**CSCE 420 - Spring 2023  
Homework 3 (HW3)  
due: Tues, Apr 4, 5:00pm - Late written homeworks will not receive credit.**

**Turn-in answers as a Word document (HW3.docx or .pdf) and commit/push it to your class github repo.**

**1. Translate** the following sentences into First-Order Logic. Remember to break things down to simple concepts (with short predicate and function names), and make use of quantifiers. For example, “tasteDelicious(someRedTomatos)”, is not broken down enough; instead we would be looking for a formulation such as: “∃x tomato(x)^red(x)^ taste(x,delicious)”. See the lecture slides for more examples and guidance.

* **bowling balls are sporting equipment** 
  + x bowlingBalls(x) -> equipment(x) ^ forSports(x)
* **all domesticated horses have an owner** 
  + x horse(x) ^ domesticated(x) -> y owner(y)
* **the rider of a horse can be different than the owner** 
  + x,y,z rider(x) ^ horse(y) ^ owner(z,y) ^ rides(x,y) ^ -(x=z)
* **horses move faster than frogs** 
  + x,y horse(x) ^ frog(y) -> speed(x) > speed(y)
* **a finger is any digit on a hand other than the thumb** 
  + x,y finger(x) ^ hand(y) <-> digit(x) ^ partOf(x, y) ^ -thumb(x)
* **an isosceles triangle is defined as a polygon with 3 edges, which are connected at 3 vertices, where 2 (but not 3) edges have the same length** 
  + x triangle(x) ^ isosceles(x) <-> polygon(x) ^ Eq(numEdges(x), 3) ^ Eq(numVertices(x), 3) ^ [ a,b,c edge(a,x), edge(b.x), edge(c,x) -> a != b ^ b != c ^ connectedTo(a,b) ^ connectedTo(a,c) ^ connectedTo(b,c) ^ Eq(length(a), length(b)) ^ -Eq(length(a),length(c)) ^ -Eq(length(b),length(c)) ]

2. Convert the following first-order logic sentence into CNF:

∀ **x person(x) ^ [**∃**z petOf(x,z) ^** ∀**y petOf(x,y)**→**dog(y)]** → **doglover(x)**

∀x – (person(x) ^ [∃z petOf(x,z) ^ ∀y petOf(x,y)→dog(y)] )v doglover(x)

∀x -person(x) v -[∃z petOf(x,z) ^ ∀y petOf(x,y)→dog(y)] v doglover(x)

∀x -person(x) v [∀z –(petOf(x,z) ^ ∀y petOf(x,y) -> dog(y))] v doglover(x)

∀x -person(x) v [∀z –(petOf(x,z) ^ ∀y -petOf(x,y) v dog(y))] v doglover(x)

∀x -person(x) v [∀z –petOf(x,z) v ∃y - (-petOf(x,y) v dog(y)))] v doglover(x)

∀x -person(x) v [∀z –petOf(x,z) v ∃y (petOf(x,y) ^ -dog(y)))] v doglover(x)

∀x -person(x) v [(∀z –petOf(x,z) v ∃y petOf(x,y)) ^ (∀z –petOf(x,z) v -dog(y))] v doglover(x)

∀x -person(x) v [(∀z –petOf(x,z) v ∃y petOf(x,y)) ^ (∀z –petOf(x,z) v -dog(y))] v doglover(x)

[ ∀x -person(x) v (∀z –petOf(x,z) v ∃y petOf(x,y)) ] ^ [∀x -person(x) v (∀z –petOf(x,z) v -dog(y)) ] v doglover(x)

3. Determine whether or not the following pairs of predicates are **unifiable**. If they are, give the most-general unifier and show the result of applying the substitution to each predicate. If they are not unifiable, indicate why. Capital letters represent variables; constants and function names are lowercase. For example, ‘loves(A,hay)’ and ‘loves(horse,hay)’ are unifiable, the unifier is u={A/horse}, and the unified expression is ‘loves(horse,hay)’ for both.

* **owes(owner(X), citibank, cost(X)), owes(owner(ferrari), Z, cost(Y))**
  + u = {X/Ferrari, Z/citibank, Y/ferrari}
* **gives(bill, jerry, book21), gives(X, brother(X),Z)**
  + Cannot unify because we cannot unify the constant of jerry to the function of brother(X)
* **opened(X, result(open(X),s0))), opened(toolbox, Z)**
  + u = {X/toolbox, Z/result(open(toolbox),s0))}

4. Consider the following situation:

*Marcus is a Pompeian.  
All Pompeians are Romans.  
Ceasar is a ruler.  
All Romans are either loyal to Caesar or hate Caesar (but not both). Everyone is loyal to someone.  
People only try to assassinate rulers they are not loyal to.  
Marcus tries to assassinate Caesar.*

a) Translate these sentences to First-Order Logic.

1. pompeian(Marcus)

2. ∀p pompeian(p) -> roman(p)

3. ruler(Caesar)

4. ∀r roman(r) -> (loyalTo(r, Caesar) v hate(r, Caesar)) ^ (-loyalTo(r, Caesar) v -hate(r, Caesar))

5. ∀r roman(r) -> ∃s roman(s) ^ loyalTo(r,s)

6. ∀p,r roman(p) ^ ruler(r) ^ tryToAssassinate(p,r) -> -loyalTo(p,r)

7. tryToAssassinate(Marcus, Caesar)

b) Prove that ***Marcus hates Caesar*** using Natural Deduction. In the same style as  
the examples in the lecture slides, label all derived sentences with the ROI used, which prior sentences were used, and what unifier was used.

8. roman(Marcus) [Modus Ponens, 1, 2, u = {p/Marcus}]

9. -loyalTo(Marcus, Caesar) [Modus Ponens, 6, 7, u = {p/Marcus, r/Caesar}]

10. ∀r -roman(r) v [ (loyalTo(r, Caesar) v hate(r, Caesar)) ^ (-loyalTo(r, Caesar) v -hate(r, Caesar)) ] [Implication Elimination, 4]

11. [∀r -roman(r) v (loyalTo(r, Caesar) v hate(r, Caesar)) ] ^ [∀r -roman(r) v (-loyalTo(r, Caesar) v -hate(r, Caesar)) ] [Distributivity, 10]

12. [∀r -roman(r) v (loyalTo(r, Caesar) v hate(r, Caesar)) ] [And Elimination, 11]

13. (loyalTo(r, Caesar) v hate(r, Caesar)) [Resolution, 8, 12]

14. (loyalTo(Marcus, Caesar) v hate(Marcus, Caesar)) [u = {r/Marcus}, 8, 13]

15. hate(Marcus, Caesar)) [Resolution, 9, 14]

c) Convert all the sentences into CNF

1. pompeian(Marcus)

2. -pompeian(p) v roman(p)

3. ruler(Caesar)

4a. [-roman(r) v (loyalTo(r, Caesar) v hate(r, Caesar)) ]

4b. [-roman(r) v (-loyalTo(r, Caesar) v -hate(r, Caesar)) ]

5a. (-roman(r) v roman(F(r))

5d. (-roman(r) v loyalTo(r, F(r)))

6. –roman(p) v -ruler(r) v -tryToAssassinate(p,r) v -loyalTo(p,r)

7. tryToAssassinate(Marcus, Caesar)

d) Prove that ***Marcus hates Caesar*** using Resolution Refutation.

Ordering starts from end of a)

8. -hate(Marcus, Caesar)

9. roman(Marcus) [Resolution, 1, 2, u = {p/Marcus}]

10. -roman(Marcus) v -ruler(Caesar) v -loyalTo(Marcus, Caesar) [Resolution, 6, 7, u = {p/Marcus, r/Caesar}]

11. loyalTo(Marcus, Caesar) v hate(Marcus, Caesar) [Resolution, 4a, 9, u = {p/Marcus}]

12. loyalTo(Marcus, Caesar) [Resolution, 8, 11, u = {p/Marcus}]

13. -roman(Marcus) v -ruler(Caesar) [Resolution, 10, 12, u = {p/Marcus, r/Caesar}]

14. -roman(Marcus) [Resolution, 3, 13]

15. null [Resolution, 9, 14]

5. Write a KB in First-Order Logic with rules/axioms for...

1. **Map-coloring** – every state must be exactly 1 color, and adjacent states must be different colors. Assume possible colors are states are defined using unary predicate like color(red) or state(WA). To say a state has a color, use a binary predicate, e.g. ‘color(WA,red)’.
   1. s,a,b state(s) ^ hasColor(s, a) ^ hasColor(s, b) -> a = b
   2. s,t,a state(s) ^ state(t) ^ neighbor(s, t) ^ hasColor(s, a) -> -hasColor(t, a)
2. **Sammy’s Sport Shop** – include sentences for specific facts like obs(1,W) or label(2,B), as well as for general constraints about the boxes and colors. Use binary predicate ‘contains(x,q)’ to represent that box x contains tennis balls of color q (where q could be W, Y, or B).
   1. color(yellow), color(white)
   2. label(yellow), label(white), label(both)
   3. contains(yellow), contains(white), contains(both)
   4. b,x,y box(b) ^ labelType(b, x) ^ labelType(b, y) -> x = y
   5. b box(b) ^ obs(b, yellow) -> containType(b, yellow) v containType(b, both)
   6. b box(b) ^ obs(b, white) -> containType(b, white) v containType(b, both)
   7. b,x,y box(b) ^ labelType(b, x) -> -containType(b, y)
   8. For all boxes that contain one type, they cannot contain other types
3. **Wumpus World** - Write rules for which rooms are ‘stenchy’, ‘breezy’, and ‘safe’. (hint: define a helper concept called ‘adjacent(x,y,p,q)’ which defines when a room at coordinates (x,y) is adjacent to another room at (p,q), and x,y,p,q are integers 1-4.
   1. xCoor(1), …, xCoor(4)
   2. yCoor(1), …, yCoor(4)
   3. type(stenchy), type(breezy), type(safe)
   4. contains(wumpus), contains(pit)
   5. x,y,p,q xCoor(x) ^ yCoor(y) ^ xCoor(p) ^ yCoor(q) ^ adjacent(x,y,p,q) ^ roomContains(x,y,wumpus) -> roomType(p,q,stenchy)
   6. x,y,p,q xCoor(x) ^ yCoor(y) ^ xCoor(p) ^ yCoor(q) ^ adjacent(x,y,p,q) ^ roomContains(x,y,pit) -> roomType(p,q,breezy)
   7. x,y xCoor(x) ^ yCoor(y) ^ -roomContains(x,y,pit) ^ -roomContains(x,y,wumpus) -> roomType(p,q,safe)
4. **4-Queens** – assume row(1)...row(4) and col(1)...col(4) are facts; write rules that describe configurations of 4 queens such that none can attack each other, using ‘queen(r,c)’ to represent that there is a queen in row r and col c.
   1. x,y,p,q row(x) ^ col(y) ^ row(p) ^ col(q) ^ queen(x,y) ^ queen(p,q) -> -sameRow(x,p) ^ -sameCol(y,q)
   2. x,y,p,q row(x) ^ col(y) ^ row(p) ^ col(q) ^ queen(x,y) ^ queen(p,q) -> -sameDiagonal(x,p,y,q)

Don’t forget to quantify all your variables.