MEM6810 Engineering Systems Modeling and Simulation

Sino-US Global Logistics Institute
Shanghai Jiao Tong University

Spring 2023 (full-time)

Assignment 2

Due Date: April 11 (in class)

Instruction

(a) You can answer in English or Chinese or both.
(b) Show **enough** intermediate steps.
(c) Write your answers **independently**.
(d) If you copy the solutions from somewhere, you must **indicate the source**.

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Question 1 (10 points)

If $X_1, X_2, \ldots, X_n$ are n independent random variables, and $X_i \sim \text{Exp}(\lambda_i)$, $i = 1, \ldots, n$, prove that

$$\min\{X_1, \ldots, X_n\} \sim \text{Exp}(\lambda_1 + \cdots + \lambda_n).$$

Question 2 (10 points)

Prove $\widehat{L}_Q(T) = \widehat{\lambda}\widehat{W}_Q(T)$ on lecture note Lec 3 page 31/64. Use similar argument for proving $\widehat{L}(T) = \widehat{\lambda}\widehat{W}(T)$ and the same illustration figures on that page.

Question 3 (20 points)

Prove Theorem 5 (Limiting Distribution of $M/M/s$ Queue) on lecture note Lec 3 page 41/64.

Question 4 $(12 + 3 = 15$ points)

Consider two systems. System 1: There are two *independent* $M/M/1$ queues, each with arrival rate λ and service rate μ. System 2: There is one $M/M/2$ queue with arrival rate 2λ and service rate μ for each server.

(1) Which system will perform better (i.e., more efficiently) in terms of the following four measures: L, W, L_Q, W_Q?
(2) This problem reflects a well known effect in resource management. What is it? Briefly explain it.

Question 5 ($3+3+3+3+3=15$ points)

Compute L, W, L_Q, W_Q for the following three queueing models:

(1) $M/M/1$, $\lambda = 0.6$, $\mu = 1$.

(2) $M/M/2$, $\lambda = 0.6$, $\mu = 0.5$.

(3) $M/G/1$, $\lambda = 0.6$, service time follows $\text{Unif}(0, 2)$.

Based on the above results, answer the following two questions:

(4) Compare models (1) and (2), which one has higher efficiency (in terms of customer flow)? Based on which quantity? Provide an *intuitive explanation* for such difference.

(5) Compare models (1) and (3), which one has higher efficiency (in terms of customer flow)? Based on which quantity? Provide an *intuitive explanation* for such difference.

Question 6 ($2+8=10$ points)

If a machine produces some products one by one. The time length of producing one piece of product follows $\text{Exp}(a)$. The finished products are stored in a warehouse, whose capacity limit is b pieces of products. When the warehouse is full, the machine will pause; once there is available space in the warehouse, the machine will continue the previous job immediately. Customers arrive to the warehouse following a Poisson process with rate c. Each customer will take away one piece of product (ignoring the time length of picking the product). If an arriving customer finds the warehouse is empty, he/she leaves immediately.

(1) If we are interested in the number of products in the warehouse, can this problem be represented by one of the queueing models introduced in Lec 3? (Only need to answer Yes or No.)

(2) If Yes, write down the queueing model and the corresponding parameters, and explain the reason; If No, add or modify some assumptions of this problem so that it can be represented by one of the queueing models introduced in Lec 3, and write down the queueing model and the corresponding parameters.

Question 7 (20 points)

Consider such a station. Customers arrive from outside according to a poisson process with rate $\lambda = 10$/hour. There is only one server, and the service time is exponentially distributed with rate $\mu = 15$/hour. When a customer finishes service, with probability 0.2, he will re-enter the station immediately. (For example, he may realize he just forgot to do something.) For this station, find out

(1) The long-run average number of customers in the station;

(2) The long-run average sojourn time in the station. (Caution! When a customer re-enters the station, his previous sojourn time will accumulate.)