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(b) $f^{-1} * c$

$f^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 6 & 2 & 5 & 1 \end{pmatrix}$ $\quad c = (R, B, B, R, R, R)$

$f^{-1} * c = (R, R, B, R, R, B)$

(d) $g \circ f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 6 & 2 & 4 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 4 & 2 & 1 & 5 & 3 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 5 & 3 & 4 & 6 \end{pmatrix}$

$\therefore (g \circ f) * c = (R, B, R, R, B, R)$

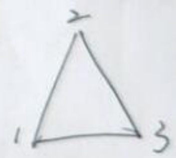
$f \circ g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 4 & 2 & 1 & 5 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 6 & 2 & 4 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 3 & 4 & 1 & 6 \end{pmatrix}$

$(f \circ g) * c = (R, R, B, R, \overset{B}{\cancel{R}}, R)$

20. The permutation group has two permutation

$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}.$



$\therefore N(G, C) = \frac{1}{2}(2^3 + 2^2) = 6.$

2∴ with p colors $N(G, C) = \frac{1}{2}(p^3 + p^2)$

26. D_7 Cycle factorization type monomial

ρ_7^0	$[1]\circ[2]\circ[3]\circ[4]\circ[5]\circ[6]\circ[7]$	$(7,0,0,0,0,0,0)$	z_1^7
ρ_7^1	$[1,2,3,4,5,6,7]$	$(0,0,0,0,0,0,1)$	z_7^1
ρ_7^2	$[1,3,5,7,2,4,6]$	$(0,0,0,0,0,0,1)$	z_7^1
ρ_7^3	$[1,4,7,3,6,2,5]$	$(0,0,0,0,0,0,1)$	z_7^1
ρ_7^4	$[1,5,2,6,3,7,4]$	$(0,0,0,0,0,0,1)$	z_7^1
ρ_7^5	$[1,6,4,2,7,5,3]$	$(0,0,0,0,0,0,1)$	z_7^1
ρ_7^6	$[1,7,6,5,4,3,2]$	$(0,0,0,0,0,0,1)$	z_7^1
r_1	$[1]\circ[2,7]\circ[3,6]\circ[4,5]$	$(1,3,0,0,0,0,0)$	$z_1^1 z_2^3$
r_2	$[1,3]\circ[2]\circ[4,7]\circ[5,6]$	$(1,3,0,0,0,0,0)$	$z_1^1 z_2^3$
r_3	$[1,5]\circ[2,4]\circ[3]\circ[6,7]$	$(1,3,0,0,0,0,0)$	$z_1^1 z_2^3$
r_4	$[1,7]\circ[2,6]\circ[3,5]\circ[4]$	$(1,3,0,0,0,0,0)$	$z_1^1 z_2^3$
r_5	$[1,2]\circ[3,7]\circ[4,6]\circ[5]$	$(1,3,0,0,0,0,0)$	$z_1^1 z_2^3$
r_6	$[1,4]\circ[2,3]\circ[5,7]\circ[6]$	$(1,3,0,0,0,0,0)$	$z_1^1 z_2^3$
r_7	$[1,6]\circ[2,5]\circ[3,4]\circ[7]$	$(1,3,0,0,0,0,0)$	$z_1^1 z_2^3$

$$G = \frac{1}{14}\left(z_1^7 + 6z_7 + 7z_1^1 z_2^3\right).$$

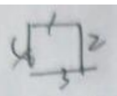
$$P_{D_7}(r+b,\ r^2+b^2,\ r^3+b^3,\ r^4+b^4,\ r^5+b^5,\ r^6+b^6,\ r^7+b^7)$$

$$= \frac{1}{14}\left((r+b)^7 + 6(r^7+b^7) + 7(r+b)(r^2+b^2)^3\right)$$

$$= \frac{1}{14}\left(C_7^4 + 7\times C_3^2\right) = 4$$

$\therefore$ There are 4 different types.

so D_4



D_4	Cycle factorization	type	monomial
e_4^0	$[1]\circ[2]\circ[3]\circ[4]$	$(4,0,0,0)$	z_1^4.
e_4^1	$[1,2,3,4]$	$(0,0,0,1)$	z_4^1
e_4^2	$[1,3]\circ[2,4]$	$(0,2,0,0)$	z_2^2.
e_4^3	$[1,4,3,2]$	$(0,0,0,1)$	z_4^1
r_1	$[1]\circ[3]\circ[2,4]$	$(2,1,0,0)$	$z_1^2 z_2^1$
r_2	$[2]\circ[4]\circ[1,3]$	$(2,1,0,0)$	$z_1^2 z_2^1$
r_3	$[1,4]\circ[2,3]$	$(0,2,0,0)$	z_2^2
r_4	$[1,2]\circ[3,4]$	$(0,2,0,0)$	z_2^2

$$P_{D_4} b = \frac{1}{8}\left(z_1^4 + 2z_4^1 + 3z_2^2 + 2z_1^2 z_2^1\right)$$

$$= \frac{1}{8}\left((r+b)^4 + 2(r^4+b^4) + 3(r^2+b^2)^2 + 2(r+b)^2(r^2+b^2)\right)$$

if we use k colors.

$$P_{D_4} = \frac{1}{8}\left[\left(\sum_{i=1}^{k} c_i\right)^4 + 2\sum_{i=1}^{k} c_i^4 + 3\left(\sum_{i=1}^{k} c_i^2\right)^2 + 2\left(\sum_{i=1}^{k}\right.\right.$$

$$= \frac{1}{8}\left(k^4 + 2k + 3k^2 + 2k^3\right).$$