Instructions: All assignments are due by **midnight** on the due date specified. Every student must write up their own solutions in their own manner.
Please present your solutions in a clean, understandable manner. Use the provided files that give mathematical notation in Word, Open Office, Google Docs, and LaTeX.
Assignments should be typed and submitted as a PDF.
You should complete all problems, but only a subset will be graded (which will be graded is not known to you ahead of time).

Sequences

1. (12 points) Rosen Ch 2.4 # 16(c,e), p. 168.
2. (6 points) A mortage loan is paid off in periodic (monthly) installments, while interest is also charged each period. A mortage with an annual interest rate of r has a monthly interest rate of $i = \frac{r}{12}$. A mortage of M dollars at monthly interest rate i has payments of P dollars. At the end of the month, interest is added to the previous balance, and then the payment P is subtracted from the result. Let m_n be the balance due after n months, where $m_0 = M$, $m_1 = M(1+i)-P, \ldots$
   (a) Let $M = 10,000$, $i = 0.03$ and $P = 105.13$, determine m_2 and m_3.
   (b) Find a recursive formula for the mortage balance, m_n

Summations

3. (2 points) Write out in sigma notation the sum of the first 40 terms of the series $3+6+9+12+\ldots$.
4. (12 points) What are the values of the sums:
   (a) $\sum_{j\in S}(2j-1)$, where $S = \{1,2,4,6\}$ (b) $\sum_{i=1}^{8} 4$ (c) $\sum_{k=0}^{4}(-3)^k$
   (d) $\sum_{j=0}^{7}(2^{j+1}-2^{j-1})$ (e) $\sum_{i=1}^{4}\sum_{j=1}^{3}(2i+3j)$ (f) $\sum_{j=0}^{3}\sum_{k=1}^{3} j * 2^k$

A *closed form* of a summation is an equation in which no summation symbol appears. The classic example is $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$. The fraction on the right is the *closed form* of the summation.

Arithmetic Properties of Summations

Fact 1:	$\sum_{i=1}^{n} c = nc$ when c is a constant	Fact 2:	$\sum_{i=j}^{n} c = (n-j+1)c$ when c is a constant
Fact 3:	$\sum_{i=j}^{n}(f(i)\pm g(i)) = \sum_{i=j}^{n} f(i) \pm \sum_{i=j}^{n} g(i)$	Fact 4:	$\sum_{i=j}^{n} cf(i) = c\sum_{i=j}^{n} f(i)$ when c is a constant

Closed-form Summation Formulae

Name	Sum	Closed Form	Name	Sum	Closed Form
Table 2.1	$\sum_{k=0}^{n} ar^k \ (r \neq 0)$	$\frac{ar^{n+1} - a}{r-1}$, if $r \neq 1$	Table 2.2	$\sum_{k=1}^{n} k$	$\frac{n(n+1)}{2}$
Table 2.3	$\sum_{k=1}^{n} k^2$	$\frac{n(n+1)(2n+1)}{6}$	Table 2.4	$\sum_{k=1}^{n} k^3$	$\frac{n^2(n+1)^2}{4}$
Table 2.5	$\sum_{k=0}^{\infty} x^k, \ \lvert x\rvert < 1$	$\frac{1}{1-x}$	Table 2.6	$\sum_{k=1}^{\infty} kx^{k-1}, \ \lvert x\rvert < 1$	$\frac{1}{(1-x)^2}$

5. (8 points) Find a closed form for the summation $\sum_{i=0}^{n}(2 \cdot 3^i - 3 \cdot 2^i)$. Show how you find the closed form solution; justify each step using the four facts of the arithmetic properties of summations and Table 2 above (p. 166 in the book).

6. (8 points) Find a closed form for the summation $\sum_{i=4}^{n} 7 \cdot 5^i$. Show how you find the closed form solution; justify each step using the four facts of the arithmetic properties of summations and Table 2 above (p. 166 in the book).

Induction

Follow the template for inductive proofs given on p. 329 of the book.

7. (10 points) Let $P(n)$ be the statement $\sum_{j=1}^{n} 2j = n + n^2$ for $n \geq 1$.
   (a) (1 pt) What is the statement $P(1)$?
   (b) (1 pt) Show that P(1) is true, completing the basis step of the proof.
   (c) (1.5 pts) What is the inductive hypothesis?
   (d) (1.5 pts) What do you need to prove in the inductive step?
   (e) (4 pts) Complete the inductive step.
   (f) (1 pt) Explain why these steps show that this formula is true whenever n is a positive integer.

8. (8 points) Prove using mathematical induction that

$$1 + 5 + 5^2 + 5^3 + \cdots + 5^n = \frac{5^{n+1} - 1}{4} \text{ for all } n \geq 0.$$

9. (8 points) Rosen Ch 5.1 # 32, p. 330

Bonus Questions

10. (4 points (bonus)) Find a closed form for the summation $\sum_{i=1}^{n}\sum_{j=1}^{n}(6i^2 - 2j)$. Show how you find the closed form solution; justify each step using the four facts of the arithmetic properties of summations and Table 2, p. 166 in the book.

11. (4 points (bonus)) Let $\{s_n\}$ be the sequence defined as,

$$s_1 = 4 \quad \text{and} \quad s_n = 3s_{n-1} - 2, \forall n \geq 2.$$

Show $\forall n \geq 1, s_n = 3^n + 1$.