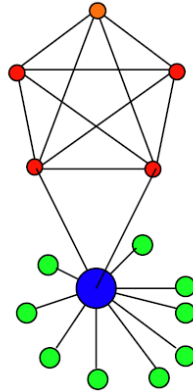


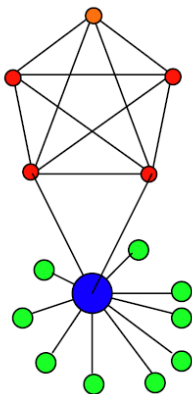
HOMEWORK PROBLEMS #4

4-1 Find the 1-core, 2-core, ... of the following network, and the coreness of each colored node:

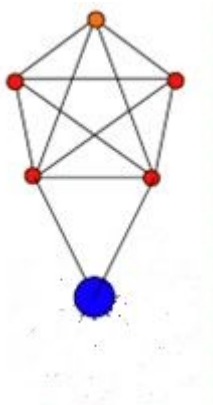


ANSWER:

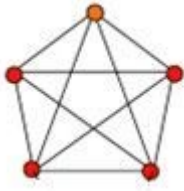
1-core:



2-core:



3-core and 4-core:

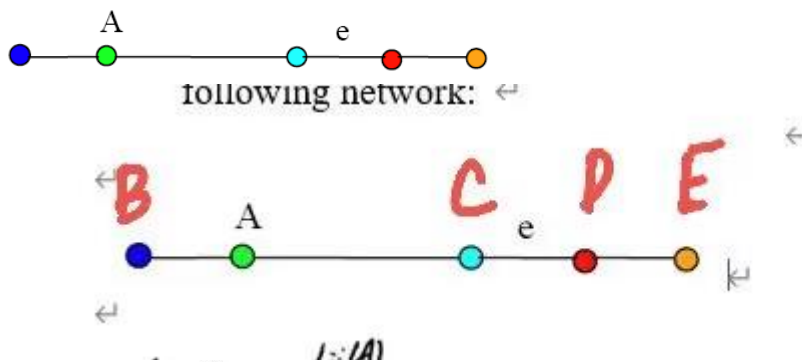


coreness 1: green nodes

coreness 2: blue nodes

coreness 4: red and orange nodes

4-2 Find the node-betweenness of Node A and edge-betweenness of Edge e in the following network:



$$\begin{aligned}
 B(A) &= \sum_{i \neq j \neq A} \frac{L_{ji}(A)}{L_{ji}} \\
 &= \frac{(B, A, C)}{(B, A, C)} + \frac{(B, A, D)}{(B, A, D)} + \frac{(B, A, E)}{(B, A, E)} \\
 &= 3 \\
 \text{Normalized: } B(A) &= \frac{3}{\frac{(N-1)(N-2)}{2}} = \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 B(e) &= \frac{(B, A, C, D)}{(B, A, C, D)} + \frac{(A, C, D)}{(A, C, D)} + \frac{(C, D, E)}{(C, D, E)} \\
 &\quad + \frac{(B, A, C, D, E)}{(B, A, C, D, E)} + \frac{(A, C, D, E)}{(A, C, D, E)} \\
 &= 5 \\
 \text{Normalized: } B(e) &= \frac{5}{\frac{(N-1)(N-2)}{2}} = \frac{5}{9}
 \end{aligned}$$

4-3

- (a) Assume that the Internet is a scale-free non-hierarchical network, and you have all real data about the topology of such an Internet model stored on your computer. You may write a program to compute the degrees of all nodes (Routers and PCs) and then identify a hub (Router) by comparing their degrees, which is a straightforward exhaustive search scheme but is obviously very expensive using global information. Design a simple and cost-effective algorithm that utilizes only local information but can very likely find a hub in just two or three steps.
- (b) Assume that a social community is a small-world network, and an epidemic is now spreading over it. You may vaccinate everybody to prevent the cascading virus propagation, which is a straightforward exhaustive immunization scheme but is obviously very expensive. Design a simple and cost-effective vaccine strategy that immunize only a small number of people but may effectively block the virus spreading over the community.

ANSWER:

- a) calculate the coreness of the graph ,remove the node which of degree less than $k-1$ and keep hubs and routers the largest coreness node in the graph.
- b) we can simulate acquaintance immunization. Select a few people randomly and select their acquaintances .Let these people newly select be immunized .Then vaccinate people who has the most acquaintances from the acquaintances of each selected person.

This HW is due in class on Thursday, 20 Oct. 2022