EL2520
Control Theory and Practice

Welcome!

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School of Electrical Engineering and Computer Science
KTH

Todays Program

- Practical information, course content
- Introduction to MIMO control
- Signal norms, System gain
- The Small Gain Theorem

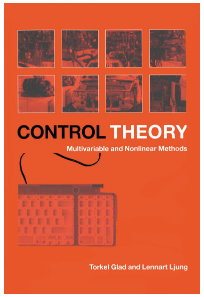
A. Practical information

Practical information

- **Course homepage** in Canvas
  https://kth.instructure.com/courses/31142
- **Lectures**: deal with the theory
  - one or more topics for each week
  - videos, lecture notes and slides for each topic available on homepage
  - lectures in some form of flipped classroom
  - one discussion group in Canvas for each week
- **Exercises**: problem solving
  - videos, and exercises w solutions, available on homepage
  - exercises in some form of flipped classroom
  - one discussion group in Canvas for each exercise
- **Computer Labs and project**: testing the theory
  - group work, supervision available on scheduled times
- **Course administration**: Student Service EECS

https://www.kth.se/en/eecs/kontakt/studentexpedition-eecs-1.21727

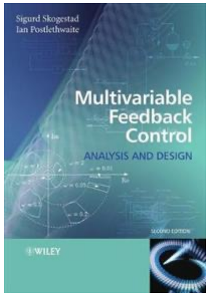
Course Book



The nominal course book is

Torkel Glad and Lennart Ljung; Control Theory - Multivariable and Nonlinear Methods, Taylor and Francis Ltd, ISBN 0748408789 (Swedish version also available)

Reading assignments under Lectures in Canvas refers to this book



An alternative recommended book is (more in-depth)

Skogestad, S. och Postlethwaite, I.: Multivariable Feedback Control, Analysis and Design, 2nd ed, John Wiley & Sons, 2005

Reading assignments for this book are available as a downloadable pdf under Lectures in Canvas

All other course material can be downloaded from course homepage

Course elements

- 16 topics/video lectures, Elling W. Jacobsen

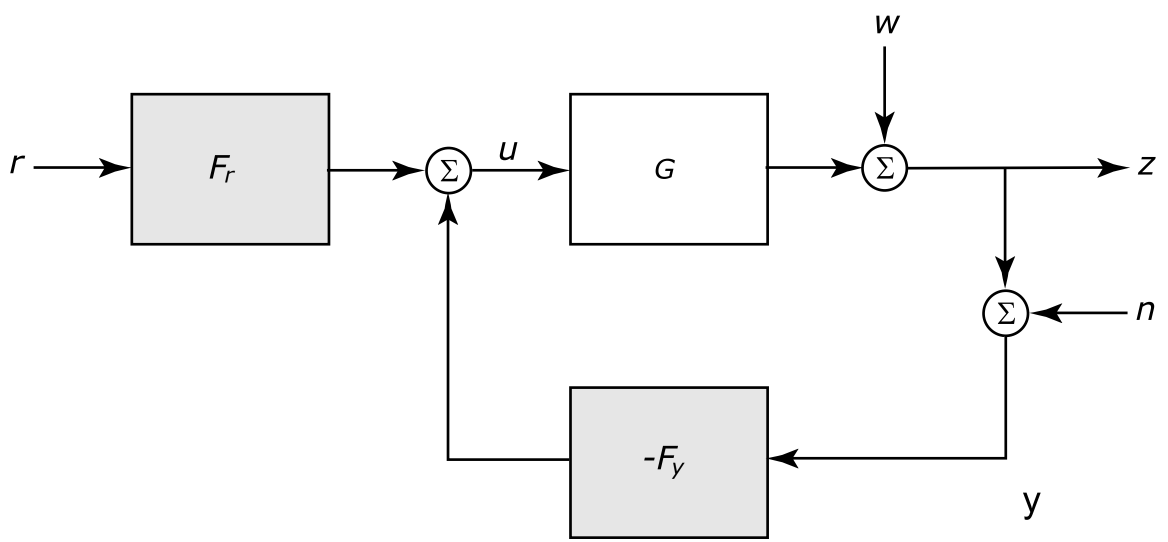
- 10 exercises, Yu Wang/Rijad Alisic

- 4 computer labs, Rijad Alisic/Yu Wang
  – in groups of 2 students
  – two alternative times for each computer lab
  – report deadlines April 7, April 21, May 5, May 19

- 1 lab project, Rijad Alisic/Yu Wang
  – in groups of 4 students
  – two sessions in laboratory
  – book times May 9-20 (not open for booking yet)
  – report deadline: May 19

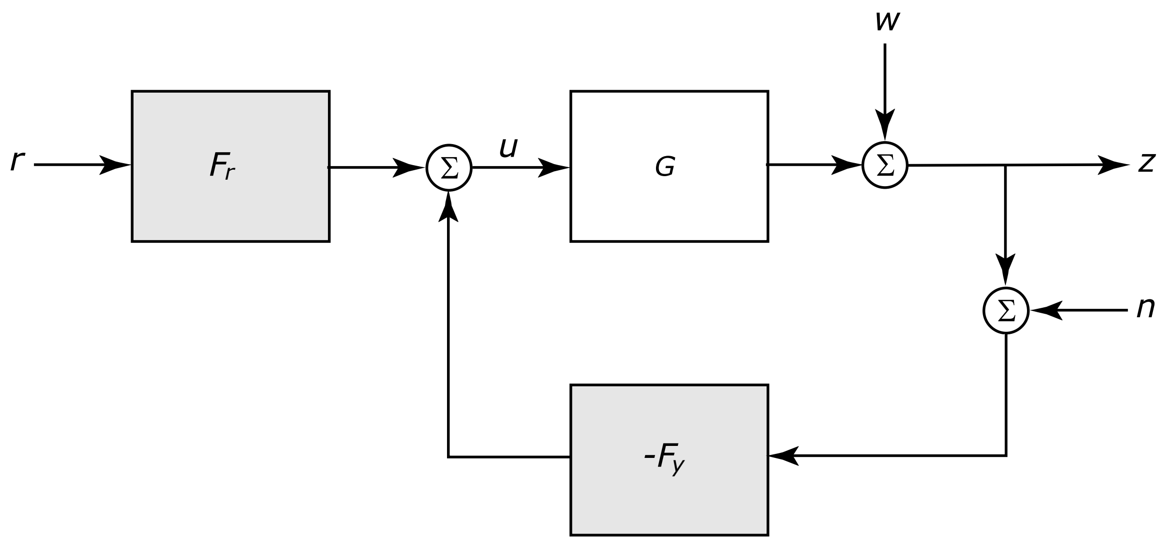
- Exam: 5 hours written exam, Tue May 31, 08-13.

Feedback Control



Given a system G, with measured signal y, determine a control input u so that the controlled variable z follows as closely as possible a reference signal r, despite disturbances w, and measurement noise n.

Why Feedback Control?



- Attenuate **unmeasured disturbances**
- Reduce impact of **uncertainty**
- Stabilize **unstable** systems

Powerful tool for tayloring system behavior!
Core of data-driven optimization!

Why EL2520?

- MIMO systems (Multi-Input-Multi-Output)
- Fundamental limitations
- Sensitivity and robustness
- Control problems cast as optimization problems
- Dealing with hard constraints
- Real-time optimal computer based control

- Applications

- Understand dynamic systems!

Course structure

1. Fundamentals of (modern) SISO control (Week 12-13)
2. Modern control of multivariable linear systems (Week 14-19)
3. Control of systems with constraints (Week 19-20)

Linear Time Invariant (LTI) Models

- State Space

$$\begin{aligned} \dot{x} &= Ax(t) + Bu(t) \ ; \ x \in \mathbb{R}^n, u \in \mathbb{R}^m \\ y(t) &= Cx(t) + Du(t) \ ; \ y \in \mathbb{R}^p \end{aligned}$$

- Transfer-function (assuming x(0)=0)

$$Y(s) = G(s)U(s) \ ; \quad G(s) = C(sI - A)^{-1}B + D$$

- Frequency response

$$Y(i\omega) = G(i\omega)U(i\omega)$$

  - Note: all variables usually deviation variables

A Brief History of Linear Control

- **Classical control**, 1930s (Bode, Nyquist, Nichols,…..)
  - frequency response methods
  - address robustness (phase and amplitude margins)
  - SISO systems only
- **Optimal control**, 1950s-60s (Bellman, Pontryagin, Kalman,……)
  - state-space methods
  - MIMO systems, control cast as optimization problem
  - do not address robustness
- **Robust control**,1980s-90s (Zames, Francis, Doyle,….)
  - frequency domain methods for MIMO systems
  - address model uncertainty explicitly
  - control cast as optimization problem in frequency domain, solved in state-space

This course will focus on Robust control, but we will also visit the other approaches

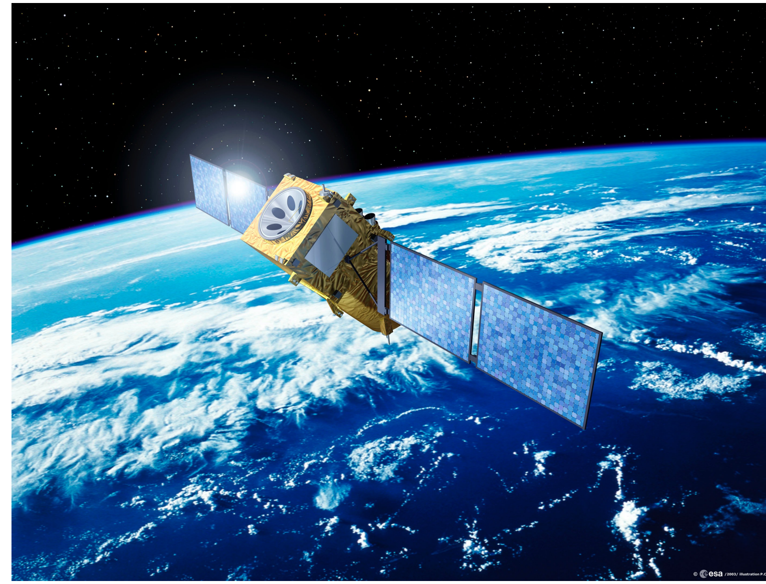
Applications

Use control theory to analyze and modify system properties!

Applications in most engineering domains:

- Aerospace
- Cars and heavy vehicles
- Autonomous systems and robots
- Process industry
- Consumer products
- Communication systems
- Economics
- Biology
- …

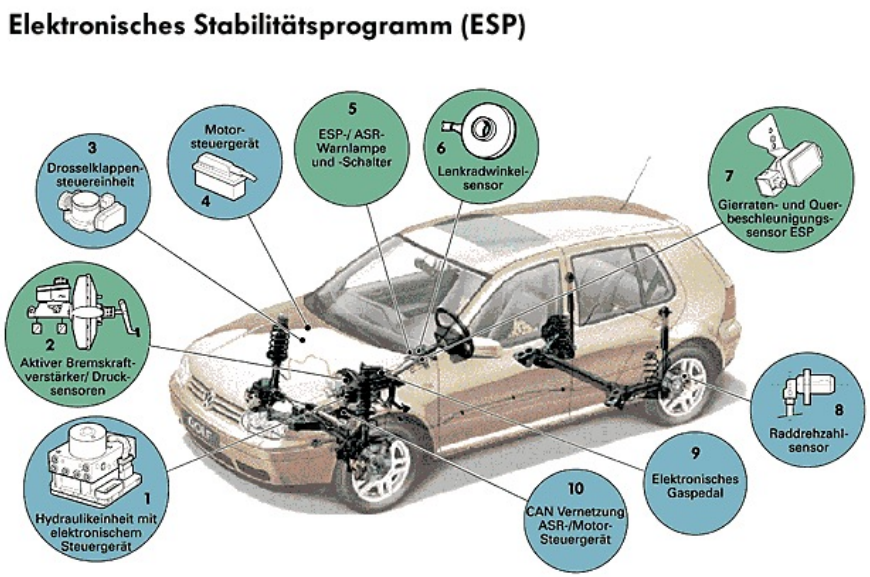
Aerospace



“Rocket Science”

- SpaceX

Vehicle control

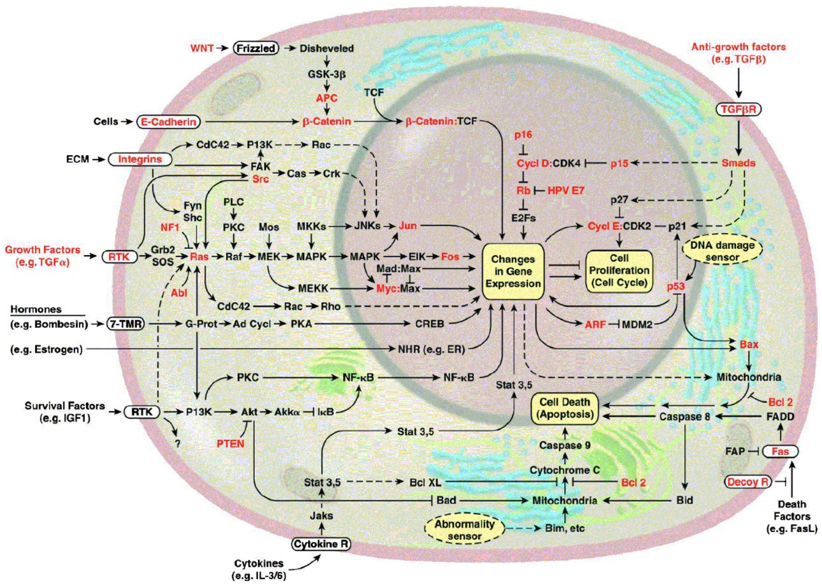


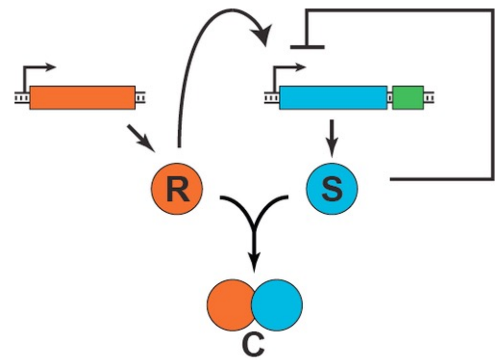
Autonomous systems and robotics

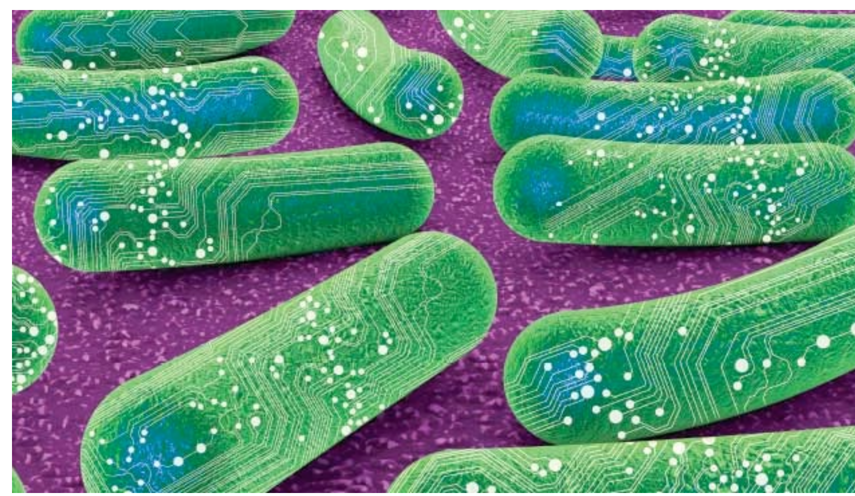
Process industry

Consumer products

Systems and Synthetic Biology







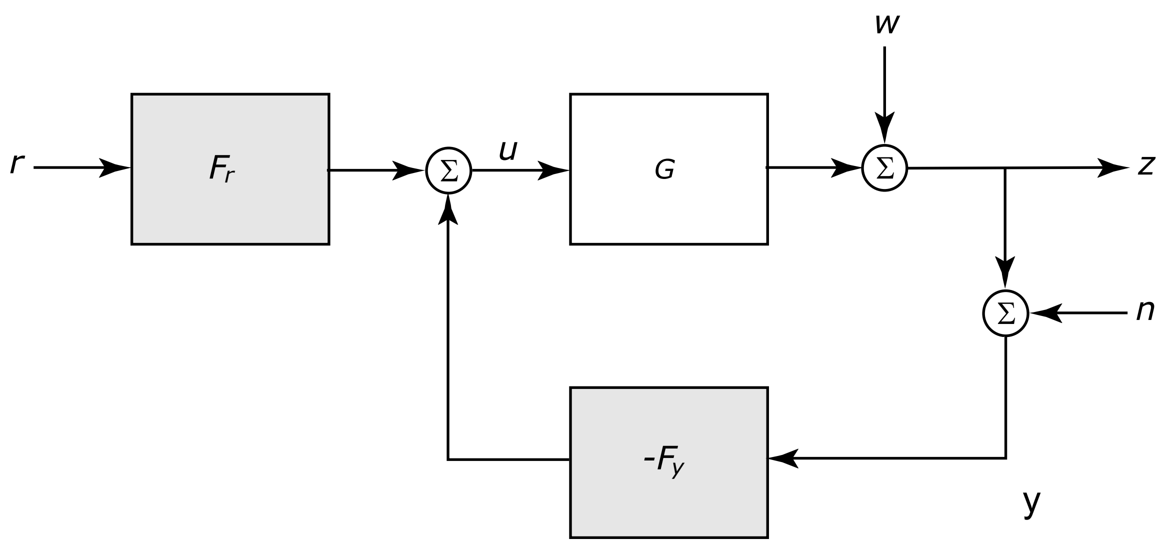
How to learn more?

- Internet
- Books (see course information, home page)
- Journals (IEEE Trans Automatic Control, Automatica, Control Systems Magazine …
- Courses (Hybrid Systems, MPC, Reinforcement Learning, Nonlinear Control, Modeling of Dynamical Systems, …)
- Software (Matlab, …)

- Interact with us!

B. Introduction to Multivariable Systems and Control

Multivariable feedback control



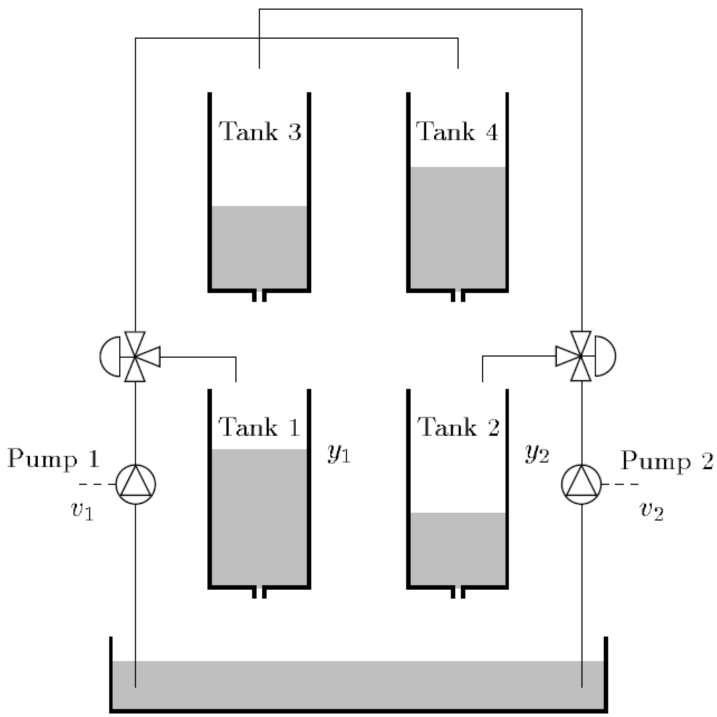
Multi-Input-Multi-Output (MIMO): all signals are vectors, all transfer-functions are matrices

Multivariable systems

Key aspects:

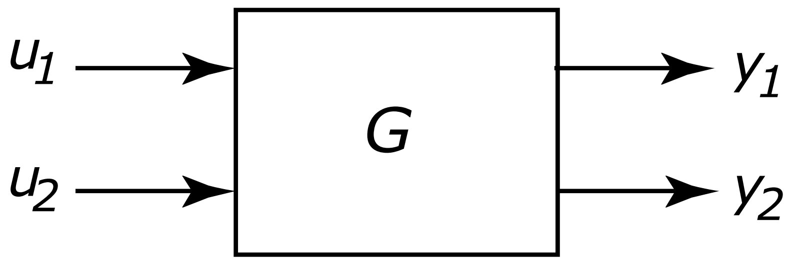
- several inputs and outputs,
- dynamics coupled (each input affects several outputs)

Laboratory project example: quadruple tank system



The need for multivariable control

Example. Consider a linear system with two inputs and two outputs

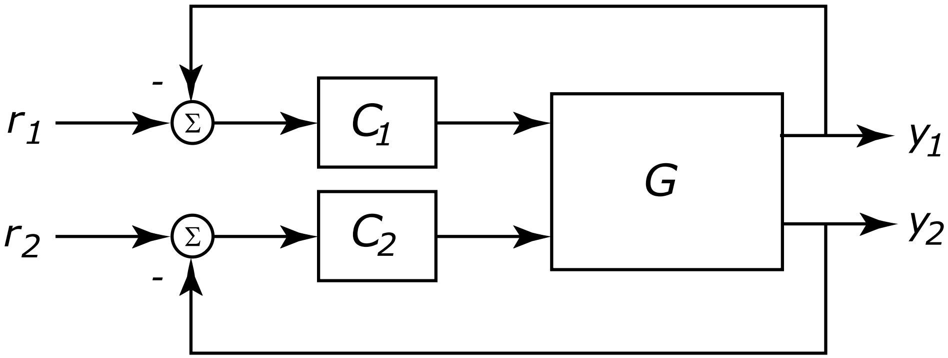


$$Y_1(s) = \frac{2}{s+1}U_1(s) + \frac{3}{s+2}U_2(s)$$
$$Y_2(s) = \frac{1}{s+1}U_1(s) + \frac{1}{s+1}U_2(s)$$

Note: both inputs influence both outputs!

Decentralized control

Simple approach: pair inputs and outputs, use SISO control



Use u_1 to control y_1 and u_2 to control y_2. Use PI-control in each loop

$$U_i(s) = \frac{K_i(s+1)}{s}(R_i(s) - Y_i(s)) \quad i = 1,2$$

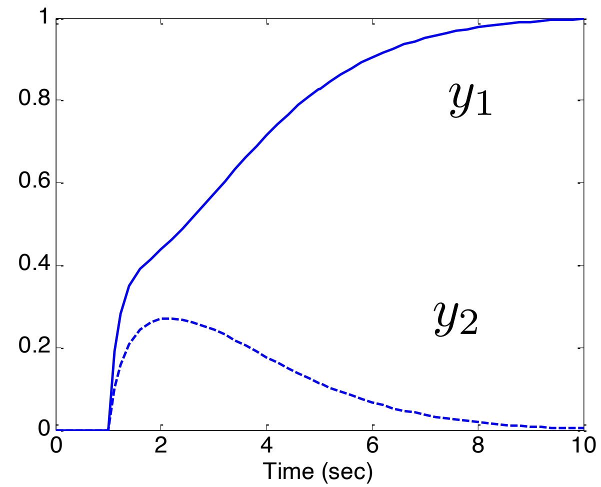
gives transfer functions for individual loops

$$G_{r_1 \to y_1} = \frac{2K_1}{s+2K_1} \; ; \quad G_{r_2 \to y_2} = \frac{K_2}{s+K_2}$$

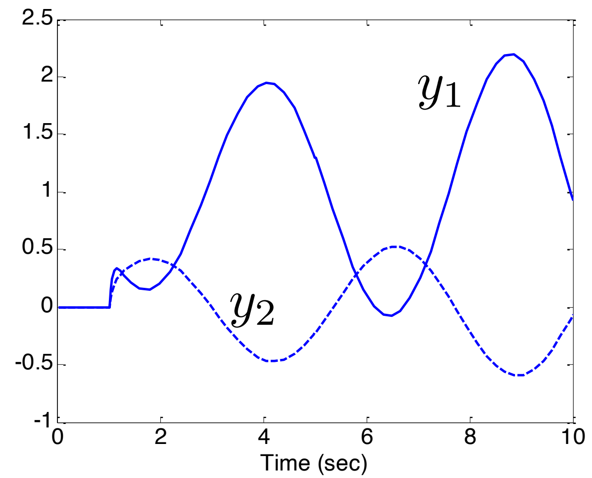
Stable for all positive values of K_1, K_2!

Decentralized control

Response to step-change in r_1, with $K_1=1$, $K_2=2$
Seems OK (but slow)

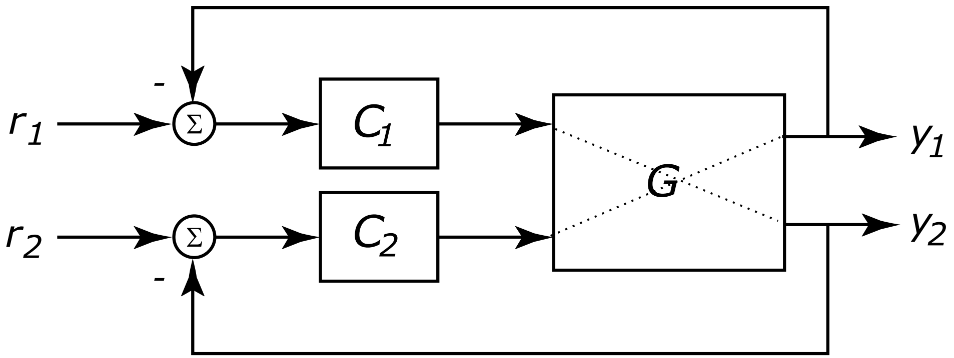


and with $K_1=4$, $K_2=8$



Multivariable system unstable, even if SISO analysis indicates stability!

What is happening?



$$Y_1(s) = \frac{2}{s+1}U_1(s) + \frac{3}{s+2}U_2(s)$$

$$Y_2(s) = \frac{1}{s+1}U_1(s) + \frac{1}{s+1}U_2(s)$$

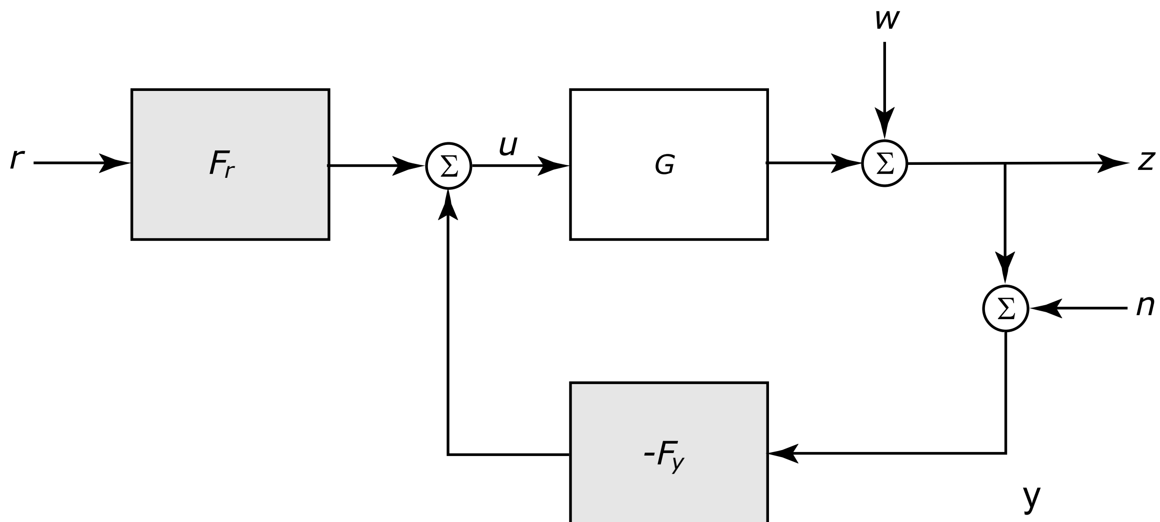
Interactions in the system make the control loops coupled!

Multivariable analysis

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \left(\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} - \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \right) \Rightarrow Y = (I + GC)^{-1} GC\, R$$

Elements of closed-loop transfer matrix very different from SISO analysis!

What to do?



- Design MIMO controllers, i.e., for vector signals / transfer-matrices
- Formulate it as an optimization problem, e.g., minimize control error and input usage
- Need to quantify size of signals (and systems): norms

Video 1: Signal norms and System gain