lecture 6

3

Supposed P1 is the perfect square numbers which are no more than 10000, P2 is the Cubes which are no more than 10000. The set A1, A2 are the two sets of those numbers. For 100^2 = 10000, the A1 has 100 elements, similarly the A2 has 21 elements.

The solution S = U - ∑Ai + A1 ∩ A2 = 10000 - 100 - 21 + 4 = 9883.

8

The equation is $x_1 + x_2 + x_3 + x_4 + x_5 = 9$,
$x_1 <= 4, x2 <= 4, x_3 <= 4, x_4 <= 4, x_5 <= 4$;

The problem can be seen there are five things which are the number of the is all four, we select 9 from them. So can be soluted the number is C(9 + 5 - 1, 9) - 5 * C(4 + 5 - 1, 4) = C(13, 9) - 5 * C(8, 4) = 715 - 5 * 70 = 365

11

Let S is the permutations of {1, 2, 3, 4, 5, 6, 7, 8}, let Ai denote the set of permutations in S for which i is in its natural position, i ∈ {2， 4， 6， 8};

|S| = 8!, |Ai| = 7!, |Ai ∩ Aj| = 6!, |Ai ∩ Aj ∩ Ak| = 5!, |A2 ∩ A4 ∩ A6 ∩ A8| = 4!

The number = 8! - 4 * 7! + 6 * 6! - 4 * 5! + 4! = 40320 - 20160 + 4320 - 480 + 24 = 24024

25

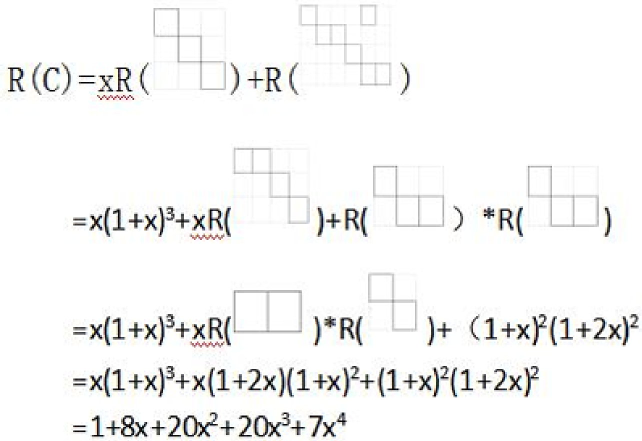
Let n = 6, $X_1 = \{1, 5\}, X_3 = \{2, 3, 5\}, X_4 = \{4\}, X_6 = \{5, 6\}$, then we get:

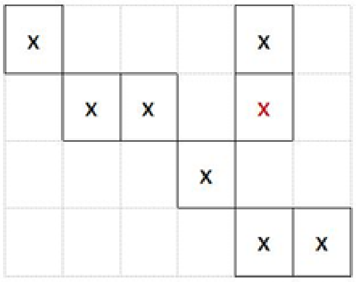
×				×	
	×	×		×	
			×		
				×	×

Then $P(i_1, i_2, i_3, i_4, i_5, i_6)$are in one-to-one correspondence with the placement of 6 non-attacking rooks on the board with forbidden positions as shown.

We interpret this problem in terms of placing six nonattacking rooks on a 6×6 chessboard. The answer is ∑6 k=0 rk (-1)k (6-k)!

Chessboard polynomial:





$$R(C) = x(1+x)^3 + x(1+2x)(1+x)^2 + (1+x)^2(1+2x)^2 = 1+8x+20x^2+20x^3+7x^4$$

31

Let S denote the set of circular permutations of {2 * a, 3 * b, 4 * c, 5 * d}.

Let A denote the set of elements of in S such that occurrences of a are consecutive, similar to B, C, D.

|S| = 13! / (2! * 3! * 4! * 5!)

|A| = 12! / (3! * 4! * 5!)

|B| = 11! / (2! * 4! * 5!)

|C| = 10! / (2! * 3! * 5!)

|D| = 9! / (2! * 3! * 4!)

|AB| = 10! / (4! * 5!)

|AC| = 9! / (3! * 5!)

|AD| = 8! / (3! * 4!)

|BC| = 8! / (2! * 5!)

|BD| = 7! / (2! * 4!)

|CD| = 6! / (2! * 3!)

|ABC| = 7! / 5!

|ABD| = 6! / 4!

|ACD| = 5! / 3!

|BCD| = 4! / 2!

|ABCD| = 3!

The number of the answer is = 13! / (2! * 3! * 4! * 5!) -

[12! / (3! * 4! * 5!) + 11! / (2! * 4! * 5!) + 10! / (2! * 3! * 5!) + 9! / (2! * 3! * 4!)] +

[10! / (4! * 5!) + 9! / (3! * 5!) + 8! / (3! * 4!) + 8! / (2! * 5!) + 7! / (2! * 4!) + 6! / (2! * 3!)] -

[7! / 5! + 6! / 4! + 5! / 3! + 4! / 2!] + 3!