Glossary of Symbols

Logic

Notation	Meaning	Example
$=$	equals	$x = x$
$\neq$	is distinct from	$x \neq x + 1$
$\Box$	end of an example or proof	
$P \wedge Q$	P and Q (both true)	$x \leq x + 1 \wedge x \neq x + 1$
$P \vee Q$	P or Q (one or both true)	$x \leq y \vee y \leq x$
$\neg P$	not P (P is not true)	$\neg\ 3 \geq 5$
$P \Rightarrow Q$	if P then Q	$x < y \Rightarrow x \leq y$
$P \equiv Q$	P if and only if Q	$x < y \equiv y > x$
$\exists x \bullet P$	there exists an x such that P	$\exists x \bullet x > y$
$\forall x \bullet P$	forall x, P	$\forall x \bullet x < x + 1$
$\exists x : A \bullet P$	there exists an x in set A such that P	
$\forall x : A \bullet P$	for all x in set A, P	

Sets

Notation	Meaning	Example
$\in$	is a member of	$Napoleon \in mankind$
$\notin$	is not a member of	$Napoleon \notin Russians$
$\{\}$	the empty set (with no members)	$\neg\ Napoleon \in \{\}$

$\{a\}$	the singleton set of a; a is the only member	$x \in \{a\} \equiv x = a$
$\{a, b, c\}$	the set with members a, b, and c	$c \in \{a, b, c\}$
$\{ x \mid P(x) \}$	the set of all x such that $P(x)$	$\{a\} = \{ x \mid x = a \}$
$A \cup B$	A union B	$A \cup B = \{ x \mid x \in A \vee x \in B \}$
$A \cap B$	A intersect B	$A \cap B = \{ x \mid x \in A \wedge x \in B \}$
$A - B$	A minus B	$A - B = \{ x \mid x \in A \wedge \neg\, x \in B \}$
$A \subseteq B$	A is contained in B	$A \subseteq B \equiv \forall\, x : A \bullet x \in B$
$A \supseteq B$	A contains B	$A \supseteq B \equiv B \subseteq A$
$\{ x : A \mid P(x) \}$	the set of x in A such that $P(x)$	
$\mathbb{N}$	the set of natural numbers	$\{0, 1, 2, \ldots\}$
$\mathbb{P}\, A$	the power set of A	$\mathbb{P}\, A = \{ X \mid X \subseteq A \}$
$\bigcup_{n \geq 0} A_n$	union of a family of sets	$\bigcup_{n \geq 0} A_n = \{ x \mid \exists\, n \geq 0 \bullet x \in A \}$
$\bigcap_{n \geq 0} A_n$	intersection of a family of sets	$\bigcap_{n \geq 0} A_n = \{ x \mid \forall\, n \geq 0 \bullet x \in A \}$

Functions

Notation	Meaning	Example
$f : A \rightarrow B$	f is a function which maps each member of A to a member of B	$square : \mathbb{N} \rightarrow \mathbb{N}$
$f(x)$	that member of B to which f maps x (in A)	
injection	a function f which maps each member of A to a distinct member of B	$x \neq y \Rightarrow f(x) \neq f(y)$
f^{-1}	inverse of an injection f	$x = f(y) \equiv y = f^{-1}(x)$
$\{ f(x) \mid P(x) \}$	the set formed by applying f to all x such that $P(x)$	
$f(C)$	the image of C under f	$\{ y \mid \exists\, x \bullet y = f(x) \wedge x \in C \}$ $square(\{3, 5\}) = \{9, 15\}$
$f \circ g$	f composed with g	$f \circ g(x) = f(g(x))$
$\lambda\, x \bullet f(x)$	the function which maps each value of x to $f(x)$	$(\lambda\, x \bullet f(x))(3) = f(3)$

Traces

Section	Notation	Meaning	Example
1.5	$\langle\rangle$	the empty trace	
1.5	$\langle a\rangle$	the trace containing only a (singleton sequence)	
1.5	$\langle a, b, c\rangle$	the trace with three symbols, a then b, then c	
1.6.1	$\frown$	(between traces) followed by	$\langle a, b, c\rangle = \langle a, b\rangle \frown \langle\rangle \frown \langle c\rangle$
1.6.1	s^n	s repeated n times	$\langle a, b\rangle^2 = \langle a, b, a, b\rangle$
1.6.2	$s \upharpoonright A$	s restricted to A	$\langle b, c, d, a\rangle \upharpoonright \{a, c\} = \langle c, a\rangle$
1.6.5	$s \leq t$	s is a prefix of t	$\langle a, b\rangle \leq \langle a, b, c\rangle$
4.2.2	$s \leq^n t$	s is like t with up to n symbols removed	$\langle a, b\rangle \leq^2 \langle a, b, c, d\rangle$
1.6.5	s in t	s is in t	$\langle c, d\rangle$ in $\langle b, c, d, a, b\rangle$
1.6.6	$\#s$	the length of s	$\#\langle b, c, b, a\rangle = 4$
1.6.6	$s \downarrow b$	the count of b in s	$\langle b, c, b, a\rangle \downarrow b = 2$
1.9.6	$s \downarrow c$	the communications on channel c recorded in s	$\langle c.1, a.4, c.3, d.1\rangle \downarrow c = \langle 1, 3\rangle$
1.9.2	$\frown / s$	flatten s	$\frown / \langle\langle a, b\rangle, \langle\rangle$
1.9.7	$s \,;\, t$	s successfully followed by t	$(s \frown \langle\checkmark\rangle) \,;\, t = s \frown t$
1.6.4	A^*	set of sequences with elements in A	$A^* = \{\, s \mid s \upharpoonright A = s \,\}$
1.6.3	s_0	the head of s	$\langle a, b, c\rangle_0 = a$
1.6.3	s'	the tail of s	$\langle a, b, c\rangle' = \langle b, c\rangle$
1.9.4	$s[i]$	the ith element of s	$\langle a, b, c\rangle[1] = b$
1.9.1	$f^*(s)$	f star of s	$square^*(\langle 1, 5, 3\rangle) = \langle 1, 25, 9\rangle$
1.9.5	$\overline{s}$	reverse of s	$\overline{\langle a, b, c, \rangle} = \langle c, b, a\rangle$

Special Events

Section	Notation	Meaning
1.9.7	$\checkmark$	success (successful termination)
2.6.2	$l.a$	participation in event a by a process named l

4.1	$c.v$	communication of value v on channel c
4.5	$l.c$	channel c of a process named l
4.5	$l.c.v$	communication of a message v on channel $l.c$
5.4.1	$\lightning$	catastrophe (lightning)
5.4.3	$\circledR$	exchange
5.4.4	$\circledC$	checkpoint for later recovery
6.2	*acquire*	acquisition of a resource
6.2	*release*	release of a resource

Processes

Section	Notation	Meaning
1.1	αP	the alphabet of process P
4.1	αc	the set of messages communicable on channel c
1.1.1	$a \rightarrow P$	a then P
1.1.3	$(a \rightarrow P \mid b \rightarrow Q)$	a then P choice b then Q (provided $a \neq b$)
1.1.3	$(x : A \rightarrow P(x))$	(choice of) x from A then $P(x)$
1.1.2	$\mu X : A \bullet F(X)$	the process X with alphabet A such that $X = F(X)$
1.8	$P \,/\, s$	P after (engaging in events of trace) s
2.3	$P \parallel Q$	P in parallel with Q
2.6.2	$l : P$	P with name l
2.6.4	$L : P$	P with names from set L
3.2	$P \sqcap Q$	P or Q (non-deterministic)
3.3	$P \Box Q$	P choice Q
3.5	$P \setminus C$	P without C (hiding)
3.6	$P \mid\mid\mid Q$	P interleave Q
4.4	$P \gg Q$	P chained to Q
4.5	$P \,//\, Q$	P subordinate to Q
6.4	$l :: P \,//\, Q$	remote subordination
5.1	$P \,;\, Q$	P (successfully) followed by Q
5.4	$P \triangle Q$	P interrupted by Q
5.4.1	$P \hat{\lightning} Q$	P but on catastrophe Q
5.4.2	$\hat{P}$	restartable P

5.4.3	$P \circledast Q$	P alternating with Q
5.5	$P \lhd b \rhd Q$	P if b else Q
5.1	$^*P$	repeat P
5.5	$b * P$	while b repeat P
5.5	$x := e$	x becomes (value of) e
4.2	$b!e$	on (channel) b output (value of) e
4.2	$b?x$	on (channel) b input to x
6.2	$l!e?x$	call of shared subroutine named l with value parameter e and results to x
1.10.1	P sat S	(process) P satisfies (specification) S
1.10.1	tr	an arbitrary trace of the specified process
3.7	ref	an arbitrary refusal of the specified process
5.5.2	$x^\checkmark$	the final value of x produced by the specified process
5.5.1	$var(P)$	set of variables assignable by P
5.5.1	$acc(P)$	set of variables accessible by P
2.8.2	$P \sqsubseteq Q$	(deterministic) Q can do at least as much as P
3.9	$P \sqsubseteq Q$	(nondeterministic) Q is as good as P or better
5.5.1	$\mathcal{D}\, e$	expression e is defined

Algebra

Term	Meaning
reflexive	a relation R such that $x\,R\,x$
antisymmetric	a relation R such that $x\,R\,y \wedge y\,R\,x \Rightarrow x = y$
transitive	a relation R such that $x\,R\,y \wedge y\,R\,z \Rightarrow x\,R\,z$
partial order	a relation $\leq$ that is reflexive, antisymmetric, and transitive
bottom	a least element $\bot$ such that $\bot \leq x$
monotonic	a function f that respects a partial order: $x \leq y \Rightarrow f(x) \leq f(y)$
strict	a function f that preserves bottom: $f(\bot) = \bot$
idempotent	a binary operator f such that $x\,f\,x = x$
symmetric	a binary operator f such that $x\,f\,y = y\,f\,x$
associative	a binary operator f such that $x\,f\,(y\,f\,z) = (x\,f\,y)\,f\,z$

distributive	f distributes through g if $x\,f\,(y\,g\,z) = (x\,f\,y)\,g\,(x\,f\,z)$ and $(y\,g\,z)\,f\,x = (y\,f\,x)\,g\,(z\,f\,x)$
unit	of f is an element 1 such that $x\,f\,1 = 1\,f\,x = x$
zero	of f is an element 0 such that $x\,f\,0 = 0\,f\,x = 0$

Graphs

Term	Meaning
graph	a relation drawn as a picture
node	a circle in a graph representing an element in the domain or range of a relation
arc	a line or arrow in a graph connecting nodes between which the pictured relation holds
undirected graph	graph of a symmetric relation
directed graph	graph of an asymmetric relation often drawn with arrows
directed cycle	a set of nodes connected in a cycle by arrows all in the same direction
undirected cycle	a set of nodes connected in a cycle by arcs or arrows in either direction