Instructions: All assignments are due by **midnight** on the due date specified. Every student must write up their own solutions in their own manner.
Please present your solutions in a clean, understandable manner. Use the provided files that give mathematical notation in Word, Open Office, Google Docs, and LaTeX.
Assignments should be typed and submitted as a PDF.
You should complete all problems, but only a subset will be graded (which will be graded is not known to you ahead of time).

Propositional Logic

1. (10 points) Use the rules of inference to show that the hypotheses imply the conclusion:
   - "If I graduate in four years, then I will have completed the CS courses", and
   - "If I do not work on CS for 10 hours a week, then I will not complete the CS courses", and
   - "If I work on CS for 10 hours a week, then I can not procrastinate."

   Conclusion: "If I procrastinate, then I will not graduate in four years."

   Let $w =$ "I work on CS for 10 hours a week",
   $g =$ "I graduate in four years",
   $c =$ "I will complete the CS courses", and
   $p =$ "I procrastinate."

   (a) (4 pts) Translate the hypotheses and conclusion to logical statements.

   (b) (6 pts) Construct a valid argument (justify each step).

   Hint: Remember you can also use the logical equivalences as a step in the argument.

2. (12 points) Use the same propositional variables of problem 1 and the rules of inference to show the hypotheses imply the conclusion:
   - "If I work on CS for 10 hours a week and I don't procrastinate, then I will complete the CS courses",
   - "I will not graduate in four years and I worked on CS for 10 hours a week.", and
   - "If I complete the CS courses, then I graduate in four years."

   Conclusion: "I procrastinated."

   First, translate the hypotheses and conclusion in to logical statements (4 points). Then, show the valid argument (8 points).

3. (12 points) Use the rules of inference to show that the hypotheses imply the conclusion:

   > If you eat carefully then you will have a healthy digestive system. If you exercise regularly you will be very fit. If you have a healthy digestive system or you are very fit, you will live to a ripe old age. You do not live to a ripe old age. Therefore, you did not eat carefully and you did not exercise regularly.

   Let $c =$ "you eat carefully",
   $h =$ "you have a healthy digestive system",
   $e =$ "you excercise regularly",
   $f =$ "you will be very fit",
   $l =$ "you will live to a ripe old age"

   (a) (4 pts) Translate the hypotheses and conclusion to logical statements.

   (b) (8 pts) Construct a valid argument (justify each step).

   Hint: Remember you can also use the logical equivalences as a step in the argument.

Predicate Logic

4. (10 points) Consider the domain of discourse to be the following set $\{duck, monkey, walleye, dog\}$. Let $F(x)$ be true when x can fly, let $H(x)$ be true when x has hair/fur, let $T(x)$ be true when x has a tail, and let $S(x)$ be true if x can swim.

   (a) (3 pt) Complete the following truth table

x	$F(x)$	$H(x)$	$T(x)$	$S(x)$
duck		F	T	T
monkey			T	T
walleye	F		T	
dog				T

   Express the following predicates, (b-d), using the universal quantifier:

   (b) (1 pt) $F(duck) \wedge F(monkey) \wedge F(walleye) \wedge F(dog)$

   (c) (1 pt) $\neg T(duck) \wedge \neg T(monkey) \wedge \neg T(walleye) \wedge \neg T(dog)$

   (d) (1 pt) $S(duck) \vee S(monkey) \vee S(walleye) \vee S(dog)$

   Determine the truth values of the following expressions

   (e) (1 pt) $\forall x\, S(x)$

   (f) (1 pt) $\forall x\, (H(x) \rightarrow T(x))$

   (g) (1 pt) $\exists x\, (\neg H(x) \wedge F(x))$

   (h) (1 pt) $\exists x\, (F(x) \wedge T(x) \wedge (\exists y\, ((x \neq y) \rightarrow (\neg H(y) \vee \neg T(y)))))$

5. (2 points) Rosen Ch 1.4 #8(a,d) p. 53

6. (3 points) Rosen Ch 1.4 #28(c, d, e), p. 54
   Use the following predicates:
   - $C(x)$ be "x is in the correct place"
   - $T(x)$ be "x is a tool"
   - $E(x)$ be "x is in excellent condition"

7. (2 points) Translate the logical expressions into English sentences using the same predicates as problem 6.

   (a) $\forall x\ (E(x) \rightarrow C(x))$

   (b) $\exists x\ (T(x) \wedge \neg E(x) \wedge \neg C(x))$

8. (3 points) Rosen Ch 1.4 # 38(d,e), p. 55

9. (3 points) Rosen Ch 1.4 #42(a-c), p. 55
   Use the following predicates:
   - $A(x)$ be "User x has access to an electronic mailbox"
   - $R(x, y)$ be "Group member x can access resource y"
   - $S(x, y)$ be "System x is in state y"

10. (3 points) For this problem, use the predicates: $F(x)$ is "x is a Freshman", $S(x)$ be "x is a student at MTU", $C(y)$ is "y is a CS course", and $T(x, y)$ is "x is taking y", where x has the domain of all students at MTU and y has the domain of all CS courses.

    (a) Translate the logical expression into English: $\forall x\ (F(x) \rightarrow T(x, CS1000))$.

(b) Translate English into logic: "Some freshman at MTU are taking CS1121."
(c) Translate English into logic: "Every freshman at MTU is taking a CS course."

11. (3 points) Repeat problem 10 with the domain of x is all people and y is all courses.

12. (2 points) Rosen Ch 1.5 #4(a,c), p. 64

13. (5 points) Rosen Ch 1.5 #10(a,c,d,e,i), p. 65

14. (4 points) Rewrite the statements so that the negations appear only on predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

    (a) $\neg\forall x\, \exists y\, P(x, y)$
    (b) $\neg\forall x\, (\exists y\, \forall z\, T(x, y, z) \rightarrow \forall y\, \forall z\, U(x, y, z))$

15. (8 points) Consider the following statements (the domain of x is all people):

    (a) Each student in the class has a computer.
    (b) Everyone with a computer can program.
    (c) Mary is a student in the class.
    (d) Therefore, Mary can program.

    Let $S(x)$ mean "x is a student in the class", $C(x)$ mean "x has a computer", and $P(x)$ mean "x can program." You will first translate the statements into logical expressions (2 pts). Then, show the conclusion (d) can be drawn from the prior statements (a-c) (6 pts). Justify each step of the argument.

16. (8 points) Rosen Ch 1.6 #14 (d), p. 79
    Let $c(x)$ be "x is in this class", $f(x)$ be "x has been to France", and $l(x)$ be "x has visited the Louvre."

17. (6 points) For each argument determine whether it is valid or not and explain why (in a sentence).

    (a) "Some math majors left the campus for the weekend" and "All seniors left the campus for the weekend" implies the conclusion "Some seniors are math majors."
    (b) "Everyone who left campus for the weekend is a senior" and "All math majors left campus for the weekend" implies the conclusion "All math majors are seniors."
    (c) "No juniors left campus for the weekend" and "Some math majors are not juniors" implies the conclusion "Some math majors left campus for the weekend."

18. (2 points) Rosen Ch 1.6 # 16(c,d), p. 79

19. (2 points) Rosen Ch 1.6 # 18, p. 79

Bonus Questions

20. (2 points (bonus)) Using the same predicates as problem 10, translate the following statement into logic assuming the domain of x is all students at MTU and the domain of y is all CS courses. "Some freshman at MTU is taking two CS courses."

21. (2 points (bonus)) Rosen Ch 1.5 #10(h,j), p. 65