第二次作业反馈

第二次作业参考

整理自部分同学作业

L3 题 4(a). How many distinct positive divisors does each of the following numbers have?

The number $3, 5, 7, 11$ are prime numbers. Hence the factor is of the form

$$3^i \times 5^j \times 7^k \times 11^l$$

where $0 \le i \le 4$, $0 \le j \le 2$, $0 \le k \le 6$, and $0 \le l \le 1$.

There are 5 choices for i, 3 for j, 7 for k, and 2 for l. By the multiplication principle, the number of the factors is $5 \times 3 \times 7 \times 2 = 210$.

L3 题 7. In how many ways can four men and eight women be seated at a round table if there are to be two women between consecutive men around the table?

First, arrange the 4 men with two empty seats between each of them in a circular arrangement, which gives $\frac{P(4,4)}{4} = 3!$ways. Once these positions are fixed (viewed as a linear arrangement), the 8 women can sit in the remaining seats with $P(8,8)$ arrangements. In total, there are 3!×8! possible arrangements.

L3 题 19(b). We are given eight rooks, five of which are red and three of which are blue. In how many ways can the eight rooks be placed on a 12-by-12 chessboard so that no two rooks can attack one another?

- Firstly, select 8 rows and 8 columns from 12*12 chessboard: $C(12,8) * C(12,8)$;
- Secondly, combine selected 8 rows and 8 columns: $P(8,8)$;
- Thirdly, consider the rooks' permutation with the same color: $\frac{P(8,8)}{P(3,3) * P(5,5)}$;
- In summary, there are $C(12,8) * C(12,8) * P(8,8) * \frac{P(8,8)}{P(3,3) * P(5,5)} = 553246848000$ ways to put rooks under the case that no two rooks can attack one another.

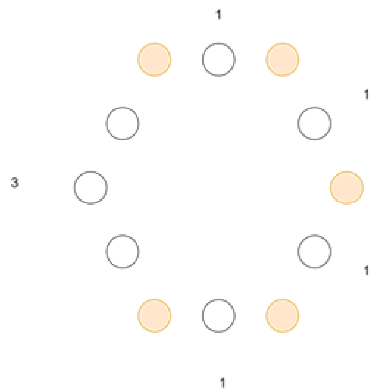
L3 题 30. We are to seat five boys, five girls, and one parent in a circular arrangement around a table. a)In how many ways can this be done if no boy is to sit next to a boy and no girl is to sit next to a girl? b)What if there are two parents?

a)

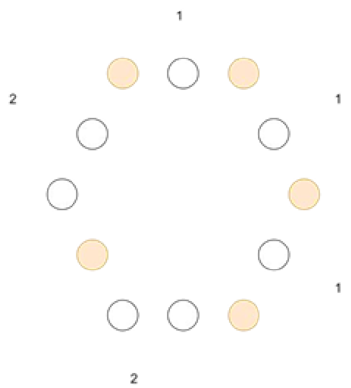
- Firstly, arrange for 5 boys and 1 parent to seat and consider circular permutation: $\frac{P(6,6)}{6}$;
- Secondly, insert 5 girls between the seated parent and the boys, and the parent has only one girl by his side to ensure that all boys are not next to each other: $P(5,5)*2$;
- In summary, there are $\frac{P(6,6)}{6}*P(5,5)*2=28800$ ways to arrange these boys, girls and the parent.

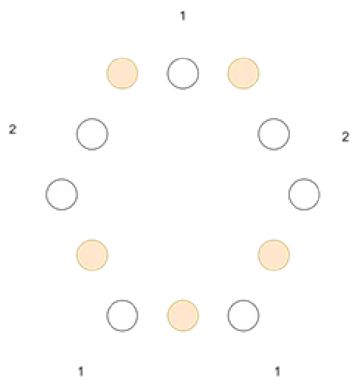
b)

- Firstly, arrange for 5 boys to sit in these 12 seats. We number the boys clockwise with 1, 2, ,,,5, and then use a_i to represent the number of seats between the i-th boy and the (i+1)-th boy (the 6-th boy and the 1-st boy are the same). Therefore, all possible seats in the set $\{a_i\}$ only include$\{1,1,1,1,3\}$ and $\{1,1,1,2,2\}$ these two situations;
- For $\{1,1,1,1,3\}$, the three adjacent seats either have only one parent who seats in the middle, or two parents who seat at will, so the result is $\frac{12*P(5,5)*P(5,5)*(C(4,1)*2+P(3,2))}{12}=P(5,5)*P(5,5)*14$;



- For $\{1,1,1,2,2\}$, we just need to ensure that there is and only one parent in each two adjacent seats, so the result is $\frac{12*P(5,5)*P(5,5)*C(2,1)*C(2,1)*2}{12}*2=P(5,5)*P(5,5)*16$.





- In summary, there are $P(5,5)*P(5,5)*14+P(5,5)*P(5,5)*16=P(5,5)*P(5,5)*30=432000$ ways to arrange these boys, girls and the two parents.

L3 题61 : Consider an 9-by-9 board and nine rooks of which five are red and four are blue. Suppose you place the rooks on the board in nonattacking positions at random. What is the probability that the red rooks are in rows 1,3,5,7, 9? What is the probability that the red rooks are both in rows 1,2,3,4,5 and in columns 1,2,3,4,5?

The total number of the random placement events $N=9!*C_4^9=(9!)^2/(5!*4!)$(The number of non-attacking positions on the board for rooks to be placed in is 9!.)

For first case, the number of placing red rooks firstly is 9*8*7*6*5. Then, the number of placing blue rooks is 4*3*2*1.$N_1 = 9!$.

For second case, the red rooks are both in rows 1,2,3,4,5 and in columns 1,2,3,4,5, so the blue rooks are both in rows 6,7,8,9 and in columns 6,7,8,9.$N_2 = 5! * 4!$

The probability that the red rooks are in rows 1,3,5,7,9:

$P_1 = N_1/N = 1/126$

The probability that the red rooks are both in rows 1,2,3,4,5 and in columns 1,2,3,4,5:$P_2 = N_2/N = 1/15876$

L4题6（a）：Determine the inversion sequences of the following permutations of {I, 2, ... ,8}: (a) 35168274

There are two numbers greater than 1 in front of the position where 1 is located in the sequence, so a1=2 . In a similar fashion, the inversion sequences of 35168274 = 24040010

L4 题 7：Construct the permutations of {I, 2, ... ,8} whose inversion sequences are (a) 2,5,5,0,2,1,1,0

Use Algorithm I, the result is 48165723.

L4 题 15. For each of the following subsets of {X7, X6, ... , XI, xo}, determine the subset that immediately follows it by using the base 2 arithmetic generating scheme: (b) {X7,X5,X3}

Use the base 2 arithmetic generating scheme 10101000, next one 10101001. The subset that immediately follows it is {x7,x5,x3,x0}.

L4 题 29. Determine the 7-subset of {I, 2, … , 15} that immediately follows 1,2,4,6,8,14,15 in the lexicographic order. Then determine the 7-subset that immediately precedes 1,2,4,6,8,14,15.

14 15 are two biggest numbers. Find a_k = 13 ,k = 6, the next one is 1, 2, 4, 6, 8,14, 15 . The immediate predecessor is 1, 2, 4, 6, 8, 13, 15.

L4 题 33. In which position does the subset 2489 occur in the lexicographic order of the 4-subsets of {1, 2, 3, 4, 5, 6, 7, 8, 9}?

The position of 2489 is calculated as follows： From 1ABC to 2DEF there are C(8,3) combinations. From 23AB to 24CD there are C(6,2) combinations. From 24AB to 25CD there are C(5,2) combinations. 8,9 are the biggest numbers, so 2489 is the last one in 24AB. Theresult is C(8,3) + C(6,2) + C(5,2) = 81.

L5 题 15：

15. Prove, that for every integer $n > 1$,

$$\binom{n}{1} - 2\binom{n}{2} + 3\binom{n}{3} + \cdots + (-1)^{n-1}n\binom{n}{n} = 0.$$

答案：

For variable x, we can get:

$$(1-x)^n = \sum_{k=0}^{n} \binom{n}{k} (-1)^k x^k$$

Taking the derivative of x on both sides of the equation simultaneously, we can get:

$$-n(1-x)^{n-1} = \sum_{k=0}^{n} \binom{n}{k} (-1)^k k x^{k-1}$$

Let $x = 1$, we can get:

$$0 = \sum_{k=0}^{n} \binom{n}{k} (-1)^k k$$

So

$$\binom{n}{1} - 2\binom{n}{2} + 3\binom{n}{3} + \cdots + (-1)^{n-1}n\binom{n}{n} = 0$$

L5 题 37：

37. Use the multinomial theorem to show that, for positive integers n and t,

$$t^n = \sum \binom{n}{n_1\, n_2 \cdots n_t},$$

where the summation extends over all nonnegative integral solutions $n_1, n_2, \ldots, n_t$ of $n_1 + n_2 + \cdots + n_t = n$.

答案：

In the multinomial theorem, we have:

$$(x_1 + x_2 + \cdots + x_t)^n = \sum \binom{n}{n_1 n_2 \dots n_t} x_1^{n_1} x_2^{n_2} \dots x_t^{n_t}$$

So let $x_1 = x_2 = \cdots = x_t = 1$, we can get:

$$t^n = \sum \binom{n}{n_1 n_2 \dots n_t}$$

L5 题 41：

41. Expand $(x_1 + x_2 + x_3)^n$ by observing that

$$(x_1 + x_2 + x_3)^n = ((x_1 + x_2) + x_3)^n$$

and then using the binomial theorem.

答案：

According to binomial theorem,

$$\begin{aligned}
(x_1 + x_2 + x_3)^n &= ((x_1 + x_2) + x_3)^n \\
&= \sum_{k=0}^{n} \binom{n}{k} (x_1 + x_2)^k x_3^{n-k} \\
&= \sum_{k=0}^{n} \binom{n}{k} \left(\sum_{i=0}^{k} \binom{k}{i} x_1^i x_2^{k-i}\right) x_3^{n-k} \\
&= \sum_{k=0}^{n} \sum_{i=0}^{k} \frac{n!}{(n-k)!k!} \frac{k!}{i!(k-i)!} x_1^i x_2^{k-i} x_3^{n-k} \\
&= \sum_{k=0}^{n} \sum_{i=0}^{k} \frac{n!}{(n-k)!i!(k-i)!} x_1^i x_2^{k-i} x_3^{n-k}
\end{aligned}$$

set $n_1 = i, n_2 = k - i, n_3 = n - k$, so $i = n_1, k = n_1 + n_2, n = n_1 + n_2 + n_3$, so

$$\begin{aligned}
(x_1 + x_2 + x_3)^n &= \sum_{n_1 + n_2 = 0}^{n} \sum_{n_1 = 0}^{n_1 + n_2} \frac{n!}{n_1! n_2! n_3!} x_1^{n_1} x_2^{n_2} x_3^{n_3} \\
&= \sum_{n_1 + n_2 + n_3 = n} \binom{n}{n_1 n_2 n_3} x_1^{n_1} x_2^{n_2} x_3^{n_3}
\end{aligned}$$