Last Name: ______________________

First Name: ______________________

Section: ______________________

Instructions:
The use of notes, lecture material, hw, quizzes, electronic devices, etc. is not allowed.

Question	Points	Score
1	8	
2	8	
3	20	
4	8	
5	12	
6	16	
7	9	
8	11	
9	8	
10	0	
Total:	100	

1. (8 points) Let $S = \{\emptyset, a, \{1\}, \{a, \emptyset\}, \{a, 1\}\}$. State whether each item below, x, is an element of S ($x \in S$), a subset of S ($x \subseteq S$), neither, or both.

(a) $\emptyset$	(b) $\{a, \{\emptyset\}\}$
(c) $\{1\}$	(d) $\{\{1, a\}\}$

(a) $\emptyset \in S$, $\emptyset \subseteq S$ - both

(b) neither

(c) $\{1\} \in S$

(d) $\{\{1, a\}\} \subseteq S$

2. (8 points) Construct a truth table for the expression $(\neg p \wedge r) \rightarrow (r \leftrightarrow \neg q)$

p	q	r	$\neg p$	$\neg q$	$\neg p \wedge r$	$r \leftrightarrow \neg q$	
T	T	T	F	F	F	F	T
T	T	F	F	F	F	T	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	F	T	F	F
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	F	F	T

3. (20 points) Let $A = \{b, \emptyset, \{c\}\}$, $B = \{b, \{c, a\}\}$, and $C = \{\emptyset, \{c, a\}\}$. Let $U = A \cup B \cup C$.

(a) (2 points) $|B \cup C| =$

3

(b) (2 points) $A \cap C$

$A \cap C = \{\emptyset\}$

(c) (2 points) $A \cup B$

$A \cup B = \{\ b, \emptyset, \{c\}, \{c, a\}\ \}$

(d) (2 points) $C - B$

$C - B = \{\ \emptyset\}$

(e) (2 points) $\overline{A}$

$\overline{A} = \{\{c, a\}\}$

(f) (3 points) $B - (A \cup C)$

$B - (A \cup C) = B - \{b, \emptyset, \{c\}, \{c, a\}\} = \{\}$

(g) (4 points) $P(A)$

$P(A) = \{\emptyset, \{b\}, \{\emptyset\}, \{\{c\}\}, \{b, \emptyset\}, \{b, \{c\}\}, \{\emptyset, \{c\}\}, \{b, \emptyset, \{c\}\}\}$

(h) (3 points) $A \times C$

$A \times C = \{(b\emptyset), (b, \{c, a\}), (\emptyset, \emptyset), (\emptyset, \{c, a\}), (\{c\}, \emptyset), (\{c\}, \{c, a\})\}$

4. (8 points) For each function given determine is it one-to-one? is it onto? **If not, explain why for each question**.

   (a) (4 points) $f : \mathbf{R} - \{0\} \rightarrow \mathbf{R}$, where $f(n) = \dfrac{1}{n} + 1$

(i) one-to-one?	(ii) onto?

(i) f is one-to-one
(ii) f is not onto, f never maps to 1, ...

   (b) (4 points) $f : \mathbf{Z} \times \mathbf{Z} \rightarrow \mathbf{Z} \times \mathbf{Z}$, where $f(n, m) = (m + n, m + 2m)$

(i) one-to-one?	(ii) onto?

(i) f is one-to-one.
(ii) f is not onto.

5. (12 points) Determine whether the following binary relations are reflexive, symmetric, antisymmetric, and transitive?

$$R = \{(1, 1), (2, 3), (3, 2)\}, \text{ where } R \text{ is a relation on the set } \{1, 2, 3\}$$
$$S \text{ on the set } \mathbb{Z} \text{ where } x\,S\,y \text{ if and only if } x^2 = y^2$$
$$T \text{ on the set } \mathbb{Z} \text{ where } x\,T\,y \text{ if and only if } x^2 + y^2 \text{ is even}$$

Relation	Reflexive (Yes/No)	Symmetric (Yes/No)	Antisymmetric (Yes/No)	Transitive (Yes/No)
R	No	Yes	No	No
S	Yes	Yes	No	Yes
T	Yes	Yes	No	Yes

6. (16 points) Let $A = \{a, b, c, d, e\}$, $B = \{v, w, x, y, z\}$, $f : A \to B$, and $g : B \to A$.

$$f(a) = y,\ f(b) = w,\ f(c) = z,\ f(d) = v,\ f(e) = x$$
$$g(v) = a,\ g(w) = d,\ g(x) = c,\ g(y) = b,\ g(z) = a$$

(a) (2 points) What is the domain of f? of g?

$A = \{a, b, c, d, e\}$ $B = \{v, w, x, y, z\}$.

(b) (2 points) What is the co-domain of f? of g?

$B = \{v, w, x, y, z\}$ $A = \{a, b, c, d, e\}$

(c) (2 points) What is the range of f? of g?

$\{v, w, x, y, z\}$ $\{a, b, c, d\}$

(d) (3 points) List all of the following properties that apply to the function f: onto, one-to-one, one-to-one correspondence, none-of-the-above.

one-to-one, onto, one-to-one correspondence

(e) (3 points) List all of the following properties that apply to the function g: onto, one-to-one, one-to-one correspondence, none-of-the-above.

none-of-the-above

(f) (4 points) Define the inverse function or state "Not Defined" for:
(i) f (ii) g

(i) $f^{-1} = \{(y, a), (w, b), (z, c), (v, d), (x, e)\}$
(ii) not defined

7. (9 points) Investigate composition of functions.
   Let f, g, and h be functions mapping from $\mathbf{R}$ to $\mathbf{R}$ where

$$f(x) = x^3, \qquad g(x) = \sqrt{x^2+2}, \qquad h(x) = x^2 - 3.$$

   Calculate the following. Express answer in reduced algebraic form.

   (a) (2 points) $(g \circ f)(x)$

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(x^3) \\ &= \sqrt{(x^3)^2+2} \\ &= \sqrt{x^6+2}\end{aligned}$$

   (b) (2 points) $(h \circ g)(x)$

$$\begin{aligned}(h \circ g)(x) &= h(g(x)) \\ &= h(\sqrt{x^2+2}) \\ &= (\sqrt{x^2+2})^2 - 3) \\ &= x^2 - 1\end{aligned}$$

   (c) (3 points) $((h \circ g) \circ f)(x)$

$$\begin{aligned}((h \circ g) \circ f)(x) &= h(g(f(x))) \\ &= h(g(x^2)) \\ &= h(\sqrt{(x^3)^2+2}) \\ &= (\sqrt{(x^3)^2+2})^2 - 3 \\ &= x^6 - 1\end{aligned}$$

   (d) (2 points) True or False. Are $(h \circ (g \circ f))(x)$ and $((h \circ g) \circ f)(x)$ equal.

TRUE

8. Consider the relations R and S on the set $\{1, 2, 3, 4\}$ with

$$R = \{(1,1), (1,2), (2,4), (3,2), (3,4), (4,3)\}$$
$$S = \{(1,1), (1,2), (2,1), (3,2), (4,3)\}$$

(a) (3 points) Find $R - S$ (list all ordered pairs in the relation).

$R - S = \{(2,4), (3,4)\}$

(b) (4 points) $S \circ R$ (list all ordered pairs in the relation).

$S \circ R = \{(1,1), (1,2), (2,3), (3,1), (3,3), (4,2)\}$

(c) (4 points) Compute S^2 (list all ordered pairs)

$S^2 = \{(1,1), (1,2), (2,1), (2,2), (3,1), (4,2)\}$

9. Consider a music library. The universal set U refers to all songs in the library. Songs are also classified by their genre (a song may receive more than one classification) as: P - pop, L - latin, C - classical, R - rock, H - hip hop, I - instrumental, J - jazz, E - English, O - other language. Each set is a subset of U.

   Translate the following expressions from English to mathematical notation or vice versa.

   (a) (2 points) $J \cap R = \emptyset$

   No song is both jazz and rock.

   (b) (2 points) The set of instrumental and jazz songs excluding those that are classical.

   $(I \cup J) - C$

   (c) (2 points) All rock songs are also pop songs.

   $R \subseteq P$

   (d) (2 points) $\overline{P \cup C \cup J} = \emptyset$

   All songs are either pop, classical, or jazz.

10. (2 points (bonus)) Given sets A, B, and C. Identify a simple equivalent expression to: $(B - A) \cup (C - A) \cup \overline{(A \cup B \cup C)}$

$\overline{A}$

Identity	Name	Identity	Name
$A \cup \emptyset = A$	Identity	$A \cup U = U$	Domination
$A \cap U = A$	laws	$A \cap \emptyset = \emptyset$	laws
$A \cup A = A$	Idempotent		Complementation
$A \cap A = A$	laws	$\overline{((A))} = A$	law
$A \cup B = B \cup A$	Commutative	$\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's
$A \cap B = B \cap A$	laws	$\overline{A \cap B} = \overline{A} \cup \overline{B}$	laws
$A \cup (A \cap B) = A$	Absorption	$A \cup \overline{A} = U$	Complement
$A \cap (A \cup B) = A$	laws	$A \cap \overline{A} = \emptyset$	laws

Identity	Name
$A \cup (B \cup C) = (A \cup B) \cup C$	Associative
$A \cap (B \cap C) = (A \cap B) \cap C$	laws
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	laws