

HW8 MATLAB code

1.

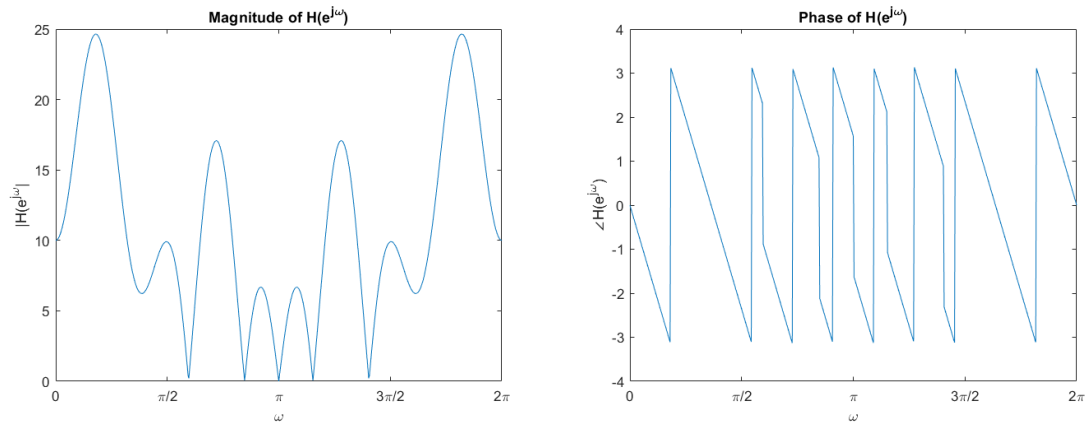
```
hn = [-4, 1, -1, -2, 5, 6, 6, 5, -2, -1, 1, -4];
N = length(hn);
n = 0:N-1;
w = 0:2*pi/600:2*pi;
H = sum(hn'.*exp(-1i*w.*n'),1);
figure(1);
plot(w,abs(H));
title('Magnitude of H(e^{j\omega})');
xlabel('\omega');
ylabel('|H(e^{j\omega})|');
xlim([0,2*pi]);
set(gca,'XTick',0:pi/2:2*pi);
set(gca,'XTickLabel',{'0','\pi/2','\pi','3\pi/2','2\pi'});
figure(2);
plot(w,angle(H));
title('Phase of H(e^{j\omega})');
xlabel('\omega');
ylabel('\angle H(e^{j\omega})');
xlim([0,2*pi]);
set(gca,'XTick',0:pi/2:2*pi);
set(gca,'XTickLabel',{'0','\pi/2','\pi','3\pi/2','2\pi'});

theta = -(N-1)/2*w;
A = H./exp(1i*theta);
figure(3);
plot(w,A);
title('Amplitude function A(\omega)');
xlabel('\omega');
ylabel('A(\omega)');
xlim([0,2*pi]);
set(gca,'XTick',0:pi/2:2*pi);
set(gca,'XTickLabel',{'0','\pi/2','\pi','3\pi/2','2\pi'});
figure(4);
plot(w,theta);
title('Phase function \theta(\omega)');
xlabel('\omega');
ylabel('\theta(\omega)');
xlim([0,2*pi]);
set(gca,'XTick',0:pi/2:2*pi);
set(gca,'XTickLabel',{'0','\pi/2','\pi','3\pi/2','2\pi'});

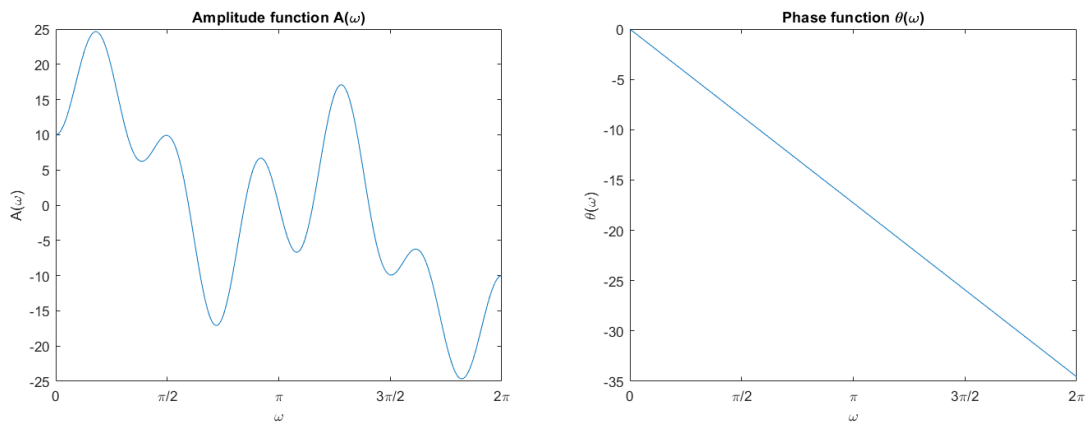
Hz = tf(hn,1,-1,'Variable','z^-1');
[p,z] = pzmap(Hz);
figure(5);
pzmap(Hz);
axis equal;

M = (N-1)/2;
Ak = A(1+(600/N)*n);
h_n = (Ak(1) + sum(2*Ak(2:N/2).*cos(2*pi*(n-M)'.*(1:N/2-1)/N),2))/N;
```

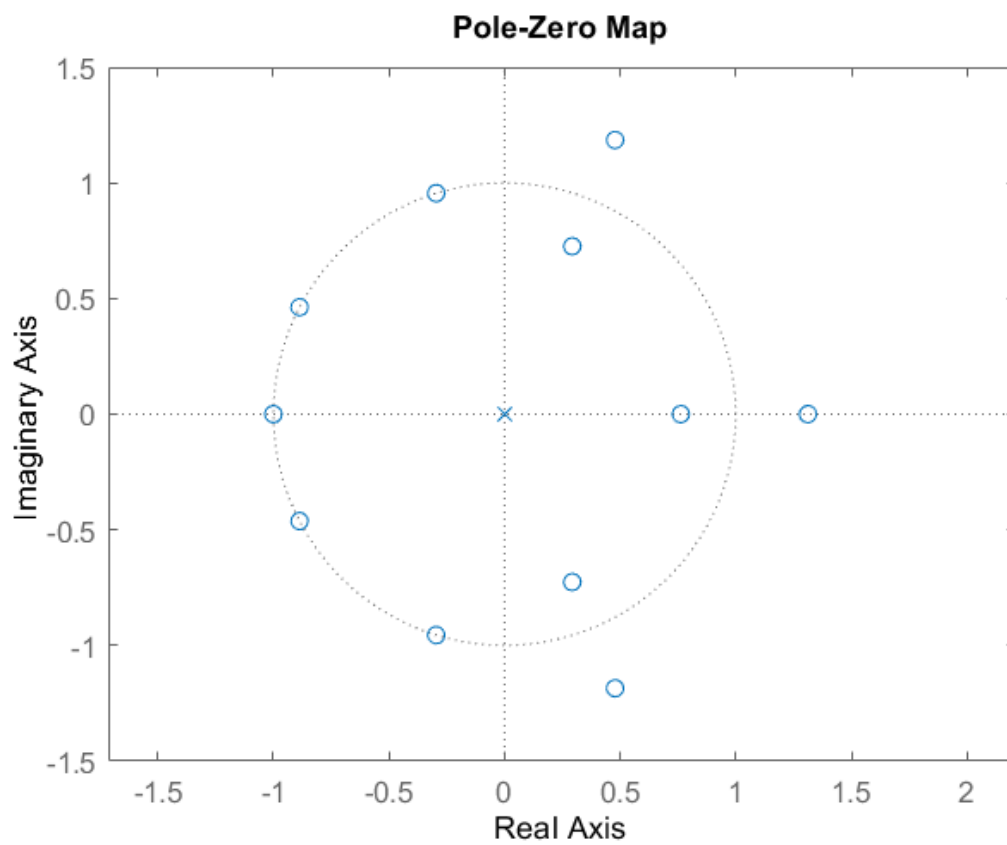
(a)



(b)



(c)



(d)

$$A(\omega) = \sum_{n=0}^{N-1} h[n] \cdot 2 \cdot \cos\left[\omega\left(\frac{N-1}{2} - n\right)\right]$$

$$\begin{aligned} A(\omega_k) &= \sum_{n=0}^{N-1} h[n] \cdot 2 \cdot \cos\left[\frac{2\pi k}{N}\left(\frac{N-1}{2} - n\right)\right] \\ &= \sum_{n=0}^{N-1} h[n] \cdot 2 \cdot \cos\left[\frac{\pi k(N-1)}{N} - \frac{2\pi kn}{N}\right] \\ &= \sum_{n=0}^{N-1} 2h[n] \cos\left[\frac{2\pi k\left(\frac{N-1}{2} - n\right)}{N}\right] \\ A_k &= \sum_{n=0}^{N-1} 2h[n] \cos\left[\frac{2\pi k(M-n)}{N}\right] \end{aligned}$$

$$\text{Let } \theta_k = -\frac{2\pi kM}{N},$$

$$\begin{aligned} A_k &= \sum_{n=0}^{N-1} 2h[n] \cos\left(-\theta_k - \frac{2\pi kn}{N}\right) \\ &= \sum_{n=0}^{N-1} 2h[n] \cos\left(\frac{2\pi kn}{N} + \theta_k\right) \end{aligned}$$

$$\begin{aligned} h[n] &= \frac{1}{N} \sum_{k=0}^{N-1} H[k] e^{j2\pi nk/N} \\ &= \frac{1}{N} \sum_{k=0}^{N-1} A_k e^{j\theta_k} e^{j2\pi nk/N} \end{aligned}$$

$$A_k = A_{N-k} \text{ (even w.r.t } \frac{N}{2} + 1) \quad k = 1, 2, \dots, N-1$$

$$\therefore h[n] = \frac{1}{N} \left[A_0 + \sum_{k=1}^{N-1} 2A_k \cos\left(\frac{2\pi(n-M)k}{N}\right) \right]$$

$$\begin{aligned} h_n &= \\ &-4.0000 - 0.0000i \\ &1.0000 + 0.0000i \\ &-1.0000 + 0.0000i \\ &-2.0000 - 0.0000i \\ &5.0000 + 0.0000i \\ &6.0000 - 0.0000i \\ &6.0000 - 0.0000i \\ &5.0000 + 0.0000i \\ &-2.0000 - 0.0000i \\ &-1.0000 + 0.0000i \\ &1.0000 + 0.0000i \\ &-4.0000 - 0.0000i \end{aligned}$$

2.

```
N = 41;
M = (N-1)/2;
k = 0:M;
w = k*2*pi/N;
Ak = 1*(w>=pi/3 & w<=2*pi/3);

hn = (Ak(1) + sum(2*Ak(2:end)'.*cos(2*pi*((0:N-1)-M).*k(2:end)'/N),1))/N;
omega = linspace(0,2*pi,1e3);
H = sum(hn'.*exp(-1i*(0:N-1)'.*omega),1);
figure(1);
stem((0:N-1),hn,'.');
title('h[n]');
xlabel('n');
ylabel('h[n]');
figure(2);
subplot(2,1,1);
stem(w,Ak,'.');
hold on;
plot(omega,abs(H));
title('A_k and |H(e^{j\omega})|, no transition point');
xlabel('\omega');
ylabel('|H(e^{j\omega})|');
xlim([0,2*pi]);
legend('A_k', '|H(e^{j\omega})|');
set(gca, 'XTick', 0:pi/3:2*pi);
```

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set(gca,'XTickLabel',{'0','\pi/3','2\pi/3','\pi','4\pi/3','5\pi/3','2\pi'});
subplot(2,1,2);
plot(omega,angle(H));
title('\angle H(e^{j\omega}), no transition point');
xlabel('\omega');
ylabel('\angle H(e^{j\omega})');
xlim([0,2*pi]);
set(gca,'XTick',0:pi/3:2*pi);
set(gca,'XTickLabel',{'0','\pi/3','2\pi/3','\pi','4\pi/3','5\pi/3','2\pi'});

```

```

Ak(find(w<pi/3,1,"last")) = 0.3904;
Ak(find(w>2*pi/3,1,"first")) = 0.3904;
hn = (Ak(1) + sum(2*Ak(2:end)'.*cos(2*pi*((0:N-1)-M).*k(2:end)'/N),1))/N;
H = sum(hn'.*exp(-1i*(0:N-1)'.*omega),1);
figure(3);
stem((0:N-1),hn,'.');
title('h[n]');
xlabel('n');
ylabel('h[n]');
figure(4);
subplot(2,1,1);
stem(w,Ak,'.');
hold on;
plot(omega,abs(H));
title('A_k and |H(e^{j\omega})|, with one transition point');
xlabel('\omega');
ylabel('|H(e^{j\omega})|');
xlim([0,2*pi]);
legend('A_k', '|H(e^{j\omega})|');
set(gca,'XTick',0:pi/3:2*pi);
set(gca,'XTickLabel',{'0','\pi/3','2\pi/3','\pi','4\pi/3','5\pi/3','2\pi'});
subplot(2,1,2);
plot(omega,angle(H));
title('\angle H(e^{j\omega}), with one transition point');
xlabel('\omega');
ylabel('\angle H(e^{j\omega})');
xlim([0,2*pi]);
set(gca,'XTick',0:pi/3:2*pi);
set(gca,'XTickLabel',{'0','\pi/3','2\pi/3','\pi','4\pi/3','5\pi/3','2\pi'});

```

```

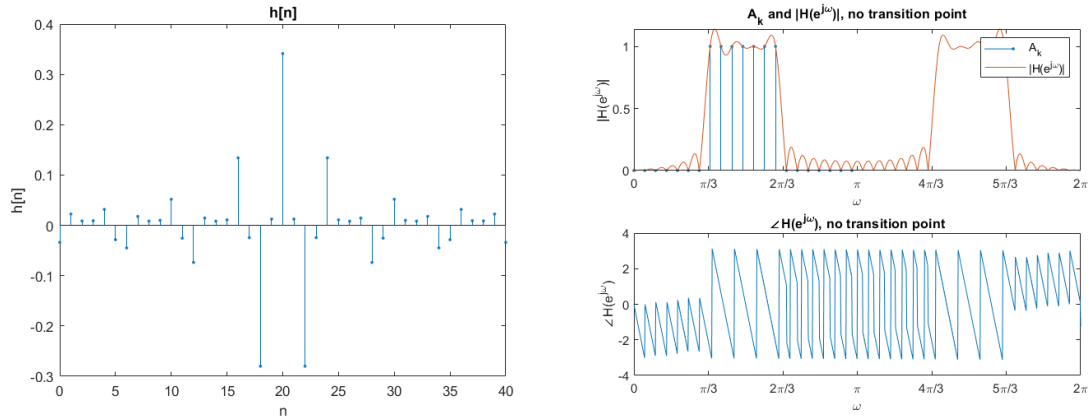
Ak(find(w<pi/3,2,"last")) = [1/4 3/4];
Ak(find(w>2*pi/3,2,"first")) = [3/4 1/4];
hn = (Ak(1) + sum(2*Ak(2:end)'.*cos(2*pi*((0:N-1)-M).*k(2:end)'/N),1))/N;
H = sum(hn'.*exp(-1i*(0:N-1)'.*omega),1);
figure(5);
stem((0:N-1),hn,'.');
title('h[n]');
xlabel('n');
ylabel('h[n]');
figure(6);
subplot(2,1,1);
stem(w,Ak,'.');
hold on;
plot(omega,abs(H));
title('A_k and |H(e^{j\omega})|, with one transition point');
xlabel('\omega');
ylabel('|H(e^{j\omega})|');
xlim([0,2*pi]);
legend('A_k', '|H(e^{j\omega})|');

```

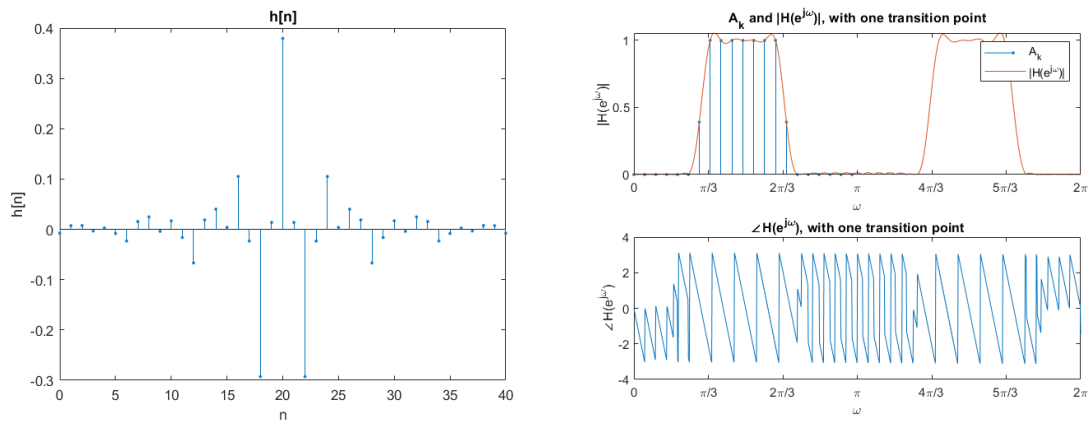
```

set(gca,'XTick',0:pi/3:2*pi);
set(gca,'XTickLabel',{'0','\pi/3','2\pi/3','\pi','4\pi/3','5\pi/3','2\pi'});
subplot(2,1,2);
plot(omega,angle(H));
title('\angle H(e^{j\omega}), N=41 with two transition points');
xlabel('\omega');
ylabel('\angle H(e^{j\omega})');
xlim([0,2*pi]);
set(gca,'XTick',0:pi/3:2*pi);
set(gca,'XTickLabel',{'0','\pi/3','2\pi/3','\pi','4\pi/3','5\pi/3','2\pi'});
(a)

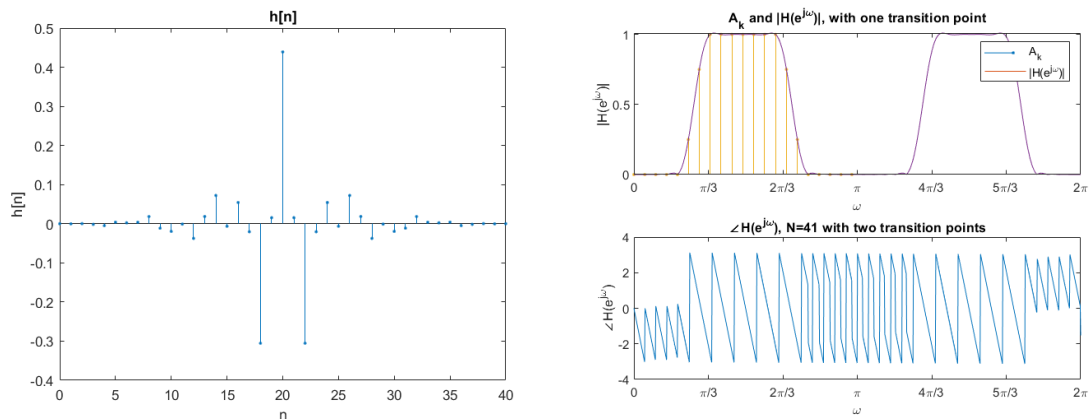
```



(b)



(c)



根據以上三種情況比較，取樣時加入 transition point 可以讓 ripple 變小，同時防止 Gibb's phenomenon 發生。

3.

```
N = 40;
alpha = (N-1)/2;
k = 0:N/2-1;
w = k*2*pi/N;
Ak = 1*(w >= 0 & w <= pi/2);
hd = sin(pi/2*(k-alpha))./(pi*(k-alpha));

win1 = rectwin(N)';
hn = hd.*win1(1:N/2);
omega = linspace(0,2*pi,1e3);
hn = cat(2,hn,hn(end:-1:1));
H = sum(hn'.*exp(-1i*(0:N-1)'.*omega),1);
figure(1);
stem((0:N-1),hn, '.');
title('h[n]');
xlabel('n');
ylabel('h[n]');
figure(2);
subplot(2,1,1);
stem(w,Ak, '.');
hold on;
plot(omega,abs(H));
title('A_k and |H(e^{j\omega})|, Rectangular window');
xlabel('\omega');
ylabel('|H(e^{j\omega})|');
xlim([0,2*pi]);
legend('A_k', '|H(e^{j\omega})|');
set(gca, 'XTick', 0:pi/3:2*pi);
set(gca, 'XTickLabel', {'0', '\pi/3', '2\pi/3', '\pi', '4\pi/3', '5\pi/3', '2\pi'});
subplot(2,1,2);
plot(omega,angle(H));
title('\angle H(e^{j\omega}), Rectangular window');
xlabel('\omega');
ylabel('\angle H(e^{j\omega})');
xlim([0,2*pi]);
set(gca, 'XTick', 0:pi/2:2*pi);
set(gca, 'XTickLabel', {'0', '\pi/2', '\pi', '3\pi/2', '2\pi'});

win2 = hanning(N)';
hn = hd.*win2(1:N/2);
hn = cat(2,hn,hn(end:-1:1));
H = sum(hn'.*exp(-1i*(0:N-1)'.*omega),1);
figure(3);
stem((0:N-1),hn, '.');
title('h[n]');
xlabel('n');
ylabel('h[n]');
figure(4);
subplot(2,1,1);
stem(w,Ak, '.');
hold on;
plot(omega,abs(H));
```

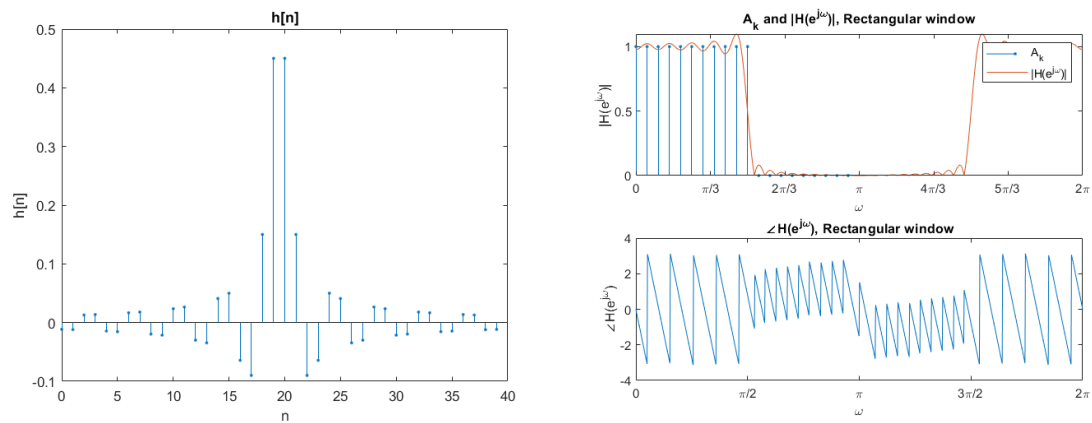
```

title('A_k and |H(e^{j\omega})|, Hanning window');
xlabel('\omega');
ylabel('|H(e^{j\omega})|');
xlim([0,2*pi]);
legend('A_k', '|H(e^{j\omega})|');
set(gca,'XTick',0:pi/3:2*pi);
set(gca,'XTickLabel',{'0','\pi/3','2\pi/3','\pi','4\pi/3','5\pi/3','2\pi'});
subplot(2,1,2);
plot(omega,angle(H));
title('\angle H(e^{j\omega}), Hanning window');
xlabel('\omega');
ylabel('\angle H(e^{j\omega})');
xlim([0,2*pi]);
set(gca,'XTick',0:pi/2:2*pi);
set(gca,'XTickLabel',{'0','\pi/2','\pi','3\pi/2','2\pi'});

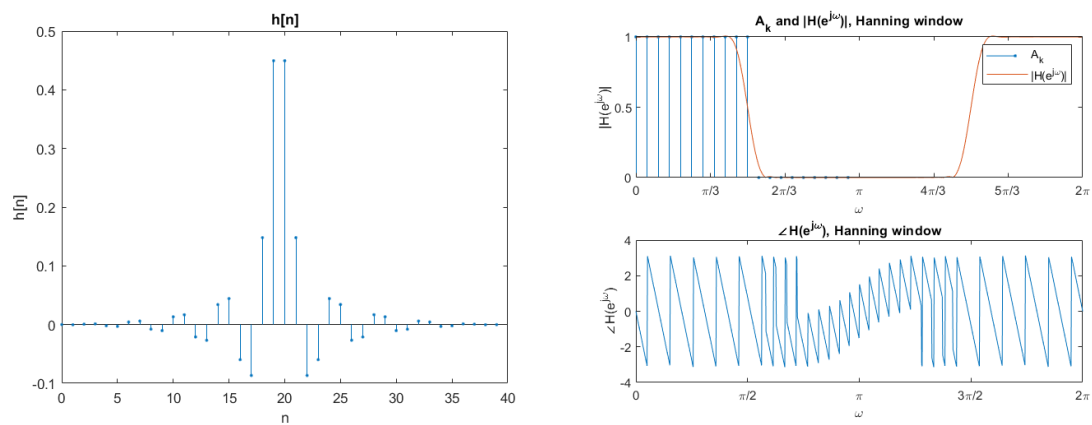
win3 = blackman(N)';
hn = hd.*win3(1:N/2);
hn = cat(2,hn,hn(end:-1:1));
H = sum(hn'.*exp(-1i*(0:N-1)'.*omega),1);
figure(5);
stem((0:N-1),hn,'.');
title('h[n]');
xlabel('n');
ylabel('h[n]');
figure(6);
subplot(2,1,1);
stem(w,Ak,'.');
hold on;
plot(omega,abs(H));
title('A_k and |H(e^{j\omega})|, Blackman window');
xlabel('\omega');
ylabel('|H(e^{j\omega})|');
xlim([0,2*pi]);
legend('A_k', '|H(e^{j\omega})|');
set(gca,'XTick',0:pi/3:2*pi);
set(gca,'XTickLabel',{'0','\pi/3','2\pi/3','\pi','4\pi/3','5\pi/3','2\pi'});
subplot(2,1,2);
plot(omega,angle(H));
title('\angle H(e^{j\omega}), Blackman window');
xlabel('\omega');
ylabel('\angle H(e^{j\omega})');
xlim([0,2*pi]);
set(gca,'XTick',0:pi/2:2*pi);
set(gca,'XTickLabel',{'0','\pi/2','\pi','3\pi/2','2\pi'});

```

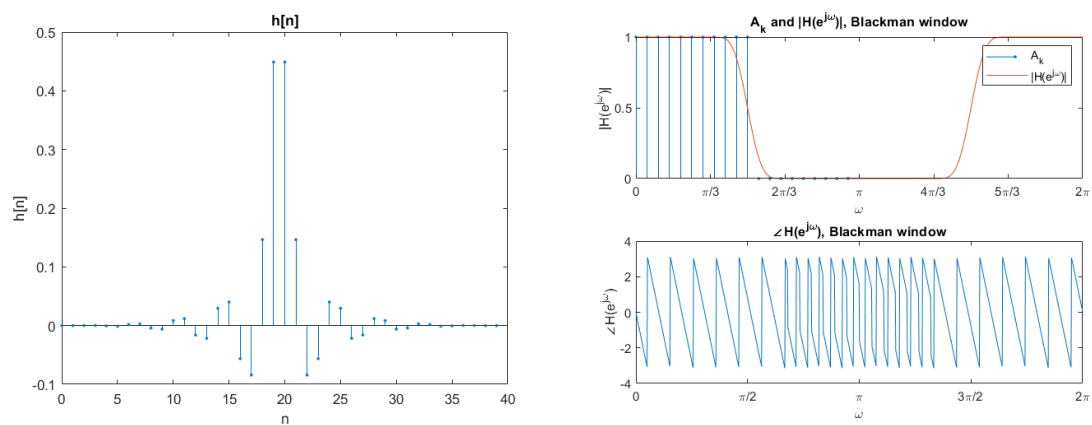
(a)



(b)



(c)



利用 windowing 取樣可以有效減少 ripple 和防止 Gibb's phenomenon, 但 transition band 會變寬。