

Final project for modern optimization methods

B06702064 會計五 林聖硯

1. A revision on Traveler Salesman Problem

(a) What is TSP?

The traveling salesman problem's statement is as follows. Given a set of cities and distance between every pair of cities, which can be viewed as an adjacency matrix, we need to find the shortest possible path that visits every city exactly once and returns to the starting point.

(b) What are the difficulties in TSP?

Although the TSP it's so easy to describle, it's rather difficult to solve it. In computational complexity theory, the TSP is clasified as **NP-hard** since it has no quick solution and the complexity of calculating the best route will increase when we add more destinations to the problem.

(c) What are the applications of TSP in real life?

Logistic planning, microchips manufacturing, DNA sequencing, etc.

2. Data Collection

Distance matrix collected from google map:

中坡	中研	中貨	玉成	玉德	向捷	庄研	佳樂	忠寵	昆陽	林坊	研究	胡達	重陽	香誠	凱松	港泰	港嘉捷	港捷	港勝	港德	港興	港環球	港匯	華技	羅達	慈要	新福玉	經貨	聯成	聯坊	華標	鵬龍	耀港	鑫貨		
中坡	0.00	5.80	4.80	2.50	0.35	2.90	5.8	3.40	1.700	1.700	0.65	5.10	5.30	2.80	4.30	1.50	2.90	3.300	4.70	4.10	1.00	1.00	3.40	3.300	2.80	6.4	1.20	3.00	0.50	4.00	1.90	0.65	4.50	2.00	4.10	4.60
中研	5.50	0.00	2.50	4.40	5.30	4.80	1.5	2.80	4.00	3.800	4.90	1.20	0.65	3.90	2.20	5.50	2.80	2.600	2.40	2.00	5.20	5.40	3.10	2.600	3.00	1.9	4.40	3.70	5.80	2.40	3.90	4.80	2.70	4.60	1.90	2.40
中貨	5.90	3.00	0.00	3.20	5.60	3.00	2.9	1.30	3.000	3.000	4.10	2.30	2.50	1.90	0.75	4.90	1.90	1.500	0.45	0.40	4.20	4.60	1.40	1.500	2.10	3.6	3.50	1.70	5.00	0.65	3.10	3.90	0.25	3.50	1.30	0.13
玉成	2.20	4.60	3.10	0.00	2.00	0.40	4.5	1.60	0.450	0.500	1.90	3.90	4.10	1.30	2.90	1.30	1.70	1.800	3.20	2.40	1.60	2.10	1.60	1.800	1.60	5.2	1.80	1.00	2.30	2.20	0.65	1.80	2.50	0.26	2.90	3.00
玉德	0.35	5.50	4.50	2.30	0.00	2.70	5.5	3.20	1.500	1.500	0.40	4.90	5.00	2.50	4.10	1.00	2.70	3.000	4.40	3.90	0.55	1.20	3.20	3.100	2.50	6.1	1.00	2.80	0.65	3.80	1.40	0.50	4.20	1.80	3.80	4.40
向捷	2.40	4.50	3.30	0.40	2.20	0.00	4.4	1.50	1.000	1.000	1.80	3.80	4.00	0.55	2.80	1.70	1.60	2.000	3.00	2.30	2.00	2.30	1.40	2.000	1.50	5.0	1.30	0.75	2.70	2.30	1.10	1.70	2.30	1.10	2.80	2.80
庄研	5.60	1.30	2.60	4.40	5.30	4.80	0.0	2.90	4.100	3.900	5.00	1.10	0.75	4.00	2.30	5.60	2.80	2.700	2.50	2.00	5.30	5.50	3.20	2.700	3.10	0.6	4.40	3.80	5.90	2.40	4.00	4.90	2.80	4.70	2.00	2.40
佳樂	3.40	2.90	1.50	1.60	3.10	2.30	2.9	0.0	1.700	1.700	2.70	2.30	2.40	1.30	0.90	2.90	0.45	0.200	1.40	1.10	2.90	3.20	0.40	0.210	0.60	3.5	2.20	1.10	3.60	1.00	1.70	2.60	2.10	1.90	1.20	1.30
忠寵	2.00	4.10	3.00	0.45	1.80	1.30	4.0	1.70	0.000	0.026	1.50	3.40	3.60	1.10	2.60	1.90	1.20	1.600	3.00	2.40	1.70	2.00	1.70	1.600	1.10	4.7	0.85	1.30	2.10	2.30	0.19	1.10	2.80	2.00	2.40	2.90
昆陽	1.70	4.70	3.60	0.50	1.40	1.00	4.6	2.30	0.026	0.000	1.30	4.00	4.20	1.70	3.20	1.70	1.80	2.200	3.50	3.00	1.40	2.30	2.30	2.200	1.70	5.2	0.80	1.90	1.80	2.90	0.16	1.10	3.30	0.75	3.00	3.50
林坊	0.65	5.20	4.20	1.90	0.40	2.30	5.2	2.80	1.800	1.200	0.00	4.60	4.70	2.20	3.70	1.10	2.40	2.700	4.10	3.60	1.30	1.70	2.90	2.700	2.20	5.8	0.55	2.40	0.80	3.40	1.00	0.16	3.90	2.20	3.50	4.00
研究	4.50	1.00	1.30	3.30	4.20	3.70	1.2	1.80	3.000	2.800	3.90	0.00	0.40	2.90	1.20	4.50	1.70	1.600	1.40	1.00	4.30	4.40	2.10	1.600	1.90	1.8	3.30	2.60	4.80	1.30	2.90	3.70	1.70	3.60	0.65	1.30
胡達	4.90	0.65	1.80	3.70	4.60	4.10	0.9	2.20	3.400	3.200	4.30	0.40	0.00	3.30	1.60	4.80	2.10	2.000	1.70	1.30	4.60	4.80	2.40	2.000	2.30	2.2	3.70	3.00	5.20	1.70	3.20	4.10	2.10	3.90	1.30	1.70
重陽	2.80	4.20	2.30	1.00	2.50	0.55	4.2	1.30	1.100	1.100	2.20	3.60	3.70	0.00	2.20	2.10	2.00	1.400	2.40	2.10	2.30	2.70	1.10	1.400	1.20	4.8	1.60	0.23	3.10	1.80	1.20	2.10	1.70	1.00	2.50	2.60
香城	3.90	2.40	0.75	2.20	3.70	2.90	2.4	0.80	2.300	2.200	3.30	1.80	1.90	1.80	0.00	3.50	1.20	0.950	0.50	0.45	3.50	3.80	1.10	1.000	1.40	3.0	2.80	1.60	4.20	0.15	2.30	3.20	0.55	2.40	0.65	0.90
凱松	1.00	5.70	4.50	1.30	0.95	2.00	5.7	2.90	1.600	1.600	1.30	5.10	5.20	2.00	4.00	0.00	2.90	3.200	4.80	3.90	0.35	2.10	2.80	3.200	2.70	6.3	1.20	2.20	1.60	4.20	1.80	1.40	4.10	1.00	4.00	4.40
港泰	3.30	2.80	1.80	2.10	3.00	2.70	2.8	0.45	1.800	1.600	2.70	2.20	2.30	1.70	1.60	3.20	0.00	0.350	1.70	1.20	3.00	3.20	0.65	0.350	0.16	3.4	2.10	1.00	3.60	1.10	1.70	2.60	1.50	2.40	1.10	1.70
港嘉捷	3.10	3.20	1.80	1.80	2.90	2.20	3.2	0.20	1.500	1.500	2.50	2.60	2.70	1.30	1.00	2.90	0.35	0.000	2.10	1.20	2.70	3.00	0.45	0.014	0.40	3.8	2.00	1.40	4.30	1.30	1.50	2.40	1.40	2.10	1.50	1.70
港捷	4.40	2.20	0.45	3.00	4.10	3.40	2.2	1.30	2.700	2.700	3.80	1.60	1.70	2.50	0.50	4.10	1.60	1.200	0.000	0.24	3.90	4.30	1.70	1.200	1.80	2.8	3.20	2.30	4.70	0.60	2.80	3.60	1.10	3.30	0.50	
港勝	4.30	2.10	0.40	2.90	4.00	3.30	2.1	1.20	2.600	2.600	3.70	1.50	1.60	2.60	4.00	1.50	1.100	0.24	0.00	3.80	4.20	1.60	1.100	1.80	2.7	3.10	2.40	4.60	0.35	2.70	3.60	0.45	3.20	0.60	0.35	
港通	1.20	5.70	4.30	1.70	0.55	2.20	5.7	2.90	1.600	1.400	1.10	5.10	5.20	2.30	3.80	0.35	2.90	3.200	4.20	3.60	0.00	1.80	3.00	3.200	2.70	6.3	1.20	3.50	1.60	1.50	4.00	1.20	4.00	4.20		
港德	1.00	5.70	4.70	2.50	1.20	2.90	5.7	3.30	2.100	1.700	1.20	5.10	5.20	2.70	4.20	2.10	2.90	3.200	4.60	4.10	1.80	0.00	3.40	3.200	2.70	6.3	2.20	3.00	0.45	4.00	1.60	1.50	4.40	2.70	4.00	4.60
港興	3.40	3.40	1.90	1.60	3.20	1.90	3.3	0.40	1.800	1.800	2.90	2.70	2.90	0.90	1.10	2.90	0.65	0.450	1.90	1.20	3.00	3.30	0.45	0.040	0.50	3.8	2.00	1.20	3.70	1.60	1.80	2.70	2.10	1.90	1.70	
港環球	3.10	3.20	1.80	1.80	2.90	2.20	3.2	0.21	1.500	1.400	2.50	2.60	2.70	1.30	1.00	2.90	0.35	0.014	1.70	1.20	2.70	3.00	0.45	0.000	0.40	3.8	2.00	1.20	3.40	1.10	1.50	2.40	1.40	2.00	1.50	1.70
港匯	3.00	3.00	2.00	1.80	2.70	2.20	3.0	0.60	1.500	1.300	2.40	2.40	2.50	1.30	1.50	2.90	0.16	0.400	1.90	1.40	2.70	2.80	0.70	0.400	0.000	3.6	1.80	1.00	3.20	2.20	2.20	1.70	2.10	1.30	1.80	
華技	6.20	1.90	3.20	5.00	5.90	5.40	0.6	3.50	4.700	4.500	5.60	1.70	1.30	4.60	2.90	6.20	3.40	3.300	3.10	2.70	6.00	6.10	3.80	3.300	3.70	0.0	5.00	4.40	6.50	3.10	4.60	5.50	3.40	5.30	2.60	3.00
華強	1.20	5.10	4.10	1.60	1.10	2.30	5.1	2.70	0.800	0.500	0.55	4.50	4.60	2.10	3.60	1.10	2.30	2.600	4.00	3.50	1.60	1.80	2.80	2.600	2.10	5.7	0.00	2.30	1.20	3.40	0.70	3.40	4.00	0.70	3.40	4.00
慈要	3.00	4.00	2.00	1.00	2.70	1.40	3.9	1.00	1.300	1.300	2.40	3.30	3.50	0.23	2.00	2.30	1.00	1.200	2.20	1.80	2.50	2.90	0.80	1.200	1.00	4.6	1.80	0.00	3.30	1.60	1.40	2.30	1.20	2.30	2.40	
新福玉	0.50	5.70	4.70	2.50	1.00	2.90	5.7	3.30	2.100	1.700	0.60	5.10	5.20	2.70	4.20	1.60	2.90	3.200	4.60	4.10	1.30	0.55	3.40	3.200	2.70	6.8	1.60	2.90	0.00	3.90	1.60	0.65	4.40	2.10	4.00	4.60
經貨	4.30	2.60	0.35	2.60	4.00	3.10	2.5	1.10	2.300	2.600	3.70	2.00	2.10	2.10	4.00	3.50	1.20	1.100	0.60	0.35	3.90	1.40	1.20	1.100	1.40	3.1	3.0	0.00	4.70	0.45	2.80	3.70	0.00	3.50	1.00	0.40
聯成	1.80	4.20	3.10	0.																																

3. An Implementation of Known Metaheuristic Methods

My implementation stores in the following two files:

- B06702064.ipynb (main execution file)
- HeuristicsAlgo.py (<http://HeuristicsAlgo.py>) (handcrafted algo.)

4. Method Comparison

(a)

- Hill Climbing

Parameter settings:

Param	value
maximum iteration time	500
Termination criteria	1. # of iteration > max_iter 2. the solution is not updating

Since the problem is hard and it do not converge to (local/global) optimum quickly, I set the maximum iteration time to 50.

- Random Walk

Parameter settings:

Param	value
maximum iteration time	500

Since the problem is hard and it do not converge to (local/global) optimum quickly, I set the maximum iteration time to 50.

- Genetic Algorithm

Parameter settings:

Param	value
maximum iteration time	20
population size	25
mutation rate	0.01
cross over method	ordered cross over (specifically used for TSP)

In the genetic algorithm, I set the population size to 25, which is suggested by the author of the paper (the population size should be 20 to 30). Also, I implemented a special cross over method called **ordered cross over** to ensure a feasible solution for the TSP problem. Finally, the mutation rate is set to be 0.01 by try and error method. (I've searched the value in the range from 0.005 to 0.1).

- Simulated Annealing

Parameter settings:

Param	value
maximum iteration time	500
initial temperature	100
stopping temperature	1
temperature scheduling	exponential scheduling with epsilon = 0.025
maximum patience (no update count)	3
stopping criteria	1. # of iteration > max iteration OR 2. patience > max patience OR 3. current temperature < stopping temperature

In the SA algo, the most important mechanism is the temperature scheduling. Here, I set the initial temperature to 100, the stopping temperature to 1 and exponential scheduling with epsilon equal to 0.025 to prevent the temperature from decreasing too fast. Since the problem is rather hard, I think that I need to add more randomness to the solution to prevent from sticking in the local optimum and be able to reach the global minimum.

- Tabu Search

Parameter settings:

Param	value
maximum iteration time	500
maximum tabu list length	10
aspiration criteria	if the best solution in tabu list is better than current solution update with it

In the tabu search algorithm, the maximum tabu list length is 5 by try and error. I found that the longer the tabu list, the more the probability that the solution will stuck in local optimum. Also, I use an aspiration criteria to prevent the tabu search alogrithm from sticking in local optimum.

- Particle Swarm Optimization

Parameter settings:

Param	value
maximum iteration time	20
population size	25
w (intertia weight)	1
c_1 (importance of global solution)	1
c_2 (importance of local solution)	1

Since the particle swarm optimization is not suitable to solve a non-continuous problem, I design an a similar algorithm in a discrete nature. The steps are as follows:

1. initiliazation

- swarm size
- no lower and upper bound (for simplicity)
- randomly assign routes to each particle
- compute route length
- get local and global optimal (use global optimal as velocity)
- initalize velocity to 0 for each particle (j)

2. iteration

- $c_1 = c_2 = w = 1$
- randomly crop global subroutes
- do ordered cross over on local subroutine
- do ordered cross over on combined subroutine as velocity

- randomly crop velocity
- do ordered cross over on current subroutine

3. termination

- converge to same route
- # of iteration > given threshold.

- Ant Colony Optimization

Parameter settings:

Param	value
maximum iteration time	20
population size	25
Q (initial pheromone)	100
rho (evaporation rate)	0.1
alpha (pheromone scaling factor)	10
beta (distance scaling factor)	20

The population size and maximum iteration size are 25 and 20 respectively for comparability reason. The value of rho, alpha, beta are set by try and error.

(b) Provide the results of 7 methods on TSP of 7-Eleven convenient stores. You need to provide 7 lists of store names in order, and provide 7 maps with the routes.

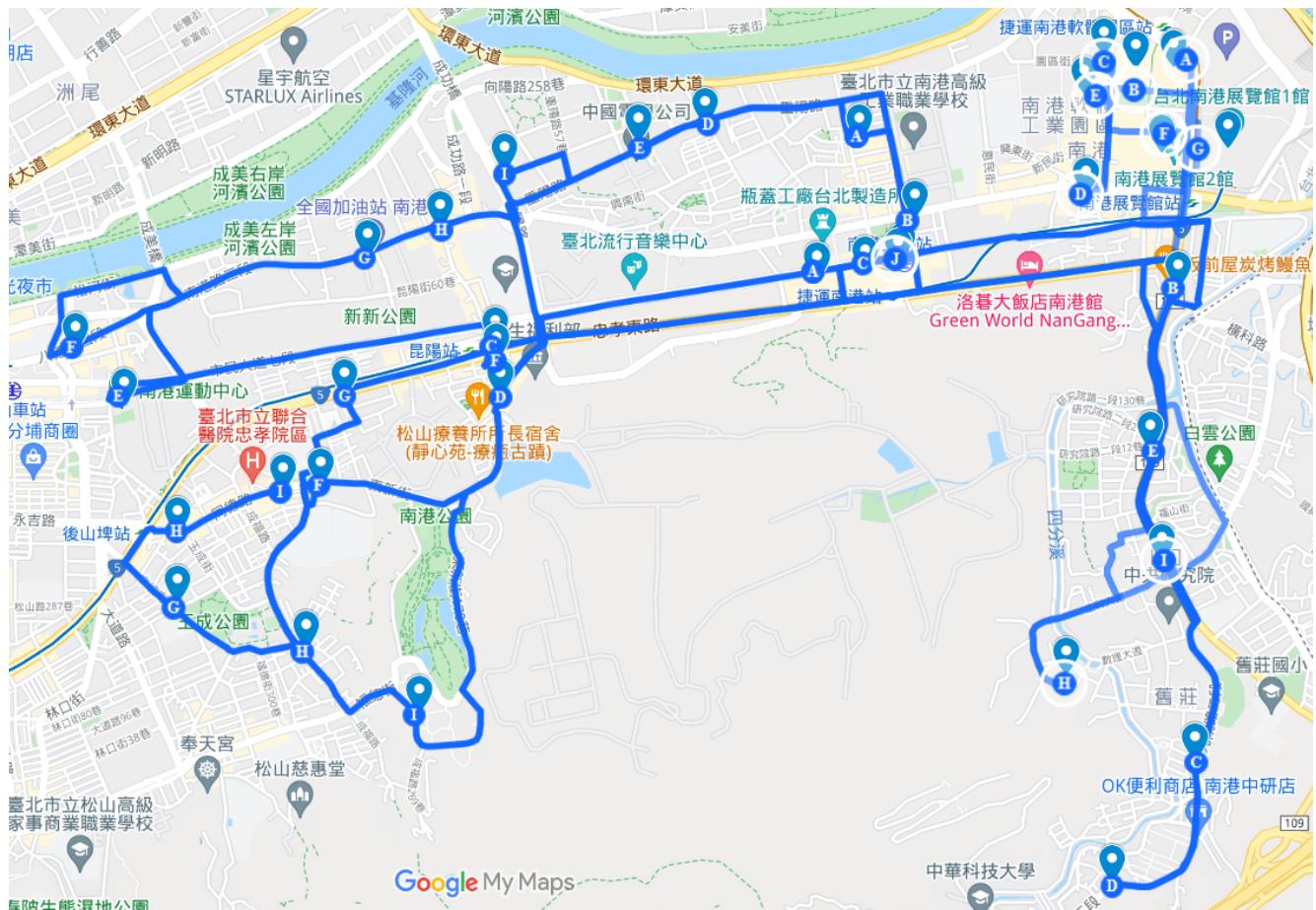
- Hill climbing

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The best solution of hill climbing algorithm is
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The best route length is 35.964
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Best route:
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鑫貿->中貿->馥樺->香城->經貿->港勝->港捷->中研->胡適->港環球->港高鐵->忠陽->聯成->港運->凱松->鵬馳->玉成->向揚->港興->佳樂->港泰->慈愛->重陽->昆陽->雄強->新福玉->港德->港麗->耀港->庄研->華技->研究->聯坊->中坡->玉德->林坊
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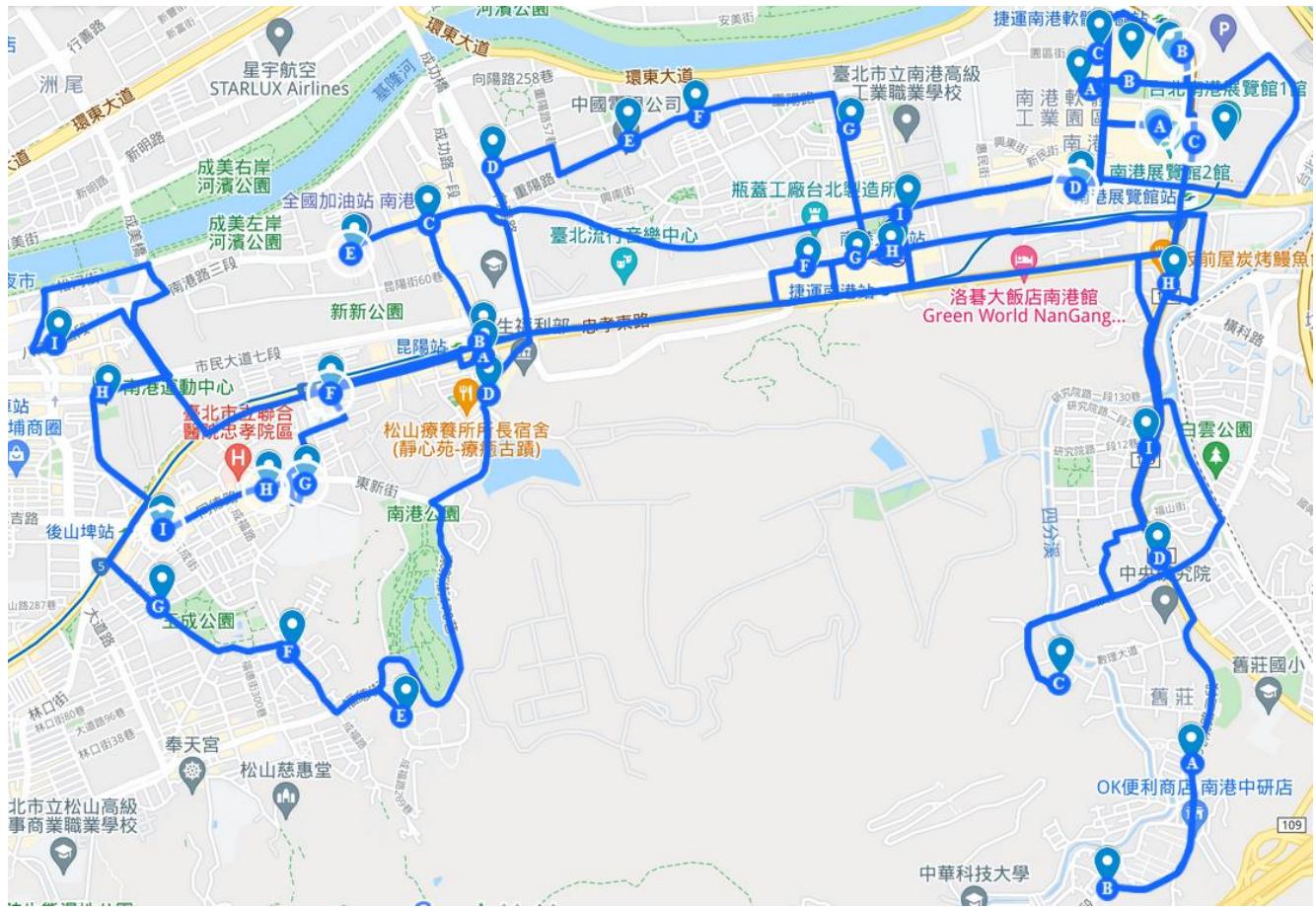
- Random Walk

The best solution of random walk algorithm is

The best route length is 32.526

Best route:

庄研->華技->中研->胡適->港環球->港麗->港泰->耀港->研究->經貿->中貿->馥樺->聯成->港德->新福玉->中坡->港運->凱松->昆陽->忠陽->玉成->向揚->重陽->慈愛->港興->港高鐵->佳樂->港勝->鑫貿->港捷->香城->鵬馳->雄強->聯坊->林坊->玉德

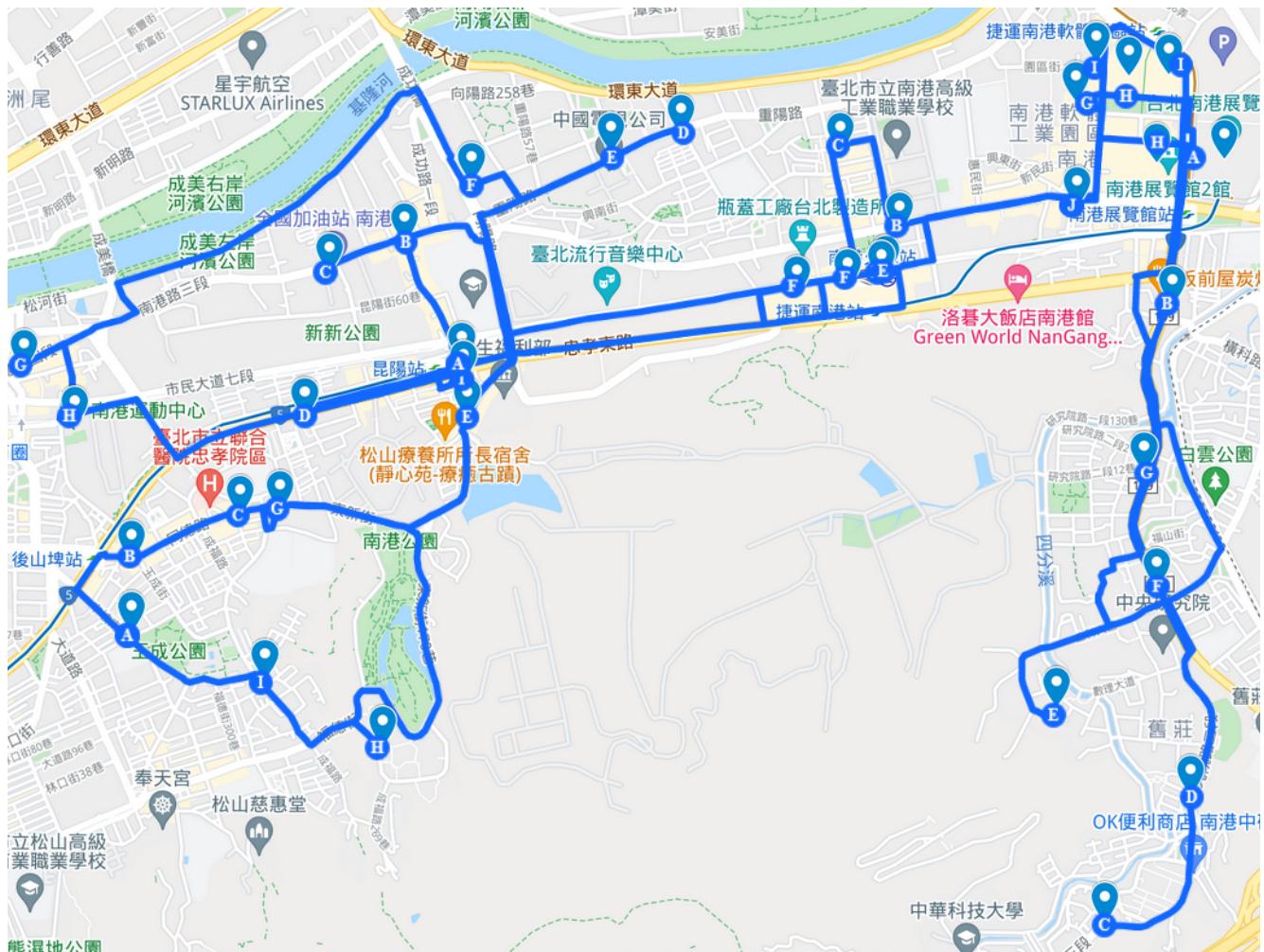


- Simulated Annealing

The best solution of simulated annealing algorithm is
The best route length is 27.01

Best route:

港捷->耀港->華技->庄研->中研->胡適->研究->港勝->馥樺->香城->佳樂->港興->港環球->港高鐵->港麗->聯坊->港德->新福玉->中坡->玉德->林坊->慈愛->重陽->向揚->凱松->港運->昆陽->忠陽->玉成->鵬馳->雄強->聯成->港泰->經貿->中貿->鑫貿



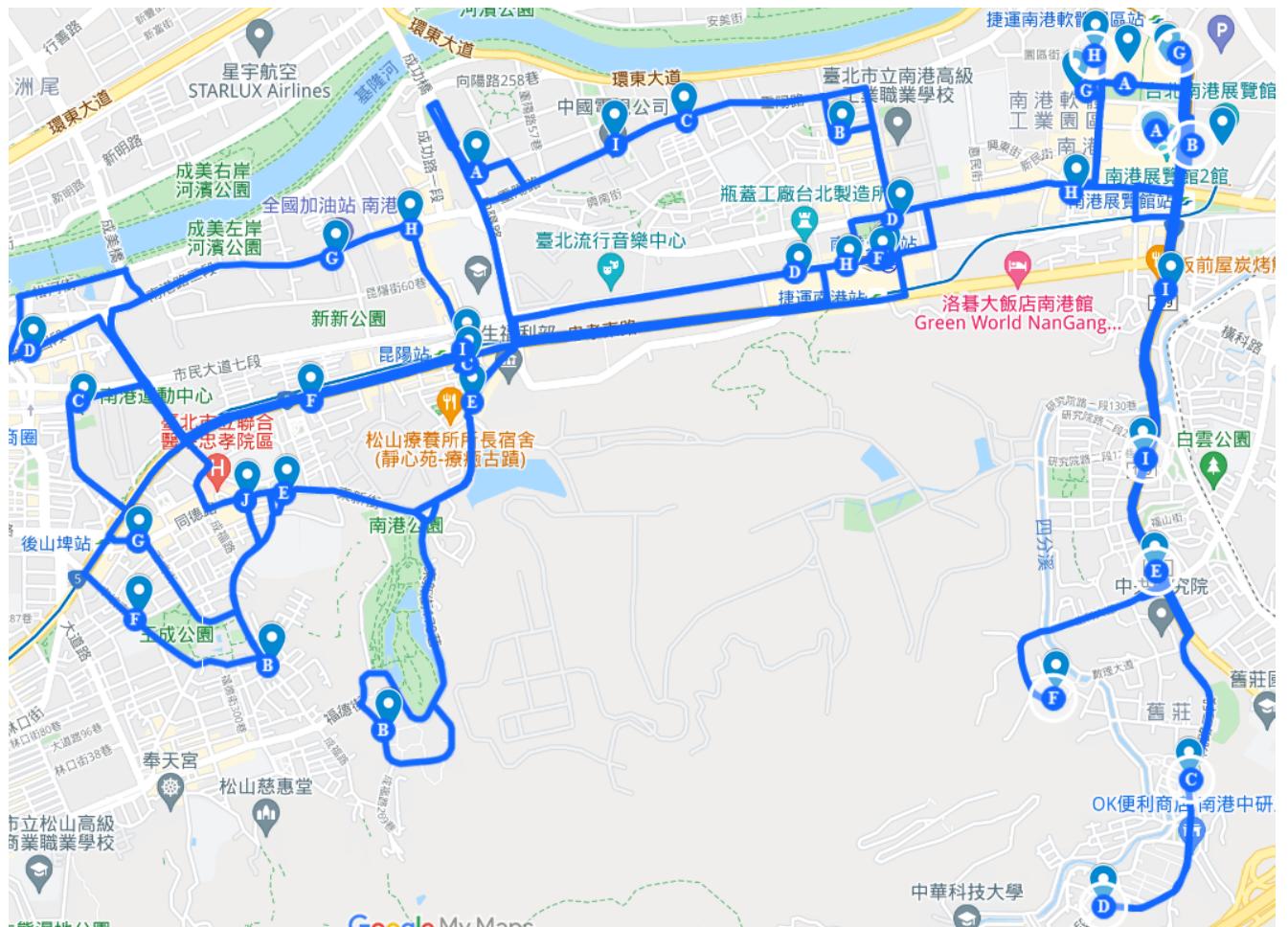
- Genetic Algorithm

The best solution of genetic algorithm is

The best route length is 39.164

Best route:

中貿->港興->慈愛->港麗->聯成->雄強->鵬馳->玉成->忠陽->林坊->新福玉->港運->凱松->聯坊->中坡->玉德->港泰->重陽->向揚->港德->昆陽->佳樂->港高鐵->港環球->經貿->香城->耀港->港勝->港捷->庄研->華技->胡適->中研->鑫貿->馥樺->研究



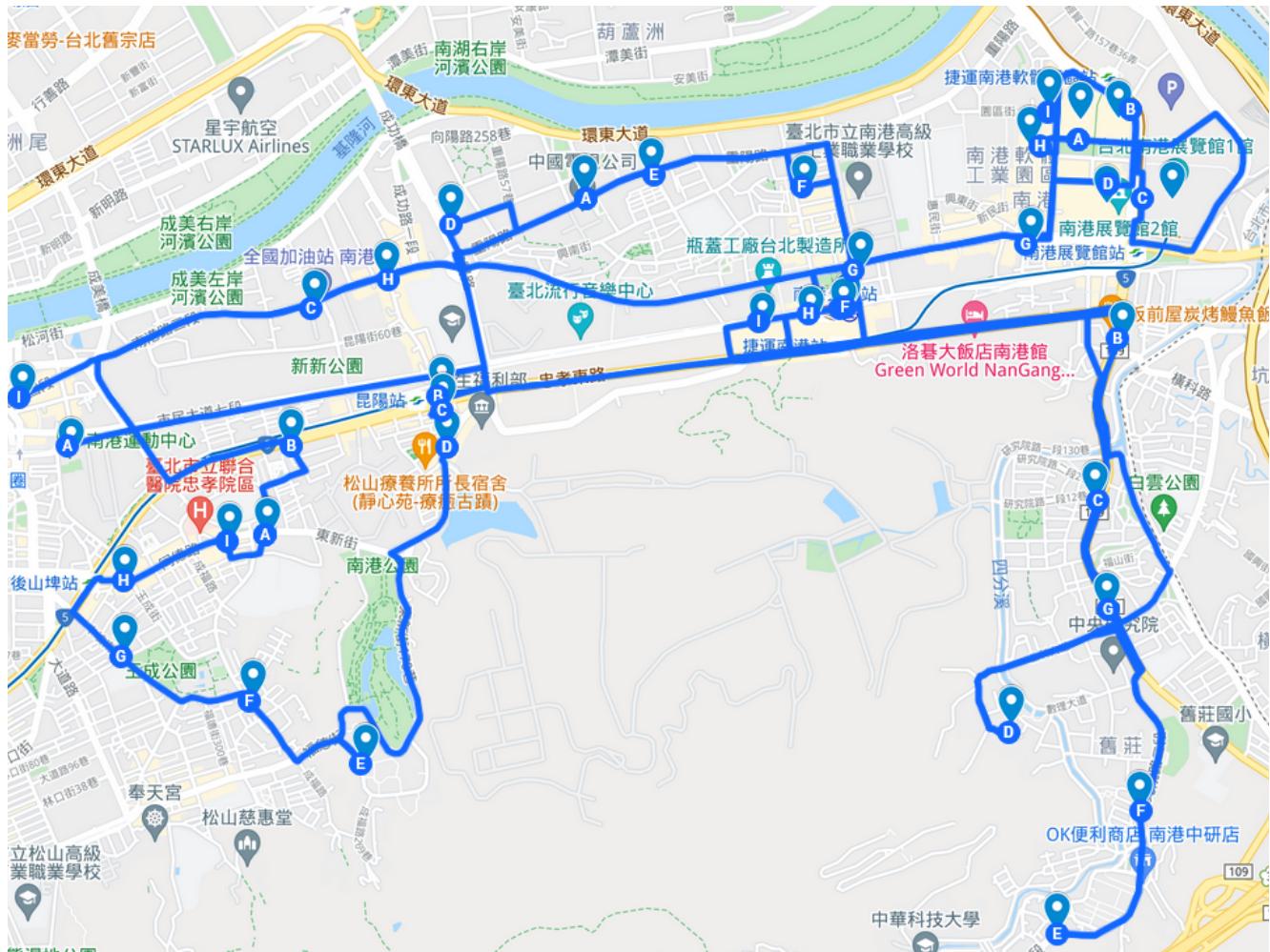
- Tabu search

The best solution of tabu search is

The best route length is 27.67

Best route:

港運->耀港->研究->中研->華技->庄研->胡適->港泰->港麗->重陽->忠陽->昆陽->聯成->港德->新福玉->中坡->玉德->林坊->聯坊->雄強->鵬馳->向揚->慈愛->港興->香城->經貿->馥樺->中貿->鑫貿->港捷->港勝->港環球->港高鐵->佳樂->玉成->凱松



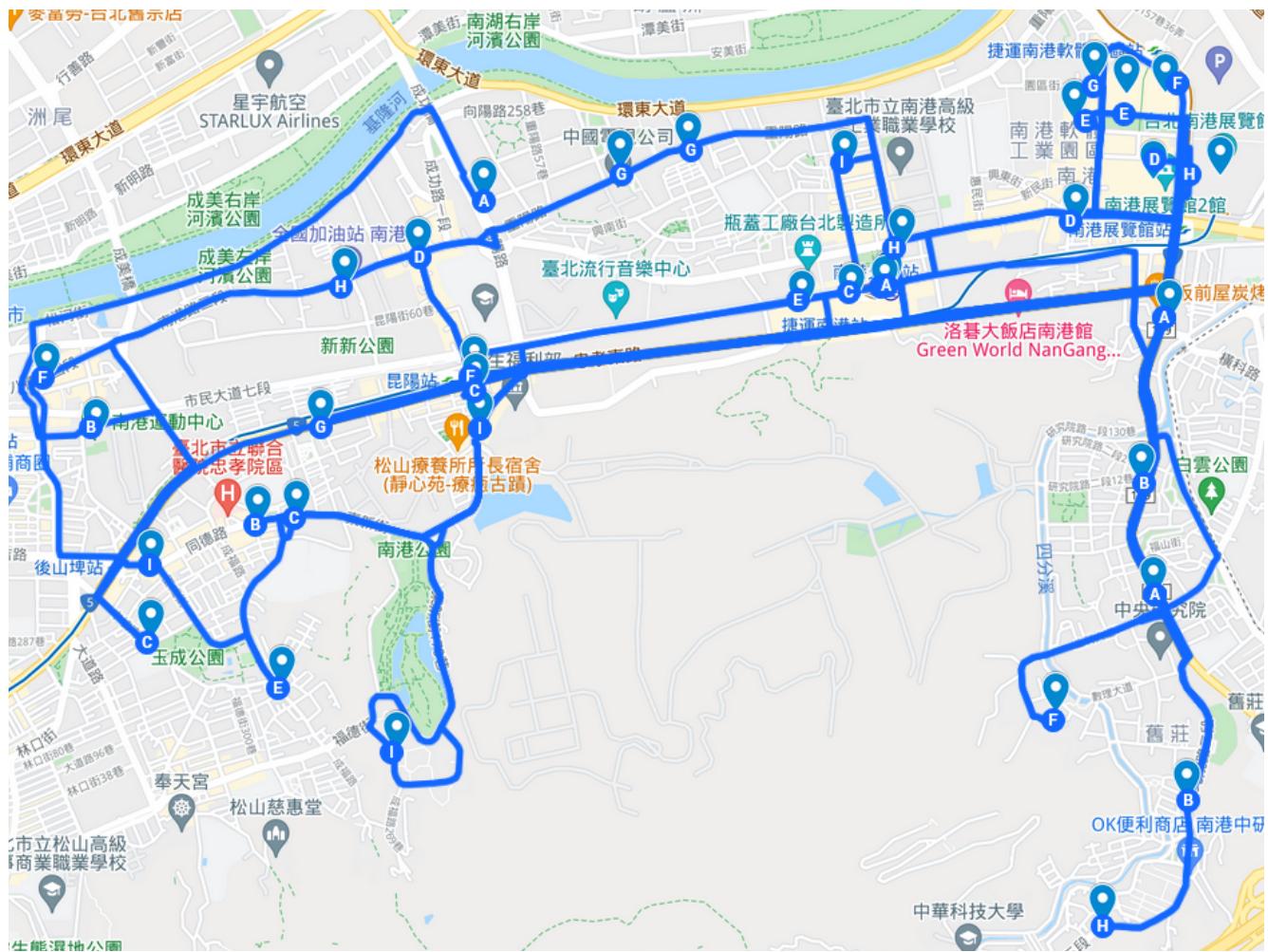
- Partical Swarm Optimization

The best solution of particle swarm optimization is

The best route length is 74.35

Best route:

向揚->港運->中坡->港勝->中貿->忠陽->雄強->港捷->聯成->耀港->庄研->聯坊->玉成->港麗->中研->馥樺->華技->港德->胡適->研究->港泰->港環球->新福玉->凱松->重陽->佳樂->港興->港高鐵->林坊->昆陽->香城->經貿->鑫貿->慈愛->鵬馳->玉德



- Ant Colony Optimization

The best solution of ant colony optimization is

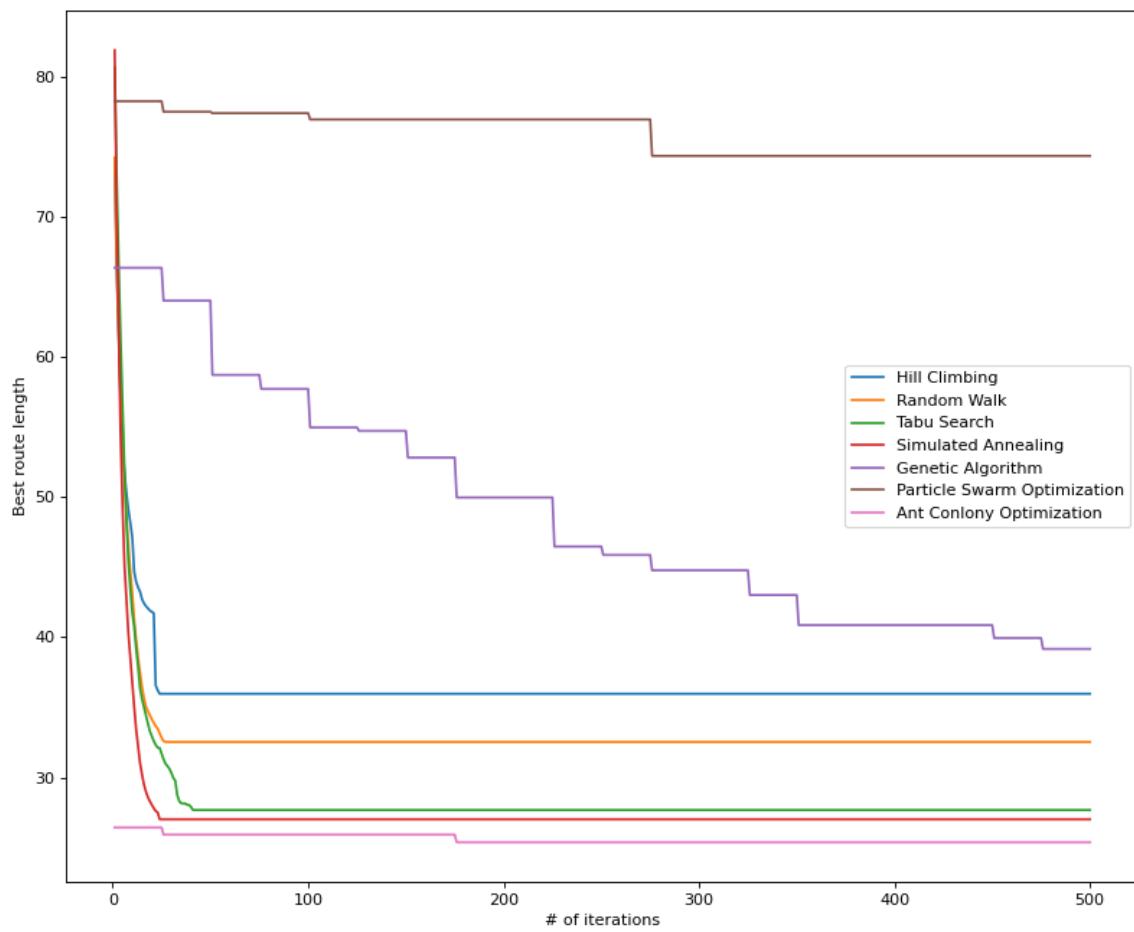
The best route length is 25.37

Best route:

昆陽->忠陽->聯成->玉成->鵬馳->雄強->聯坊->林坊->玉德->中坡->新福玉->港德->港運->凱松->向揚->重陽->慈愛->港興->佳樂->港環球->港高鐵->港泰->港麗->經貿->馥樺->中貿->鑫貿->港勝->港捷->香城->耀港->研究->胡適->中研->庄研->華技



(c) Draw the progress plot by overlapping the results of all 7 methods.



5. Conclusion

(a) Summarize your comparison results on 7 methods

From the FE plot we can find that, by setting proper hyperparameters, the ACO is the best and the most effective algorithm among all 7 algorithms. Also, I found that the population-based algorithm doesn't always converge to the optimal solutions, especially the PSO algorithm which is not suitable to solve discrete problems. Also, the genetic algorithm doesn't reach the best solution because of lack of iteration times.

The SA and TS algorithms are two simple yet effective algorithms in solving the PSO problem. Their implementations are much easier than the population-based methods, while their effectivenesses are the same as those population-based methods (or even better).

Surprisingly, the RW and HC algorithms can achieve good results (though not global optima) and their execution time are rather fast than the others.

(b) State the advantages and disadvantages of all 7 methods

Algorithm	Advantage	disadvantages
Hill climbing	Fast	Easily stuck in local optimum
Random walk	Fast	Easily stuck in local optimum
Tabu search	Can effectively find global optimum with aspiration criteria	Slow
Simulated annealing	Can effectively find global optimum with randomness provided by temperature scheduling	Hyperparameter tuning is time-consuming
Genetic algorithm	Exhibit randomness in each generation, may avoid sticking in local optimum	Too much randomness may lead to suboptimal solutions
Particle swarm optimization	Can consider global and local optimal solution simultaneously	Not suitable for discrete problem
Ant colony optimization	Suitable for solving TSP problem (consider the length of each route internally in the algorithm)	Slow Hyperparameter tuning is time-consuming

(c) State at least one potential improvement on the best method to make the algorithm even better on this TSP application.

Design more complicated pheromone rules rather than simply added a constant pheromone on each path, such as

$$\Delta\tau_{i,j} = \frac{Q}{L^S}, \text{ if } (i, j) \text{ is contained in current path of ant } S, \text{ where } L^S \text{ is the total route length that ant } S \text{ has gone through.}$$

(d) Suggest a potential memetic algorithm by combining two good methods from the 7 methods that is suitable for improving the result of TSP.

1. Use population-based algorithm such as PSO, ACO, GA to find a best solution (may be really close to global optimum)
2. Use local search method such as, hill climbing, random walk, simulated annealing algorithm and initialize them with best solution obtained in step 1.
3. Obtain best solution from step 2.

6. Find New Method by Yourself

I implemented a meta-heuristic algorithm called "Cuckoo Search" from the paper below. The

cacukoo search is a population-based algorithm which is designed primarily for the traveling salesman problem and thus it's efficient to solve TSP.

Parameter settings:

Param	value
maximum iteration time	50
num_nests(population size)	10

Ouaarab, A., Ahiod, B. & Yang, XS. Discrete cuckoo search algorithm for the travelling salesman problem. Neural Comput & Applic 24, 1659–1669 (2014). <https://doi.org/10.1007/s00521-013-1402-2> (<https://doi.org/10.1007/s00521-013-1402-2>).

From the FE plot, we can find that the cuckoo search algorithm can achieved an local optimum as genetic algorithm. Although it's really fast and efficient (total run time is less than 0.001 sec), it's not able to reach the global optimum in the end.

```
The best solution of cacukoo search is
The best route length is 39.75
Best route:
港興->重陽->向揚->玉成->凱松->港運->玉德->雄強->鵬馳->忠陽->研究->胡適->庄研->華技->中買->港捷->港高
鐵->港泰->耀港->馥樟->佳樂->港麗->中研->香城->昆陽->聯坊->港德->新福玉->中坡->林坊->聯成->慈愛->港環
球->港勝->經貿->鑫貿
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