AUTOMATIC CONTROL
KTH

<u>EL2520 Control Theory and Practice - Advanced Course</u>

Exam 08.00–13.00 May 29, 2019

Aid:

Course book *Glad and Ljung, Control Theory / Reglerteori* **or** *Skogestad and Postlethwaite, Multivariable Control*, basic control course book *Glad and Ljung, Reglerteknik*, or equivalent if approved by examiner beforehand, copies of slides from lectures 2019, lecture notes from 2019, mathematical tables, pocket calculator (graphing, not symbolic). Any notes related to solutions of problems are not allowed.

Note that separate notes, exercise material and old exams etc are NOT allowed.

Observandum:

- Name and social security number (*personnummer*) on every page.
- Only one solution per page.
- Do only write on one side per sheet.
- Each answer has to be motivated.
- Specify the total number of handed in pages on the cover.
- The exam consists of five problems worth a total of 50 credits

Grading:

Grade A: ≥ 43, Grade B: ≥ 38

Grade C: ≥ 33, Grade D: ≥ 28

Grade E: ≥ 23, Grade Fx: ≥ 21

Results: The results will be available about 3 weeks after the exam on "My Pages".

Responsible: Elling W. Jacobsen 0703722244

Good Luck!

1. (a) Determine the poles and zeros of

$$G(s) = \frac{1}{s+1}\begin{pmatrix} \frac{1}{s+2} & -1 \\ 1 & 1 \end{pmatrix}$$

What is the number of states in a minimal state-space realization of the model?
. (4p)

(b) Determine the $\mathcal{H}_\infty$-norm of

$$S(s) = \frac{s}{s+1}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

(2p)

(c) Consider the 2×2 system

$$G(s) = \frac{1}{10s+1}\begin{pmatrix} 1 & \frac{-0.9}{s+1} \\ 1 & -1 \end{pmatrix}$$

Use the RGA to determine the preferrable pairing of inputs and outputs if decentralized control is to be used with a closed-loop bandwidth $\omega_B \approx 1$ rad/s. Would you recommend decentralized control for this system? Motivate! (4p)

2. (a) An electromechanical system with two inputs and two outputs is described by the model

$$G(s) = \frac{1}{(0.5s+1)^2} \begin{pmatrix} 2 & s+1 \\ 3 & 2 \end{pmatrix}$$

An engineer has proposed to use $\mathcal{H}_\infty$-optimization to shape the sensitvity function S such that

$$\bar{\sigma}(S(i\omega)) \leq |W_p^{-1}(i\omega)| \; \forall \omega$$

where

$$W_p(s) = 100\frac{0.05s+1}{10s+1}$$

Determine whether this is feasible, that is, whether there exist an internally stabilizing controller such that the bound is met. (3p)

(b) Given the system

$$Y(s) = \frac{2-s}{s}U(s)$$

We shall design a P-controller $U(s) = -K_c Y(s)$ so that

$$\|w_P S\|_\infty \; ; w_P = \frac{0.5(s+1)}{s}$$

is minimized. Determine the minimum value of $\|w_P S\|_\infty$ and the corresponding controller gain K_c. Also determine the lower bound on $\|w_P S\|_\infty$ when we allow any internally stabilizing controller. (4p)

(c) Consider the 1×2 system

$$G(s) = \begin{pmatrix} \frac{1-s}{10s+1} & \frac{s-2}{(10s+1)^2} \end{pmatrix}$$

What is the maximum frequency for which we can attenuate disturbances at the output using feedback with i) input u_1 only, ii) input u_2 only and iii) using both inputs simultaneously? (3p)

3. (a) A biochemical reactor with two control inputs, a feed flow u_1 and a cooling flow u_2, and two controlled outputs, the biomass y_1 and a protein concentration y_2, is to be controlled to attenuate disturbances. The most serious disturbance is changes in the cooling flow due to upsets in other parts of the process. A model has been developed for the reactor

$$\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = \frac{1}{(3s+1)^2} \begin{pmatrix} s+1 & -1 \\ 4 & -2 \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \end{pmatrix}$$

where $u_1 = u_{1c}$ and $u_2 = u_{2c} + u_d$ where u_{1c}, u_{2c} are the inputs computed by the controller and u_d is a disturbance on input 2. Assume the disturbance $|u_d| < 0.1$ and that the aim is to keep $|y| < 0.01$ at all frequencies. Determine if this is feasible. You can assume there are no constraints on the inputs u_1, u_2.

(4p)

(b) We shall design a feedback controller for a system with three inputs and three outputs. The aim of the control is to attenuate disturbances at the output by a factor of at least 100 at frequency $\omega = 0$ rad/s, and by a factor at least 10 for frequencies up to $\omega = 0.1$ rad/s. Measurement noise should be attenuated in the output at least by a factor of 10 for frequencies above $\omega = 10$ rad/s.

(i) Formulate requirements on the sensitivity S and complementary sensitivity T, reflecting the requirements above.

(ii) Translate the requirements on S and T into approximate requirements on the loop-gain L. Draw a figure showing the bounds imposed on L.

(iii) Can it be hard to meet the requirements? Motivate!

(iv) Determine weights W_S and W_T on S and T so that the performance requirements are met if $\|W_S S\|_\infty < 1$ and $\|W_T T\|_\infty < 1$.

(6p)

4. (a) We shall design an LQG controller for the system

$$G(s) = \frac{1}{s-1}$$

On state-space form, with disturbances and noise added, we get

$$\begin{aligned} \dot{x} &= x(t) + u(t) + v_1(t) \\ y(t) &= x(t) + v_2(t) \end{aligned}$$

where v_1 and v_2 are independent white noise with covariance $R_1, R_2 > 0$, respectively. The control objective is to minimize the objective function

$$J = \int_0^\infty Q_1 x^2 + Q_2 u^2 dt$$

where the weights $Q_1, Q_2 > 0$.

(i) Show that the LQG controller is a function of the ratios $r = R_1/R_2$ and $q = Q_1/Q_2$ only. (4p)

(ii) Determine the poles of the closed-loop system as a function of r and q. (2p)

(iii) Discuss how we can increase the robustness of the system with respect to relative input uncertainty through the choice of the parameters r and q. (2p)

(b) The discrete time system

$$x_{k+1} = \begin{pmatrix} -1 & 1 \\ 2 & 1 \end{pmatrix} x_k + \begin{pmatrix} 1 \\ 2 \end{pmatrix} u_k$$

is controlled by the state feedback

$$u_k = -\begin{pmatrix} 1 & 1 \end{pmatrix} x_k$$

Is the closed-loop system stable? (2p)

5. Decentralized control can be seen as neglecting off-diagonal elements in the transfer-matrix of the plant when designing the controller. It is then of course important that the presence of these off-diagonal elements in the true system does not cause problems with stability. One way to ensure this is to treat the off-diagonal terms as model uncertainty and then design the decentralized controller for robust stability with respect to this uncertainty.

   Consider again the 2×2 system from problem 1c

$$G(s) = \frac{1}{10s+1} \begin{pmatrix} 1 & \frac{-0.9}{s+1} \\ 1 & -1 \end{pmatrix}$$

   Assume we decide to use decentralized control and pair inputs and outputs on the diagonal, i.e., y_1 with u_1 and y_2 with u_2. We employ IMC based control which yields the controller

$$F_y(s) = \frac{10s+1}{s} \begin{pmatrix} K_{c1} & 0 \\ 0 & K_{c2} \end{pmatrix}$$

   Model the off-diagonal terms of $G(s)$ as relative input uncertainty and then use the corresponding robust stability criterion to determine for which values of K_{c1} and K_{c2} the closed-loop is robustly stable. (10p)