain Array - RGA

- The matrix of all relative gains can be computed from

$$RGA(G) = \Lambda(G) = G \times (G^{-1})^T$$

where x denotes Hadamard (element-by-element) product

- If $\lambda_{ij} \approx 1$ then interactions have small impact on transfer-function from u_j to y_i
- Most important is $|\lambda_{ij}(i\omega_c)|$ since small changes in loop gain at crossover has large impact on closed-loop behavior
- If $\lambda_{ij}(0) < 0$ then loop gain changes sign as other loops closed $\rightarrow$ stability problems!
- Pairing rules:
  1. Never pair y_i and u_j if $\lambda_{ij}(0) < 0$
  2. Prefer pairings with $|\lambda_{ij}(i\omega_c)| \approx 1$

Examples

- Ex.1: $$G(s) = \frac{1}{s+1}\begin{bmatrix} 1 & 1 \\ 0.4 & -0.1 \end{bmatrix} \quad \Rightarrow \quad \Lambda = \begin{bmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{bmatrix}$$
  - Rule 2 suggests pair $u_1 - y_2$ and $u_2 - y_1$
  - Weak interactions with this pairing, i.e., decentralized control should work OK

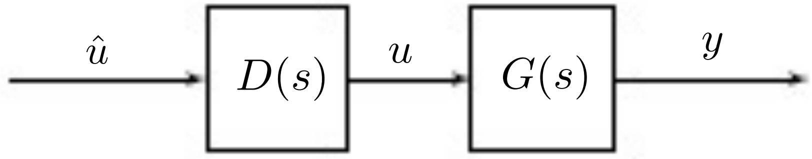
- Ex.2: $$G = \frac{1}{s+1}\begin{bmatrix} \frac{1}{s+1} & -1 \\ 2 & -2.1 \end{bmatrix}$$

$$\Rightarrow \quad \Lambda(0) = \begin{bmatrix} 21 & -20 \\ -20 & 21 \end{bmatrix}, \quad |\Lambda(i1)| = \begin{bmatrix} 1.05 & 1.41 \\ 1.41 & 1.05 \end{bmatrix}$$

  - Rule 1 suggests pairing on diagonal
  - Weak interactions if bandwidth is chosen around frequency 1
  - Strong interactions if bandwidth significantly less than 1

Decoupling

If there are strong interactions (large RGA elements), then one option is to design a *decoupler*



- Design $D(s)$ so that $G(s)D(s)$ is diagonal $\forall s$ or for some frequency, e.g., $\omega = 0$ (static decoupling)
- There may be problems with
  - non-realizable D, due to improperness and non-causality
  - internal stability, due RHP pole-zero cancellations
  - model uncertainty

Example - Decoupling and Model Uncertainty

- No model uncertainty

$$G = \begin{pmatrix} 1 & -1 \\ 1.1 & -1 \end{pmatrix} \; ; \quad D = \begin{pmatrix} 1 & -1 \\ 1.1 & -1 \end{pmatrix}^{-1} \quad \Rightarrow \quad GD = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- 10% uncertainty in elements of G

$$G = \begin{pmatrix} 1 & -1 \\ 1.1 & -1 \end{pmatrix} \; ; \quad D = \begin{pmatrix} 1.1 & -0.9 \\ 1.2 & -0.9 \end{pmatrix}^{-1} \quad \Rightarrow \quad GD = \begin{pmatrix} 3.3 & -2.2 \\ 2.3 & -1.2 \end{pmatrix}$$

  - small uncertainty results in poor decoupling
  - can show: decoupling most sensitive to uncertainty when RGA elements are large!
  - better is to design multivariable controller taking model uncertainty into account; later