Lecture10

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A	B	
15	46	46 = 15 * 3 + 1
15	1	15 = 15 * 1 + 0
0	1	1

1 = 46 - 15 * 3

$15^{-1} = 46 - 3 = 43$

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b' = v' = 7, k' = 4, r' = 4, λ'=2.

Suppose the starter block is B = {2, 4, 5, 6};

-	2	4	5	6
2	0	5	4	3
4	2	0	6	5
5	3	1	0	6
6	4	2	1	0

Examining this table we see that each of the non-zero integers 1, 2, 3, 4, 5, 6 in Z_7 occurs exactly twice in the off-diagonal positions and hence exactly twice as a difference. Hence, B is a difference set mod 7.

Then the blocks developed from B as a starter block,we have: B+0={2,4,5,6}, B+1={3,5,6,0}, B+2={4,6,0,1}, B+3={5,0,1,2}, B+4={6,1,2,3}, B+5={0,2,3,4}, B+6={1,3,4,5}

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-	0	1	3	9
0	0	12	10	4
1	1	0	11	5
3	3	2	0	7
9	9	8	6	0

Examining this table we see that each of the non-zero integers 1, 2, 3, 4, 5, 6,7,8,9,10,11,12 in Z13 occurs exactly once in the off-diagonal positions and hence exactly once as a difference. Hence, B is a difference set mod 13.

For that is a SBIBD, so the b = v = 13, k = r = 4, λ = 1;

B+0={0,1,3,9};

B+1={1,2,4,10};

B+2 = {2,3,5,11};

B+3 = {3,4,6,12};

B+4 = {4,5,7,0};

B+5={5,6,8,1};

B+6={6,7,9,2};

B+7={7,8,10,3};

B+11={11,12,1,7};

B+12={12,0,2,8}

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We find two Steiner triple systems for a three one and a seven one;

Let X={a0,a1,a2}, Y={b0,b1,b2,b3,b4,b5,b6} are the sets of varieties. Let B1={(a0,a1,a2)}, B2={(b0,b1,b3),(b1,b2,b4),(b2,b3,b5),(b3,b4,b6),(b4,b5,b0),(b5,b6,b1),(b6,b0,b2)};

Then:

1. r = s = t, (0,5,3),(5,9,12),(9,3,15),(3,12,18),(12,15,0),(15,18,5),(18,0,9) (1,6,8),(6,10,13),(10,8,16),(8,13,19),(13,16,1),(16,19,6),(19,1,10). (2,7,4),(7,11,14),(11,4,17),(4,14,20),(14,17,2),(17,20,7),(20,2,11)
2. i = j = k, (0,1,2),(5,6,7),(9,10,11),(3,8,4),(12,13,14),(15,15,17),(18,19,20)
3. i,j,k is differnt from each other, r,s,t is different from each other,hence there are 42 triples. Including: (0,6,4),(0,7,8),(1,5,4),(1,7,3),(2,5,8),(2,6,3)...

So there are 70 triples in this system.

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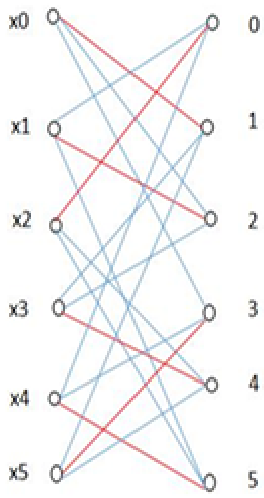
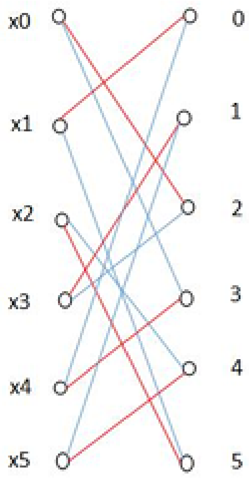
(1)Let L be an 3-by-6 Latin rectangle based on Z6.Define a bigraph G =(X, △, Y), X = {x0, x1, ..., x5} corresponds to columns 0, 1, ..., 5 of the rectangle L, Y = {0, 1, ..., 5} is the elements on which L is based. △ = {(xi, j): j does not occur in column i of L}. G has a perfect matching. Suppose the edges of a perfect matching are{(x0, i0), (x1, i1), ..., (x5, i5)}. Then 4-by-6 array obtained by adjoining i0, i1, ...,i5 as a new row is a Latin rectangle. Continue the process until the 6-by-6 Latin square is completed.



0	1	2	3	4	5
4	3	1	5	2	0
5	4	3	0	1	2
1	2	0	4	5	3



0	1	2	3	4	5
4	3	1	5	2	0
5	4	3	0	1	2
1	2	0	4	5	3
2	0	5	1	3	4

0	1	2	3	4	5
4	3	1	5	2	0
5	4	3	0	1	2
1	2	0	4	5	3
2	0	5	1	3	4
3	5	4	2	0	1

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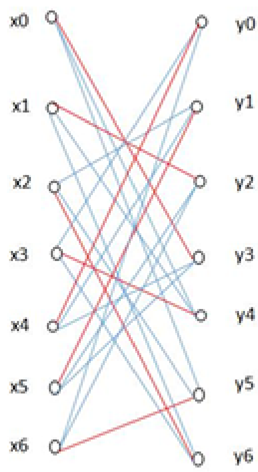
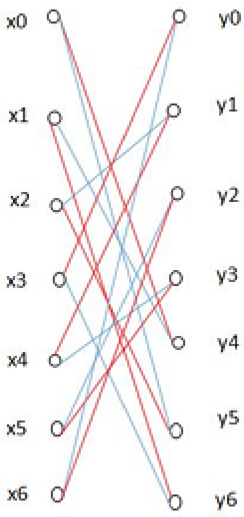
(1)Let L be a semi-Latin square of order 7 and index 4.

Define a bigraph G =(X, △, Y), X = {x0, x1, ..., x6} correspond to rows 0, 1, ..., 6 of the rectangle L, Y = {y0, y1, ..., y6} correspond to columns of L. △ = {(xi, yj): the position at row i column j is unoccupied. Then G is 3-regular and has a perfect matching. This matching identifies the desired position for number 4. Continue to place other numbers 5, 6.... until L is completed.



0	2	1	4			3
2	0	4	1		3	
3		0	2	1		4
	3	2	0	4	1	
4		3		0	2	1
1	4			3	0	2
	1		3	0	4	0



0	**2**	**1**	**4**	**5**		**3**
2	0	4	1		3	5
3		0	2	1	5	4
5	3	2	0	4	1	
4	5	3		0	2	1
1	4		5	3	0	2
	1	5	3	0	4	0

0	**2**	**1**	**4**	**5**	**6**	**3**
2	0	4	1	6	3	5
3	6	0	2	1	5	4
5	3	2	0	4	1	6
4	5	3	6	0	2	1
1	4	6	5	3	0	2
6	1	5	3	0	4	0