Instructions: All assignments are due by **midnight** on the due date specified. Every student must write up their own solutions in their own manner.

Please present your solutions in a clean, understandable manner. Use the provided files that give mathematical notation in Word, Open Office, Google Docs, and LaTeX.

Assignments should be typed and submitted as a PDF.

You should complete all problems, but only a subset will be graded (which will be graded is not known to you ahead of time).

Counting

1. (6 points) How many different functions are there from a set with 4 elements to sets with the following numbers of elements: (a) 2? (b) 8? (c) How many different functions are there from a set with 8 elements to a set with 5 elements?

2. (4 points) Every US coin is stamped with the year it was minted.
   (a) How many coins do you need to have in your pocket to be assured that at least two have the same last digit?
   (b) How many do you need to be assured that at least two have the same first digit?

3. (6 points) Consider a bowl with 10 red chips, 10 blue chips, 10 green chips, and 10 yellow chips.
   (a) How many chips must be selected to ensure that at least 3 chips have the same color?
   (b) How many chips must be selected to ensure that at least 3 chips are green?

4. (4 points) The game TENZI comes with 40 six-sided dice. (a) If you roll all 40 dice, show that there will be at least seven dice that land on the same number. (b) How many dice do you need to roll before you were guaranteed that four of them would all match or be all different?

5. (3 points) Let $S = \{1, 3, 5, 7, 9, 11, 13, 15\}$. How many numbers must be selected from S so that at least one pair add up to 16?

6. (7 points) Seating arrangements:
   (a) (3pts) How many ways are there to arrange 8 people at a round table? Two arrangements are called the same if one can be obtained from another by rotation.
   (b) (4pts) How many ways are there to seat 6 CS majors and 8 ME majors in a row of chairs so that no two CS majors are seated next to each other.

7. (3 points) Suppose a password is four characters long with the following restrictions. It must contain exactly two instances of one symbol from the set $\{@, \%, \#, \$\}$, where these two identical characters can appear anywhere in the string. The remaining characters are from the sets $\{a..z\}$ or $\{0..9\}$. Determine the number of possible passwords.

8. (15 points) Consider strings of length 10 consisting of letters from the English alphabet (only uppercase letters will be used). How many strings could be created
   (a) (1 pt) if letters can be repeated?
   (b) (1 pt) if letters can not be repeated?
   (c) (1 pt) that start with AA, if letters can be repeated?
   (d) (2 pt) that start with AA, if no other letters can be repeated?
   (e) (2 pt) that start and end with AB, if letters can be repeated?

(f) (2 pt) that start or end with ABC, if letters can be repeated?

(g) (2 pt) that contain the letter A? (*Hint: use strategy of complements*)

(h) (2 pt) that contain the letters Y and Z?

(i) (2 pt) that contain B and C in consecutive positions as BC, with no repetition of letters?

9. (6 points) How many bit strings of length 8 have

   (a) (2 pt) exactly 3 zeros?

   (b) (2 pt) at least three 1s?

   (c) (2 pt) at least 6 1s?

10. (8 points) Consider the standard deck of 52 playing cards.
    How many 5-card hands

    (a) (2 pt) contain exactly one ace?

    (b) (3 pt) contain at least one ace?

    (c) (3 pt) contain 4 cards of the same value?

11. (11 points) A bowl contains Skittles with at least 100 of each color: red, purple, orange, yellow and green. Consider Skittles of the same color to be identical.

    (a) (2pts) How many different handfuls of candies can be formed containing 16 Skittles.

    (b) (3pts) How many different handfuls of candies can be formed containing 20 Skittles that have at least 1 of each color.

    (c) (3pts) How many different handfuls of candies can be formed containing 20 Skittles with exactly 3 red candies and at least 4 purple candies.

    (d) (3pts) How many different handfuls of 20 Skittles can be formed with no more than 4 yellow candies?

    (e) (2 points (bonus)) How many different handfuls of 16 Skittles have at least 2 red, purple, and green candy, at least 3 yellow candies, and no more than 2 orange candies.

12. (4 points) Consider the following words:

    (a) (2 pt) How many ways are there to arrange the letters in the word: "prosopopopeia" (a figure of speech in which an imaginary or absent person is speaking or acting).

    (b) (2 pt) How many ways are there to arrange the letters in the word: "kartoffelpuffer" (german for potato pancakes or hash brown).

    (c) (2 points (bonus)) For the word, "seeress", how many strings with 5 or more characters can be formed?

Bonus Questions

13. (10 points (bonus)) Powerball Example
    Consider a lottery game, SuperDuperBall, that is similar to Powerball or MegaMillions. The format is to draw five white balls from a set of balls labeled from 1 to 60 and pick a sixth green (superduperball) from a set of balls labeled 1 to 45. To win the jackpot, the first five numbers must match the number of the white balls (in any order), and the sixth number must match the green superduperball. Consider a number of different counting problems.

    (a) (1 pt) What are the number of ways to select a single white or single green ball.

(b) (1 pt) The white and green balls are in separate containers. What are the number of ways to select both a single white or single green ball.

(c) (2 pts) Consider selecting the 5 white balls from the single container, where the order the balls appears does not matter. What are the number of ways 5 white balls can be selected?

(d) (2 pts) The 5 white balls continue to be selected from a single container, now the green ball is also selected from its own container. To win a player must match the white balls (order does not matter) and the green superduperball. Determine the number of different patterns that could win?

(e) (2 pts) Now consider that order does mater for the white balls. What is the number of ways to select the white balls only?

(f) (2 pts) For the Superduperball lottery all 5 white balls must match (order matters) and the green superduperball is selected last. How many ways are there to select numbers for a ticket?

14. (6 points (bonus)) Consider the card game Euchre. Euchre is a trick game involving 4 players and a 24 card deck (the 9, 10, J, Q, K, and A of each suit). You do not need to understand how the card game is played to answer the following questions, you will just use information about the cards to include and count in a hand. Let's ignore the bidding process, and consider that a suit of cards is selected to be trump. The order of precedence in the trump cards of Euchre are:

Jack of Trump Suit,   Jack of same color as Trump Suit,   Ace,   King,   Queen,   10,   9

where the Jack of the trump suit is known as the Right Bower and the Jack of the same color as trump suit is known as the Left Bower.

For instance, if trump is hearts, then the set of cards considered "trump" ordered from highest to lowest trump is

$$J\heartsuit, J\diamondsuit, A\heartsuit, K\heartsuit, Q\heartsuit, 10\heartsuit, 9\heartsuit.$$

(a) (1 pts) How many different 5-card hands are possible in Euchre?

(b) (1 pts) Suppose hearts are trump, how many different hands can be selected with each card being a trump card?

(c) (2 pts) When a player has the Right and Left Bower and the Ace of the trump suit, they are guaranteed to win the trick. Suppose diamonds are trump, how many hands have this property?

(d) (2 pts) A farmer's hand refers to a hand with no face-cards or aces (that is, only 9s and 10s). How many ways may a farmer's hand be dealt?