1. **Solution**

(a) The utilization factor is given by

$$U = \sum_{i=1}^{3} \frac{C_i}{T_i} = \frac{1}{4} + \frac{2}{8} + \frac{4}{12} = \frac{20}{24},$$

and the schedule length is given by

$$\mathrm{lcm}(4, 8, 12) = 24.$$

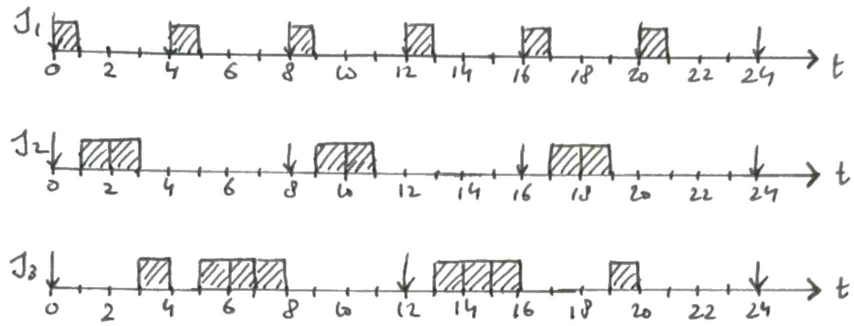

Figure 1: Schedule when task J_2 is prioritized at $t = 17$.

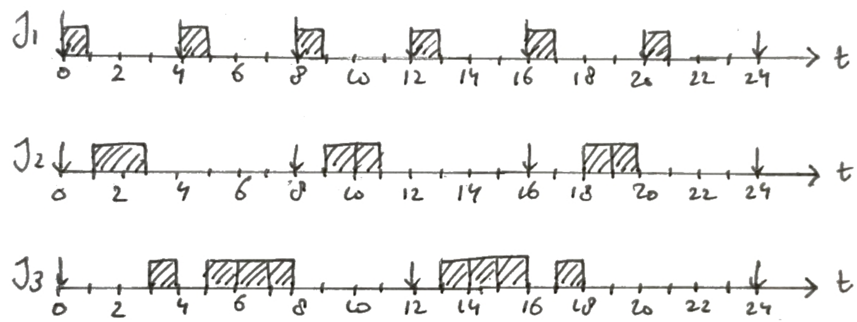

Figure 2: Schedule when task J_3 is prioritized at $t = 17$.

(b) It holds that $U \leq 1$, so the tasks are schedulable under EDF.

In order to determine the worst-case response times we need to draw the schedule. It is important to note here that at $t = 17$ both task J_2 and J_3 have the same time until deadline, so an assumption needs to be made regarding their priority. Figures 1 and 2 show the schedules when respectively either task J_2 and J_3 is prioritized. Let R_i denote the worst-case response time for task J_i. If task J_2 is prioritized we obtain

$$R_1 = 1, R_2 = 3, R_3 = 8,$$

and if task J_3 is prioritized we obtain

$$R_1 = 1, R_2 = 4, R_3 = 8.$$

(c) Since the utilization factor $U = \frac{20}{24} \approx 0.83$ is higher than $3(2^(1/3) - 1) = 0.78$ we cannot determine if the tasks are schedulable by RM. In this task you are therefore asked to use the worst-case response times to determine schedulability. Let R_i denote the worst-case response time for task J_i. The fixed priority is assigned such that J_1 has the highest priority, J_2 has the middle priority, and J_3 has the lowest priority.

For task J_1:

$$R_1 = C_1 = 1 \leq D_1 = 4$$

For task J_2:

$$\begin{aligned}
R_2^0 &= C_2 = 2 \\
R_2^1 &= C_2 + \lceil \frac{R_2^0}{T_1} \rceil C_1 = 2 + \lceil \frac{2}{4} \rceil = 3 \\
R_2^2 &= C_2 + \lceil \frac{R_2^1}{T_1} \rceil C_1 = 2 + \lceil \frac{3}{4} \rceil = 3 \leq D_2 = 8
\end{aligned}$$

For task J_3:

$$\begin{aligned}
R_3^0 &= C_3 = 4 \\
R_3^1 &= C_3 + \lceil \frac{R_3^0}{T_1} \rceil C_1 + \lceil \frac{R_3^0}{T_2} \rceil C_2 = 4 + \lceil \frac{4}{4} \rceil + 2\lceil \frac{4}{8} \rceil = 7 \\
R_3^2 &= C_3 + \lceil \frac{R_3^1}{T_1} \rceil C_1 + \lceil \frac{R_3^1}{T_2} \rceil C_2 = 4 + \lceil \frac{7}{4} \rceil + 2\lceil \frac{7}{8} \rceil = 8 \\
R_3^3 &= C_3 + \lceil \frac{R_3^2}{T_1} \rceil C_1 + \lceil \frac{R_3^2}{T_2} \rceil C_2 = 4 + \lceil \frac{8}{4} \rceil + 2\lceil \frac{8}{8} \rceil = 8 \leq D_3 = 12
\end{aligned}$$

Thus the three tasks are schedulable by RM, since all tasks meet their deadlines.

(d) The RM scheduling (sufficient) condition gives

$$\frac{1}{4} + \frac{2}{8} + \frac{C_3}{12} \leq 3(2^{1/3} - 1),$$

which can written as

$$C_3 \leq 36(2^{1/3} - 1) - 6 \approx 3.36.$$

2. **Solution**

(a) (1) By the change of variables, we obtain $\dot{x}_1 = \dot{y} = x_2$, and $\dot{x}_2 = \ddot{y}$, which becomes

$$\left.\begin{array}{c}\dot{x}_1 = x_2 \\ \dot{x}_2 = \ddot{y} = 2y + \dot{y} + u = 2x_1 + x_2 + u\end{array}\right\} \text{ if } \|x\| \geq 1$$
$$\left.\begin{array}{c}\dot{x}_1 = x_2 \\ \dot{x}_2 = \ddot{y} = -4y - \frac{1}{2}\dot{y} - u = -4x_1 - \frac{1}{2}x_2 - u\end{array}\right\} \text{ if } \|x\| < 1$$

which becomes

$$\dot{x} = A_1 x + B_1 u = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, \quad \|x\| \geq 1$$
$$\dot{x} = A_2 x + B_2 u = \begin{bmatrix} 0 & 1 \\ -4 & -\frac{1}{2} \end{bmatrix} x + \begin{bmatrix} 0 \\ -1 \end{bmatrix} u, \quad \|x\| < 1$$

(2) The switching system can be modeled as a Hybrid Automaton

$$H = (Q, X, \text{Init}, f, E, D, G, R),$$

where

- $Q = \{q_1, q_2\}$,
- $X = \mathbb{R}^2$,
- $\text{Init} = (q_1, [2, 0])$,
- $f(q_1, x) = A_1 x + B_1 u$, $f(q_2, x) = A_2 x + B_2 u$,
- $E = \{(q_1, q_2), (q_2, q_1)\}$,
- $D(q_1) = \{x \in \mathbb{R}^2 : \|x\| \geq 1\}$, $D(q_2) = \{x \in \mathbb{R}^2 : \|x\| < 1\}$,
- $G((q_1, q_2)) = \{x \in \mathbb{R}^2 : \|x\| < 1\}$, $G((q_2, q_1)) = \{x \in \mathbb{R}^2 : \|x\| = 1\}$,
- $R((q_1, q_2), x) = R((q_2, q_1), x) = x$

(b) (1) By substituting $u = -k_1 x_1 - k_2 x_2$ in the first subsystem, we obtain

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 2 - k_1 & 1 - k_2 \end{bmatrix} x,$$

whose characteristic polynomial is $\lambda^2 + (k_2 - 1)\lambda + k_1 - 2$. By using Routh's criterion, we obtain that $k_2 > 1$ and $k_1 > 2$.

(2) By substituting $u = -k_1 x_1 - k_2 x_2$ in the second subsystem, we obtain

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -4 + k_1 & -\frac{1}{2} + k_2 \end{bmatrix} x,$$

whose characteristic polynomial is $\lambda^2 + (\frac{1}{2} - k_2)\lambda + 4 - k_1$. By using Routh's criterion, we obtain that $k_1 < 4$ and $k_2 < \frac{1}{2}$.

(3) No, in view of the aforementioned derivations for k_2.

(c) (1) The desired charact. polynomial is $\lambda^2 + 2\lambda + 1$, which implies that $k_1 = k_2 = 3$.

(2) The closed loop dynamics are

$$\dot{x}_1 = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} x, \quad \|x\| \geq 1$$
$$\dot{x}_1 = \begin{bmatrix} 0 & 1 \\ -1 & \frac{5}{2} \end{bmatrix} x, \quad \|x\| < 1.$$

The second subsystem's eigenvalues are $\frac{1}{2}$ and 2, and hence it is unstable, and hence there is no common Lyapunov function.

(3) Since the first subsystem is asymptotically stable and the second one unstable, there is arbitrarily fast switching at the switching surface $\|x\| = 1$ and hence the system exhibits Zeno behavior.

(d) A switching feedback protocol consists of two different control laws for the two subsystems, i.e., $u = -Kx$, when $\|x\| \geq 1$, and $u = -Lx$, when $\|x\| < 1$. According to the results above, a choice would be $k_1 = k_2 = 3$ and $l_1 = l_2 = 0$.

3. **Solution:**

(a) By assumption we know that all elements of $x(0)$ are nonnegative and $u(k) \geq 0$ for all k. Writing down the dynamics of $x_i(k)$, we obtain

$$x_i(k+1) = \sum_{j=1}^{n} a_{ij} x_j(k) + b_i u(k),$$

Hence, for all $k \geq 0$ and $i \in \{1, \cdots, n\}$, $x_i(k) \geq 0$ if $a_{ij} \geq 0$ and $b_i \geq 0$. Now knowing that for all $k \geq 0$ all elements of $x(k)$ are nonnegative, we look at the output equation,

$$y(k) = Cx(k) = \sum_{i=1}^{n} c_i x_i(k).$$

Hence, we see that if $c_i \geq 0$ for all $i \in \{1, \cdots, n\}$ then $y(k) \geq 0$ for all $k \geq 0$.

(b) Assume $x(0)$ is such that the ith element is greater than zero, i.e. $x_i(0) > 0$. The dynamics of $x_i(k)$ are then given by

$$x_i(k) = a_{ii}^k x_i(0) + \cdots.$$

Here, we see that $a_{ii}^k \to \infty$ as $k \to \infty$ when $a_{ii} > 1$. If $a_{ii} > 1$ the system is unstable. Hence, if there exists any $a_{ii} > 1$ the system is unstable.

(c) The observability matrix is given by

$$W_O = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 3 & \alpha \end{bmatrix},$$

from which we can immediately see that $rank(W_O) = 2$ if $\alpha \neq 3$.

(d) The error dynamics are given by $A - KC$, which is given by

$$\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 - k_1 & 1 - k_1 \\ 1 - k_2 & -k_2 \end{bmatrix}.$$

The characteristic polynomial of the error dynamics is

$$\begin{aligned} P_e(z) = \det(zI - A + KC) &= \det\left(\begin{bmatrix} z - 2 + k_1 & k_1 - 1 \\ k_2 - 1 & z + k_2 \end{bmatrix} \right) \\ &= (z - 2 + k_1)(z + k_2) + (1 - k_1)(k_2 - 1) \\ &= z^2 + (k_1 + k_2 - 2)z - 2k_2 + k_1 k_2 + k_2 - 1 - k_1 k_2 + k_1 \\ &= z^2 + (k_1 + k_2 - 2)z - k_2 - 1 + k_1. \end{aligned}$$

Having a deadbeat observer means $P_e(z) = z^2$ which results in the equation system

$$\begin{aligned} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} &= \begin{bmatrix} 2 \\ 1 \end{bmatrix} \\ \Leftrightarrow \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} &= -\frac{1}{2} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 0.5 \end{bmatrix}. \end{aligned}$$

The observer dynamics are given by

$$\begin{aligned}\hat{x}(k+1) &= A\hat{x}(k) + K(y(k) - C\hat{x}(k)) = (A - KC)\hat{x}(k) + Ky(k) \\ &= \begin{bmatrix} 0.5 & -0.5 \\ 0.5 & -0.5 \end{bmatrix} \hat{x}(k) + \begin{bmatrix} 1.5 \\ 0.5 \end{bmatrix} y(k).\end{aligned}$$

We see that the observer does not represent a positive system!

(e) The closed-loop dynamics are given by

$$A - BL = \begin{bmatrix} 2 - l_1 & 1 - l_2 \\ 1 & 0 \end{bmatrix},$$

from which one can readily see that

$$L = \begin{bmatrix} 2 & 1 \end{bmatrix}$$

leads to a deadbeat controller. Alternatively, we can determine the characteristic polynomial of the closed-loop system as

$$P(z) = \det(zI - A + BL) = z^2 + (2 - l_1)z + 1 - l_2.$$

Having a deadbeat controller means that $P(z) = z^2$ from which the above L follows.

4. **Solution**

(a) Consider the relations:

$$\sim = \{(s_1, s_1'), (s_2, s_3'), (s_2, s_4'), (s_3, s_2'), (s_4, s_5'), (s_5, s_6')\} \subseteq S \times S',$$

The relation $\sim$ is a simulation relation from $\mathcal{T}_1$ to $\mathcal{T}_2$ since the following hold:

(a) For the state $s_1 \in S$, there exists the state $s_1' \in S'$ such that $(s_1, s_1') \in \sim$.

(b) We have that $(s_5, s_6') \in \sim$ with $s_5 \in S_F$ and $s_6' \in S_F'$.

(c)
- $(s_1, s_1') \in \sim, (s_1, a, s_2) \in \rightarrow$, and at the same time $(s_1', a, s_3') \in \rightarrow'$, and $(s_2, s_3') \in \sim$.
- $(s_1, s_1') \in \sim, (s_1, b, s_3) \in \rightarrow$, and at the same time $(s_1', b, s_2') \in \rightarrow'$, and $(s_3, s_2') \in \sim$.
- $(s_2, s_3') \in \sim, (s_2, a, s_4) \in \rightarrow$, and at the same time $(s_3', a, s_5') \in \rightarrow'$, and $(s_4, s_5') \in \sim$.
- $(s_2, s_3') \in \sim, (s_2, c, s_5) \in \rightarrow$, and at the same time $(s_3', c, s_6') \in \rightarrow'$, and $(s_5, s_6') \in \sim$.
- $(s_2, s_4') \in \sim, (s_2, a, s_4) \in \rightarrow$, and at the same time $(s_4', a, s_5') \in \rightarrow'$, and $(s_4, s_5') \in \sim$.
- $(s_2, s_4') \in \sim, (s_2, c, s_5) \in \rightarrow$, and at the same time $(s_4', c, s_6') \in \rightarrow'$, and $(s_5, s_6') \in \sim$.
- $(s_3, s_2') \in \sim, (s_3, b, s_4) \in \rightarrow$, and at the same time $(s_2', b, s_5') \in \rightarrow'$, and $(s_4, s_5') \in \sim$.
- $(s_3, s_2') \in \sim, (s_3, c, s_5) \in \rightarrow$, and at the same time $(s_2', c, s_6') \in \rightarrow'$, and $(s_5, s_6') \in \sim$.
- $(s_4, s_5') \in \sim, (s_4, a, s_4) \in \rightarrow$, and at the same time $(s_5', a, s_5') \in \rightarrow'$, and $(s_4, s_5') \in \sim$.
- $(s_4, s_5') \in \sim, (s_4, b, s_4) \in \rightarrow$, and at the same time $(s_5', b, s_5') \in \rightarrow'$, and $(s_4, s_5') \in \sim$.
- $(s_4, s_5') \in \sim, (s_4, c, s_5) \in \rightarrow$, and at the same time $(s_5', c, s_6') \in \rightarrow'$, and $(s_5, s_6') \in \sim$.
- $(s_5, s_6') \in \sim, (s_5, a, s_1) \in \rightarrow$, and at the same time $(s_6', a, s_1') \in \rightarrow'$, and $(s_1, s_1') \in \sim$.
- $(s_5, s_6') \in \sim, (s_5, b, s_1) \in \rightarrow$, and at the same time $(s_6', b, s_1') \in \rightarrow'$, and $(s_1, s_1') \in \sim$.
- $(s_5, s_6') \in \sim, (s_5, c, s_5) \in \rightarrow$, and at the same time $(s_6', c, s_6') \in \rightarrow'$, and $(s_5, s_6') \in \sim$.

It can be shown analogously, that the three conditions for the relation $\sim^{-1}$ hold. Therefore, $\sim$ is a *bisimulation relation* and the Transition Systems $\mathcal{T}_1, \mathcal{T}_2$ are *bisimilar*.

(b) We start with the partition $S_{/\sim} = \{\{s_1, s_2, s_3, s_4\}, \{s_5\}\}$.

Step 1: Set $P = \{s_1, s_2, s_3, s_4\}$ and $P' = \{s_5\}$. We have that

$$\emptyset \neq P \cap \mathrm{Pre}_c(P') = \{s_1, s_2, s_3, s_4\} \cap \{s_2, s_3, s_4, s_5\} = \{s_2, s_3, s_4\} \neq P.$$

Then, set:

$$\begin{aligned} P_1 &= P \cap \mathrm{Pre}_c(P') = \{s_2, s_3, s_4\}, \\ P_2 &= P \backslash \mathrm{Pre}_c(P') = \{s_1\}, \\ S_{/\sim} &= \{\{s_5\}, \{s_1\}, \{s_2, s_3, s_4\}\}. \end{aligned}$$

Step 2: Set $P = P' = \{s_2, s_3, s_4\}$. We have that

$$\emptyset \neq P \cap \mathrm{Pre}_b(P') = \{s_2, s_3, s_4\} \cap \{s_1, s_3, s_4\} = \{s_3, s_4\} \neq P.$$

Then, set:

$$\begin{aligned} P_1 &= P \cap \mathrm{Pre}_b(P') = \{s_3, s_4\}, \\ P_2 &= P \backslash \mathrm{Pre}_b(P') = \{s_2, s_3, s_4\} \backslash \{s_1, s_3, s_4\} = \{s_2\}, \\ S_{/\sim} &= \{\{s_1\}, \{s_2\}, \{s_3, s_4\}, \{s_5\}\}. \end{aligned}$$

Step 3: Set $P = P' = \{s_3, s_4\}$. We have that

$$\emptyset \neq P \cap \mathrm{Pre}_a(P') = \{s_3, s_4\} \cap \{s_2, s_4\} = \{s_4\} \neq P.$$

Then, set:

$$\begin{aligned} P_1 &= P \cap \mathrm{Pre}_a(P') = \{s_4\}. \\ P_2 &= P \backslash \mathrm{Pre}_a(P') = \{s_3, s_4\} \backslash \{s_2, s_4\} = \{s_3\} \\ S_{/\sim} &= \{s_1, s_2, s_3, s_4, s_5\}. \end{aligned}$$

The algorithm terminates since no other partitions can be performed. Thus the minimal quotient Transition System is the same as the original transition system.

(c) (i) The state is reachable with the following sequence of transitions:

$$\begin{aligned}(q_1, 0, 0, 0) &\stackrel{Time=1}{\longrightarrow} (q_1, 1, 1, 1)\\ &\stackrel{Action=left}{\longrightarrow} (q_2, 1, 1, 0)\\ &\stackrel{Time=1}{\longrightarrow} (q_2, 2, 2, 1)\\ &\stackrel{Action=up}{\longrightarrow} (q_3, 0, 2, 1)\end{aligned}$$

(ii) The state is reachable with the following sequence of transitions:

$$\begin{aligned}(q_1, 0, 0, 0) &\stackrel{Time=1}{\longrightarrow} (q_1, 1, 1, 1)\\ &\stackrel{Action=left}{\longrightarrow} (q_2, 1, 1, 0)\\ &\stackrel{Time=1}{\longrightarrow} (q_2, 2, 2, 1)\\ &\stackrel{Action=up}{\longrightarrow} (q_3, 0, 2, 1)\\ &\stackrel{Action=down}{\longrightarrow} (q_2, 0, 0, 1)\end{aligned}$$

(iii) The state is not reachable. In q_1, x_1 and x_2 have the same value and the transition from q_1 to q_2 resets x_3 to zero so in that case the states in q_2 will have the form (x, x, y), where $x \leq 2$ and $y \leq 1$. The transition from q_3 to q_2 will reset x_2 so there needs to be a time transition of 2 time instants for x_2 to reach 2. This cannot happen, since x_3 must be less than 1 in q_2.

(iv) The state is reachable with the following sequence of transitions:

$$\begin{aligned}(q_1, 0, 0, 0) &\stackrel{Time=1}{\longrightarrow} (q_1, 1, 1, 1)\\ &\stackrel{Action=left}{\longrightarrow} (q_2, 1, 1, 0)\\ &\stackrel{Action=up}{\longrightarrow} (q_3, 0, 1, 0)\\ &\stackrel{Time=1}{\longrightarrow} (q_3, 1, 2, 1)\\ &\stackrel{Action=down}{\longrightarrow} (q_2, 1, 0, 1)\\ &\stackrel{Action=right}{\longrightarrow} (q_1, 0, 0, 1)\\ &\stackrel{Time=2}{\longrightarrow} (q_1, 2, 2, 3)\end{aligned}$$

(v) The state is not reachable. The transition to q_3 resets x_1 to zero so there needs to be a time transition of 3 time instants for x_1 to reach 3. This contradicts the fact that $x_3 \leq 1$ in q_3.

(vi) The state is not reachable. The transition to q_3 needs x_2 to be greater or equal to 1.

5. **Solution**

(a) By writing the system in vector form we have that $\dot{x} = Ax + Bu$ where

$$A = \begin{bmatrix} 1 & -1 \\ 3 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

The characteristic equation is given by

$$\begin{aligned}
\psi_o(\lambda) &= \det\left[\lambda I_2 - A\right] \\
&= \det \begin{bmatrix} \lambda - 1 & 1 \\ -3 & \lambda - 1 \end{bmatrix} \\
&= (\lambda - 1)^2 + 3 \\
&= [(\lambda - 1) - j\sqrt{3}][(\lambda - 1) - j\sqrt{3}] \qquad (1)
\end{aligned}$$

which has two roots: $1 + j\sqrt{3}, 1 - j\sqrt{3}$ to the RHP. Hence the system is *unstable*.

(b) By substituting the given controller in the system (S) we have the closed loop form

$$\dot{x}(t) = Ax(t) + B\left[Kx(t)\right] = (A + BK)x(t)$$

The closed loop characteristic polynomial is given by

$$\begin{aligned}
\psi_c(\lambda) &= \det\left[\lambda I_2 - A - BK\right] \\
&= \det \left[\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 3 & 1 \end{bmatrix} - \begin{bmatrix} K_1 & K_2 \\ K_1 & K_2 \end{bmatrix} \right] \\
&= \det \begin{bmatrix} \lambda - K_1 - 1 & -K_2 + 1 \\ -K_1 - 3 & \lambda - K_2 - 1 \end{bmatrix} \\
&= \lambda^2 + \left(-K_1 - K_2 - 2\right)\lambda + \left(2K_1 - 2K_2 + 4\right)
\end{aligned}$$

The desired characteristic polynomial is given by $\psi_{\text{des}}(\lambda) = \lambda^2 + 3\lambda + 2 = 0$. Hence, the following holds

$$\begin{cases} -K_1 - K_2 - 2 = 3 \\ 2K_1 - 2K_2 + 4 = 2 \end{cases} \iff \boxed{\begin{cases} K_1 = -3 \\ K_2 = -2 \end{cases}}$$

(c) We have that $u(t) = Kx(t_k) = K\left(e(t) + x(t)\right)$. By substituting it to the clossed loop system we get

$$\begin{aligned}
\dot{x}(t) &= Ax(t) + BK\left[e(t) + x(t)\right] \\
&= \left(A + BK\right)x(t) + (BK)e(t) \\
&= \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} -3 & -2 \\ -3 & -2 \end{bmatrix} e(t)
\end{aligned}$$

or

$$\boxed{(S') : \begin{cases} \dot{x}_1(t) = -2x_1(t) - 3x_2(t) - 3e_1(t) - 2e_2(t) \\ \dot{x}_2(t) = -x_2(t) - 3e_1(t) - 2e_2(t) \end{cases}, \ t \geq 0}$$

(d) By taking the time derivative of the given Lyapunov function we have that

$$\begin{aligned}
\dot{V}(x) &= x_1\dot{x}_1 + 2x_2\dot{x}_2 \\
&= x_1\left\{-2x_1 - 3x_2 - 3e_1 - 2e_2\right\} + 2x_2\left\{-x_2 - 3e_1 - 2e_2\right\} \\
&= -2x_1^2 - 3x_1x_2 - 3x_1e_1 - 3x_1e_2 - 2x_2^2 - 6x_2e_1 - 4x_2e_2 \\
&= -2\left(x_1^2 + x_2^2\right) - 3x_1x_2 - 3x_1e_1 - 2x_1e_2 - 6x_2e_1 - 4x_2e_2 \\
&= -2\|x\|^2 - 3x_1x_2 - 3x_1e_1 - 3x_1e_2 - 6x_2e_1 - 4x_2e_2
\end{aligned}$$

By using the elementary inequalities

$$-x_1x_2 \leq \frac{1}{2}(x_1^2 + x_2^2), -ab \leq |a||b|, \ \forall \ x_1, x_2, a, b \in \mathbb{R}$$
$$|x_1| \leq \|x\|, |x_2| \leq \|x\|, |e_1| \leq \|e\|, |e_2| \leq \|x\|, \ \forall \ x, e \in \mathbb{R}^2$$

we get

$$\begin{aligned}
\dot{V}(x) &\leq -2\|x\|^2 + \frac{3}{2}(x_1^2 + x_2^2) + 3|x_1||e_1| + 2|x_1||e_2| + 6|x_2||e_1| + 4|x_2||e_2| \\
&= -2\|x\|^2 + \frac{3}{2}\|x\|^2 + 3|x_1||e_1| + 2|x_1||e_2| + 6|x_2||e_1| + 4|x_2||e_2| \\
&= -\frac{1}{2}\|x\|^2 + 3|x_1||e_1| + 2|x_1||e_2| + 6|x_2||e_1| + 4|x_2||e_2| \\
&\leq -\frac{1}{2}\|x\|^2 + 3\|x\|\|e\| + 2\|x\|\|e\| + 6\|x\|\|e\| + 4\|x\|\|e\| \\
&\leq -\frac{1}{2}\|x\|^2 + 15\|x\|\|e\| \\
&= -\frac{1}{2}\|x\|\left(\|x\| - 30\|e\|\right)
\end{aligned}$$

Thus, for $\|x\| - 30\|e\| > 0 \Leftrightarrow \boxed{\|e\| < \frac{1}{30}\|x\|}$ the system remains asymptotically stable.