Instructions: All assignments are due by **midnight** on the due date specified.

Every student must write up their own solutions in their own manner.

Please present your solutions in a clean, understandable manner. Use the provided files that give mathematical notation in Word, Open Office, Google Docs, and LaTeX.

Assignments should be typed and submitted as a PDF.

Do Not Crowd Your Answers

Good Example

(a) $p \vee q$
(b) $(r \rightarrow \neg p) \wedge \neg q$
(c) $(\neg p \wedge q) \rightarrow r$

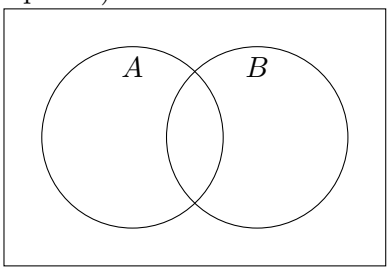
Bad Example

(a) $p \vee q$ (b) $(r \rightarrow \neg p) \wedge \neg q$ (c) $(\neg p \wedge q) \rightarrow r$

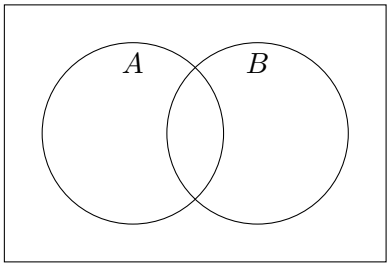
You should complete all problems, but only a subset will be graded (which will be graded is not known to you ahead of time).

Sets

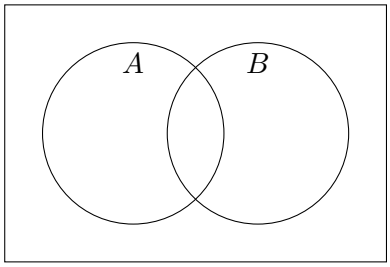
1. (3 points) For each of the following sets, shade the corresponding region of the Venn diagram.



(a) $\bar{A}$

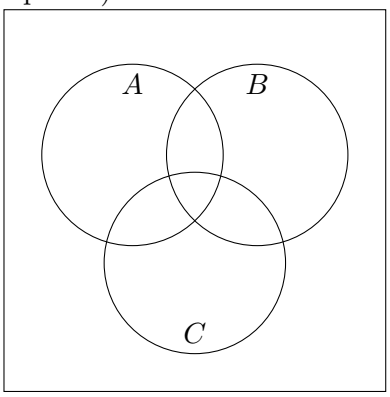


(b) $A \cap \bar{B}$ or $A - B$

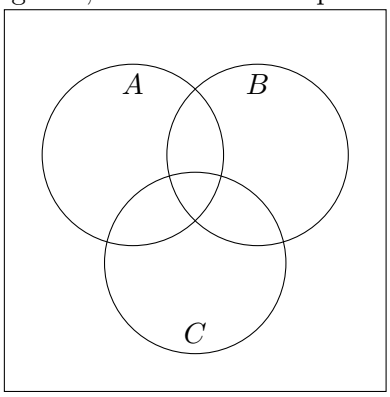


(c) $\overline{(A \cup B)}$

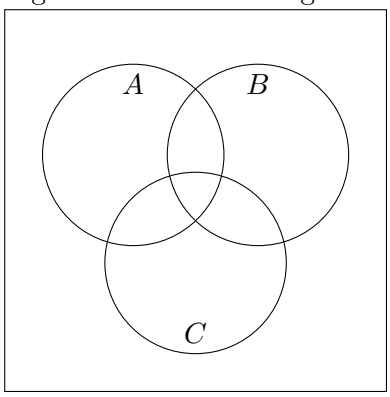
2. (6 points) For each of the following sets, shade the corresponding region of the Venn diagram.



(a) $(A \cap B) - C$

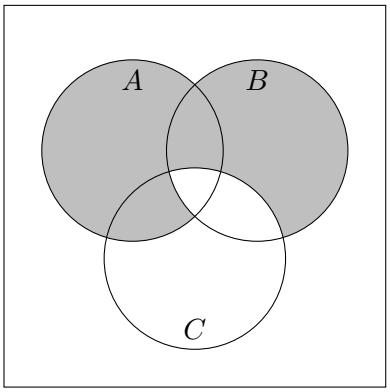


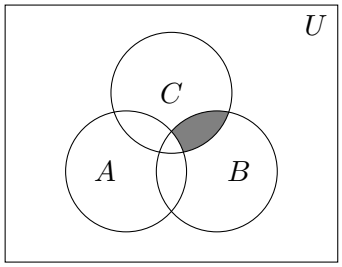
(b) $(C - B) \cup (B - C)$

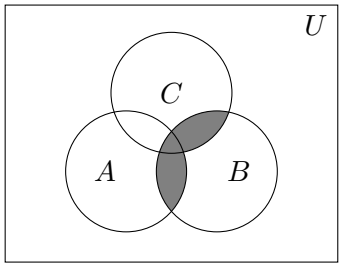


(c) $(A - C) \cup ((B \cap C) - A)$

3. (3 points) For each of the following sets, state what the corresponding shaded region of the Venn diagram represents.







4. (6 points) Show the following sets are equal, $\overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}$, using
   (a) set membership table
   (b) set identities

5. (4 points) Show the following sets are equal, $A \cap (\overline{A} \cup B) = A \cap B$, using
   (a) set membership table
   (b) set identities

6. (8 points) Consider a collection of books in the library. The universal set U refers to all books in the library. In addition, books are classified by their subject (each book may receive more than one classification) into the following sets: F - fiction, B - biography, H - historical, P - poetry, N - non-fiction, S - scientific, E - English, O - other language, L - literary, T - travel, R - reference. Each set is a subset of U.

   Express each of the following statements using set expressions, e.g., set operators ($\cup$, $\cap$, -, $\overline{A}$, etc.), relations ($=$, $\subseteq$, $\subset$), etc.:
   (a) The set of scientific references.
   (b) No book is both fiction and non-fiction.
   (c) A scientific book is not literary.
   (d) The set of historical fiction books excluding those in other languages.
   (e) Any biography is also a historical book.
   (f) All books are either reference, literary, or poetry.
   (g) All travel books are reference books.
   (h) All poetry in other languages are literary works.

Function

7. (6 points) Let $S = \{a, b, c, d, e\}$ and $T = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Determine whether the following are functions (Yes / No).
   (a) $f : S \to T$, $f = \{(a, 5), (b, 4), (c, 1), (d, 3), (e, 2)\}$
   (b) $g : S \to T$, $g = \{(a, 4), (d, 3), (c, 3), (b, 5)\}$
   (c) $h : S \to T$, $h = \{(a, 7), (b, 4), (c, 1), (d, 2), (e, 3), (b, 3)\}$
   (d) $i : S \to T$, $i = \{(a, 2), (b, 2), (c, 1), (d, 5), (e, 8)\}$
   (e) $j : \mathbb{Z} \to \mathbb{Z} = \{(x, y) \mid 3x + y^2 = 8\}$

(f) $k : \mathbb{R} \to \mathbb{R} = \{(x, x^2) \mid x \in \mathbb{R}\}$

8. (4 points) Rosen, Ch 2.3 # 6 (a-d), p. 152. (p. 161 for 8th ed)

9. (8 points) Find these values:

   (a) $\lfloor 8.6 \rfloor$ (b) $\lceil 4.3 \rceil$ (c) $\lfloor -3.6 \rfloor$ (d) $\lfloor 10.5\bar{3} \rfloor$

   (e) $\lceil -2.1 \rceil$ (f) $\left\lfloor \frac{-3}{4} \right\rfloor$ (g) $\left\lceil \frac{13}{3} + \left\lfloor \frac{-5}{4} \right\rfloor \right\rceil$ (h) $\lfloor\ 3.2 - \lceil 10.4 \rceil\ \rfloor$

10. (14 points) Determine whether each of the following functions are (i) one-to-one and (ii) onto.

    (a) Rosen Ch 2.3, #10(a), p. 153 (p. 162 for 8th ed)
    (b) Rosen Ch 2.3, #10(b), p. 153 (p. 162 for 8th ed)
    (c) $f : \mathbb{Z} \to \mathbb{Z}$ where $f(x) = x^2 - 5x + 5$.
    (d) $g : \mathbb{N} \to \mathbb{N}$ where $g(n) = n + 1$
    (e) $h : \mathbb{N} \to \mathbb{N}$ where $h(n) = \lfloor \frac{n}{2} \rfloor$
    (f) $i : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ where $i(m, n) = 2n - 4m$
    (g) $j : \mathbb{R} \to \mathbb{R}$ where $j(x) = \sqrt{x}$

11. (4 points) Let A and B be finite sets, and f be a function is $f : A \to B$. Determine which of the following statements are true.

    (a) If $f : A \to B$ is onto, then the domain and range are not only the same size, but the same set.
    (b) If $f : A \to B$ is both one-to-one and onto, then A and B have the same cardinality.
    (c) If $f : A \to B$ is one-to-one, then $|A| \leq |B|$.
    (d) If $f : A \to B$ is onto, then $|A| \geq |B|$.

12. (12 points) Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, d, f, g\}$. Let $f : A \to B$ and $g : B \to B$ with

$$f = \{(a, b), (b, d), (c, g), (d, a), (e, b)\} \text{ and}$$
$$g = \{(a, f), (b, d), (d, a), (f, g), (g, b)\}$$

    For each of the following compositions, define the function or explain why it is not defined.

    (a) $f \circ g$ (b) $g \circ f$ (c) $f \circ f$ (d) $g \circ g$

    (e) Find f^{-1} if it exists. If it doesn't, explain why not.
    (f) Find g^{-1} if it exists. If it doesn't, explain why not.

13. (8 points) Let f, g, and h all be functions mapping from A to $\mathbb{R}$ where $A = \{x \in \mathbb{R} \mid x > 0\}$,

$$f(x) = \frac{1}{x+1}, \qquad g(x) = \frac{x+1}{x}, \text{ and} \qquad h(x) = x - 1.$$

    Compute (a) $(f \circ g)(x)$, (b) $(g \circ f)(x)$, (c) $(h \circ g \circ f)(x)$, (d) $(f \circ g \circ h)(x)$

14. (6 points) (a) A hospital maintains a set of *Patients*, P, who have ever been admitted to the hospital at some time. (No patient is ever deleted from this set, even after they leave the hospital.)

    There is also a set *Beds*, B, describing the beds available for patients.

    Consider the function *currentBed* that maps *Patients* to *Beds*. The function relates a patient to the bed that he/she is currently occupying in the hospital.

    (a) Is this function total?

(b) Is the function one-to-one?

(c) Is the function onto?

Explain your answers.

(b) Let *Dates* be a set of dates.

Consider the function *date1stAdmitted* that maps *Patients* to *Dates*, relating a patient to the date he/she was first admitted to the hospital.

(a) Is this function total?

(b) Is the function one-to-one?

(c) Is the function onto?

Explain your answers.

Bonus

15. (1 point (bonus)) For the following sets, state what the corresponding shaded region of the Venn diagram represents.