Instructions: All assignments are due by **midnight** on the due date specified. Assignments should be typed or scanned and submitted as a PDF in Canvas.

Every student or student group must write up their own solutions in their own manner.

You should complete all problems, but only a subset will be graded (which will be graded is not known to you ahead of time).

Logic

1. (4 points) **Graded (a-d)** Determine which of the following statements are propositions? What are the truth values of those that are propositions?
   (a) $x - 4 = 7$ (b) When in Winter Carnival? (c) 14 is prime.
   (d) 37 is odd. (e) Read Chapter 2.1 before class. (f) $8 - 5 = -3$

(a) $x - 4 = 7$	not a proposition, depends on x	–
(b) When is Winter Carnival?	not a proposition, question	–
(c) 14 is prime.	proposition	F
(d) 37 is odd.	proposition	T
(e) Read Chapter 2.1 before class.	not a proposition, command	–
(f) $8 - 5 = -3$	proposition	F

2. **Ungraded** Rosen Ch 1.1, #10 (a,c,f,h), p. 14

   (a) I did not buy a lottery ticket this week.
   (c) If I bought a lottery ticket this week, then I won the million dollar jackpot on Friday.
   (f) If I did not buy a lottery ticket this week, then I did not win the million dollar jackpot on Friday.
   (h) Either I did not buy a lottery ticket this week, or else I did buy one and won the million dollar jackpot on Friday.

3. (4 points) **Graded (c-f)** Rosen Ch 1.1, #16, p. 14

   (a) $r \wedge \neg q$
   (b) $p \wedge q \wedge r$
   (c) $r \rightarrow p$
   (d) $p \wedge \neg q \wedge r$
   (e) $(p \wedge q) \rightarrow r$
   (f) $r \leftrightarrow (q \vee p)$

4. (3 points) **Graded (a-c)** Rosen Ch 1.1 #24 (a-f), p. 15.

   - If you get promoted, then you wash the boss's car.
   - If the winds are from the south, then there will be a spring thaw.
   - If you bought the computer less than a year ago, then the warranty is good.
   - If Willy cheats, then he gets caught.
   - If you access the website, then you must pay a subscription fee.
   - If you know the right people, then you will be elected.

5. (3 points) **Graded (b)** State the converse, contrapositive, and inverse of each of the following statements.

   (a) If it is summer, then I take a vacation.
   (b) To be able to go on the trip, it is necessary that you get written permission.

   (a) Converse: If I take a vacation, then it is summer.
   Contrapositive: If I do not take a vacation, then it is not summer.
   Inverse: If it is not summer, then I do not take a vacation.

   (b) If you go on the trip, then you get written permission.
   Getting written permission is a necessary condition for going on the trip.
   Converse: If you get written permission, then you go on the trip
   Contrapositive: If you do not get written permission, then you are not able to go on the trip.
   Inverse: If you do not go on the trip, then you did not get written permission.

6. (9 points) **Graded (b,c,e)** Construct a truth table for each compound proposition.

   (a) (2 pt) $p \to \neg p$
   (b) (2 pt) $(p \vee q) \wedge \neg p$
   (c) (3 pt) $(\neg q \to p) \oplus (p \wedge \neg q)$
   (d) (4 pt) $p \vee (q \to r)$
   (e) (4 pt) $(p \leftrightarrow \neg r) \to (q \vee \neg(p \vee \neg r))$

(a) $p \to \neg p$

p	$\neg p$	$p \to \neg p$
T	F	F
F	T	T

(b) $(p \vee q) \wedge \neg p$

p	q	$\neg p$	$p \vee q$	$(p \vee q) \wedge \neg p$
T	T	F	T	F
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

(c) $(\neg q \to p) \oplus (p \wedge \neg q)$

p	q	$\neg q$	$\neg q \to p$	$p \wedge \neg q$	(c)
T	T	F	T	F	T
T	F	T	T	T	F
F	T	F	T	F	T
F	F	T	F	F	F

(d) $p \vee (q \to r)$

p	q	r	$q \to r$	(d)
T	T	T	T	T
T	T	F	F	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	F
F	F	T	T	T
F	F	F	T	T

(e) $(p \leftrightarrow \neg r) \to (q \vee \neg(p \vee \neg r))$

p	q	r	$\neg r$	$p \vee \neg r$	$\neg(p \vee \neg r)$	$(q \vee \neg(p \vee \neg r))$	$p \leftrightarrow \neg r$	(e)
T	T	T	F	T	F	T	F	T
T	T	F	T	T	F	T	T	T
T	F	T	F	T	F	F	F	T
T	F	F	T	T	F	F	T	F
F	T	T	F	F	T	T	T	T
F	T	F	T	T	F	T	F	T
F	F	T	F	F	T	T	T	T
F	F	F	T	T	F	F	F	T

Sets

7. (3 points) **Graded (all)** List the members of the sets.
   (a) $\{x \mid x \in \mathbb{N} \text{ and } -4 < x < 4\}$
   (b) $\{2x+1 \mid x \in \mathbb{N} \text{ and } x < 4\}$
   (c) $\{\{a, a^2-1\} \mid a \in \mathbb{N} \text{ and } -2 \leq a < 3\}$

   (a). $\{0, 1, 2, 3\}$
   (b). $\{1, 3, 5, 7\}$
   (c). $\{\{0, -1\}, \{1, 0\}, \{2, 3\}\}$

8. (3 points) **Graded (c-e)** Write each of the following sets in set-builder notation.
   (a) $S_a = \{2, 4, 8, 16, 32, \ldots\}$
   (b) $S_b = \{-3, -2, -1, 0, 1, 2\}$
   (c) $S_c = \{\ldots, \frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27 \ldots\}$
   (d) The natural numbers divisible by 4, S_d
   (e) The square roots of the natural numbers, S_e

   (a) $S_a = \{2n \mid n \in \mathbb{Z}^+\}$
   (b) $S_b = \{x \in \mathbb{Z} \mid -4 < x < 3\}$
   (c) $S_c = \{3^x \mid x \in \mathbb{Z}\}$
   (d) $S_d = \{4n \mid n \in \mathbb{N}\}$
   (e) $S_e = \{\sqrt{n} \mid n \in \mathbb{N}\}$

9. (6 points) **Graded (all)** Let $A = \{b, \{b, a\}, \emptyset\}$ and $B = \{\emptyset, \{\emptyset\}\}$. Determine whether each of the statements are *True* or *False*.

   (a) $\emptyset \in A$ (b) $\emptyset \subseteq A$ (c) $a \in A$ (d) $a \subseteq A$ (e) $\{\emptyset\} \in A$ (f) $\{\emptyset\} \subseteq A$
   (g) $\emptyset \in B$ (h) $\emptyset \subseteq B$ (i) $\{\emptyset\} \in B$ (j) $\{\emptyset\} \subseteq B$ (k) $\{\emptyset\} \subset A$ (l) $\{\emptyset\} \subset B$

   (a) True
   (b) True
   (c) False
   (d) False
   (e) False
   (f) True
   (g) True
   (h) True
   (i) True
   (j) True
   (k) True
   (l) True

10. **Ungraded** Let $A = \{1, \{2, 3\}, 2\}$, $B = \{1, 2, 4\}$, $C = \{3, 4, \{2\}\}$, $D = \emptyset$, $E = \{\emptyset\}$, where the universal set is $U = A \cup B \cup C \cup D \cup E$. Determine (write out members of the set explicitly, that is do not use set builder notation):

    (a) $A \cup B$ (b) $C \cup E$ (c) $A \cap B$ (d) $C \cap B$
    (e) $A - C$ (f) $C - A$ (g) $\overline{A}$ (h) $(A \cup C) - D$
    (i) $E \times A$ (j) $A \times B$ (k) $\mathcal{P}(B)$ (l) $\mathcal{P}(D)$
    (m) $|C|$ (n) $|E \cup A|$ (o) $(A \cap U) \cup \overline{U}$

11. (4 points) **Graded (a-d,g-h,k-l)** Using the sets from problem 10, determine the truth value of the following statements.

(a) $A \subseteq B$	(b) $B \subseteq A$	(c) $D \subseteq D$	(d) $E \subseteq B$
(e) $A \subset B$	(f) $D \subset D$	(g) $E \subset B$	(h) $D \subset E$
(i) A disjoint from B	(j) E disjoint from C	(k) E disjoint from E	(l) $\emptyset$ disjoint from D

(a) False
(b) False
(c) True
(d) False
(e) False
(f) False
(g) False
(h) True
(i) False
(j) True
(k) False
(l) True

12. (1 point) **Graded (all)** How many elements are in $\mathcal{P}(A) \times A$ when $A = \{a, b, c, d, e, f\}$? Explain your answer.

The number of elements in $\mathcal{P}(A)$ is $2^{|A|} = 2^6 = 64$. Therefore, the number of elements in $\mathcal{P}(A) \times A$ is all pairs of the elements in the sets or $64 * 6 = \mathbf{384}$.

Bonus Questions

Note, the Bonus questions can at times be more challenging or lengthy in crafting your solution. First, complete the main problems of the assignment before considering doing the bonus questions.

13. (1 point (bonus)) Many restaurants have the sign, "No shoes, no shirt, no service." Write this sentence as a conditional proposition.

If you do not wear shoes and a shirt, then you can not enter the restaurant for service.

14. (2 points (bonus)) When considering the case of two propositions, p and q, there are are 16 truth tables. All sixteen possibilities are listed in the table below (columns a-p). For each column, express the proposition represented using standard operators ($\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$, $\oplus$). Try to keep the expressions simple (use as few logical operators and propositional variables).

p	q	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)	(k)	(l)	(m)	(n)	(o)	(p)
T	T	T	T	T	T	T	T	T	T	F	F	F	F	F	F	F	F
T	F	T	T	T	T	F	F	F	F	T	T	T	T	F	F	F	F
F	T	T	T	F	F	T	T	F	F	T	T	F	F	T	T	F	F
F	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F

(a) $\mathbf{T} \equiv p \vee \neg p$
(b) $p \vee q$
(c) $q \rightarrow p$
(d) p
(e) $p \rightarrow q$
(f) q
(g) $p \leftrightarrow q$
(h) $p \wedge q$
(i) $\neg(p \wedge q)$
(j) $p \oplus q \equiv \neg(p \leftrightarrow q)$
(k) $\neg q$
(l) $\neg(p \rightarrow q)$
(m) $\neg p$
(n) $\neg(q \rightarrow p)$
(o) $\neg(p \vee q)$
(p) $\mathbf{F} \equiv p \wedge \neg p$