§10-1

1. 满足 $F=-kx$ 的称为简谐运动.

$\rightarrow a=\frac{d^2x}{dt^2}=\frac{F}{m}=-\frac{kx}{m}$ let $\frac{k}{m}=w^2$

$\rightarrow \frac{d^2x}{dt^2}+w^2x=0$

$\rightarrow$
$$\begin{cases} x=A\cos(wt+\phi_0) \\ v=\frac{dx}{dt}=-\underline{wA}\sin(wt+\phi_0)=-v_m\sin(wt+\phi_0) \\ w=\frac{dv}{dt}=-w^2A\cos(wt+\phi_0)=-a_m\sin(wt+\phi_0) \end{cases}$$

let $t=0$: $\begin{cases} x_0=A\cos\phi_0 \\ v_0=-wA\sin\phi_0 \end{cases} \Rightarrow \begin{cases} A=\sqrt{x_0^2+\frac{v_0^2}{w^2}} \\ \phi_0=\arctan\left(-\frac{v_0}{wx_0}\right) \end{cases}$

2. $T=\frac{2\pi}{w}$, $\nu=\frac{1}{T} \Rightarrow w=\frac{2\pi}{T}=2\pi\nu$

3. 对弹簧振子, $w=\sqrt{\frac{k}{m}}$.

$\therefore T=2\pi\sqrt{\frac{m}{k}}$, $\nu=\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

4. 设有 $\begin{cases} x_1=A_1\cos(wt+\phi_{01}) \\ x_2=A_2\cos(wt+\phi_{02}) \end{cases}$

$\Delta\phi=(wt+\phi_{02})-(wt+\phi_{01})$
$=\phi_{02}-\phi_{01}$

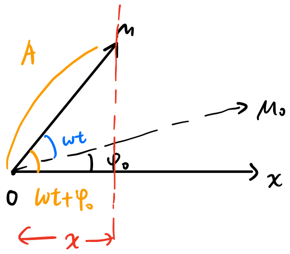
当 $\Delta\phi$ 为 0 或 2π 整数倍时 → "同相"

为 π 或 π 奇数倍 → "反相"

$\Delta\phi=\phi_{02}-\phi_{01}>0$ → "振动 2 超前振动 1"

并有 $\Delta t=\frac{\Delta\phi}{w}$

5. 旋转矢量:

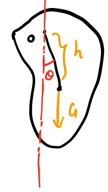


$x=A\cos(wt+\phi_0)$

相位角 ⇔ 旋转矢量之夹角

6. 单摆: $T=\frac{2\pi}{w}=2\pi\sqrt{\frac{L}{g}}$

复摆: $T=\frac{2\pi}{w}=2\pi\sqrt{\frac{J}{mgh}}$



7. 能量: $\begin{cases} E_k=\frac{1}{2}mv^2=\frac{1}{2}mw^2A^2\sin^2(wt+\phi_0) \\ E_p=\frac{1}{2}kx^2=\frac{1}{2}kA^2\cos^2(wt+\phi_0) \end{cases}$

$\rightarrow E=\frac{1}{2}kA^2$

§10-2

1. 阻尼振动: 除线性回复力 F 之外, 受到一个阻力

$F_f=-\gamma v=-\gamma\frac{dx}{dt}$

$\rightarrow m\frac{d^2x}{dt^2}=-kx-\gamma\frac{dx}{dt}$

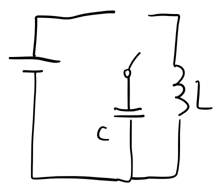
$\frac{d^2x}{dt^2}+2\delta\frac{dx}{dt}+w_0^2x=0$

$\therefore x=A_0e^{-\delta t}\cos(w't+\phi_0')$,

其中 $w'=\sqrt{w_0^2-\delta^2}=\sqrt{\frac{k}{m}-\frac{\gamma^2}{4m^2}}$

§10-4

1.



电磁振荡:

$-L\frac{di}{dt}=\frac{q}{C}$, $i=\frac{dq}{dt}$

$\Rightarrow \frac{d^2q}{dt^2}=-\frac{1}{LC}q$

设 $w^2=\frac{1}{LC}$. 代入解得 $\Rightarrow w=\frac{1}{\sqrt{LC}}$

$q=Q_0\cos(wt+\phi_0)$

电磁振荡频率: $\nu=\frac{w}{2\pi}=\frac{1}{2\pi\sqrt{LC}}$. $T=2\pi\sqrt{LC}$

最大电荷: Q_0

q 对 t 求导得 i:

$i=-\underline{wQ_0}\sin(wt+\phi_0)=I_0\cos(wt+\phi_0+\frac{\pi}{2})$

$I_m=wQ_0$

§10-5.

1. 同一直线上两个同频率的谐振动合成:

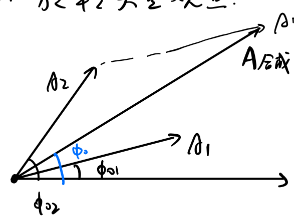
$$\begin{cases} x_1 = A_1\cos(\omega t+\phi_{01}) \\ x_2 = A_2\cos(\omega t+\phi_{02}) \end{cases} \Longrightarrow x' = A\cos(\omega t+\phi_0)$$

其中:

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_{01}-\phi_{02})}$$

$$\tan\phi_0 = \frac{A_1\sin\phi_{01} + A_2\sin\phi_{02}}{A_1\cos\phi_{01} + A_2\cos\phi_{02}}$$

2. 旋转矢量观点:



3. 同一直线上两个不同频率的谐运动合成 "拍"

$$\begin{cases} x_1 = A_1\cos(\omega_1 t+\phi_{01}) \\ x_2 = A_2\cos(\omega_2 t+\phi_{02}) \end{cases}$$

$$\rightarrow x = x_1 + x_2 = \underbrace{\boxed{2A\cos\left(\frac{\omega_1-\omega_2}{2}t\right)}}_{A'}\cos\left(\underbrace{\frac{\omega_1+\omega_2}{2}}_{\omega'}t + \phi_0\right)$$

$$★\begin{cases} \omega' = \frac{\omega_1+\omega_2}{2} \\ \nu' = \frac{1}{T} = \left|\frac{\omega_1-\omega_2}{2\pi}\right| = |\nu_1-\nu_2| \end{cases}$$