Scientific Computing: HW2 Solution

March 27, 2021

Professor Lai

Xu Shengze 3190102721

Problem 1

According to Lagrange interpolation formula,we have $l_0(x) = \frac{(x-2)(x-4)}{3}$,$l_1(x) = -\frac{(x-1)(x-4)}{2}$,$l_2(x) = \frac{(x-1)(x-2)}{6}$.
Thus,we get $\phi_2(x) = y_0l_0(x) + y_1l_1(x) + y_2l_2(x) = \frac{x^2-3x+8}{6} = f(x)$.
Finally,we substitute $x = 1.5$,$f(1.5) = 0.95833$

Problem 2

(a) Take $f(x)$ as 1, $f(x_1) = f(x_2) = ... = f(x_n) = 1$, the degree of interpolation function is n, then $\phi_n(x) = f(x) = 1$, therefore we have $\sum_{i=0}^{n} l_i(x) = 1$.

(b) Take $f(x)$ as x^j, $f(x_i) = x_i^j$, because the degree of $f(x) \leq n$, then $\sum_{i=0}^{n} x_i^j l_i(x) = x^j, j = 1, 2, ..., n$.

(c) Take $f(x)$ as 0, and regard $f(x)$ as a polynomial on $(x_i - x)^n$, then $\sum_{i=0}^{n}(x_i - x)^j l_i(x) = 0$.

(d) When $j = 0$, it's easy to find that $\sum_{i=0}^{n} l_i(0)x^j = \sum_{i=0}^{n} l_i(0) = 1$.

When $j = 1, 2, ..., n$, according to (b), we find that $\sum_{i=0}^{n} l_i(0)x^j = 0$.

When $j = n + 1$, consider the function $g(x) = f(x) - \phi_n(x)$, $g(x_i) = 0, i = 0, 1, 2, ..., n$, because the degree of $g(x)$ is $n + 1$, so we can write $g(x)$ as $(x - x_0)(x - x_1)...(x - x_n)$.Therefore, take $x = 0$, we easily get $\sum_{i=0}^{n} l_i(0)x^j = (-1)^n x_0x_1...x_n$.

Problem 3

We have already konwn that $|R(x)| \leq \frac{h^2}{8} max|I_0^{''}(x)|$.
$I_0^{'}(x) = -\frac{1}{\pi}\int_0^{\pi} \sin t \sin(x \sin t)dt$, $I_0^{''}(x) = -\frac{1}{\pi}\int_0^{\pi} \sin^2 t \cos(x \sin t)dt$
We havr $I_0^{''}(x) \leq \frac{1}{\pi}\int_0^{\pi} \sin^2 tdt = \frac{1}{2}$,therefore, we just need $\frac{h^2}{16} \leq 10^{-6}$, so $h \leq 4 \times 10^{-3}$.

Problem 4

$\phi_k(x) = f(x_0) + (x - x_0)f[x_0, x_1] + (x - x_0)(x - x_1)f[x_0, x_1, x_2] + ... + (x - x_0)...(x - x_{k-1})f[x_0, ..., x_k]$.
When $k > n$, the largest degree of $\phi_k(x)$ is n, so $f[x_0, ..., x_k]$ should be 0.

Problem 5

(1) $P_{10}(-0.56) = 1.3683$, $P_{10}(0.15) = 1.0228$, $P_{10}(0.98) = 2.6128$.

```
 function p= lagrange(x,X,Y)
L1=length(X);
L2=length(Y);

for i=1:L1
        a=x(i);
        sum=0.0;
        for j=1:L2
                b=1.0;
                for k=1:L2
                        if k~=j
                        b=b*(a-X(k))/(X(j)-X(k));
```

```
                        end
                    end
                    sum=b*Y(j)+sum;
            end
            p(i)=sum;
end
```

```
X=-1:0.2:1;
Y=exp(X.^2);
x=[-0.56 0.15 0.98];
truevalue=exp(x.^2);
caculatedvalue=lagrange(x,X,Y);
disp(caculatedvalue);
```

(2) The table following contains the answers:

	absolute error	bounds of absolute error	bounds of relative error
$P_{10}(-0.56)$	2.7438×10^{-7}	$\frac{1}{2} \times 10^{-11}$	$\frac{1}{4} \times 10^{-4}$
$P_{10}(0.15)$	2.1209×10^{-8}	$\frac{1}{2} \times 10^{-12}$	$\frac{1}{4} \times 10^{-4}$
$P_{10}(0.98)$	1.1870×10^{-5}	$\frac{1}{2} \times 10^{-9}$	$\frac{1}{4} \times 10^{-4}$

(3) The first figure is the curve on $[-1, 1]$.

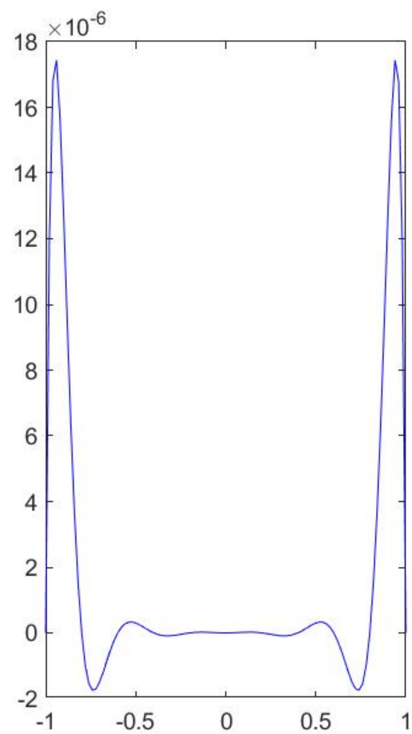

Figure 1: [-1,-1]

The second figure is the curve on $[-2, 2]$.

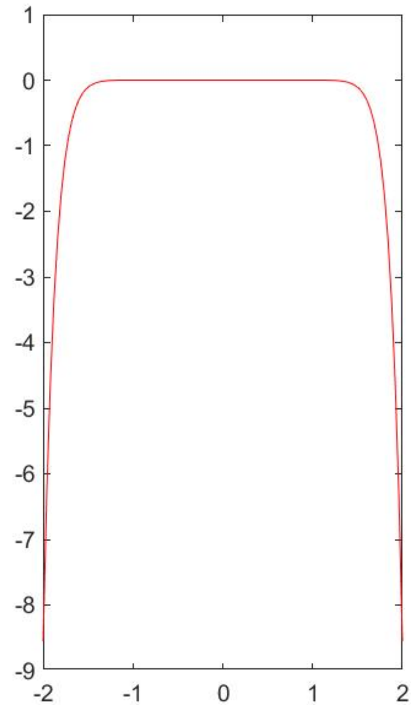

Figure 2: [-2,2]