Instructions: All assignments are due by **midnight** on the due date specified. Assignments should be typed and submitted as a PDF in Canvas.

Every student must write up their own solutions in their own manner.

You should complete all problems, but only a subset will be graded (which will be graded is not known to you ahead of time).

Graphs

1. (2 points) **Graded (all)** Rosen Ch 10.2 # 54, p. 667.

   The graphs $K_{m,n}$ are regular if and only if $m = n$.

2. (3 points) **Graded (all)** Rosen Ch 10.3 #36, p. 676.

   First check the invariants. Graph U and V both have 5 vertices. Graph U and V both have 7 edges.

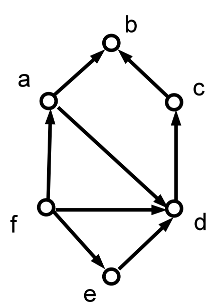
   Graph U has 1 vertex of degree 2 and 4 vertices of degree 3. Graph V has 2 vertices of degree 2, 2 vertices of degree 3 and one vertex of degree 4.

   The graphs are **not isomorphic**.

3. **Ungraded** Rosen Ch 10.3 #38, p. 676.

   First check the invariants. Graph U and V both have 5 vertices. Graph U and V both have 8 edges. Graph U has 1 vertex with degree=2, 2 vertices with degree=3, and 2 vertices with degree=4. Graph V has 1 vertex with degree=2, 2 vertices with degree=3, and 2 vertices with degree=4. Several possible isomorphisms are listed below.

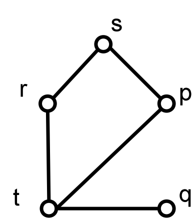
   $f(u_1) = v_1, f(u_2) = v_3, f(u_3) = v_2, f(u_4) = v_5, f(u_5) = v_4$
   $f(u_1) = v_4, f(u_2) = v_5, f(u_3) = v_2, f(u_4) = v_3, f(u_5) = v_1$
   $f(u_1) = v_1, f(u_2) = v_5, f(u_3) = v_2, f(u_4) = v_3, f(u_5) = v_4$
   $f(u_1) = v_4, f(u_2) = v_3, f(u_3) = v_2, f(u_4) = v_5, f(u_5) = v_1$



Graph 1

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

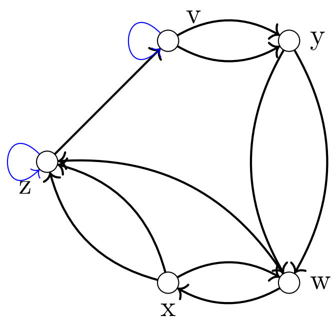
Graph 2



Graph 3

4. **Ungraded** Consider Graphs 1-3 above. Determine the adjacency list for Graph 1. Draw the graph represented by the adjacency matrix for Graph 2 (label the nodes: v, w, x, y, z). Determine the adjacency matrix for Graph 3.

Node	List
a	b, d
b	
c	b
d	c
e	d
f	a, d, e

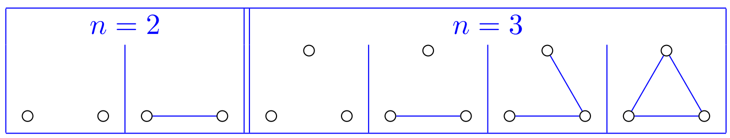


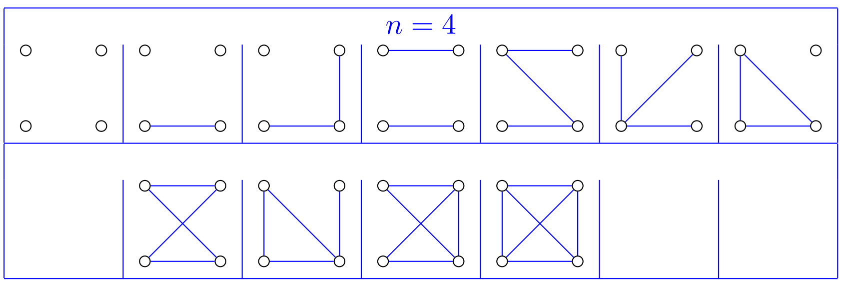
node ordering: p, q, r, s, t

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

5. (6 points) **Graded (b-c)** Rosen Ch 10.3 #54, p. 677.

   (a) 2 (b) 4 (c) 11





6. **Ungraded** Rosen Ch 10.3 #68(a), p. 678.

   3 graphs,

7. (2 points) **Graded (b)** Rosen Ch 10.4 # 2a,b,c, p. 689.

Vertices	Path?	Simple?	Circuit?	Length?
a) a, b, e, c, b	Y	Y	N	4
b) a, d, a, d, a	Y	N	Y	4
c) a, d, b, e, a	N	–	–	–

8. (2 points) **Graded (a)** Rosen Ch 10.4 # 12, p. 690.
   (a) not strongly connected, weakly connected.
   (b) strongly connected and weakly connected.
   (c) neither strongly nor weakly connected.

9. **Ungraded** Rosen Ch 10.4 # 14(a,b), p. 690
   (a) $\{a, b, e\}, \{c\}, \{d\}$
   (b) $\{a\}, \{b\}, \{c, d, e\}, \{f\}$
   (c) $\{a, b, c, d, f, g, h, i\}, \{e\}$

10. **Ungraded** Determine the cut vertices and edges for the graphs of Rosen Ch 10.4, Exercise 31 & 32, p. 691.

For the graph of Ex. 31, the cut vertex is c and there are no cut edges.

For the graph of Ex. 32, the cut vertices are c and d, the cut edge is $\{c, d\}$.

11. (6 points) **Graded (b-d)** For each graph, determine whether it has an Euler circuit and an Euler path. If a circuit or path exists, construct an example.
    (a) Rosen Ch 10.1, Exercise 4, p.650
    (b) Rosen Ch 10.2, Exercise 22, p. 665
    (c) Rosen Ch 10.2, Exercise 23, p. 666
    (d) Rosen Ch 10.2, Exercise 24, p. 666
    For the directed graphs, read the conditions expressed in Rosen Ch 10.5, #16-17.
    (e) Rosen Ch 10.1, Exercise 7, p. 650
    (f) Rosen Ch 10.5, Exercise 18, p. 704
    (g) Rosen Ch 10.5, Exercise 20, p. 704

	Euler Circuit?	Euler Path?	Example Circuit	Example Path
(a)	No	Yes	—	a, b, d, b, d, c, a, b
(b)	No	Yes	—	a, b, c, e, a, d, c
(c)	No	Yes	—	a, b, c, d, a, e, c, f, b
(d)	Yes	Yes	a, c, b, f, d, c, e, f, a	a, c, b, f, d, c, e, f, a
(e)	No	No	—	—
(f)	No	Yes	—	a,b,d,b,c,d,c,a,d
(g)	Yes	Yes	a,d,b,d,e,b,e,c,b,a	a,d,b,d,e,b,e,c,b,a

12. (6 points) **Graded (a-c)** Rosen Ch 10.5, # 26(a,b,c), p. 705.
    (a) There is a Euler circuit if n is odd and $n > 1$
    (b) For all $n \geq 3$, C_n has a Euler circuit
    (c) No wheel graph has a Euler circuit

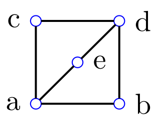
13. (6 points) **Graded (a-c)** For each graph, determine whether it has a Hamilton circuit and a Hamilton path. If a circuit or path exists, construct an example.
    (a) Rosen Ch 10.3, Figure 1, p. 668
    (b) Rosen Ch 10.3, Figure 10, graph H, p. 673
    (c) Rosen Ch 10.4, Exercise 33, p. 691
    (d) Rosen, Ch 10.4, Exercise 14(b), p. 690
    (e) Rosen, Ch 10.5, Exercise 6, p. 704

    Present results in a table like Problem 11.

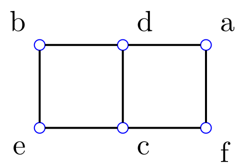
	Hamilton Circuit?	Hamilton Path?	Example Circuit	Example Path
(a)	No	Yes	—	b, a, c, d, e
(b)	Yes	Yes	$s, w, x, y, z, v, u, t, s$	s, w, x, y, z, v, u, t
(c)	No	No	—	—
(d)	No	Yes	—	c, d, e, b, a, f
(e)	No	Yes	—	$e, f, d, g, i, h, a, b, c.$

14. (4 points) **Graded (b,c)** Determine whether each graph is planar, if it is give a planar representation.
    (a) Rosen Ch 10.7, Exercise 2, p. 725
    (b) Rosen Ch 10.7, Exercise 6, p. 725
    (c) Rosen Ch 10.7, Exercise 8, p. 725

(a) Planar



(b) Planar



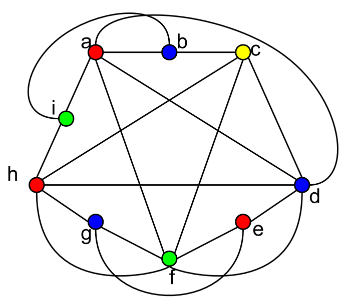
(c) Not Planar

The graph contains a subgraph homeomorphic to $K_{3,3}$ therefore it can not be planer. In fact, it has a subgraph of directly $K_{3,3}$ with vertices $\{a, c, e\}$ and $\{b, d, f\}$

15. (3 points) **Graded (c)** For each graph, determine the chromatic number.
    (a) Rosen Ch 10.2, Exercise 24, p. 666
    (b) Rosen Ch 10.3, Exercise 2, p. 675
    (c) Rosen Ch 10.7, Exercise 24, p. 726

    (a) 2 colors
    (b) 3 colors
    (c) 4 colors, one coloring shown below

Exercise 24, p. 726



Bonus

16. (1 point (bonus)) Rosen Ch 10.3 # 44, p. 676

The two graphs are not isomorphic. To see this, consider the complement of both graphs. The complement of the graph on the left consists of two 4-cycles. The complement of the graph on the right is an 8-cycle. The complements are not isomorphic, therefore the original graphs are not isomorphic.

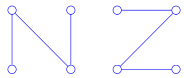
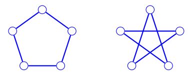
17. (1 point (bonus)) Rosen Ch 10.5, # 28(a), p. 705.

The graph has a Euler circuit if and only if both positive integers m and n are even.

18. (3 points (bonus)) The **complementary graph** G is describe in Rosen Ch 10.2, # 59, p. 667. A graph G is called **self-complementary** if it is isomorphic to G.

(a) Give an example of a self-complementary graph with $|V| = 4$.

(b) Give an example of a self-complementary graph with $|V| = 5$.

(a)  (b) 

19. (10 points (bonus)) Consider the set of all non-isomorphic simple undirected graphs (but do allow self-loops) of 3 vertices.

    (a) Draw all such graphs.
    (b) How many are connected? <u>10</u>
    (c) How many are bipartite? <u>3</u>
    (d) How many have a Euler circuit? <u>6</u>
    (e) How many have a Hamiltonian circuit? <u>4</u>
    (f) How many have a Euler path? <u>15</u>
    (g) How many have a Hamiltonian path? <u>10</u>
    (h) How many are planar? <u>20</u>

    Consider the set of graphs: