Int J Adv Manuf Technol (2006) 28: 930–935
DOI 10.1007/s00170-004-1884-2

ORIGINAL ARTICLE

Jun Hu · Lingjian Xiao · Yuhan Wang · Zuyu Wu

An optimal feedrate model and solution algorithm for a high-speed machine of small line blocks with look-ahead

Received: 19 June 2003 / Accepted: 22 July 2003 / Published online: 8 March 2006

Abstract In complex high-speed machining of consecutive small line blocks, the tool path segments can be so short that a machining center moving at a high feedrate cannot accelerate or decelerate fast enough to make direction changes accurately. Aiming at adjusting the feedrate automatically to achieve maximum productivity, this paper present a novel mathematical model considering the key and representative factors, and then based on it, propose an algorithm to seek the approximate optimal feedrate by evaluating the toolpath ahead. Simulation results demonstrate the machine using the proposed model and algorithm can go fast where possible and to slow down just enough where needed and the productivity can be improved dramatically.

Keywords Acceleration and deceleration · Feedrate · Look-ahead · High-speed machine · Small line blocks

1 Introduction

The 1980s and especially the 1990s have brought about a proliferation of very good CAD/CAM techniques, with the ability to easily produce 3D contours of all sorts. Generally the machining program generated by the CAD/CAM system composes a large number of small line blocks, which creates new problems for the CNC control. When used in the conventional way, the CNC has to stop at the end of one move before continuing on to the next to meet the accuracy requirement. Otherwise, corners may be rounded off and the workpiece surface may be gouged. Usually each line block is too small for the CNC to accelerate to a high enough speed, so feedrate is heavily compromised.

In recent years a few researchers have been addressing this problem. Farouki et al. [1] developed a method to determine the maximum safe fixed feedrate due to fixed upper bounds of motor torque and power available to each axis. But in fact there are many other physical factors such as tool chatter, flexure, heat generation, tool wear, machined finish surface and so on that influence the selection of feedrates.

Cao et al. [2] proposed a method in an effort to get a large feedrate by eliminating the redundant data while they are in a straight line. Such a method, however, is applicable only in special circumstances when the angle between two consecutive line blocks equals zero degrees. In fact the angle can vary from 0 to 180 degrees, so this way is not universal and can't improve the feed rate greatly.

In order to find the optimal feedrate, many factors should be taken into account. Most researchers put emphasis on only one or two influences on feedrates, so it is impossible for them to get the optimal feedrate. Considering all factors is a huge work load and often results in something too complicated to use. So in this paper, we are going to take only the key and representative factors into account.

In part 2, we explain the kinematic model with specified starting point speed and end point speed on the base of linear acceleration and deceleration and then analyze the key and representative constrains on the feedrate. At last a recursive inequality model for feedrates is deduced. In part 3, aiming at seeking the optimal feedrate, an algorithm is proposed by adding a buffer to look a preset-number of line blocks ahead. In part 4, a simulation is presented.

2 Model formulation

For the easier illustration, we carry out our research on the base of linear acceleration and deceleration.

2.1 Kinematic model with specified starting point speed and end point speed

Suppose L_{i+1} is the $(i+1)$th line block's distance, V_i, V_{i+1} are the starting point speed and end point speed, S_1, S_2, S_3 are distances in the speed acceleration phase, constant speed phase and

J. Hu (✉) · L. Xiao · Y. Wang · Z. Wu
School of Mechanical Engineering,
Shanghai Jiao Tong University,
Shanghai 200030, P.R. China
E-mail: hujun@sjtu.edu.cn
Tel.: +86-21-62937610
Fax: +86-21-62932610

speed deceleration phase, respectively, t_a, t_l, t_d are the time for each corresponding phase, $V_{\max}$ is the maximum speed and V_m is the practical maximum speed, is the programmed acceleration, as shown in Fig. 1.

First let us find the practical speed V_m.

In the case of Fig. 1a, in which the distance is long enough for the CNC to accelerate to the maximum speed, we get

$$V_m = V_{\max}$$

In the case of Fig. 1b, in which the distance is too short for the CNC to accelerate to the maximum speed, we get

$$V_m^2 - V_i^2 = 2a_m S1 \,,$$
$$V_m^2 - V_{i+1}^2 = 2a_m S3 \,,$$
$$S1 + S3 = L_{i+1} \,.$$

So,

$$V_m^2 - V_i^2 + V_m^2 - V_{i+1}^2 = 2a_m L_{i+1} \,.$$

Simplifying it we get,

$$V_m = \sqrt{\frac{V_i^2 + V_{i+1}^2 + 2a_m L_{i+1}}{2}} \,.$$

Considering the two cases, we have

$$V_m = \min\left\{\sqrt{\frac{V_i^2 + V_{i+1}^2 + 2a_m L_{i+1}}{2}},\ V_{\max}\right\} \,.$$

If V_m is determined, the other parameters can be easily specified as

$$\begin{cases} V_m = \min\left\{\sqrt{\frac{V_i^2+V_{i+1}^2+2a_m L_{i+1}}{2}},\ V_{\max}\right\} \\ S_1 = \frac{V_m^2 - V_i^2}{2a_m} \\ S_3 = \frac{V_m^2 - V_{i+1}^2}{2a_m} \\ S_2 = L_{i+1} - S_1 - S_3 \\ t_a = \frac{(V_m - V_i)}{a_m} \\ t_d = \frac{(V_m - V_{i+1})}{a_m} \\ t_l = \frac{S_2}{V_m} \end{cases} \tag{1}$$

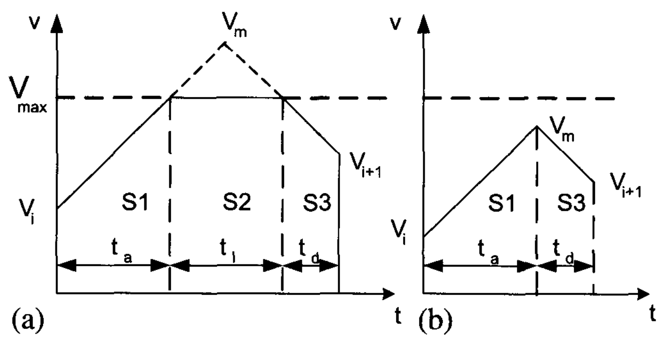


Fig. 1. Velocity profile with linear acceleration/deceleration

From Eq. 1, we know that the velocity profile can be determined by specifying the starting point speed and the end point speed for any given line block. So if we can find the speed at each turning point on the toolpath, the whole velocity profile is determined so that the interpolation can be executed and the machine can be controlled. The conventional way is to force the speed at each turning point to be zero and then interpolate. Of course such a way is conservative enough to be accurate, but the productivity is too low especially in the case of machining small line blocks.

The speed at the turning point is subjected to some constraints, but it is not necessary to be zero. In order to achieve maximum feedrate, we should find the maximum speed, namely the optimal feedrate, at each point of the toolpath. It is impractical and unnecessary, however, to consider all of the constraints on feedrates. In this paper, we are going to simplify and generalize the constraints and only consider the key and representative ones.

2.2 Constrains on feedrates

First we assume the starting point speed of the ith line block is as large as the end point speed of the $i+1$th line block, but varies in direction. Suppose V_{i-1}, V_i, L_i are the starting point speed, the end point speed and the distance of the ith line block, respectively, as shown in Fig. 2.

1. In case of acceleration from V_{i-1} to V_i to cover distance L_i,

$$V_i^2 \le V_{i-1}^2 + 2a_m L_i \,. \tag{2}$$

2. In case of deceleration from V_i to V_{i+1} to cover distance L_{i+1},

$$V_i^2 \le V_{i+1}^2 + 2a_m L_{i+1} \,. \tag{3}$$

3. Constraints from the angle between two consecutive line blocks. In the simplest case, if the next point simply extends

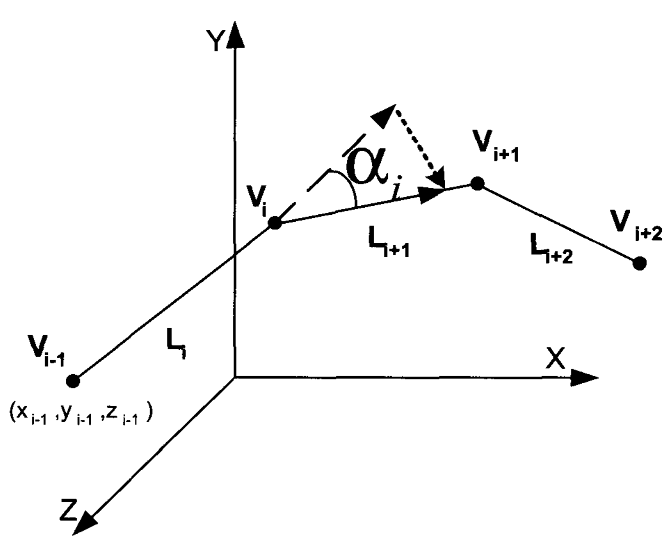


Fig. 2. Toolpath of small line blocks

the current path further along a line and there is no deviation, no deceleration is required. Likewise, if the next point is at a 180-degree deviation from the current path, the machine must be stopped completely. So there must be certain constraints due to the angle. Suppose α denotes the angle, as shown in Fig. 2, and we get

$$2V_i \sin\frac{\alpha_i}{2} \le a_{\max}T\ .$$

So,

$$V_i \le \frac{Ta_{\max}}{2\sin\frac{\alpha_i}{2}} \tag{4}$$

where $a_{\max}$ is the maximum allowed acceleration and T is the interpolation time.
From inequality Eq. 4, we can see when $\alpha_i = 0$, we get $V_i \le +\infty$, which means no constraint is needed when the next point is in the extension of the current line block. When $\alpha_i = 180$, we get $V_i \le \frac{Ta_{\max}}{2}$ (usually a very small number), which means the feedrate should be lowered to almost zero when the next point is at a 180-degree deviation from the current line blocks. So inequality Eq. 4 can exactly explain the constraints due to deviation of consecutive line blocks.

4. Physical limits on the maximum velocity. Of course, the feed rate is subject to many physical factors, such as torque/power of motors, tool chatter, flexure, breakage, heat generation, tool wear and so on [3]. Usually it would be too complicated or make the model too complex to study their effects on feedrate in detail. In most cases, however, they only influence the choosing of maximum speed. So we define a safe maximum feedrate $V_{\max}$ that the machine cannot exceed allowing for all of the physical factors. So we get

   $$V_i \le V_{\max}\ .$$

5. V_i is a scalar quantity and is always greater than zero, so

   $$V_i \ge 0\ .$$

6. For a toolpath having totally N line blocks, it is obvious that the start point speed of the first line block is zero, and the end point speed of the last line block is also zero. So,

   $$V_0 = 0; \quad V_N = 0\ .$$

   For a toolPath having N number of line blocks, it is obvious that $V_{N+1} = 0$ and $L_{N+1} = 0$. Substituting them to inequality Eq. 3, we get

   $$V_N \le V_{N+1} + 2a_m L_{N+1} = 0\ .$$

   So $V_N = 0$ is a redundant condition. In summary, the constraints on the feedrate can be formulated as

$$\begin{cases} V_i^2 \le V_{i-1}^2 + 2a_m L_i \\ V_i^2 \le V_{i+1}^2 + 2a_m L_{i+1} \\ V_i \le \frac{Ta_{\max}}{2\sin\frac{\alpha_i}{2}} \\ V_i \le V_{\max} \\ V_i \ge 0 \\ V_0 = 0 \end{cases} \tag{5}$$

In order to gain maximum productivity, the optimal feedrate is the maximum V_i of the inequality Eq. 5. From the inequality Eq. 5 we can see V_i is limited not only by V_{i-1} and L_i, but also by V_{i+1} and L_{i+1}. So seeking the optimal feedrate would be a recursive calculation. For a toolpath having N line blocks, it would need calculation for at most $N - i$ times in theory to find the optimal V_i, which would be impractical in terms of time expense or memory expense for the CNC in case of a very large N. Aiming at finding the optimal feedrate with a low and practical cost, we propose the following algorithm.

3 Algorithm to find the approximate optimal feedrate

Our algorithm assumes that the maximum number of look-ahead blocks is preset, which can be adjusted according to your requirement and the capability of the CNC, so the maximum times of the recursive calculation would not exceed it. Of course, such a solution will not always be an exact optimal answer, but it is large enough for productivity and practical for the CNC.

First, as we can see in inequality Eq. 5, there are many square calculations and trigonometric calculations that would take a large amount of time for the CNC. So we should simplify inequality Eq. 5.

Let

$$\begin{cases} a_i = V_i^2 \\ b_i = 2a_m L_i \\ e_i = \left(\frac{Ta_{\max}}{2\sin\frac{\alpha_i}{2}}\right)^2 = \frac{(Ta_{\max})^2}{2(1-\cos\alpha_i)} \\ d = V_{\max}^2 \\ c_i = \min\{b_i, d\} \end{cases} \tag{6}$$

For in the sub-item e_i of Eq. 6 includes time-consuming trigonometric calculations, we are going to rewrite it. As shown in Fig. 2, the angle can be evaluated by

$$\cos\alpha = \frac{(x_{i+1}-x_i)(x_i-x_{i-1}) + (y_{i+1}-y_i)(y_i-y_{i-1})}{L_i L_{i+1}} + \frac{(z_{i+1}-z_i)(z_i-z_{i-1})}{L_i L_{i+1}}\ .$$

So

$$e_i = \frac{(Ta_{\max})^2}{2(1-\cos\alpha_i)} = \frac{(Ta_{\max})^2}{2\left(1 - \frac{(x_{i+1}-x_i)(x_i-x_{i-1})+(y_{i+1}-y_i)(y_i-y_{i-1})+(z_{i+1}-z_i)(z_i-z_{i-1})}{L_i L_{i+1}}\right)}\ .$$

Substituting Eq. 6 into inequality Eq. 5 yields

$$\begin{cases} a_i \le a_{i-1} + b_i & \text{(a)} \\ a_i \le a_{i+1} + b_{i+1} & \text{(b)} \\ a_i \le c_i & \text{(c)} \\ a_i \ge 0 & \text{(d)} \\ a_0 = 0 & \text{(e)} \end{cases} \tag{7}$$

Suppose the maximum number of look-ahead blocks is N_t and the speed at the end point of the look-ahead blocks is zero. We are going to discuss the solution for a_i in three different circumstances.

1. In the case that there exists such a least number $K_1 \in [1, Nt]$, satisfying $c_{i+K_1-1} \le b_{i+K_1}$, which means there is a small corner in somewhere in the next K_1 number of line blocks to limit the maximum speed, combined with the assumption that the last point's speed $a_{i+k} = 0$ and inequality Eq. 7b, we get

$$\begin{aligned} &a_{i+K_1-1} \le c_{i+K_1-1} \\ &a_{i+K_1-2} \le a_{i+K_1-1} + b_{i+K_1-1} \\ &\vdots \\ &a_i \le a_{i+1} + b_{i+1} \end{aligned}$$

So,

$$a_i = \sum_{j=i+1}^{i+K_1-1} b_j + c_{i+K_1-1} .$$

Considering inequality Eq. 7a,c, a_i can be determined by

$$a_i = \min \left\{ \sum_{j=i+1}^{i+K_1-1} b_j + c_{i+K_1-1}, a_{i-1} + b_i, c_i \right\} . \tag{8}$$

2. In cases other than the above, there exist such least numbers $K_2 \in [1, Nt]$, satisfying $\sum_{j=i+1}^{K_2} b_j \ge c_i$, which means the next N_t number of line blocks is long enough and have no limitation on the maximum speed. The optimal feedrate is only subjected to inequality Eq. 7a,b. So a_i is determined by

$$a_i = \min\{a_{i-1} + b_i, c_i\} . \tag{9}$$

3. In cases other than the above two cases, which means the next N_t number of line blocks has no small corner but is not long enough, considering inequality Eq. 7b we get

$$\begin{aligned} &a_{i+N_t-1} \le b_{i+N_t} \\ &a_{i+N_t-2} \le a_{i+N_t-1} + b_{i+N_t-1} \\ &\vdots \\ &a_i \le a_{i+1} + b_{i+1} \end{aligned}$$

Combined with inequalities Eqs. 7a,c a_i can be determined by

$$a_i = \min \left\{ \sum_{j=i+1}^{i+N_t} b_j, a_{i-1} + b_i, c_i \right\} . \tag{10}$$

Suppose K is the practical calculation times. As we can see from the above analysis, K does not necessarily equal N_t. In fact, K is determined by

$$K = \min\{N_t, K_1, K_2\} \tag{11}$$

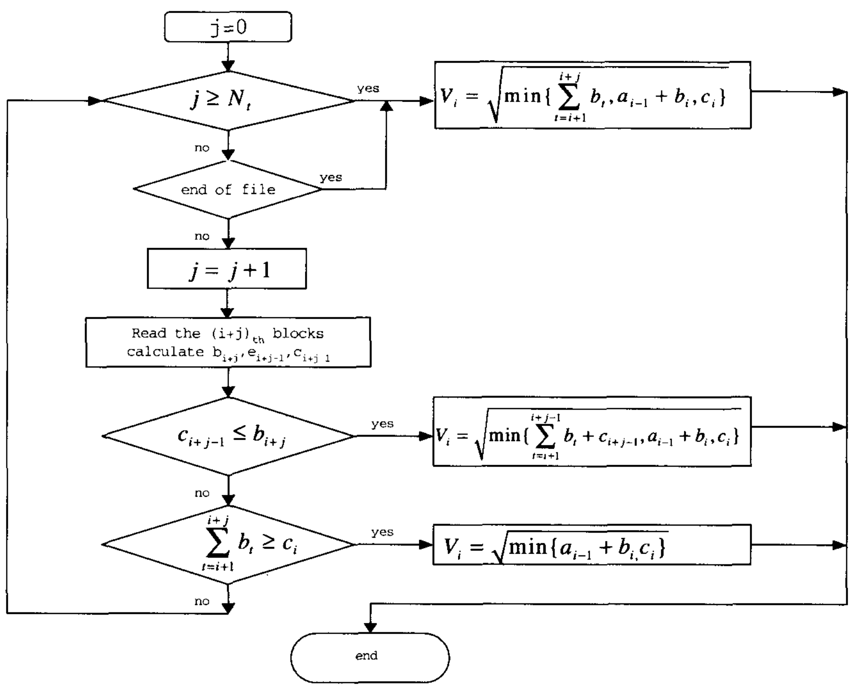
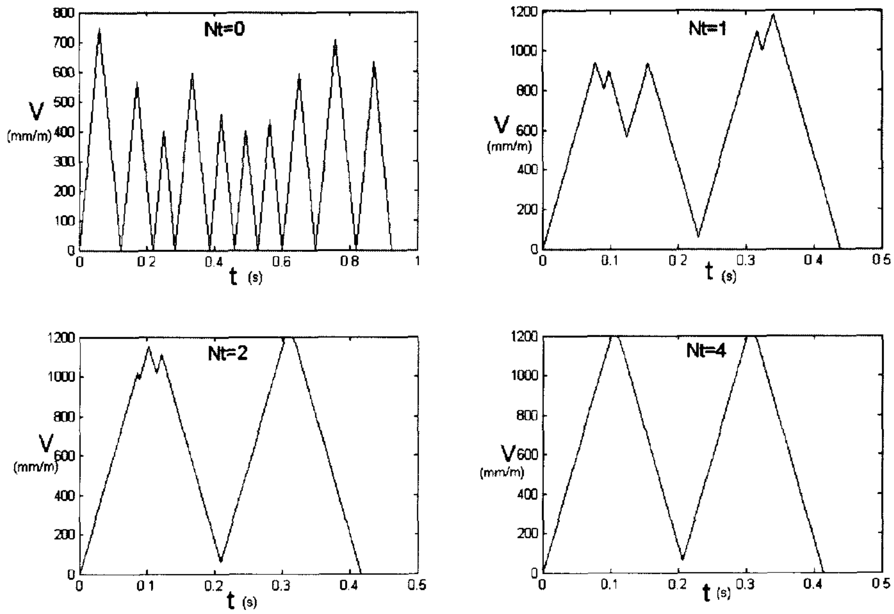


Fig. 3. Flow chart of seeking the approximate optimal feedrate for the point

Fig. 4. Velocity profile



In summary, the optimal feedrate is formulated by

$$V_i = \sqrt{a_i} = \begin{cases} \sqrt{\min\left\{ \sum\limits_{j=i+1}^{i+K_1-1} b_j + c_{i+K_1-1}, a_{i-1} + b_i, c_i \right\}} \\ \sqrt{\min\{a_{i-1} + b_i, c_i\}} \\ \sqrt{\min\left\{ \sum\limits_{j=i+1}^{i+N_t} b_j, a_{i-1} + b_i, c_i \right\}} \end{cases} \tag{12}$$

The initialization condition is given as Eq. 7e, the optimal feedrate given by Eq. 12 is always solvable. As we can see in Eq. 12, the calculation for the optimal feedrate is at most N_t times of addition and one square root, so it is easy for the computer to realize. The flow chart of calculating the optimal feedrate at the ith point can be shown in Fig. 3.

4 Simulation

Let us consider a CNC, in which the maximum acceleration, the programmed acceleration, the maximum velocity and the interpolation time are 200 mm/s^2, 200 mm/s^2, 1200 mm/m and 10 ms, respectively. In order to make the simulation more representative, the toolpath to be machined is designed to be a to-and-fro straight line that has a total of ten small line blocks as shown in Table 1. When the maximum number of look-ahead blocks is set to 0, 1, 2 and 4, respectively, the velocity profiles are shown in Fig. 4. The time required corresponding to the number of look-ahead blocks is shown in Fig. 5.

As shown in Fig. 4, at about the first one fourths and three fourths of the toolpath, when the machine looks ahead and finds the next N_t number of line blocks is long enough, the feedrate is determined by Eq. 9 just as case ii of Sect. 3. At about the two fourths part, when the machine foresees a sharp corner somewhere in the next N_t number of line blocks, the feedrate is determined by Eq. 8 just as case i. At about the four fourths part when the machine foresees the end of the toolpath somewhere in the next N_t number of line blocks, the feedrate is determined

Table 1. The to-and-pro straight line to be machined (in absolute coordinate)

x(mm)	0	0.35	0.55	0.65	0.87	1	0.9	0.78	0.56	0.25	0
y(mm)	0	0.7	1.1	1.3	1.74	2	1.8	1.56	1.12	0.5	0
z(mm)	0	0	0	0	0	0	0	0	0	0	0

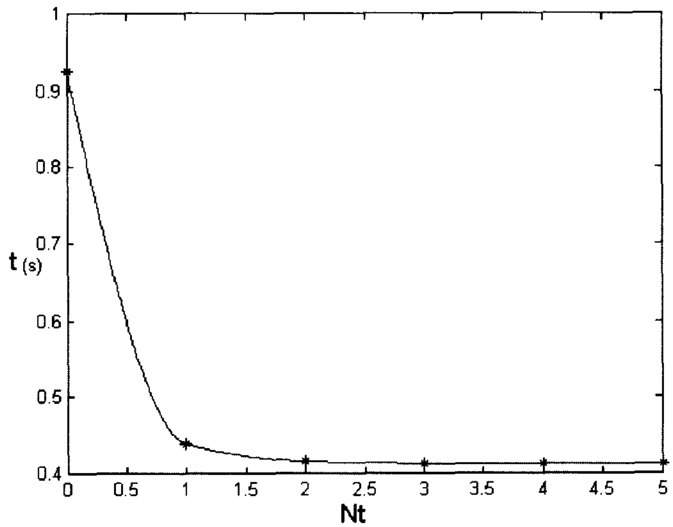


Fig. 5. Machining time corresponding to the number of pre-read blocks

by Eq. 10 just as case iii. This toolpath is deliberately chosen to include three cases, which can be seen especially clearly when $N_t = 4$.

With a closer look at Fig. 4, the velocity is forced to zero at every point when the number of look-ahead blocks $N_t = 0$, which is just the same as the conventional controls. Such a way has the lowest productivity along with frequent changing motions. As the number of look-ahead blocks increases, the motion becomes more fluid and the feedrate is becoming steadier and higher with a result of less machining time. This is something like a car with longer vision that can go faster when driving on a zigzag road. It is not necessary to set a very large number of look-ahead blocks for there is no obvious effect on the productivity and more calculation time and more memory are needed instead as it becomes too large as shown in Fig. 5.

5 Conclusion

This paper successfully builds up a mathematical model for feeedrates by considering only the key and representative factors, and then proposes a feasible way to find the approximate feedrate by reading a preset number of line blocks ahead. Simulations show this algorithm provides the CNC the ability to anticipate sudden direction changes and react accordingly. If the look-ahead check reveals that there may be a problem, feedrate for intermediate points along the toolpath can slow down, otherwise it will maintain a high speed. Generally the more look-ahead blocks there are, the greater productivity is. However, when the number of look-ahead blocks is set too large, the calculation time and memory cost is also increased. So more blocks don't necessarily mean better performance.

Although the model and algorithm are based on linear acceleration/deceleration, the method can be applied to other types of acceleration/deceleration. In fact some ways proposed in this paper have been applied to the research of CNC Key Technology under the grant number 2002AA424042 sponsored by The National High Technology Research and Development Program of China. As the function of look-ahead has been a critical criterion of today's advanced CNC system especially for high-speed machining, the idea in this paper has some significance on manufacturing.

References

1. Farouki RT, Tsai YF, Wilson CS (2000)Physical constrains on federates and feed acceleration along curved tool paths. Comput Aided Geom Des 17:337-359
2. Zhong Q, Li J, Huang SK (2000) Interpolation of the micro-line at high continuum speed with high accuracy in RP system. Chin J Huazhong Univ Sci Technol 28(3):39–41
3. Schuett T (2003) A closer look at look-ahead, http://www.mmsonline.com/articals/039603.html