

MEM6810 Engineering Systems Modeling and Simulation

Sino-US Global Logistics Institute
Shanghai Jiao Tong University

Spring 2023 (full-time)

Assignment 4

Due Date: June 13, 14:00

Instruction

- (a) You can answer in English or Chinese or both.
 - (b) Show **enough** intermediate steps.
 - (c) Write your answers **independently**.
 - (d) If you copy the solutions from somewhere, you must **indicate the source**.
-

Question 1 (30 points)

We have $k > 2$ different (system) designs, and their mean performances are θ_i , $i = 1, 2, \dots, k$. We want to select the one with the largest mean performance. Bechhofer's Procedure (Lec 9 page 19/29) can ensure that when Assumptions 1-4 (Lec 9 page 18/29) are satisfied, $\mathbb{P}\{\text{select the target } \theta_i\} \geq 1 - \alpha$. Now we relax Assumption 3. Give a rigorous proof that, when Assumptions 1, 2, and 4 (Lec 9 page 18/29) are satisfied,

$$\mathbb{P}\left\{\left|\text{selected } \theta_i - \max_{1 \leq i \leq k} \theta_i\right| < \delta\right\} \geq 1 - \alpha.$$

Question 2 (20 points)

Explain why the Paulson's Procedure (Lec 9 page 24/29), under Assumptions 1-3 (Lec 9 page 18/29) and common known variance assumption, will stop almost surely (i.e., with probability one). Try to be as rigorous as possible.

Question 3 (50 points)

Consider the simulation optimization problem,

$$\min_{\mathbf{x} \in \mathcal{X}} g(\mathbf{x}),$$

where $g(\mathbf{x}) := \mathbb{E}[G(\mathbf{x})]$ and $G(\mathbf{x})$ is the output of a simulation replication conducted at \mathbf{x} . Let \mathbf{x}^* be a global optimal solution. Grid search is often used to find a global optimal solution to the problem. It first chooses m grid points, $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_m$, in \mathcal{X} .

It then takes r i.i.d. observations from each of the m grid points and calculates the sample means, $\bar{G}(\mathbf{x}_1), \bar{G}(\mathbf{x}_2), \dots, \bar{G}(\mathbf{x}_m)$. Let

$$\hat{\mathbf{x}}_m^* = \arg \min \{\bar{G}(\mathbf{x}_1), \bar{G}(\mathbf{x}_2), \dots, \bar{G}(\mathbf{x}_m)\} \text{ and } \mathbf{x}_m^* = \arg \min \{g(\mathbf{x}_1), g(\mathbf{x}_2), \dots, g(\mathbf{x}_m)\}.$$

Suppose that the grid points are chosen such that $g(\mathbf{x}_m^*) \rightarrow g(\mathbf{x}^*)$ as $m \rightarrow \infty$. (How to ensure the above condition is of course an important question in practice. Here we simply assume we can do it.) We further assume that $\sup_{\mathbf{x} \in \mathcal{X}} \text{Var}[G(\mathbf{x})] = \sigma^2 < \infty$. In order to ensure that $g(\hat{\mathbf{x}}_m^*) \rightarrow g(\mathbf{x}^*)$ almost surely as $m \rightarrow \infty$, r and m need to satisfy some relationship. Prove that, if r will increase when m increases (that is to say $r = r(m)$ is an increasing function on m) and

$$\sum_{m=1}^{\infty} \frac{m}{r(m)} < \infty,$$

then the above almost sure convergence holds.

Hint: You may need to use the following fact: $|\min_{i=1, \dots, k} \{a_i\} - \min_{i=1, \dots, k} \{b_i\}| \leq \max_{i=1, \dots, k} \{|a_i - b_i|\}$, for any given $\{a_1, \dots, a_k\}$ and $\{b_1, \dots, b_k\}$. You can find some idea from [L. Jeff Hong, Barry L. Nelson (2006). Discrete optimization via simulation using COMPASS. *Operations Research* **54**(1):115-129. <https://doi.org/10.1287/opre.1050.0237>]