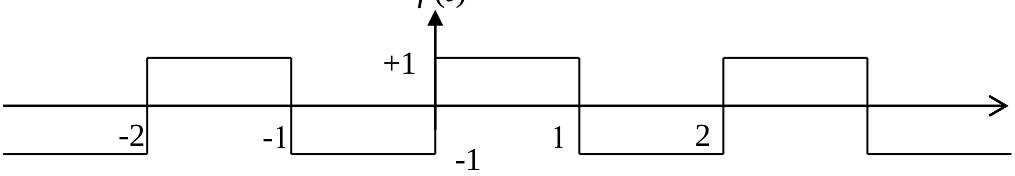
數位信號處理 (DSP) 2021/09/24

Homework #1 Due: Oct. 1, 2021 (Friday)

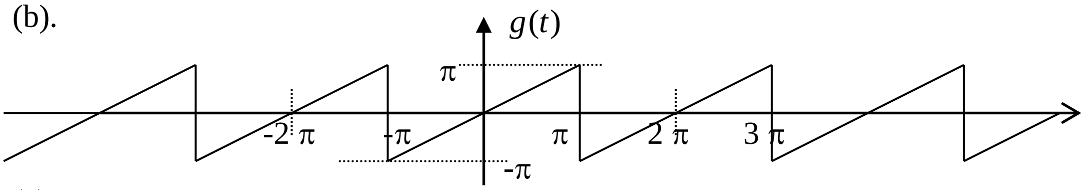
Fourier Series Expansion and Fourier Transform

1. Please determine the Fourier series coefficients for the following periodic functions,

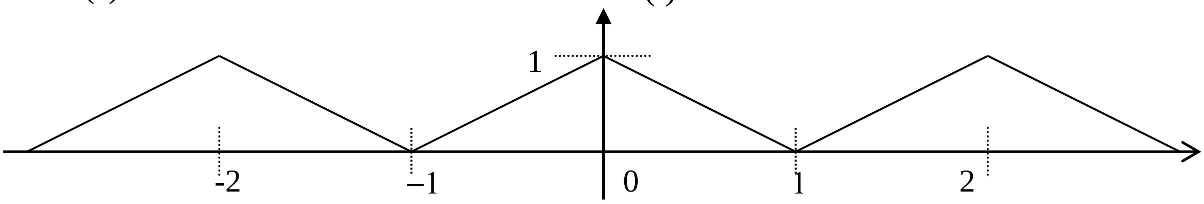
(a).



(b).

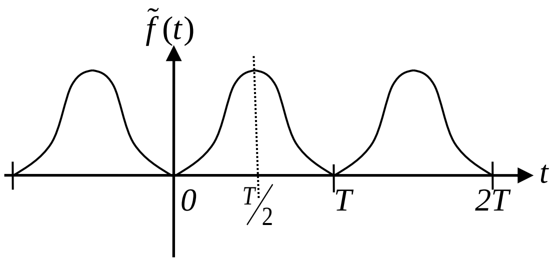


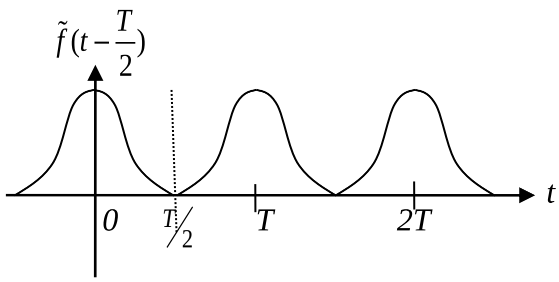
(c)



2. For the three functions ($f(t), g(t), h(t)$) discussed in Problem 1, please use their Fourier series expansion to approximate the functions by increasing the number of terms step by step. Please calculate with a PC and plot the results. You should make comparison between the true functions and their Fourier series approximation. Can you see Gibb's phenomenon? Can you verify if the overshot of Gibb's phenomenon is about 9 % of the discontinuity jump.

3. As show in the following figure, $\tilde{f}(t)$ is a periodic function with a period of T. $\tilde{f}(t)$ is symmetrical with respect to $t=0$ and $t=\frac{T}{2}$. (a) What can you expect about the Fourier Series Expansion of $\tilde{f}(t)$? (b) What can you expect about the Fourier Series Expansion of $\tilde{f}(t-\frac{T}{2})$? Please mathematically prove your conclusions.





4. Please first write a program to compute the functions,

$$H(f) = \frac{\sin\left[\pi(2N+1)f \big/ f_o\right]}{\sin\left[\pi f \big/ f_o\right]}$$, for N = 3, 5, 10, 30 using $f_o = 1$, and then plot these functions in the f-domain between (-1.5, 1.5). You should hand in the print-out of your code along with the figures.

5. Prove the following Fourier Transform properties,

   (a). Time-scaling property, i.e., if $h(t) \Leftrightarrow H(f)$ then $h(kt) \Leftrightarrow \frac{1}{|k|} H(\frac{f}{k})$.

   (b). Time-shifting property, i.e., if $h(t) \Leftrightarrow H(f)$ then $h(t - t_o) \Leftrightarrow H(f) \cdot e^{-j2\pi f t_o}$.

   (c). Symmetry property, i.e., if $h(t) \Leftrightarrow H(f)$, then $H(t) \Leftrightarrow h(-f)$.

   (d). Show that, if *h(t)* is a real function, then the magnitude of its Fourier transform, |*H(f)*|, is an even function, and the phase, $\angle H(f)$, is an odd function.

   (e). Show that, if *h(t)* is real and even function, then its Fourier transform, *H(f)*, is also real and even.