Modeling of Complex Networks

Lecture 6: Cascading Reactions

S8101003Q-01(Sem A, Fall 2021)

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Cascading Reactions

- **The phenomenon of *cascading reactions (failures)*, is quite similar to virus spreading over various complex networks**
- **Similarity: They typically lead to collapse of a large portion of the network or even the entire network, called *avalanche***
- **Differences:**
  - **As long as the network falls apart into pieces, the network is considered failed, while in virus spreading infected individuals are the ultimate concerns**
  - **In cascading failures, loads (weights) on nodes are taken into main consideration, but usually not so in virus spreading**

Real Examples of Cascading Failures

Internet

Power grid

October 1986:
the first documented Internet collapse due to congestion

↓

Drop in speed of a factor 100

August 1996:
sag of just one electric line in Oregon USA

↓

Blackout affected 4 million people in 9 different states

August 2003:
local failure in Ohio USA

↓

Largest blackout in the US history

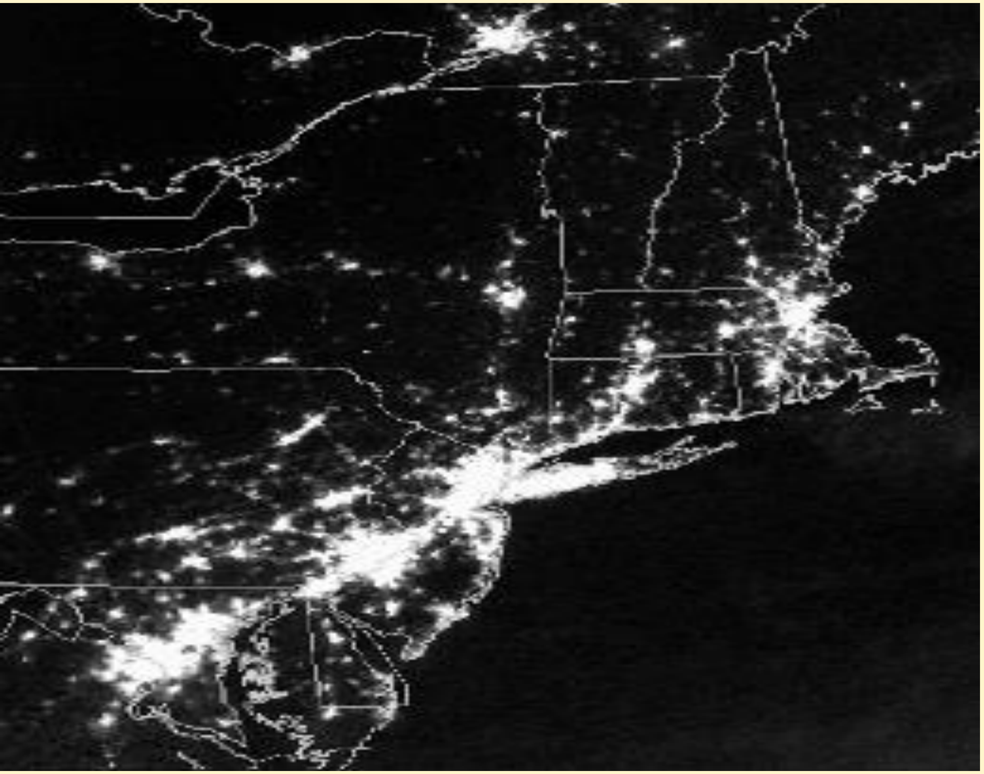
Some Typical Incidences of Power Blackouts

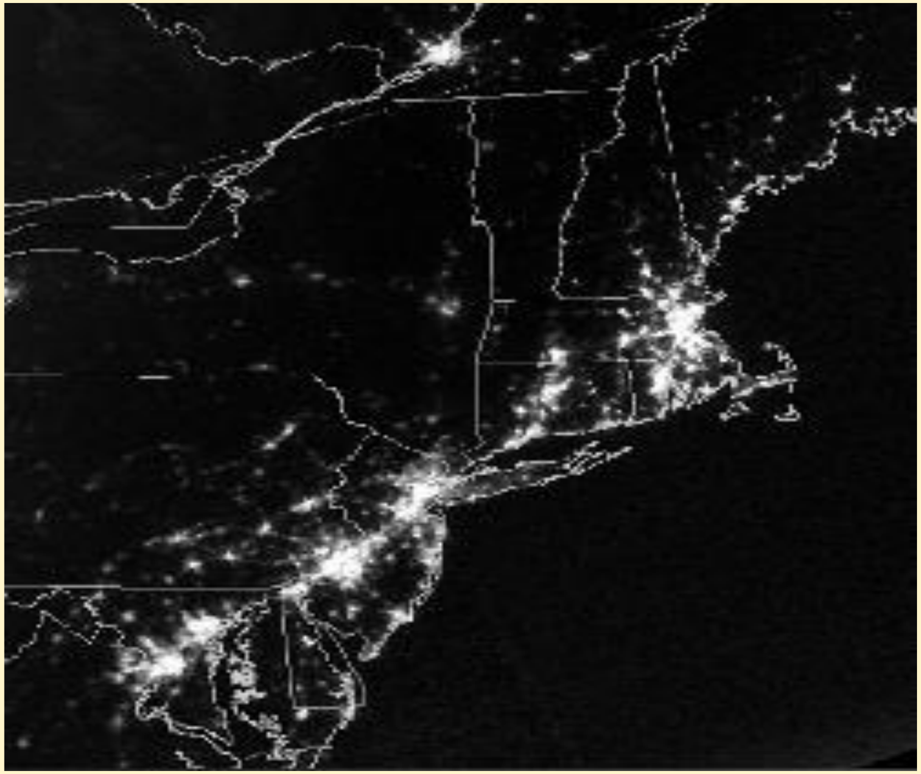
Data Source: http://en.wikipedia.org/wiki/List_of_power_outages

Largest [edit]

article	millions of people affected	location	date
July 2012 India blackout	620	India	30 July 2012-31 July 2012
January 2001 India blackout	230	India	2 January 2001
November 2014 Bangladesh blackout	150	Bangladesh	1 November 2014
2015 Pakistan blackout	140	Pakistan	26 January 2015
2005 Java–Bali blackout	100	Indonesia	18 Aug 2005
1999 Southern Brazil blackout	97	Brazil	11 March 1999
2009 Brazil and Paraguay blackout	87	Brazil, Paraguay	10–11 Nov 2009
2015 Turkey blackout	70	Turkey	31 March 2015
Northeast blackout of 2003	55	United States, Canada	14–15 Aug 2003
2003 Italy blackout	55	Italy, Switzerland, Austria, Slovenia, Croatia	28 Sep 2003
Thailand Nationwide blackout of 1978	40	Thailand	18 Mar 1978
Northeast blackout of 1965	30	United States, Canada	9 Nov 1965

Snapshot: 15 August 2003 **Power Blackout**



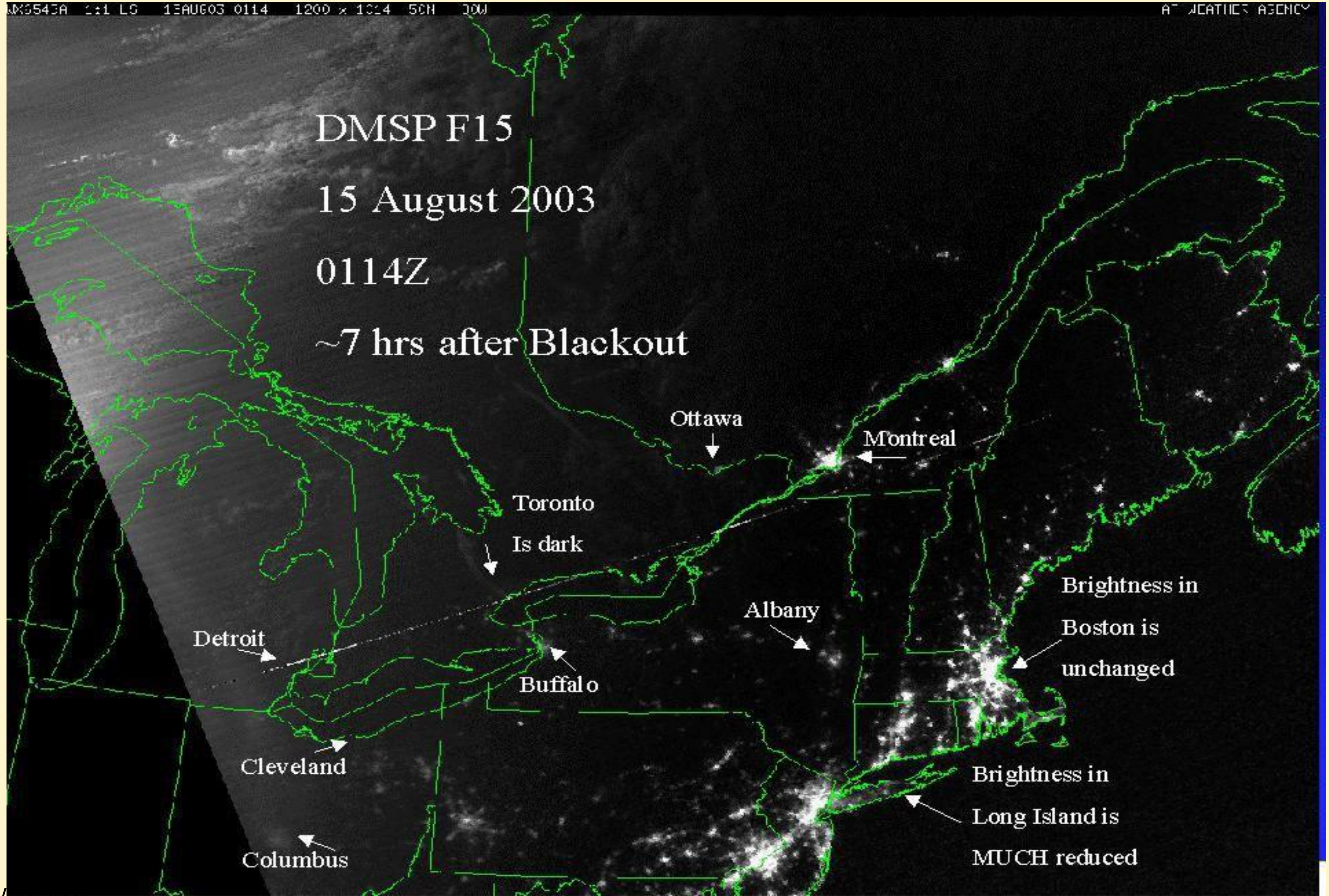


Satellite images

Before blackout **During blackout**

Reference: http://www.anl.gov/Media_Center/logos22-1/electricity.htm

A detailed map



Will It Happen Again ?

…1959, 1961, 1965, 1977, 2003, … …2020, 2021?

Major Power Blackout Records:
http://en.wikipedia.org/wiki/List_of_power_outages

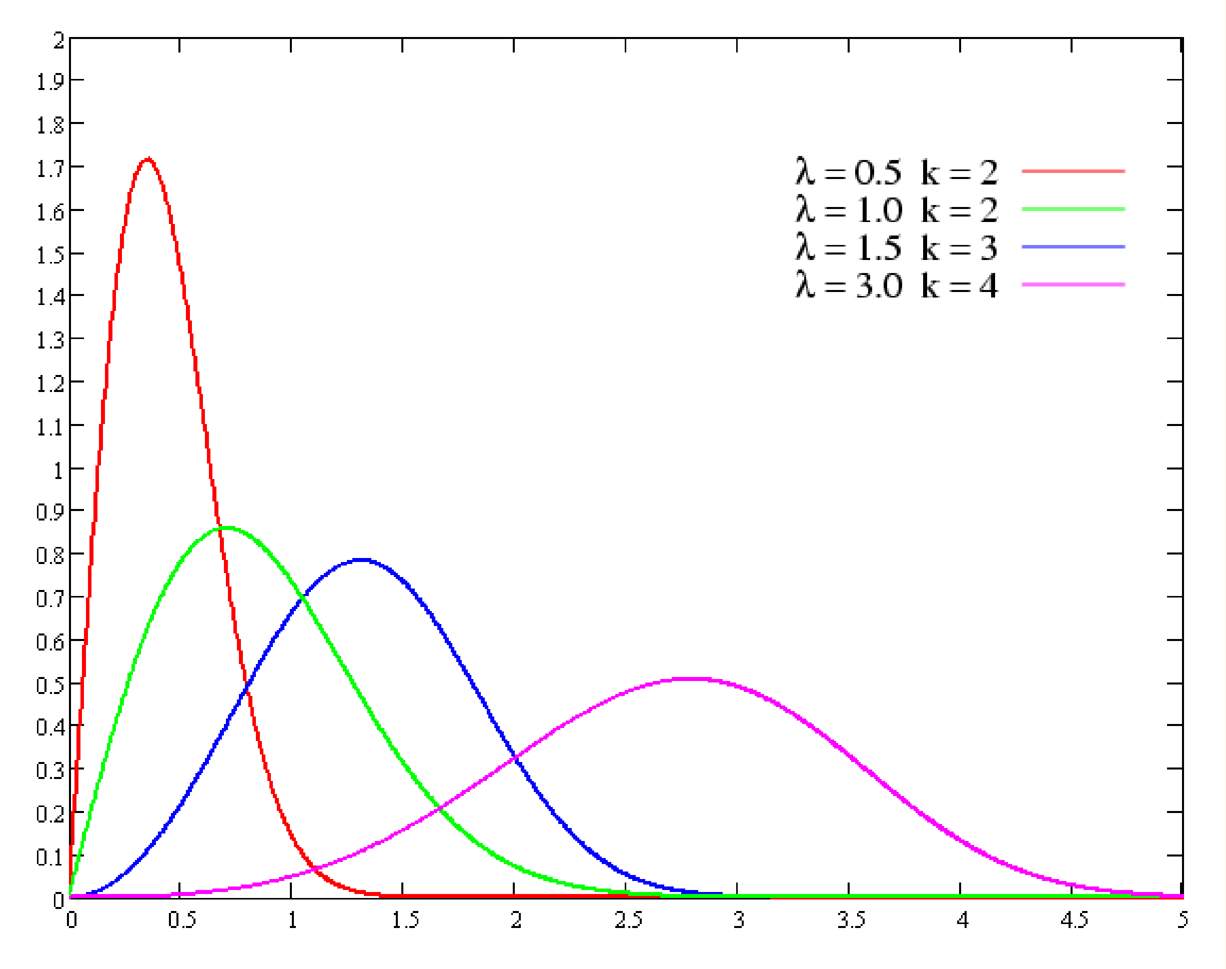
Cascading Failures: Models and Analysis

Models Based on Node Dynamics

- The Fiber-Bundle Model (FBM) is a conceptual graph framework, typically a BA model, for cascading failure analysis
- In the FBM, a set of large "fibers" (clusters of nodes) are located on the sites of a lattice, and each element is randomly assigned a security threshold, sampled from a given probability $\rho(\sigma_i) = 1 - e^{-(\sigma_i)^{\rho}}$ distribution — typically the Weibull (cumulative) distribution
- Then, this set of nodes are loaded against their loading security thresholds $\sigma_i > 0$
- For a network load σ, any node with $\sigma_i < \sigma$ is considered failed
- The individual load carried by each failed node is then equally redistributed through connections (edges) to their neighbors
- This redistribution may induce secondary failures, which in turn may trigger tertiary failures, and so on, leading to collapse

Weibull distribution

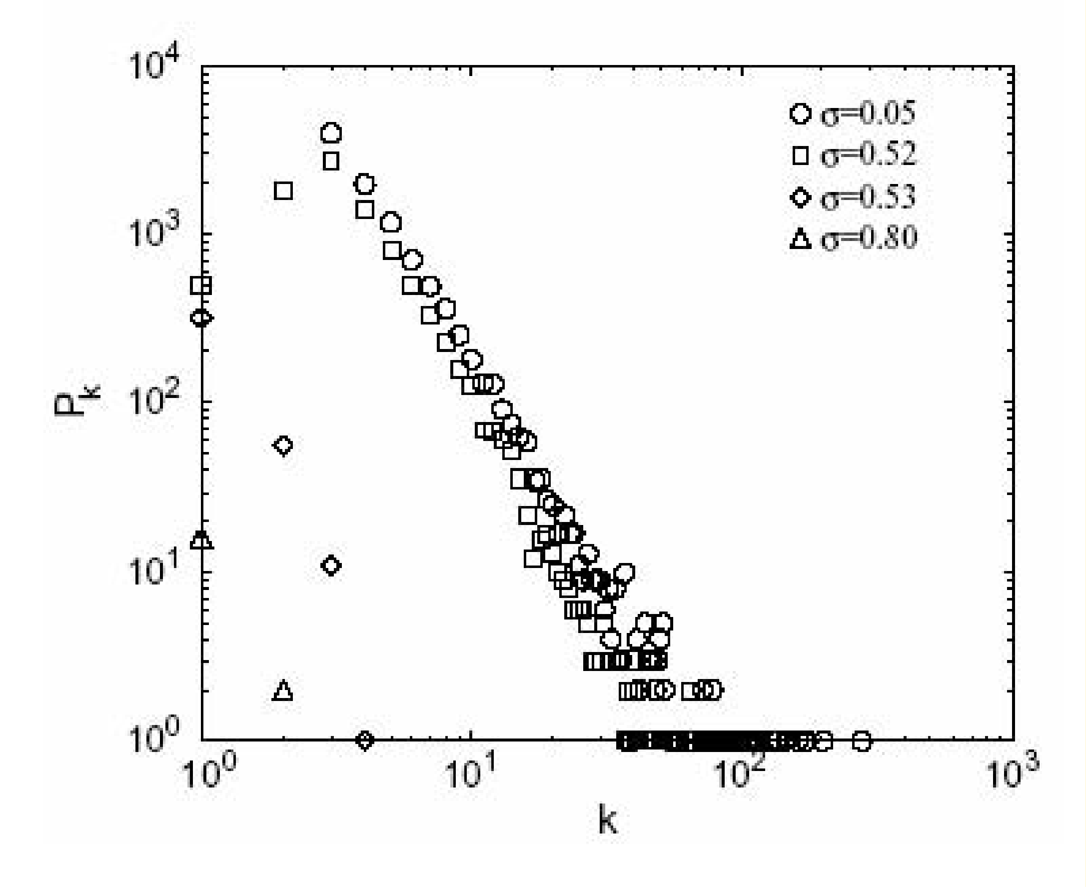
$$f_k(x) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k}$$



k and λ are constant parameters

Cumulative distribution function: $1-e^{-(x/\lambda)^k}$

Fiber-Bundle Model based on BA Network



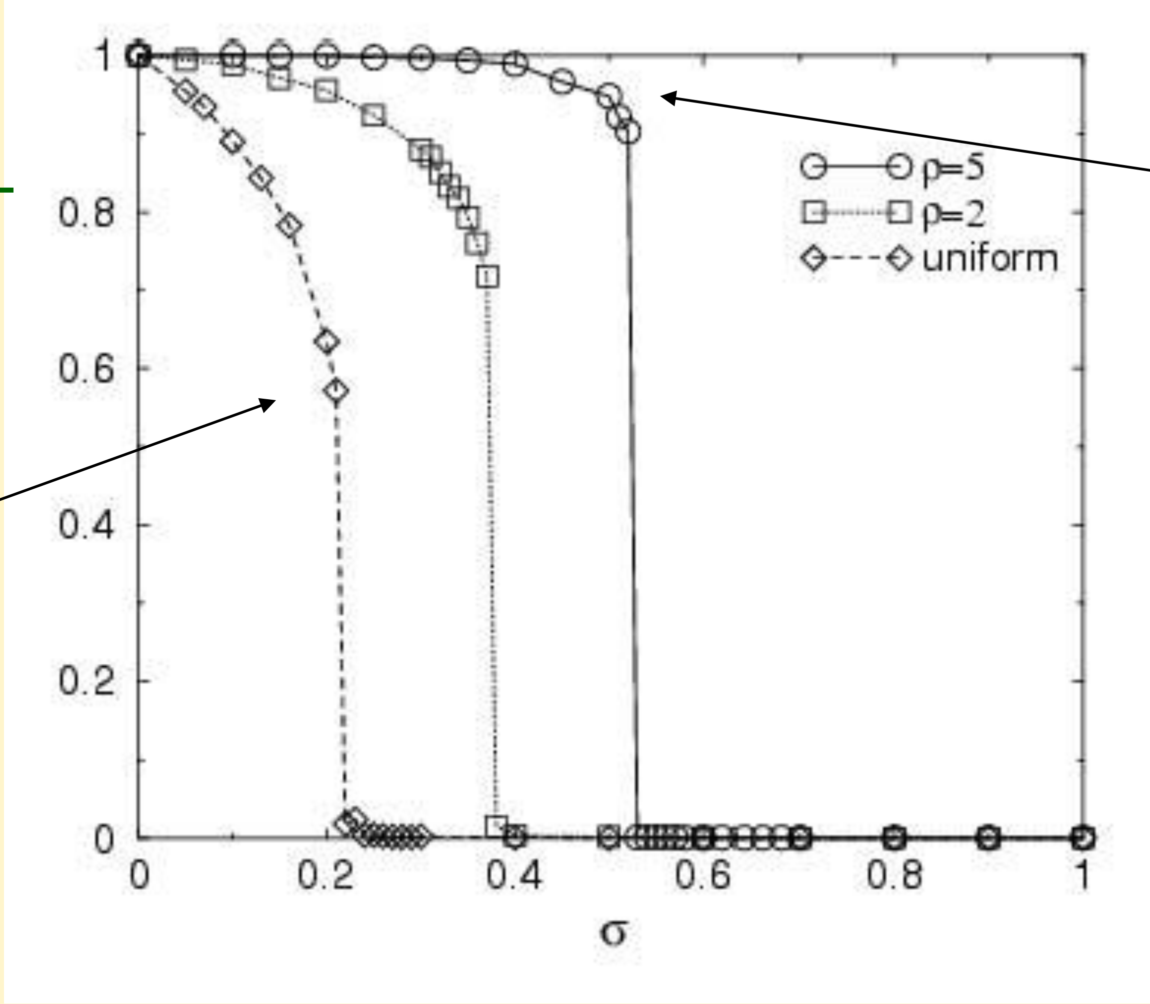
Power-law distribution of degree- κ nodes for small σ

For small delta, it does not change the network topology too much

Moreno et al. (2002)

Fiber-Bundle Model

Fraction of connected sub-nets



more severe (whole network collapses even when only 10% nodes are disconnected)

less severe (network not collapses unless 40% nodes are disconnected)

BA scale-free network: N **= 10,000** $\rho(\sigma_i) = 1 - e^{-(\sigma_i)^\rho}$

Moreno et al. (2002)

Node-Betweenness Model

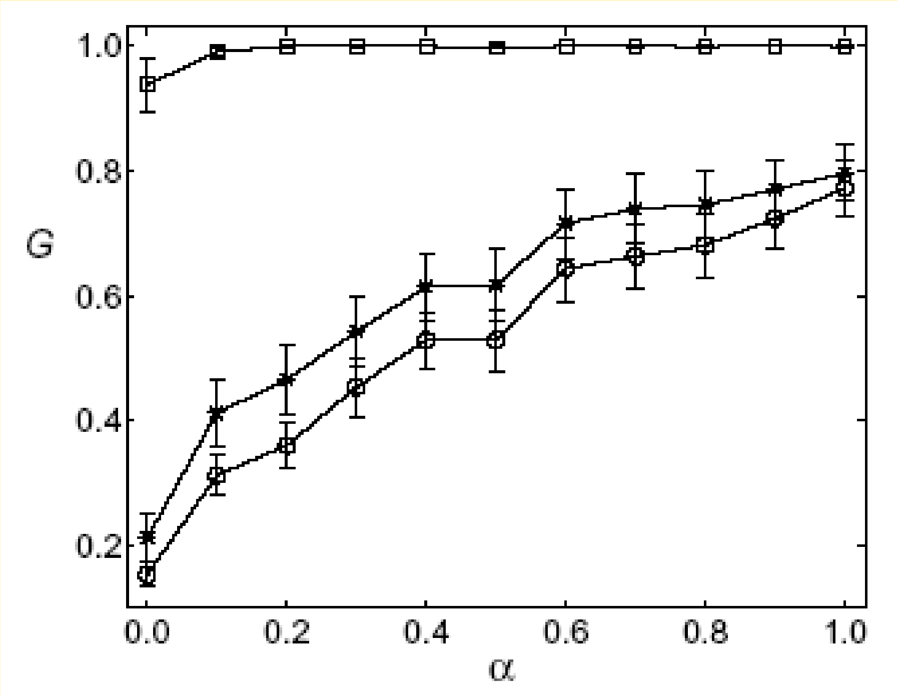
- Let the maximum-load capacity of node i be proportional to its initial loading L_i :

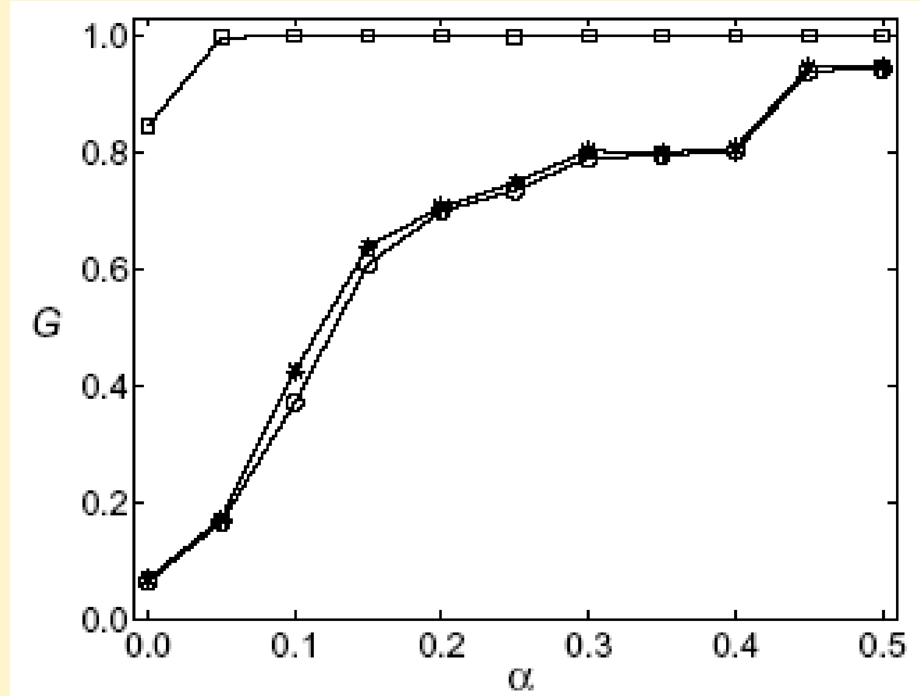
$$C_i = (1+\alpha)L_i \qquad i = 1, 2, \cdots, N$$

where constant $\alpha \geq 0$ is the tolerance parameter

- Normally, there are free flows traversing on the network
- When a node fails to work, traffic cannot go through it, so the node is considered being separated therefore will be removed
- Then, the total traffic flows are uniformly redistributed over the rest of the network
- This generally changes the loads of the remaining nodes and the distribution of the shortest paths

Node-Betweenness Model





Scale-free network model **AS-level real Internet**

G **is the fraction of remaining connected sub-nets over the whole network**

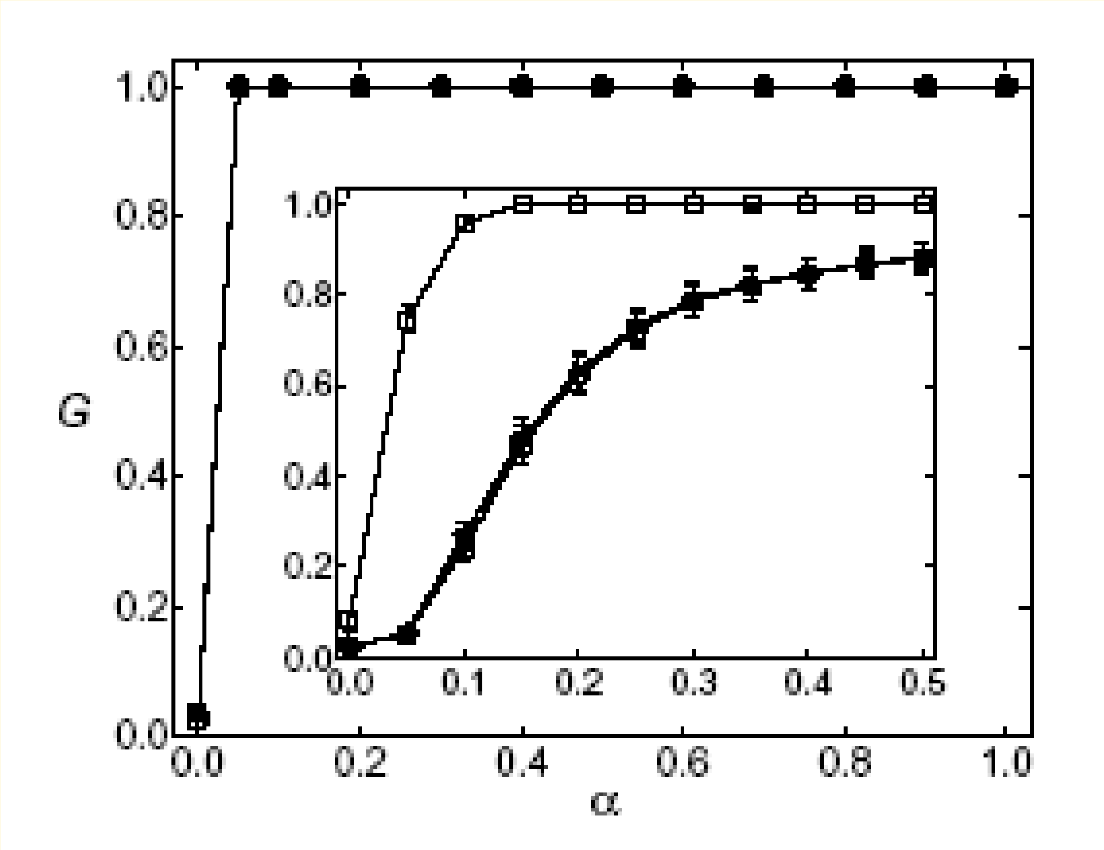
[square curves correspond to random node failures, star curves to largest-degree node failures, and circle curves to biggest-betweenness node failures]

Observation 1: smaller tolerance → more severe failure

Observation 2: largest-degree/betweenness failures is more severe than random failures

Motter et al. (2002)

Node-Betweenness Model



Large picture:

Random network

Small picture

Scale-free network

Comparison of cascading failures on random and scale-free networks

square – random attack, circles – intentional attack. $N = 5{,}000$, $\langle k \rangle \approx 3$

Observation 1: For random graphs, both random and intentional attacks perform similarly

Observation 2: For scale-free networks, intentional attacks are more severe than random attacks

Cascading Failures: Models and Analysis

Models Based on Edge Dynamics

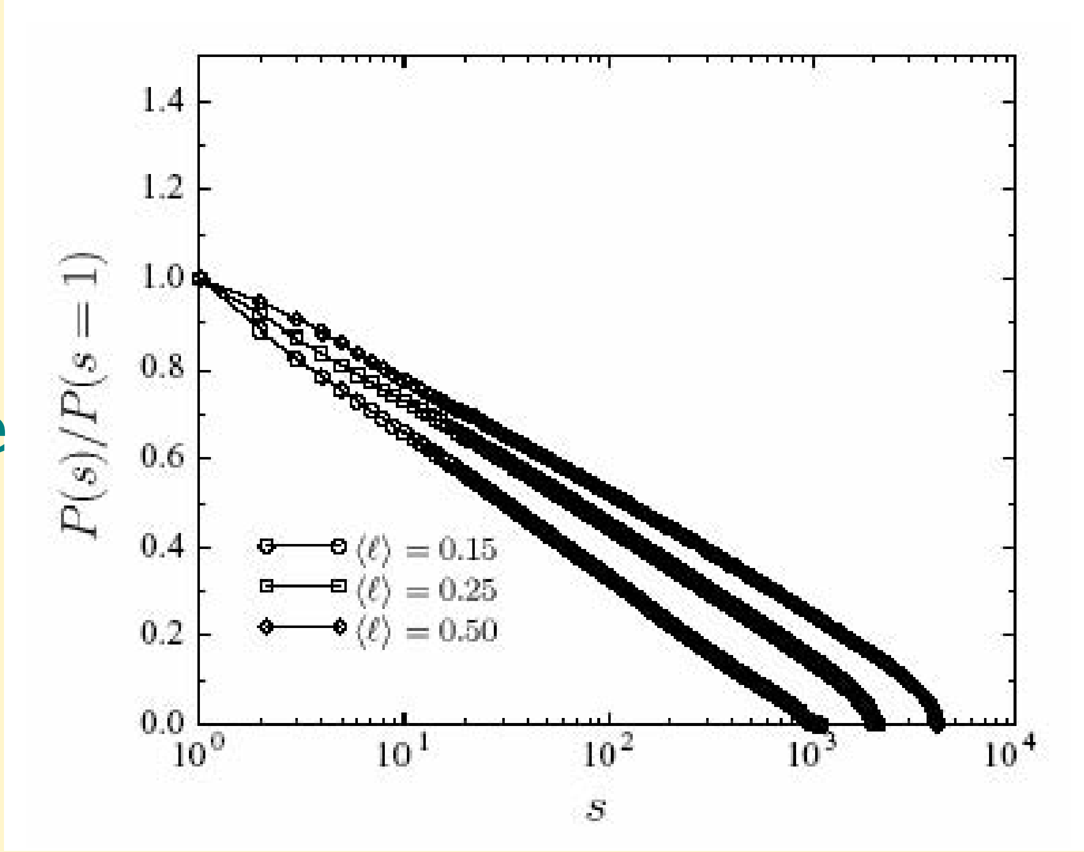
Consider loads on edges

Let s be the number of overloaded edges

Let $P(s)$ be the cumulated failures (normalized)

Let $\langle \ell \rangle$ be the average edge loading over the whole network

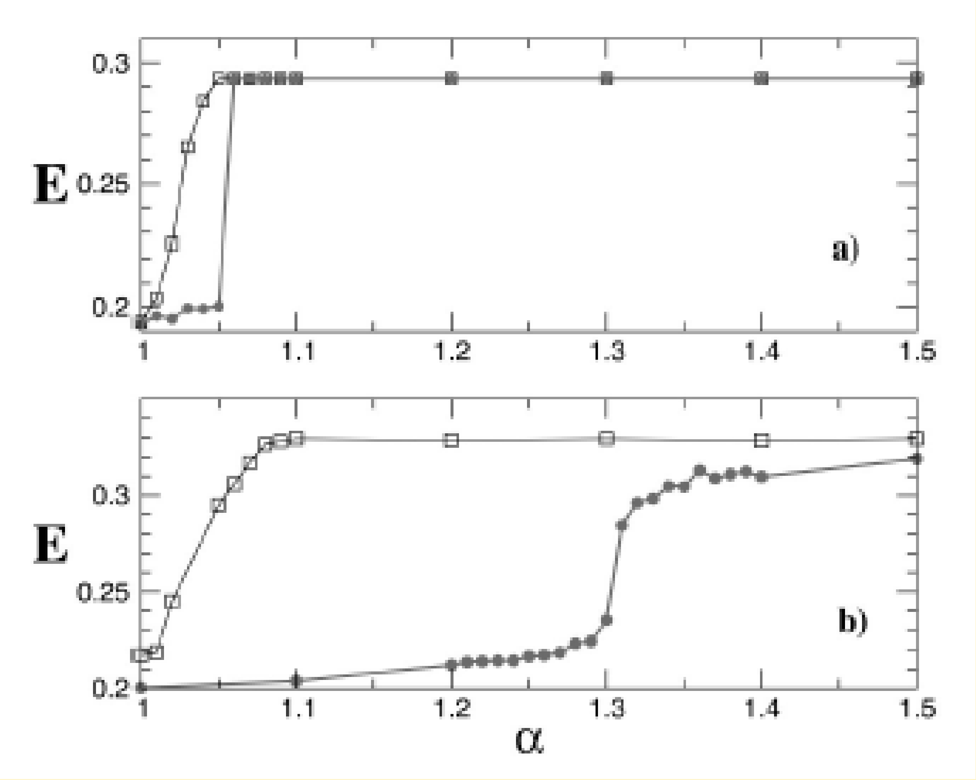
Observation: it is basically independent of average edge loading (because the three curves are similar and yet 0.15<<0.25<<0.5)



Cascading Failures: Models and Analysis

A model based on both node and edge dynamics

$e_{ij} \in [0,1]$ – efficiency of edge (i,j), e.g., bandwidth



E – average efficiency:

$$E(G) = \frac{1}{N(N-1)} \sum_{i \neq j} e_{ij}$$

$L_i(t)$ -- loading

$C_i = \alpha \cdot L_i(0)$ -- capacity

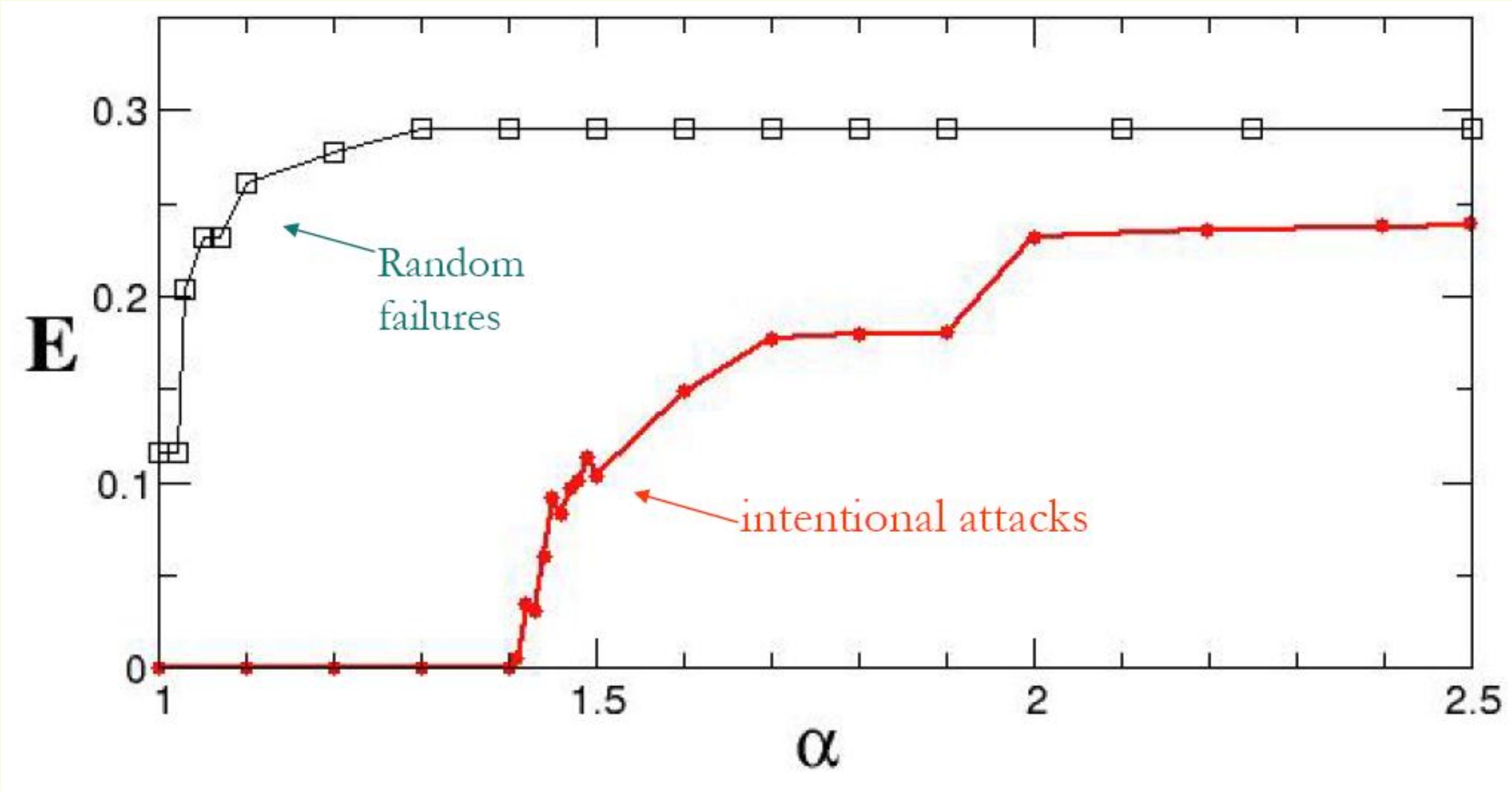
$\alpha \geq 0$ -- tolerance

a) ER network, b) BA network. [squares -- random node failures, circles -- biggest node failures]

Observations

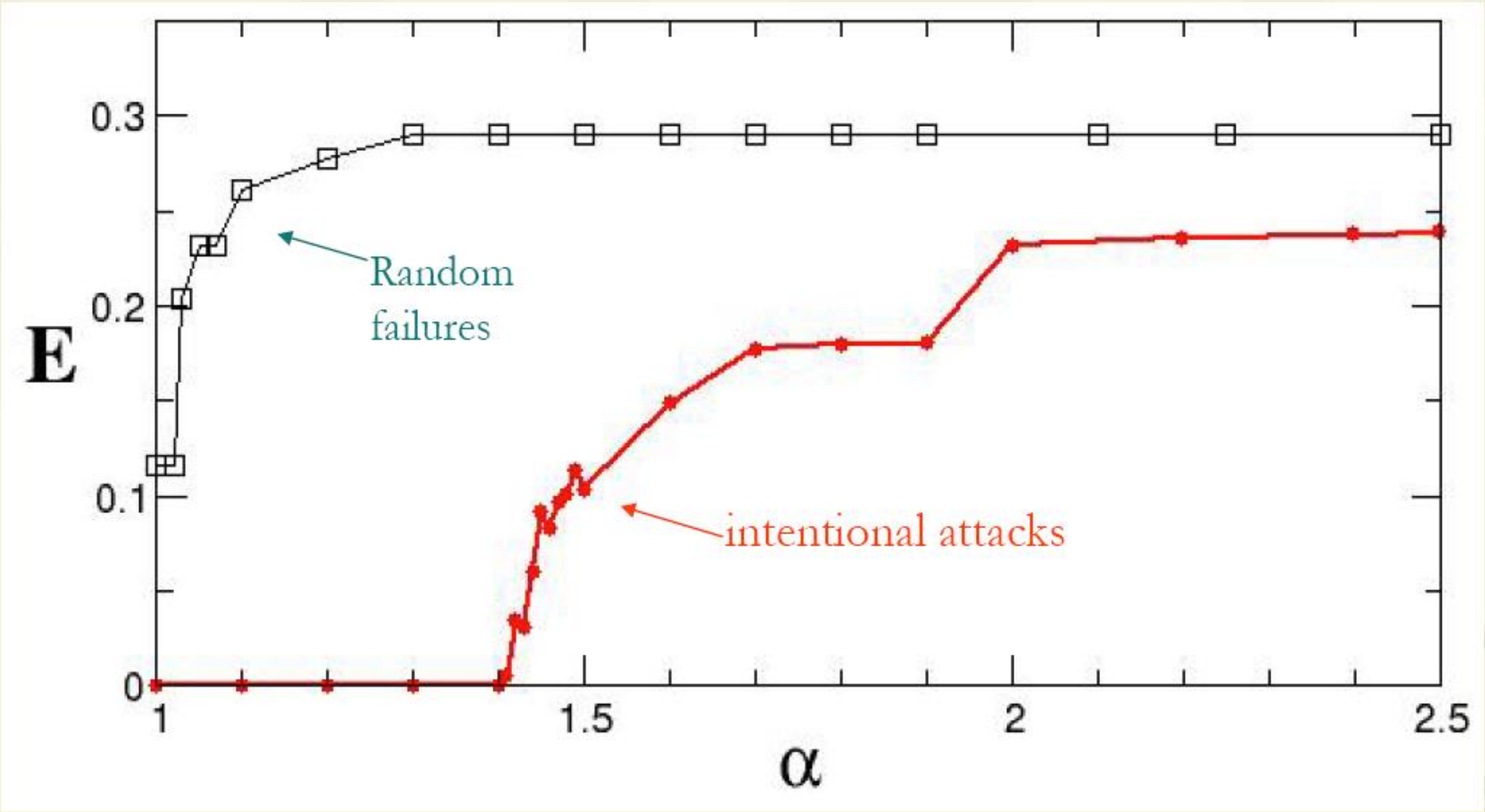
- It can be seen that both networks have their threshold values α_c : when $\alpha < \alpha_c$, *the network* collapses, with E(G)< 0.25 . But the threshold of the ER network is much smaller than that of the BA network, showing that ER random-graph networks are more robust than the BA networks in resisting cascading failures.
- Besides, for both ER and BA networks, the intentional node-removal scheme based on loadings is easier to create cascading failures than the random removal scheme.

AS-level Internet (Real Data)



Crucitti et al. (2004)

US Power Grid (Real Data)



Crucitti et al. (2004)

Models based on Load Redistributions

Consider a BA network with N nodes

Initialize node i with a load $L_i = ak_i^{\alpha}$, where a and α are tunable constant parameters.

When node i fails during a dynamical process, then its initial load will be redistributed to every of its neighboring nodes.

The probability that a load from node i is redistributed to node j is given by $\Pi_j = \frac{k_j^{\alpha}}{\sum_{l\in\Omega_i} k_l^{\alpha}}$, where Ω_i is the neighborhood of node i.

Thus, the additional load received by node j is

$$\Delta L_{ji} = L_i \Pi_i = L_i \frac{k_j^{\alpha}}{\sum_{l\in\Omega_i} k_l^{\alpha}}$$

Wang *et al.,* Physica A (2008)

Models based on Load Redistributions

Assume: Each node has a capacity $C_i = TL_i$ of holding loads, with a constant $T \geq 1$.

Thus, if the accumulated loads of a node j is over its capacity, namely, $L_j + \sum_{i \in \Omega_j} \Delta L_{ji} > C_j$, then this node j will fail.

As a result, its total load will be redistributed to all its neighbors, which may cause cascading failures.

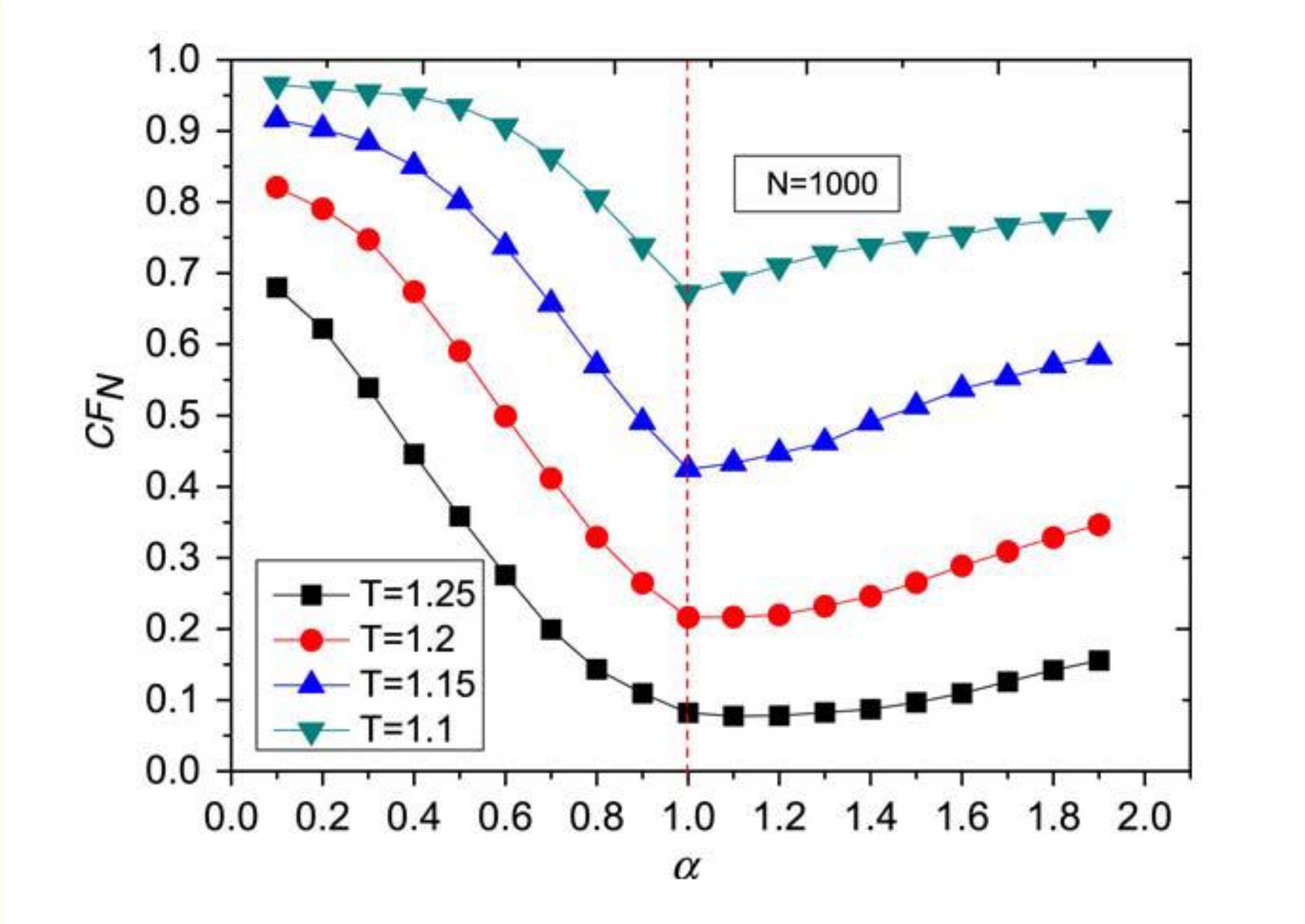
To measure the ability of the network against cascading failures, define an index CF_i for each node i, to be the total number of failed nodes after the failing process is ended.

The average index over the whole network:

$$CF_N = \frac{1}{N(N-1)} \sum_i CF_i$$

-- the smaller the better

Wang *et al.*, Physica A (2008)



Simulation Results: when $\alpha = 1$, in the case of the same T, all CF_N reach the minimum, implying the strongest robustness of the network against cascading failures.

Wang *et al.,* Physica A (2008)

Interdependent Networks

A large-scale complex network in the real world typically is a network of networks: an integrated network of many different kinds of interdependent networks.

Example: human mobility networks consist of short-range interconnected flows by cars, trains and other means of transportation networks, interconnected with long-range commuting flows like airline flights, on top of which are communication networks (mobile phone calls and Internet message exchanges).

Interdependent Networks of human mobility in the North America
blue – short-range
brown – long-range

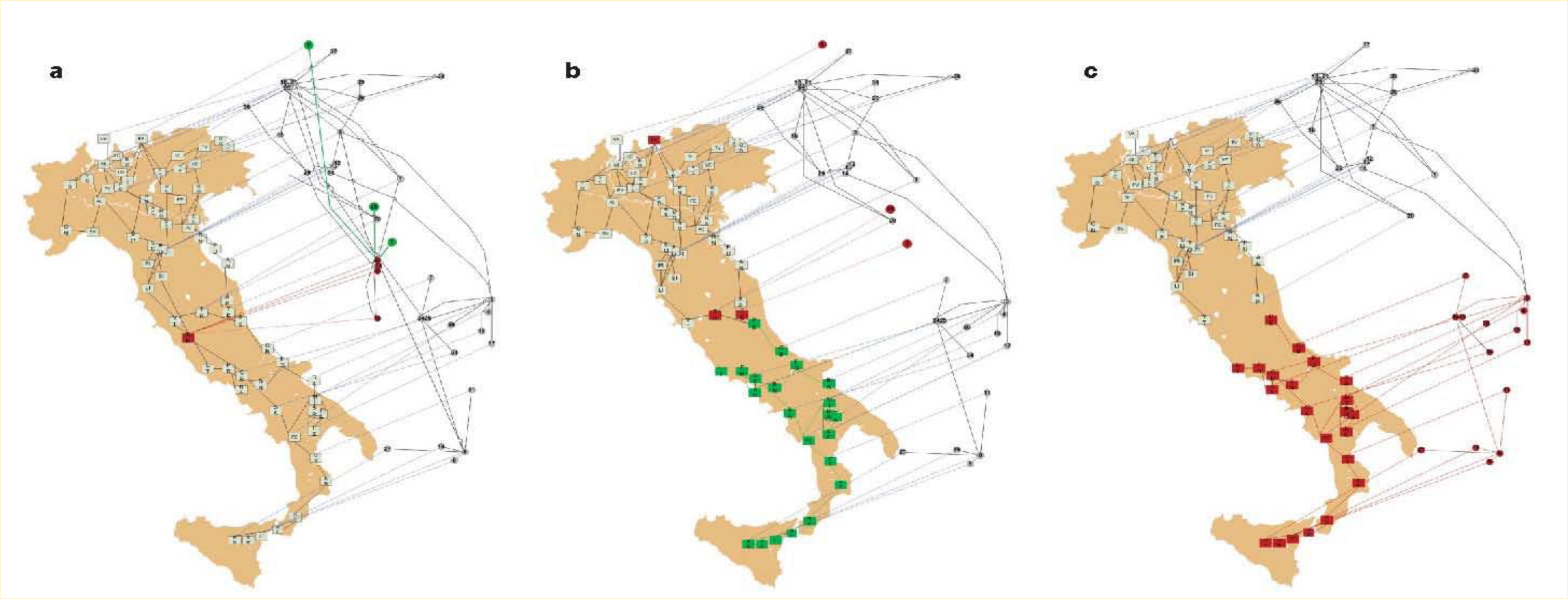
Buldyrev et al. Nature (2010) 464: 1025-1028

Cascading Failures on Interdependent Networks

- **In a tightly interconnected network infrastructure, the failure of nodes in one network not only can lead to consequent failures of some other nodes in the same network but also can lead to failures of nodes in another network, which in turn can cause further failures in the first network, ultimately leading to collapse of the entire networked system.**
- **A broader degree distribution increases the vulnerability of interdependent networks to random failures of nodes, which is opposite to the scenario in a single heterogeneous network.**
- **Need to consider interdependence among different networks in an integrated network in designing its strong and robust ability against random attack and failures.**

Buldyrev et al. Nature (2010) 464: 1025-1028

A Real-World Example: Italy Blackout in 2003



a One power station failed (red) →a few associated Internet servers failed (red), and some Internet servers are isolated (green)

b The above failed Internet servers (red) and associated edges are removed→ related power stations failed (red), and some power stations became isolated (green)

c The above failed power stations (red) and Internet servers (red) are removed →both networks failed (both red) →system collapsed

Buldyrev et al. Nature (2010) 464: 1025-1028

Cascading Failures: Models and Analysis

- Other Models:
  - Binary influence model
  - Sand-pile model
  - OPA model -- ORNL-PSerc-Alaska model used to study the blackout dynamics in power transmission grids, where ORNL stands for Oak Ridge National Laboratory; PSerc stands for Power Systems Engineering Research Center; both in USA
  - CASCADE model
  - ……

- Cascading Failures in Coupled Map Lattices

Naming Game with Finite Memories

An Example of Cascading Reactions

Outline

- Brief review of the naming game with <u>infinite</u> memories: theory and simulations on different networks

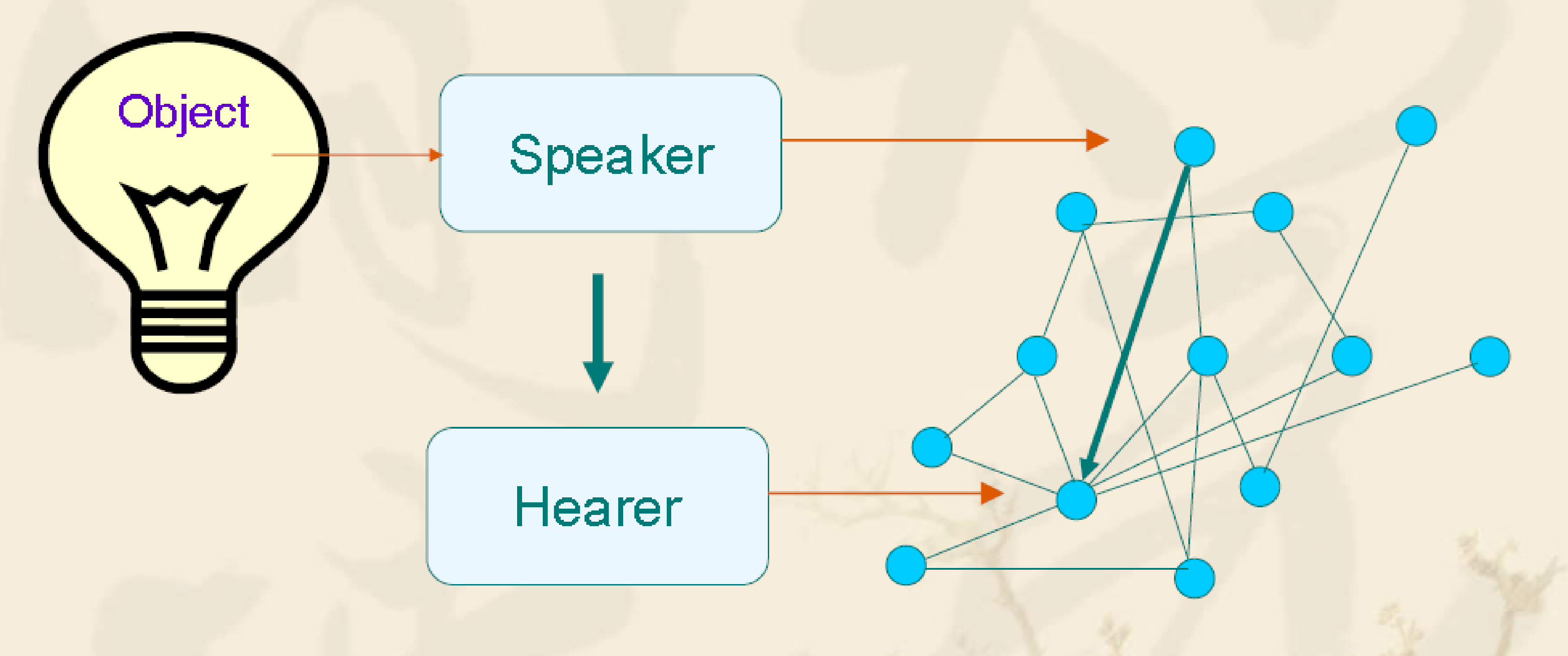
- Introducing the naming game with <u>finite</u> memories: theory and simulations on different networks

Naming Game: Introduction

Naming Game is an experiment using computer simulations to explore the emergence of shared lexicons in a population of agents about some objects they observed.
In the simplest (minimal) naming game, agents interact with each other, where repeated interactions lead to the development of a common word for naming a single object.

By varying experimental parameters, it is possible to explore the effects of environmental issues (such as memory limitations, uncertainties) and the contacts between different language groups.

Minimal Naming Game



Here, consider the simplest experiment:

A population of agents are trying to learn one common word

Minimal Naming Game

Consider:

a population of agents who are communicating on giving a name to a single observed object

Assume:

- only one object is being observed
- all agents have infinite memories
- all agents can keep different words in their memories

Minimal Naming Game

- Initially, each agent has an empty memory. The speaker invents a name for the object he observed, and tells the name to a hearer

At each time step:

- Two agents, a speaker and a hearer, are randomly selected from the population
- The speaker randomly picks a name from his memory and tells the name to the hearer (if the speaker has nothing in his memory, then he invents a name, or picks a name from a vocabulary, for the object)
- If the hearer already has the same name in his memory

  $\rightarrow$success

  else $\rightarrow$failure

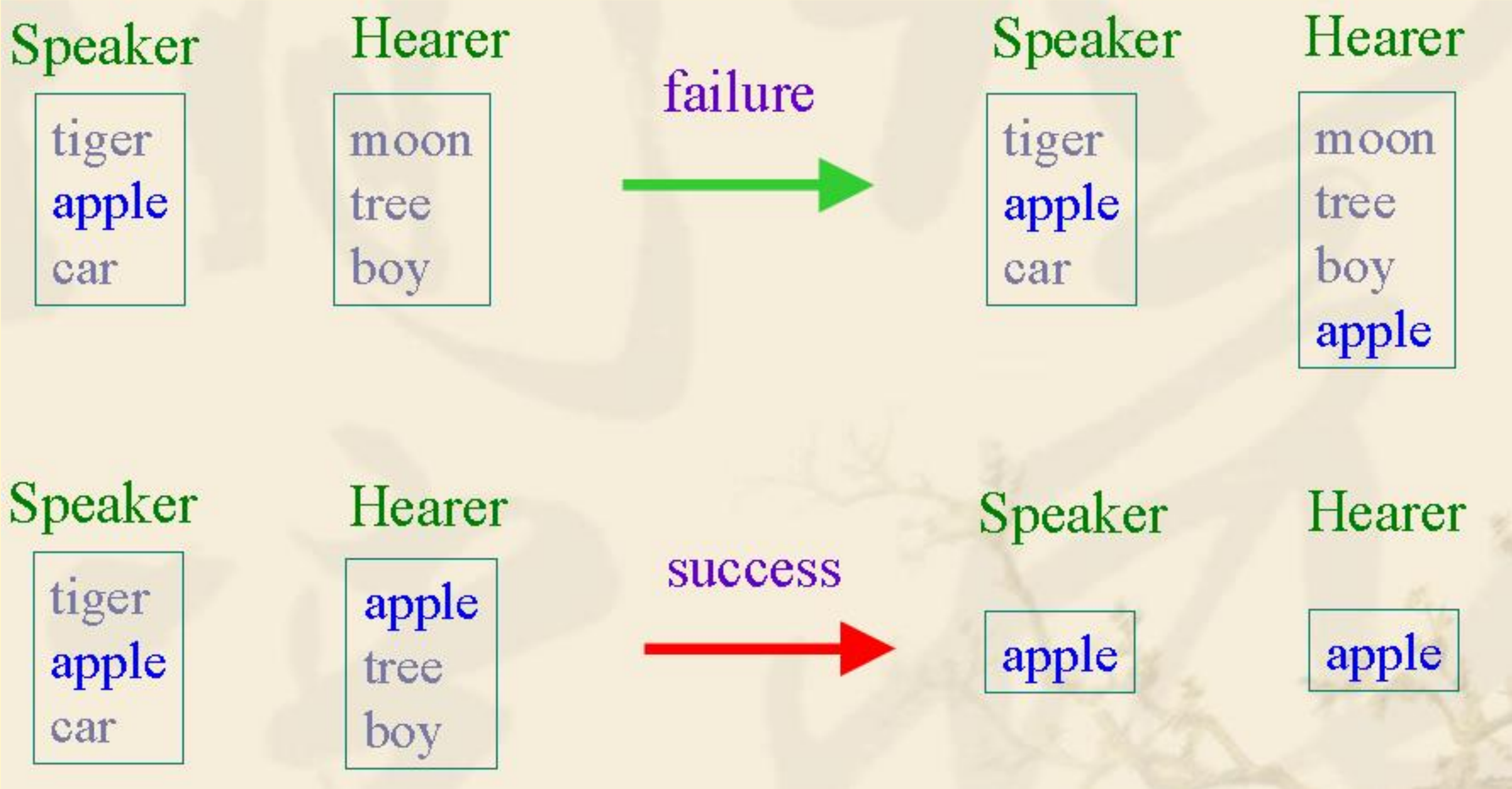
Minimal Naming Game
(with two agents)

If success → speaker and hearer retain the uttered word as the correct name and cancel all other words from their memories

If failure →hearer adds to his memory the word given by the speaker

Minimal Naming Game

(with two agents, one name)



Naming Game

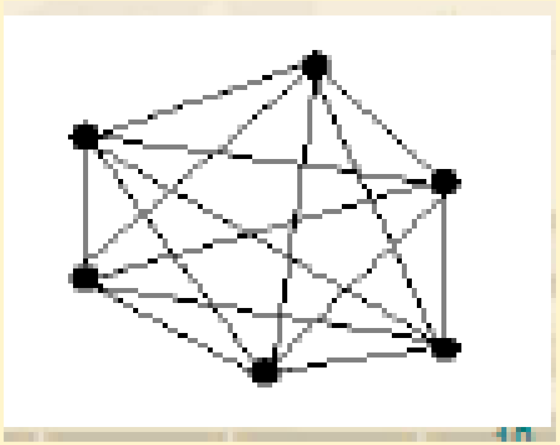
Mutual interactions among N agents in a network:

Agent $\longleftrightarrow$ node

interaction $\longleftrightarrow$ edge

Simplest community case:
A fully connected network

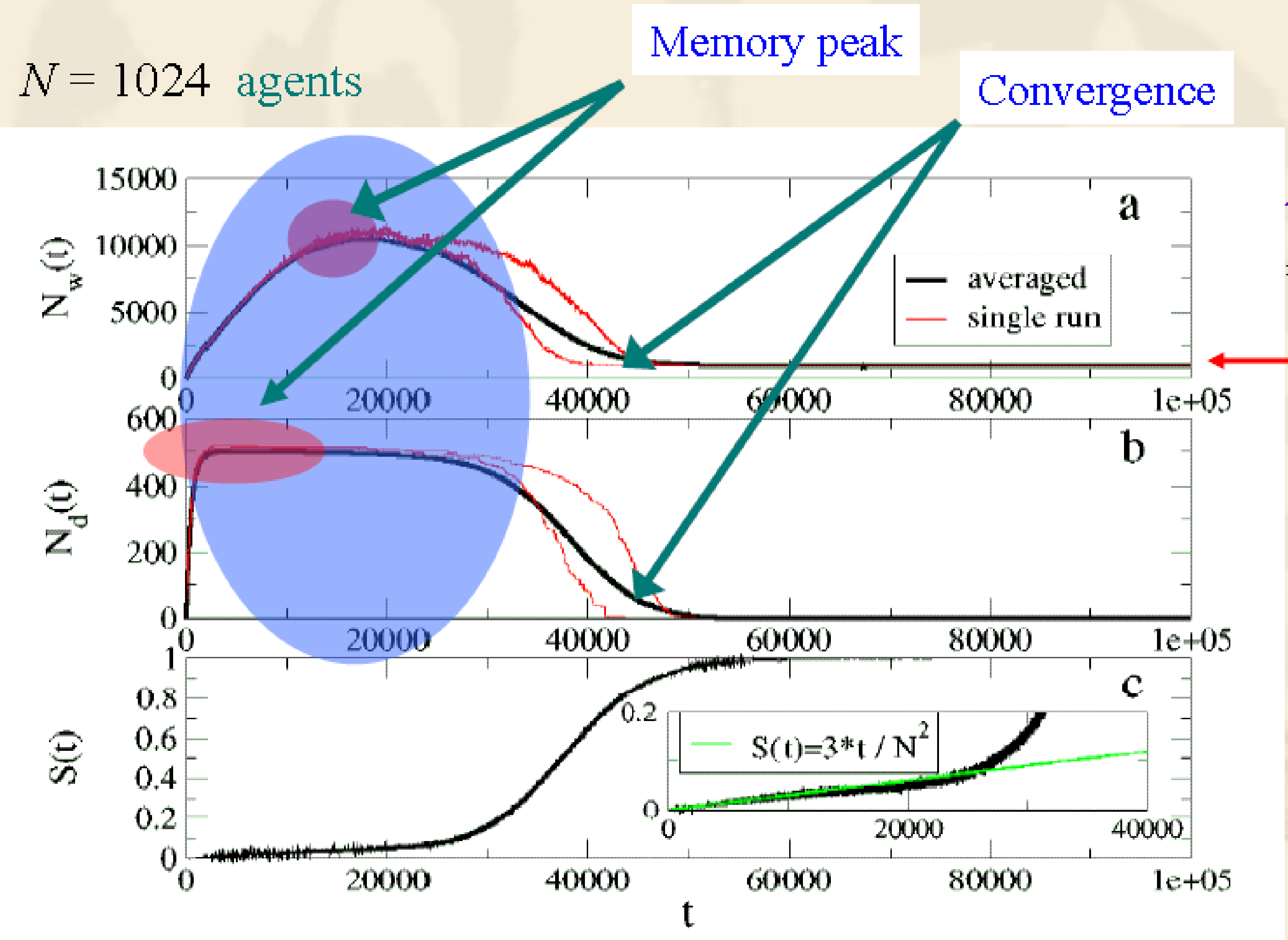
Every node interacts equally with all the others



Simulation Results - I

Agents on a fully-connected network

$N = 1024$ agents



total number of words = total memory Nw

This is 1 (magnified)

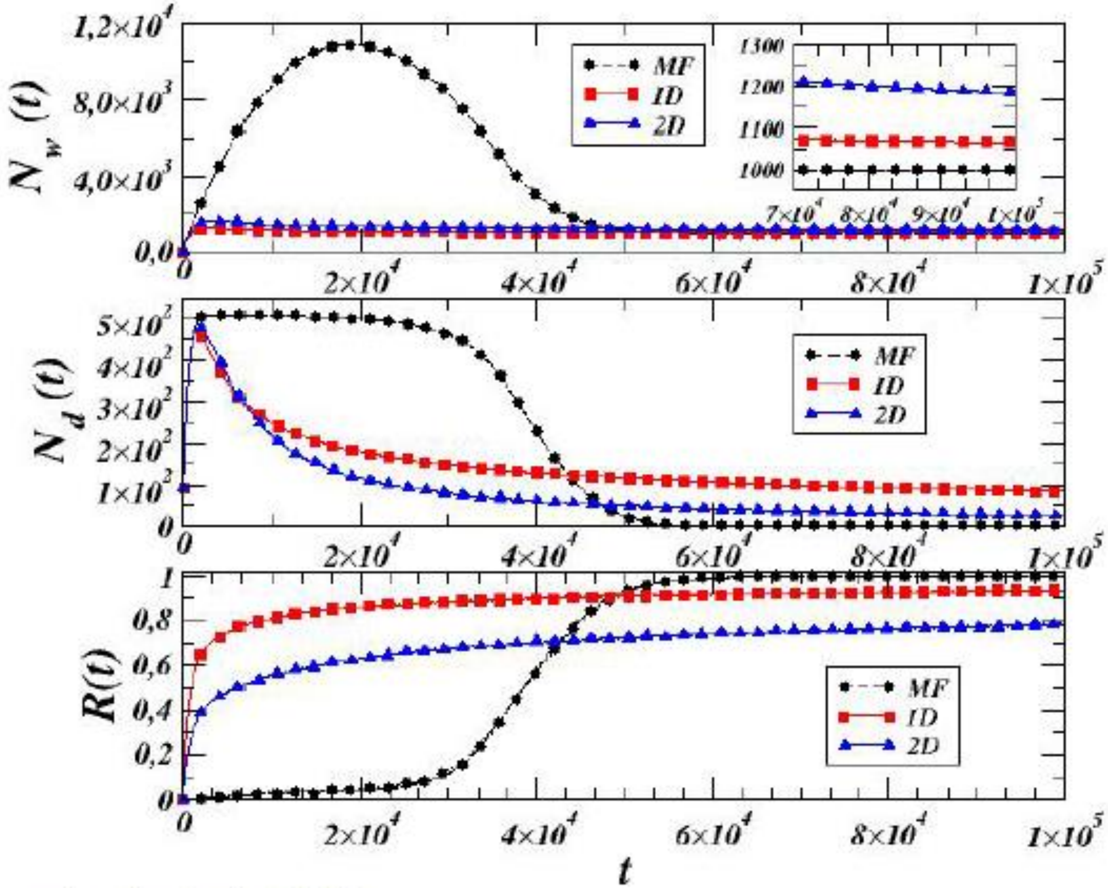
total number of different words Nd

success rate $S(t)$

(Baronchelli *et al.*, 2005)

Simulation Results – II

Agents on a regular lattice



(Baronchelli *et al.*, 2005)

N = 1000 agents

MF = fully-connected network

1D, 2D: agents on regular lattices

N_w = total number of words

N_d = total number of distinct words

R = success rate

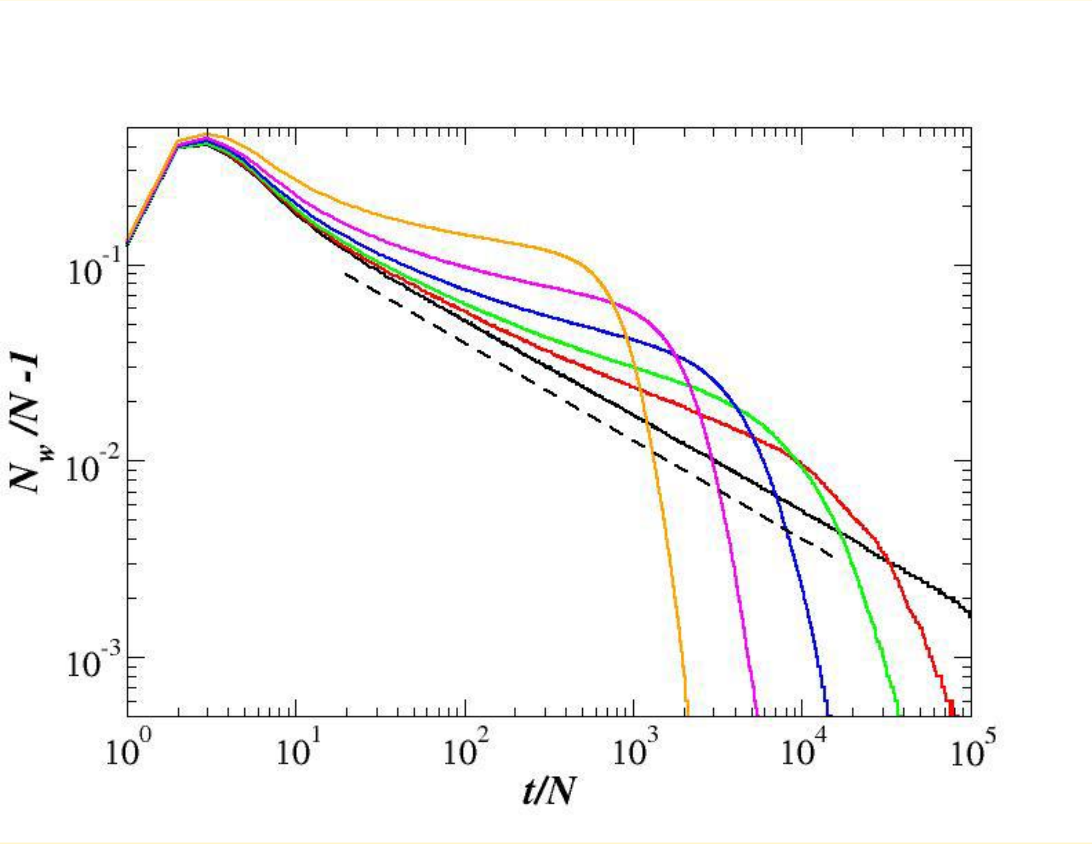
Naming Game

Comparison

	Fully Connected Networks	Regular Lattices
Maximum Memory	$N^{1.5}$	N
Convergence Time	$N^{1.5}$	N^3

N – number of agents

Naming Game
Agents on a small-world network



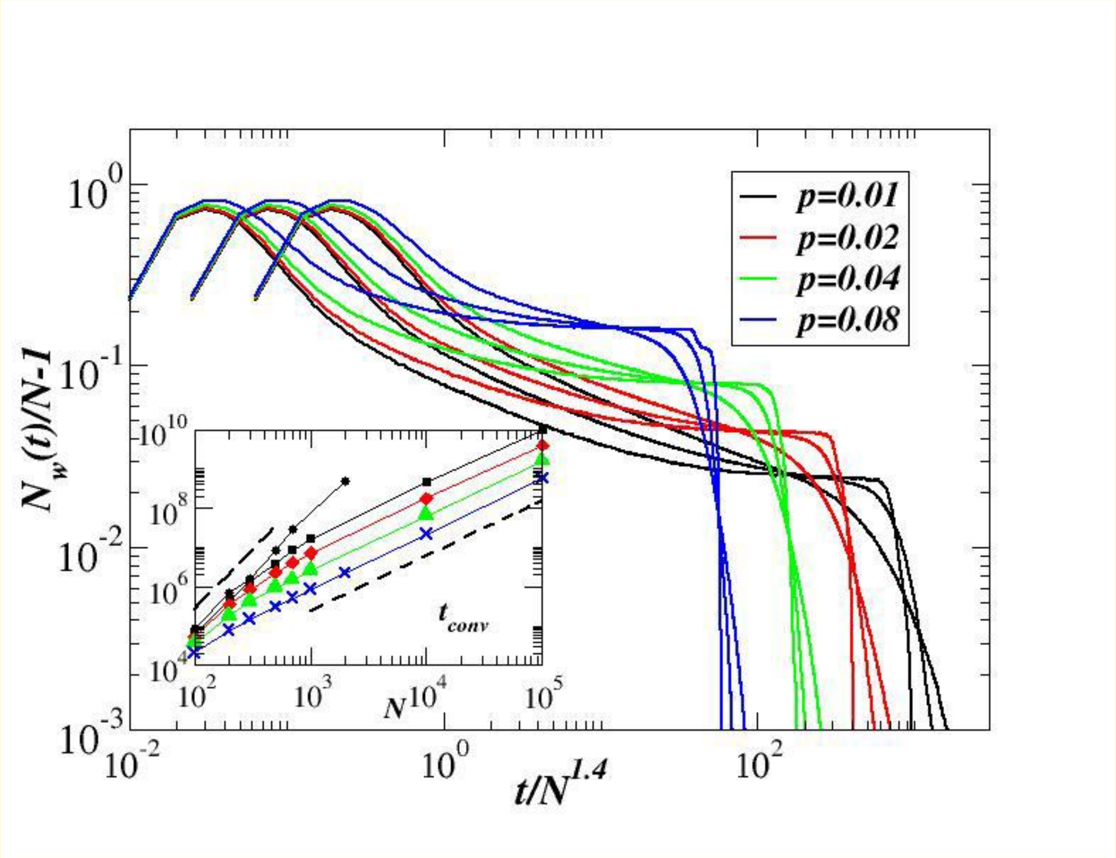
(Dall'Asta *et al.*, 2006)

N$_w$ = total number of words
p = 0: linear chain
other *p*: small-world

←increasing *p* (in different colors)

Naming Game

Agents on a small-world network



maximum memory: ~ N

convergence time: ~ $N^{1.4}$

N_w = total number of words

p = 0: linear chain

other p: small-world

Naming Game

Comparison

	Fully Connected Networks	Regular Lattices	Small-World Networks
Maximum Memory	$N^{1.5}$	N	N
Convergence Time	$N^{1.5}$	N^3	$N^{1.4}$

→Better not to have all-to-all communications
not to have a too-regular communication network topology

Naming Game
Agents on a heterogeneous network

Recall the original model:

→Select a speaker and a hearer *randomly from among all nodes*

Possible modifications:

- First select a speaker and then a hearer from speaker's neighbors
- First select a hearer and then a speaker from hearer's neighbors
- ……

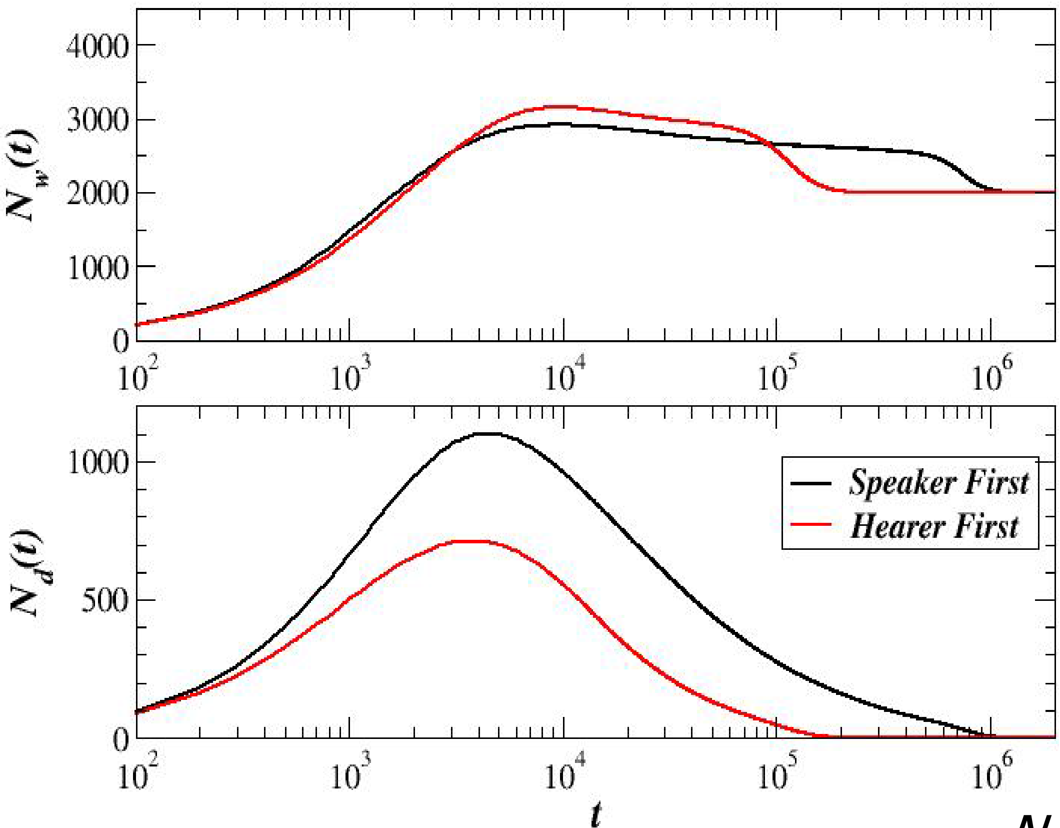
These modifications can be important in heterogeneous networks:

- a randomly selected node typically has a small degree
- in the neighborhood of a randomly selected node, there typically is a large node

Naming Game

Agents on a heterogeneous network

BA scale-free network:



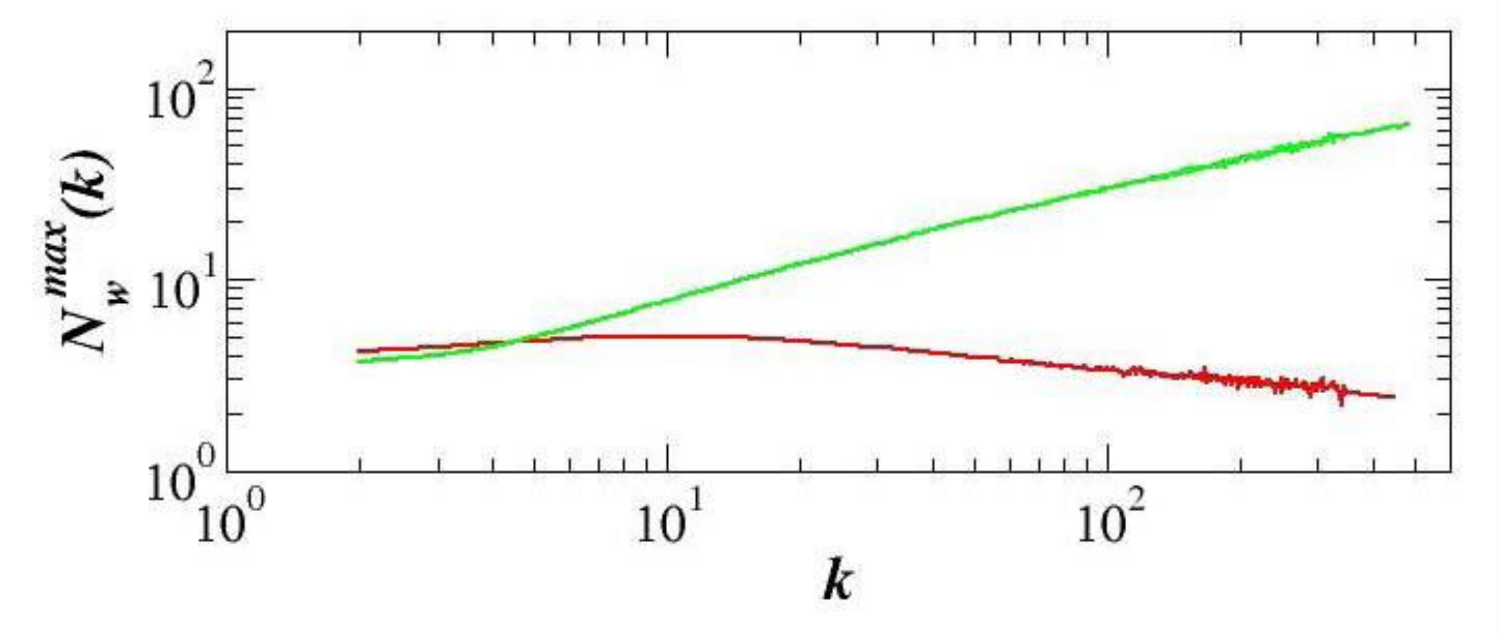
Different behaviors on the BA network

The importance of understanding the role of hubs on heterogeneous networks

N_w – total number of words
N_d – number of different words

Naming Game

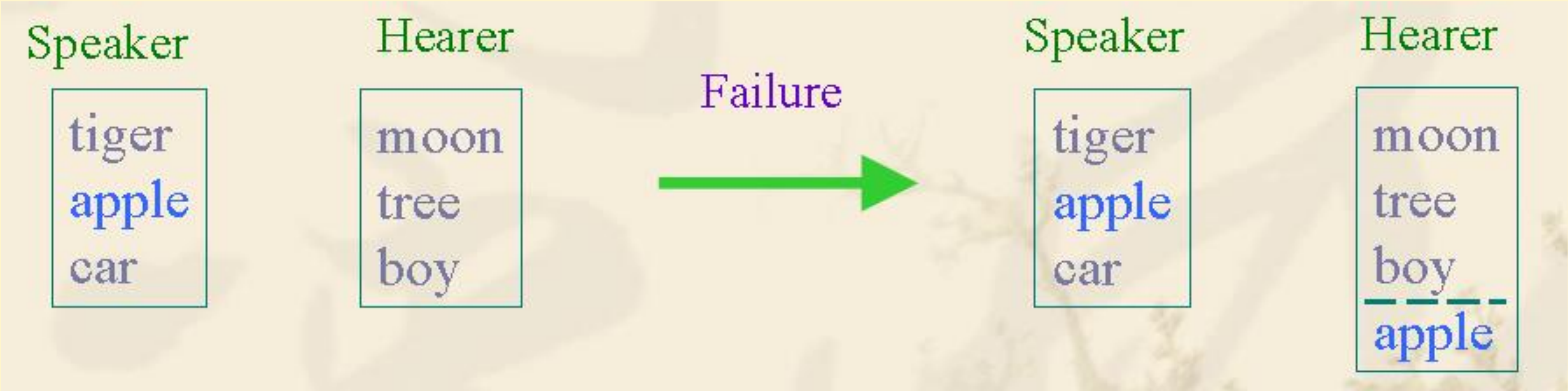
Agents on a heterogeneous network



Speaker first —hubs accumulate more words
Hearer first —hubs have less words
→faster spreading over the network

What happens if agents' memories are limited?

- It has been shown that the topology of a communication network can be quite important for the success of (fast) reaching a consent in naming an object by the whole population of agents
- One basic assumption therein is that all agents have infinite memories, which allows hearers to unlimitedly accumulate

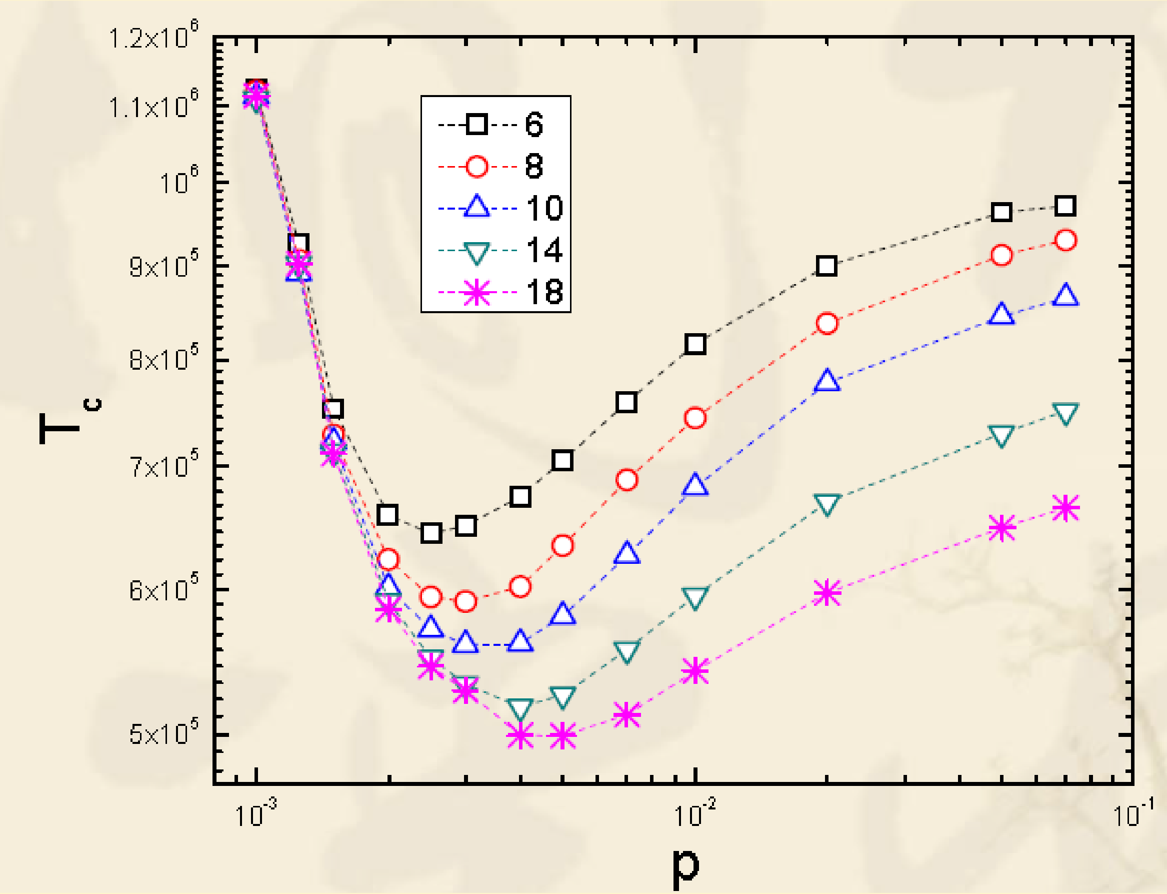


- So, the question is: What happens if agents' memories are limited?

Naming Game with Finite Memories

- Every agent has a limited memory of length L
  (when $L=\infty$ this new model reduces to the old one)
- At each time step:
- First, a speaker is randomly selected; then, the speaker randomly selects a hearer from its neighbors
- The speaker randomly picks a name from his memory and tells the name to the hearer:
  - if the hearer already has the name in his memory → success → both agents keep this name and discard all other names
  - else → failure →
    - if hearer's memory is not full then add the new name to his memory
    - else, with probability 0.5 a randomly selected old name is replaced by the new one (i.e., with probability 0.5 nothing is done)

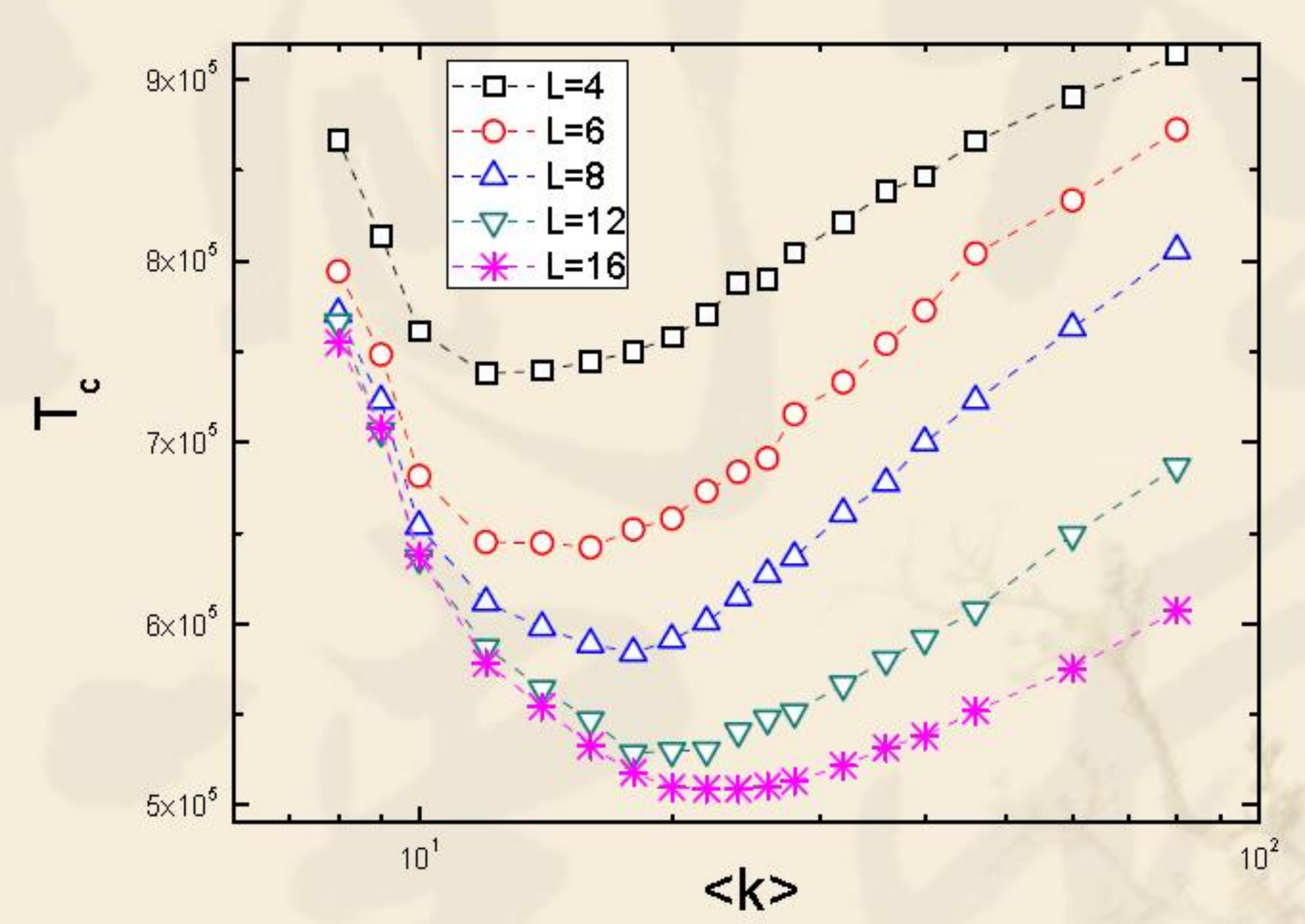
Simulations on Random Networks -I



- N = 5,000
- p – random network generation probability
- Each curve is the average of 1000 runs
- For memory lengths L = 6, 8, 10, 14, 18

Tc – convergence time →
There are optimal convergence times

Simulations on Small-World Networks -I

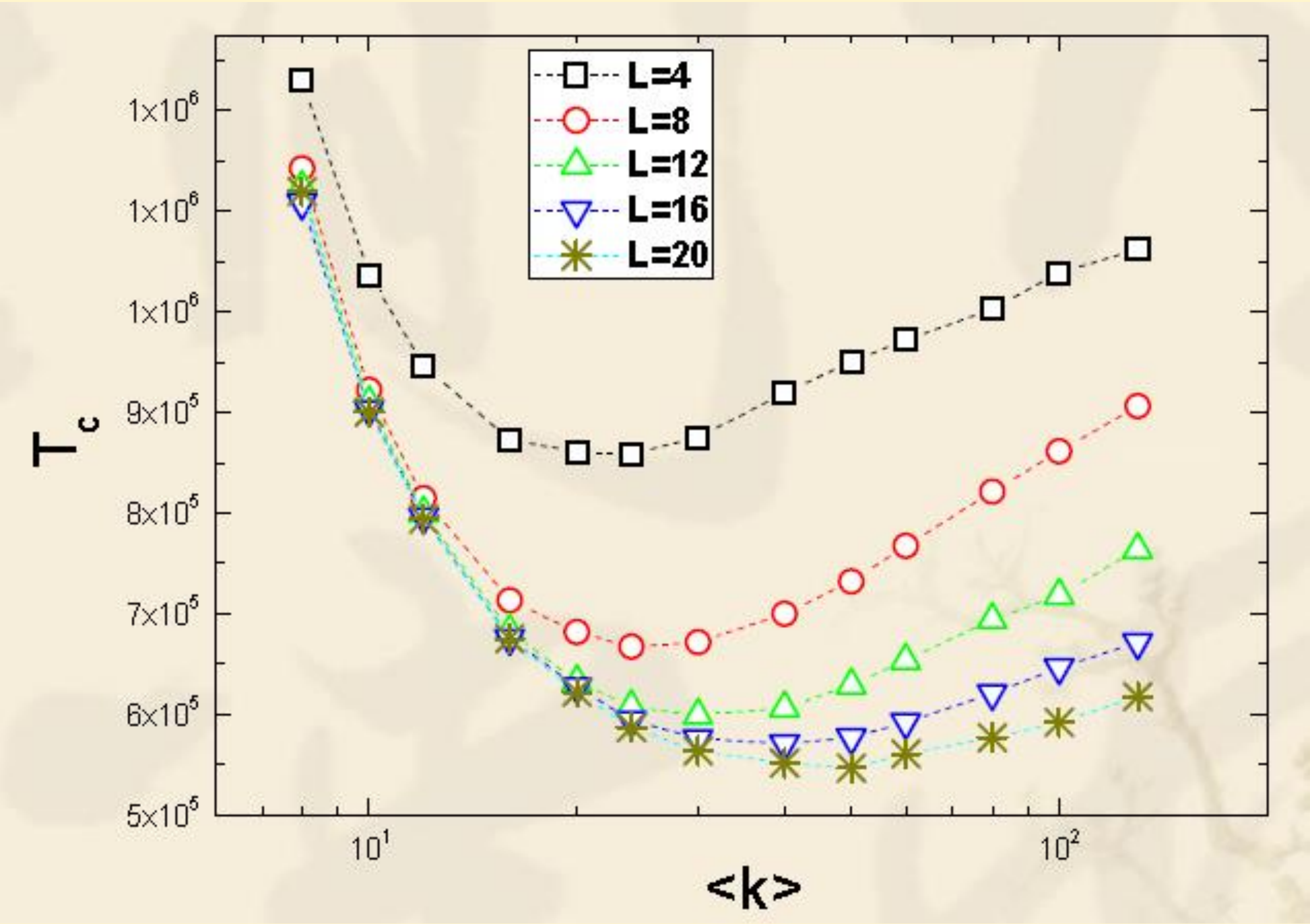


$<k>$ - average degree

Wang *et al.* (2007)

Tc – convergence time →
There are optimal convergence times

Simulations on Scale-Free Networks -I

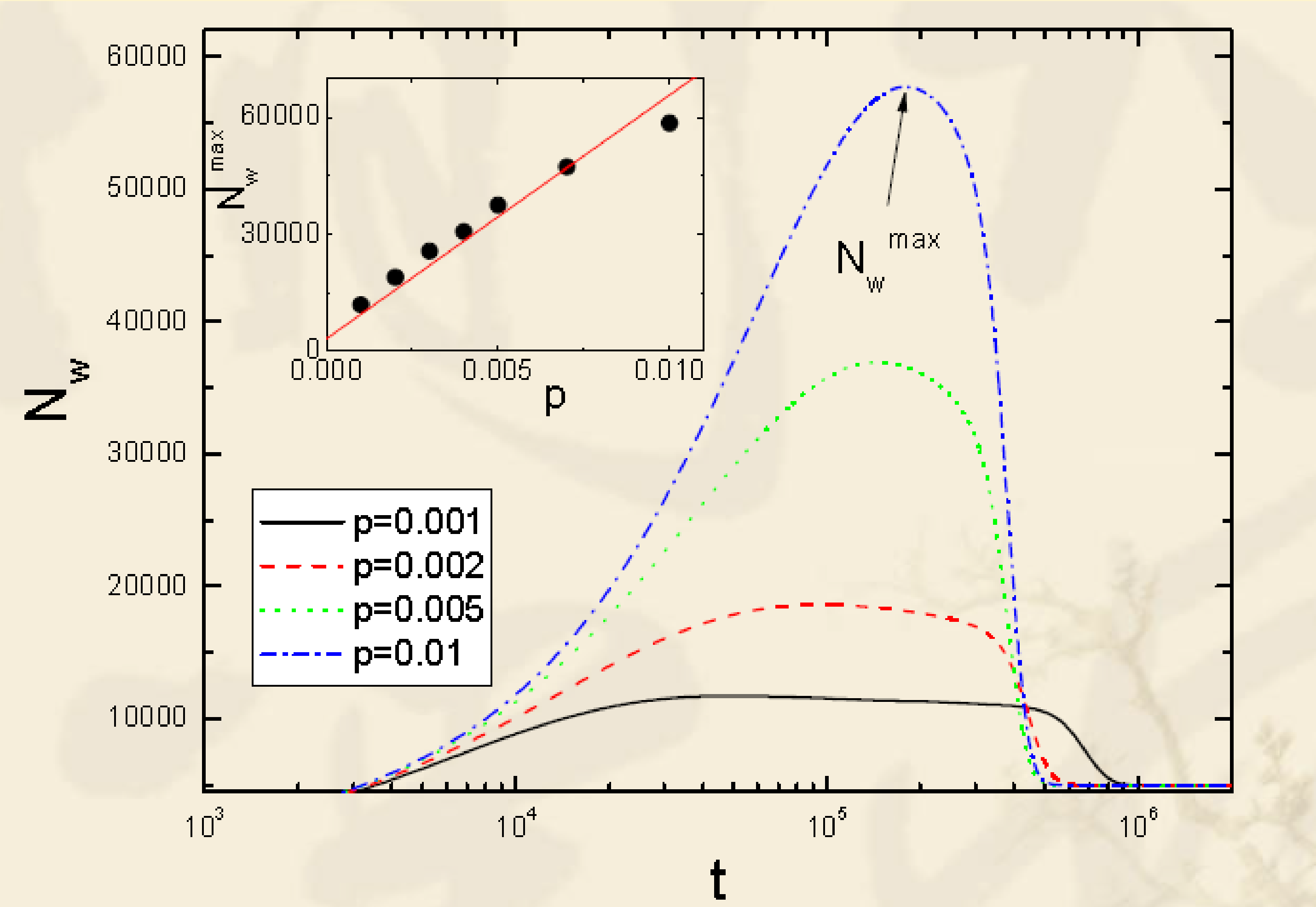


$<k>$ - average degree

Wang *et al.* (2007)

Tc – convergence time →
There are optimal convergence times

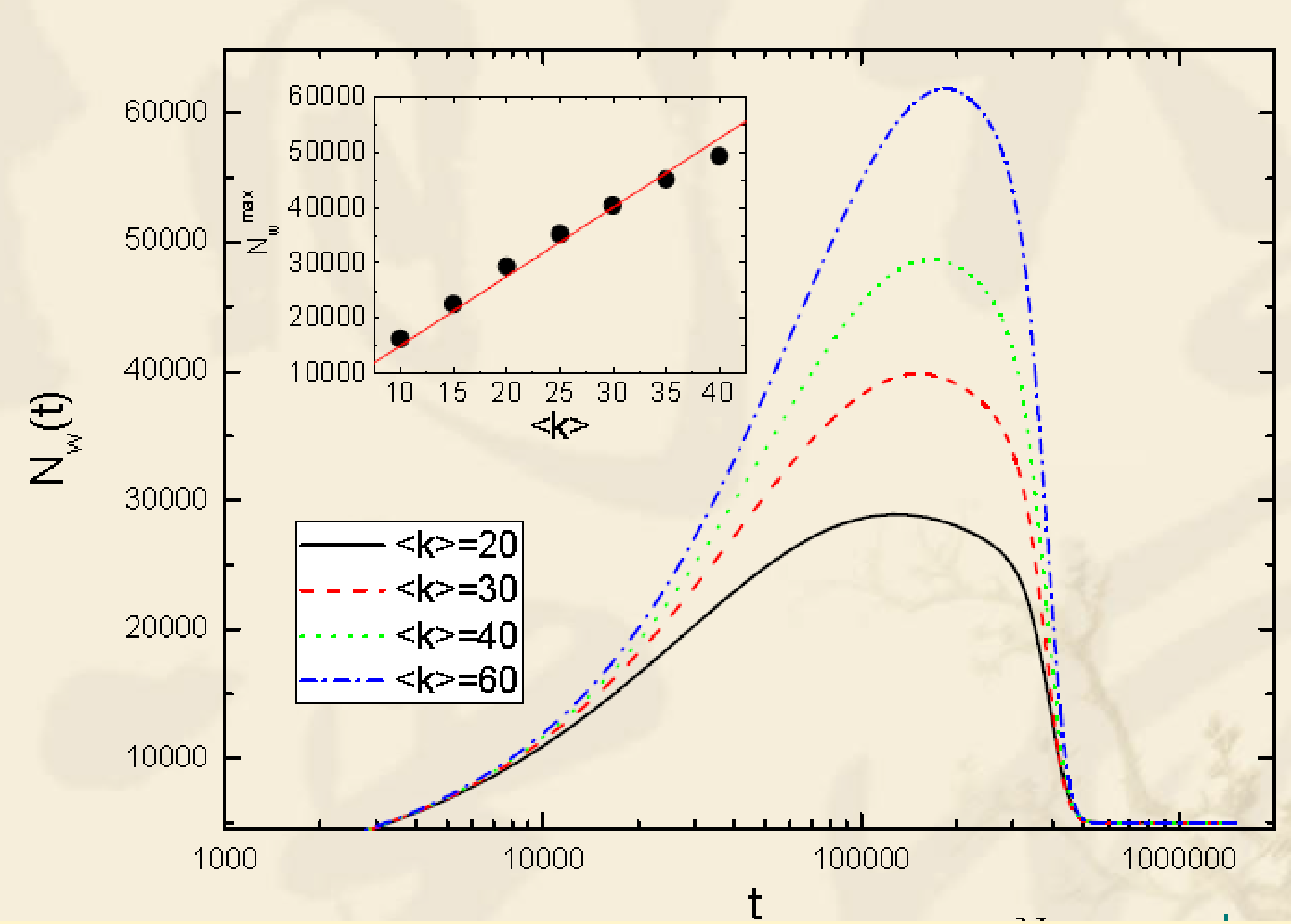
Simulations on Random Networks -II



Wang *et al.* (2007)

N_W – number of new words →
There are maximum numbers

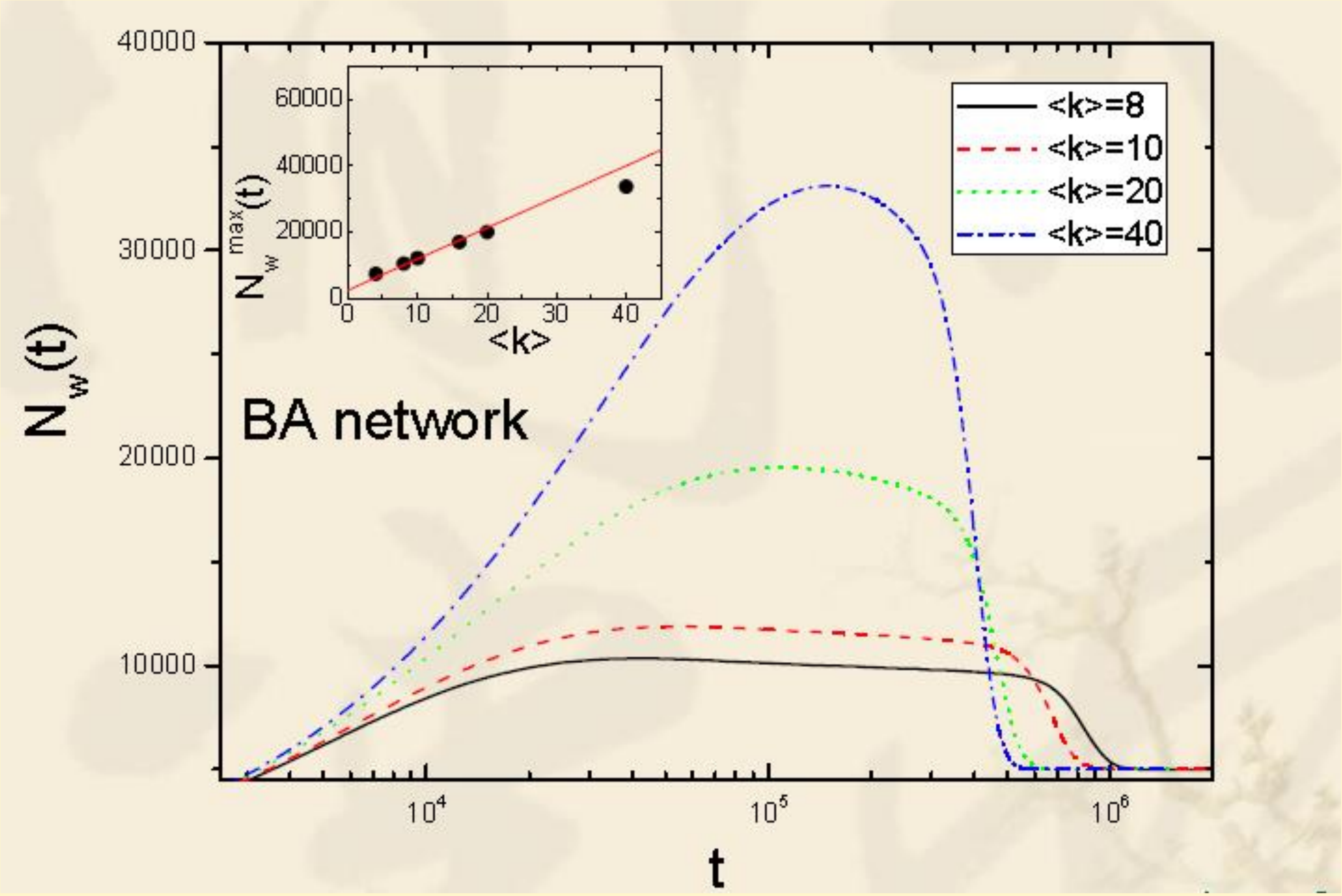
Simulations on Small-World Networks -II



N_w – number of new words →

There are maximum numbers

Simulations on Scale-Free Networks -II



Wang *et al.* (2007)

N_W – number of new words →
There are maximum numbers

Concluding Remarks

- ❖ Naming Game with infinite memory
- ❖ Naming Game with finite memory
- ❖ Related Topics:
  - ➢ Language evolution
  - ➢ Opinion formation
  - ➢ Business cooperation and competition
  - ➢ ……
- ❖ Future Topics:
  - ➢ Naming games with multiple objects
  - ➢ Naming games with multiple hearers
  - ➢ Naming games on networks with community or hierarchical structures
  - ➢ Applications to language studies
  - ➢ ……

References

- Infinite Memory:
  - Fully-connected networks (MF), regular lattices and random networks (ER)
  - A. Baronchelli, L. Dall'Asta, A. Barrat, and V. Loreto, Phys. Rev. E 73, 015102(R) (2006)
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