复杂网络部分知识点复习

walk: a sequence of edges

Trail: edges distinct

Path: nodes distinct

Isomorphism 同构

if there is a one-one correspondence between the nodes of G1 and those of G2, with the property that the number of edges joining any two nodes of G1 is equal to the number of edges joining the two corresponding nodes of G2

homeomorphism 同胚 add nodes of degree 2

A graph is planar if and only if it contains no subgraphs homeomorphic to K_5 or $K_{3,3}$

Euler graph. close trail(edge distinct

methords: all nodes has even degree

Hamiltonian graph: closed trail(nodes distince)

methords: $k_i + k_j \geq N$ for all i, j in nodes set, and i != j

G with N nodes has K comonents.

N - K <= M <= 1/2(N-K)(N-K+1)

when K = 1, N - 1 <= M <= N(N-1)

disconnecting set: A set of edges, $E_0(G)$, after it is being removed, the graph G will become unconnected.

cut-set: minimum disconnecting set

corenest. K-core, remove k-1 degree

node betweenness:

$$\frac{B'(1)}{(N-1)(N-2)/2}$$

edge betweenness:

$$\frac{B'(1)}{N(N-1)/2}$$

homeogeneous 同质
hetergeneous　异质

Chinese postman. odd degree -> even degree, then add loop which is not larger than the half of all corresponding existing loops.

两点最短距离 dijstra, 最小生成树 kruskal

random network. probability P
WS small-world　k-ring-shape, probability p rewiring edges
NW small-world　k-ring-shape, probability p add edges
Kleinberg's Navigable Networks consider distance $P(u,v)=\beta d_{uv}^{-\alpha}$

BA Scale-Free Network Model
add new nodes:
Add 1 new node into the network:
This node is connected to m (m ≤m0) existing nodes simultaneously
Add new edges:
The way to add the m new edges into the network: Every existing node is to be chosen with probability ki/k
EBA
probability p. add one new node
Re-wiring:
With probability q, m (m ≤m0) edges are rewired
add new edges the same as BA

Casell:Sync region S =(α_1, ∞)
condition: $c\lambda_2 > \alpha_1$
The synchronizability is determined by the smallest nonzero eigenvalue λ_2 of its Laplacian matrix L.
The larger the λ_2 ,the small the c is needed, so the better or stronger the synchronizability of the network.

case III: Sync region S=(α_2, α_3)
condition: $0<\frac{\lambda_N}{\lambda_2}<c\alpha$
The synchronizability is characterized by the ratio λ_N/λ_2 of the largest and smallest nonzero eigenvalues of the Laplacian matrix L.
The smaller the ratio λ_N/λ_2 the smaller the c is needed, so the better the synchronizability of the network.

Synchronizability of small-world and scale-free networks will increase as the node betweenness decreases

homogeneous networks — there is a clear correlation between betweenness and synchronizability. Betweenness up, $|\frac{\lambda_N}{\lambda_2}|$ up.

heterogeneous networks — there is no clear correlation between betweenness and synchronizability

Lemma 1: For any given connected undirected graph G, all its nonzero eigenvalues increase monotonically with the number of added edges, i.e., by adding any edge e, one has $\lambda_i(G+e) \geq \lambda_i(G)$

If the synchronized region is unbounded, then adding edges never decreases the synchronizability.

Main reason: $\lambda_i(G+e) \geq \lambda_i(G)$

Lemma 2: For any given graph G:

(i) the largest eigenvalue of G, $\lambda_N(G)$, satisfies $\lambda_N(G) \leq N$,

(ii) $\lambda_N(G) = N$ if and only if G^c is disconnected. Moreover, if G^c has (exactly) q connected components, then the multiplicity of $\lambda_N(G) = N$ as an eigenvalue of G is q-1

(iii) $\lambda_i(G^c) = N - \lambda_{N-i+2}(G), 2 \leq i \leq N$

example. (G has 6 nodes.)

G^c is disconnected. The largest eigenvalue of G_1 is 6, which remains the same for the graph with any more edges being added. Hence, Lemma 1→the synchronizability of all the networks built on graph G_1 never decrease with edge-adding.

recalled lemma1:If the synchronized region is unbounded, then adding edges never decreases the synchronizability.

think about bounding, $\frac{\lambda_N}{\lambda_2}$. $\lambda_N = N$ 因此不会更大了，而 λ_2 up, 于是 $\frac{\lambda_N}{\lambda_2}$ down, and synchronizability up.

Remark 4: The multiplicity of the largest eigenvalue of a graph G is related to the number of connected components of its complement G^c . In order to reduce the number of edges needed to enhance the synchronizability, the multiplicity of the largest eigenvalue of G^c (i.e., the multiplicity of the least nonzero eigenvalue of G) should be large.

Therefore, better understanding and careful manipulation of complementary graphs are useful for enhancing the network synchronizability; and, at least for dense networks, the complementary graphs are easier to analyze than the original graphs, e.g., G_1^c is simpler than G_1 .