第一次作业反馈

作业提交说明：

第一次作业已全部收齐。

作业出现较多文件命名错误，第二次作业（题目对应 lecture3、lecture4、lecture5，截止时间为 11 月 13 日上课前），命名按照课程群文件中 lecture1 的 ppt 要求，作业号按次数累加，如果一周内布置多章作业需要整理为一个 PDF 文件提交。

第二次作业命名为 学号_姓名_第二次作业，请命名错误的同学重新提交。

第一次作业参考

整理自部分同学的作业

第 1 题

题目：Show that if n+1 distinct integers are chosen from the set {1, 2, .., 3n}, then there are always two which differ by at most 2.

Proof: Consider subsets $S_k = \{3k+1,\ 3k+2,\ 3k+3 \mid k \in N\}$ of $S = \{1, 2, \ldots, 3n\}$, there are n of such disjoint subsets $(0 \le k < n)$ in total. In each of these subsets, the elements differ by at most 2.

By the Pigeonhole Principle, selecting $n+1$ distinct integers from S results in at least one of the n subsets containing at least two integers being selected, differing by at most 2.

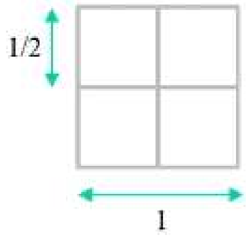
Hence, there always exist two chosen integers which differ by at most 2.

第 2 题

题目：Prove that of any five points chosen within a square of side length 1, there are two whose distance apart is √2/2 at most.

Proof:

We can divide the square into 4 identical smaller squares with side length of $\frac{1}{2}$ as shown below. According to the Pigeonhole Principle, at least 2 of the five points will fall into the same smaller square (including the borders), and obviously the farthest distance between 2 points in the smaller square is the length of diagonal, which is $\frac{\sqrt{2}}{2}$, so any five points chosen within a square of side length 1, there are two among these 5 points whose distance apart is $\frac{\sqrt{2}}{2}$ at most.



第 3 题

题目： In a room there are 10 people with integer ages [1, 60]. Prove that we can always find two groups of people (with no common person) the sum of whose ages is the same.

Selection of a group of 10 people in which the number of people in the group ranges from 1 to 9. There exist $C_{10}^1 + C_{10}^2 + \cdots + C_{10}^9 = 1022$ ways to form a group from 10 people. And the sum ages of a group ranges from $1 * 1 = 1$ to $60 * 9 = 540$. According to the pigeonhole principle, there must be two groups the sum of whose ages is the same. Even if there are some common members of the two groups, removing them won't change the conclusion.