

实验报告

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实验名称 实验数据处理

实验目的 熟练掌握数据处理的相关方法并处理实际问题

实验一:用最小偏向法测三棱镜的折射率

实验数据

表 1 用分光计测量三棱镜的顶角

	θ_1	θ_2	θ_1'	θ_2'
第一次	23° 43'	203° 40'	263° 40'	83° 43'
第二次	24° 43'	204° 40'	144° 41'	324° 40'
第三次	263° 37'	83° 40'	143° 40'	323° 47'

表 2 用分光计测量最小偏向角

	θ_1	θ_2	θ_1'	θ_2'
第一次	79° 52'	259° 51'	28° 24'	208° 20'
第二次	200° 49'	20° 49'	149° 22'	329° 23'
第三次	319° 47'	139° 48'	268° 17'	88° 20'

数据处理

$$\text{有 } A = 180^\circ - \frac{1}{2} [|\theta_1 - \theta_1'| + |\theta_2 - \theta_2'|], \quad \delta = \frac{1}{2} [|\theta_1 - \theta_1'| + |\theta_2 - \theta_2'|]$$

依次计算，得

$$A_1 = 180^\circ - \frac{1}{2} [|\theta_1 - \theta_1'| + |\theta_2 - \theta_2'|] = 60^\circ 00'$$

$$A_2 = 180^\circ - \frac{1}{2} [|\theta_1 - \theta_1'| + |\theta_2 - \theta_2'|] = 60^\circ 01'$$

$$A_3 = 180^\circ - \frac{1}{2} [|\theta_1 - \theta_1'| + |\theta_2 - \theta_2'|] = 60^\circ 05'$$

$$\delta_1 = \frac{1}{2} [|\theta_1 - \theta_1'| + |\theta_2 - \theta_2'|] = 51^\circ 29' 30''$$

$$\delta_2 = \frac{1}{2} [|\theta_1 - \theta_1'| + |\theta_2 - \theta_2'|] = 51^\circ 26' 30''$$

$$\delta_3 = \frac{1}{2} [|\theta_1 - \theta_1'| + |\theta_2 - \theta_2'|] = 51^\circ 29' 00''$$

$$\text{故 } \bar{A} = 60^\circ 02', \bar{\delta} = 51^\circ 28' 20''$$

将数据列成表格

表 3 关于三棱镜顶角和最小偏向角的表格

	第一次	第二次	第三次	平均值
三棱镜顶角A	60°00'	60°01'	60°05'	60°02'
最小偏向角 δ	51°29'30"	51°26'30"	51°29'00"	51°28'20"

$$\text{故折射率 } n = \frac{\sin \frac{\bar{\delta}_{\min} + \bar{A}}{2}}{\sin \frac{\bar{A}}{2}} = 1.6524$$

不确定度分析

$$\text{由 } n = \frac{\sin \frac{\bar{\delta}_{\min} + \bar{A}}{2}}{\sin \frac{\bar{A}}{2}}, \text{ 两边取对数, 得 } \ln n = \ln \sin \frac{\bar{\delta}_{\min} + \bar{A}}{2} - \ln \sin \frac{\bar{A}}{2}$$

$$\text{两边取微分, } \frac{dn}{n} = \frac{\cos \frac{\bar{\delta}_{\min} + \bar{A}}{2}}{\sin \frac{\bar{\delta}_{\min} + \bar{A}}{2}} \left(\frac{d\bar{\delta}}{2} + \frac{d\bar{A}}{2} \right) - \frac{\cos \frac{\bar{A}}{2}}{\sin \frac{\bar{A}}{2}} \frac{d\bar{A}}{2},$$

$$\text{将微分符号变为不确定度, 整理得 } \frac{U_n}{n} = \frac{U_A}{2} \left(\frac{\cos \frac{\bar{\delta}_{\min} + \bar{A}}{2}}{\sin \frac{\bar{\delta}_{\min} + \bar{A}}{2}} - \frac{\cos \frac{\bar{A}}{2}}{\sin \frac{\bar{A}}{2}} \right) + \frac{U_\delta}{2} \frac{\cos \frac{\bar{\delta}_{\min} + \bar{A}}{2}}{\sin \frac{\bar{\delta}_{\min} + \bar{A}}{2}}$$

$$\text{故 } U_n = \sqrt{\left(U_A \frac{\cos \frac{\bar{\delta}_{\min} + \bar{A}}{2} \sin \frac{\bar{A}}{2} - \sin \frac{\bar{\delta}_{\min} + \bar{A}}{2} \cos \frac{\bar{A}}{2}}{2 \sin^2 \frac{\bar{A}}{2}} \right)^2 + \left(U_\delta \frac{\cos \frac{\bar{\delta}_{\min} + \bar{A}}{2}}{2 \sin \frac{\bar{A}}{2}} \right)^2}$$

$$\text{化简得, } U_n = \sqrt{\left(U_A \cdot \frac{\sin \frac{\bar{\delta}_{\min}}{2}}{2 \sin^2 \frac{\bar{A}}{2}} \right)^2 + \left(U_\delta \frac{\cos \frac{\bar{\delta}_{\min} + \bar{A}}{2}}{2 \sin \frac{\bar{A}}{2}} \right)^2}$$

$$\sigma_A = \sqrt{\frac{1}{n-1} \sum (A - \bar{A})^2} = \sqrt{\frac{(60^\circ 00' - 60^\circ 02')^2 + (60^\circ 01' - 60^\circ 02')^2 + (60^\circ 05' - 60^\circ 02')^2}{2}} = 0.044^\circ = 0.0008 \text{ rad}$$

$$\text{有 } U_A = \sqrt{(t_{0.95} \frac{\sigma_A}{\sqrt{n}})^2 + (k_{0.95} \frac{\Delta}{C})^2} = \sqrt{(4.30 \cdot \frac{0.044^\circ}{\sqrt{3}})^2 + (1.645 \cdot \frac{1'}{\sqrt{3}})^2} = 0.11^\circ = 0.0019 \text{ rad}, P = 0.95$$

$$\sigma_\delta = \sqrt{\frac{1}{n-1} \sum (\delta - \bar{\delta})^2} = \sqrt{\frac{(51^\circ 29' 30'' - 51^\circ 28' 20'')^2 + (51^\circ 26' 30'' - 51^\circ 28' 20'')^2 + (51^\circ 29' 00'' - 51^\circ 28' 20'')^2}{2}} = 0.027^\circ = 0.0005 \text{ rad}$$

$$\text{有 } U_\delta = \sqrt{(t_{0.95} \frac{\sigma_\delta}{\sqrt{n}})^2 + (k_{0.95} \frac{\Delta}{C})^2} = \sqrt{(4.30 \cdot \frac{0.027^\circ}{\sqrt{3}})^2 + (1.645 \cdot \frac{1'}{\sqrt{3}})^2} = 0.07^\circ = 0.0012 \text{ rad}, P = 0.95$$

$$\text{故 } U_n = \sqrt{\left(U_A \cdot \frac{\sin \frac{\bar{\delta}_{\min}}{2}}{2 \sin^2 \frac{\bar{A}}{2}} \right)^2 + \left(U_\delta \frac{\cos \frac{\bar{\delta}_{\min} + \bar{A}}{2}}{2 \sin \frac{\bar{A}}{2}} \right)^2} = \sqrt{\left(0.0019 \cdot \frac{\sin \frac{51^\circ 28' 20''}{2}}{2 \sin^2 \frac{60^\circ 02'}{2}} \right)^2 + \left(0.0012 \frac{\cos \frac{51^\circ 28' 20'' + 60^\circ 02'}{2}}{2 \sin \frac{60^\circ 02'}{2}} \right)^2} = 0.0018$$

折射率 $n = \bar{n} \pm U_n = 1.6524 \pm 0.0018 (P = 0.95)$

相对不确定度为 0.109%

实验二、计算黄铜盘在空气中的自由散热速率

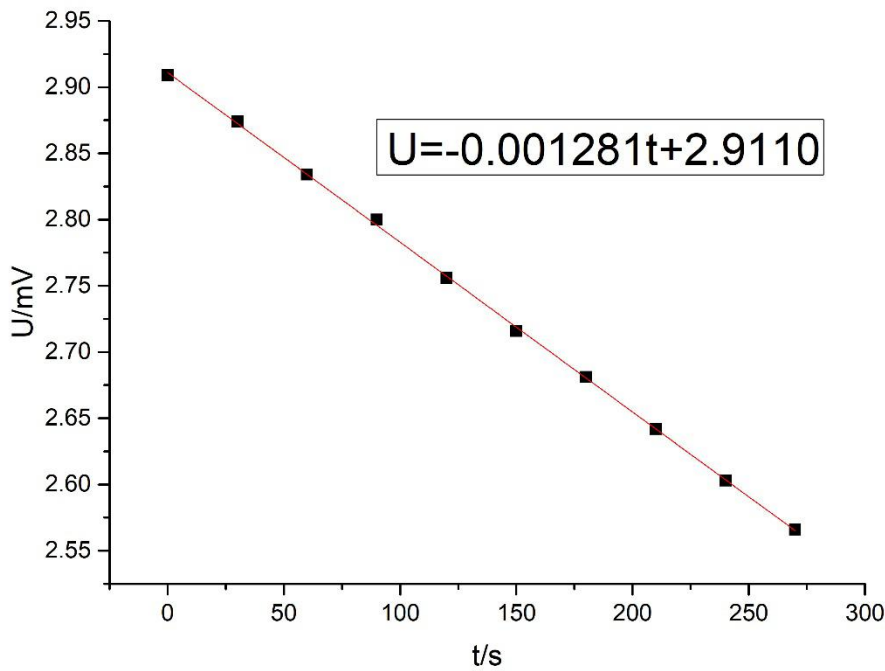
实验数据

黄铜盘温差电动势与时间的关系

$t(s)$	0	30	60	90	120	150	180	210	240	270
$U(mV)$	2.909	2.874	2.834	2.8	2.756	2.716	2.681	2.642	2.603	2.566

数据处理

图表 2 黄铜盘温差电动势随时间的变化曲线



有 $\bar{t} = 135s$, $\bar{U} = 2.7381mV$

为了计算斜率 k , 先计算 $\sum t_i U_i, \sum t_i, \sum U_i, \sum t_i^2$

$$\begin{aligned}\sum t_i U_i &= (0 \cdot 2.909 + 30 \cdot 2.874 + 60 \cdot 2.834 + 90 \cdot 2.8 + 120 \cdot 2.756 + 150 \cdot 2.716 + 180 \cdot 2.681 + 210 \cdot 2.642 + 240 \cdot 2.603 + 270 \cdot 2.566)mV \cdot s \\ &= 3601.32mV \cdot s\end{aligned}$$

$$\sum t_i = (0 + 30 + 60 + 90 + 120 + 150 + 180 + 210 + 240 + 270)s = 1350s$$

$$\sum U_i = (2.909 + 2.874 + 2.834 + 2.8 + 2.756 + 2.716 + 2.681 + 2.642 + 2.603 + 2.566)mV = 27.381mV$$

$$\sum t_i^2 = (0 + 30^2 + 60^2 + 90^2 + 120^2 + 150^2 + 180^2 + 210^2 + 240^2 + 270^2)s^2 = 256500s^2$$

$$\sum U_i^2 = (2.909^2 + 2.874^2 + 2.834^2 + 2.8^2 + 2.756^2 + 2.716^2 + 2.681^2 + 2.642^2 + 2.603^2 + 2.566^2)mV^2 = 75.093795mV^2$$

$$\text{故相关系数 } r = \frac{\bar{t}\bar{U} - \bar{t}\bar{U}}{\sqrt{(\bar{t}^2 - \bar{t}^2)(\bar{U}^2 - \bar{U}^2)}} = \frac{\frac{3601.32}{10} \cdot \frac{1350}{10} \cdot \frac{27.381}{10}}{\sqrt{\left[\frac{256500}{10} - \left(\frac{1350}{10}\right)^2\right] \left[\frac{75.093795}{10} - \left(\frac{27.381}{10}\right)^2\right]}} = -0.99985$$

r 极接近 1, 可见可以直线拟合, 且线性关系较好。

$$\text{下求斜率 } k = \frac{n \sum t_i U_i - \sum t_i \sum U_i}{n \sum t_i^2 - (\sum t_i)^2} = \frac{10 \cdot 3601.32 - 1350 \cdot 27.381}{10 \cdot 256500 - 1350^2} mV/s = -0.0012810 mV/s$$

$$\text{截距 } b = \bar{U} - m\bar{t} = \left(\frac{27.381}{10} - (-0.001281) \cdot \frac{1350}{10} \right) mV = 2.91104 mV$$

$$\text{故斜率标准差 } s_k = |k| \sqrt{\frac{\left(\frac{1}{r^2} - 1\right)}{n-2}} = 0.0012810 \sqrt{\frac{\frac{1}{0.99985^2} - 1}{10-2}} mV/s = 8 \times 10^{-6} mV/s$$

$$\text{截距标准差为 } s_b = s_k \sqrt{t^2} = 8 \times 10^{-6} \sqrt{\frac{256500}{10}} mV = 0.0013 mV$$

故拟合直线应写为 $U = -0.001281t + 2.9910$

有斜率 $k = (-0.001281 \pm 0.000006) mV/s$

截距 $b = (2.9910 \pm 0.0013) mV$