數位信號處理 (DSP) 2021/11/26

Homework #7 Due: Dec. 10, 2021 (Friday)

** Reading Assignment: Chap. 3 and Chap.5 in A.V. Oppenheim and R.W. Schafer, 1999.

Homework Problems in the Text book by A.V. Oppenheim and R.W. Schafer, 2nd edition, 1999.

1. Problem 3.8

3.8. The system function of a causal linear time-invariant system is

$$H(z) = \frac{1 - z^{-1}}{1 + \frac{3}{4}z^{-1}}.$$

The input to this system is

$$x[n] = \left(\tfrac{1}{3}\right)^n u[n] + u[-n-1].$$

(a) Find the impulse response of the system, $h[n]$.
(b) Find the output $y[n]$.
(c) Is the system stable? That is, is $h[n]$ absolutely summable?

2. Problem 3.9

3.9. A causal LTI system has impulse response $h[n]$, for which the z-transform is

$$H(z) = \frac{1 + z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{4}z^{-1}\right)}.$$

(a) What is the region of convergence of $H(z)$?
(b) Is the system stable? Explain.
(c) Find the z-transform $X(z)$ of an input $x[n]$ that will produce the output

$$y[n] = -\tfrac{1}{3}\left(-\tfrac{1}{4}\right)^n u[n] - \tfrac{4}{3}(2)^n u[-n-1].$$

(d) Find the impulse response $h[n]$ of the system.

3. Problem 3.18

3.18. A causal LTI system has the system function

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{\left(1 + \frac{1}{2}z^{-1}\right)(1 - z^{-1})}.$$

(a) Find the impulse response of the system, $h[n]$.
(b) Find the output of this system, $y[n]$, for the input

$$x[n] = e^{j(\pi/2)n}.$$

4. Problem 3.23

3.23. An LTI system is characterized by the system function

$$H(z) = \frac{\left(1 - \frac{1}{2}z^{-2}\right)}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)}, \qquad |z| > \frac{1}{2}.$$

(a) Determine the impulse response of the system.
(b) Determine the difference equation relating the system input $x[n]$ and the system output $y[n]$.

5. Problem 3.28

3.28. Determine the inverse z-transform of each of the following. You should find the z-transform properties in Section 3.4 helpful.

(a) $X(z) = \dfrac{3z^{-3}}{\left(1 - \frac{1}{4}z^{-1}\right)^2}$, $x[n]$ left sided

(b) $X(z) = \sin(z)$, ROC includes $|z| = 1$

(c) $X(z) = \dfrac{z^7 - 2}{1 - z^{-7}}$, $|z| > 1$

6. Problem 3.32

3.32. The pole–zero diagram in Figure P3.32-1 corresponds to the z-transform $X(z)$ of a causal sequence $x[n]$. Sketch the pole–zero diagram of $Y(z)$, where $y[n] = x[-n+3]$. Also, specify the region of convergence for $Y(z)$.

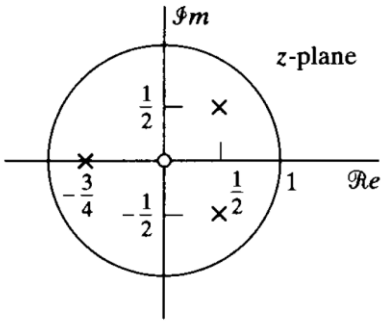

Figure P3.32-1

7. Problem 3.34

3.34. Consider an LTI system that is stable and for which $H(z)$, the z-transform of the impulse response, is given by

$$H(z) = \frac{3 - 7z^{-1} + 5z^{-2}}{1 - \frac{5}{2}z^{-1} + z^{-2}}.$$

Suppose $x[n]$, the input to the system, is a unit step sequence.
(a) Find the output $y[n]$ by evaluating the discrete convolution of $x[n]$ and $h[n]$.
(b) Find the output $y[n]$ by computing the inverse z-transform of $Y(z)$.

8. Problem 3.44

3.44. When the input to a causal LTI system is

$$x[n] = -\frac{1}{3}\left(\frac{1}{2}\right)^n u[n] - \frac{4}{3}2^n u[-n-1],$$

the z-transform of the output is

$$Y(z) = \frac{1+z^{-1}}{\left(1-z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)\left(1-2z^{-1}\right)}.$$

(a) Find the z-transform of $x[n]$.
(b) What is the region of convergence of $Y(z)$?
(c) Find the impulse response of the system.
(d) Is the system stable?