MEM6810 Engineering Systems Modeling and Simulation

Sino-US Global Logistics Institute
Shanghai Jiao Tong University

Spring 2025 (full-time)

Assignment 2

Due Date: April 25 (in class)

Instruction

(a) You can answer in English or Chinese or both.
(b) Show **enough** intermediate steps.
(c) Write by hand.

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Question 1 (10 points)

Prove $\widehat{L}_Q(T) = \widehat{\lambda}\widehat{W}_Q(T)$ on lecture note Lec 3 page 31/64. (Hint: Use similar argument for proving $\widehat{L}(T) = \widehat{\lambda}\widehat{W}(T)$ and illustration figure on that page.)

Question 2 (15 points)

Prove Theorem 5 (Limiting Distribution of $M/M/s$ Queue) on lecture note Lec 3 page 41/64. (Hint: Use similar argument for proving Theorem 4.)

Question 3 ($12 + 3 = 15$ points)

Consider two systems. System 1: There are two *independent* $M/M/1$ queues, each with arrival rate λ and service rate μ. System 2: There is one $M/M/2$ queue with arrival rate 2λ and service rate μ for each server.

(1) Which system will perform better (i.e., more efficiently) in terms of the following four measures?
  (1.1) The long-run average number of customers in the system;
  (1.2) The long-run average sojourn time in the system;
  (1.3) The long-run average number of customers waiting in the system;
  (1.4) The long-run average waiting time in the system.

(2) This problem reflects a well known effect in resource management. What is it? Briefly explain it.

Question 4 ($3 + 4 + 3 + 4 + 4 = 18$ points)

Compute L, W, L_Q, W_Q for the following three queueing models:

(1) $M/M/1$, $\lambda = 0.6$, $\mu = 1$.
(2) $M/M/2$, $\lambda = 0.6$, $\mu = 0.5$.
(3) $M/G/1$, $\lambda = 0.6$, service time follows $\mathrm{Unif}(0, 2)$.

Based on the above results, answer the following two questions:

(4) Compare models (1) and (2), which one has higher efficiency (in terms of customer flow)? Based on which quantity? Provide an *intuitive explanation* for such difference.
(5) Compare models (1) and (3), which one has higher efficiency (in terms of customer flow)? Based on which quantity? Provide an *intuitive explanation* for such difference.

Question 5 ($4 + 4 + 4 = 12$ points)

Consider an $M/M/1/5$ queue with arrival rate $\lambda = 10$/hour and service rate $\mu = 8$/hour.

(1) What is the probability that an arrival customer finds the station is full?
(2) What is the expected amount of time a customer who enters the station will spend in it?
(3) What is the expected amount of time an arrival customer will spend in the station? (Note: If a customer doesn't enter the station, the amount of time he spends in the station is 0.)

Question 6 (10 points)

Consider a Jackson queueing network with external arrival rate $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \lambda_3]^\intercal = [6, 2, 4]^\intercal$, service rate $\boldsymbol{\mu} = [\mu_1, \mu_2, \mu_3]^\intercal = [6, 4, 12]^\intercal$, server number $\boldsymbol{s} = [s_1, s_2, s_3]^\intercal = [2, 3, 1]^\intercal$, and routing matrix

$$\boldsymbol{P} = \begin{bmatrix} 0 & 0.6 & 0.2 \\ 0 & 0 & 0.4 \\ 0 & 0.5 & 0.1 \end{bmatrix}.$$

Calculte the expected number of customers in the entire network in steady state.

Question 7 ($10 + 10 = 20$ points)

Consider such a station. Customers arrive from outside according to a poisson process with rate $\lambda = 10$/hour. There is only one server, and the service time is exponentially distributed with rate $\mu = 15$/hour. When a customer finishes service, with probability 0.2, he will re-enter the station immediately. (For example, he may realize he just forgot to do something.) For this station, find out

(1) The long-run average number of customers in the station;
(2) The long-run average sojourn time in the station. (Caution! When a customer re-enters the station, his previous sojourn time will accumulate.)