AUTOMATIC CONTROL

KTH

EL2450 Hybrid and Embedded Control Systems

Exam –, March , 2017

Aid:

Lecture notes (slides) from the course, compendium ("reading material") and textbook from basic course ("Reglerteknik" by Glad & Ljung or similar text approved by course responsible). Mathematical handbook (e.g., "Beta Mathematics Handbook" by Råde & Westergren). Other textbooks, handbooks, exercises, solutions, calculators etc. may **not** be used unless previously approved by course responsible.

Observandum:

- Name and social security number (*personnummer*) on every page.
- Only one solution per page and write on one side per sheet.
- Each answer has to be motivated.
- Specify the total number of handed in pages on the cover.
- Each subproblem is marked with its maximum credit.

Grading:

Grade A: $\geq$ 43, Grade B: $\geq$ 38

Grade C: $\geq$ 33, Grade D: $\geq$ 28

Grade E: $\geq$ 23, Grade Fx: $\geq$ 21

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Lycka till!

1. Consider the hybrid automaton $H = (Q, X, Init, f, D, E, G, R)$ displayed in Figure 1 and defined by:

   - discrete state space $Q = \{q_1, q_2\}$
   - continuous state space $X = [0, 3]$
   - initial states $Init = \{(q_1, 0)\}$
   - domains $D(q_1) = D(q_2) = X$
   - edges $E = \{(q_1, q_2), (q_2, q_1)\}$
   - guards $G((q_1, q_2)) = [1, 3]$ and $G((q_2, q_1)) = [0, 1)$
   - resets $R((q_1, q_2), x) = R((q_2, q_1), x) = \emptyset$

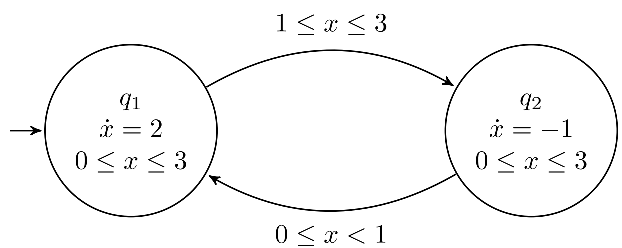

Figure 1: Hybrid automaton H

The goal of this problem is to do a safety verification on the hybrid automaton H with respect to the bad set $B = \{q_2\} \times [2, 3]$ by using the bisimulation quotient algorithm.

(a) [3p] Provide the equivalent representation of the hybrid automaton H as a transition system $T_H = (S, \Sigma, \rightarrow, S_0, S_F)$ by defining its elements S, Σ, S_0, S_F and sketching the transition system T_H.

(b) [5p] Obtain the quotient transition system $\hat{T} = (\hat{S}, \Sigma, \hat{\rightarrow}, \hat{S}_0, \hat{S}_F)$ of T_H in the following steps:

   - apply the bisimulation quotient algorithm to T_H to obtain the set of equivalence classes $\hat{S}$
   - prove that the algorithm has terminated
   - define the elements $\hat{S}$, $\hat{S}_0$, $\hat{S}_F$ of $\hat{T}$ and draw the resulting transition system $\hat{T}$.

(c) [2p] Compute the reach set $Reach(\hat{S}_0)$ of $\hat{T}$. Use this result to conclude on the safety verification of the hybrid automaton H with respect to the bad set $B = \{q_2\} \times [2, 3]$.

Solution

(a) $T_H = (S, \Sigma, \rightarrow, S_0, S_F)$ with:

- $S = Q \times X = \{q_1, q_2\} \times [0, 3]$
- $\Sigma = \{g_1, g_2, t\}$, where g_1 corresponds to the jumps with guard $x \in [1, 3]$, g_2 to the jumps with guard $x \in [0, 1)$ and t to the continuous evolution
- $S_0 = Init = \{(q_1, 0)\}$
- $S_F = B = \{q_2\} \times [2, 3]$
- the transition relation of T_H can be sketched as in Figure 2.

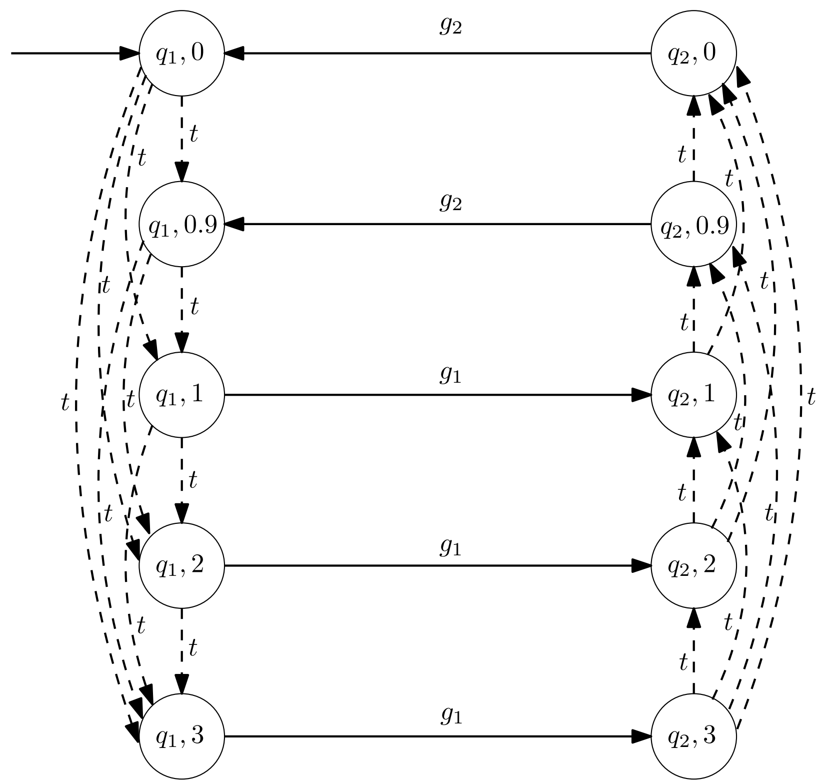

Figure 2: Transition system T_H

(b) Bisimulation quotient algorithm

- Initialization: $\hat{S} = (P_1, P_2)$,
  - $\mathbf{P_1 = S_F = \{q_2\} \times [2, 3]}$
  - $P_2 = S \backslash S_F = \{q_1\} \times [0, 3] \cup \{q_2\} \times [0, 2)$
- $Pre_{g_1}(P_1) = \{q_1\} \times [2, 3]$ and $Pre_{g_1}(P_1) \cap P_2 = \{q_1\} \times [2, 3]$
  - $\mathbf{P_{21} = \{q_1\} \times [2, 3]}$
  - $P_{22} = \{q_1, q_2\} \times [0, 2)$
- $Pre_t(P_{21}) = \{q_1\} \times [0, 3]$ and $Pre_t(P_{21}) \cap P_{22} = \{q_1\} \times [0, 2)$
  - $P_{221} = \{q_1\} \times [0, 2)$
  - $P_{222} = \{q_2\} \times [0, 2)$

- $Pre_{g_2}(P_{221}) = \{q_2\} \times [0,1)$ and $Pre_{g_2}(P_{221}) \cap P_{222} = \{q_2\} \times [0,1)$
  - $\mathbf{P_{2221} = \{q_2\} \times [0,1)}$
  - $\mathbf{P_{2222} = \{q_2\} \times [1,2)}$
- $Pre_{g_1}(P_{2222}) = \{q_1\} \times [1,2)$ and $Pre_{g_1}(P_{2222}) \cap P_{221} = \{q_1\} \times [1,2)$
  - $\mathbf{P_{2211} = \{q_1\} \times [1,2)}$
  - $\mathbf{P_{2212} = \{q_1\} \times [0,1)}$
- $\hat{S} = \{P_1, P_{21}, P_{2221}, P_{2222}, P_{2211}, P_{2212}\}$

Confirm that the algorithm terminated by checking that for all $\sigma \in \Sigma$ and all $P \in \hat{S}$, $Pre_\sigma(P)$ is either empty or equal to the union of elements in $\hat{S}$ (i.e. for all $P' \in \hat{S}$, $Pre_\sigma(P) \cap P' = \emptyset$ or $Pre_\sigma(P) \cap P' = P'$).

- $\sigma = g_1$:
  - $Pre_{g_1}(P_1) = P_{21}$
  - $Pre_{g_1}(P_{2222}) = P_{2211}$
  - $Pre_{g_1}(P) = \emptyset$ for all other P
- $\sigma = g_2$:
  - $Pre_{g_2}(P_{2212}) = P_{2221}$
  - $Pre_{g_2}(P) = \emptyset$ for all other P
- $\sigma = t$:
  - $Pre_t(P_1) = P_1$
  - $Pre_t(P_{21}) = P_{2212} \cup P_{2211} \cup P_{21}$
  - $Pre_t(P_{2221}) = P_{2221} \cup P_{2222} \cup P_1$
  - $Pre_t(P_{2222}) = P_{2222} \cup P_1$
  - $Pre_t(P_{2211}) = P_{2212} \cup P_{2211}$
  - $Pre_t(P_{2212}) = P_{2212}$

Resulting quotient transition system $\hat{T} = (\hat{S}, \Sigma, \hat{\rightarrow}, \hat{S}_0, \hat{S}_F)$ with:

- $\hat{S} = \{P_1, P_{21}, P_{2221}, P_{2222}, P_{2211}, P_{2212}\}$
- $\Sigma = \{g_1, g_2, t\}$
- $\hat{S}_0 = \{P_{2212}\}$
- $\hat{S}_F = \{P_1\}$
- transition relation as in Figure 3.

(c) Reach set algorithm

- $R_{-1} = \emptyset$, $R_0 = \hat{S}_0 = \{P_{2212}\}$
- $R_1 = \{P_{2212}, P_{2211}, P_{21}\}$
- $R_2 = \{P_{2212}, P_{2211}, P_{21}, P_{2222}, P_1\}$
- $R_3 = \hat{S}$, $R_4 = R_3$, so $Reach(\hat{S}_0) = R_3 = \hat{S}$.

Safety verification:

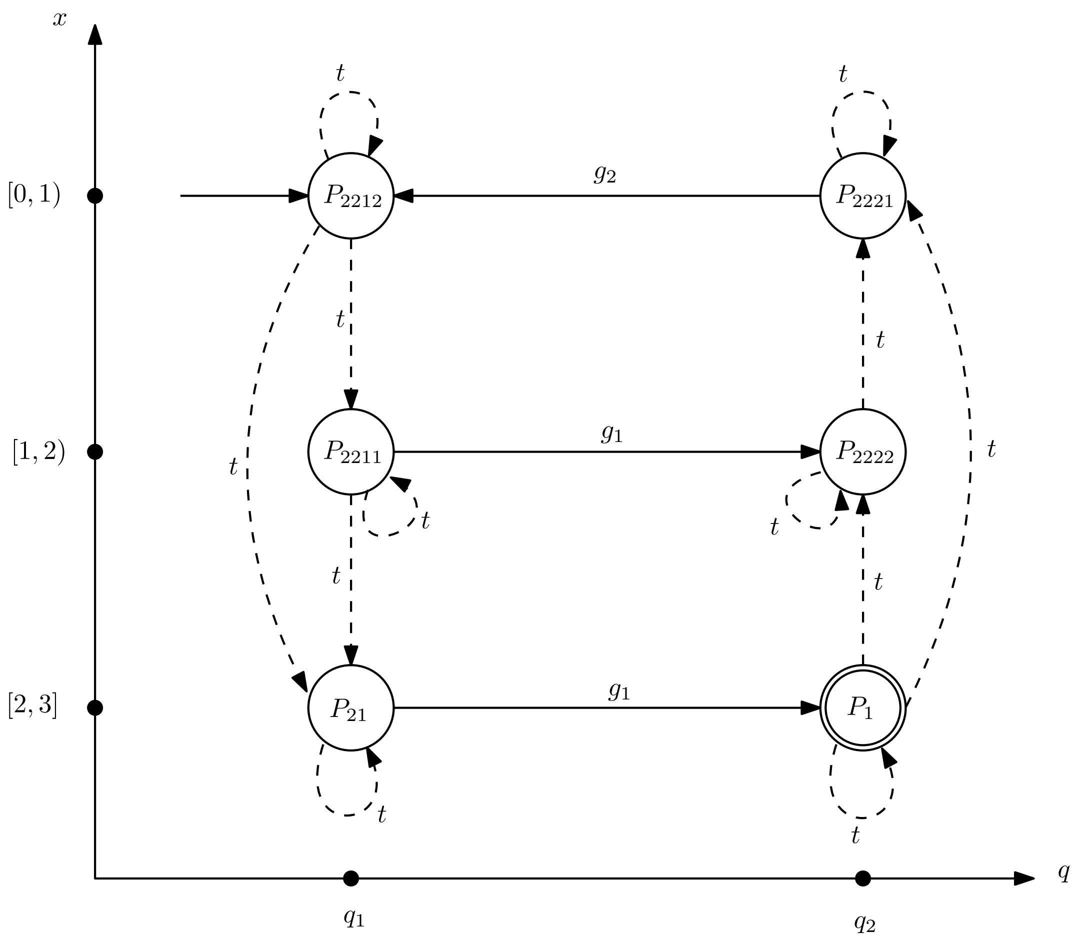

Figure 3: Quotient transition system $\hat{T}$

- Quotient transition system $\hat{T}$: $Reach(\hat{S}_0) \cap \hat{S}_F \neq \emptyset$.
- Due to bisimulation between T_H and $\hat{T}$, we also have $Reach(S_0) \cap S_F \neq \emptyset$.
- Since T_H is an equivalent representation of H, we have $Reach_H(Init) \cap B \neq \emptyset$.