數位信號處理導論 (DSP) 2021/10/08

Homework #3 Due: Oct. 20, 2021 (Wednesday)

Sampling theory

1. Please write a computer program which can carry out the signal reconstruction formula as given by $h^r(t) = \sum_{n=N1}^{N2} h_n \frac{\sin[\pi(t-nT)/T]}{\pi(t-nT)/T}$, where T is the sampling interval and $N1$ and $N2$ are truncation limits. The inputs for this program is h_n , $N1$, $N2$, and T, and the output is $h(t)$ for a given t.

2. If $h(t)= \cos(2\pi t)$ have been sampled at $t = n/6$, n=0, ± 1, ± 2, …, ± 6, please use these data, h_n , n=-6 ~+6, and the program of the previous problem to reconstruct $h(t)$. You should plot the followings in one single xy-plot, (a). a continuous curve of $h(t)= \cos(2\pi t)$ for t = -1~1, (b). h_n data points (c). a continuous curve of the reconstructed $h^r(t)$ function for t = -1~1,. Please use solid-line, dash-line and symbols to distinguish them. Does the reconstructed curve completely match the original curve? Explain why.

3. Redo problem 3, with $h(t)$ sampled at t = n/1.5, n=-9, -8, -7, ….0, +1, +2, ….+8, and plot those curves for t=-3~3. What is the approximated frequency of the reconstructed curve? Can you explain why and what is happening?

4. For a signal, $s(t) = \cos(100t) + 2 \cdot \sin(200t)$, and t is time in second. What is the lowest sampling frequency you can use to sample this signal without any aliasing?

DTFT and Inverse DTFT

5. Prove the following properties of DTFT

   (a) Time-Shifting；If $x[n] \Leftrightarrow X(e^{j\omega})$ then $x[n-n_o] \Leftrightarrow X(e^{j\omega}) \cdot e^{-jn_o\omega}$

   (b) Frequency Shifting；If $x[n] \Leftrightarrow X(e^{j\omega})$, then $x[n]e^{jn\omega_o} \Leftrightarrow X(e^{j(\omega-\omega_o)})$

   (c) Linear Convolution；If $x[n] \Leftrightarrow X(e^{j\omega})$ and $y[n] \Leftrightarrow Y(e^{j\omega})$, then $x[n]*y[n] \Leftrightarrow X(e^{j\omega})\, Y(e^{j\omega})$

6. Prove the following symmetry properties of DTFT

   (a). If $x[n] \Leftrightarrow X(e^{j\omega})$, then $x[-n] \Leftrightarrow X(e^{-j\omega})$

   (b). If $x[n]$ is real and even, then $X(e^{j\omega})$ is a real function

   (c). If $x[n]$ is real and odd, then $X(e^{j\omega})$ is a pure imaginary function

Digital Convolution

7. $x[n]$ and $y[n]$ are both finite-length digital signals of length 5 (n=0,1,2,3,4). Please calculate (a). linear convolution of $x[n]$ and $y[n]$; (b). 5-point circular convolution of $x[n]$ and $y[n]$; (c).

9-point circular convolution of zero-padded $x[n]$ and zero-padded $y[n]$ to N=9; (d). 12-point circular convolution of zero-padded $x[n]$ and zero-padded $y[n]$ to N=12.