

MEM6810 Engineering Systems Modeling and Simulation

工程系统建模与仿真

Theory Analysis

Lecture 1: Introduction to Simulation

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What is Simulation?

- *Simulation* (仿真) is the imitation of the operation of a real-world process or system over time.
 - Done by hand or (usually) on a computer;
 - Involves the generation and observation of an artificial history of a system;
 - Draw inferences about the characteristics of the real system.
- Simulation is EVERYWHERE!

What is Simulation?

Figure: Physical Simulation of Solid-Fluid Interaction (*from Ruan et al. (2021)*)



What is Simulation?



Figure: Pilot Training in Boeing 787 Flat Panel Trainer (from [Boeing](#))

What is Simulation?

Figure: Airport Simulation (*by Vancouver Airport Services*)
[Video: <https://www.youtube.com/watch?v=JuXwEbAvk2Q>]



What is Simulation?

Figure: Typhoon Simulation ([image](#) by [Atmoz](#) / [CC BY 3.0](#))



What is Simulation?

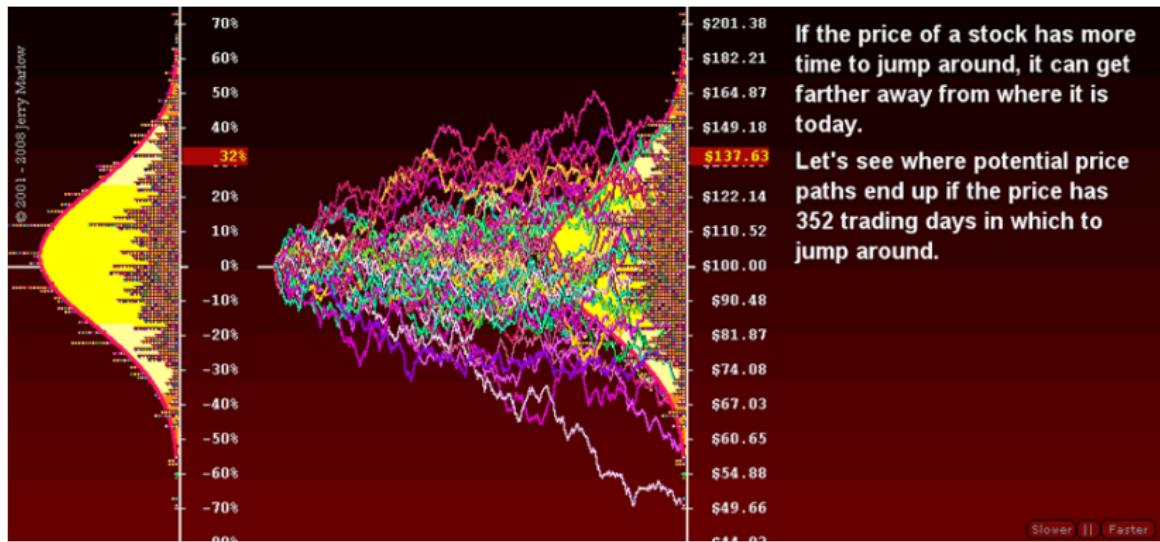


Figure: Financial Analysis

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Why Simulation?

- It is often too costly or even impossible to do physical studies in reality with the actual system.
 - May be *disruptive*, *expensive*, *dangerous*, or *rare*.
- The mathematical **model** (will be defined shortly) which can well represent the real problem, may be very *difficult* to solve.
 - You can only solve it with high *simplification*.
- With simulation technique, we can easily make change and observe the effect, while keeping high fidelity.

Why Simulation?

- Simulation can be used as both an *analysis tool* and a *design tool*.
- ① An analysis tool: To answer “**what if**” questions about the existing real-world system.
 - E.g., try alternative layout of a production line, try other staff shifts of a service center, test a financial system in some extreme situation, etc.
- ② A design tool: To study systems in the design stage, before they are built.
 - E.g., evaluate designs and operations for new transportation facilities, service organizations, manufacturing systems, etc.
- Simulation is also an important type of numerical methods.

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How to Do Simulation?

- This is the focus of the course!

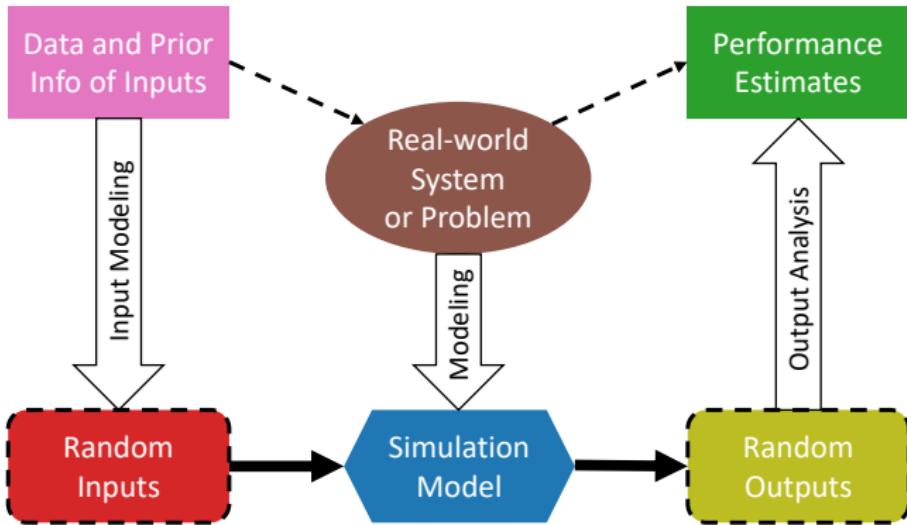


Figure: Basic Paradigm of A Simulation Study

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- A **model** is a representation of a system or problem.
 - A set of **assumptions** and/or **approximations** about how the system works will often be imposed.
 - It is only necessary to consider those aspects that affect the problem under investigation.
 - However, the model should be sufficiently **detailed** to draw **valid** conclusions about the real system or problem.
 - The trade-off: **simplicity** vs. **accuracy**.
- **Physical model** vs. **Mathematical model**
 - ① Physical model is a scaled-down (or -up) version of the system.
 - ② Mathematical model uses symbolic notation and mathematical equations to represent the system.
- Instead of doing physical studies with the actual system in real world, we can study the model.
 - It will be much easier, faster, cheaper, and safer!
- A **simulation model** is a particular type of **mathematical model**.





“ All models are wrong,
but some are useful. ”

— George E. P. Box

George E. P. Box (1919.10 – 2013.03) was a British statistician, who worked in the areas of quality control, time-series analysis, design of experiments, and Bayesian inference. He has been called “one of the great statistical minds of the 20th century”.

- When a mathematical model is simple enough, we can **solve** it
 - *analytically*, with mathematical tools like algebra, calculus, probability theory;
 - *numerically*, with computational procedures (e.g., solving a quintic equation).
- But not all mathematical models can be “**solved**”.
- In simulation, the mathematical models (more specifically, simulation models) are **run** rather than **solved**:
 - Artificial history of the system is *generated* from the model assumptions;
 - Observations of system status are *collected* for analysis;
 - System performance measures are *estimated*.
- Essentially, running simulation is still one type of numerical methods.
 - Real-world simulation models can be large, and such runs are usually conducted with the aid of a computer.

- Simulation models may be classified as being *static* or *dynamic*.

① Static: Time does not play a **natural** role.

- Example 1 – Finance: evaluate portfolio return and risk.
- Example 2 – Project Management: evaluate projects payoff in different scenarios.
- Sometimes called **Monte Carlo (蒙特卡洛) simulation**.
- Often used in the complex numerical calculation in financial engineering (金融工程), computational physics, etc.

② Dynamic: Time does play a **natural** role.

- Example 1 – Logistics Management: evaluate the efficiency of a terminal.
- Example 2 – Service Management: evaluate waiting time of customers under different staff shifts.
- Often used to simulate the logistics/transportation/service systems, whose status naturally changes over time.

- Simulation models may be classified as being *deterministic* or *stochastic*.
- ① Deterministic: Everything is known with **certainty**.
- E.g., patients arrive at a hospital precisely on schedule, the service time is precisely fixed, the transfer among different units is pre-determined.
- ② Stochastic: **Uncertainty** exists.
- E.g., arrival times and service times of patients have random variations, the transfer is random.
 - Used much more often (uncertainty is more or less involved in a real-world system).

- Simulation models may be classified as being *discrete* or *continuous*.
- ① Discrete: System states change only at **discrete** time points.
- E.g., the number of customers in the bank, changes only when a customer arrives or leaves after service (*left fig*).
- ② Continuous: System states change **continuously** over time.
- E.g., the head of water (水位) behind a dam changes continuously during a period of time (*right fig*).

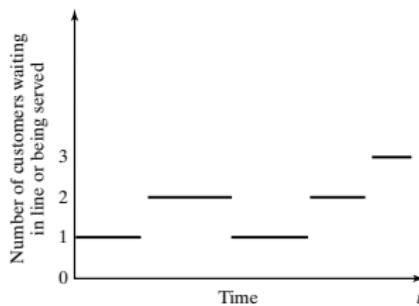


Figure: Discrete State (from Banks et al. (2010))

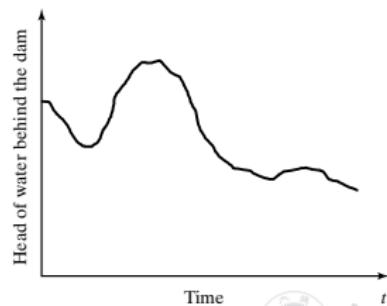


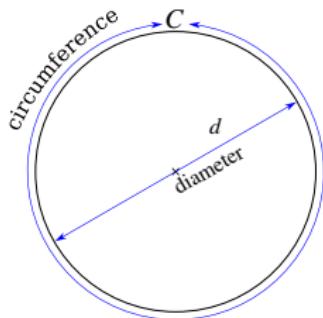
Figure: Continuous State (from Banks et al. (2010))

- In summary, simulation models may be classified as being *static* or *dynamic*, *deterministic* or *stochastic*, and *discrete* or *continuous*.
- For most operational decision-making problems, the suitable simulation models are *dynamic*, *stochastic* and *discrete*.
 - The simulation is called **Discrete-Event System Simulation** (离散事件系统仿真).
 - It is the main **focus** of this course.

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- The mathematical constant π , is originally defined as the ratio of circle's circumference to its diameter.



$$\pi = \frac{\text{circumference}}{\text{diameter}} = 3.14159\ 26535\dots$$

- It was considered as a quite difficult problem in the history of mankind to find the value of π .

- The earliest written approximations of π :
 - Babylon: A clay tablet (1900–1600 BC), $\pi \approx \frac{25}{8} = 3.125$;
 - Egypt: The Rhind Papyrus (莱因德纸草书, 1650 BC, 1850 BC), $\pi \approx (\frac{16}{9})^2 = 3.160 \dots$



Figure: Archimedes of Syracuse
(287–212 BC) ([Source/Photographer](#))



Figure: Liu Hui (刘徽, 魏晋时期,
225–295 AD)



Figure: Zu Chongzhi
(祖冲之, 南北朝时期, 429–500 AD)
([statue image](#) by [三猎](#) / [CC BY 4.0](#))

$$\frac{223}{71} < \pi < \frac{22}{7}$$

$$\frac{223}{71} = 3.1408\dots$$

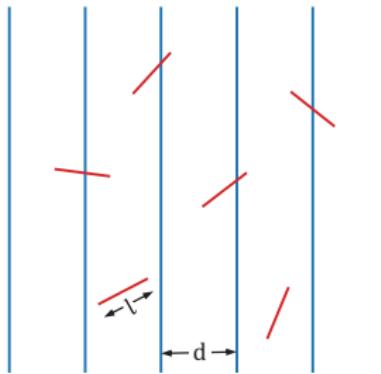
$$\frac{22}{7} = 3.1428\dots$$

$$\pi \approx 3.1416$$

$$\pi \approx \frac{355}{113} = 3.14159\ 292\dots$$

- Buffon's Needle (布丰投针)

- Buffon, a French mathematician, in 1733 (1777) did a static simulation (by hand), which can be used to estimate π .
- Drop a needle of length l onto the floor with parallel lines d apart, where $l < d$.
- Suppose the needle is *equally likely* to fall anywhere.

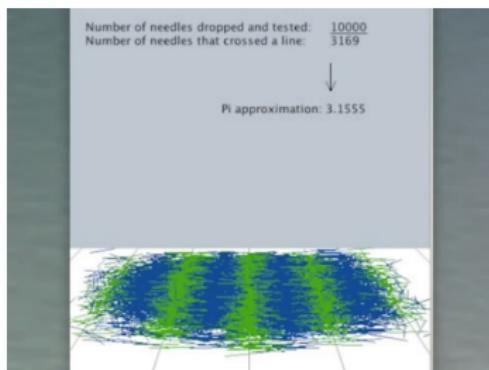


- $\mathbb{P}(\text{needle crosses a line}) = \frac{2l}{\pi d}.$

- If Buffon repeats the experiment for n times (i.e., drops n needles), and let h denote the number of needles crossing a line, then,

$$\mathbb{P}(\text{needle crosses a line}) = \frac{2l}{\pi d} \approx \frac{h}{n}.$$

- So, $\pi \approx \frac{2ln}{dh}$.
- Let $d = 2l$, then $\pi \approx n/h$.
- The approximation gets more and more accurate when n increases.



- Try it out!

<https://mste.illinois.edu/activity/buffon>

<http://datagenetics.com/blog/may42015/index.html>

Figure: A Computer Simulation (by Jeffrey Ventrella)
[Video: <https://www.youtube.com/watch?v=kazgQXae0Hk>]

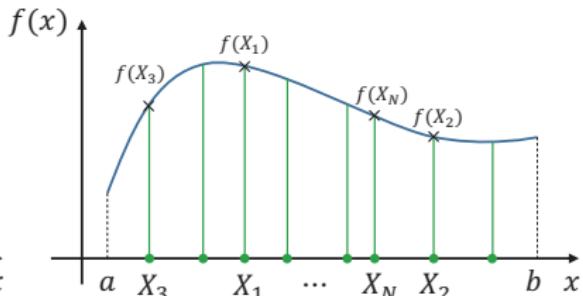
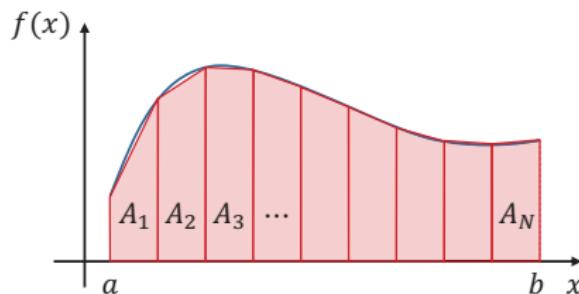
- Now consider another simulation to estimate π .
 - Randomly throw n dots to a square.
 - Suppose the dots are *equally likely* to fall anywhere inside the square.
 - Let h denote the number of dots in the circular sector.



Figure: Animation ([image](#) by [nicoguardo](#)) / [CC BY 3.0](#)

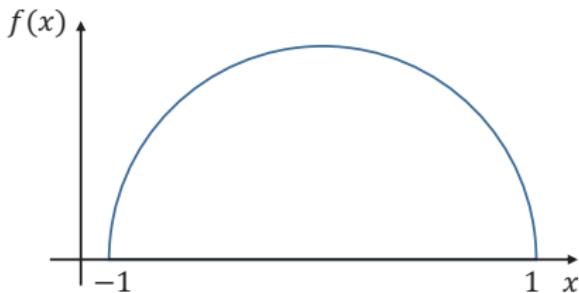
- $\mathbb{P}(\text{dot in sector}) = \frac{\text{sector area}}{\text{square area}} = \frac{\pi d^2/4}{d^2} \approx \frac{h}{n} \Rightarrow \pi \approx \frac{4h}{n}$.
- Visit <https://xiaoweiz.shinyapps.io/calPi> for interaction.

- Consider a numerical integration (数值积分) $\int_a^b f(x)dx$.



- Trapezoidal rule (梯形法) (*left fig*):
 - Divide the area into N parts.
 - $\int_a^b f(x)dx \approx A_1 + A_2 + \dots + A_N$.
- Monte Carlo method (*right fig*):
 - Randomly sample N points on $[a, b]$ from $\text{uniform}(a, b)$.
 - $\int_a^b f(x)dx \approx \frac{b-a}{N} [f(X_1) + f(X_2) + \dots + f(X_N)]$.
- Monte Carlo method will be much more **efficient** when the dimension is high! (E.g., $\int_{[a, b]^d} f(x)dx$ for large d .)

- Recall the numerical integration problem $\int_a^b f(x)dx$.
- Let $f(x) = \sqrt{1 - x^2}$, $a = -1$, $b = 1$.



- Then, $\int_{-1}^1 \sqrt{1 - x^2} dx = \pi/2$.
- So we have another way to estimate π using Monte Carlo simulation (provided we know how to compute square root).

- There is a system:
 - Two components work as active and spare, so the system fails if both components are failed.
 - Suppose the time to next component failure is random (when there is at least one functional components), which follows a known distribution, and we know how to generate it.
 - To make it simple, suppose the time to next failure is equally likely 1, 2, 3, 4, 5 or 6 days (no memory).
 - Repair takes exactly 2.5 days (only one at a time).
- What can we say about the time to failure for this system?
- Let's run a simulation by hand!
 - Let the system **state** denote the number of functional components.
 - The **events** are the failure of a component and the completion of repair.

Event Calendar			
Clock	System State	Next Failure	Next Repair
0	2	$0 + \textcolor{red}{5} = 5$	∞
5	1	$5 + \textcolor{red}{3} = 8$	$5 + 2.5 = 7.5$
7.5	2	8	∞
8	1	$8 + \textcolor{red}{6} = 14$	$8 + 2.5 = 10.5$
10.5	2	14	∞
14	1	$14 + \textcolor{red}{1} = 15$	$14 + 2.5 = 16.5$
15	0	∞	16.5

- We can observe:
 - Time to failure = 15
 - Average number of functional components =
$$\frac{1}{15-0} [2(5-0) + 1(7.5-5) + 2(8-7.5) + 1(10.5-8) + 2(14-10.5) + 1(15-14)] = \frac{24}{15}$$
- Some questions:
 - How to deal with the randomness?
 - How to generate the time interval of component failure?

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- Introduction to Simulation
- Elements of Probability and Statistics
- Queueing Models
- Random Variate Generation
- Input Modeling
- Verification and Validation of Simulation Models
- Output Analysis I: Single Model
- Simulation in Excel and FlexSim
- Output Analysis II: Comparison
- Output Analysis III: Optimization