Problem Chosen	2021	Team Control Number
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	Summary Sheet	

Summary

The higher education system of a country affects the basic literacy and knowledge level of its citizens to a large extent. At the same time, the higher education system also makes important contributions to the output of talents and the improvement of the scientific research level of a country. Therefore, it is very important for a country to have a healthy and sustainable higher education system. In this paper, we mainly evaluate the status of the higher education system in various countries, provide policy suggestions for improving the status of the higher education system in China. Then we analyze and explain the difficulty of policy implementation.

In the first part, we use a suite of 3 models, the TOPSIS model, linear weighted model with base period data, DEA model to evaluate the higher education systems in China, the United States, Germany, the United Kingdom and France. We assess a country's higher education system from 5 aspects including fiscal education investment, education access, education equality, education quality and research level. We use analytic hierarchy process (AHP) to determine the weight of the 5 aspects.

In the second part, we choose Chinese higher education system for improvement. By making a few reasonable assumptions, we build a nonlinear programming model. Finding out the optimal solution, we are able to give a healthier and sustainable future vision of Chinese higher education. Our models in the first part confirm the superiority of the future vision.

With the help of the result from nonlinear programming model and an economic model, we give a few policies and policy implementing timeline. Our policies include the Chinese government should focus on increasing the number of teachers in Chinese higher education system to increase the teacher-student ratio and improve the level of teaching in higher education. We use DEA-Malmquist index analysis to test the effectiveness of our proposal.

Finally, we explain some difficulties in implementing our policies in the real world from the perspective of students, teachers and the government respectively.

Keywords: TOPSIS, AHP, DEA, nonlinear programming, total factor product

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1.Introduction

1.1 Background

Many countries attract large numbers of foreign students each year to attend local higher education, and the higher education system is an important factor in a country's efforts to equip its citizens with further education beyond the necessary primary and secondary education.The education system of each country has both advantages and disadvantages. In the context of the epidemic, we need to reflect on the disadvantages of the existing education system. Our goal is to evaluate the existing education model of each country and improve it so as to provide a healthy and sustainable state for the education system.

1.2The Problem Repeat

As mentioned in the question, we need to design a model suitable for the evaluation of the higher education system of various countries, and select a country with room for improvement based on the evaluation results of the model, so as to help its higher education system find a more sustainable and healthy state.At the same time, we need to develop policies and timelines to help transform the existing higher education systems in our selected countries to a state that we believe is healthier and more sustainable.

We need to design another model to evaluate the effectiveness of our proposed policies and schedules, as well as to explore the real-world implications of the plan.

2. Assumption and Symbol

2.1 Assumption

- In the TOPSIS model, the indicators for evaluating the status of higher education can be divided into five categories: educational input, educational access, educational equity, educational quality and research level. Their respective scoring weights can be calculated by analytic hierarchy process (AHP).
- Analytic Hierarchy Process (AHP) is divided into target layer, criterion layer and scheme layer. We assume that the criterion layer for evaluating the health of higher education system can be divided into five dimensions: educational input, educational access, educational equity, educational quality and research level.

- In order to quantify the five evaluation indexes of topsis model, we select (x_1) on behalf of the national higher education fiscal spending education enrolment situation, the national higher education enrolment (x_2) on behalf of the education for men and women of national higher education scale (x_3) on behalf of education fairness, national higher education student-faculty ratios (x_4) on behalf of the education quality and the number of papers published every year from all over the world (x_5) represent the research levels.

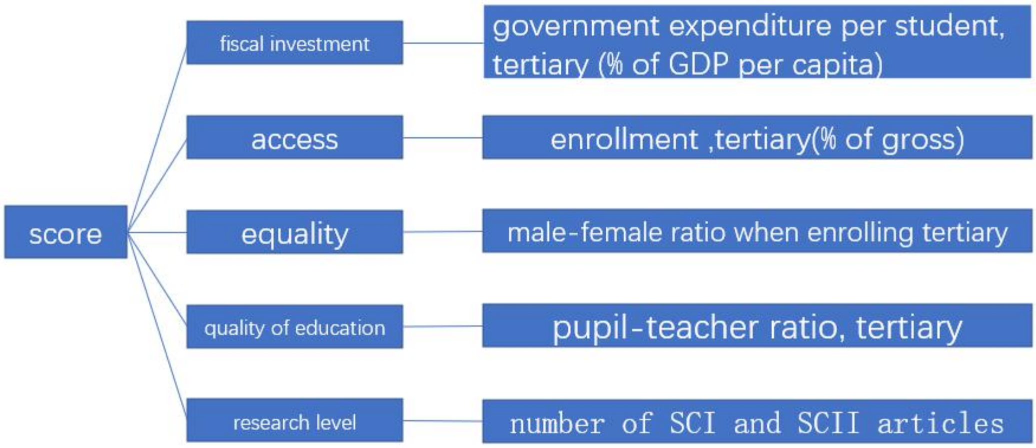

Figure 1:TOPSIS

2.2 Symbol

Table 1:symbol

symbol	meaning
x_1	Government expenditure per student, tertiary (% of GDP per capita)
x_2	enrolment ,tertiary(% of gross)
x_3	Female-male ratio when enrolling, tertiary
x_4	pupil-teacher ratio, tertiary
x_5	Number of SCI and SCII articles

3 Higher Education Evaluation System

3.1Topsis Model

3.1.1 Brief Description of Model

TOPSIS, short for Technique for Order Preference by Similarity to Ideal

Solution,is a evaluation method which can make full use of the original data. When evaluating the health of higher education, the TOPSIS evaluates N subjects by assuming that they are compared with each other (there are m=5 indicators), which are scored by TOPSIS in the following four steps.

（1） Of these five indicators, there may be some that are bigger is better, some that are smaller is better, some that are closer to a certain value is better.We first do a simple mathematical transformation to make them all bigger is better. In the five indicators we selected, we believe that the ratio of males to females (x_3) is closer to 1, the better. Therefore, the following transformation can be made:

$$x_3^{'} = 1 - |x_3 - 1|$$

（2） Eliminate dimensional effects. For the matrix after the transformation of the previous step:

$$\begin{pmatrix} x_{11} & \cdots & x_{15} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{n5} \end{pmatrix}$$

Its normalized matrix is denoted by Z, where

$$z_{ij} = x_{ij} / \sqrt{\sum_{i=1}^{n} x_{ij}^2}$$

（3） Define:

$$Z_{max} = (\max\{z_{11},z_{21},\ldots z_{n1}\},\ldots,\max\{z_{15},\ldots,z_{n5}\})$$
$$Z_{min} = (\min\{z_{11},z_{21},\ldots z_{n1}\},\ldots,\min\{z_{15},\ldots,z_{n5}\})$$

Then the distance between the ith object and the maximum and minimum value is respectively:

$$D_i^+ = \sqrt{\sum_{j=1}^{5} w_j \left(Z_{max_j} - z_{ij}\right)^2}$$

$$D_i^- = \sqrt{\sum_{j=1}^{5} w_j \left(Z_{min_j} - z_{ij}\right)^2}$$

Where W is the weight.We choose Analytic Hierarchy Process (AHP) to calculate the weight in TOPSIS.

（4） The final score is

$$S_i = \frac{D_i^-}{D_i^+ + D_i^-}$$

We believe that the higher the score, the healthier the higher education.

3.1.2Weight:Analytic Hierarchy Process (AHP)

By using analytic hierarchy process (AHP) modeling, the target problem is generally divided into three levels: the target level, the criterion level and the scheme level.However, we need to use AHP to calculate the weights of five evaluation indexes in TOPSIS model, and we only divide the target problem into the target layer and the criterion layer.

3.1.2.1 Target Stratification

（1） The goal layer, with only one element, is the predetermined goal or desired outcome of the actual problem, here "the health of the education system".

（2） Criteria layer. In order to achieve the intermediate links involved in the goal, the criterion layer can be composed of several levels, including the criteria to be considered, sub-criteria, etc., we only consider the case of one criterion (hypothesis).Here, the criteria are "education input", "education access", "education equity", "education quality" and "research level".

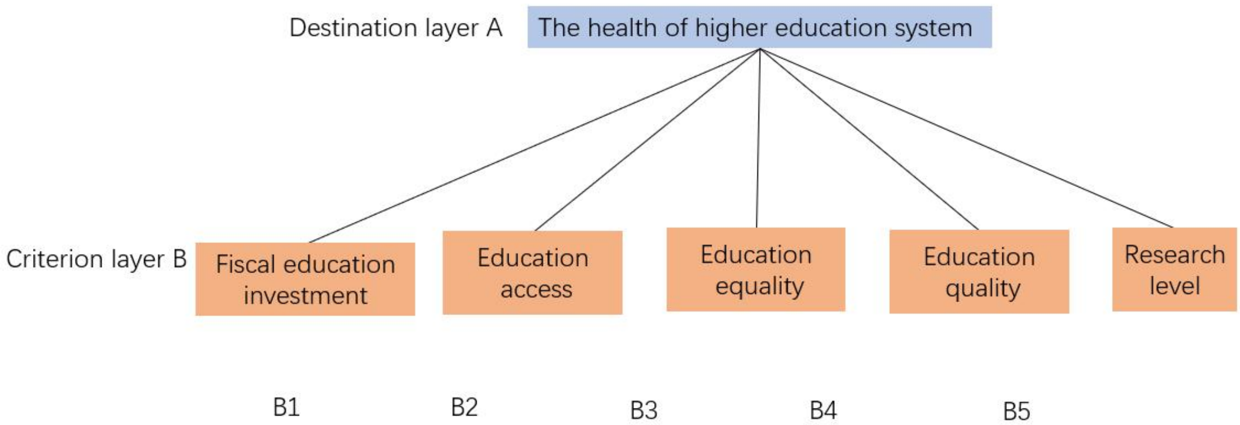

Figure 2:AHP

3.1.2.2 Construct Judgment Matrix of Criterion Layer

AHP is largely influenced by human subjective factors. In order to determine the influence degree of each abstract factor and calculate the index weight in TOPSIS, we need to quantify the qualitative analysis. First, we construct the judgment matrix of the criterion layer.Here we use the scale 1-9 to construct the judgment matrix.

Table 2:scale of AHP

scale	meaning
1	It means that the two factors are of equal importance compared to each other
3	It indicates that the former is slightly more important than the latter
5	It indicates the former is obviously more important than the latter

7	It indicates that the former is much more important than the latter
9	It indicates that the former is extremely more important than the latter
2,4,6,8	Represents the median value of the above adjacent judgments
1,1/2...1/9	If the importance ratio of factor i to factor j is a_{ij}, then the importance ratio of factor j to factor i is $\frac{1}{a_{ij}}$.

The comparison judgment matrix of criterion layer constructed by 1-9 scale table is as follows:

Table 3:comparison judgment matrix

A	B1	B2	B3	B4	B5
B1	1	1/5	1/3	1/7	1/5
B2	5	1	2	1/2	1
B3	3	1/2	1	1/3	1/2
B4	7	2	3	1	2
B5	5	1	2	1/2	1

3.1.2.3 Consistency test of judgment matrix

Although the construction of comparison judgment matrix can reduce the interference of other factors and objectively reflect the difference of the influence of a bunch of factors, it may contain a certain degree of inconsistency when integrating all the comparison results. If the comparison results are completely consistent, then the elements of the criterion comparison judgment matrix should also satisfy:

$$a_{ij}a_{jk}=,a_{ik}\forall i,j,k = 1，2，3，4，5$$

The steps of consistency test of judgment matrix are as follows:

（1） The consistency index CI was calculated: CI=$\frac{\mu_{max}-n}{n-1}$, where $\mu_{max} = 5.28672$, calculated by MATLAB,is the maximum eigenvalue of the judgment matrix of the criterion layer.n=5 is the total number of factors of the criterion layer.

（2） The consistency ratio CR is calculated in this way: CR $=\frac{CI}{RI}$, where RI is the average random consistency index. When n=5, the corresponding RI is 1.12, so we find out CR =0.064.

（3） We generally believe that the consistency of the judgment matrix is acceptable when CR<0.10. Here CR=0.064 meets the condition of our consistency. Therefore, we believe that the judgment matrix of the criterion layer we set has some rationality.

When the consistency test of criterion layer judgment matrix passes, we use linear algebra knowledge and MATLAB program to find out all the features of criterion judgment matrix, and find out the eigenvector W corresponding to the maximum eigenvalueμ_{max}, We standardize W and get:

$$W' = [0.0459,0.2210,0.1216,0.3904,0.2210]^T$$

In our opinion, the weight of educational input, educational access, educational equity, educational quality and research level obtained by AHP in the TOPSIS model is given by W'.

3.1.3 Model Application

3.1.3.1 Country selection and data collection

We selected five countries -- China, the United States, the United Kingdom, Germany and France -- as evaluation objects. Firstly, we collected data, and the data obtained are shown in the table below. All the data are downloaded from the official site of World Bank.

Table 4:data

country	investment	access	Equality	quality	research
china	55.245	50.60%	1.17811	0.051313866	528263.3
USA	20.705	88.30%	1.26827	0.083169213	422807.7
Britain	38.181	61.38%	1.26931	0.046972063	97680.9
Germany	34.55	70.34%	1.03077	0.131685682	104396.1
France	33.677	67.62%	1.19771	0.131685682	66352.18

3.1.3.2 Model results

Table 5:TOPSIS results

country	score
China	0.5542
USA	0.6073
Britain	0.1237
Germany	0.4677
France	0.4382

3.2 Linear Weighted Model with Base Period Data

3.2.1 Model Description

TOPSIS can make full use of data information to score, but there is a disadvantage: since each index of each country needs to be compared with the maximum and minimum value of the index of all selected countries to give the score, the score of each country will be different depending on the other countries selected.To overcome this shortage, we can build another model, comparing the data of a country (no matter which year the data is of) with those of the USA in table 3.3.1-1, to give a score.

（1） Get rid of the dimensions.Suppose the index of the selected country is :[x_1,x_2,x_3,x_4,x_5], the index of the United States in the base period (in table 3.3.1-1) is :[x_{10},x_{20},x_{30},x_{40},x_{50}], and the index of the country after dimensionality removal is [z_1,z_2,z_3,z_4,z_5], where:

$$z_i = \frac{x_i}{x_{i0}} \quad ,i = 1,2,3,4,5$$

（2） Determine the weights. We take the weight obtained by AHP,$[0.0459,0.2210,0.1216,0.3904,0.2210]^T$,as the weight of this model.

（3） Give score. The linear weighted form of the score is:

$$score = 0.0459z_1 + 0.221z_2 + 0.1216z_3 + 0.3904z_4 + 0.221z_5$$

3.2.2 Model Application

Also score the data in Table 3.3.1.The results are as follows:

Table 6:Linear weighted model results

country	score
china	0.90
USA	1.00
Britain	0.63
Germany	1.09
France	1.03

3.3 DEA Model

3.3.1 Model Description

In order to judge the effectiveness of higher education systems in different countries, DEA-CCR model is adopted here.DEA is a statistical model that evaluates the relative efficiency of separable entities (what we call "decision making units").DEA efficiency is defined as the ratio of input to output, and its calculation formula is as follows:

$$DEA_k = \frac{\sum_{i=1}^{s} u_i y_{ik}}{\sum_{j=1}^{m} v_j x_{jk}},\text{K} = 1, 2,\ldots 5$$

There are s output indicators, m input indicators in the formula above. we selected five indicators in the TOPSIS model "fiscal education investment", "education access", "education equality ", "quality " and "the level of research". We selected "level of research", "equality" and "access" as the output variable, namely s = 3, and determined the " investment" and "quality" as the input variables, namely m = 2. In our model, the decision unit (DMU) is considered to be the education system of the five selected countries, that is, the range of k is [1,5].

3.3.2 Model Application

After the decision unit (DMU), output variables and input variables are determined, DEAP2.1 is used for calculation. The calculation results are shown in the table below:

Table 7:DEA model results

Country	crste	scale	rs
China	0.771	0.771	drs
USA	1.000	1.000	crs
Britain	0.567	0.567	drs
Germany	1.000	1.000	crs
France	0.471	0.766	irs

Table 8:Tips for DEA results

symbol	Interpretation
crste	technical efficiency under constant return to scale assumption
scale	Scale efficiency
rs	Returns to scale
drs	Decreasing returns to scale
crs	Constant returns to scale
irs	Increasing returns to scale

Firm1-5 represents the decision units we selected, and the order is China, the United States, the United Kingdom, Germany, and France. CRSTE represents the DEA technical efficiency calculated by us, and its maximum value is 1. It is found that the DEA efficiency of both the United States and Germany is 1, indicating that the efficiency of input to the output of the higher education system in the United States and Germany is relatively high. The DEA efficiency of China, the United Kingdom, and France are 0.771, 0.567, and 0.471 respectively, indicating that the higher education systems of these three countries still have room for improvement.

The DEA model not only helps us to analyze the technical efficiency of each country's higher education system but also helps us to analyze the scale effect of each country's higher education system. 'drs' stands for diminishing returns to scale, while 'irs' stands for increasing returns to scale. According to the analysis results of our model, the higher education system in the United States and Germany is in a state of constant returns to scale. Although DEA efficiency is not high in Germany, it is in a state of increasing returns to scale. Increasing input will increase output correspondingly. However, the higher education system in China and the UK is in the state of diminishing returns to scale, which needs further improvement and perfection to make the output of the higher education system more effective.

4. Improvement of Higher Education System

4.1 Choosing a country

We choose china to propose a healthier and sustainable higher education system.

4.2 Nonlinear programming model prediction

4.2.1Aussumptions and Model Description

According to the AHP model and the TOPSIS model above, the relationship between evaluation scores and evaluation indexes was obtained,$y = 0.0459x_1 + 0.2210x_2 + 0.1216x_3 + 0.3904x_4 + 0.2210x_5$, where x_1 reflects per capita educational input and can be obtained from Government expenditure per student, tertiary (% of GDP per capita). x_2 represents the proportion of students who have obtained an education, which can be obtained from the enrolment rate. x_3 represents the fairness of education, which can be obtained from the number of female students to that of male students in higher education system. x_4 refers to the quality of education, is decided by the Pupil-teacher ratio, tertiary. We use the teacher-pupil ratio to reflect the teaching quality of the education system, because when a teacher teaches a large number of students, it is difficult for him to pay attention to the learning of each student, and vice versa. x_5 indicates the research strength of the higher education system, which is reflected by the number of papers of each country available on SCI and SCII.

Next, we form a series of hypothesis, to establish a relatively ideal mathematical model, to propose a reasonable and relatively feasible education system construction plan, from above we have grades of five national education systems as you can see, China, with the world's largest population, and score not good enough. So, it is necessary and significant to provide a feasible solution for the construction of China's education system.

(1) Assumption 1: the educational expenditure provided by the national government in a certain period grows at a rate of 6% and the country's school-age population is relatively stable, so we assume that the relationship between the product of per capita educational investment and enrolment rate and time is :

$$x_1x_2 = k_1 \times (1+0.06)^t$$

($k_1$is the product of the current per capita educational investment and enrolment rate, τ is time, and the unit of t is year)

(2) Assumption 2: The salary of higher education teachers does not change in a certain period, that is, combining with Hypothesis 1, the total number of teachers does not change. That is,

$$x_2x_4 = k_2 \times (1+0.06)^t$$

($k_2$is the product of current enrolment rate and teacher-pupil ratio)

(3) Assumption 3: Since a proper teacher-student ratio can maximize the quality of teaching quality, teacher-pupil ratio of a higher education should be kept within a certain range, i.e.

$$0.05 \le x_4 \le 0.2$$

(4) Assumption 4:assume that student-faculty ratios (education) and enrolment are into positive correlation with level of research, and the relationship are as follows:

$$x_5 = k_3[\frac{1}{1+e^{-x_2}} + \frac{1}{1+e^{-x_4}}]$$

(5) Assumption 5: Our goal is to maximize the health index of the national education system after 20 years, that is, t=20.

(6) Assumption 6: The enrolment rate after 20 years should not be lower than the current enrolment rate, otherwise it will cause the problem of educational inequality, so we require

$$x_2 \ge k_4$$

(k_4 is the current enrolment rate).

From the above assumptions, it can be concluded that to find a optimal higher education program, the essence is to solve the planning problem:

$$Max \quad y = 0.0459x_1 + 0.2210x_2 + 0.1216x_3 + 0.3904x_4 + 0.2210x_5$$

$$\begin{cases} x_1x_2 = k_1 \times (1+0.06)^t \\ x_2x_4 = k_2 \times (1+0.06)^t \\ 0.05 \le x_4 \le 0.2 \\ x_5 = k_3[\frac{1}{1+e^{-x_2}} + \frac{1}{1+e^{-x_4}}] \\ t = 20 \\ x_2 \ge k_4 \end{cases}$$

4.2.2Proposing Future Vision

MATLAB is used to solve the above optimization problems, and the results are as follows:

Table 9:Nonlinear programming results

x_1	x_2	x_3	x_4	x_5
177.1782	0.5060	1.0000	0.1646	543204.6728

Table 10:A healthier and sustainable vision of Chinese higher education in 20 years

Government expenditure per student, tertiary (% of GDP per capita)	177.1782
enrolment ,tertiary(% of gross)	50.60
Female-male ratio when enrolling, tertiary	1.0000
pupil-teacher ratio, tertiary	6.0753
Number of SCI and SCII articles	543204.6728

4.2.3Assessing the Future Vision

We use the TOPSIS model to assess our future higher education vision , and compare with the present score. The result shows the future vison has a higher score.

Table 11:Score of future vision

Future vision	Present
0.8679	0.5542

4.3 Targeted Policies and Implication Timeline

4.3.1Proposing Policies from Nonlinear Programming Results

Table 12:comparison of the future and now

	20 years from now	now
Government expenditure per student, tertiary (% of GDP per capita)	177.18	55.25
enrolment ,tertiary(% of gross)	50.60	50.60
Female-male ratio when enrolling, tertiary	1.00	1.18
pupil-teacher ratio, tertiary	6.08	19.49
Number of SCI and SCII articles	543204.6728	528263.30

By comparing the current and future Chinese higher education system, we find that some indicators have changed significantly, while others have not. Therefore, we can make the following policy recommendations:

(1) Policy 1:The Chinese government should focus on increasing the number of

teachers in Chinese higher education system to increase the teacher-student ratio and improve the level of teaching in higher education.

(2) Policy 2: The Chinese government should optimize the structure of majors, so that the ratio of male to female in higher education could be as balanced as possible.

4.3.2 Proposing Policies from an Economic View

Policy: Chinese government should focus on expanding the scale of teachers and researchers in higher education and introducing high-level researchers.(similar to policy 1 in 4.3.1)

The following shows our analysis.

Let's first assume that the government balances its payments before the new policy is implemented. We have to be aware that if the government spends more on higher education, then it can only spend less on other things, or the government will have a fiscal deficit. To understand the government's trade-off, consider the following simple economy.

In this economy, there are three parts, the government, the manufacturing enterprises and the research universities. Manufacturing enterprises play the role of producing, while universities play the role of developing new technologies and training people.

Assume that the production function of the manufacturing enterprises is a C-D function:

$$Y = AK^{\wedge\alpha}L^{1-\alpha}$$

Y represents output, K represents the capital stock, and L represents the quantity of labor. And most importantly, A is the technological level of the economy.

A technique called GDP growth rate accounting tells us the following:

$$\frac{\Delta Y}{Y} = \alpha\frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L} + \frac{\Delta A}{A}$$

That is, economic growth depends on an increase in the capital stock, an increase in the labor force, and technological progress. Known as Solow's residual or total factor product, $\frac{\Delta A}{A}$ it is often used in real world statistics to measure a country's technological level.

At the same time, universities also have their own production function, which is assumed to be of the following form:

$$A = f(gf,t,s)$$

A represents technology, gf represents government funding, t represents the number of teachers and researchers, and s represents the number of students. The function means in universities, through the use of government funding, faculty, researchers and students drive the technological progress of the economy. There are two reasons why s is also an explaining variable of A. For one thing, many students who are still in the stage of higher education will also engage in research activities. The other is that by receiving higher education, students have improved their own labor productivity, and when they put themselves to work, they will bring more output with greater efficiency. An economic theory called human capital theory underpins this. The theory holds that education can directly improve people's labor productivity. Education improves cognitive abilities and encourages individuals to redistribute their resource endowments (time, assets, money) to maximize labor productivity for higher wages.

Suppose one day the government runs up a debt pile in order to spend more on education. As the government increases its investment in higher education, it hopes that universities will develop new technologies, train more talents and raise their technical standards. One of the many benefits is that the economy will produce more. And, for the government, tax revenues will be guaranteed, allowing it to build more without having to worry about defaulting on its debts. So the government will be very concerned about how the investment in higher education can better drive technological progress.

Due to the form of university production function, the government can choose to increase the number of teachers and researchers (t), or expand the enrolment scale (s), or invest in equipment, establish research funds and other measures that can be included in the variable gf. However, these three choices have different promoting effects on total factor product. In order to select the better one, we can make a comparison through grey correlation analysis.

Grey correlation analysis can judge whether the relationship is close through the similarity degree of geometric shapes of sequence curves. The data symbol meanings are shown in the following table.

Table 13:symbols

Symbol	Interpretation
y_j	Total factor product of China($\frac{\Delta A}{A}$)
x_{1j}	Chinese government expenditure in education
x_{2j}	The number of teachers and researchers in Chinese higher education
x_{3j}	Cumulative number of students who graduated from Chinese higher education since 2001

j= 1, 2,...15, Meaning the year of 2001,2002... , 2015.

Data source: Total factor product of China are form reference[1].Another Data are form reference[2].

We first eliminate the dimensional effect. After taht, we draw a statistical chart to preliminarily judge the correlation of the trend of each index.

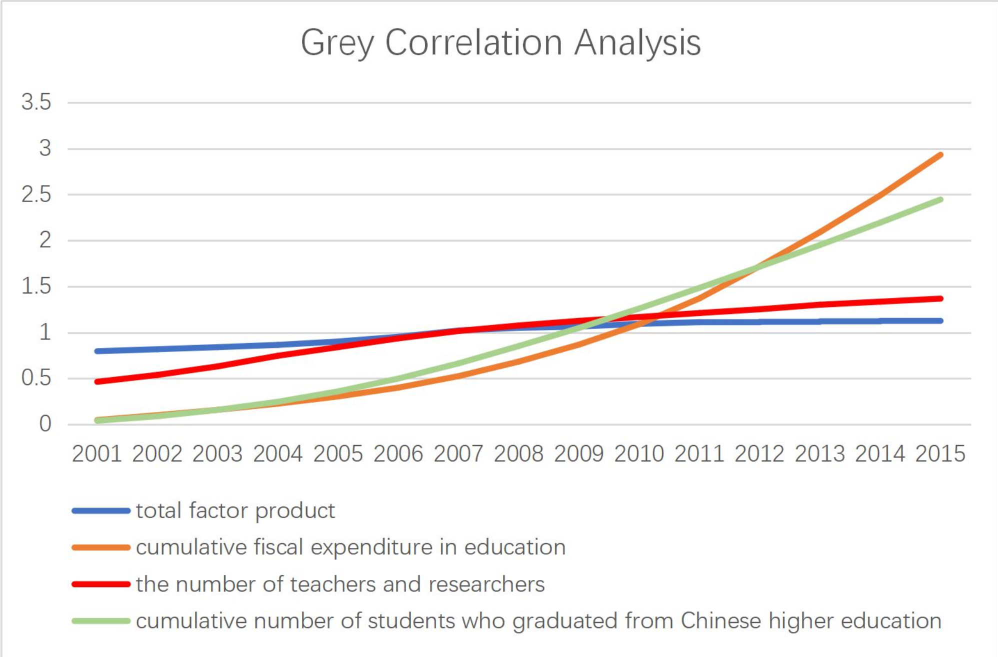

Figure 3:Grey correlation analysis

The above chart shows that the four indicators we selected have similar trends from 2001 to 2015, and there may be a correlation.

Determine parent sequence and subsequence. Since we want to get the promotion effect of fiscal expenditure, the number of teachers and researchers, and the number of graduates on total factor productivity, we choose the total factor product from 2001 to 2015 as the parent series, and the other three as the sub-series.

Then we can calculate the grey correlation coefficients of each sub-sequence and parent sequence .The correlation coefficient between the parent sequence and the ith subsequence in the jth year is defined as:

$$\gamma(y_j,x_{ij}) = \frac{a+\rho b}{|y_j - x_{ij}| + \rho b}$$

,in which

$$a = \min_i \min_j |y_j - x_{ij}|$$

$$b = \max_i \max_j |y_j - x_{ij}|$$

$$\rho = 0.5$$

We define $(n = 1,\ldots,5)$:

$$\gamma(y,x_i) = \frac{1}{n}\sum_{j=1}^{n} \gamma(y_j,x_{ij})$$

as the grey correlation between y and x_i. Fit in our data and the results are shown in the table below.

Table 14:Grey correlation analysis result

Symbol	Grey correlation with total factor product
x_{1j}	2.0416
x_{2j}	3.5657
x_{3j}	2.7174

We see that the number of teachers and researchers has the highest grey correlation with TFP. Therefore, from the perspective of the Chinese government, in order to maximize the role of higher education in improving social labor productivity, we should focus on expanding the scale of teachers and researchers in higher education and introducing high-level researchers.

4.3.3Timeline

Under the assumption of nonlinear programming model, the Chinese government should increase its fiscal input in higher education by 6% a year. To make a timeline for the government, we let the time parameter t in nonlinear programming model be 5,10,15,thus find out the goal of every 5 years for the government to reach the ideal system after 20 years.

Table 15:Goal of every 5 years

t/year	x_1	x_2	x_3	x_4	x_5
5	73.9302	0.5060	1.0000	0.0687	530555.3974
10	98.9354	0.5060	1.0000	0.0919	533621.9876
15	132.3979	0.5060	1.0000	0.1230	537723.4585
20	177.1782	0.5060	1.000	0.1646	543204.6728

4.4 Testing Policy Efficiency

For the prediction results of our "improved model based on nonlinear programming", we used DEA-Malmquist exponential analysis method to test the effectiveness of our improved system. The Malmquist index is an approach to measure the productivity change, also known as the "efficiency change index". The efficiency change index is the product of comprehensive efficiency change and pure technical change, its formula is as follows:

$$M_0(x^t,y^t,x^{t+1},y^{t+1}) = \frac{D_0^{t+1}(x^{t+1},y^{t+1})}{D_0^t(x^t,y^t)} \cdot [\frac{D_0^t(x^{t+1},y^{t+1})}{D_0^{t+1}(x^{t+1},y^{t+1})} \cdot \frac{D_0^t(x^t,y^t)}{D_0^{t+1}(x^t,y^t)}]^{1/2}$$

In the formula above, $M_0(x^t,y^t,x^{t+1},y^{t+1})$represents the efficiency change index, $\frac{D_0^{t+1}(x^{t+1},y^{t+1})}{D_0^t(x^t,y^t)}$ represents the comprehensive efficiency change, and $[\frac{D_0^t(x^{t+1},y^{t+1})}{D_0^{t+1}(x^{t+1},y^{t+1})} \cdot \frac{D_0^t(x^t,y^t)}{D_0^{t+1}(x^t,y^t)}]^{1/2}$ represents the technical change. The comprehensive efficiency change represents the realization degree of the Chinese higher education system to the optimal frontier from T period to T +1 period. The value greater than 1 indicates the improvement of comprehensive efficiency, equal to 1 indicates the unchanged comprehensive efficiency, and less than 1 indicates the decrease of comprehensive efficiency. The technological change measures the movement of technological boundary from T to T +1 period. If the value is greater than 1, it means technological progress; if equal to 1, it means technological unchanged; and if less than 1, it means technological regression.

We regard the prediction result of our "improved model based on nonlinear programming" as the data of T +1 period, that is, the state we recommend that China's higher education system achieve in the future, and the current state of China's higher education system as the data of T period. We use stata to analyze the DEA-Malmquist index model, the results are as follows:

Table 16:DEA-Malmquist result

Row	Newcou-y	Pdwise	TFPCH	TECH	TECCH
2	china	2020-2021	1.0142	1.0000	1.0142

The five indicators of China's current higher education system are recorded as the state in 2020, and the five indicators of China's higher education system predicted by our nonlinear programming model (Table 9) are recorded as the state in 2021. From the figure above, we can learn that TFPCH represents efficiency change index, TFCH represents comprehensive efficiency change, TECCH represents technological change. Since TFPCH>1,TECCH >1, it can be seen that the proposed education system is more efficient and progressive than the existing Chinese higher education system.

5.Real World Impacts

Here we intend to discuss the probable impact of our proposed policies to increase the number of teachers and researchers in the higher education system

During the implementation process:

Students: In the short term, some of the cost of hiring teachers may be shared with students, leading to an increase in economic pressure on students' families. However, the improvement of teaching quality will benefit students in the long term.

Teachers: An increase in the teacher-pupil ratio will appropriately reduce the teaching pressure of individual teachers and enable them to focus more on the academic performance of students. At the same time, the increase in the number of teachers can increase the competition among teachers and promote them to improve their teaching levels.

Schools: The improvement of education quality makes it easier for schools to attract excellent students. The technological innovation of schools will more easily integrate with the surrounding industries, and the willingness of the society to invest in the school will also greatly increase.

The nation: The employment of the increasing number of teachers will lie hard upon the finance of the government, in the meantime, provide a large number of jobs for the society and reduce social unemployment rate. The improvement of higher education quality will improve the average quality of the social labor force, which is conducive to the long-term development of the national economy and improve the ability of the society to transform advances of science and technology into productive forces.

After the implementation:

Students: The improvement of teaching quality will improve students' abilities, make them more competitive, and the social and international reputation of the school they attend will increase their return on education, and increase their sense of honor and achievement in education.

Teachers: The larger number of teachers will transform into a great politic power to make advice on the improvement of education, politics, society. They can also struggle for the right of staffs of the higher education system in a larger number.

Schools: Not only higher prestige can be obtained in the society, but also employers in the international community will give a better evaluation of the schools, and students' sense of identity with the school will be strengthen.

The nation: Help build a healthier and better education system in which relevant industries and corresponding schools can work together to promote the development and upgrading of national industries, enhance innovation in science and technology, effectively improve the quality of the workforce, and thus enhance the vitality and sustainability of economic development.

There also will be plenty of difficulties in the implementation process, such as:

1.The fiscal investment from the government is require to increase by 6% per year, which may bring pressure on the finance of the government.

2.The cost student pay for education will increase, the students' family may not support the policy out of the short-term interest, some students from poor family even can't afford higher education.

3.The recruitment of additional teacher may lower the hiring standard of high school teacher, which may even cause the temporary descent of teaching quality.

4.A policy raising the number of teachers may not be support by some taxpayers, if it doesn't raise the enrolment rate correspondingly.

5.the enhancement of quality of higher education may widen the gap between highly educated people and those who haven't received higher education, which cause the inequality in distribution of education resource. It may even enlarge the gap between the poor and the rich, speeding the social polarization. From this perspective, the public acceptance for the policy may not be promising.

6. Pros and Cons

6.1 Advantages

(1) To assess a country's higher education system,we use a suite of 3 models, the TOPSIS model, linear weighted model with base period data, DEA model, complementing each other, making the evaluation more comprehensive and objective. After the construction of the TOPSIS model, considering that the scores of a country in TOPSIS is affected by other chosen countries, the linear weighted model is established. With base period data as the object of comparison for all countries, it overcomes the shortcomings of TOPSIS. At the same time, since AHP is subjective to a certain extent, a more objective DEA model is constructed as a supplementary comparison.

(2) In the design of China's future higher education system, the nonlinear programming model is used to make rational quantitative analysis by fully considering the constraint relationship among various indicators, the limit of government financial expenditure and the declining marginal output of inputs. Using the results of the nonlinear programming model, the policy is formulated with a clear purpose.

(3) When formulating policies, we consider the role of higher education in economic development by combining economic theories and models.

(4) Sensitivity analysis is used to evaluate the effectiveness of policies, which is objective and reasonable.

6.2 Disadvantages

The data quality used by the evaluation system can be further improved. It can be considered to use the penetration rate of financial aid as an indicator of equity, the average income gap between those who have received and those who have not received higher education as an indicator of the value of a degree,the number of SCI papers in the top 10% of citation volume as the index to evaluate the research level.

7.Reference

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