Scientific Computing: HW5 Solution

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Problem 1

According to the coefficient formula, we know that $C_{n-i}^{(n)} = \frac{(-1)^i}{n \cdot i! \cdot (n-i)!} \int_0^n \frac{t(t-1)\cdots(t-n)}{t-(n-i)} dt$.
Let $t = n - x$, then $C_{n-i}^{(n)} = \frac{(-1)^i}{n \cdot i! \cdot (n-i)!} \int_n^0 - \frac{(n-x)(n-x-1)\cdots(-x)}{(n-x)-(n-i)} dx = \frac{(-1)^i}{n \cdot i! \cdot (n-i)!} \cdot (-1)^n \int_0^n \frac{x(x-1)\cdots(x-n)}{x-i} dx$.
So $C_{n-i}^{(n)} = \frac{(-1)^{n+i}}{n \cdot i! \cdot (n-i)!} \int_0^n \frac{t(t-1)\cdots(t-n)}{t-i} dt$, and we know that $C_i^{(n)} = \frac{(-1)^{n-i}}{n \cdot i! \cdot (n-i)!} \int_0^n \frac{t(t-1)\cdots(t-n)}{t-i} dt$.
It's obvious that $(-1)^{n+i} = (-1)^{n-i}$, therefore $C_i^{(n)} = C_{n-i}^{(n)}$, $i = 0, 1, 2, \cdots, n$.

Problem 2

According to Taylor's theorem, we have $f(x) = f(\frac{a+b}{2}) + (x - \frac{a+b}{2}) f'(\frac{a+b}{2}) + \frac{(x-\frac{a+b}{2})^2}{2} f''(\xi)$, where $\xi \in (a, b)$.
So $\int_a^b f(x)dx = \int_a^b f(\frac{a+b}{2})dx + \int_a^b (x - \frac{a+b}{2}) f'(\frac{a+b}{2})dx + \int_a^b \frac{(x-\frac{a+b}{2})^2}{2} f''(\xi)dx$, where $\xi \in (a, b)$.
And we could easily konw that, $\int_a^b (x - \frac{a+b}{2}) f'(\frac{a+b}{2})dx = f'(\frac{a+b}{2}) \int_a^b (x - \frac{a+b}{2})dx = 0$.
Therefore, $\int_a^b f(x)dx = f(\frac{a+b}{2}) + \int_a^b \frac{(x-\frac{a+b}{2})^2}{2} f''(\xi)dx = f(\frac{a+b}{2}) + \frac{(b-a)^3}{24} f''(\xi)$, where $\xi \in (a, b)$.

Problem 3

$I_2(f) = \int_{-1}^1 f(x)dx = A_1 f(-1) + A_2 f(-\frac{1}{3}) + A_3 f(\frac{1}{3})$.
Take $f(x)$ as $1, x, x^2$, then we get the relation below,

$$\begin{cases} 2 = A_1 + A_2 + A_3 \\ 0 = -A_1 - \dfrac{1}{3} A_2 + \dfrac{1}{3} A_3 \\ \dfrac{2}{3} = A_1 + \dfrac{1}{9} A_2 + \dfrac{1}{9} A_3 \end{cases} \tag{1}$$

From the equation set, we can get that

$$\begin{cases} A_1 = \dfrac{1}{2} \\ A_2 = 0 \\ A_3 = \dfrac{3}{2} \end{cases} \tag{2}$$

Problem 4

$I_2(f) = \frac{1}{3}(f(-1) + 2f(x_1) + 3f(x_2))$.
Take $f(x)$ as $1, x, x^2$, then we get the relation below,

$$\begin{cases} 2 = \dfrac{1}{3} \times 6 \\ 0 = \dfrac{1}{3}(-1 + 2x_1 + 3x_2) \\ \dfrac{2}{3} = \dfrac{1}{3}(1 + 2x_1^2 + 3x_2^2) \end{cases} \tag{3}$$

From the equation set, we can get that

$$\begin{cases} x_1 = \dfrac{1 \pm \sqrt{6}}{5} \\ x_2 = \dfrac{3 \mp 2\sqrt{6}}{15} \end{cases} \tag{4}$$

Problem 5

Firstly, we define two functions.

```
function ans= trap(f,a,b)
        ans=1/2*(b-a)*(f(a)+f(b));
end
```

```
function ans= simpson(f,a,b)
        ans=1/6*(b-a)*(f(a)+4*f((a+b)/2)+f(b));
end
```

(a)

```
format long e
syms x1;
f1=exp(x1);
f=inline('exp(x)','x');
ans1=trap(f,1.1,1.8);
ans2=simpson(f,1.1,1.8);
true=int(f1,x1,1.1,1.8);
disp(ans1);
disp(ans2);
disp(vpa(true));
disp(vpa(true-ans1));
disp(vpa(true-ans2));
```

Trapezoid:3.168834720925783

Simpson:3.045731680720709

Eaxct value:3.045481440466512

(b)

```
syms x2;
f2=(sin(x2))^2;
f=inline('(sin(x))^2','x');
ans1=trap(f,0,pi/2);
ans2=simpson(f,0,pi/2);
true=int(f2,x2,0,pi/2);
disp(ans1);
disp(ans2);
disp(vpa(true));
disp(vpa(true-ans1));
disp(vpa(true-ans2));
```

Trapezoid:$7.853981633974483 \times 10^{-1}$

Simpson:$7.853981633974481 \times 10^{-1}$

Eaxct value:$7.85398163397448309 \times 10^{-1}$

(a)

```
format long e
```

```
syms x3;
f3=exp(-(x3)^2);
f=inline('exp(-x^2)','x');
ans1=trap(f,1,2);
ans2=simpson(f,1,2);
disp(ans1);
disp(ans2);
error1=1/12*max(abs(subs(diff(f3,x3,2),x3,[1:0.01:2])));
error2=1/2880*max(abs(subs(diff(f3,x3,4),x3,[1:0.01:2])));
fprintf('%e\n',error1);
fprintf('\n');
fprintf('%e\n',error2);
```

Trapezoid:$1.930975400300882 \times 10^{-1}$

Simpson:$1.346319963846057 \times 10^{-1}$

Trapezoid-error:7.437168×10^{-2}

Simpson-error:2.554718×10^{-3}

(b)

```
syms x4;
f4=sin(x4)/(x4);
f=inline('sin(x)/x','x');
ans1=pi/2*(1+f(pi/2))/2
ans2=pi/12*(1+4*f(pi/4)+f(pi/2))
disp(ans1);
disp(ans2);
error1=1/12*(pi/2)^3*max(abs(subs(diff(f4,x4,2),x4,[0.01:0.01:pi/2])));
error2=1/2880*(pi/2)^5*max(abs(subs(diff(f4,x4,4),x4,[0.01:0.01:pi/2])));
fprintf('%e\n',error1);
fprintf('\n');
fprintf('%e\n',error2);
```

Trapezoid:1.285398163397448

Simpson:1.371275096047879

Trapezoid-error:1.076575×10^{-1}

Simpson-error:6.640815×10^{-4}