Instructions: All assignments are due by **midnight** on the due date specified. Assignments should be typed and submitted as a PDF. Every student must write up their own solutions in their own manner.

You should complete all problems, but only a subset will be graded (which will be graded is not known to you ahead of time).

Propositional Logic

1. **Ungraded** Use the rules of inference to show that the hypotheses imply the conclusion:
   - "If I graduate in four years, then I will have completed the CS courses", and
   - "If I do not work on CS for 10 hours a week, then I will not complete the CS courses", and
   - "If I work on CS for 10 hours a week, then I can not procrastinate."

   Conclusion: "If I procrastinate, then I will not graduate in four years."

   Let $w =$ "I work on CS for 10 hours a week",
   $g =$ "I graduate in four years",
   $c =$ "I will complete the CS courses", and
   $p =$ "I procrastinate."

   (a) (4 pts) Translate the hypotheses and conclusion to logical statements.

   (b) (6 pts) Construct a valid argument (justify each step).

   Hint: Remember you can also use the logical equivalences as a step in the argument.

   (a) The hypotheses are:
   - $g \to c$
   - $\neg w \to \neg c$
   - $w \to \neg p$.

   The conclusion is: $p \to \neg g$.

   (b)

Step	Reason
1. $g \to c$	hypothesis
2. $\neg w \to \neg c$	hypothesis
3. $w \to \neg p$	hypothesis
4. $\neg\neg p \to \neg w$	Table 7, rule 2, with (3)
5. $p \to \neg w$	Double Negation with (4)
6. $p \to \neg c$	Hyp. syl., with (2) & (5)
7. $\neg c \to \neg g$	Table 7, rule 2 with (1)
8. $p \to \neg g$	Hyp. syl. with (6) & (7)

   Note, this is one possible valid argument; many others exists.

Step	Reason
1. $g \to c$	premise
2. $\neg w \to \neg c$	premise
3. $w \to \neg p$	premise
4. $\neg g \lor c$	Table 7.1 with (1)
5. $\neg\neg w \lor \neg c$	Table 7.1 with (2)
6. $\neg w \lor \neg p$	Table 7.1 with (3)
7. $w \lor \neg c$	Double Negation with (5)
8. $\neg p \lor \neg c$	Resolution with (6) and (7)
9. $\neg c \lor \neg p$	Commutative with (8)
10. $c \lor \neg g$	Commutative with (9)
11. $\neg p \lor \neg g$	Resolution with (9) and (10)
12. $p \to \neg g$	Table 7.1 with (11)

2. (12 points) **Graded (all)** Use the same propositional variables of problem 1 and the rules of inference to show the hypotheses imply the conclusion:

   - "If I work on CS for 10 hours a week and I don't procrastinate, then I will complete the CS courses",
   - "I will not graduate in four years and I worked on CS for 10 hours a week.", and
   - "If I complete the CS courses, then I graduate in four years."

   Conclusion: "I procrastinated."

   First, translate the hypotheses and conclusion in to logical statements (4 points). Then, show the valid argument (8 points).

   The hypotheses are:

   - $(w \land \neg p) \to c$
   - $\neg g \land w$
   - $c \to g$

   The conclusion is: p.

Step	Reason
1. $(w \land \neg p) \to c$	hypothesis
2. $\neg g \land w$	hypothesis
3. $c \to g$	hypothesis
4. $\neg g$	Simplification with (2)
5. $\neg c$	modus tollens with (3) & (4)
6. $\neg(w \land \neg p)$	modus tollens with (1) & (5)
7. $\neg w \lor \neg\neg p$	DeMorgans with (6)
8. $\neg w \lor p$	Double neg. with (7)
9. w	Simplifiation with (2)
10. p	Disj. syl. with (8) & (9)

3. **Ungraded** Use the rules of inference to show that the hypotheses imply the conclusion:

   > If you eat carefully then you will have a healthy digestive system. If you exercise regularly you will be very fit. If you have a healthy digestive system or you are very fit, you will live to a ripe old age. You do not live to a ripe old age. Therefore, you did not eat carefully and you did not exercise regularly.

Let $c =$ "you eat carefully",
$h =$ "you have a healthy digestive system",
$e =$ "you excercise regularly",
$f =$ "you will be very fit",
$l =$ "you will live to a ripe old age"

(a) (4 pts) Translate the hypotheses and conclusion to logical statements.

(b) (8 pts) Construct a valid argument (justify each step).

Hint: Remember you can also use the logical equivalences as a step in the argument.

(a) The hypotheses are:

- $c \to h$
- $e \to f$
- $(h \lor f) \to l$
- $\neg l$

The conclusion is: $\neg c \land \neg e$.

(b)

Step	Reason
1. $c \to h$	hypothesis
2. $e \to f$	hypothesis
3. $(h \lor f) \to l$	hypothesis
4. $\neg l$	hypothesis
5. $\neg(h \lor f)$	modus tollens (3) & (4)
6. $\neg h \land \neg f$	DeMorgans (5)
7. $\neg h$	Simplification (6)
8. $\neg c$	modus tollens (7) & (1)
9. $\neg f$	Simplification (6)
10. $\neg e$	modus tollens (2) & (9)
11. $\neg c \land \neg e$	Conjunction (8) & (10)

Note, this is one possible valid argument; many others exists.

Predicate Logic

4. (3 points) **Graded (d-f)** Consider the domain of discourse to be the following set $\{duck, monkey, walleye, dog\}$. Let $F(x)$ be true when x can fly, let $H(x)$ be true when x has hair/fur, let $T(x)$ be true when x has a tail, and let $S(x)$ be true if x can swim.

(a) (3 pt) Complete the following truth table

x	$F(x)$	$H(x)$	$T(x)$	$S(x)$
duck	**T**	F	T	T
monkey	**F**	**T**	T	T
walleye	F	**F**	T	**T**
dog	F	**T**	T	T

Express the following predicates, (b-d), using the universal quantifier:

(b) (1 pt) $F(duck) \land F(monkey) \land F(walleye) \land F(dog)$

(c) (1 pt) $\neg T(duck) \wedge \neg T(monkey) \wedge \neg T(walleye) \wedge \neg T(dog)$
(d) (1 pt) $S(duck) \vee S(monkey) \vee S(walleye) \vee S(dog)$

(b) $\forall x\ F(x)$ (c) $\forall x\ \neg T(x)$ (d) $\neg\forall x\ \neg S(x)$

Determine the truth values of the following expressions

(e) (1 pt) $\forall x\, S(x)$
(f) (1 pt) $\forall x\, (H(x) \to T(x))$
(g) (1 pt) $\exists x\, (\neg H(x) \wedge F(x))$
(h) (1 pt) $\exists x\, (F(x) \wedge T(x) \wedge (\exists y\, ((x \neq y) \to (\neg H(y) \vee \neg T(y)))))$

(e) True (f) True (g) True (h) False

5. **Ungraded** Rosen Ch 1.4 #8(a,d) p. 53
   (a) "For every animal, if it is a rabbit then then that animal hops."
   Or, "Every rabbit hops."
   (d) "There exists an animal that is a rabbit and it hops."
   Or, "Some rabbits hop."
   Or, "Some hopping animals are rabbits."

6. (2 points) **Graded (c-d)** Rosen Ch 1.4 #28(c, d, e), p. 54
   Use the following predicates:
   - $C(x)$ be "x is in the correct place"
   - $T(x)$ be "x is a tool"
   - $E(x)$ be "x is in excellent condition"

   (c) $\forall x\ (C(x) \wedge E(x))$
   (d) $\neg\exists x\ (C(x) \wedge E(x)) \equiv \forall x\ \neg(C(x) \wedge E(x)) \equiv \forall x\ (\neg C(x) \vee \neg E(x))$
   (e) $\exists x\ (T(x) \wedge \neg C(x) \wedge E(x))$

7. **Ungraded** Translate the logical expressions into English sentences using the same predicates as problem 6.
   (a) $\forall x\ (E(x) \to C(x))$
   (b) $\exists x\ (T(x) \wedge \neg E(x) \wedge \neg C(x))$

   (a) Everything that is in excellent condition, is in its correct place.
   (b) There exists a tool that is not in excellent condition and is not in the correct place.

8. **Ungraded** Rosen Ch 1.4 # 38(d,e), p. 55
   (d) Some system is unavailable.
   (e) No system is working. Or, Every system is not working

9. (1 point) **Graded (b)** Rosen Ch 1.4 #42(a-c), p. 55
   Use the following predicates:
   - $A(x)$ be "User x has access to an electronic mailbox"
   - $R(x, y)$ be "Group member x can access resource y"
   - $S(x, y)$ be "System x is in state y"

- $\forall x\, A(x)$
- $S(filesystem, locked) \to \forall x\, A(x, systemmailbox)$
- $S(firewall, diagnostic) \to S(proxyserver, diagnostic)$

10. (1 point) **Graded (b)** For this problem, use the predicates: $F(x)$ is "x is a Freshman", $S(x)$ be "x is a student at MTU", $C(y)$ is "y is a CS course", and $T(x, y)$ is "x is taking y", where x has the domain of all students at MTU and y has the domain of all CS courses.
    (a) Translate the logical expression into English: $\forall x\, (F(x) \to T(x, CS1000))$.
    (b) Translate English into logic: "Some freshman at MTU are taking CS1121."
    (c) Translate English into logic: "Every freshman at MTU is taking a CS course."

    (a) "For every student x at MTU, if x is a freshman, then x is taking CS1000."
    Conversationally, "All freshman at MTU take CS1000."
    (b) $\exists x\, (F(x) \wedge T(x, CS1121))$
    (c) $\forall x\, (F(x) \to \exists y\, T(x, y))$ or $\forall x\, \exists y\, (F(x) \to T(x, y))$

11. **Ungraded** Repeat problem 10 with the domain of x is all people and y is all courses.

    (a) "For every person x, if x is a freshman then x is taking CS1000."
    Conversationally, "All freshman take CS1000."
    (b) $\exists x\, (S(x) \wedge F(x) \wedge T(x, CS1121))$
    (c) $\forall x\, ((S(x) \wedge F(x)) \to \exists y(C(y) \wedge T(x, y)))$ or
    $\forall x\, \exists y\, ((S(x) \wedge F(x)) \to (C(y) \wedge T(x, y)))$

12. **Ungraded** Rosen Ch 1.5 #4(a,c), p. 64

    (a) There is a student in your class who has taken some computer science class at your school.
    (c) Every student in your class has taken a computer science class at your school.

13. (3 points) **Graded (c-e)** Rosen Ch 1.5 #10(a,c,d,e,i), p. 65

    (a) $\forall x\, F(x, Fred)$
    (c) $\forall x\, \exists y\, F(x, y)$
    (d) $\neg \exists x\, \forall y\, F(x, y)$
    (e) $\forall y\, \exists x\, F(x, y)$
    (i) $\neg \exists x F(x, x)$

14. **Ungraded** Rewrite the statements so that the negations appear only on predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).
    (a) $\neg \forall x\, \exists y\, P(x, y)$
    (b) $\neg \forall x\, (\exists y\, \forall z\, T(x, y, z) \to \forall y\, \forall z\, U(x, y, z))$

    Repeatedly apply DeMorgans law of quantifiers, DeMorgans law, Double Negation, and other logical equivalence laws.

    (a) $\neg \forall x\, \exists y\, P(x, y)$
    $\equiv \exists x\, \neg \exists y\, P(x, y)$
    $\equiv \exists x\, \forall y\, \neg P(x, y)$

(b) $\neg\forall x\ (\exists y\ \forall z\ T(x,y,z) \rightarrow \forall y\ \forall z\ U(x,y,z))$
$\equiv \exists x\ \neg(\exists y\ \forall z\ T(x,y,z) \rightarrow \forall y\ \forall z\ U(x,y,z))$
$\equiv \exists x\ (\exists y\ \forall z\ T(x,y,z) \wedge \neg\forall y\ \forall z\ U(x,y,z))$
$\equiv \exists x\ (\exists y\ \forall z\ T(x,y,z) \wedge \exists y\ \neg\forall z\ U(x,y,z))$
$\equiv \exists x\ (\exists y\ \forall z\ T(x,y,z) \wedge \exists y\ \exists z\ \neg U(x,y,z))$

15. (8 points) **Graded (all)** Consider the following statements (the domain of x is all people):
   (a) Each student in the class has a computer.
   (b) Everyone with a computer can program.
   (c) Mary is a student in the class.
   (d) Therefore, Mary can program.

   Let $S(x)$ mean "x is a student in the class", $C(x)$ mean "x has a computer", and $P(x)$ mean "x can program." You will first translate the statements into logical expressions (2 pts). Then, show the conclusion (d) can be drawn from the prior statements (a-c) (6 pts). Justify each step of the argument.

   For the problem, the translated statements are as follows:
   (a) $\forall x\ (S(x) \rightarrow C(x))$
   (b) $\forall x\ (C(x) \rightarrow P(x))$
   (c) $S(Mary)$
   (d) $P(Mary)$

   The argument is as follows:

Step	Reason
1. $\forall x\ (S(x) \rightarrow C(x))$	Given
2. $\forall x\ (C(x) \rightarrow P(x))$	Given
3. $S(Mary)$	Given
4. $S(Mary) \rightarrow C(Mary)$	Univ. instantiation with (1)
5. $C(Mary) \rightarrow P(Mary)$	Univ. instantiation with (2)
6. $C(Mary)$	Modus ponens with (3) and (4)
7. $P(Mary)$	Modus ponens with (5) and (6)

16. (8 points) **Graded (all)** Rosen Ch 1.6 #14 (d), p. 79
   Let $c(x)$ be "x is in this class", $f(x)$ be "x has been to France", and $l(x)$ be "x has visited the Louvre."

   (d) Premises:
      1. $\exists x\ (c(x) \wedge f(x))$
      2. $\forall x\ (f(x) \rightarrow l(x))$

   The conclusion is: $\exists x\ (c(x) \wedge l(x))$.

Step	Reason
1. $\exists x\ (c(x) \wedge f(x))$	Hypothesis
2. $c(a) \wedge f(a)$	Exist. Inst. with (1)
3. $f(a)$	Simplification with (2)
4. $c(a)$	Simplification with (2)
5. $\forall x\ (f(x) \rightarrow l(x))$	Hypothesis
6. $f(a) \rightarrow l(a)$	Univ. Inst. with (5)
7. $l(a)$	Modus ponens with (3) and (6)
8. $c(a) \wedge l(a)$	Conjunction with (4) and (7)
9. $\exists x\ (c(x) \wedge l(x))$	Existential generalization with (8)

17. (2 points) **Graded (a)** For each argument determine whether it is valid or not and explain why (in a sentence).
    (a) "Some math majors left the campus for the weekend" and "All seniors left the campus for the weekend" implies the conclusion "Some seniors are math majors."
    (b) "Everyone who left campus for the weekend is a senior" and "All math majors left campus for the weekend" implies the conclusion "All math majors are seniors."
    (c) "No juniors left campus for the weekend" and "Some math majors are not juniors" implies the conclusion "Some math majors left campus for the weekend."

    (a) Not Valid argument. The two premises do not imply the conclusion.
    (b) Valid argument. The argument uses Univ. instantiation (x2), hyp. syllogism, followed by Univ. generalization.
    (c) Not Valid argument. The two premises do not imply the conclusion.

18. **Ungraded** Rosen Ch 1.6 # 16(c,d), p. 79

    (c) Incorrect, first apply universal instantiation then fallacy of denying the hypothesis.
    (d) Correct, using universal instantiation and modus ponens.

19. **Ungraded** Rosen Ch 1.6 # 18, p. 79

    It is true that there is some s in the domain s.t. $S(s, Max)$; however, there is no guarantee that Max is the s. This first step of the proof is invalid.

Bonus Questions

20. (2 points (bonus)) Using the same predicates as problem 10, translate the following statement into logic assuming the domain of x is all students at MTU and the domain of y is all CS courses. "Some freshman at MTU is taking two CS courses."

    $\exists x\ \exists y\ \exists z\ (F(x) \wedge T(x, y) \wedge T(x, z) \wedge (y \neq z))$

21. (2 points (bonus)) Rosen Ch 1.5 #10(h,j), p. 65

    (h) $\exists y\ (\forall x\ F(x, y) \wedge \forall z\ (\forall x\ F(x, z) \rightarrow z = y))$
    (j) $\exists x\ \exists y\ (x \neq y \wedge F(x, y) \wedge \forall z\ ((F(x, z) \wedge z \neq x) \rightarrow z = y))$