Modeling of Complex Networks

Lecture 8: Human Opinion Dynamics

S8101003Q-01(Sem A, Fall 2021)

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Introduction

- Human opinion dynamics refer to a kind of human behavioral dynamics
- Study the behaviors, actions and interactions of human individuals, groups, communities, and various social organizations
- Concern with opinions formation, evolution, separation and consensus, as well as other related issues such as effects and impacts of media and government policies on opinion-based decision making and behavioral consequences

Introduction

- Human opinion dynamics are driven and influenced by many important factors like cultures, economics, politics and religions
- Human opinion dynamics involve many invisible and immeasurable human emotional and psychological factors
- Human opinion dynamics create order from disorder, and preserves order after it is in place
- Human opinion dynamics theory concerns questions like: how can an initially disordered opinion distribution eventually become orderly through interactions among the large number of interrelated agents?

Social Network Topologies

- Social networks are generally assortative
- Technological networks are disassortative

- Social networks are hierarchical networks
- Social networks are scale-free networks
- Social networks are small-world networks

Sociodynamics

Master Equation of sociodynamics:

$$\frac{dP(\mathrm{x,t})}{dt} = \sum_{y \in S} \left\{ W_{y,x} P(\mathrm{y,t}) - \mathrm{W}_{x,y} P(x,\mathrm{t}) \right\}$$ (gain-loss)

Here, S is the state space of all possible states x and y, which only take discrete-values x_k and y_k, and $P(x,t)$ is the probability that the system occupies state x at time t, and $\boldsymbol{W_{x,y}}$ represents the transition rate from state x to state y in the state space S

This is a gain-loss equation, in which the first term on the right-hand side is the transition "gain" from the other state y to the present state x, while the second term is the opposite, namely the transition loss from x to y, both occurring in the state space S.

Helbing D. *Quantitative Sociodynamics*. Berlin: Springer, 1991

Sociodynamics

Actually, a closed-form sociodynamical system model with an analytic solution like the above simple master equation is rare in the study of realistic large-scale complex social networks. As an alternative, one of the most successful approaches based on sociodynamics is agent-based modeling. The idea of this agent-based model is to study how the global behaviors emerge from the interactions of local dynamics of connected agents within a population under a noisy and uncertain environment, how such global behaviors may be managed and controlled by tuning local characteristic parameters of individual agents using local information

$$\bar{x}(\mathrm{t})=\sum_{x\in S} xP(x,t)$$

Approximate solution

$$P(x,t)\approx \bar{P}(x,t)\approx \left\langle P(\bar{x}(\mathrm{t}),\mathrm{t})\right\rangle_t$$

Helbing D. *Quantitative Sociodynamics*. Berlin: Springer, 1991

Social Opinion Formation

To describe human opinion formation, change, split, evolution, opposition or consensus, mathematical model is useful:

Every opinion is a variable

In discrete setting: binary-valued $s = \pm 1$

agree or disagree, yes or no, support or against

In continuous setting: real-valued $s \in [0,1]$

strongly agree, somewhat oppose, …, not sure

Voter Model

Assume: Every agent (voter) has opinion +1 (support) or -1 (against)

$$s = \pm 1$$

Initialization: Randomly distributed disordered opinions of +1 and -1

At each step: Randomly select an agent i and one of its neighbors, j.

If these two agents have same opinion then no change; if different then

Direct voter model: $s_j = s_i$

Reverse voter model: $s_i = s_j$

Objective: **Find out whether and how a global consensus of opinions will gradually emerge from a process of such random local interactions, leading to all +1 or all -1 in the end**

Voting is an important social activity in human life, in which voters' opinions determine the outcome of an election or a decision. Voters' behavioral dynamics were first studied in

Clifford P, Sudbury A. *Biometrika* 1973, 60: 581-588

Voter Model

Let $S=\{S_i\}$ be the set of all possible states of voters, $i = 1,2\ldots, N$.
Rate of transition in changing opinions for agent i is

$$W_i(S)=W(s_i \to -s_i)=\frac{d}{4}\left(1-\frac{1}{2d}s_i\sum_{j\in B(i)} s_j\right)$$

$B(i)$ – neighborhood of agent i and d – dimension of the lattice

Master Equation →probability distribution function satisfies:

$$\frac{dP(S,t)}{dt}=\sum_i\left[W_i(S-\{i\})P(S-\{i\},t)-W_i(S)P(S,t)\right]$$

where S-$\{i\}$ means S excluding i who had changed its original opinion. Further analysis shows that :

$d = 1,2$ → converging to one opinion;

$d > 2$ → converging to communities(different opinions co-exist in the form of clusters on the lattice network)

Galam Majority-Rule Model

A fully-connected network of N agents

Initialization: a fraction $p+$ of agents have opinion +1

a fraction $p-$ of agents have opinion -1

At each step: A group of r agents are randomly

selected for discussion about their opinions.

Majority Rule: In the discussion group, majority stay unchanged,while minority change opinions to the opposite

"Majority" - can be a number >1/2 , or

a number larger than a threshold value

Galam Majority-Rule Model

Initialization: p_+^0 have opinion +1, the average opinion of the whole population is given by

$$s = \frac{1}{N}\sum_{k=1}^{N} s_k = p_+ - p_-$$

Critical threshold: p_c

Result: if r is odd and $p_c = 1/2$, or

if r is even and $P_c < 1/2$

then opinion dynamics converge to the initially dominant one

Latané Social Impact Theory

Social Impact Theory:

- addresses the concerns of how a person feels his/her peers and how he/she influences the others as a return
- predicts that, as the strength and immediacy increase within a group of agents, conformity of every of its members will also increase
- explains that, the more important a group is and the more involved an individual into the group, the more likely the individual will conform to the normative pressures of the group
- shows that, as the size of a group increases, an individual has less effect on the others and received less influence from the addition of new members if the group is large

Latané B. The psychology of social impact, American Psychologist 1981, 36: 343-356

Latané Social Impact Theory

Latané model:

A networked population of N agents

Agent i has opinion $v_i = \pm 1$ and a real-valued strength on its neighbors:

persuasiveness p_i (capability to persuade other people to change their opinions)

and supportiveness s_i (capability to convince other people to keep their viewpoints)

Total impact of agent i on other agents: $I_i = [\sum_{j=1}^{N} \frac{p_j}{d_{ij}^{\alpha}}(1 - v_i v_j)] - [\sum_{j=1}^{N} \frac{s_j}{d_{ij}^{\alpha}}(1 + v_i v_j)]$

Dynamics:

$$v_i(t+1) = -\mathrm{sgn}[s_i(t) I_i(t) + h_i]$$

Agent i will change its opinion, namely, flip its sign from +1 to -1, or vice versus, if one kind of pressure overturns the other kinds of pressure.

d_{ij} -distance between; h_i -external disturbance (e.g., media); α - parameter

Sznajd Model

In the simplest chain or lattice network setting with N agents, assumes that a pair of neighboring agents determine the opinions of their nearest neighbors:

On a chain:

$$\begin{aligned} &if \quad s_i = s_{i+1} \quad then \quad s_{i-1} = s_i = s_{i+1} = s_{i+2} \\ &if \quad s_i \neq s_{i+1} \quad then \quad unchanged \end{aligned}$$

On a lattice:

$$if \quad s_{i,j} = s_{i+1,j} \quad then \quad \begin{aligned} &s_{i-1,j} = s_{i,j} = s_{i+1,j} = s_{i+2,j} \\ &= s_{i,j-1} = s_{i,j+1} = s_{i+1,j-1} = s_{i+1,j+1} \end{aligned}$$

$$if \quad s_{i,j} \neq s_{i+1,j} \quad then \quad unchanged$$

The process starts with a randomized initial configuration, in which every agent is randomly given an opinion. At each step, randomly pick an agent and perform operation above on it and also on its nearest neighbors.

Initialization: $s_i(0) = \pm 1 \quad or \quad \rho = \langle s_{i,j}(0) \rangle \quad \to \quad concensus$

The reactions will always converge to +1 (or -1).

Note: On a fully-connected network, the result can be complicated

F Slanina et al., European Phys. J. B 2003, 35: 279-288

Online Social Opinion Formation

To describe the effect of social activities on opinion formation within a community, suppose that the state $S_i = \pm 1$ of each agent i is influenced by its neighbors depending on the interactions, which is affected by some external perturbation input I (e.g., media or policies) in such a way that

$$h_i = -s_i(t) A_i \left(\sum_{j=1}^{k_i} A_j s_j(t) + I \right)$$

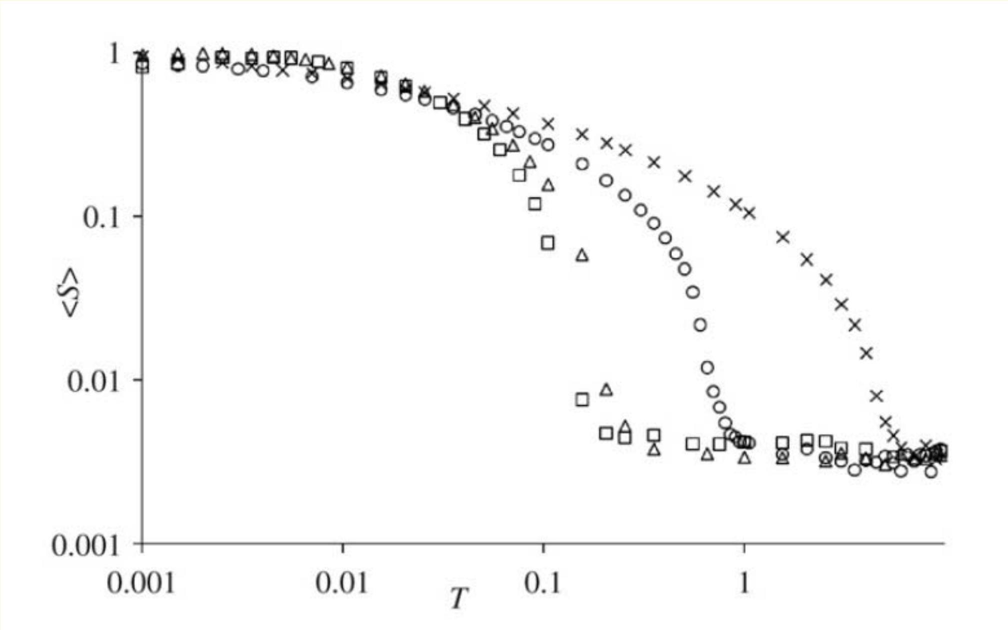
k_i - node degree; A_i- node activity

Opinion dynamics:

$$s_i(t+1) = \begin{cases} s_i(t) & \textit{with probability} \quad p = \dfrac{e^{-h_i/T}}{e^{-h_i/T} + e^{h_i/T}} \\ -s_i(t) & \textit{with probability} \quad 1-p \end{cases}$$

T – level of randomness of individual behaviors

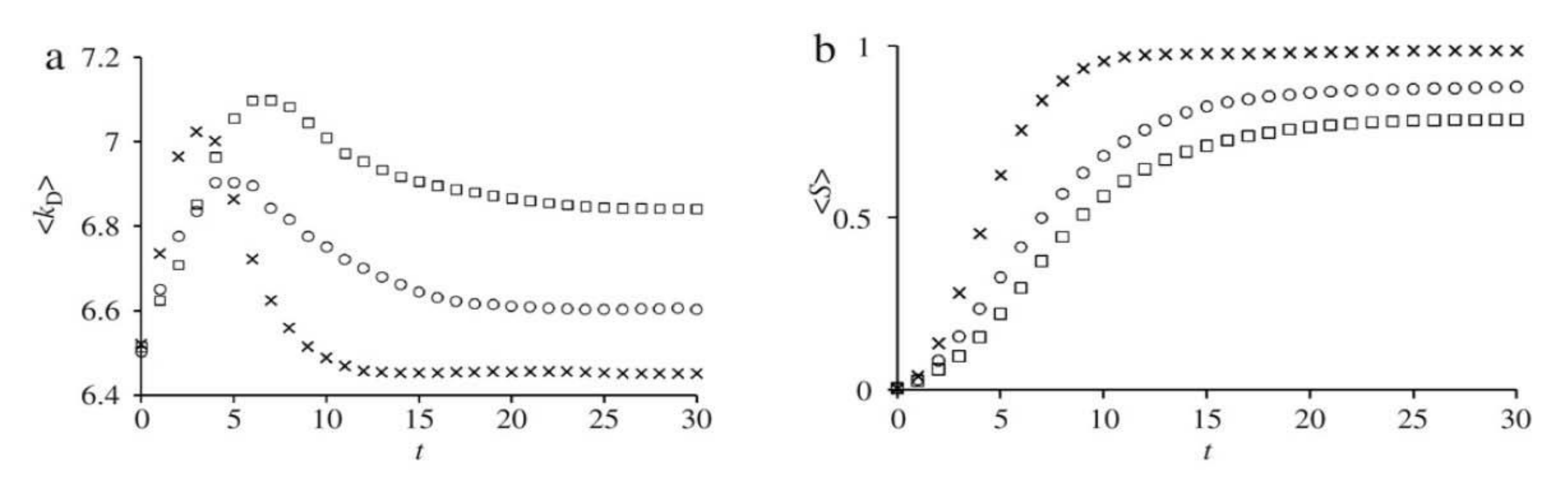
Online Social Opinion Formation



Relation between individual randomness *T* and synchronous initial condition $\langle s \rangle = \frac{1}{N}\sum_i s_i(0)=0$, *N* = 28011, and set $A_t = a \in [k^{0.1}, k^{1.0}]$ for all

It indicates that for lower T (individuals behave well) dominant opinion will always emerge in the community. However, when T>Tc, a certain threshold, dominant opinion disappears abruptly, and stronger correlation between degree and activity emerges for larger threshold Tc.

Online Social Opinion Formation



Time evolution on a scale-free social network: $P(k) \sim k^{-\alpha}$

The average degree $\langle k_D \rangle$ of individuals who dominate the common opinion and the average opinion $\langle s \rangle$

From (a), one can see that *$<k_D>$* first increases and then decreases, implying that hub nodes took a common state initially and then switches to affect more and more lower-degree nodes (less active individuals) gradually.

(b) Shows that the correlation between activity and degree also influences the time evolution of *$<s>$*: the larger the power-law exponent alpha, the faster the *$<s>$* increases, showing that the dominant opinion emerges faster in heterogeneous than in uniform distributions of activities.

Bounded Confidence Models

Recall: **To model human opinion formation, changes,**

split, evolution, opposition or consensus:

Opinion is a variable

In discrete setting: binary-valued $s = \pm 1$

agree or disagree, yes or no, support or against

In continuous setting: real-valued $s \in [0,1]$

strongly agree, somewhat oppose, …, not sure

All the opinion formation models discussed above considered binary values. The bounded confidence model is a representative model used to study the continuous opinion dynamics.

Deffuant Model

Consider a network of N agents, where agent i initially had opinion x_i

Opinion Dynamics:

$$\begin{cases} x_i(t+1) = x_i(t) + \alpha[x_j(t) - x_i(t)] \\ x_j(t+1) = x_j(t) + \alpha[x_i(t) - x_j(t)] \end{cases} \qquad \alpha \in [0, 0.5]$$

Here, tolerance $\varepsilon > 0$ such that $[x_i - \varepsilon,\ x_i + \varepsilon] \subseteq [0,1]$

If $\alpha = 0.5$ then any two in any cluster agents will converge to the average of their original opinions, giving raise to the formation of clusters with similar opinions within the network

G Deffuant et al. Adv. Complex Syst. 2000, 3: 87-98

Deffuant Model

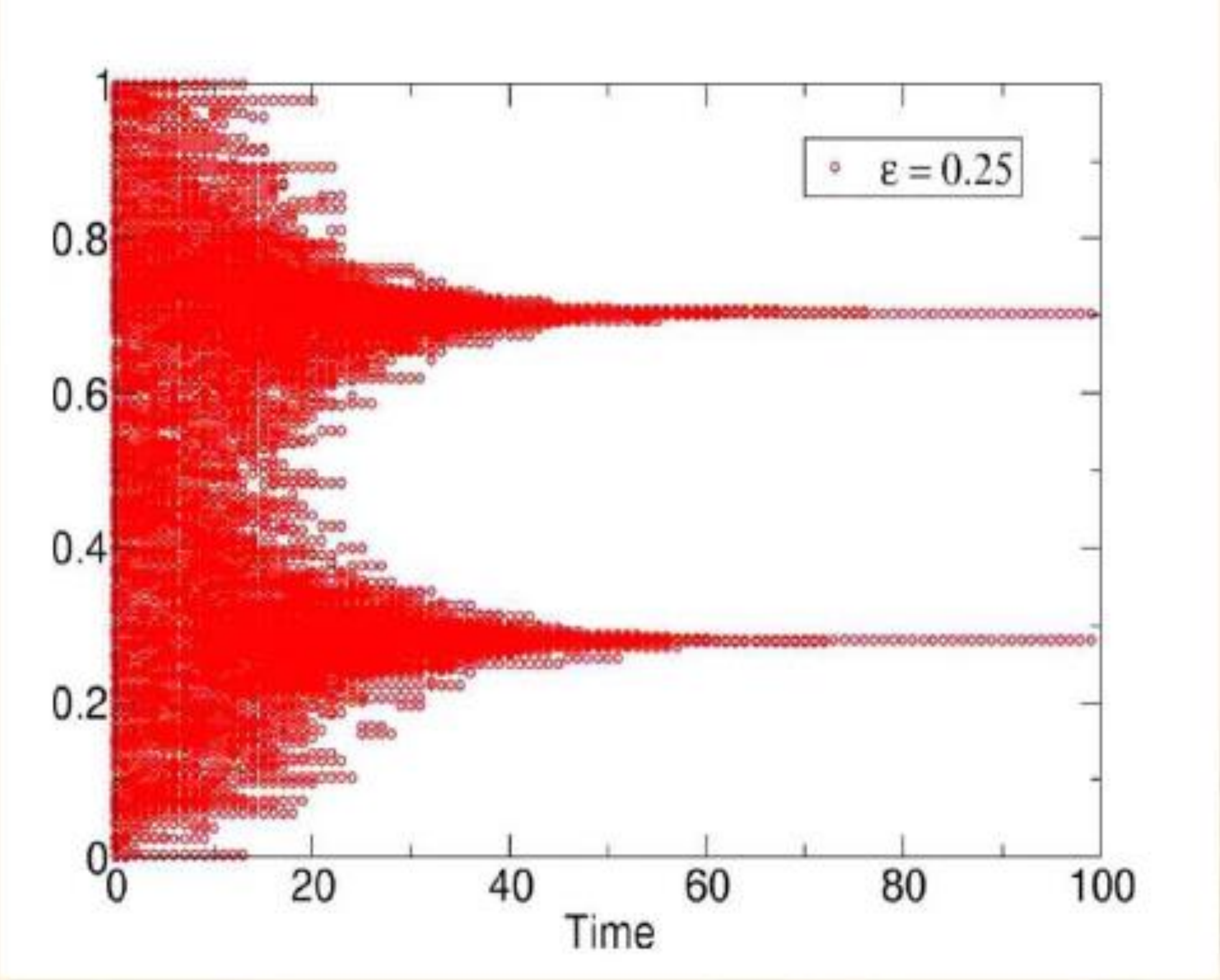
Example:

Fully-connected network

$N = 500$

$\alpha = 0.5$

Opinion converging to two clusters



Castellano C, Fortunato S, Loreto V. Rev. Modern Physics 2009, 81: 591-646

Summary

Human Opinion Dynamics

share many interesting common characteristics and features with

- **Epidemic spreading**
- **Cascading reactions**
- **Language formation**
- **Consensus**
- **Synchronization**
- **……**