Lecture 3

4(a)

Since 3, 5, 7, 11 are primes, the divisors can be consisted by the different combination of the four numbers. So we can choose the number 3 range from 0 to 4, then we have five choices. Similarly, the 5, 7, 11 have three, seven, two choice. So the number of distinct positive divisors is 5*3*7*2.

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We should set the four men sitting around the table. So this time it is a circle permutaion, there are 3! ways to set them. Then we arrange the women in pairs to sit between two different men, there are P(8, 8) ways to set the women.

So the total number of the ways is 3! * 8!.

19(b)

For that we can first convert this problem into the 8*8 scale, so choose 8 rows and 8 lines from 12* 12 is C(12, 8) * C(12, 8). Then we arrange the 8 cars, there is 8! ways to set them for that no two cars can attack one another. But the five red car can be seen the same, and similarly the blue ones. So we need to divide the repeated solution, the final solution is C(12, 8) * C(12, 8) * 8! * 8! / 5! / 3!;

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First we set the five boys sitting around the table. The number of the ways is 4!; Then we arrange th five girls to sit between two different boys. There exactly five internals. So there are 5! ways to set the five girls. Finally, put the parent into one of the 10 holes. So the solution is 4! * 5! * 10.

If there are the two parents are either next to each other or not. The final sum of entries is (2 + 2 + 4 + 2 + 4 + 2 + 4 + 2 + 4 + 2 + 2) * (5!) * 5! = 30 * 5! * 5!

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So the first probability is 1/C(9, 5).

The second probability is 5! / 5! * C(9, 5) * C(9, 5) = 1 / [C(9, 5) * C(9, 5)]

Lecture 4

6

The inversions in permutations are:

1	2	3	4	5	6	7	8
2	4	0	4	0	0	1	0

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We can use the algorithm in PPT,

1	8
2	87
3	867
4	8657
5	48657
6	486573
7	4865723
8	48165723

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the subset is 10101000, add 1 = 10101001 so the next subset is $\{x_7, x_5, x_3, x_0\}$

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Use the algorithm in PPT, the next is $\{1, 2, 4, 6, 9, 10, 11\}$, the pre is $\{1, 2, 4, 6, 8, 13, 15\}$

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the location is C(9, 4) - C(7, 4) - C(5, 3) - C(1, 2) - C(0, 1) = 126 - 35 - 10 - 0 - 0 = 81

Lecture 5

15

(1 + x)^n = 1 + C(n, 1) * x^1+ C(n, 2) * x^2 + ... + C(n, n) * x^n;

After differentiation,

equaltion = 1 *C(n, 1) + 2 * C(n, 2) * x^1 + ... + n * C(n, n) * x^(n - 1)

so let x = -1 , the solution = 0

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For the multinomial theorem, let all xi = 1, then we get the equaltion in the question.

t^n = ∑ (n, n1n2...nt)

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let x1 + x2 = x, then we get = ∑C(n, i) * x^i * x3^(n - i);

then ∑∑C(i, j) * C(n, i) * x1^j * x2^(i - j) * x3^(n - i);