1028 Glossary of Symbols

## **Glossary of Symbols**

symbol	meaning
::=	is defined to be
	end of proof symbol
$\neq$	not equal
$\wedge$	and, AND
V	or, OR
$\longrightarrow$	implies, if, then $\cdots$ , IMPLIES
$\longrightarrow$	state transition
$\neg P, \overline{P}$	not $P$ , NOT $(p)$
$\longleftrightarrow$	iff, equivalent, IFF
$\oplus$	xor, exclusive-or, XOR
3	exists
$\forall$	for all
$\in$	is a member of, is in
$\subseteq$	is a (possibly =) subset of
⊈	is <b>not</b> a (possibly =) subset of
⊆ ⊈ ⊂ ⊄	is a proper (not =) subset of
$\not\subset$	is <b>not</b> a proper (not =) subset of
U	set union
$\bigcup_{i\in I} S_i$	union of sets $S_i$ where $i$ ranges over set $I$ of indices
$\cap$	set intersection
$\bigcap_{i\in I} S_i$	intersection of sets $S_i$ where $i$ ranges over set $I$ of indices
Ø	the empty set, {}
$\overline{A}$	complement of set A
_	set difference
pow(A)	powerset of set, A
$A \times B$	Cartesian product of sets A and B
$S^n$	Cartesian product of $n$ copies of set $S$
$\mathbb{Z}$	integers
$\mathbb{N},\mathbb{Z}^{\geq 0}$	nonnegative integers
$\mathbb{Z}^+, \mathbb{N}^+$	positive integers
$\mathbb{Z}^-$	negative integers
Q	rational numbers
$\mathbb{R}$	real numbers
$\mathbb{C}$	complex numbers
$\lfloor r \rfloor$	the <i>floor</i> of $r$ : the greatest integer $\leq r$
$\lceil r \rceil$	the <i>ceiling</i> of $r$ : the least integer $\geq r$
r	the absolute value of a real number $r$

## 1029 Glossary of Symbols

$R(X)$ image of set $X$ under binary relation $R$ $R^{-1}$ inverse of binary relation $R$ $R^{-1}(X)$ inverse image of set $X$ under relation $R$ surj $A$ surj $B$ iff $\exists f: A \rightarrow B. f$ is a surjective functioninj $A$ hij $B$ iff $\exists f: A \rightarrow B. f$ is a bijectioninj $A$ bij $B$ iff $\exists f: A \rightarrow B. f$ is a bijection $[\leq 1$ in]injective property of a relation $[\leq 1$ in]surjective property of a relation $[\leq 1$ out]function property of a relation $[= 1$ out, $= 1$ in]bijection relation0relational composition operator $\lambda$ the empty string/list $A^*$ the finite strings over alphabet $A$ rev(s)the reversal of string $s$ $s \cdot t$ concatenation of strings $s, t$ ; append( $s, t$ ) $\#_c(s)$ number of occurrences of character $c$ in string $s$ $m \mid n$ integer $m$ divides $n; m$ is a factor of $n; \exists k \in \mathbb{Z}. km = n$ gcdgreatest common divisorlogthe base 2 logarithm, $log_2$ lnthe natural logarithm, $log_2$ lcmleast common multiple $[kn]$ $\{i \mid k \leq i \leq n\}$ $(kn]$ $[kn] = \{k\}$ $[kn]$ $[kn] = \{n\}$ $(kn)$ $[kn] = \{n\}$ $(kn)$ $[kn] = \{n\}$ $(m)$	symbol	meaning
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$\begin{array}{lll} \lambda & \text{the empty string/list} \\ A^* & \text{the finite strings over alphabet } A \\ A^{\omega} & \text{the infinite strings over alphabet } A \\ \text{rev}(s) & \text{the reversal of string } s \\ s \cdot t & \text{concatenation of string } s, t; \text{ append}(s,t) \\ \#_c(s) & \text{number of occurrences of character } c \text{ in string } s \\ m \mid n & \text{integer } m \text{ divides } n; m \text{ is a factor of } n; \exists k \in \mathbb{Z}. km = n \\ \text{gcd} & \text{greatest common divisor} \\ \log & \text{the base 2 logarithm, } log_2 \\ \ln & \text{the natural logarithm, } log_2 \\ \ln & \text{least common multiple} \\ [kn] & \{i \mid k \leq i \leq n\} \\ (kn] & [kn] - \{k\} \\ [kn] & [kn] - \{k\} \\ [kn] & [kn] - \{k,n\} \\ \sum_{i \in I} r_i & \text{sum of numbers } r_i \text{ where } i \text{ ranges over set } I \text{ of indices} \\ \prod_{i \in I} r_i & \text{product of numbers } r_i \text{ where } i \text{ ranges over set } I \text{ of indices} \\ \text{qcnt}(n,d) & \text{quotient of } n \text{ divided by } d \\ \equiv & \text{(mod } n) & \text{congruence modulo } n \\ \not\equiv & \text{(mod } n) & \text{congruence modulo } n \\ \not\equiv & \text{not congruent} \\ \mathbb{Z}_n & \text{the ring of integers modulo } n \\ +n, \cdot n & \text{addition and multiplication operations in } \mathbb{Z}_n \\ \not\equiv & \text{the set of numbers in } [0,n) \text{ relatively prime to } n \\ \phi(n) & \text{Euler's totient function } ::=  \mathbb{Z}_n^*  \\ \langle u \rightarrow v \rangle & \text{directed edge from vertex } u \text{ to vertex } v \\ \end{array}$	-	
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lem least common multiple $ [kn] \qquad \{i \mid k \leq i \leq n\} $ $ (kn] \qquad [kn] - \{k\} $ $ [kn] \qquad [kn] - \{n\} $ $ (kn) \qquad [kn] - \{n\} $ $ (kn) \qquad [kn] - \{k, n\} $ $ \sum_{i \in I} r_i \qquad \text{sum of numbers } r_i \text{ where } i \text{ ranges over set } I \text{ of indices } $ $ \prod_{i \in I} r_i \qquad \text{product of numbers } r_i \text{ where } i \text{ ranges over set } I \text{ of indices } $ $ \text{quotient of } n \text{ divided by } d $ $ \text{rem}(n, d) \qquad \text{remainder of } n \text{ divided by } d $ $ \equiv \pmod{n} \qquad \text{congruence modulo } n $ $ \not\equiv \pmod{n} \qquad \text{congruent } $ $ \not\equiv \pmod{n} \qquad \text{addition and multiplication operations in } \mathbb{Z}_n $ the set of numbers in $[0, n)$ relatively prime to $n $ $ \phi(n) \qquad \text{Euler's totient function } ::=  \mathbb{Z}_n^*  $ $ (u \rightarrow v) \qquad \text{directed edge from vertex } u \text{ to vertex } v $	log	the base 2 logarithm, $log_2$
	ln	the natural logarithm, $log_e$
	lcm	least common multiple
$ [kn] \qquad [kn] - \{n\} $ $ (kn) \qquad [kn] - \{k,n\} $ $ \sum_{i \in I} r_i \qquad \text{sum of numbers } r_i \text{ where } i \text{ ranges over set } I \text{ of indices} $ $ \prod_{i \in I} r_i \qquad \text{product of numbers } r_i \text{ where } i \text{ ranges over set } I \text{ of indices} $ $ \text{quntient of } n \text{ divided by } d $ $ \text{rem}(n, d) \qquad \text{remainder of } n \text{ divided by } d $ $ \equiv \pmod{n} \qquad \text{congruence modulo } n $ $ \not\equiv \pmod{n} \qquad \text{congruent} $ $ \mathbb{Z}_n \qquad \text{the ring of integers modulo } n $ $ +_n, \cdot_n \qquad \text{addition and multiplication operations in } \mathbb{Z}_n $ $ \text{the set of numbers in } [0, n) \text{ relatively prime to } n $ $ \phi(n) \qquad \text{Euler's totient function } ::=  \mathbb{Z}_n^*  $ $ \langle u \rightarrow v \rangle \qquad \text{directed edge from vertex } u \text{ to vertex } v $	[kn]	$\{i \mid k \le i \le n\}$
$ \begin{array}{lll} (kn) & [kn] - \{k,n\} \\ \sum_{i \in I} r_i & \text{sum of numbers } r_i \text{ where } i \text{ ranges over set } I \text{ of indices} \\ \prod_{i \in I} r_i & \text{product of numbers } r_i \text{ where } i \text{ ranges over set } I \text{ of indices} \\ \text{qcnt}(n,d) & \text{quotient of } n \text{ divided by } d \\ \text{rem}(n,d) & \text{remainder of } n \text{ divided by } d \\ \equiv & (\text{mod } n) & \text{congruence modulo } n \\ \not\equiv & \text{not congruent} \\ \mathbb{Z}_n & \text{the ring of integers modulo } n \\ +_n, \cdot_n & \text{addition and multiplication operations in } \mathbb{Z}_n \\ \mathbb{Z}_n^* & \text{the set of numbers in } [0,n) \text{ relatively prime to } n \\ \phi(n) & \text{Euler's totient function } ::=  \mathbb{Z}_n^*  \\ \langle u \rightarrow v \rangle & \text{directed edge from vertex } u \text{ to vertex } v \\ \end{array} $	(kn]	$[kn] - \{k\}$
	[kn)	$[kn] - \{n\}$
$ \begin{array}{ll} \prod_{i \in I} r_i & \text{product of numbers } r_i \text{ where } i \text{ ranges over set } I \text{ of indices} \\ \text{qcnt}(n,d) & \text{quotient of } n \text{ divided by } d \\ \text{rem}(n,d) & \text{remainder of } n \text{ divided by } d \\ \equiv \pmod{n} & \text{congruence modulo } n \\ \not\equiv & \text{not congruent} \\ \mathbb{Z}_n & \text{the ring of integers modulo } n \\ +_n,\cdot_n & \text{addition and multiplication operations in } \mathbb{Z}_n \\ \mathbb{Z}_n^* & \text{the set of numbers in } [0,n) \text{ relatively prime to } n \\ \phi(n) & \text{Euler's totient function } ::= \mathbb{Z}_n^*  \\ \langle u \rightarrow v \rangle & \text{directed edge from vertex } u \text{ to vertex } v \\ \end{array} $	(kn)	$[kn] - \{k, n\}$
$\operatorname{qcnt}(n,d)$ quotient of $n$ divided by $d$ $\operatorname{rem}(n,d)$ remainder of $n$ divided by $d$ $\equiv \pmod{n}$ congruence modulo $n$ $\not\equiv \pmod{n}$ the ring of integers modulo $n$ $+_n,\cdot_n$ addition and multiplication operations in $\mathbb{Z}_n$ $\mathbb{Z}_n^*$ the set of numbers in $[0,n)$ relatively prime to $n$ $\phi(n)$ Euler's totient function $:= \mathbb{Z}_n^* $ $\langle u \to v \rangle$ directed edge from vertex $u$ to vertex $v$	$\sum_{i \in I} r_i$	sum of numbers $r_i$ where $i$ ranges over set $I$ of indices
rem $(n, d)$ remainder of $n$ divided by $d$ $\equiv \pmod{n}$ congruence modulo $n$ $\not\equiv \pmod{n}$ not congruent $\mathbb{Z}_n$ the ring of integers modulo $n$ $+_n, \cdot_n$ addition and multiplication operations in $\mathbb{Z}_n$ $\mathbb{Z}_n^*$ the set of numbers in $[0, n)$ relatively prime to $n$ $\phi(n)$ Euler's totient function ::= $ \mathbb{Z}_n^* $ $\langle u \rightarrow v \rangle$ directed edge from vertex $u$ to vertex $v$	$\prod_{i\in I} r_i$	product of numbers $r_i$ where $i$ ranges over set $I$ of indices
$\equiv \pmod{n}$ $ congruence modulo n  not congruent  \mathbb{Z}_n  the ring of integers modulo n  +_n, \cdot_n  addition and multiplication operations in \mathbb{Z}_n  \mathbb{Z}_n^*  the set of numbers in [0, n) relatively prime to n  \phi(n)  Euler's totient function :=  \mathbb{Z}_n^*   \langle u \rightarrow v \rangle  directed edge from vertex u to vertex v$	qent(n, d)	quotient of $n$ divided by $d$
$\not\equiv$ not congruent $\mathbb{Z}_n$ the ring of integers modulo $n$ $+_n, \cdot_n$ addition and multiplication operations in $\mathbb{Z}_n$ $\mathbb{Z}_n^*$ the set of numbers in $[0, n)$ relatively prime to $n$ $\phi(n)$ Euler's totient function $:=  \mathbb{Z}_n^* $ $\langle u \rightarrow v \rangle$ directed edge from vertex $u$ to vertex $v$	rem(n, d)	remainder of $n$ divided by $d$
$\mathbb{Z}_n$ the ring of integers modulo $n$ $+_n, \cdot_n$ addition and multiplication operations in $\mathbb{Z}_n$ $\mathbb{Z}_n^*$ the set of numbers in $[0, n)$ relatively prime to $n$ $\phi(n)$ Euler's totient function $:=  \mathbb{Z}_n^* $ $\langle u \rightarrow v \rangle$ directed edge from vertex $u$ to vertex $v$	$\equiv \pmod{n}$	congruence modulo n
$+_n, \cdot_n$ addition and multiplication operations in $\mathbb{Z}_n$ $\mathbb{Z}_n^*$ the set of numbers in $[0, n)$ relatively prime to $n$ $\phi(n)$ Euler's totient function $:=  \mathbb{Z}_n^* $ $\langle u \to v \rangle$ directed edge from vertex $u$ to vertex $v$		not congruent
$\mathbb{Z}_n^*$ the set of numbers in $[0, n)$ relatively prime to $n$ $\phi(n)$ Euler's totient function ::= $ \mathbb{Z}_n^* $ $\langle u \to v \rangle$ directed edge from vertex $u$ to vertex $v$	$\mathbb{Z}_n$	
$\phi(n)$ Euler's totient function ::= $ \mathbb{Z}_n^* $ $\langle u \rightarrow v \rangle$ directed edge from vertex $u$ to vertex $v$		addition and multiplication operations in $\mathbb{Z}_n$
$\langle u \rightarrow v \rangle$ directed edge from vertex $u$ to vertex $v$	• •	
	• ` '	the state of the s
Id. identity relation on set A: $a Id \cdot a'$ iff $a = a'$	$\langle u \rightarrow v \rangle$	
$a_A$ identity relation on set $A$ . $a_{1}a_Aa$ in $a=a$	$\mathrm{Id}_A$	identity relation on set A: $a \operatorname{Id}_A a'$ iff $a = a'$

## 1030 Glossary of Symbols

symbol	meaning
$R^*$	path relation of relation $R$ ; reflexive transitive closure of $R$
$R^+$	positive path relation of $R$ ; transitive closure of $R$
$\mathbf{f}  \widehat{\mathbf{x}}  \mathbf{g}$	merge of walk $\mathbf{f}$ with end vertex $x$
	and walk $\mathbf{g}$ with start vertex $x$
f^g	merge of walk <b>f</b> and walk <b>g</b>
	where $\mathbf{f}$ 's end vertex equals $\mathbf{g}$ 's start vertex
$\langle u-v\rangle$	undirected edge connecting vertices $u \neq v$
E(G)	the edges of graph $G$
V(G)	the vertices of graph G
$C_n$	the length- <i>n</i> undirected cycle
$L_n$	the length- <i>n</i> line graph
$K_n$	the <i>n</i> -vertex complete graph
$H_n$	the <i>n</i> -dimensional hypercube
L(G)	the "left" vertices of bipartite graph G
R(G)	the "right" vertices of bipartite graph G
$K_{n,m}$	the complete bipartite graph with $n$ left and $m$ right vertices
$\chi(G)$	chromatic number of simple graph G
$H_n$	the <i>n</i> th Harmonic number $\sum_{i=1}^{n} 1/i$
~	asymptotic equality
n!	<i>n</i> factorial ::= $n \cdot (n-1) \cdots 2 \cdot 1$
$\binom{n}{m}$	:= n!/m!((n-m)!; the binomial coefficient
o()	asymptotic notation "little oh"
0()	asymptotic notation "big oh"
$\Theta()$	asymptotic notation "Theta"
$\Omega()$	asymptotic notation "big Omega"
$\omega()$	asymptotic notation "little omega"
Pr[A]	probability of event A
$Pr[A \mid B]$	conditional probability of A given B
$\mathcal{S}$	sample space
$I_A$	indicator variable for event A
PDF	probability density function
CDF	cumulative distribution function
$\operatorname{Ex}[R]$	expectation of random variable $R$
$\operatorname{Ex}[R \mid A]$	conditional expectation of $R$ given event $A$
$\operatorname{Ex}^2[R]$	abbreviation for $(Ex[R])^2$
Var[R]	variance of R
$\operatorname{Var}^2[R]$	the square of the variance of $R$
$\sigma_R$	standard deviation of R