EL2520– Control Theory and Practice
Laboratory experiment: The four-tank process
Process A

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Abstract

This report is a summary of the experiment regarding the control of four-tank process, which can be shaped into minimum phase structure and non-minimum phase structure respectively. The first part of the experiment is concerning theoretical model specification as well as parameters calculations of the four-tank system. In the second experiment, decentralized control and Glover-McFarlane robust control were conducted for the system. Performance analysis was also done to further evaluate the differences between two different approaches.

For performance analysis we analysed the step response and behaviour after external disturbances. We found out that the Glover-McFarlane has robustified the controller as it supposed to since we get a stable and quite fast response to external disturbances. In contrast, for the minimum phase system the response with decentralized control was not asymptotically stable. In all experiments we experienced that the non-minimum phase system is slower in step response and disturbance attenuation. This is due to the RHP zero which limits the bandwidth of the sensitivity.

1 Introduction

This report is about the project lab in the course EL2520 Control Theory and Practice. In the project, we are asked to design controllers for a four-tank process. The work starts with the modeling part where we derive the system equations in state space and input-output space. Besides, we have to measure using suitable experiments to obtain some of the system parameters.

The four-tank process is depending on the valve setting either minimum-phase or non-minimum-phase. Controllers have to be calculated separately for each case. In total we have designed four different controllers. For each case one as a decentralized controller and a robustified one using the Glover-McFarlane method. The performance results are analyzed and compared together.

2 Laboratory occasion 1

2.1 Modeling

Before the controllers can be applied on the real plant, some modeling exercises have to be done. Firstly, we derive the system equations. After calculating equilibrium points, we are able to derive the linear state space model. The linear model is transformed into the frequency domain to analyze zeros and cross-coupling using RGA. Lastly, several parameters of the system are measured.

Exercise 2.1.1

To get the describing system equations we start with the given differential equation

$$A\frac{\mathrm{d}h}{\mathrm{d}t} = q_{in} - q_{out}.$$

There are two inflows for tank 1. One is generated by the pump $q_{\mathrm{L},1} = \gamma_1 k u_1$. The second is the outflow of tank 3 which is according to Bernoulli's law $q_{\mathrm{out},3} = a_3\sqrt{2gh_3}$. The outflow of tank 1 is respectively $q_{\mathrm{out},1} = a_1\sqrt{2gh_1}$. Putting all of them together, one gets

$$\frac{\mathrm{d}h_1}{\mathrm{d}t} = \frac{1}{A_1}(-q_{\mathrm{out},1} + q_{\mathrm{out},3} + q_{\mathrm{L},1}) \tag{1}$$

The second equation has the same structure but taking the outflow of tank 4. Tank 3 and 4 have only the inflow by the pump. Therefore, the differential equation for tank 3 is

$$\frac{\mathrm{d}h_3}{\mathrm{d}t} = \frac{1}{A_3}(-q_{\mathrm{out},3} + (1-\gamma_2)k_2u_2) \tag{2}$$

For tank 4 is used pump 1. Thus, the indices are interchanged correspondingly.

Exercise 2.1.2

The equilibrium values are obtained when the derivative of h with regard to t equals 0, i.e.

$$\left.\begin{aligned}
\frac{dh_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}u_1 = 0\\
\frac{dh_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}u_2 = 0\\
\frac{dh_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}u_2 = 0\\
\frac{dh_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}u_1 = 0
\end{aligned}\right\} \Rightarrow \left\{\begin{aligned}
u_1^0 &= \frac{a_4\sqrt{2gh_4^0}}{(1-\gamma_1)k_1}\\
u_2^0 &= \frac{a_3\sqrt{2gh_3^0}}{(1-\gamma_2)k_2}\\
h_1^0 &= \frac{\left[(1-\gamma_2)k_2u_2^0+\gamma_1k_1u_1^0\right]^2}{2ga_1^2}\\
h_2^0 &= \frac{\left[(1-\gamma_1)k_1u_1^0+\gamma_2k_2u_2^0\right]^2}{2ga_2^2}\\
h_3^0 &= \frac{(1-\gamma_2)^2k_2^2u_2^0}{2ga_3^2}\\
h_4^0 &= \frac{(1-\gamma_1)^2k_1^2u_1^0}{2ga_4^2}\\
y_1^0 &= k_ch_1^0\\
y_2^0 &= k_ch_2^0
\end{aligned}\right.$$

where u_i^0, h_i^0, y_i^0 denote the steady state values.

Exercise 2.1.3

According to the definition of the equations which describe the water levels in the four tanks, we have

$$\frac{d\Delta h_i}{dt} = \frac{d(h_i - h_i^0)}{dt} = \frac{dh_i}{dt}$$

For the formula $\sqrt{2gh_i}$, we perform a Taylor series expansion at equilibrium value h_i. Neglecting the high-order terms, we have

$$\begin{aligned}\sqrt{2gh_i} &= \frac{\sqrt{2gh_i^0}}{0!} + \frac{\frac{g}{\sqrt{2gh_i^0}}}{1!}(h - h_i^0) + o(h - h_i^0) \\ &\approx \sqrt{\frac{g}{2h_i}}h + \sqrt{\frac{gh_i^0}{2}}\end{aligned}$$

Therefore, we have

$$\begin{aligned}\frac{d\Delta h_i}{dt} &= -\frac{a_i}{A_i}\sqrt{\frac{g}{2h_i^0}}\Delta h_i + \frac{a_j}{A_i}\sqrt{\frac{g}{2h_j^0}}\Delta h_j + \frac{\gamma_i k_i}{A_i}\Delta u_i (i = 1,2; j = 3,4) \\ \frac{d\Delta h_i}{dt} &= -\frac{a_i}{A_i}\sqrt{\frac{g}{2h_i^0}}\Delta h_i + \frac{(1-\gamma_j)k_j}{A_i}\Delta u_j (i = 3,4; j = 2,1)\end{aligned}$$

By introducing

$$u = \begin{bmatrix} \Delta u_1 \\ \Delta u_2 \end{bmatrix}, \quad x = \begin{bmatrix} \Delta h_1 \\ \Delta h_2 \\ \Delta h_3 \\ \Delta h_4 \end{bmatrix}, \quad y = \begin{bmatrix} \Delta y_1 \\ \Delta y_2 \end{bmatrix}, \quad T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_i^0}{g}}$$

we obtain

$$\begin{aligned}\frac{dx}{dt} &= \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} x + \begin{bmatrix} \frac{\gamma_1k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} u \\ y &= \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} x\end{aligned}$$

Exercise 2.1.4

Calculating the transfer function matrix, the general formula $G(s) = C \cdot (sI - A)^{-1} \cdot B$ is used. Plugging in the matrices from Exercise 2.1.3 we get

$$G(s) = \begin{bmatrix} \frac{\gamma_1k_1c_1}{1+sT_1} & \frac{(1-\gamma_2)k_2c_1}{(1+sT_1)(1+sT_3)} \\ \frac{(1-\gamma_1)k_1c_2}{(1+sT_4)(1+sT_2)} & \frac{\gamma_2k_2c_2}{1+sT_2} \end{bmatrix}$$

The given transfer function matrix is obtained by using $c_i = T_ik_c/A$.

Exercise 2.1.5

Since all the four poles of $G(s)$ are always in the LHP, the zeros of $G(s)$ are the only factor to determine whether it is minimum phase system or vice versa.

In order to get the zeros of a system, one has to solve

$$\det G(s) = \frac{\gamma_1\gamma_2k_1k_2c_1c_2}{\Pi_{i=1}^4(1+sT_i)}\left[(1+sT_3)(1+sT_4) - \frac{(1-\gamma_1)(1-\gamma_2)}{\gamma_1\gamma_2}\right] = 0$$

The given determinant can only be zero when

$$
\begin{aligned}
(1+sT_3)(1+sT_4) - \frac{(1-\gamma_1)(1-\gamma_2)}{\gamma_1\gamma_2} &= 0 \\
T_3T_4s^2 + (T_3+T_4)s + \frac{\gamma_1+\gamma_2-1}{\gamma_1\gamma_2} &= 0
\end{aligned}
$$

Therefore, combined with the constrain that $\gamma \in [0,1]$, we have

$$
\begin{cases}
G(s) \text{ is minimum phase} \Rightarrow & \frac{\gamma_1+\gamma_2-1}{\gamma_1\gamma_2T_3T_4} > 0 \quad \Rightarrow 1 < \gamma_1+\gamma_2 \leq 2 \\
G(s) \text{ is non-minimum phase} \Rightarrow & \frac{\gamma_1+\gamma_2-1}{\gamma_1\gamma_2T_3T_4} \leq 0 \quad \Rightarrow 0 < \gamma_1+\gamma_2 \leq 1
\end{cases}
$$

Exercise 2.1.6

From the equation of $G(s)$ and the definition of RGA of $G(0)$, we have

$$\mathrm{RGA(G(0))} = G(0).*G(0)^{-1T}$$

Where

$$G(0) = \begin{bmatrix} \gamma_1 k_1 c_1 & (1-\gamma_2)\,k_2 c_1 \\ (1-\gamma_1)\,k_1 c_2 & \gamma_2 k_2 c_2 \end{bmatrix}$$

$$G(0)^{-1T} = \left(\frac{1}{\gamma_1\gamma_2 k_1 k_2 c_1 c_2 - (1-\gamma_1)(1-\gamma_2)\,k_1 k_2 c_1 c_2}\right) \begin{bmatrix} \gamma_2 k_2 c_2 & -(1-\gamma_1)\,k_1 c_2 \\ -(1-\gamma_2)\,k_2 c_1 & \gamma_1 k_1 c_1 \end{bmatrix}$$

Therefore,

$$\mathrm{RGA(G(0))} = \begin{bmatrix} \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2-1} & \frac{\gamma_1+\gamma_2-1-\gamma_1\gamma_2}{\gamma_1+\gamma_2-1} \\ \frac{\gamma_1+\gamma_2-1-\gamma_1\gamma_2}{\gamma_1+\gamma_2-1} & \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2-1} \end{bmatrix} = \begin{bmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{bmatrix} (\lambda = \gamma_1\gamma_2/(\gamma_1+\gamma_2-1))$$

Exercise 2.1.7

The following parameters were determined twice. One in lab occasion 1 and the other in lab occasion 2. This was done because in lab occasion two we had several problems with the tank sensor that did not work correctly and the pumps that were not plugged in correctly. Afterwards, when everything was running again, the values did not match the ones from lab occasion 1 anymore and we discovered a calculation error done earlier. Therefore, we had to recalculate the parameters. In the following sections, we present the values found on occasion two, for which all the controllers were calculated. In order to determine the values k_1 and k_2, we decided to only look at the upper tanks of both processes. If we can eliminate the outflow of these tanks and know the voltage of the pump (u_1 or u_2), according to equation (2) we only need to determine the change in water level ($\frac{dh}{dt}$) to calculate k_1 and k_2.

$$
\begin{aligned}
k_1 &= \frac{\frac{dh_4}{dt}A_4}{(1-\gamma_1)u_1} \\
k_2 &= \frac{\frac{dh_3}{dt}A_3}{(1-\gamma_2)u_2}
\end{aligned}
$$

u_1 and u_2 were held constant at 7.5 V and the slope ($\frac{dh}{dt}$) was calculated by measuring the time it took to fill the tank from 0 to 20cm. These resulted into the slope $\frac{dh_4}{dt} = 0.615$ cm/s and $\frac{dh_3}{dt} = 0.714$ cm/s. This leads to the following values for k_1 and k_2:

$$
\begin{aligned}
k_1 &= 3.4 \quad \mathrm{cm^3/(V\,s)} \\
k_2 &= 3.95 \quad \mathrm{cm^3/(V\,s)}
\end{aligned}
$$

Exercise 2.1.8

To determine the six effective hole outlets ($a_1, a_2, a_{3,mp}, a_{4,mp}, a_{3,nmp}$ and $a_{4,nmp}$) we decided to use the equilibrium properties when to filling the tank from 0 to 20 cm. At equilibrium, the slope $\frac{dh}{dt} = 0$. For a_3 and a_4 in the minimum and non-minimum phase case this is straightforward. Using the formulas from Exercise 2.1.2 we directly get:

$$a_3 = \frac{(1-\gamma_2)k_2 u_2}{\sqrt{2gh_3}}$$
$$a_4 = \frac{(1-\gamma_1)k_1 u_1}{\sqrt{2gh_4}}$$

For a_1 and a_2 the equation has an additional inflow term, coming from the upper tanks. We decided to get rid of this by simply closing the upper tanks and getting the equilibrium of only the lower two tanks. This then leads to:

$$a_1 = \frac{\gamma_1 k_1 u_1}{\sqrt{2gh_1}}$$
$$a_2 = \frac{\gamma_2 k_2 u_2}{\sqrt{2gh_2}}$$

By measuring the water level height at the equilibrium level for the lower tanks and for all four tanks in both the minimum and non-minimum phase case, we obtained:

$$\begin{aligned} a_1 &= 0.131 \quad \text{cm}^2 \\ a_2 &= 0.143 \quad \text{cm}^2 \\ a_{3,mp} &= 0.063 \quad \text{cm}^2 \\ a_{4,mp} &= 0.056 \quad \text{cm}^2 \\ a_{3,nmp} &= 0.167 \quad \text{cm}^2 \\ a_{4,nmp} &= 0.148 \quad \text{cm}^2 \end{aligned}$$

2.2 Manual Control

During Lab Occasion 1, the parameters mentioned in 2.1 were calculated and we attempted to control the tanks manually. In the second lab, we had some problems with the sensors and the system overall. Furthermore, we discovered an error in our parameter calculations from Lab 1. This forced us to recalculate the values. However, we did not have time to do the manual controlling again, so in this section we present our findings from Lab occasion 1 with the corresponding, wrongly calculated parameters.

Exercise 2.2.1 - Minimum phase case

In lab occasion 1, we calculated the following parameters: $k_1 = 7.15\ \text{cm}^3/(\text{V s})$, $k_2 = 7.21\ \text{cm}^3/(\text{V s})$, $a_1 = 0.24\ \text{cm}^2$, $a_2 = 0.255\ \text{cm}^2$, $a_{3,mp} = 0.102\ \text{cm}^2$ and $a_{4,mp} = 0.14\ \text{cm}^2$. This leads to the calculated values of the equilibrium water level in the tanks displayed on the left in Table 1. The measured water level, reached with the pumps running at 50% (e.g. 7.5 Volts) are shown in Table 1 on the right.

Table 1: Comparison between the measured and the calculated water tank levels

Calculated water levels		Measured water levels	
$h_{1,calc}$	25.6 cm	$h_{1,measured}$	23 cm
$h_{2,calc}$	22.77 cm	$h_{2,measured}$	16 cm
$h_{3,calc}$	20.14 cm	$h_{3,measured}$	20.5 cm
$h_{4,calc}$	10.52 cm	$h_{2,measured}$	10.5 cm

These calculated values are relatively close to the one we measured. The only big difference is to be seen in Tank 2, which should be filled around 6-7 cm higher. This is surprising, as our calculations were made with faulty parameters.

Exercise 2.2.1 - Non - minimum phase case

In lab occasion 1, we calculated the following parameters: $k_1 = 7.15\ \mathrm{cm}^3/(\mathrm{V\,s})$, $k_2 = 7.21\ \mathrm{cm}^3/(\mathrm{V\,s})$, $a_1 = 0.24\ \mathrm{cm}^2$, $a_2 = 0.255\ \mathrm{cm}^2$, $a_{3,nmp} = 0.34\ \mathrm{cm}^2$ and $a_{4,nmp} = 0.34\ \mathrm{cm}^2$. This leads to the calculated values of the equilibrium water level in the tanks displayed on the left in Table 2. The measured water level, reached with the pumps running at 50% (e.g. 7.5 Volts) are shown in Table 2 on the right.

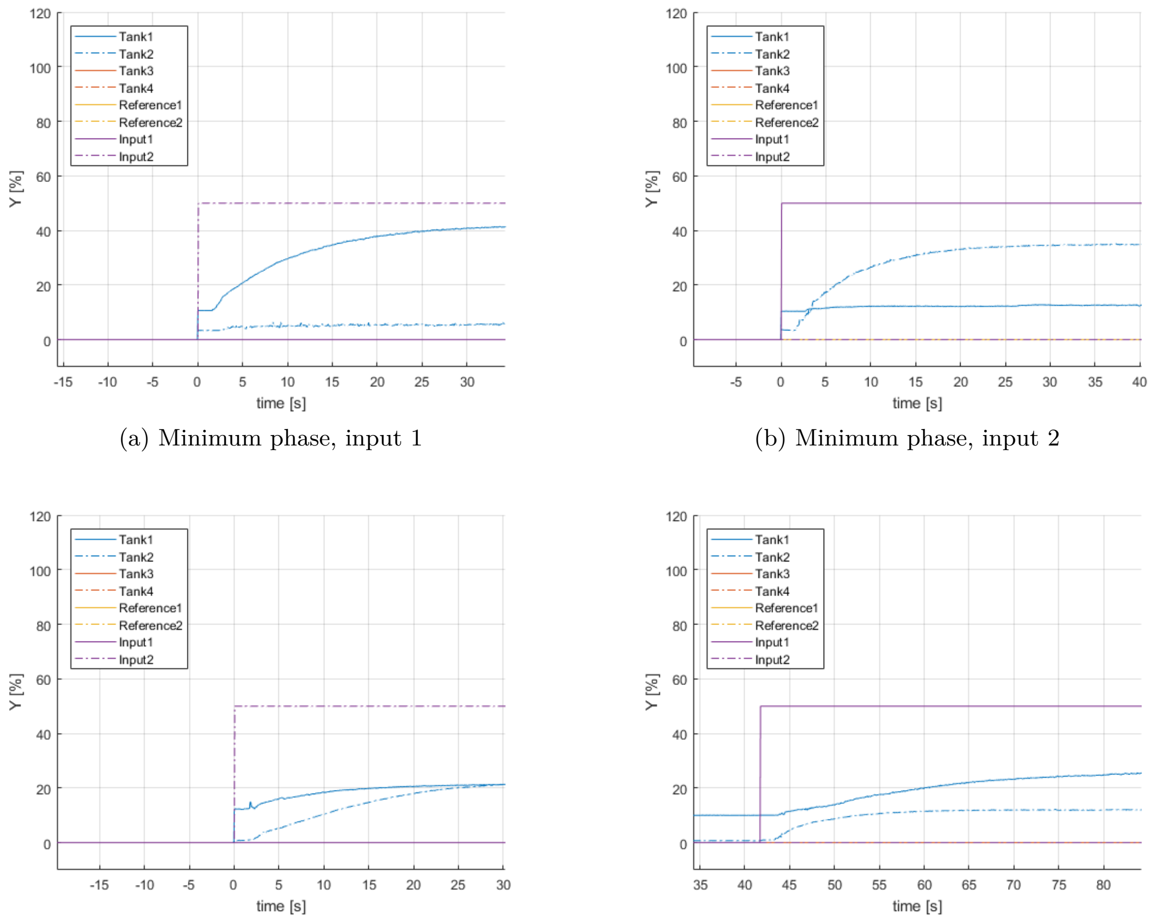
Table 2: Comparison between the measured and the calculated water tank levels

Calculated water levels		Measured water levels	
$h_{1,calc}$	25.71 cm	$h_{1,measured}$	20 cm
$h_{2,calc}$	22.68 cm	$h_{2,measured}$	20 cm
$h_{3,calc}$	5.04 cm	$h_{3,measured}$	5 cm
$h_{4,calc}$	4.95 cm	$h_{2,measured}$	5 cm

These calculated values are relatively close to the one we measured. The only big difference is to be seen in Tank 1, which should be filled around 5 cm higher. This is surprising, as our calculations were made with faulty parameters.

Exercise 2.2.2

In this exercise, one input at a time was set to 50% and the step response of both the minimum phase and the non-minimum phase system were recorded. They can be seen in Fig. 1



(a) Minimum phase, input 1

(b) Minimum phase, input 2

(c) Non-minimum phase, input 1

(d) Non-minimum phase, input 2

Figure 1: Response of the minimum phase system to a step in input 1 (a) and (b). Response of non-minimum phase system to step in input 1 (c) and input 2 (d)

For the non-minimum phase case (Fig. 1 c) and d)), the system is clearly coupled, as both outputs (e.g. tank levels) are showing a response to the step in only one input. One can also see that in this case the coupling is offdiagonal. That means the first input influences rather the second output and vice-versa. This is also what we expected looking at the stationary RGA

$$\mathrm{RGA}(G(0)) = \begin{bmatrix} -0.56 & 1.56 \\ 1.56 & -0.56 \end{bmatrix}$$

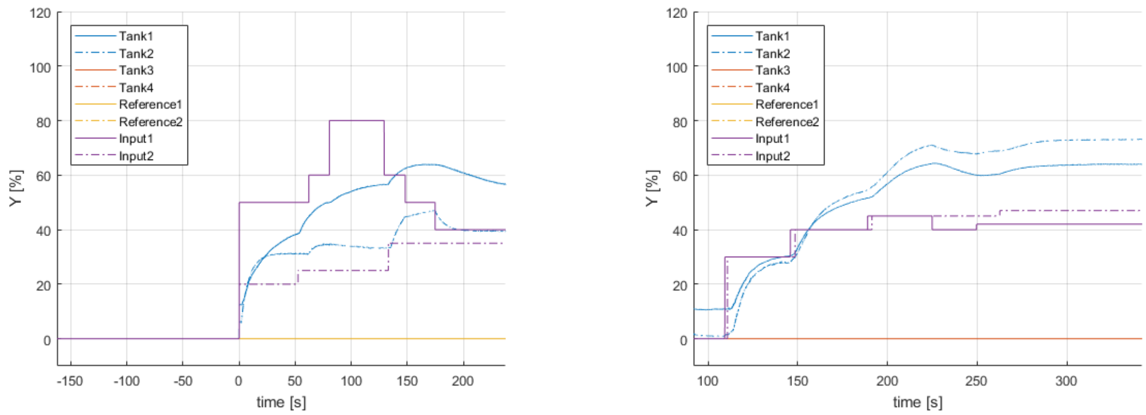
In the minimum phase case (Fig. 1 a) and b)), the system seems to be decoupled, as only one tank level is rising in response of the change in one input. In contrast, the first and input connected to the first output and corresponding for the second input. However, during the experiment it was observed, that water was flowing into tank 2 when input 1 was turned on and into tank 1 when input 2 was turned on. It is not visible in the graphs of Fig. 1, as the outflow of those tanks are larger than the inflows. Should the inflow be larger, e.g. the pump turned on with more than 50% power, the system would show that it is in fact coupled. So we can conclude, that the minimum phase system is almost decoupled if the inputs are small, but coupled if they are large. The RGA does not show this strict decoupling. The static RGA

$$\mathrm{RGA}(G(0)) = \begin{bmatrix} 1.56 & -0.56 \\ -0.56 & 1.56 \end{bmatrix}$$

indicates a that there should be visible a coupling like we have seen in the simulations in the fourt computer lab exercise.

Exercise 2.2.3

For the minimum phase case it was tried to achieve the reference level of 60% for Tank 1 and 40% for Tank 2 by tuning the pump voltages manually. The same was done with the non-minimum phase system and reference level of 65% for Tank 1 and 75% for Tank 2. Those values were chosen arbitrarily.



(a) Minimum phase, Manual control

(b) Non-Minimum phase, Manual control

Figure 2: Manual control of minimum phase case (a), with reference level of 60% for Tank 1 and 40% for Tank 2. In b), manual control of non-minimum phase case with reference level 70% for Tank 1 and 80% for Tank 2

The transient time for the minimum phase system was around 220 seconds, the one for the non-min phase system around 300 s, whereby a value within 10% of the target reference was accepted. The minimum phase system was more challenging to control, because even small changes in one input had a big impact on both outputs. This was surprising, as we thought that the system is almost decoupled for small inputs. However, once some water was in the tanks, this assumptions did not hold anymore, as the inflow in the lower tanks got bigger than the outflow out of them.

Exercise 2.2.4

In both the minimum and non minimum phase cases, stationarity can be reached in the four tanks and the levels are fairly close in accordance with the ones calculates in exercise 2.1.2 excepted for one tank (Tank 2 in the former, Tank 1 in the latter). According to the calculations, the two lower tanks should have had a similar stationary level, around 25cm for tank 1 and 22 cm for tank 2.

Furthermore, the coupling of the two outputs in both case is different. In the non-minimum phase case the outputs are coupled for any input, whereas in the minimum phase case they are only coupled for a large input. The given results show also that the coupling was diagonal for the minimum phase system and rather off-diagonal for the other.

Finally, obtaining a precise reference level with manual control was difficult, especially for the minimum phase system which was surprising.

3 Controller calculations

Exercise 3.1.1

(1) Decentralized & dynamic decoupling control

Minimum phase case

With the obtained values in Section 2 for k_1, k_2 and the six different outlet holes, we repeated the calculation of both the decentralized and the dynamically decoupled PID controller from the Computer Labs of this course. In Fig. 3 and 4 you can see the simulated performance of the two controllers for the minimum phase case.

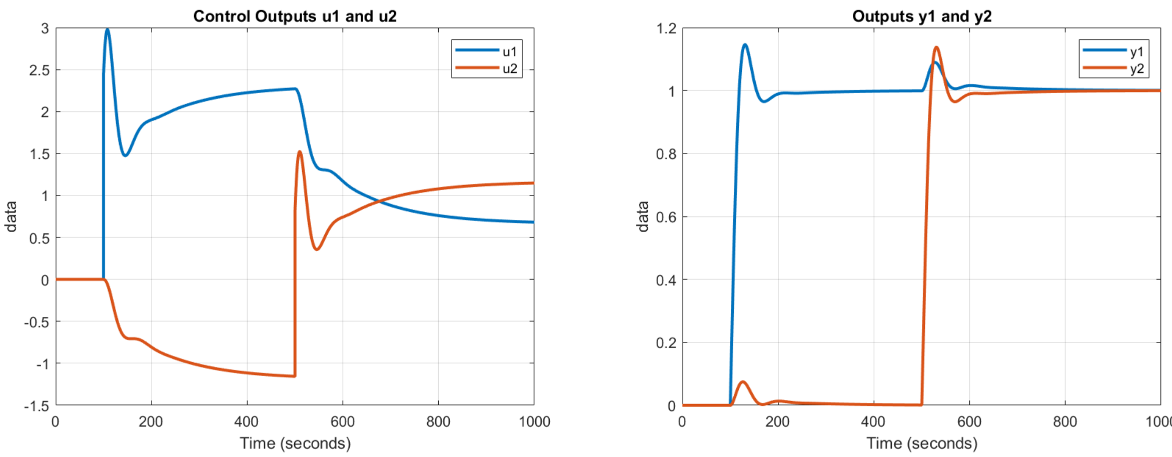


Figure 3: This figure shows the results for the inputs a), and the outputs b) of the decentralized controller for the minimum phase case.

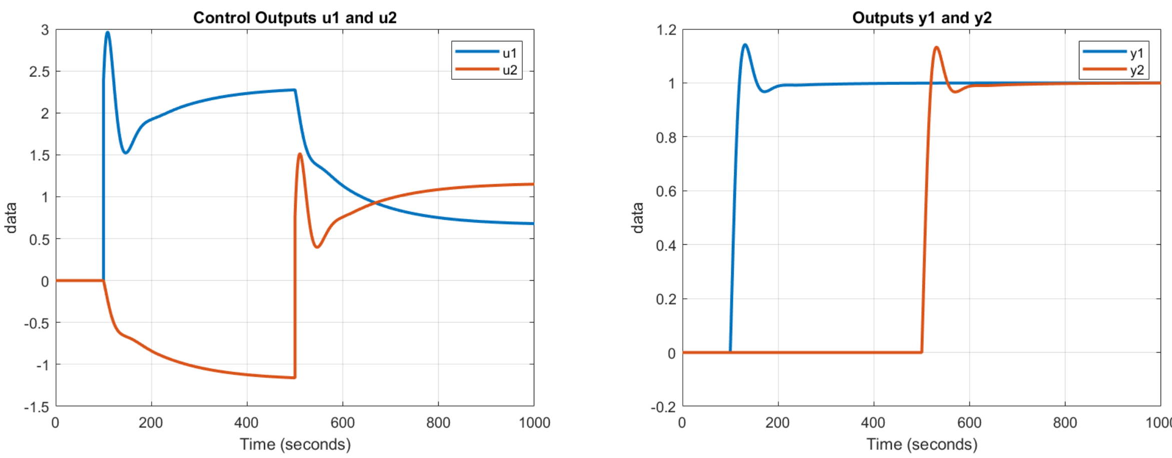


Figure 4: This figure shows the results for the inputs a), and the outputs b) of the dynamic decoupled controller for the minimum phase case.

It can be clearly seen, that the performance is similar. Both controllers have a similar rise time and an overshoot of around 15%. The input usage is also similar, with a maximum usage of 3. The biggest difference can be seen in the decoupling of the outputs. In the decentralized controller we see that y2 has a peak of around 10% of the maximum value, even though it should not react to a step in reference 1. This means we have cross-coupling. However, in the dynamic decoupled control, we do not have cross-coupling, which is why we choose this controller in the subsequent experiments.

Non-Minimum phase case

Also for the non-minimum phase case, we recalculated the decentralized and the dynamic decoupled controller. The results in the performances can be seen in Fig. 5 and 6.

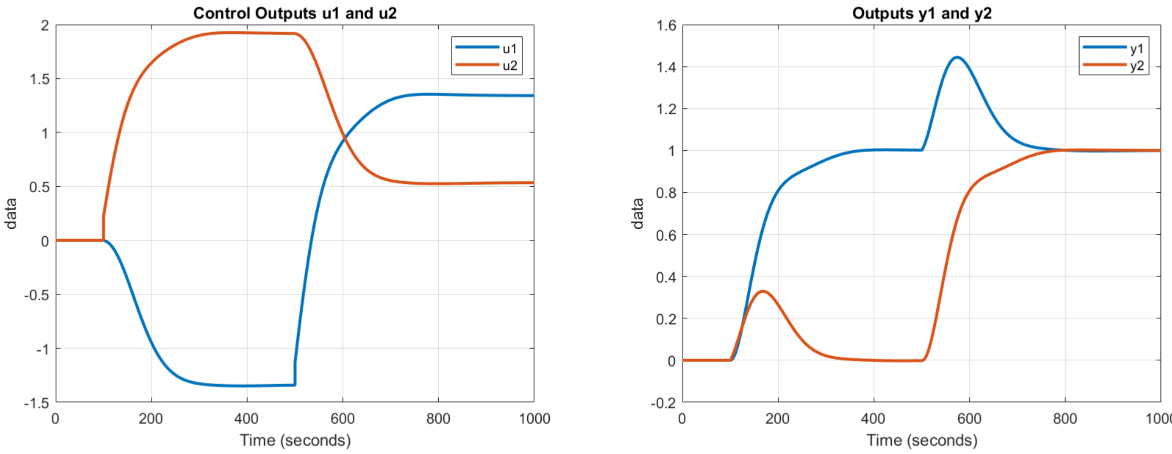


Figure 5: This figure shows the results for the inputs a), and the outputs b) of the decentralized controller for the non-minimum phase case.

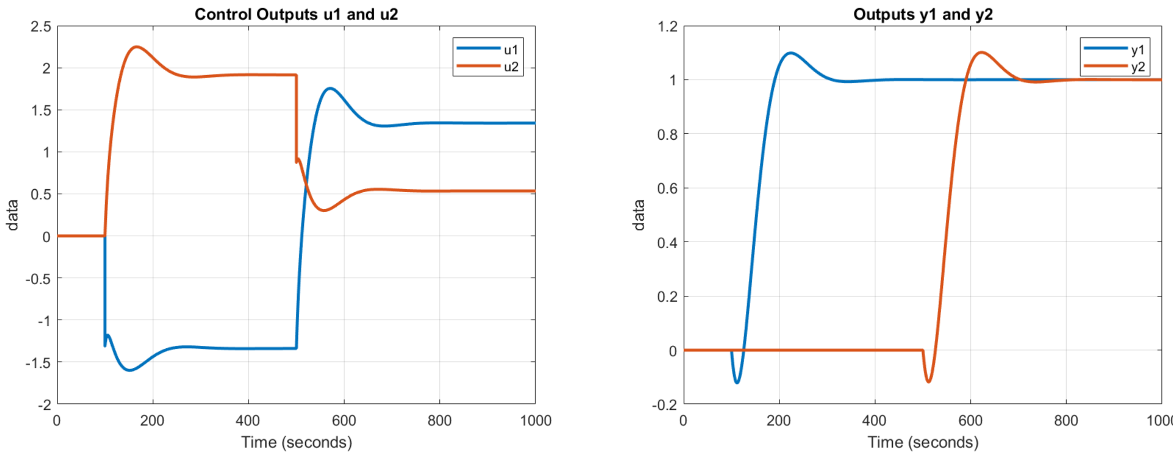


Figure 6: This figure shows the results for the inputs a), and the outputs b) of the dynamic decoupled controller for the non-minimum phase case.

The difference in the performance between those two controllers is bigger than in the minimum phase case. The decentralized control has no under- and no overshoot, but the rise time is clearly longer than in the dynamic decoupled control. The dynamic decoupled system has an undershoot of around 7% and an overshoot of around 5-7 %. The biggest difference here is again the cross-coupling. In the decentralized control, output y2 has a peak of around 20% to a step in reference 1. On the other hand, after the dynamic decoupling, the outputs have no cross-coupling. Therefore, the dynamic controller is chosen to perform the rest of the laboratory project with.

(2) Glover-McFarlane control

We also calculated the Glover-McFarlane controllers based on the results we achieved in Sections 2 and 3.

Minimum phase case

In the minimum phase case, the performance of the controller is very similar to the dynamic decoupled controller. The rise time is similar, the overshoot could be reduced to 5% and the Glover-McFarlane controller should also be more stable when it comes to robust stability. The simulation result is given in fig. 7.

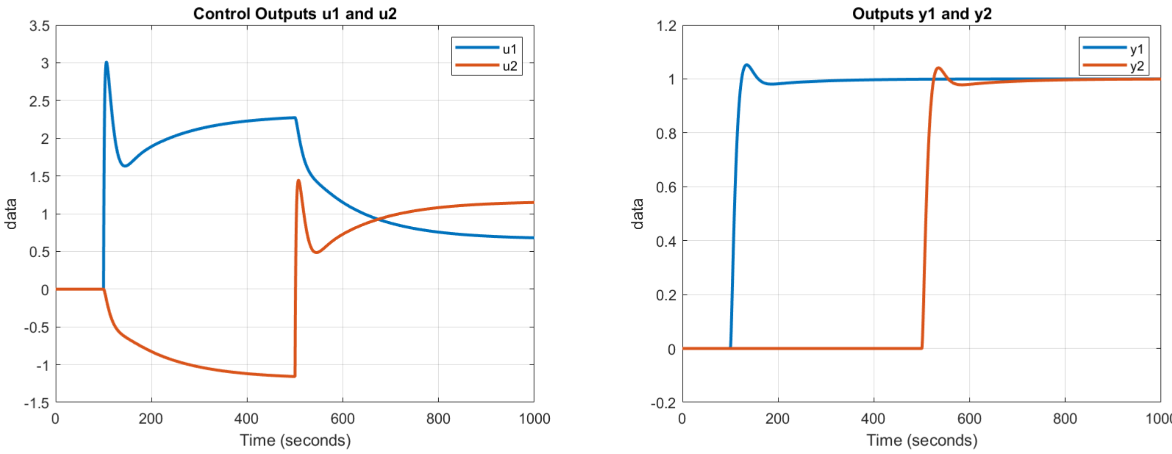


Figure 7: This figure shows the results for the inputs a), and the outputs b) of the Glover-McFarlane controller for the minimum phase case.

Non-minimum phase case

In the non-minimum phase case, the performance of the Glover-McFarlane controller is a bit different to the dynamic decoupled one. The undershoot is reduced to around 2-3% and there is no overshoot at all. However, the rise time was around 50s for the dynamic decoupled controller, in the Glover-McFarlane it is now around 500s.

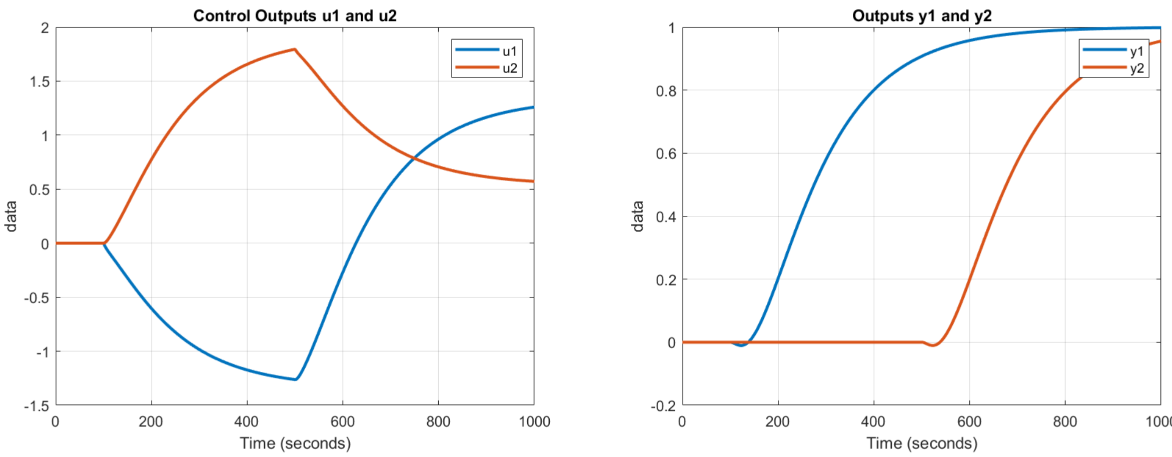


Figure 8: This figure shows the results for the inputs a), and the outputs b) of the Glover-McFarlane controller for the non-minimum phase case.

4 Laboratory occasion 2

In the second laboratory occasion we applied our calculated controller to both, the minimum phase and non-minimum phase system. We examined a step in the reference and an external disturbance. We used a decentralized controller and a robustified controller with the Glover-McFarlane method. All the different experiments and their corresponding results are described in the following sections.

4.1 Results Decentralized control

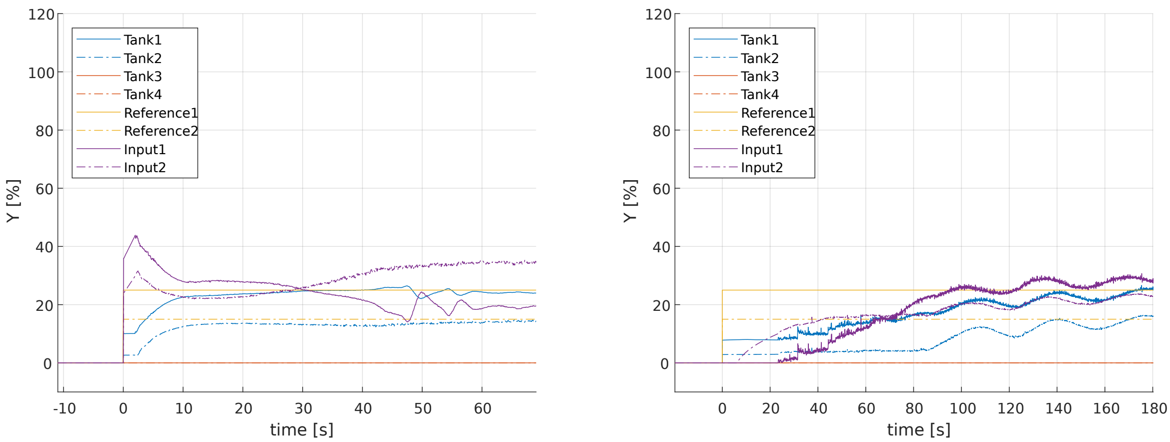
In the following section, the response of a step in reference as well as the response to an external disturbance, e.g. adding a cup of water to one tank, are examined for the system with the dynamic decoupled controller. Both cases, minimum and non-minimum phase, were examined.

Unfortunately, both references were set at the same time during lab 2, as opposed to one at the time, and therefore nothing can be said about the decoupling. However the different systems can still be compared in terms of rise time, overshoot and the transient time on external disturbances.

Step response

Exercise 4.1.1

To examine the step response of the minimum and non-minimum phase system, a step of 25% in reference 1 and 15% in reference 2 was chosen. The results can be seen in Fig. 9



(a) Step response minimum phase

(b) Step response non-minimum phase

Figure 9: The response of the system to a step of 25% in reference 1 and 15% in reference 2, with the dynamic decoupled controller in the minimum phase case a) and the non-minimum phase case b).

Exercise 4.1.2

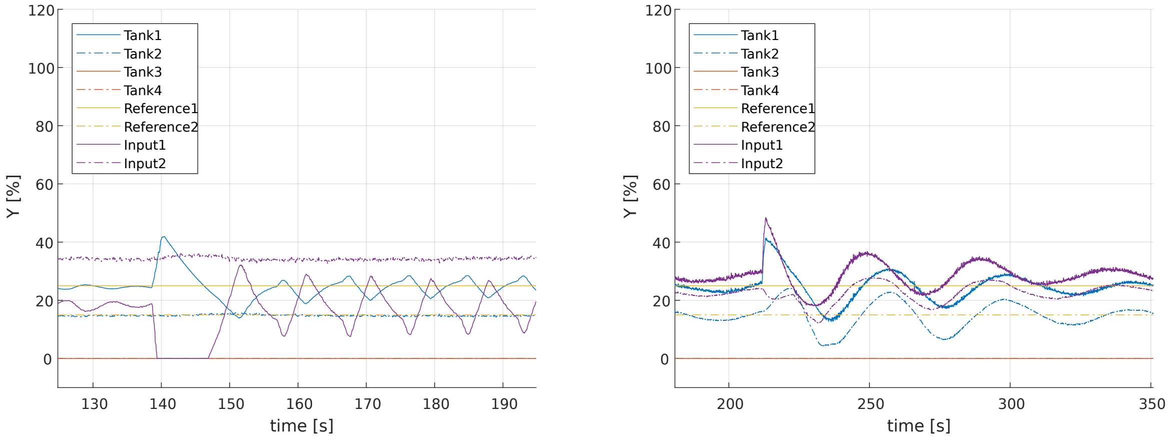
The first thing that can be seen on the results is the fact that the response in the minimum phase case is faster than in the non-minimum one. It takes for Tank 1 around 10 seconds to reach 90% of the reference in the minimum phase case against 130s in the other one. In both cases the evolution of the water level in tanks 1 and 2 is similar.

The non-minimum phase case also takes longer to stabilize. There are bigger oscillations and some overshoot (around 1%) which are not present in the other case.

Response on external disturbance

Exercise 4.1.1

To examine the response on an external disturbance, a glass of water (around 2dl) was added to tank one. The time to get back to steady state was measured.



(a) Response minimum phase

(b) Response non-minimum phase

Figure 10: The response of the system to external disturbances with the dynamic decoupled controller in the minimum phase case a) and the non-minimum phase case b).

Exercise 4.1.2

In the minimum phase case on the left in fig. 10, the system does not stabilize asymptotically after the disturbance. While the level in tank 2 remains constant, the water level in tank 1 starts oscillating with a period time of about 9 seconds, between $+10\%$ and -20% . Input 1 is also periodic at the same period while input 2 remains constant. However, these oscillations are not damped and the system remains unstable. Regarding the non minimum phase case, the response to the disturbance is smoother. There are still oscillations with a period time of 45 seconds, but those ones are damped and the system is back at setpoint after 200 seconds. The other main difference is that this time, the water level in tank 2 that had not additional water, also starts to oscillate.

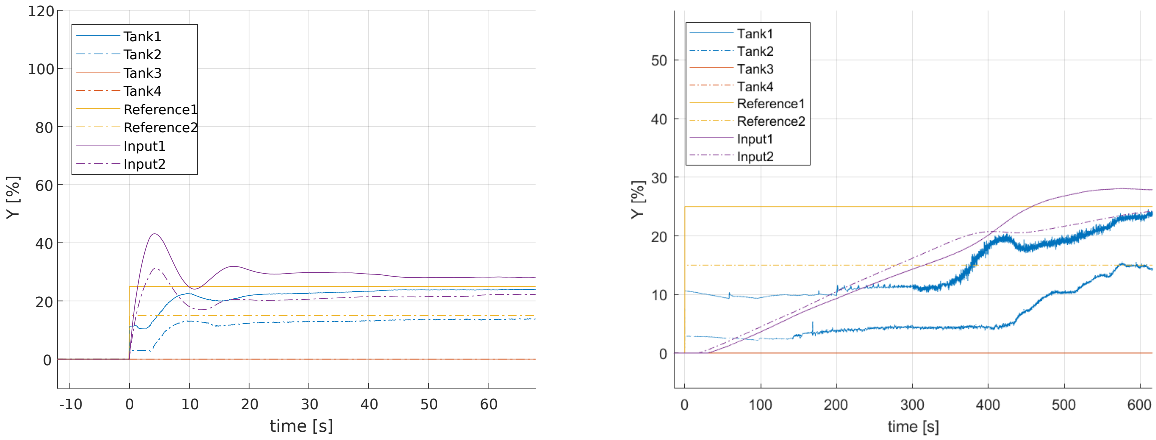
4.2 Results Robust control

In the following section, the response of a step in reference as well as the response to an external disturbance, e.g. adding a cup of water to one tank, are examined for the minimum and the non-minimum phase case. We use the robust controller, e.g. the Glover-McFarlane controller in order to do so.

Step response

Exercise 4.2.1

To examine the step response of the minimum and non-minimum phase system, a step of 25% in reference 1 and 15% in reference 2 were chosen. The results can be seen in Fig. 11



(a) Step response minimum phase

(b) Step response non-minimum phase

Figure 11: The response of the system to a step of 25% in reference 1 and 15% in reference 2, with the Glover-McFarlane controller in the minimum phase case a) and the non-minimum phase case b).

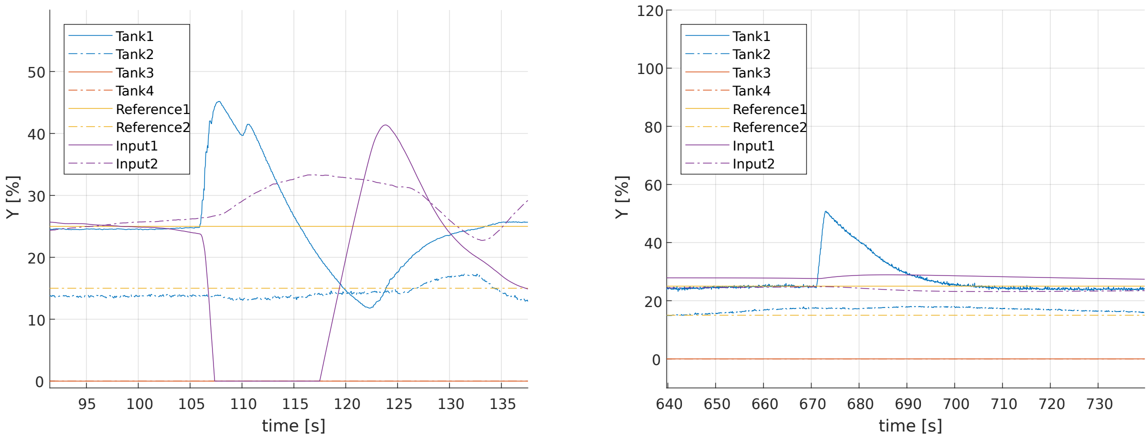
Exercise 4.2.2

In the minimum phase case, the Glover-McFarlane controller seems to be working well. For both tanks 1 and 2 there is no overshoot as well as no undershoot. The time to steadily reach 90% of the reference is 30 seconds and there are no long-term oscillations. The behavior of the system is different in the non-minimum phase case. It is way slower, it takes around 560 seconds, i.e. more than 9 minutes to reach 90% of the reference value. Whereas it took the input 5 seconds to reach its maximum value in the first case, it takes 10 minutes in the second one. As a result, the water level in tank 1 doesn't change much for more than 300 seconds, in tank 2 for more than 400 seconds. There is also no overshoot.

Response on external disturbance

Exercise 4.2.1

To examine the response on an external disturbance, a glass of water (around 2dl) was added to tank one. The time to get back to steady state was measured.



(a) Response minimum phase

(b) Response non-minimum phase

Figure 12: The response of the system to external disturbances with the Glover-McFarlane controller in the minimum phase case a) and the non-minimum phase case b).

Exercise 4.2.1

In the minimum phase case, it takes the controller 130 seconds to make the system return to its original state, with a small overshoot. The water level in tank 2 also changes for 10 seconds, 20 seconds after the perturbation occurred, reaching 115% of its original level. The response for the non-minimum phase case is quite different. The water level in tank 2 does not change and the one in tank 1 goes back to its original state in 30 seconds, without any under- or overshoot. The system is really robust against perturbations.

5 Conclusion

In conclusion, in this laboratory experiment we controlled the four-tank process, which is a multivariable system, based on advanced control theory. We first modeled the process mathematically according to minimum phase case and non-minimum phase case respectively. Based on the obtained equations of the four-tank process, we designed different experiments to do the system identification through manual control. Next, based on RGA theory, we applied a dynamic decoupled controller and Glover-McFarlane controller respectively to obtain a system process with good dynamic and robust performances. For the controller design we used our measured parameters of the system. The experimental results demonstrate that both controllers were successful to achieve a good performance. However, the latter controller worked better in terms of robustness to external disturbances in both minimum and non-minimum phase case.