數位信號處理 (DSP) 2021/10/21

Homework #5 Due: Nov. 03, 2021 (Wednesday)

Window Kernel Function

1. $h(t)=cos(2\pi t)$ is sampled with $T=1/4$ to get $h[n]=h(nT)$, n=0,1,2,3,…,15. Let H_k be the DFT of $h[n]$, please analytically derive the H_k based on Rectangular Window Kernel Function. Compare your theoretical results of H_k with the calculated results of H_k using DFT or FFT algorithm.

Leakage, and Frequency Resolution

2. Please first calculate and then re-plot the DTFT functions appeared in Figure 10.3 (a) to (e), page 823-824, in the book by A.V. Oppenheim and R.W. Schafer, 3rd Ed., 2010, ***Example 10.3, "Effect of Windowing on Fourier Analysis of Sinusoidal Functions"***

3. Please first calculate and then re-plot the DFT and IDFT appeared in Figure 10.5 (a) to (f), page 828-829, in the book by A.V. Oppenheim and R.W. Schafer, 3rd Ed., 2010, ***Example 10.4, "Illustration of the Effect of Spectral Sampling"***

Windowing

4. $h(t) = cos(2\pi t*1.1)+0.07cos(2\pi t*2.9)$, sampling $h(t)$ with $T=0.1$ and get $h[n] = h(nT)$, n = 0, 1, 2, 3,…,31.
   (a). Using a rectangular window to calculate the DFT of $h[n]$, that is H_k. Plot the $|H_k|$ with respect to k (k= 0, 1, 2, …, 31) and frequency f (Hz);
   (b). Redo (a), but using a Hamming window, $w_{hm}[n]$, please plot $h[n]$, $w_{hm}[n]$, and $h[n]\ w_{hm}[n]$ with respect to n, and the $|H_k|$ with respect to k (k= 0, 1, 2, …, 31) and frequency f (Hz);
   (c). Redo (b), but using a Blackman window, $w_b[n]$,
   (d). Please explain the difference in $|H_k|$ calculated from (a), (b), (c).

5. Signal $v(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t)$ where $f_1 = 1$ kHz，$f_2 = 1.01$ kHz. One decides to first sample the signal $v(t)$ with a sampling frequency f_S and totally N sampled data points, { $x[n]$, n = 0, 1, 2, …, N-1 }. If Bartlett window is used for the spectrum analysis, please determine the smallest sampling frequency (f_S) and minimum number of data points (N) which are sufficient to distinguish the frequency components at frequencies f_1 and f_2.

6. Please verify your answers to Problem #5 by calculating the DTFT and DFT of related digital data.