Scientific Computing: HW12 Solution

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Problem 1

Let $f(x) = 3x^2 - e^x$, then $f(x)$ is continuous on R. $f(0) = -1 < 0$, $f(1) = 3 - e > 0$, so $3x^2 - e^x = 0$ has root on $[0, 1]$, so we can take $a = 0$ and $b = 1$ to satisfy the problem.
From $f(x) = 0$ we get $x = \sqrt{\frac{1}{3}e^x} = \varphi(x)$, so we do iterative $x_{n+1} = \varphi(x_n)$, i.e.$x_{n+1} = \sqrt{\frac{1}{3}e^{x_n}}$. Take the derivative of $\varphi(x)$, we get $\varphi'(x) = \frac{1}{2}\sqrt{\frac{1}{3}e^x}$. It's obvious that $\varphi'(x)$ is increasing on $[0, 1]$, so $|\varphi'(x)| < \max |\varphi'(0)|, |\varphi'(1)| < 1$, therefore the iterative method converges.

Problem 2

Assume that $x_n = \sqrt{2 + \sqrt{2 + \cdots + \sqrt{2}}}$, than we know that $x_{n+1} = \sqrt{2 + x_n}$ and $x_1 = \sqrt{2}$. Take the derivative of $\sqrt{2 + x}$, we can easily know that $|(\sqrt{2 + x})'| = |\frac{1}{2\sqrt{2+x}}| < 1$ when $x > 0$, so the iteration coverges.
Assume that $\lim\limits_{n\to\infty} = \xi$, then we have $\xi = \sqrt{2 + \xi}$, so $\xi = 2$, i.e. $\lim\limits_{n\to\infty} \sqrt{2 + \sqrt{2 + \cdots + \sqrt{2}}} = 2$.

Problem 3

First, we write the function program of bisection method.

```
function xc=bisection(f,a,b,eps)
        while (b-a)/2>eps
                c=(a+b)/2;
                if f(c)==0
                        break
                end
                if f(a)*f(c)<0
                        b=c;
                else
                        a=c;
                end
        end
        xc=(a+b)/2;
end
```

Then we substitute the functional equation to solve the answer.

```
format long e;
syms x;
f=inline("x-2^(-x)","x");
g=inline("exp(x)-x^2+3*x-2","x");
xc1=bisection(f,0,1,10^(-10));
xc2=bisection(g,0,1,10^(-10));
fprintf('%.11f\n',xc1);
fprintf('%.11f\n',xc2);
```

Run the program and get the following solution:

(1) $x = 0.6411857445$.

(2) $x = 0.2575302855$.

Problem 4

First, write the program according to the title.

```
format long e;
x=1.5;
eps=1e-12;
N=10000;
cnt=0;
while cnt<N
        x1=func(x);
        cnt=cnt+1;
        if abs(x1-x)<eps
                break;
        end
        x=x1;
end
fprintf(' %d ,%.11f\n',cnt,x1);
```

The iterative function program used in the above program is as follows.

```
function x = func(x)
        x=1+1/(x*x);
        %x=(1+x*x)^(1/3);
        %x=(1/(1-x))^(1/2);
end
```

According to the related mathematical theories and the results of program operation, we have the following conclusions.

(a) Converges at $x_0 = 1.5$, a total of 56 iterations, root is $x = 1.4655712319$.

(b) Converges at $x_0 = 1.5$, a total of 32 iterations, root is $x = 1.4655712319$.

(c) Does not converge at $x_0 = 1.5$.