MEM6810 Engineering Systems Modeling and Simulation

Sino-US Global Logistics Institute
Shanghai Jiao Tong University

Spring 2022 (full-time)

Assignment 2

Due Date: April 27

Instruction

(a) You can answer in English or Chinese or both.
(b) Show **enough** intermediate steps.
(c) Write your answers **independently**.

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Question 1 (15 points)

For a Poisson process $\{N(t),\ t \geq 0\}$ with rate λ, given that $N(t) = n$, the n arrival times $S_1, \ldots, S_n$ have the same distribution as the order statistics corresponding to n independent RVs uniformly distributed on the interval $(0, t)$. (See Lec 3 page 20/64.) Prove it rigorously.

Hint: If RVs Y_i, $i = 1, \ldots, n$, are uniformly distributed over $(0, t)$, then the joint pdf of the order statistics $Y_{(1)}, Y_{(2)}, \ldots, Y_{(n)}$ is

$$f(y_1, y_2, \ldots, y_n) = \frac{n!}{t^n}, \quad 0 < y_1 < y_2 < \cdots < y_n < t.$$

Question 2 $(10 + 15 = 25$ points)

For an $M/M/\infty$ queue, solve its limiting (steady-state) distribution (see Theorem 6 on Lec 3).

(1) Use the result of $M/M/s$ queue and let $s \to \infty$.
(2) Derive the result using the state space diagram.

Question 3 (15 points)

Consider a Jackson queueing network with external arrival rate $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \lambda_3]^\intercal = [5, 2, 4]^\intercal$, service rate $\boldsymbol{\mu} = [\mu_1, \mu_2, \mu_3]^\intercal = [6, 4, 12]^\intercal$, server number $\boldsymbol{s} = [s_1, s_2, s_3]^\intercal = [2, 3, 1]^\intercal$, and routing matrix

$$\boldsymbol{P} = \begin{bmatrix} 0 & 0.6 & 0.2 \\ 0 & 0 & 0.4 \\ 0 & 0.5 & 0.1 \end{bmatrix}.$$

Calculte the expected number of customers in the entire network in steady state.

Question 4 (15 + 10 = 25 points)

Fibonacci Generator is a simple Multiple Recursive Generator (MRG). It extends the Linear Congruential Generator (LCG) in the following way:

$$x_i = (x_{i-1} + x_{i-2}) \bmod m.$$

But Fibonacci Generator has a serious flaw in statistics.

(1) Prove that Fibonacci Generator can never successively generate three pseudo-random numbers u_i, u_{i+1}, u_{i+2} satisfying $u_i < u_{i+2} < u_{i+1}$.

(2) Prove that for a perfect RNG, the probability of the event stated in (1) should be 1/6.

Question 5 (20 points)

Rigorously prove the Box–Muller method (see Lec 4 page 35/38).