Lecture10-Homework

16. 将 10.1 节计算 GCD 的算法应用于 15 和 46，然后用这个结果去确定 15 在 Z_{46} 中的乘法逆元。

Answer:

A	B	
15	46	46=15*3+1
15	1	15=15*1+0
0	1	d=1

1=46-15*3

Therefore ,15^{-1}=-3=43

21. 确定有 10.2 节中给出的参数 $b=v=7$，$k=r=3$，$\lambda=1$ 的 BIBD 的补设计。

Answer:

Suppose B is a BIBD with parameters b=v=7, k=r=3,λ=1 , B^C is a block design with parameters

b'=v'=7,k'=4,r'=4,λ'=2. Suppose the starter block is B={2,4,5,6},

-	2	4	5	6
2	0	5	4	3
4	2	0	6	5
5	3	1	0	6
6	4	2	1	0

Examining this table we see that each of the non-zero integers 1, 2, 3, 4, 5, 6 in Z_7 occurs exactly twice in the off-diagonal positions and hence exactly twice as a difference. Hence, B is a difference set mod 7.

Then the blocks developed from B as a starter block,we have:

B+0={2,4,5,6}, B+1={3,5,6,0}, B+2={4,6,0,1}, B+3={5,0,1,2}, B+4={6,1,2,3}, B+5={0,2,3,4}, B+6={1,3,4,5}

28. 证明：$B=\{0,1,3,9\}$ 是 Z_{13} 中的差分集，并用该差分集作为初始区组构造一个 SBIBD。确定这个区组设计的各参数。

Answer:

-	0	1	3	9
0	0	12	10	4
1	1	0	11	5
3	3	2	0	7
9	9	8	6	0

Examining this table we see that each of the non-zero integers 1, 2, 3, 4, 5, 6,7,8,9,10,11,12 in Z_{13} occurs exactly once in the off-diagonal positions and hence exactly once as a difference. Hence, B is a difference set mod 13.

Using B as a starter block we obtain the following blocks for a SBIBD with parameters b = v = 13, k = r = 4 and λ= 1.Then we have:

B+0={0,1,3,9}; B+1={1,2,4,10}; B+2 = {2,3,5,11}; B + 3 = {3,4,6,12}; B+ 4 = {4,5,7,0}; B+5={5,6,8,1}; B+6={6,7,9,2}; B+7={7,8,10,3}; B+8={8,9,11,4}; B+9={9,10,12,5}; B+10={10,11,0,6}; B+11={11,12,1,7}; B+12={12,0,2,8}

32. 用定理 10.3.2 构造一个指数为 1 且有 21 个样品的 Steiner 三元系。

Answer:

$$
\begin{array}{c}
\\
b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6
\end{array}
\begin{array}{c}
\begin{array}{ccc} a_0 & a_1 & a_2 \end{array} \\
\begin{pmatrix}
0 & 1 & 2 \\
5 & 6 & 7 \\
9 & 10 & 11 \\
3 & 8 & 4 \\
12 & 13 & 14 \\
15 & 16 & 17 \\
18 & 19 & 20
\end{pmatrix}
\end{array}
$$

Let X={a0,a1,a2}, Y={b0,b1,b2,b3,b4,b5,b6} are the sets of varieties. Let B1={(a0,a1,a2)}, B2={(b0,b1,b3),(b1,b2,b4),(b2,b3,b5),(b3,b4,b6),(b4,b5,b0),(b5,b6,b1),(b6,b0,b2)}

If we choose (Cir,Cjs,Ckt) as a triple of,then :

(1) r=s=t, we have 7 different triples in every column,hence there are 21 triples.Including:

(0,5,3),(5,9,12),(9,3,15),(3,12,18),(12,15,0),(15,18,5),(18,0,9)

(1,6,8),(6,10,13),(10,8,16),(8,13,19),(13,16,1),(16,19,6),(19,1,10).

(2,7,4),(7,11,14),(11,4,17),(4,14,20),(14,17,2),(17,20,7),(20,2,11)

(2) i=j=k, we have 7 different triples in every row, hence there are 7 triples.Including:

(0,1,2),(5,6,7),(9,10,11),(3,8,4),(12,13,14),(15,15,17),(18,19,20)

(3) i,j,k is differnt from each other, r,s,t is different from each other,hence there are 42 triples. Including:

(0,6,4),(0,7,8),(1,5,4),(1,7,3),(2,5,8),(2,6,3)...

So there are 70 triples in this system.

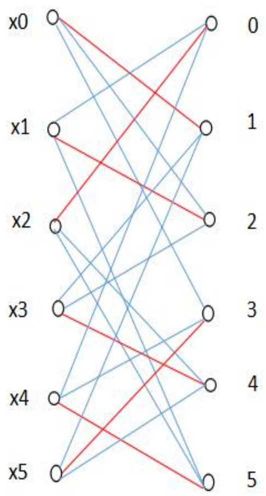
52. 构造 3 行 6 列拉丁矩形

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 4 & 3 & 1 & 5 & 2 & 0 \\ 5 & 4 & 3 & 0 & 1 & 2 \end{bmatrix}$$

的一个完备化。

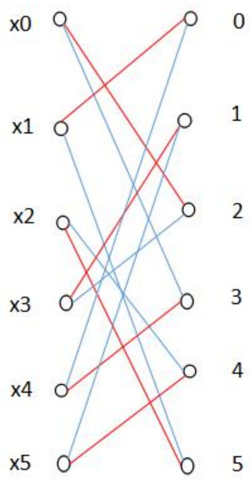
Answer:

(1)Let L be an 3-by-6 Latin rectangle based on Z6.Define a bigraph G =(X, △, Y), X = {x0, x1, ⋯, x5} corresponds to columns 0, 1, ⋯, 5 of the rectangle L, Y = {0, 1, ⋯, 5} is the elements on which L is based. △ = {(xi, j): j does not occur in column i of L}. G has a perfect matching. Suppose the edges of a perfect matching are{(x0, i0), (x1, i1), ⋯., (x5, i5)}. Then 4-by-6 array obtained by adjoining i0, i1, ⋯,i5 as a new row is a Latin rectangle. Continue the process until the 6-by-6 Latin square is completed.



0	1	2	3	4	5
4	3	1	5	2	0
5	4	3	0	1	2
1	2	0	4	5	3

(2))Let L be an 4-by-6 Latin rectangle based on Z6.



0	1	2	3	4	5
4	3	1	5	2	0
5	4	3	0	1	2
1	2	0	4	5	3
2	0	5	1	3	4

(3) Finally fill in the last line.

0	1	2	3	4	5
4	3	1	5	2	0
5	4	3	0	1	2
1	2	0	4	5	3
2	0	5	1	3	4
3	5	4	2	0	1

56. 构造半拉丁方

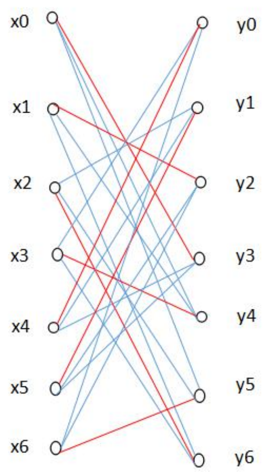
$$\begin{bmatrix} 0 & 2 & 1 & & & & 3 \\ 2 & 0 & & 1 & & 3 & \\ 3 & & 0 & 2 & 1 & & \\ & 3 & 2 & 0 & & 1 & \\ & & 3 & & 0 & 2 & 1 \\ 1 & & & & 3 & 0 & 2 \\ & 1 & & 3 & 2 & & 0 \end{bmatrix}$$

的一个完备化。

Answer:

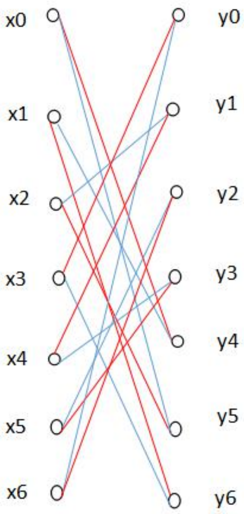
(1)Let L be a semi-Latin square of order 7 and index 4.

Define a bigraph G =(X, △, Y), X = {x0, x1, …, x6} correspond to rows 0, 1, …, 6 of the rectangle L, Y = {y0, y1, …, y6} correspond to columns of L. △ = {(xi, yj): the position at row i column j is unoccupied. Then G is 3-regular and has a perfect matching. This matching identifies the desired position for number 4. Continue to place other numbers 5, 6…. until L is completed.



0	2	1	4			3
2	0	4	1		3	
3		0	2	1		4
	3	2	0	4	1	
4		3		0	2	1
1	4			3	0	2
	1		3	0	4	0

(2)Remove the red edge, find the new perfect matching. Let L be a semi-Latin square of order 7 and index 5.



0	2	1	4	5		3
2	0	4	1		3	5
3		0	2	1	5	4
5	3	2	0	4	1	
4	5	3		0	2	1
1	4		5	3	0	2
	1	5	3	0	4	0

(3)Finally place 6 on the empty position.

0	2	1	4	5	6	3
2	0	4	1	6	3	5
3	6	0	2	1	5	4
5	3	2	0	4	1	6
4	5	3	6	0	2	1
1	4	6	5	3	0	2
6	1	5	3	0	4	0