Scientific Computing: HW4 Solution

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Problem 1

Assume that x_1,x_2 are two least square solutions of $Ax = b$, therefore we have $A^T Ax_1 = A^T b = A^T Ax_2$. From the equation above, we have $A^T A(x_1 - x_2) = 0$, so $(x_1 - x_2)^T A^T A(x_1 - x_2) = 0$, i.e.$(A(x_1 - x_2))^T (A(x_1 - x_2)) = 0$, then $||A(x_1 - x_2)|| = 0$, so $A(x_1 - x_2) = 0$.
Therefore, we have $Ax_1 = Ax_2$.

Problem 2

$\sum_{i=1}^r ||Ax - b_i||^2 = \sum_{i=1}^r (Ax - b_i)^T (Ax - b_i)$
$= \sum_{i=1}^r (x^T A^T Ax + b_i^T b_i - x^T A^T b_i - b_i^T Ax)$
$= r(||Ax||^2 - (Ax)^T (\frac{1}{r} \sum_{i=1}^r b_i) - (\frac{1}{r} \sum_{i=1}^r b_i)^T (Ax)) + \sum_{i=1}^r ||b_i||^2$
$= r||Ax - \frac{1}{r} \sum_{i=1}^r b_i||^2 + \sum_{i=1}^r ||b_i||^2 - \frac{1}{r} || \sum_{i=1}^r b_i||^2$
Therefore, when $\sum_{i=1}^r ||Ax - b_i||^2$ take minimum, x is also the least square solution of $Ax = \frac{1}{r} \sum_{i=1}^r b_i$, vice versa.

Problem 3

Assume that the $m \times m$ coefficient matrix is X, where $X_{i,j} = x_i^{j-1}, i, j = 1, 2, ..., m$.
$\alpha = [a_0, a_1, ..., a_{m-1}]^T$, $y = [y_1, y_2, ..., y_m]^T$.
It's easy to find that $|X| \neq 0$, so X is invertible and $X\alpha = y$ has the unique solution $\alpha = X^{-1} y$.
For Lagrange interpolation polynomial $f(x)$, its degree is $m - 1$.
Let $g(x) = p(x) - f(x)$, so $g(x_i) = p(x_i) - f(x_i) = 0$. It's obvious that the degree of $g(x) \leq m - 1$. Meanwhile, $g(x)$ has m roots, therefore $g(x)$ must be 0, further, we have $p(x) = f(x)$.

Problem 4

```
format rat
x=[-3;-2;-1;0;1;2;3];
y=[4;2;3;0;-1;-2;-5];
X=ones(7,3);
X(:,2)=x;
X(:,3)=x.^2;
a=(X'*X)\(X'*y);
disp(a);
```

According to the program, we have $a_0 = \frac{2}{3} = 0.667, a_1 = -\frac{39}{28} = -1.393, a_2 = -\frac{11}{84} = -0.131$.

Problem 5

```
format rat
x=[1.02;0.95;0.87;0.77;0.67;0.56;0.44;0.30;0.16;0.01];
y=[0.39;0.32;0.27;0.22;0.18;0.15;0.13;0.12;0.13;0.15];
X=ones(10,5);
X(:,1)=y.^2;
X(:,2)=x.*y;
```

```
X(:,3)=x;
X(:,4)=y;
a=(X'*X)\(X'*(x.^2));
disp(a);
```

According to the program, we have $a = -\frac{651}{247} = -2.636, b = \frac{3511}{24442} = 0.144, c = \frac{343}{622} = 0.551, d = \frac{1995}{619} = 3.223, e = -\frac{1216}{2809} = -0.433$.