DECISIONS AND CONTROL
KTH

<u>EL2520 Control Theory and Practice - Advanced Course</u>

Exam 08.00–13.00 May 31, 2022

Aid:

Printed course book *Glad and Ljung, Control Theory / Reglerteori* **or** *Skogestad and Postlethwaite, Multivariable Control*, basic control course book *Glad and Ljung, Reglerteknik*, or equivalent if approved by examiner beforehand, printed copies of slides from lectures 2022, printed lecture notes, mathematical tables, pocket calculator (graphing, not symbolic). **Any notes related to solutions of problems are not allowed**. **No digital aids allowed.**

Note that separate notes, exercise material and old exams etc are NOT allowed.

Observandum:

- Name and social security number (*personnummer*) on every page.
- Only one solution per page.
- Each answer has to be motivated.
- The exam consists of five problems worth a total of 50 credits

Grading:

Grade A: ≥ 43, Grade B: ≥ 38

Grade C: ≥ 33, Grade D: ≥ 28

Grade E: ≥ 23, Grade Fx: ≥ 20

All grades require at least 18p on problems 1-3

Results: The results will be available about 3 weeks after the exam on ”My Pages”.

Responsible: Elling W. Jacobsen 0703722244

Good Luck!

1. (a) Consider the system

$$G(s) = \frac{1}{(s+1)(s-1)} \begin{pmatrix} (s-1) & (s-2) \end{pmatrix}$$

Determine the poles and zeros, and the corresponding input direction for those in the RHP. (3p)

(b) Consider control of the system

$$y = \frac{10e^{-5s}}{10s+1} u + \frac{8e^{-s}}{10s+1} d$$

Assume the system is scaled such that disturbances $|d| < 1$, allowed inputs $|u| < 1$ and acceptable control corresponds to $|y| < 1$. Determine if it is possible to achieve acceptable performance using feedback control. (3p)

(c) Consider the discrete time system

$$x_{k+1} = 0.5x_k + u_k$$

Determine the input sequence U that minimizes the criterion

$$J(U) = \sum_{k=0}^{1} (Q_1 x_k^2 + Q_2 u_k^2)$$

Also determine the number of samples it takes to reach steady-state if $Q_2 = 0$ and $x_0 \neq 0$. (4p)

2. (a) A controller has been designed for the system

$$G(s) = \frac{s-1}{s(s+1)}$$

such that the sensitivity function is

$$S(s) = \frac{s(s+1)}{s^2 + 0.5s + 0.5}$$

Determine if the closed-loop system is internally stable. (3p)

(b) Consider controller synthesis for the following system

$$G(s) = \frac{1}{(0.3s+1)(s+1)} \begin{pmatrix} s+2 & s+1 \\ 8 & 6 \end{pmatrix}$$

The aim is to shape the sensitivity function such that

$$\|W_p S\|_\infty \leq 1 \; ; \quad W_p = \frac{s+10}{2s+1} \tag{1}$$

(i) Is it possible to find a controller that fulfills the objective above? (3p)

(ii) Consider replacing the $\mathcal{H}_\infty$-norm by the $\mathcal{H}_2$-norm in (1). Is it possible to find a controller such that $\|W_p S\|_2 \leq 1$. (1p)

(c) A model of a heat exchanger is given by

$$\begin{pmatrix} T_h \\ T_c \end{pmatrix} = \frac{1}{(10s+1)(2s+1)} \begin{pmatrix} 2(s+1) & -3 \\ 10 & -20(s+2) \end{pmatrix} \begin{pmatrix} q_h \\ q_c \end{pmatrix}$$

The aim is to design a feedback controller such that the closed-loop system has a bandwidt around 1 rad/min. One considers using a decentralized control structure in which the cold side temperature T_c is controlled by the cold flow q_c and the hot side temperature T_h is controlled by the hot flow q_h. Use the RGA to determine if this is a reasonable choice. (3p)

3. We shall consider control design for the following system

$$Z = \frac{1}{(10s+1)(s+1)} \begin{pmatrix} s+1 & s+2 \\ s & 2 \end{pmatrix} U + \frac{1}{s+1} \begin{pmatrix} 5 \\ 3 \end{pmatrix} D$$

Assume that the disturbance $|d| < 5$ and that the aim is to keep $|z| < 0.5$ for all frequencies. Also assume 20% uncertainty in the control input u at low frequencies, and that the uncertainty exceeds 100% for frequencies above the frequency $\omega = 100\ rad/s$. You can assume that there are no constraints on the input u.

(a) The primary aim of the control system is to attenuate disturbances $d(t)$ in the output $z(t)$. Analyze the system to determine if the objective stated above is feasible using feedback control. (3p)

(b) Is it feasible to find a controller that provides robust stability in the presence of the given input uncertainty? (1p)

(c) Define a controller synthesis problem for synthesizing a controller that provides acceptable disturbance attenuation while ensuring robust stability in the presence of the given input uncertainty. That is, determine a suitable objective function J to be used in the optimization problem

$$F_y = \arg \min_{F_y} J$$

(4p)

(d) To solve the optimization problem in (c) we need to write the problem as a signal minimization problem in state-space. Determine inputs and outputs and suitable signal norms such that minimizing the norm of the output over the norm of the input reflects the objective in (c). (2p)

4. Given the multivariable system on state-space form

$$\begin{aligned}\dot{x} &= \begin{pmatrix} -2 & 0 \\ 0 & -4 \end{pmatrix} x(t) + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} u(t) \\ y(t) &= \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} x(t)\end{aligned}$$

(a) Determine a state feedback that minimizes the cost

$$J = \int_0^\infty (x^T Q_1 x + u^T Q_2 u) dt$$

Assume $Q_2 = I$ and that $Q_1 = \begin{pmatrix} q_1 & 0 \\ 0 & q_2 \end{pmatrix}$. How should q_1 and q_2 be chosen to have two closed-loop poles in $s = -5$? (4p)

(b) Determine the complementary sensitivity function at the input $T_I(s)$ when the control from (a) is used. (2p)

(c) On the basis of the answer in (b), what input can we tolerate the largest uncertainty in without risking instability? (2p)

(d) Assume now that we add white noise with covariance R_1 to $\dot{x}$ and white noise with covariance R_2 to the measurement $y(t)$. Assume we design a Kalman filter and base the feedback in (a) on the estimated state $\hat{x}(t)$. To increase robust stability, which of R_1 or R_2 should we increase in the design of the Kalman filter? (2p)

5. For SISO systems we showed in the lectures that ensuring nominal performance and robust stability, with some margin, ensured also robust performance. However, for MIMO systems this does not hold. We shall demonstrate this here using decoupling control as an example. As we shall see, there is also a big difference between uncertainty at the input side and at the output side for MIMO systems, in particular when it comes to the impact of uncertainty on performance.

   Consider the system

$$G(s) = \frac{1}{5s+1}\begin{pmatrix} 1 & -1.1 \\ 1 & -1 \end{pmatrix}$$

   The system is strongly interactive, so we try a decoupling controller

$$F_y(s) = \frac{1}{s}G^{-1}(s)$$

   (a) Determine the sensitivity function and the corresponding $\|S\|_\infty$ for the closed-loop system. (1p)

   The inputs are uncertain and we model this as

$$u_{p1} = u_1(1+\delta_1)\ ; \quad u_{p2} = u_2(1+\delta_2) \quad |\delta_i| < 0.2 \tag{2}$$

   where u is the input as computed by the controller, u_p is the input actually implemented on the system and δ_i is a real perturbation.

   (b) Show that the closed-loop is robustly stable with the input uncertainty in (2) by computing the closed-loop poles as a function of δ_1 and δ_2. *Note that the closed-loop poles can be computed from* $\det(I + L_I(s)) = 0$ *where* $L_I(s) = F_y(s)G(s)$ *is the loop-gain at the input provided there are no cancellations in the RHP.* (1p)

   (c) Now use a robust stability condition to show that the closed-loop is robustly stable for the uncertainty in (2). Discuss the assumptions made on the uncertainty relative to the uncertainty in (2). (3p)

   (d) Determine the sensitivity function S_p for the two cases $\delta_1 = \delta_2 = \delta$ and $\delta_1 = -\delta_2 = \delta$. Also determine the corresponding values of $\|S_p\|_\infty$ for $\delta = 0.2$. Would you conclude that the system displays robust performance in terms of the sensitivity function S from this result? (3p)

   (e) Repeat (c) and (d) for the case with output uncertainty instead of input uncertainty, i.e.,

$$y_{p1} = y_1(1+\delta_1)\ , \quad y_{p2} = y_2(1+\delta_2) \quad |\delta_i| < 0.2$$

   (2p)