Instructions: All assignments are due by **midnight** on the due date specified.

Every student must write up their own solutions in their own manner.

Please present your solutions in a clean, understandable manner. Use the provided files that give mathematical notation in Word, Open Office, Google Docs, and LaTeX. Do Not Crowd Your Answers!

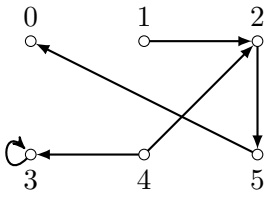
Assignments should be typed and submitted as a PDF.

You should complete all problems, but only a subset will be graded (which will be graded is not known to you ahead of time).

Relations

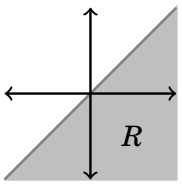
1. (6 points) Let $X = \{a, b\}$ and $Y = \{1, 2\}$.
   (a) Give the sets $X \times Y$ and $\mathcal{P}(X \times Y)$.
   (b) How many possible relations exist from X to Y?
   (c) What does $\mathcal{P}(X \times Y)$ represent with respect to relations?
   (d) How many binary relations exist on the set $C = \{1, 2, 3, 4\}$?
   You do **not** need to list all such relations

2. (10 points) For each part, describe a relation using the different representations asked for.
   (a) Let $A = \{1, 2, 3, 4, 5, 6\}$.
      (i) Write out the relation R on A that expresses $x \mid y$ (divides), that is if $x \mid y$ then $(x, y) \in R$, that is describe the relation using the set enumeration methods (list all elements of the set).
      (ii) Draw the relation as a digraph.
      (iii) Describe the relation as a zero-one matrix (assume rows/columns are ordered numerically).
   (b) Let R be a relation on a set A, illustrated below.

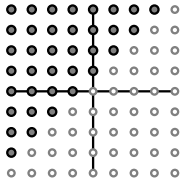


      (i) Write out the sets A and R.
      (ii) Describe the relation as a zero-one matrix (assume rows/columns are ordered numerically).
   (c) Congruence modulo 5 is a relation, R, on $\mathbb{Z}$, where $(x, y) \in R$ means $x \equiv y \ (mod\ 5)$. Write out the set R in set-builder notation.

3. (4 points) In the following figures relations R are indicated by gray shading. In figure (a), the relation is on $\mathbb{R}$ in (b) the relation is on $\mathbb{Z}$. State what familiar relation is being represented.



(a)



(b)

4. (28 points) Consider the following relations, determine whether the relation described is reflexive (R), symmetric (S), antisymmetric (AS), and transitive (T). Report the results in a table, for example,

Relation	**R?**	**S?**	**AS?**	**T?**
(a)	Yes / No	Yes / No	Yes / No	Yes / No
⋮	⋮	⋮	⋮	⋮

Also, briefly explain for any "no" answer, why the relation does not have a given property.

(a) $R_a = \{(a,a),(b,b),(c,c),(d,d),(a,b),(b,a)\}$ on $\{a,b,c,d\}$
(b) $R_b = \{(0,0),(2,0),(0,2),(2,2)\}$ on $\mathbb{R}$
(c) Rosen 9.1.4(a)
(d) Rosen 9.1.4(c)
(e) Rosen 9.1.4(d)
(f) $R_f = \{(\text{Bill Gates}, \text{Mark Zuckerberg})\}$ on the set of all people.
(g) The $>$ relation on $\mathbb{Z}$
(h) The $\leq$ relation on $\mathbb{Z}$
(i) The $\neq$ relation on $\mathbb{Z}$
(j) The $\mid$ relation on $\mathbb{Z}$.
(k) Rosen 9.1.6(b)
(l) Rosen 9.1.6(c)
(m) Rosen 9.1.6(e)
(n) Rosen 9.1.6(f)

Consider the relations R, S, T, U on the set $\{a,b,c,d\}$. Use the definitions and properties discussed in class and chapter 9.1 and the properties or operations of **irreflexive**, **asymmetric**, **inverse relations**, and **complementary relations** mentioned before Rosen 9.1 # 11 (p. 581), # 18 (p. 582), and # 26 (p. 582).

Let $R = \{(a,a),(b,c),(c,b),(c,d),(d,c),(d,d)\}$,
$S = \{(a,a),(a,d),(b,a),(b,b),(b,d),(c,a),(c,c),(d,c),(d,d)\}$,
$T = \{(a,a),(a,b),(b,c),(b,d),(c,d),(d,a),(d,b)\}$, and
$U = \{(a,a),(a,d),(b,c),(b,d),(c,a),(d,d)\}$

5. (6 points) Determine (Yes/No) whether R and S have each of the following properties: reflexive, irreflexive, symmetric, antisymmetric, asymmetric, and transitive.

6. (25 points) Find the following expressions:

a) (1pt) $R \cup S$	b) (1pt) $R \cap S$	c) (1pt) $R - S$	d) (1pt) $S - R$
e) (1pt) $\overline{S}$	f) (1pt) S^{-1}	g) (2pt) $T \circ T$	h) (2pt) $U \circ T$
i) (2pt) $T \circ U$	j) (4pt) R^3	k) (4pt) U^3	l) (5pt) $R \circ S \circ T$

Bonus

7. (2 points (bonus)) There are 16 possible relations R on the set $A = \{a,b\}$. Describe all of them as directed graphs (be sure to label the nodes in the graph). Which relations are reflexive? symmetric? transitive?