Lecture 3-homework

4. How many distinct positive divisors does each of the following numbers have? (a) $3^4 \times 5^2 \times 7^6 \times 11$

Since 3,5,7,11 both are primes,the divisors can be consisted by the different combination of the four number. We have five choices for 3,three choices for 5,seven choices for 7,two choices for 11.So the number of distinct positive divisors is 5*3*7*2.

7. In how many ways can four men and eight women be seated at a round table if there are to be two women between consecutive men around the table?

(1) First, we arrange the four men to sit around the table. The number of ways is P(4,4)/4=3!

(2) Second, we arrange the women in pairs to sit between two different men.So the second problem is just linear permutation.The number of ways is P(8,8)=8!.

(3)so the total number of ways is 3!*8!.

19. We are given eight rooks, five of which are red and three of which are blue.

(b) In how many ways can the eight rooks be placed on a 12-by-12 chessboard so that no two rooks can attack one another?

(1) we have a 1-to-1 correspondence between sets of 8 non-attacking rooks.First we choose 8 rows from 12 rows,and choose 8 columns from 12 columns.Then we get a 8-permutations. $[C(12,8)]^2*(8!)$

(2)Based on (1), decide which permutation of the 8 colors. $[C(12,8)*(8!)]^2$

(3) based on (2), we decide a permutation of colors of the multiset.
$[C(12,8)*(8!)]^2/(5!*3!)$

30. We are to seat five boys, five girls, and one parent in a circular arrangement around a table. In how many ways can this be done if no boy is to sit next to a boy and no girl is to sit next to a girl? What if there are two parents?

(1)First, we arrange the five boys to sit around the table. The number of ways is P(5,5)/5=4!

Second, we arrange the five girls to sit between two different boys. There exactly five intervals. When the five girls sit in different seats, their neighboring boys will be different. So the second problem is just linear permutation. The number of ways is P(5,5)=5!

Last,we arrange one parent to sit in one of 10 intervals.

so the total number of ways is 4!*5!*10.

(3) If have two parents,two parents(labelled P and Q)are either next to each other or not.Suppose we move clockwise around the table from P to Q. Let n denote the number of seats between the two. Thus n = 0 (n = 10) if Q sits next to P at P's left (right). We now partition the set of solutions according to the value of n.

n	The number of Possible situation
0	2*5!*5!
1	2*5!*5!
2	4*5!*5!
3	2*5!*5!
4	4*5!*5!
5	2*5!*5!
6	4*5!*5!
7	2*5!*5!
8	4*5!*5!
9	2*5!*5!
10	2*5!*5!

The sum of the entries in the right-most column, which comes to $30 \times (5!)^2$.

61. Consider an 9-by-9 board and nine rooks of which five are red and four are blue. Suppose you place the rooks on the board in nonattacking positions at random. What is the probability that the red rooks are in rows 1,3,5,7, 9? What is the probability that the red rooks are both in rows 1,2,3,4,5 and in columns 1,2,3,4,5?

(1)we choose 5 rows from 9 rows,the probability that the 5 rows exactly is 1,3,5,7,9 is 1/C(9,5).

(2)First, we choose 5 rows from 9 rows,and choose 5 columns from 9 columns.Then we get a 5-permutations. C(9,5)*C(9,5)*5!.

The total number of the special permutation (the red rooks are both in rows 1,2,3,4,5 and in columns 1,2,3,4,5)is 5!.

The probability that the red rooks are both in rows 1,2,3,4,5 and in columns 1,2,3,4,5 is 1/[C(9,5)*C(9,5)].