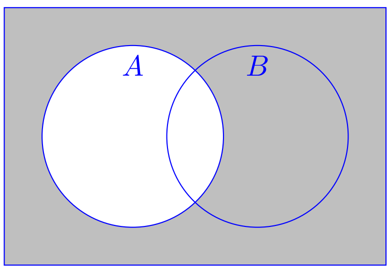
Instructions: All assignments are due by **midnight** on the due date specified. Assignments should be typed and submitted as a PDF in Canvas.

Every student must write up their own solutions in their own manner.

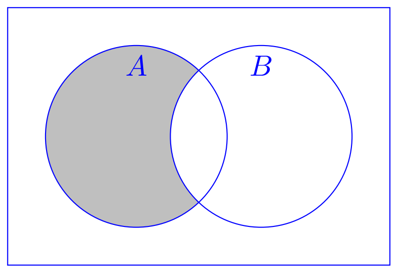
You should complete all problems, but only a subset will be graded (which will be graded is not known to you ahead of time).

Sets

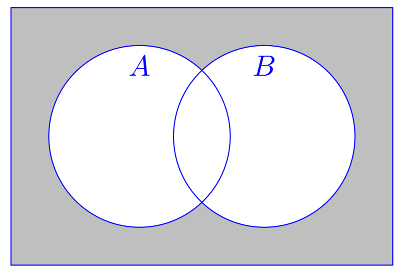
1. **Ungraded** For each of the following sets, shade the corresponding region of the Venn diagram.



(a) $\bar{A}$

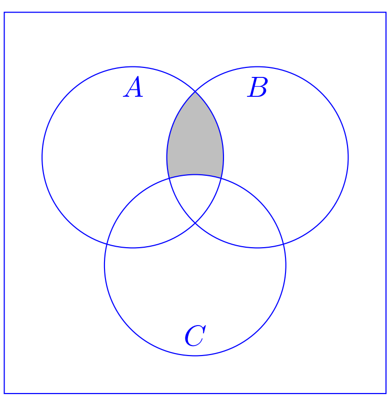


(b) $A \cap \bar{B}$ or $A - B$

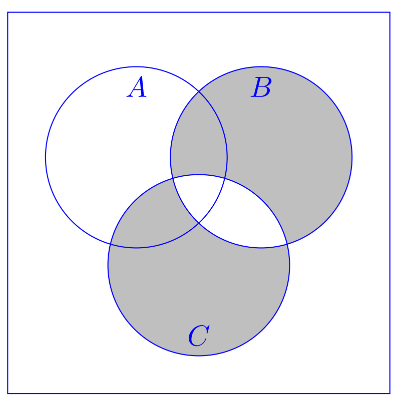


(c) $\overline{(A \cup B)}$

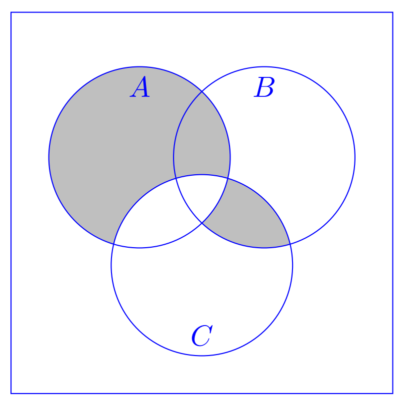
2. (4 points) **Graded (a,b)** For each of the following sets, shade the corresponding region of the Venn diagram.



(a) $(A \cap B) - C$

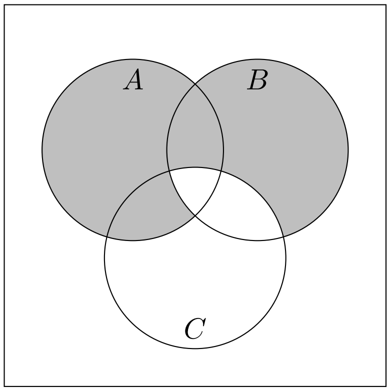


(b) $(C - B) \cup (B - C)$

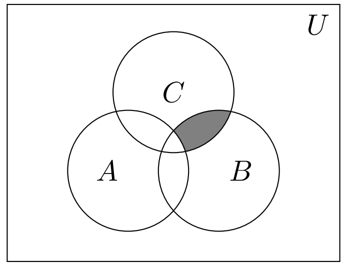


(c) $(A - C) \cup ((B \cap C) - A)$

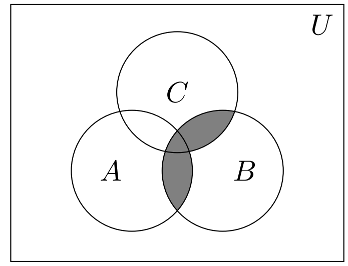
3. **Ungraded** For each of the following sets, state what the corresponding shaded region of the Venn diagram represents.



(a) $(A - B) \cup (A - C) \cup (B - C)$ or $((A \cup B) - C) \cup ((A \cap C) - B)$ or $(A - B) \cup (B - C)$



(b) $B \cap C \cap \bar{A}$



(c) $(A \cap B) \cup (B \cap C)$

4. **Ungraded** Show the following sets are equal, $\overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}$, using
   (a) set membership table
   (b) set identities

(a)

A	B	C	$B \cap C$	$A \cup (B \cap C)$	$\overline{A \cup (B \cap C)}$	$\overline{A}$	$\overline{B \cap C}$	$(\overline{C} \cup \overline{B}) \cap \overline{A}$
1	1	1	1	1	0	0	0	0
1	1	0	0	1	0	0	1	0
1	0	1	0	1	0	0	1	0
1	0	0	0	1	0	0	1	0
0	1	1	1	1	0	1	0	0
0	1	0	0	0	1	1	1	1
0	0	1	0	0	1	1	1	1
0	0	0	0	0	1	1	1	1

(b)

$$\begin{aligned}
\overline{A \cup (B \cap C)} &= \overline{A} \cap \overline{B \cap C} && \text{(DeMorgan's set identities)}\\
&= \overline{A} \cap (\overline{B} \cup \overline{C}) && \text{(DeMorgan's set identities)}\\
&= (\overline{B} \cup \overline{C}) \cap \overline{A} && \text{(Commutative)}\\
&= (\overline{C} \cup \overline{B}) \cap \overline{A} && \text{(Commutative)}
\end{aligned}$$

5. (4 points) **Graded (all)** Show the following sets are equal, $A \cap (\overline{A} \cup B) = A \cap B$, using
   (a) set membership table
   (b) set identities

(a)

A	B	$\bar{A}$	$\bar{A} \cup B$	$A \cap (\bar{A} \cup B)$	$A \cap B$
1	1	0	1	1	1
1	0	0	0	0	0
0	1	1	1	0	0
0	0	0	0	0	0

(b)

$$\begin{aligned}
A \cap (\overline{A} \cup B) &= (A \cap \overline{A}) \cup (A \cap B) && \text{(Distributive)}\\
&= \emptyset \cup (A \cap B) && \text{(Complement)}\\
&= (A \cap B) \cup \emptyset && \text{(Commutative)}\\
&= A \cap B && \text{(Identity)}
\end{aligned}$$

6. (3 points) **Graded (d-f)** Consider a collection of books in the library. The universal set U refers to all books in the library. In addition, books are classified by their subject (each book may receive more than one classification) into the following sets: F - fiction, B - biography, H - historical, P - poetry, N - non-fiction, S - scientific, E - English, O - other language, L - literary, T - travel, R - reference. Each set is a subset of U.

   Express each of the following statements using set expressions, e.g., set operators ($\cup$, $\cap$, -, $\overline{A}$, etc.), relations ($=$, $\subseteq$, $\subset$), etc.:

   (a) $S \cap R$ The set of scientific references.
   (b) $F \cap N = \emptyset$ No book is both fiction and non-fiction.
   (c) $S \cap L = \emptyset$ A scientific book is not literary.
   (d) $(H \cap F) - O$ The set of historical fiction books excluding those in other languages.
   (e) $B \subseteq H$ Any biography is also a historical book.
   (f) $\overline{R \cup L \cup P} = \emptyset$ All books are either reference, literary, or poetry.
   (g) $T \subseteq R$ All travel books are reference books.
   (h) $P \cap O \subseteq L$ All poetry in other languages are literary works.

Function

7. (6 points) **Graded (all)** Let $S = \{a, b, c, d, e\}$ and $T = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Determine whether the following are functions (Yes / No).

   (a) Yes, a function $f : S \to T,\ f = \{(a, 5), (b, 4), (c, 1), (d, 3), (e, 2)\}$
   (b) Not a function $g : S \to T,\ g = \{(a, 4), (d, 3), (c, 3), (b, 5)\}$
   (c) Not a function $h : S \to T,\ h = \{(a, 7), (b, 4), (c, 1), (d, 2), (e, 3), (b, 3)\}$
   (d) Yes, a function $i : S \to T,\ i = \{(a, 2), (b, 2), (c, 1), (d, 5), (e, 8)\}$
   (e) Not a function $j : \mathbb{Z} \to \mathbb{Z} = \{(x, y) \mid 3x + y^2 = 8\}$
   (f) Yes, a function $k : \mathbb{R} \to \mathbb{R} = \{(x, x^2) \mid x \in \mathbb{R}\}$

8. (2 points) **Graded (a-b)** Rosen, Ch 2.3 # 6 (a-d), p. 152. (p. 161 for 8th ed)

   (a) the domain is $\mathbb{Z}^+ \times \mathbb{Z}^+$, the range is $\mathbb{Z}^+$
   (b) the domain is $\mathbb{Z}^+$, the range is $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
   (c) the domain is the set of all bit strings, the range is $\mathbb{Z}$
   (d) the domain is $\mathbb{Z}^+$, the range is $\mathbb{Z}^+$

9. **Ungraded** Find these values:

   (a) $\lfloor 8.6 \rfloor = 8$
   (b) $\lceil 4.3 \rceil = 5$
   (c) $\lfloor -3.6 \rfloor = -4$
   (d) $\lfloor 10.5\bar{3} \rfloor = 10$
   (e) $\lceil -2.1 \rceil = -2$
   (f) $\lfloor \frac{-3}{4} \rfloor = -1$
   (g) $\lceil \frac{13}{3} + \lfloor \frac{-5}{4} \rfloor \rceil = 3$
   (h) $\lfloor 3.2 - \lceil 10.4 \rceil \rfloor = -8$

10. (6 points) **Graded (c-e)** Determine whether each of the following functions are (i) one-to-one and (ii) onto.

(a)	one-to-one	onto	Rosen Ch 2.3, #10(a), p. 153 (p. 162 for 8th ed)
(b)	not one-to-one	not onto	Rosen Ch 2.3, #10(b), p. 153 (p. 162 for 8th ed)
(c)	not one-to-one	not onto	$f : \mathbb{Z} \to \mathbb{Z}$ where $f(x) = x^2 - 5x + 5$.
(d)	one-to-one	not onto	$g : \mathbb{N} \to \mathbb{N}$ where $g(n) = n + 1$
(e)	not one-to-one	onto	$h : \mathbb{N} \to \mathbb{N}$ where $h(n) = \lfloor \frac{n}{2} \rfloor$
(f)	not one-to-one	not onto	$i : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ where $i(m, n) = 2n - 4m$
(g)	not one-to-one	not onto	$j : \mathbb{R} \to \mathbb{R}$ where $j(x) = \sqrt{x}$

11. (4 points) **Graded (all)** Let A and B be finite sets, and f be a function is $f : A \to B$. Determine which of the following statements are true.

    (a) False If $f : A \to B$ is onto, then the domain and range are not only the same size, but the same set.
    (b) True If $f : A \to B$ is both one-to-one and onto, then A and B have the same cardinality.
    (c) True If $f : A \to B$ is one-to-one, then $|A| \leq |B|$.
    (d) True If $f : A \to B$ is onto, then $|A| \geq |B|$.

12. (8 points) **Graded (c-f)** Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, d, f, g\}$. Let $f : A \to B$ and $g : B \to B$ with

$$f = \{(a, b), (b, d), (c, g), (d, a), (e, b)\} \text{ and}$$
$$g = \{(a, f), (b, d), (d, a), (f, g), (g, b)\}$$

For each of the following compositions, define the function or explain why it is not defined.

(a) $f \circ g$ (b) $g \circ f$ (c) $f \circ f$ (d) $g \circ g$

(e) Find f^{-1} if it exists. If it doesn't, explain why not.
(f) Find g^{-1} if it exists. If it doesn't, explain why not.

(a) $f \circ g$ is not defined, the range of g is not a subset of the domain of f
(b) $g \circ f = \{(a, d), (b, a), (c, b), (d, f), (e, d)\}$
(c) $f \circ f$ is not defined, the range of f is not a subset of the domain of f
(d) $g \circ g = \{(a, g), (b, a), (d, f), (f, b), (g, d)\}$
(e) f^{-1} does not exists, f is not one-to-one and not onto
(f) g^{-1} does exist, $= \{(f, a), (d, b), (a, d), (g, f), (b, g)\}$

13. **Ungraded** Let f, g, and h all be functions mapping from A to $\mathbb{R}$ where $A = \{x \in \mathbb{R} \mid x > 0\}$,

$$f(x) = \frac{1}{x+1}, \qquad g(x) = \frac{x+1}{x}, \text{ and} \qquad h(x) = x - 1.$$

Compute (a) $(f \circ g)(x)$, (b) $(g \circ f)(x)$, (c) $(h \circ g \circ f)(x)$, (d) $(f \circ g \circ h)(x)$

(a) $(f \circ g)(x) = f(g(x)) = f(\frac{x+1}{x}) = \frac{1}{(\frac{x+1}{x}) + 1}$

(b) $(g \circ f)(x) = g(f(x)) = g(\frac{1}{x+1}) = \frac{\left(\frac{1}{x+1}\right) + 1}{\left(\frac{1}{x+1}\right)}$

(c) $(h \circ g \circ f)(x) = h(g(f(x))) = h\left(g\left(\frac{1}{x+1}\right)\right) = h\left(\frac{\left(\frac{1}{x+1}\right)+1}{\left(\frac{1}{x+1}\right)}\right) = \frac{\left(\frac{1}{x+1}\right)+1}{\left(\frac{1}{x+1}\right)} - 1$

(d) $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(x-1)) = f\left(\frac{(x-1)+1}{(x-1)}\right) = \frac{1}{\left(\frac{(x-1)+1}{(x-1)}\right)+1}$

14. (3 points) **Graded (a)** Let P, be a set of *Patients* who have ever been admitted to the hospital at some time, B, be a set *Beds* available for patients.

The function *currentBed* maps *Patients* to *Beds*. The function relates a patient to the bed that he/she is currently occupying in the hospital.

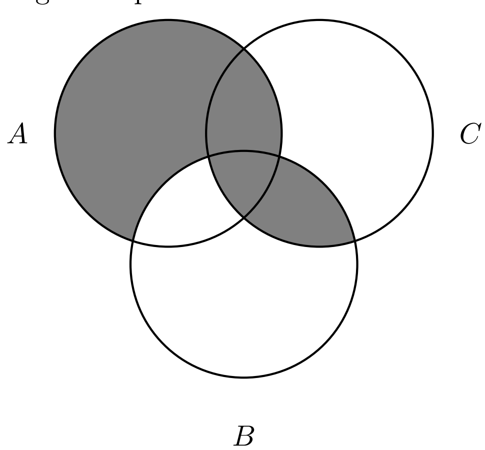
The function *date1stAdmitted* that maps *Patients* to *Dates*, relating a patient to the date he/she was first admitted to the hospital.

(a) function, $\textit{currentBed} : \textit{Patients} \rightarrow \textit{Beds}$
  (a) No, the function is not total; not every patient is currently occupying a bed.
  (b) Yes, no two patients may occupy the same bed.
  (c) No, there may be beds that are not occupied.

(b) function $\textit{date1stAdmitted} : \textit{Patients} \rightarrow \textit{Dates}$
  (a) Yes, the function is total; every patient admitted to the hospital has a date of first admittance.
  (b) No, the function is not one-to-one, multiple patients may be admitted on the same day.
  (c) No, the function is not onto, there may be a day with no patients admitted.

Bonus

15. (1 point (bonus)) For the following sets, state what the corresponding shaded region of the Venn diagram represents.



$(A - B) \cup (B \cap C)$ or $(A \cap \overline{B}) \cup (B \cap C)$