lecture 9

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No, the family does not have a SDR.

The largest number of sets in the family with an SDR is 5.

With n=6, k=1, $min_{i=1,2,...,6}|A_i|+6-1=6$;

With n=6, k=2, $min_{i_1,i_2=1,2,...,6}|A_{i1}\cup A_{i2}|+6-2=6$

With n=6, k=3, $min_{i_1,i_2,i_3=1,2,...,6}|A_{i_1}\cup A_{i_2}\cup A_{i_3}|+6-3=6$

With n=6, k=4, $min_{i_1,i_2,i_3,i_4=1,2,...,6}|A_{i_1}\cup A_{i_2}\cup A_{i_3}\cup A_{i_4}|+6-4=5$

With n=6, k=5, $min_{i_1,i_2,i_3,i_4,i_5=1,2,...,6}|A_{i_1}\cup A_{i_2}\cup A_{i_3}\cup A_{i_4}\cup A_{i_5}|+6-5=5$

With n=6, k=6, $min_{i_1,i_2,i_3,i_4,i_5,i_6=1,2,...,6}|A_{i_1}\cup Ai_2\cup Ai_3\cup A_{i_4}\cup Ai_5\cup A_{i_6}|+6-6=5$

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We can convert this problem to a matching problem that one can sit on any seat but the latest one. So, we can choose any number from the set, but the set do not have the number i. So the number of the SDR is the permutations of the matching problem, the answer is D_n;

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1.A->a,B->a,C->b,D->d,a reject B;

2.B->d, d reject D;

3.D->b, b reject C;

4.C->a, a reject A;

5.A->b, b reject A;

6.A->c;

A->c, B->d, C->a, D->b;

1.a->D, b->B, c->D, d->C,D reject a;

2.a->C, C reject d;

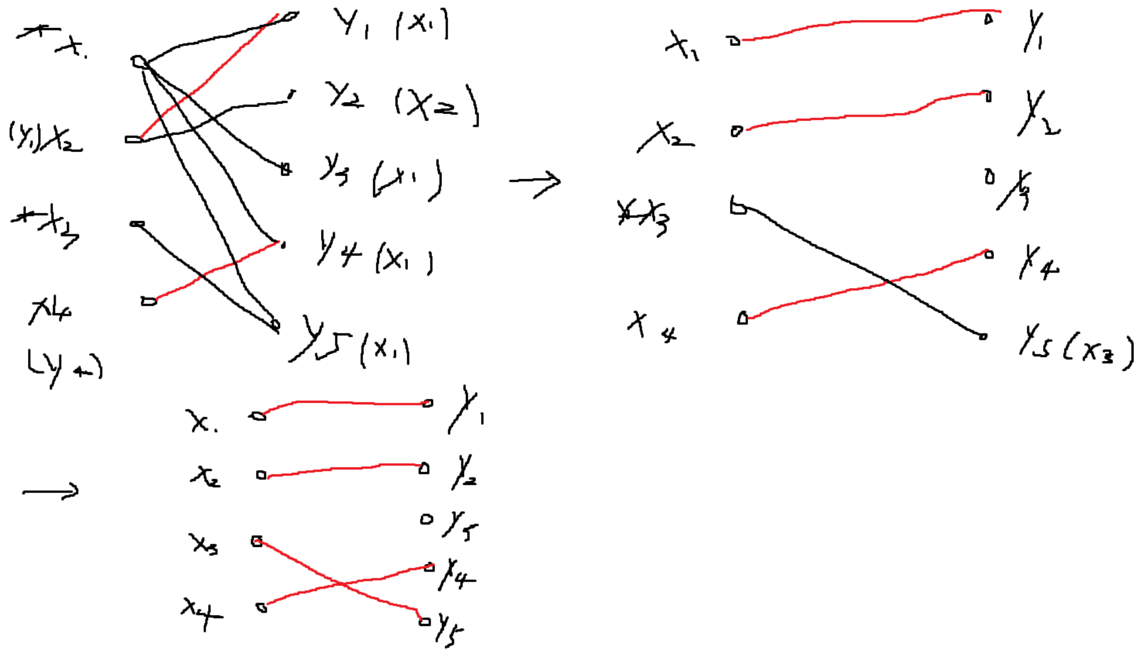
3.d->B, B reject b;

4.b->D, D reject c;

5.c->A;

A->c, B->d, C->a, D->b;

CONT"D



Here, we get the max-matching is $\{(x_1, y_1), (x_2, y_2), (x_3, y_5), (x_4, y_4)\}$;

min-cover$\{x_1, x_2, x_4, y_5\}$;

We get a max-matching $M = \{(x_3, y_5), (x_1, y_1), (x_2, y_2), (x_4, y_4)\}$, but we can find the vertex y_3 is still uncovered. And we should construct a subgraph composed of edges incident to the vertex y_3, Then we find the max-matching of the subgraph, and add it to the max-matching of M, then we get a minimum edge cover. The max-matching of the subgraph is $\{(x_4, y_3)\}$, so we get the minimum edge cover for the graph is $M = \{(x_1, y_1), (x_2, y_2), (x_3, y_5), (x_4, y_3), (x_4, y_4)\}$