Instructions: All assignments are due by **midnight** on the due date specified. Every student must write up their own solutions in their own manner.
Please present your solutions in a clean, understandable manner. Use the provided files that give mathematical notation in Word, Open Office, Google Docs, and LaTeX.
Assignments should be typed and submitted as a PDF.
You should complete all problems, but only a subset will be graded (which will be graded is not known to you ahead of time).

For many of these proof problems you will be asked to proof expressions related to even (definition p. 83), odd (definition p. 83), and divides (definition p. 238).

1. (6 points) Prove: the product of two odd integers is odd.
2. (6 points) Prove: For all natural numbers m and n, if m is divisible by 5 and n is divisible by 4, then $m \cdot n$ is divisible by 10.
3. (6 points) Prove: For any three consecutive natural numbers, the sum of the consecutive numbers is divisible by 3.
4. (6 points) Prove: If a is an even integer and b is divisible by 3, then ab is divisible by 6.
5. (12 points) Prove that if n is an integer and $n^2 - 2n + 1$ is odd, then n is even using: (a) proof by contraposition and (b) proof by contradiction.
6. (12 points) Prove that if n is an integer and $n^3 - 1$ is even, then n is odd using: (a) proof by contraposition and (b) proof by contradiction.
7. (13 points) Prove for all natural numbers n and m, nm is odd if and only if n and m are both odd.
8. (2 points) Prove that there is a positive integer that equals the sum of the positive integers not exceeding it.
9. (3 points) Prove or disprove: If a and b are rational numbers, then a^b is also rational.
10. (3 points) Prove or disprove: The sum of four consecutive integers is divisible by 4.
11. (6 points) Prove that if n is an integer that $n^3 - n$ is even.
12. (6 points) Prove: Suppose a and b are integers. If $a^2(b^2 - 2b)$ is odd, then a and b are odd.

Sequences

13. (5 points) What are the first four terms of each sequence:
    (a) $a_n = 4 - 2n \ \ \forall n \geq 0$
    (b) $b_n = 6 - 3 \cdot 2^n \ \ \forall n \geq 0$
    (c) $c_1 = 4, c_n = 3 \cdot c_{n-1} - 2 \ \ \forall n \geq 2$
    (d) $d_1 = -1, d_n = 5 \cdot d_{n-1} + n \ \ \forall n \geq 2$
    (e) $e_0 = 1, e_1 = 1, \ e_n = ne_{n-1} + n^2 e_{n-2} + 1 \ \forall n \geq 2$

14. (12 points) Find a closed formula for each sequence; assume the sequence starts with $n = 0, 1, 2, \ldots$.
    (a) $1, -4, 9, -16, 25, \ldots$
    (b) $8, 4, 2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$
    (c) $3, 4, 7, 12, 19, 28, 39 \ldots$
    (d) $6, 1, -4, -9, -14, -19 \ldots$
    (e) $3, 6, 12, 24, 48, \ldots$
    (f) $1, 0, 1, -4, 9, -16, 25, -36 \ldots$

15. (8 points) Find a recursive formula for each sequence; assume the sequence with $n = 0, 1, 2, \ldots$.
    (a) $3, 6, 12, 24, 48, \ldots$
    (b) $7, 10, 15, 22, 31, 42, 55, 70, \ldots$
    (c) $9, 4, -1, -6, -11, -16, \ldots$
    (d) $2, 5, 13, 42, 171, 858, 5151, \ldots$

16. (6 points) Determine whether each answer is a solution to the recurrence relation,

$$a_n = a_{n-1} + 2a_{n-2} + 2n - 9$$

    (a) $a_n = 0$
    (b) $a_n = -n + 2$

Bonus Questions

17. (6 points (bonus)) Prove: For an integer a if $7|4a$, then $7|a$.
    Hint: you may want to use the definition of even and knowledge of the products of even and odd integers.

18. 4 Prove the statement: For all integers a, b, and c, if $a^2 + b^2 = c^2$, then a or b is even.