Instructions: All assignments are due by **midnight** on the due date specified.

Every student must write up their own solutions in their own manner.

Please present your solutions in a clean, understandable manner. Use the provided files that give mathematical notation in Word, Open Office, Google Docs, and LaTeX. Do Not Crowd Your Answers!

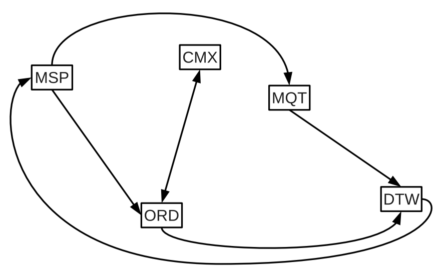
Assignments should be typed and submitted as a PDF.

You should complete all problems, but only a subset will be graded (which will be graded is not known to you ahead of time).

Relations

1. (10 points) Let R on $\{a, b, c, d, e\}$ be $\{(a, e), (b, a), (c, d), (d, b), (e, e)\}$. Determine the following:
   (a) Δ and the reflexive closure of R.
   (b) R^{-1} and the symmetric closure of R.
   (c) R^2
   (d) R^3
   (e) R^* and the transitive closure of R

2. (22 points) Given the following directed graph showing a relation R of flights between cities. Answer the following questions.



   (a) (1pt) Give the set A the relation R is on.
   (b) (1pt) Give the relation R (that is, list the ordered pairs explicitly).
   (c) (2pt) Express the relation R as a zero-one matrix (present cities in alphabetic order, e.g., CMX, DTW, MQT, ...)
   (d) (2pt) Find the diagonal relation on A (list all ordered pairs explicitly).
   (e) (2pt) Find the reflexive closure of R (list all ordered pairs explicitly).
   (f) (2pt) Find R^{-1} (list all ordered pairs explicitly).
   (g) (2pt) Find the symmetric closure of R.
   (h) (1pt) Describe what the pairs in $R \circ R$ represent.
   (i) (2pt) Find $R \circ R$ (list all ordered pairs explicitly).
   (j) (1pt) Describe what the pairs in R^3 represent.
   (k) (2pt) Find R^3 (list all ordered pairs explicitly).
   (l) (1pt) Describe what the transitive closure, R^* represents.
   (m) (3pt) Find R^* (list all ordered pairs explicitly).

3. (11 points) Let the sets *Director*, D, be a set of movie directors, *Movie*, M, be a set of movies, and *Actor*, A, be a set of actors. We define the following relations:

- $appearedIn \subseteq Actor \times Movie$ or $appearedIn \subseteq A \times M$, where a $appearedIn$ m if a was an actor who appeared in movie m
- $directedFilm \subseteq Director \times Movie$ or $directedFilm \subseteq D \times M$, where d $directedFilm$ m if d was the director of movie m
- $starredIn \subseteq Actor \times Movie$ of $starredIn \subseteq A \times M$, where a $starredIn$ m if a was an actor with top 3 billing in movie m

Express the following using only set operations and relations:

(a) There must be actors who have both appeared in and starred in a movie.
(b) Some people appear in movies they direct.
(c) Some movies are directed by people who do not star in them.

Take the following relations and express what they mean in English. Think about what sets define the elements the relations is made up of.

(d) $appearedIn \circ appearedIn^{-1}$
Is this relation (i) reflexive? (ii) symmetric? and (iii) transitive?
(e) $directedFilm \circ starredIn^{-1}$
Is this relation (i) reflexive? (ii) symmetric? and (iii) transitive?

4. (8 points) Rosen Ch 9.5 # 2(a,c-e) p. 615

5. (4 points) Let R_m be the relation on $\mathbb{Z}^+$ where $R_m = \{(a, b) \mid a \equiv b \ (mod\ m)\}$.
   (a) Give the four members of $[4]_{R_2}$ with the smallest values.
   (b) Give the four members of $[4]_{R_6}$ with the smallest values.
   (c) Give the four members of $[5]_{R_3}$ with the smallest values.
   (d) How many unique equivalence classes for R_7 exist on the members of $\mathbb{Z}^+$.

6. (3 points) Rosen Ch 9.5 # 24, p. 616.
   If it is an equivalence relation, list the equivalence classes assuming the elements are ordered $x_1, x_2, x_3, x_4, \ldots$.

7. (3 points) Rosen Ch 9.5 # 56(a,b), p.617

8. (2 points) Rosen Ch 9.6 #4(a,c), p. 630.

9. (6 points) Which of the following are posets?
   (a) $(\mathbb{Z}, =)$
   (b) $(\mathbb{Z}, \neq)$
   (c) $(\mathbb{Z}, \geq)$
   (d) $(\mathbb{R}, =)$
   (e) $(\mathbb{R}, <)$
   (f) $(\mathbb{R}, \geq)$

10. (6 points) Rosen Ch 9.6 #32(a-f), p. 631.

Graphs

11. (9 points) Use a graph to model the following problems:

    (a) Twitter "follows" - Construct a graph of who follows who on Twitter on a set of people. Ann follows Bob, Ann also follows Charlie, Bob follows Ann, Bob also follows Ed, Charlie follows Dana, Dana follows Bob, Dana also follows Flora, Ed follows Ann, Ed also follows Charlie.

    (b) Integer division -Construct a graph that models whether for a pair of integers, one integer even divides the other. Consider the set of integers $V = \{2, 3, 4, 6, 8, 9, 12, 18, 27\}$; two vertices are adjacent if one evenly divides the other.

    (c) Airline flights
    An airline has the following direct flights on their system, where each direct route between city a and b and back is listed below. Model the all the routes on the system as a graph.

City a	City b	# of flights	City a	City b	# of flights
Atlanta	Chicago	1	Chicago	San Francisco	2
Atlanta	Miami	1	Denver	Los Angeles	2
Atlanta	New York	1	Denver	San Francisco	1
Boston	New York	2	Los Angeles	New York	3
Boston	Chicago	1	Los Angeles	San Francisco	2
Chicago	Denver	1	Miami	New York	1
Chicago	New York	2	New York	San Francisco	2

12. (3 points) Determine the type of graphs depicted in each part of Problem 11: simple graph? multigraph? pseudograph? directed graph? directed multigraph?

13. (3 points) For the graphs in Problem 11(a-c), determine the number of vertices and edges.

14. (6 points) For the graphs in Problem 11(a-c), determine degree of each vertex (for directed graphs, indicate both the in-degree and out-degree of each vertex).

15. (6 points) Draw these graphs. **(a)** C_5 **(b)** W_7 **(c)** $K_{3,4}$

16. (6 points) For which values of n are (a) C_n, (b) K_n and (c) W_n bipartite? Explain your answer.

17. (4 points) (a) A simple graph with $|V| = n$ and $|E| = 17$ has one vertex of degree 1, two vertices of degree 2, one vertex of degree 3, and two vertices of degree 5. The remaining vertices of the graph have degree 4. What is n?

    (b) A graph with $|V| = 25$ and $|E| = 62$ has vertices of degree 3, 4, 5, or 6. There are two vertices of degree 4 and 11 vertices of degree 6. How many vertices of the graph have degree 3?

Bonus

18. (2 points (bonus)) List all the equivalence relations on the set $\{a, b, c\}$. (Describe the relation be describing the partition on the set).