Instructions: All assignments are due by **midnight** on the due date specified. Every student must write up their own solutions in their own manner.
Please present your solutions in a clean, understandable manner. Use the provided files that give mathematical notation in Word, Open Office, Google Docs, and LaTeX.
Assignments should be typed and submitted as a PDF.
You should complete all problems, but only a subset will be graded (which will be graded is not known to you ahead of time).

Induction and Recursion

Follow the template for inductive proofs given on p. 329 of the book.

1. (8 points) Rosen Ch 5.1 # 22, p. 330

2. (8 points) The Fibonacci numbers are defined as $F(0) = 0$, $F(1) = 1$, and $F(n) = F(n-1) + F(n-2)$ for $n \geq 2$. Use induction to prove for the positive integers n:

$$F(1)^2 + F(2)^2 + \cdots + F(n)^2 = F(n)F(n+1).$$

3. (8 points) Consider the game of football (that is, the American game of football). Let's assume teams can either score via field goal (3 points) or touchdowns (7 points, assume all point after touchdowns are made). Safeties are ignored for this problem.

   Show that it is then possible (assuming no time constraints) for a team to score any number of points from 12 on up.

4. (8 points) Let $\{s_n\}$ be the sequence defined as,

$$s_0 = 0, \quad s_1 = 1, \quad \text{and} \quad s_n = 3s_{n-1} - 2s_{n-2}, \forall n \geq 2.$$

   Show $\forall n \geq 0, s_n = 2^n - 1$.

5. (4 points) Rosen Ch 5.3 # 24(a,b), p. 358.

6. (6 points) Rosen Ch 5.3 # 26a, p. 358. (See book examples and # 27 to help with this problem)

Counting

7. (2 points) You go to your favorite sandwich shop and order a single sandwich. You have a choice of sub roll, white bread, wheat bread, wrap or pita. You also have a choice of proteins: ham, turkey, roast beef, tuna, bacon, or hummus. How many different single sandwichs (single type of bread, single protein) may be ordered?

8. (2 points) At the same sandwich shop. Kari wants to order a triple-decker sandwich. Kari believes that the order in which the sandwich is constructed matters when eating (that is, bread-ham-turkey-roast beef-bread tastes different than bread-turkey-ham-roast beef-bread). How many different triple-decker sandwiches may be ordered (assume the bread selection is constant)?

9. (2 points) At the same sandwich shop. Dylan also wants a triple-decker sandwich, but doesn't want any protein to repeat. How many different sandwiches may he order (assume order of construction still matters and bread selection is constant)?

10. (2 points) Six teams are going to an Broomball tournament. If each team plays each other team exactly once, how many games are played?

For the next several problems, consider strings representing the sequence of a DNA strand where the four letters in the sequence represent the four base nucleotides (A - adenine, C - cytosine, G - guanine, or T - thymine). A length 4 DNA sequence, would then be four of the letters in a string, e.g., AAAA, ACGT, ACTA, CGCG, etc.

11. (2 points) How many DNA sequences are there of length 10?

12. (2 points) How many DNA sequence of length 10 only contain the purines (A and G)?

13. (2 points) How many DNA sequences are there of length 10 that begin with a T and end with a T?

14. (2 points) How many DNA sequences of length 10 begin with TATA or begin with TAGC?

15. (2 points) How many DNA sequences of length 10 begin with TATA or end with GCGC?

16. (2 points) How many DNA sequences are there of length 6 or less, not counting the empty string?

For the next several problems, consider strings representing 10 digit phone numbers.

17. (2 points) How many ten-digit phone numbers begin with 906 and end with 0?

18. (2 points) How many ten-digit phone numbers only contain odd numbers?

19. (2 points) How many ten-digit phone numbers start with 555 or end with 0000?

20. (10 points) How many positive integers less than 1000
    (a) are divisible by 6?
    (b) are divisible by both 6 and 13?
    (c) are divisible by 6 but not by 13?
    (d) are divisible by either 6 or 13?
    (e) are divisible by exactly one of 6 and 13?
    (f) are divisible by neither 6 nor 13?

Bonus Questions

21. (4 points (bonus)) Rosen Ch 5.3 #26(b).

22. (2 points (bonus)) How many positive integers less than 1000 have (a) distinct digits and (b) have distinct digits and are even?