Computer Exercise 3
EL2520 Control Theory and Practice

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Suppression of disturbances

The weight is

$$W_S(s) = \frac{0.1 \cdot (s + 2 \cdot 100\pi)}{(s - (-0.5 + i\sqrt{(100\pi)^2 - 0.5^2}))((s - (-0.5 - i\sqrt{(100\pi)^2 - 0.5^2}))}$$

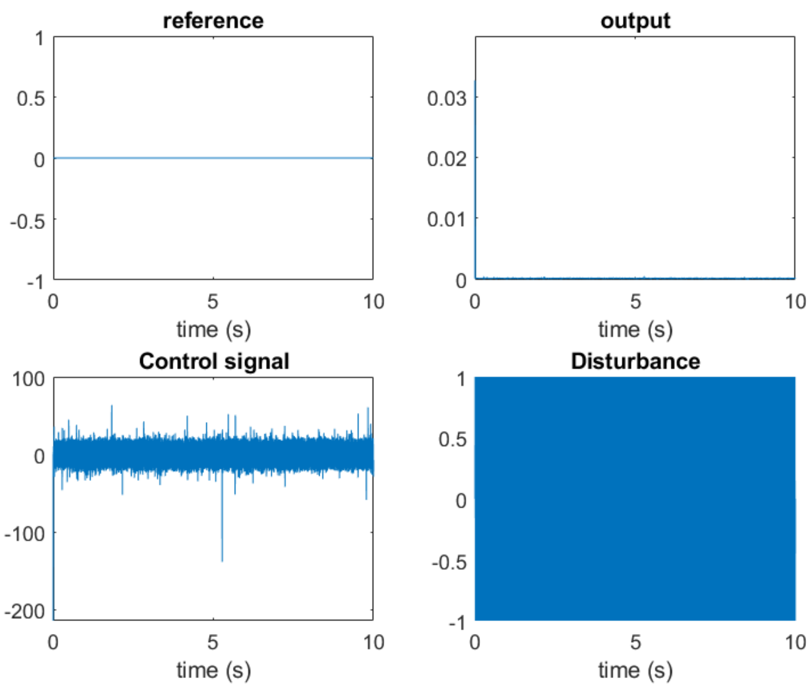

Figure 1: Simulation results with system G, using W_S.

How much is the disturbance damped on the output? What amplification is required for a P-controller to get the same performance, and what are the disadvantages of such a controller?
From fig.1 we can calculate that the maximum amplitude of the output signal is reduced to around 0.0003 in steady-state. The disturbance is attenuated more than 99%.
To design a correspondent P-controller, the gain should be $K_p = 3 \times 10^4$. Then, we get a similar attenuation. The P-controller gives just a constant proportional factor over all frequencies. It is not possible to influence the behaviour only in a particular frequency range.

Robustness

What is the condition on T to guarantee stability according to the small gain theorem, and how can it be used to choose the weight W_T?
The condition is

$$||W_{\mathrm{T}} \cdot T||_\infty < 1 \qquad \rightarrow \qquad |T| < |W_{\mathrm{T}}^{-1}| \tag{1}$$

Therefore, we choose a weight that its inverse is above the magnitude of the complementary sensitivity over all frequencies.
The weights are

$$W_S(s) = \frac{0.1 \cdot (s + 2 \cdot 100\pi)}{(s - (-0.5 + i\sqrt{(100\pi)^2 - 0.5^2}))((s - (-0.5 - i\sqrt{(100\pi)^2 - 0.5^2}))}$$
$$W_T(s) = \frac{0.01 \cdot (s + 150)}{s + 1000}$$

Is the small gain theorem fulfilled?
As one can see in fig.2, the small-gain theorem is satisfied, since the magnitude of the complementary sensitivity $|T|$ (green) is always below the magnitude of the inverse of the designed weight W_{T} (black).

Compare the results to the previous simulation.
Now, the attenuation of the disturbance is not as good as in the first part. It is only damped up to 90%.

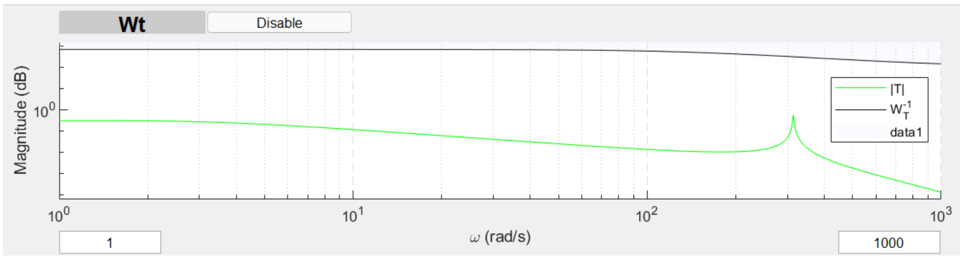

Figure 2: Bode diagram showing that the small gain theorem is satisfied.

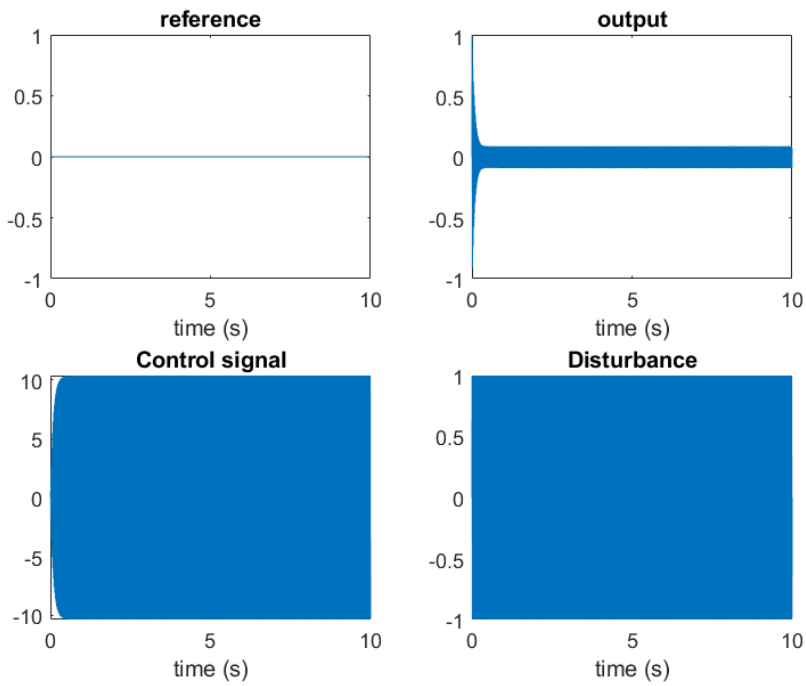

Figure 3: Simulation results with system G_0, using W_S and W_T.

Control signal

The weights are

$$W_S(s) = \frac{0.1 \cdot (s + 2 * 100\pi)}{(s - (-0.5 + i\sqrt{(100\pi)^2 - 0.5^2}))((s - (-0.5 - i\sqrt{(100\pi)^2 - 0.5^2}))}$$

$$W_T(s) = \frac{0.01 \cdot (s + 150)}{s + 1000}$$

$$W_U(s) = \frac{0.06 \cdot (s + 1)}{s + 1000}$$

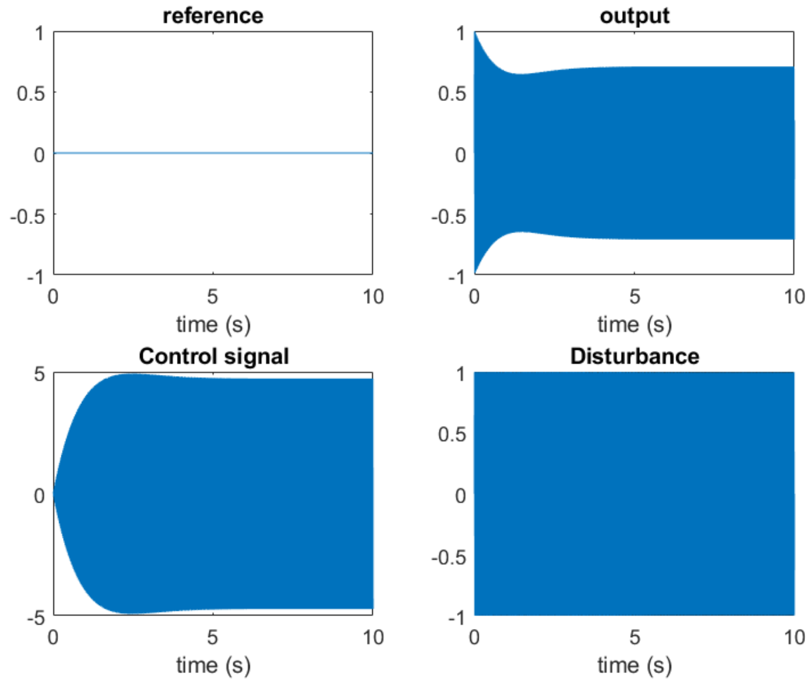

Figure 4: Simulation results with system G_0, using W_S, W_T and W_U.

Compare the results to the previous simulations.
The control input is now bounded to 5. However, the attenuation is much less compared to previous simulation. That is obvious since we use less control. In steady-state, the disturbance is only attenuated of around 35%.