DECISIONS AND CONTROL

KTH

<u>EL2520 Control Theory and Practice - Advanced Course</u>

Exam 08.00–13.40   May 25, 2020

Part I (problems 1-2) 08.00-10.00, hand in 10.00-10.20

Part II (problems 3-5) 10.35-13.35, hand in 13.35-13.55

Aid:

Printed course book *Glad and Ljung, Control Theory / Reglerteori* **or** *Skogestad and Postlethwaite, Multivariable Control*, basic control course book *Glad and Ljung, Reglerteknik*, or equivalent if approved by examiner beforehand, printed copies of slides from lectures 2020, printed lecture notes from 2019, mathematical tables, pocket calculator (graphing, not symbolic). Any notes related to solutions of problems are not allowed. No digital aids allowed.

Note that separate notes, exercise material and old exams etc are NOT allowed.

Observandum:

- Name and social security number (*personnummer*) on every page.
- Only one solution per page.
- Each answer has to be motivated.
- The exam consists of five problems worth a total of 50 credits
- The answers to Part I should be uploaded on Canvas no later than 10.20
- The answers to Part II should be uploaded on Canvas no later than 13.55

Grading:

Grade A: $\geq$ 43, Grade B: $\geq$ 38

Grade C: $\geq$ 33, Grade D: $\geq$ 28

Grade E: $\geq$ 23, Grade Fx: $\geq$ 21

Results: The results will be available about 3 weeks after the exam on "My Pages".

Responsible: Elling W. Jacobsen 0703722244

Good Luck!

1. (a) Determine the poles and zeros of

$$G(s) = \frac{1}{s+1}\begin{pmatrix} \frac{6}{s+2} & 1 \\ 2 & 1 \end{pmatrix}$$

and for those in the RHP the corresponding input and output directions. (4p)

(b) Determine the $\mathcal{H}_\infty$-norm of

$$G(s) = \frac{1}{s+1}\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

(2p)

(c) Given a 2×2 system with model

$$G(s) = \frac{1}{s+1}\begin{pmatrix} \frac{1}{10s+1} & 1 \\ 1 & 2 \end{pmatrix}$$

The aim is to design a feedback controller so that the bandwidth for the sensitivity function is around 1 rad/s. You are asked to determine if decentralized control is a reasonable choice for this system, and if so, which pairing of inputs and outputs is preferrable. Motivate your answer! (4p)

2. (a) Someone has proposed the following $\mathcal{H}_2$-optimal control problem

$$F_y = \arg\min_{F_y} \|S\|_2$$

where S is the closed-loop sensitivity function. Explain why this is an ill-posed problem. (2p)

(b) Consider the system

$$G(s) = \frac{1}{(5s+1)^2}\begin{pmatrix} 1 & -(s+2) \\ 1 & -3 \end{pmatrix}$$

(i) Someone has proposed a feedback controller $F_y(s)$ so that the closed-loop sensitivity becomes

$$S(s) = \frac{s}{s+1}\begin{pmatrix} 1 & -0.1 \\ 0.1 & 1 \end{pmatrix}$$

Is the closed-loop system stable? (3p)

(ii) We want to find a controller for the system by solving the $\mathcal{H}_\infty$-optimal control problem

$$\arg\min_{F_y} \|W_p S\|_\infty \; ; \quad W_p = 10\frac{0.5\tau s + 1}{10\tau s + 1}$$

For what values of τ is it feasible to find a controller that gives $\|W_p S\|_\infty < 1$? (2p)

(c) A commonly used model for the dynamics of virus spread is the SIR model. A discrete time linearized SIR model is

$$\begin{pmatrix} S_{n+1} \\ I_{n+1} \end{pmatrix} = \begin{pmatrix} 1 & -\frac{\beta}{N}S_e \\ 0 & 1-\gamma+\frac{\beta}{N}S_e \end{pmatrix}\begin{pmatrix} S_n \\ I_n \end{pmatrix}$$

Here S is the number of susceptible, I is the number of infected, β is the average number of susceptible contacts an infected person will have during infection, $1/\gamma$ is average number of days an individual remains infected, N is the total population considered and S_e is the number of susceptible at the point of linearization. The ratio $R_0 = \beta/\gamma$ is called the basic reproduction ratio. Assume that initially the total population is susceptible, i.e., $S_e = N$. For what values of R_0 is the model unstable, i.e., we get an (exponential) increase in the number of infected? (3p)

3. (a) An engineer who has learnt LQG is trying to design an optimal state feedback combined with a Kalman filter for the system

$$\begin{aligned}\dot{x}_1 &= x_2(t) + 2u(t) + w_1(t)\\ \dot{x}_2 &= x_1(t)\\ z(t) &= x_1(t) - 2x_2(t) + w_2(t)\end{aligned}$$

The manager of the project has required that the disturbance on the output $w_2(t)$ should never be amplified by a factor more than 1.5 in the output $z(t)$. However, the engineer is not able to meet this specification no matter how the weights are chosen in the LQG problem. Since you have taken the control theory and practice course at KTH you know why. What is the reason the specification can not be met for any LQG controller ? (4p)

(b) Consider the 2×2 process

$$z = \frac{1}{5s+1}\begin{pmatrix} 1 & s+2 \\ s+1 & s+5 \end{pmatrix} u + \frac{1}{s+1}\begin{pmatrix} 3 \\ 2 \end{pmatrix} d$$

Assume all variables have been scaled so that the aim is to keep $|z| < 1$ in the presence of disturbances $|d| < 1$. Determine if this is feasible, that is, if there exist a stabilizing controller that fulfills the aim (you can assume all variables being sinusoidal). (4p)

(c) Someone has designed a controller for the system

$$G(s) = \frac{1}{(s+1)^3}$$

such that the sensitivity function becomes

$$S(s) = 0.9\frac{s}{s+1}$$

Without determining what the controller is, explain why the controller can not be realized. (2p)

4. (a) The transfer-function for a system with a time-delay θ is

$$G(s) = G_0(s)e^{-\theta s}$$

where $G_0(s)$ is the delay-free transfer-function. To simplify the controller design we can choose to use the delay-free model $G_0(s)$ and then ensure that the system is closed-loop stable also with the delay, i.e., with $G(s)$. One way to do this is to use the model set

$$G_p(s) = G_0(s)(1 + W_I(s)\Delta(s)) \; ; \quad \|\Delta\|_\infty < 1$$

and choose the uncertainty weight $W_I(s)$ such that $G(s)$ is within the model set. By ensuring robust stability for the model set G_p we also ensure that the closed-loop system is stable with the delay included. Derive a robust stability condition which depends on the nominal closed-loop and the delay θ. Also determine the bandwidth limitation that the robust stability condition gives rise to. (5p)

(b) We shall design a controller for the multivariable system

$$z = G(s)(u + w_u)$$

using $\mathcal{H}_\infty$-optimal control. The design objectives are

(a) Disturbances w_u should be attenuated in the output z by a factor at least 10 up to a frequency $\omega = 0.5 \; rad/s$.

(b) The closed-loop system should be robustly stable for 30% uncertainty in the control inputs u.

Formulate an $\mathcal{H}_\infty$-optimal control problem that reflects the above objectives. (5p)

5. When a system has dynamic properties that put fundamental limitations on the control performance, and these limitations are in conflict with the desired control performamce, then the system itself needs to be modified. One possible modification is then to add measurements and/or control inputs. We shall consider a simple positioning system with the model

$$z = \frac{s - 0.2}{s + 1} u$$

Here z reflects a certain position and u is the voltage to a motor. The design objective is

$$\|W_P S\|_\infty < 1 \; ; \quad W_P = \frac{s + 2\omega_{Bs}}{2s + 0.01} \; ; \quad \omega_{Bs} = 1$$

(a) Show that there is no stabilizing controller that can meet the design objective. (2p)

(b) One considers first adding a second output y with

$$y = \frac{1}{s + 1} u$$

Show that the extended system

$$\begin{pmatrix} z \\ y \end{pmatrix} = \begin{pmatrix} \frac{s-0.2}{s+1} \\ \frac{1}{s+1} \end{pmatrix} u$$

has no RHP zeros. Does this solve the problem, i.e., is the design objective feasible now? Motivate! (3p)

(c) One consider next adding also an extra control input v so that the overall system is

$$\begin{pmatrix} z \\ y \end{pmatrix} = \frac{1}{s + 1} \begin{pmatrix} s - 0.2 & s - 0.5 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

Show that by closing a loop between one of the inputs and the output y, using a simple P-controller, one can make the control design objective for the output z feasible. What pairing of inputs and outputs should one choose? (5p)