1. Show that if n+1 distinct integers are chosen from the set {1, 2, …, 3n}, then there are always two which differ by at most 2.

We can divide the original set into n sets, including {1,2,3},{4,5,6},...{3n-1,3n-2,3n-3},then we get n distinct integers by choosing 1 integer from every set.The last integer must be selected from one of n sets.So,there are at least two integers from the same set.And there are always two which differ by at most 2.

2. Prove that of any five points chosen within a square of side length 1, there are two whose distance apart is at most $\frac{\sqrt{2}}{2}$.

We can divide the square into four squares of side length 1/2,then place one point into each square.The last point must be placed into one of four squares.So,there are at least two points in the same square of side length 1/2 whose the largest distance is $\frac{\sqrt{2}}{2}$. So there are two whose distance apart is at most $\frac{\sqrt{2}}{2}$.

3. In a room there are 10 people with integer ages [1, 60]. Prove that we can always find two groups of people (with no common person) the sum of whose ages is the same.

There are two free groups of 10 people.The number of all situation is 1022.

The sum of each group ages range from 10 to 600, the number of all situation is 591.

Because 1022 is larger than 591,According to the pigeonhole principle, there must be two groups the sum of whose ages is the same.