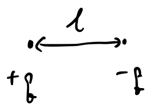
§7-1.

1. $k=\frac{1}{4\pi\varepsilon_0}$

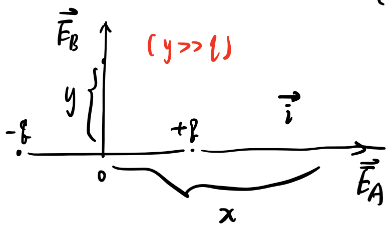
§7-2.

1. 电偶极子



定义 $\vec{p}=q\cdot\vec{l}$ → 电偶极矩 (电矩)

在 $\vec{l}$ 延长线上:



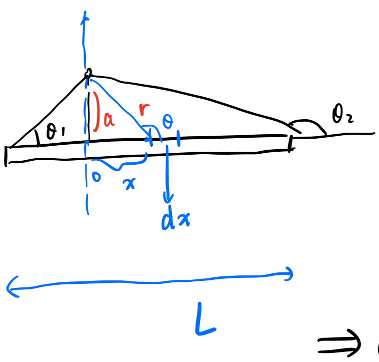
$$\vec{E}_A=\frac{1}{4\pi\varepsilon_0}\frac{2ql}{x^3}\vec{i}$$
$$=\frac{1}{4\pi\varepsilon_0}\frac{2p}{x^3}$$

$$\vec{E}_B=-\frac{1}{4\pi\varepsilon_0}\frac{ql}{y^3}\vec{i}$$
$$=-\frac{1}{4\pi\varepsilon_0}\frac{p}{y^3}$$

2. 利用微元思想

$$\begin{cases} d\vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\vec{e}_r \\ \vec{E}=\int d\vec{E} \end{cases}$$

3. 均匀带电直棒:



$\lambda=\frac{q}{L}$

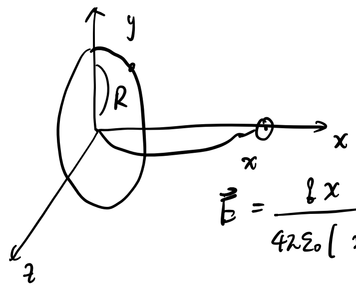
$$x=-a\cot\theta$$
$$dx=a\csc^2\theta\, d\theta$$
$$r^2=x^2+a^2=a^2\csc^2\theta$$
$$\Rightarrow d\vec{E}_x=k\cdot\frac{\lambda}{a}\cos\theta\, d\theta$$
$$d\vec{E}_y=k\frac{\lambda}{a}\sin\theta\, d\theta$$
$$\Rightarrow E_x=\int dE_x=\cdots=\frac{\lambda}{4\pi\varepsilon_0 a}(\sin\theta_2-\sin\theta_1)$$
$$E_y=\int dE_y=\cdots=\frac{\lambda}{4\pi\varepsilon_0 a}(\cos\theta_1-\cos\theta_2)$$

$$\therefore E=\frac{\lambda}{4\pi\varepsilon_0 a}\sqrt{2-2\cos(\theta_1-\theta_2)}$$

若此棒无限长 $\Longleftrightarrow$ $\theta_1=0,\ \theta_2=\pi$

$$\vec{E}=\frac{\lambda}{2\pi\varepsilon_0 a}$$

4. 带电圆环:



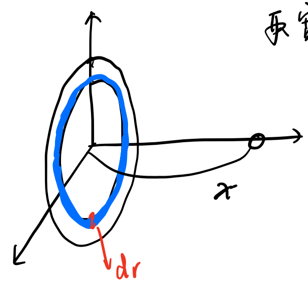
$$\vec{E}=\frac{qx}{4\pi\varepsilon_0(x^2+R^2)^{\frac{3}{2}}}$$

当 $x>>R$ 时, 有

$$\vec{E}=\frac{q}{4\pi\varepsilon_0 x^2}$$

5. 带电圆盘:

取宽为dr的细条



$$d\vec{E}=\frac{\sigma x}{2\varepsilon_0}\frac{r\,dr}{(x^2+r^2)^{\frac{3}{2}}}$$

(σ 为面密度.)

$$\vec{E}=\int dE=\frac{\sigma}{2\varepsilon_0}\left(1-\frac{x}{\sqrt{R^2+x^2}}\right)$$

6. 电场强度通量

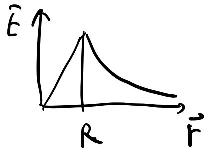
$$\Psi_E=\vec{E}\cdot\vec{S}$$
$$=\iint_S\vec{E}\cdot d\vec{S}$$
$$(\text{闭合面})=\oiint_S \vec{E}\cdot d\vec{S}$$

§7-2.

1. 静电场的Gauss定理

$$\Psi_E=\oiint\vec{E}\cdot d\vec{S}=\frac{1}{\varepsilon_0}\sum q_i$$

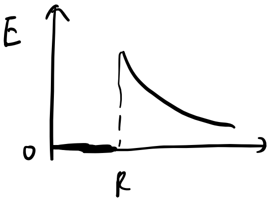
2. 带电球体电场（半径R）.



$$\vec E = \frac{qr}{4\pi\varepsilon_0 R^3}\vec e_r \quad (r<R)$$

$$= \frac{q}{4\pi\varepsilon_0 r^2}\vec e_r$$

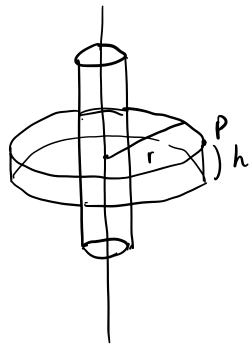
3. 带电球壳.



$r<R$. $E=0$

$r>R$ 易见同上.

4. 柱的电场



$\Phi_E = 2\pi r h E$.

又 $2\pi r h E = \frac{\lambda h}{\varepsilon_0}$

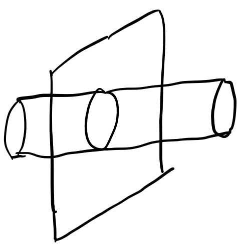
$\therefore E = \frac{\lambda}{2\pi\varepsilon_0 r}\vec e_r$.

（点P在柱外，即 $r>R$）

$r<R$ 时：

$$E = \frac{\lambda r}{2\pi\varepsilon_0 R^2} e_r.$$

5. 平面对称



$\vec E = \frac{\sigma}{2\varepsilon_0}\vec e_n$. 即均匀电场

若是一对板（如电容）

$$\vec E' = 2\vec E = \frac{\sigma}{\varepsilon_0}\vec e_n$$

§7-4.

1. 静电场的环路定理.

$$\oint_l \vec E\cdot d\vec l = 0.$$

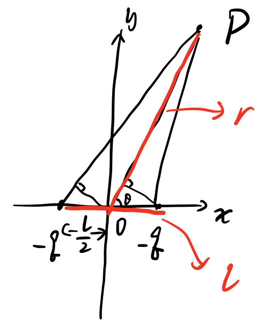
保守场. 绕一圈不做功

2. $W_{ab} = q_0(V_a - V_b)$

3. 电势：

$$V_p = \int \frac{dq}{4\pi\varepsilon_0 r}$$

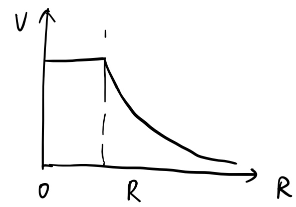
4. 电偶极子电势.



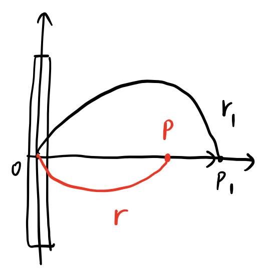
$$V_p = \frac{q}{4\pi\varepsilon_0 r_+} - \frac{q}{4\pi\varepsilon_0 r_-}$$

$$= \cdots = \frac{\vec p\cdot\vec r}{4\pi\varepsilon_0 r^3} = \frac{ql\cos\theta}{4\pi\varepsilon_0 r^3}$$

5. 带电球壳：



6. 无限长带电直导线电势：



★ 取P_1作零势能点

$\vec E = \frac{\lambda}{2\pi\varepsilon_0 r}$

$$V = \int_r^{r_1}\vec E\, d\vec r = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_1}{r}$$

§7-5

1. $E\cos\varphi = -\frac{dv}{dl}$

散度：定义：取 dS 面元，包围体积 dV.
$\vec{A}$ 为矢量场. $\vec{A}\cdot d\vec{S}$ 为通量.
散度 $\operatorname{div} A = \frac{\oint_S \vec{A}\cdot d\vec{S}}{dV}$
易记 $\operatorname{div} A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

→ 取法线方向 n.

$E = -\frac{dv}{dn}$ 即 E 为 P_1 点最大电势空间变化率

记 $\operatorname{grad} v = \frac{dv}{dn}\vec{e}_n$ $\operatorname{div} A = \nabla\cdot\vec{A}$

则 $\vec{E} = -\frac{dv}{dn}\vec{e}_n = -\operatorname{grad} V$

在空间中可写成 $\operatorname{grad} V = \frac{\partial V}{\partial x}\vec{i} + \frac{\partial V}{\partial y}\vec{j} + \frac{\partial V}{\partial z}\vec{k}$

之后 求场强便可转化为求电势分布 → 求偏导

2. 电偶极子的电势分布:

$$V = \frac{px}{4\pi\varepsilon_0 (x^2+y^2)^{\frac{3}{2}}}$$

$$E_x = -\frac{\partial V}{\partial x} = -\frac{p}{4\pi\varepsilon_0}\left[\frac{1}{(x^2+y^2)^{\frac{3}{2}}} - \frac{3x^2}{(x^2+y^2)^{\frac{5}{2}}}\right] = \frac{p(2x^2-y^2)}{4\pi\varepsilon_0 (x^2+y^2)^{\frac{5}{2}}}.$$

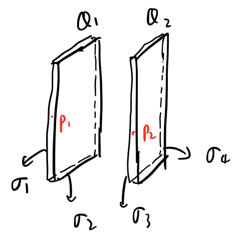
同理. $E_y = -\frac{\partial V}{\partial y} = \frac{3pxy}{4\pi\varepsilon_0 (x^2+y^2)^{\frac{5}{2}}}$

§7-6

1. 导体中无任何电荷定向运动的状态称为静电平衡态
此时导体内任一点电场强度都为0

平衡时，
2. 导体是等势体. 其表面是等势面.
3. 导体表面场强垂直于导体表面.
4. 带电导体处于静电平衡状态时，导体内处处无净电荷.
电荷只能分布在导体的外表面上.
5. 表面场强: $\vec{E} = \frac{\sigma}{\varepsilon_0}\vec{e}_n$

6. 遇到"板"类问题: 考虑面面间关系.



$(\sigma_1+\sigma_2)S = Q_1$
$(\sigma_3+\sigma_4)S = Q_2$

在板内选一点
$\vec{E}_{P_1} = \vec{E}_{P_2} = \vec{0}$

$$\begin{cases} \frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} - \frac{\sigma_3}{2\varepsilon_0} - \frac{\sigma_4}{2\varepsilon_0} = 0 \\ \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0} + \frac{\sigma_3}{2\varepsilon_0} - \frac{\sigma_4}{2\varepsilon_0} = 0 \end{cases}$$

$$\Longrightarrow \begin{cases} \sigma_1 = \sigma_4 \\ \sigma_2 + \sigma_3 = 0 \end{cases}$$

§7-7

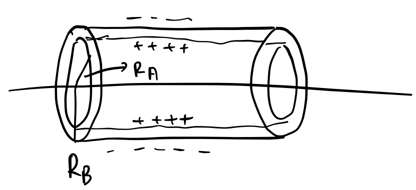
1. 电容: $C = \frac{q}{V}$

电容器 (两极电势 V_1 V_2).

$$C = \frac{q}{V_1 - V_2}.$$

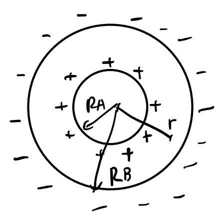
2. 平行板. $C = \frac{\varepsilon_0 S}{d}$

3. 圆柱:



$C = \frac{2\pi\varepsilon_0}{\ln\frac{R_B}{R_A}}$

4.



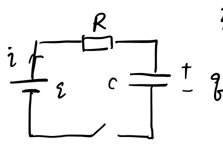
$C = \frac{q}{U_A - U_B} = \frac{4\pi\varepsilon_0 R_A R_B}{R_B - R_A}$

当 $R_B \to \infty$

即有孤立导体球电容

$C = 4\pi\varepsilon_0 R_A$

5. 动态充/放电



充电: 欧姆定律: $\varepsilon = IR + \frac{q}{C}$ 放电: $iR = \frac{q}{C}$

$i = \frac{dq}{dt}$ $i = -\frac{dq}{dt}$

→ $\varepsilon = R\frac{dq}{dt} + \frac{q}{C}$

→ 微分方程

§7-8.

1. 电极化强度/P矢量：

$$\vec{P} = \frac{\sum \vec{p}}{\Delta V}$$ ← 单位体积元中所有分子的电偶极矩矢量和

2. $\sigma' = \vec{P}\cdot\vec{e}_n = P_n$ 介质极化所产生的电荷面密度

为电极化强度沿介质表面外法线分量

3. $$\vec{E} = \vec{E}_0 + \vec{E}'$$

介质内部 二者一般反向

$\vec{E}$：空间中合场强

$\vec{E}_0$：激发外电场所有电荷所产生的场强

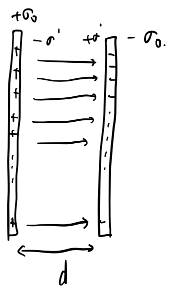
$\vec{E}'$：极化电荷激发场强

4. 介质内：$\vec{P} = \chi_e \varepsilon_0 \vec{E}$.

χ_e：电极化率 是常量

$\vec{E}$：介质内合场强

5. 平板电容器内：



$$\vec{E} = \vec{E}_0 - \vec{E}' = \frac{\sigma_0}{\varepsilon_0} - \frac{\sigma'}{\varepsilon_0}$$

$$E = E_0 - \frac{P}{\varepsilon_0} = E_0 - \chi_e E$$

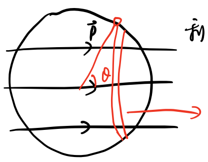
$$\Rightarrow E = \frac{E_0}{1+\chi_e}$$

$$U = Ed = \frac{\sigma_0 d}{\varepsilon_0(1+\chi_e)}$$

$$C = \frac{q}{U} = (1+\chi_e)C_0$$

$$\therefore \boxed{\varepsilon_r = 1+\chi_e}$$

6. 半径为R的均匀电介质球极化：



利用 $\boxed{\sigma' = P\cos\theta}$

取出一条环.

$$dq' = \sigma' 2\pi R^2 \sin\theta\, d\theta$$

$$= P\cdot 2\pi R^2 \sin\theta\cos\theta\, d\theta$$

在球心处激发的电场大小 $dE = \frac{dq'}{4\pi\varepsilon_0 R^2}\cos\theta = \frac{P}{2\varepsilon_0}\cos^2\theta\sin\theta\, d\theta$

↓分量

在球心处激发的总大小 $E' = \int dE' = \int_0^\pi \frac{P}{2\varepsilon_0}\sin\theta\cos^2\theta\, d\theta = \frac{P}{3\varepsilon_0}$

∴ 此时球心 $E = E_0 - \frac{P}{3\varepsilon_0}$，代入 $P = \chi_e\varepsilon_0 E_0$

$$\rightarrow E = \frac{3}{2+\varepsilon_r}E_0$$ ✓

§7-9.

1. 有电介质时，Gauss 定理

$$\oiint_S \vec{E}\cdot d\vec{S} = \frac{1}{\varepsilon_0}\left(\sum q_0 + \sum q'\right)$$

$\sum q_0$：自由电荷　$\sum q'$：极化电荷

定义 $\vec{D} = \varepsilon_0\vec{E} + \vec{P}$（电位移矢量）

则 $\oiint_S \vec{D}\cdot d\vec{S} = q_0$

电通量

代入 $P = \chi_e\varepsilon_0 E$

$$\therefore D = \varepsilon_r\varepsilon_0 E = \varepsilon E$$

§7-10

1. 电容器能量 $W_e = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}C(V_1-V_2)^2 = \frac{1}{2}Q(V_1-V_2)$

$$= \frac{1}{2}\varepsilon E^2 Sd = \frac{1}{2}\varepsilon E^2 V.$$

一般有 $w_e = \frac{1}{2}\vec{D}\cdot\vec{E} = \frac{1}{2}\varepsilon E^2$

w_e：能量密度

2. 带电均匀球体能量：

取球壳中"一层"

$dW = \rho V = \cdots$ ✓

$$W_{总} = \frac{3q^2}{20\pi\varepsilon_0 R}$$