Lecture 9-Homework

7. 设$A=(A_1, A_2, A_3, A_4, A_5, A_6)$，其中

$$A_1=\{a,b,c\},\quad A_2=\{a,b,c,d,e\},\quad A_3=\{a,b\},$$
$$A_4=\{b,c\},\quad A_5=\{a\},\quad A_6=\{a,c,e\}$$

族A有 SDR 吗？如果没有，族中有 SDR 的集合的最大个数是多少？

Answer: No,The family A doesn't have an SDR. The largest number of sets in the family with an SDR is 5.

With n=6, k=1, $\min_{i=1,2,\dots,6}|A_i|+6-1=6$;

With n=6, k=2, $\min_{i1,i2=1,2,\dots,6}|A_{i1}\cup A_{i2}|+6-2=6$

With n=6, k=3, $\min_{i1,i2,i3=1,2,\dots,6}|A_{i1}\cup A_{i2}\cup A_{i3}|+6-3=6$

With n=6, k=4, $\min_{i1,i2,i3,i4=1,2,\dots,6}|A_{i1}\cup A_{i2}\cup A_{i3}\cup A_{i4}|+6-4=5$

With n=6, k=5, $\min_{i1,i2,i3,i4,i5=1,2,\dots,6}|A_{i1}\cup A_{i2}\cup A_{i3}\cup A_{i4}\cup A_{i5}|+6-5=5$

With n=6, k=6, $\min_{i1,i2,i3,i4,i5,i6=1,2,\dots,6}|A_{i1}\cup A_{i2}\cup A_{i3}\cup A_{i4}\cup A_{i5}\cup A_{i6}|+6-6=5$

12. 考虑带有禁止落子位置的棋盘，它具有如下性质：如果一个方格是禁止落子位置，那么这个方格所在行中它右边的每一个方格都是禁止位置，而且它所在列中它下方的每一个方格也是禁止位置。证明该棋盘有多米诺骨牌的完美覆盖当且仅当允许落子的白方格的个数等于允许落子的黑方格的个数。

Answer:

→ : If the number of white squares is not equal to the number of black squares ,there is not to be a tiling.Because there are at least two edges have a common vertex.

← : Suppose the number of white squares is equal to the number of black squares .In order to get the tiling,we need take two consecutive squares,either in the same row or in the same column.We remove them,then the board become another satisfied board.We use recursive method to do next step up to empty board.So the problem is prove that there are two consecutive square to get next board.As board definition,each step we take the margin of forbidden position.For example,we can choose w2,B3 to remove.

X	X	X	X
X	W1	B1	W2
X	B2	W3	B3
X	W4	B4	X

19. 使用延迟认可算法求得下面优先排名矩阵的女士最优和男士最优的稳定完美婚姻。

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1,3 & 2,3 & 3,2 & 4,3 \\ 1,4 & 4,1 & 3,3 & 2,2 \\ 2,2 & 1,4 & 3,4 & 4,1 \\ 4,1 & 2,2 & 3,1 & 1,4 \end{bmatrix} \end{array}$$

对于这个给定的优先排名矩阵推断只存在一个稳定的完美婚姻。

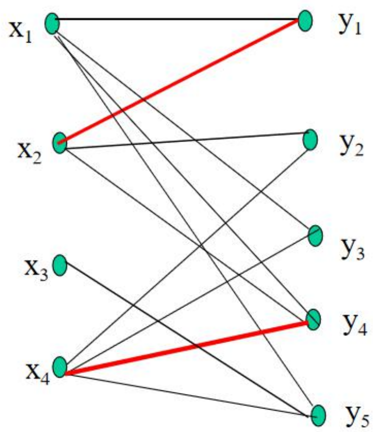
Answer:

Solutions 1: We apply the deferred acceptance to the preferential ranking matrix, the
result of the algorithm are (here we use women–optimal):
1) A->a, B->a, C->b, D->d; a rejects B.
2) B->d; d rejects D.
3) D->b; b rejects C.
4) C->a; a rejects A.
5) A->b; b rejects A.
6) A->c; there are no rejections.
Hence, (A,c), (B,d),(C,a),(D,b).

Solutions 2: We apply the deferred acceptance to the preferential ranking matrix, the result of the algorithm are (here we use men–optimal):
1) a->D, b->B, c->D, d->C; D rejects a.
2) a->C; C rejects d.
3) d->B; B rejects b.
7) b->D; D rejects c.
8) c->A; there are no rejections.
Hence, (A,c), (B,d),(C,a),(D,b).
We can conclude that, for the given preferential ranking matrix, there is only one stable complete marriage.

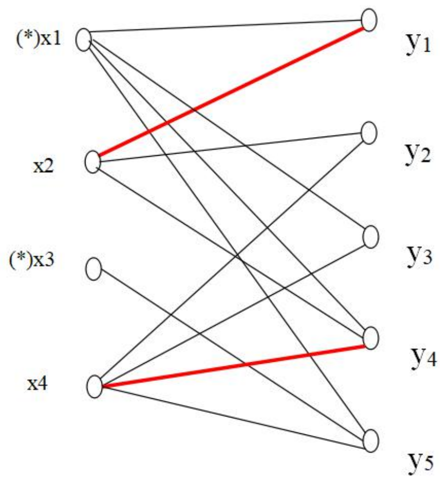
Assignments (cont'd)

- **Determine the max-matching and the min-cover of the right graph by applying the matching algorithm. We choose the red edges and obtain a matching M^1.**
- **Find a minimum edge cover for the right graph.**

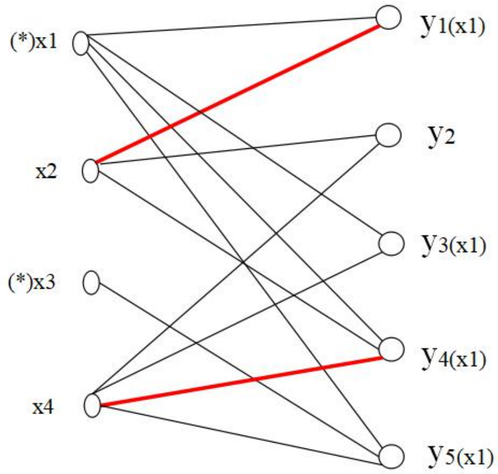


Answer:
1. the max-matching and the min-cover:

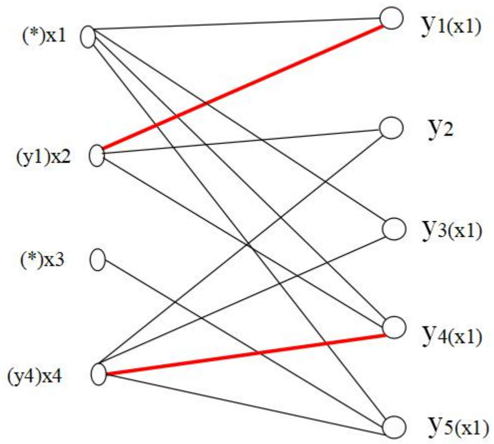
First, we get that M1 = {(x2,y1),(x4,y4)} of size 2 and U={x1,x3}
1) The vertices x1,x3 are labeled (*).



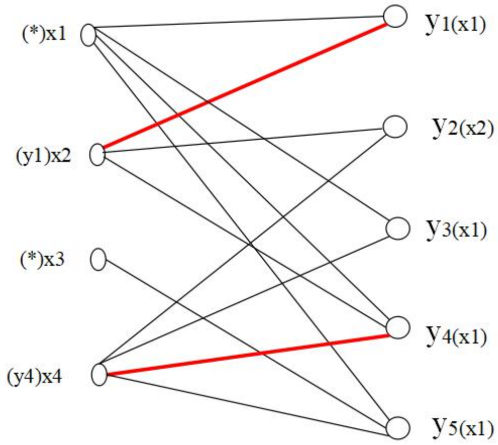
2) Scan the vertices in U in turn, and label y1, y3, y4, y5 with (x1), and since y5 has labeled by x1, so no vertices label (x3).



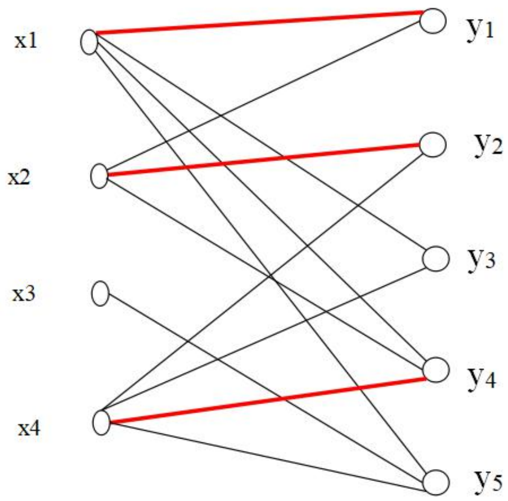
3) Scan the vertices y1,y3, y4,y5 labeled with (x1), and label x2 with (y1), label x4 with (y4).



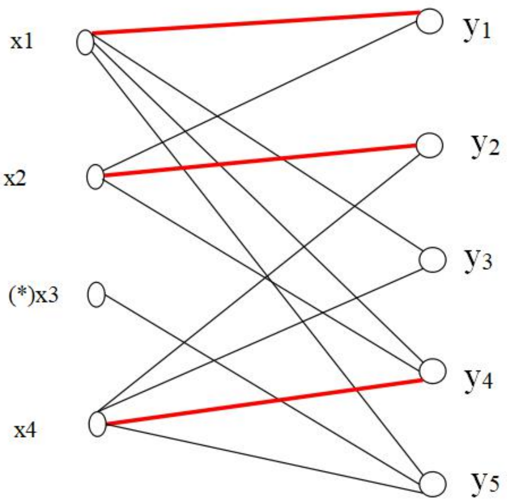
4) Scan the vertices x2 and x4, and label y2 with (x2).



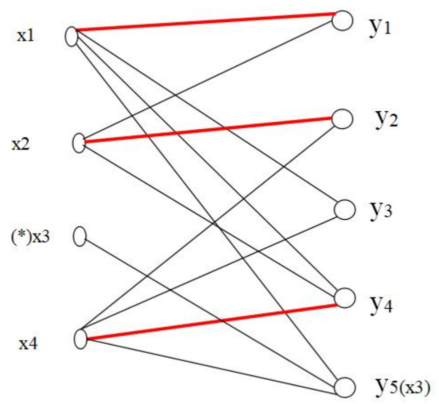
5) Scan the vertices y2, and find that no new labels are possible. So, we get a breakthrough. We find the M1-augmenting path $r_1=y_2x_2y_1x_1$ using the labels as a guide. Then M2 = {(x1,y1),(x2,y2),(x4,y4)} and U2={x3}



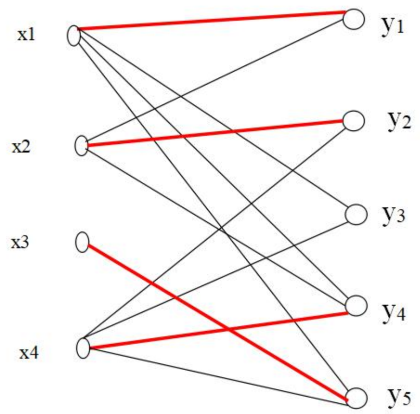
6)The vertex x3 is labeled (*).



7) Scan the vertex x3, and label y5 with (x3).



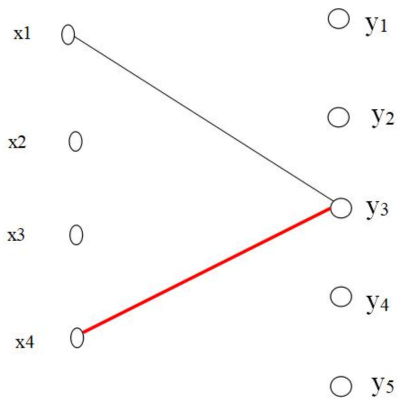
8) Scan the vertex y5, and find that no new labels are possible. So, we get a breakthrough. We find the M2-augmenting path r2=y_5x_3 using the labels as a guide. Then M3 = {(x3,y5),(x1,y1),(x2,y2),(x4,y4)} is a matching of four edges.



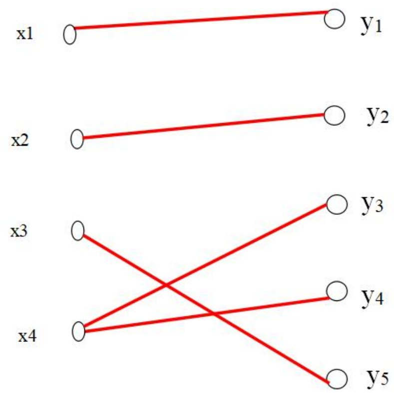
Here, we get that the max-matching M = {(x3,y5),(x1,y1),(x2,y2),(x4,y4)}, and the min-cover = {x1,x2,x4,y5}

2.Find a minimum edge cover for the right graph.

We get a max-matching M={(x3,y5),(x1,y1),(x2,y2),(x4,y4)}, but we can find the vertex y3 is still uncovered. And we should construct a subgraph composed of edges incident to the vertex y3, Then we find the max-matching of the subgraph, and add it to the max-matching of M, then we get a minimum edge cover. The max-matching of the subgraph is {(x4,y3)}, so we get the minimum edge cover for the graph is M={(x1,y1),(x2,y2),(x3,y5),(x4,y3),(x4,y4)}



x1
x2
x3
x4
y1
y2
y3
y4
y5



x1
x2
x3
x4
y1
y2
y3
y4
y5