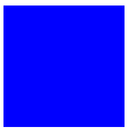
Instructions: All assignments are due by **midnight** on the due date specified. Assignments should be typed and submitted as a PDF. Every student must write up their own solutions in their own manner.

You should complete all problems, but only a subset will be graded (which will be graded is not known to you ahead of time).

Graphs

1. **Ungraded** Rosen Ch 10.5 #56, p. 707.



2. (2 points) **Graded (all)** Rosen Ch 10.5 #58(a), p. 707.

   *A **knight's tour** is a sequence of legal moves by a knight starting at some square and visiting each square exactly once. We can model knights tours using the graph that has a vertex for each square on the board, with an edge connecting two vertices if a knight can legally move between the squares represented by these vertices.*

   (a) *Show that finding a knight's tour on an $m \times n$ chessboard is equivalent to finding a Hamilton path.*
   Let the graph represent legal moves of the knights, where the vertices are the squared on the board and the edges between those squares where moves are legal.
   By definition, a knight's tour visits each square exactly once, therefore traverses a simple path that includes each vertex once; which fits the definition of a Hamilton path.

3. (2 points) **Graded (all)** Rosen Ch 10.7 #14, p. 725.

   You are given $e = 30$ and $r = 20$. Therefore, rewriting Euler's formula: $r = e - v + 2$ so $v = e - r + 2 = 30 - (20) + 2 = 12$

4. **Ungraded** Rosen Ch 10 Supplementary Exercises #19, p. 739.

   The graph does not contain K_5 or any larger complete graphs. It has the following cliques:

   - $K_4 - \{b, c, e, f\}$
   - $K_3 - \{a, b, g\}$, $\{a, d, g\}$, $\{d, e, g\}$, $\{b, e, g\}$

5. (2 points) **Graded (all)** Rosen Ch 10 Supplementary Exercises #20, p. 739.

   The graph does not contain K_6 or any larger complete graphs. It has the following cliques:

   - $K_5 - \{c, e, g, h, i\}$
   - $K_4 - \{a, b, c, e\}$, $\{c, d, e, g\}$
   - K_3 - $\{a, e, f\}, \{e, f, g\}$

6. (2 points) **Graded (all)** Rosen Ch 10 Supplementary Exercises #22, p. 739.

   The unique minimum dominating set is $\{e\}$

Logic

For each question, there are many possible solutions. I have typically provided one or two common solutions to these problems.

7. (24 points) **Graded (a-c)** Prove the following statement is a tautology without using truth tables (use the logical equivalences from Table 6-8 of the book). Justify each step with the law used. Model the solutions in the style of Examples 6-8, pp. 29-30 of the book.

   (a) (6 points) $p \to (\neg q \to r) \equiv \neg(q \vee r) \to \neg p$
   (b) (8 points) $p \to (q \vee r) \equiv (p \wedge \neg q) \to r$
   (c) (10 points) $[\neg p \wedge (p \vee q)] \to q \equiv \mathbf{T}$
   (d) (10 points) $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r \equiv \mathbf{T}$

(a) $p \to (\neg q \to r) \equiv \neg(q \vee r) \to \neg p$

$$
\begin{aligned}
& p \to (\neg q \to r) \\
& \equiv p \to (\neg\neg q \vee r) && \text{(Table 7, rule 1)} \\
& \equiv p \to (q \vee r) && \text{(Double Negation)} \\
& \equiv \neg(q \vee r) \to \neg p && \text{(Table 7, rule 2)}
\end{aligned}
$$

OR

$$
\begin{aligned}
& p \to (\neg q \to r) \\
& \equiv p \to (q \vee r) && \text{(Table 7, rule 3)} \\
& \equiv \neg(q \vee r) \to \neg p && \text{(Table 7, rule 2)}
\end{aligned}
$$

(b) $p \to (q \vee r) \equiv (p \wedge \neg q) \to r$

$$
\begin{aligned}
& p \to (q \vee r) \equiv (p \wedge \neg q) \to r \\
& \equiv \neg p \vee (q \vee r) && \text{(Table 7, rule 1)} \\
& \equiv (\neg p \vee q) \vee r) && \text{(Associative)} \\
& \equiv \neg(\neg p \vee q) \to r && \text{(Table 7, rule 1)} \\
& \equiv (\neg\neg p \wedge \neg q) \to r && \text{(DeMorgan's)} \\
& \equiv (p \wedge \neg q) \to r && \text{(Double Negation)}
\end{aligned}
$$

OR

$$
\begin{aligned}
p \to (q \vee r) &\equiv (p \wedge \neg q) \to r \\
&\equiv (p \to q) \vee (p \to r) && \text{(Table 7, rule 8)} \\
&\equiv (\neg p \vee q) \vee (p \to r) && \text{(Table 7, rule 1)} \\
&\equiv (\neg p \vee q) \vee (\neg p \vee r) && \text{(Table 7, rule 1)} \\
&\equiv \neg p \vee q \vee \neg p \vee r && \text{(drop parenthesis)} \\
&\equiv \neg p \vee \neg p \vee q \vee r && \text{(Commutative)} \\
&\equiv \neg p \vee q \vee r && \text{(Idempotent Law)} \\
&\equiv (\neg p \vee q) \vee r && \text{(add parenthesis)} \\
&\equiv \neg\neg(\neg p \vee q) \vee r && \text{(Double Negation)} \\
&\equiv \neg(\neg\neg p \wedge \neg q) \vee r && \text{(DeMorgans)} \\
&\equiv \neg(p \wedge \neg q) \vee r && \text{(Doulbe Negation)} \\
&\equiv (p \wedge \neg q) \to r && \text{(Table 7, rule 1)}
\end{aligned}
$$

(c) $[\neg p \wedge (p \vee q)] \to q \equiv \mathbf{T}$

$$
\begin{aligned}
&[\neg p \wedge (p \vee q)] \to q \\
&\equiv \neg[\neg p \wedge (p \vee q)] \vee q && \text{(Table 7, rule 1)} \\
&\equiv \neg\neg p \vee \neg(p \vee q) \vee q && \text{(DeMorgan's law)} \\
&\equiv p \vee \neg(p \vee q) \vee q && \text{(Double Negation)} \\
&\equiv p \vee q \vee \neg(p \vee q) && \text{(Commutative)} \\
&\equiv (p \vee q) \vee \neg(p \vee q) && \text{(add in parenthesis)} \\
&\equiv \mathbf{T} && \text{(Negation)}
\end{aligned}
$$

OR

$$
\begin{aligned}
&[\neg p \wedge (p \vee q)] \to q \\
&\equiv \neg[\neg p \wedge (p \vee q)] \vee q && \text{(Table 7, rule 1)} \\
&\equiv \neg\neg p \vee \neg(p \vee q) \vee q && \text{(DeMorgan's law)} \\
&\equiv p \vee \neg(p \vee q) \vee q && \text{(Double Negation)} \\
&\equiv p \vee q \vee \neg(p \vee q) && \text{(Commutative)} \\
&\equiv p \vee q \vee (\neg p \wedge \neg q) && \text{(DeMorgan's)} \\
&\equiv p \vee (q \vee \neg p) \wedge (q \vee \neg q) && \text{(Distributive)} \\
&\equiv p \vee (q \vee \neg p) \wedge \mathbf{T} && \text{(Negation)} \\
&\equiv p \vee (q \vee \neg p) && \text{(Identity)} \\
&\equiv p \vee (\neg p \vee q) && \text{(Commutative)} \\
&\equiv (p \vee \neg p) \vee q && \text{(Associative)} \\
&\equiv \mathbf{T} \vee q && \text{(Negation)} \\
&\equiv \mathbf{T} && \text{(Domination)}
\end{aligned}
$$

(d) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r \equiv \mathbf{T}$

$$
\begin{aligned}
&[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r && \\
&\equiv \neg[(p \vee q) \wedge (\neg p \vee r) \wedge (\neg q \vee r)] \vee r && \text{(Table 7, rule 1, x3)} \\
&\equiv \neg(p \vee q) \vee \neg(\neg p \vee r) \vee \neg(\neg q \vee r) \vee r && \text{(DeMorgan's)} \\
&\equiv (\neg p \wedge \neg q) \vee (\neg\neg p \wedge \neg r) \vee (\neg\neg q \wedge \neg r) \vee r && \text{(DeMorgan's, x3)} \\
&\equiv (\neg p \wedge \neg q) \vee (p \wedge \neg r) \vee (q \wedge \neg r) \vee r && \text{(Double Negation, x2)} \\
&\equiv (\neg p \wedge \neg q) \vee (p \wedge \neg r) \vee r \vee (q \wedge \neg r) && \text{(Commutative Law)} \\
&\equiv (\neg p \wedge \neg q) \vee (p \wedge \neg r) \vee ((r \vee q) \wedge (r \vee \neg r)) && \text{(Distributive Law)} \\
&\equiv (\neg p \wedge \neg q) \vee (p \wedge \neg r) \vee ((r \vee q) \wedge \mathbf{T}) && \text{(Negation Law)} \\
&\equiv (\neg p \wedge \neg q) \vee (p \wedge \neg r) \vee r \vee q && \text{(Identity Law)} \\
&\equiv q \vee (\neg p \wedge \neg q) \vee r \vee (p \wedge \neg r) && \text{(Commutative Law, x2)} \\
&\equiv ((q \vee \neg p) \wedge (q \vee \neg q)) \vee ((r \vee p) \wedge (r \vee \neg r)) && \text{(Commutative Law, x2)} \\
&\equiv ((q \vee \neg p) \wedge \mathbf{T}) \vee ((r \vee p) \wedge \mathbf{T}) && \text{(Negation Law, x2)} \\
&\equiv (q \vee \neg p) \vee (r \vee p) && \text{(Identity Law, x2)} \\
&\equiv q \vee \neg p \vee r \vee p && \text{(Associative Law)} \\
&\equiv p \vee \neg p \vee q \vee r && \text{(Commutative Law)} \\
&\equiv \mathbf{T} \vee q \vee r && \text{(Negation Law)} \\
&\equiv \mathbf{T} && \text{(Domination Law)}
\end{aligned}
$$

An alternative solution is:

$$
\begin{aligned}
&[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r && \\
&\equiv [(p \vee q) \wedge ((p \vee q) \rightarrow r)] \rightarrow r && \text{(Table 7, rule 7)} \\
&\equiv [(p \vee q) \wedge (\neg(p \vee q) \vee r)] \rightarrow r && \text{(Table 7, rule 1)} \\
&\equiv [((p \vee q) \wedge \neg(p \vee q)) \vee ((p \vee q) \wedge r)] \rightarrow r && \text{(Distributive)} \\
&\equiv [\mathbf{F} \vee ((p \vee q) \wedge r)] \rightarrow r && \text{(Negation)} \\
&\equiv [((p \vee q) \wedge r) \vee \mathbf{F}] \rightarrow r && \text{(Commutative)} \\
&\equiv [((p \vee q) \wedge r)] \rightarrow r && \text{(Identity)} \\
&\equiv [(p \vee q) \wedge r] \rightarrow r && \text{(drop extra ())} \\
&\equiv \neg[(p \vee q) \wedge r] \vee r && \text{(Table 7, rule 1)} \\
&\equiv [\neg(p \vee q) \vee \neg r] \vee r && \text{(DeMorgans)} \\
&\equiv \neg(p \vee q) \vee [\neg r \vee r] && \text{(Associative)} \\
&\equiv \neg(p \vee q) \vee [r \vee \neg r] && \text{(Commutative)} \\
&\equiv \neg(p \vee q) \vee \mathbf{T} && \text{(Negation)} \\
&\equiv \mathbf{T} && \text{(Domination)}
\end{aligned}
$$

8. (8 points) **Graded (all)** Show the following expressions are logically equivalent.

   If you are not lazy, then you work hard or you are clever and lazy.

   You work hard or you are lazy.

   Use the following propositions to represent the concepts in the expression. Let

   - l be "you are lazy"
   - w be "you work hard"
   - c be "you are clever"

First, translate the expressions into logic.

$$\neg l \to (w \vee (c \wedge l)) \equiv w \vee l$$

$$\begin{aligned}
\neg l \to & (w \vee (c \wedge l)) & \\
&\equiv l \vee (w \vee (c \wedge l)) & \text{(Table 7.3)} \\
&\equiv l \vee ((c \wedge l) \vee w) & \text{(Commutative)} \\
&\equiv (l \vee (c \wedge l)) \vee w & \text{(Associative)} \\
&\equiv l \vee w & \text{(Absorption)} \\
&\equiv w \vee l & \text{(Commutative)}
\end{aligned}$$

9. **Ungraded** Rosen Ch 1.6 #6, p. 78.
Let r be the proposition "It rains", let f be "It is foggy", let s be "The sailing race will be held", let l be "The life saving demonstration will go on", and let t be "The trophy will be awarded".

The premises are: $(\neg r \vee \neg f) \to (s \wedge l)$, $s \to t$, $\neg t$.
The conclusion we want is: r.

Step	Reason
1. $\neg t$	Hypothesis (Premise, Given)
2. $s \to t$	Hypothesis (Premise, Given)
3. $\neg s$	Modus tollens with (1) and (2)
4. $(\neg r \vee \neg f) \to (s \wedge l)$	Hypothesis (Premise, Given)
5. $(\neg(s \wedge l)) \to \neg(\neg r \vee \neg f)$	Contrapositive with (4)
6. $(\neg s \vee \neg l) \to (\neg\neg r \wedge \neg\neg f)$	De Morgans law with (5) (x2)
7. $(\neg s \vee \neg l) \to (r \wedge f)$	Double negation with (6) (x2)
8. $\neg s \vee \neg l$	Addition with (3)
9. $r \wedge f$	Modus ponens with (7) and (8)
10. r	Simplification with (9)

Another valid argument is:

Step	Reason
1. $\neg t$	Hypothesis
2. $s \to t$	Hypothesis
3. $(\neg r \vee \neg f) \to (s \wedge l)$	Hypothesis
4. $\neg s$	Modus tollens, (1), (2)
5. $\neg(\neg r \vee \neg f) \vee (s \wedge l)$	Table 7, rule 1, (3)
6. $(\neg\neg r \wedge \neg\neg f) \vee (s \wedge l)$	DeMorgans, (5)
7. $(r \wedge f) \vee (s \wedge l)$	Double Negation (x2), (6)
8. $\neg s \vee \neg l$	Addition with (4)
9. $\neg(s \wedge l)$	DeMorgans, (8)
10. $r \wedge f$	Disjunctive Syllogism, (7), (9)
11. r	Simplification, (10)

Or, another valid argument is:

Step	Reason
1. $(\neg r \vee \neg f) \rightarrow (s \wedge l)$	Hypothesis
2. $s \rightarrow t$	Hypothesis
3. $\neg t$	Hypothesis
4. $\neg s$	Modus tollens, (2), (3)
5. $\neg s \vee \neg l$	Addition with (4)
6. $\neg(s \wedge l)$	DeMorgans with (5)
7. $\neg(\neg r \vee \neg f)$	Modus tollens with (1) and (6)
8. $\neg\neg r \wedge \neg\neg f$	DeMorgans with (7)
9. $r \wedge f$	Double Negation with (8)
10. r	Simplification, (9)

10. (2 points (bonus)) Give a compound proposition with three variables p, q, and r that is true when at most one of the three variables is true, and false otherwise.

$$(p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \neg r) \vee (\neg p \wedge \neg q \wedge r) \vee (\neg p \wedge \neg q \neg r)$$