Instructions: All assignments are due by **midnight** on the due date specified.

Every student or student group must write up their own solutions in their own manner.

Please present your solutions in a clean, understandable manner. Use the provided files that give mathematical notation in Word, Open Office, Google Docs, and LaTeX.

Assignments should be typed or scanned and submitted as a PDF.

You should complete all problems, but only a subset will be graded (which will be graded is not known to you ahead of time).

Logic

1. (6 points) Determine which of the following statements are propositions? What are the truth values of those that are propositions?
   (a) $x - 4 = 7$ (b) When in Winter Carnival? (c) 14 is prime.
   (d) 37 is odd. (e) Read Chapter 2.1 before class. (f) $8 - 5 = -3$

2. (4 points) Rosen Ch 1.1, #10 (a,c,f,h), p. 14

3. (6 points) Rosen Ch 1.1, #16, p. 14

4. (6 points) Rosen Ch 1.1 #24 (a-f), p. 15.

5. (6 points) State the converse, contrapositive, and inverse of each of the following statements.
   (a) If it is summer, then I take a vacation.
   (b) To be able to go on the trip, it is necessary that you get written permission.

6. (15 points) Construct a truth table for each compound proposition.
   (a) (2 pt) $p \to \neg p$ (b) (2 pt) $(p \vee q) \wedge \neg p$
   (c) (3 pt) $(\neg q \to p) \oplus (p \wedge \neg q)$ (d) (4 pt) $p \vee (q \to r)$
   (e) (4 pt) $(p \leftrightarrow \neg r) \to (q \vee \neg(p \vee \neg r))$

Sets

7. (3 points) List the members of the sets.
   (a) $\{x \mid x \in \mathbb{N} \text{ and } -4 < x < 4\}$
   (b) $\{2x + 1 \mid x \in \mathbb{N} \text{ and } x < 4\}$
   (c) $\{\{a, a^2 - 1\} \mid a \in \mathbb{N} \text{ and } -2 \leq a < 3\}$

8. (5 points) Write each of the following sets in set-builder notation.
   (a) $S_a = \{2, 4, 8, 16, 32, \ldots\}$
   (b) $S_b = \{-3, -2, -1, 0, 1, 2\}$
   (c) $S_c = \{\ldots, \frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27 \ldots\}$
   (d) The natural numbers divisible by 4, S_d
   (e) The square roots of the natural numbers, S_e

9. (6 points) Let $A = \{b, \{b, a\}, \emptyset\}$ and $B = \{\emptyset, \{\emptyset\}\}$. Determine whether each of the statements are *True* or *False*.

   (a) $\emptyset \in A$ (b) $\emptyset \subseteq A$ (c) $a \in A$ (d) $a \subseteq A$ (e) $\{\emptyset\} \in A$ (f) $\{\emptyset\} \subseteq A$
   (g) $\emptyset \in B$ (h) $\emptyset \subseteq B$ (i) $\{\emptyset\} \in B$ (j) $\{\emptyset\} \subseteq B$ (k) $\{\emptyset\} \subset A$ (l) $\{\emptyset\} \subset B$

10. (15 points) Let $A = \{1, \{2, 3\}, 2\}$, $B = \{1, 2, 4\}$, $C = \{3, 4, \{2\}\}$, $D = \emptyset$, $E = \{\emptyset\}$, where the universal set is $U = A \cup B \cup C \cup D \cup E$. Determine (write out members of the set explicitly, that is do not use set builder notation):

    (a) $A \cup B$ (b) $C \cup E$ (c) $A \cap B$ (d) $C \cap B$
    (e) $A - C$ (f) $C - A$ (g) $\overline{A}$ (h) $(A \cup C) - D$
    (i) $E \times A$ (j) $A \times B$ (k) $\mathcal{P}(B)$ (l) $\mathcal{P}(D)$
    (m) $|C|$ (n) $|E \cup A|$ (o) $(A \cap U) \cup \overline{U}$

11. (6 points) Using the sets from problem 10, determine the truth value of the following statements.

    (a) $A \subseteq B$ (b) $B \subseteq A$ (c) $D \subseteq D$ (d) $E \subseteq B$
    (e) $A \subset B$ (f) $D \subset D$ (g) $E \subset B$ (h) $D \subset E$
    (i) A disjoint from B (j) E disjoint from C (k) E disjoint from E (l) $\emptyset$ disjoint from D

12. (1 point) How many elements are in $\mathcal{P}(A) \times A$ when $A = \{a, b, c, d, e, f\}$? Explain your answer.

Bonus Questions

Note, the Bonus questions can at times be more challenging or lengthy in crafting your solution. First, complete the main problems of the assignment before considering doing the bonus questions.

13. (1 point (bonus)) Many restaurants have the sign, "No shoes, no shirt, no service." Write this sentence as a conditional proposition.

14. (2 points (bonus)) When considering the case of two propositions, p and q, there are are 16 truth tables. All sixteen possibilities are listed in the table below (columns *a*-*p*). For each column, express the proposition represented using standard operators ($\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$, $\oplus$). Try to keep the expressions simple (use as few logical operators and propositional variables).

p	q	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)	(k)	(l)	(m)	(n)	(o)	(p)
T	T	T	T	T	T	T	T	T	T	F	F	F	F	F	F	F	F
T	F	T	T	T	T	F	F	F	F	T	T	T	T	F	F	F	F
F	T	T	T	F	F	T	T	F	F	T	T	F	F	T	T	F	F
F	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F