Scientific Computing: HW1 Solution

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Problem 1

(1) bounds of absolute error:$\frac{1}{2}\times 10^{-4}$

bounds of relative error: $\frac{1}{8}\times 10^{-2}$

number of significant figures:3

(2) bounds of absolute error:$\frac{1}{2}\times 10^{-4}$

bounds of relative error: $\frac{1}{8}\times 10^{-3}$

number of significant figures:4

(3) bounds of absolute error:$\frac{1}{2}\times 10^{-2}$

bounds of relative error: $\frac{1}{6}\times 10^{-3}$

number of significant figures:4

(4) bounds of absolute error: $\frac{1}{2}$

bounds of relative error: $\frac{1}{8}\times 10^{-3}$

number of significant figures:4

Problem 2

(1) $\tan(\arctan(x+1)-\arctan(x)) = \frac{(x+1)-(x)}{1+x(x+1)} = \frac{1}{1+x+x^2}$

therefore,we have $\arctan(x+1)-\arctan(x) = \arctan(\frac{1}{1+x+x^2})$

(2) $\ln(x-\sqrt{x^2-1}) = \frac{1}{ln(x+\sqrt{x^2-1})}$

(3) $\frac{\sin x}{x-\sqrt{x^2-1}} = \sin x(x+\sqrt{x^2-1})$

Problem 3

Assume that the approximation is ϕ^*,hence $\phi^*-\phi=\delta$.
If $\cos\phi=0$, then $\phi=\frac{\pi}{2}$or$\frac{3\pi}{2}$, now $\frac{\cos(\phi^*)-\cos(\phi)}{\cos(\phi)}$ approaches to be infinite.
If $\cos\phi\neq 0$, then $\frac{\cos(\phi^*)-\cos(\phi)}{\cos(\phi)} = \frac{\cos(\phi+\delta)-\cos(\phi)}{\cos(\phi)} = \frac{\cos\delta\cos\phi-\sin\delta\sin\phi}{\cos\phi} - 1 = \cos\delta - 1 - \sin\delta\tan\phi$
Another way, relative error is $\frac{d(\cos\phi)}{\cos\phi} = \frac{(\cos\phi)'}{\cos\phi}d\phi = -\delta\tan\phi$

Problem 4

The result is $6.127{\times}10^{-13}$

```c
#include <stdio.h>
#include <math.h>

int main(){
  double a,b;
  double result;
  scanf("%lf %lf",&a,&b);
```

```c
  result=b*b/(-a+sqrt(a*a+b*b));
  printf("%.3e",result);
  return 0;
}
```

Problem 5

The result is $x_1 = -2.824 \times 10^{11}, x_2 = 1.062 \times 10^{-11}$

```c
#include <stdio.h>
#include <math.h>

int main(){
  double a,b,c;
  double x1,x2;
  scanf("%lf %lf %lf",&a,&b,&c);
  if(b>0){
    x1=(-b-sqrt(b*b-4*a*c))/(2*a);
    x2=c/(a*x1);
  }
  else{
    x1=(-b+sqrt(b*b-4*a*c))/(2*a);
    x2=c/(a*x1);
  }//here we ignore the condition that b==0
  printf("%.3e %.3e",x1,x2);
  return 0;
}
```

Problem 6

The result is -0.00050024507964763210.
The equivalent expression is $P(x) = \frac{1-x^{100}}{1+x}$.
According to the expression, the result is -0.00050024507964746079.
The error is $-1.713039 \times 10^{-16}$.

```c
#include <stdio.h>
double horner(int coef[],int n, double x);

int main(){
  double result;
  int i=0;
  int a[100]={0};
  for(i=0;i<100;i++){
        if(i%2==0){
          a[i]=1;
        }
        else{
```

```c
			a[i]=-1;
		}
	}
	result=horner(a,100,1.00001);
	printf("%.20lf",result);
	return 0;
}

double horner(int coef[],int n,double x){
	int i;
	double result;
	result=coef[n-1];
	for(i=1;i<=n-1;i++){
			result=result*x+coef[n-1-i];
	}
	return result;
}
```