14.2) 4) Suppose the set of irrational numbers, Q, is countable.

The union of 2 countable sets is countable implies that $Q \cup Q^c = \mathbb{R}$ is countable, which is a contradiction.

Thus, Q^c is uncountable. $\boxtimes$

13.) A is infinite because $\forall n \in \mathbb{N}$, $\{n\} \in A$ and N is infinite.

Define $f: A \to \mathbb{N} \cup \{0\}$ such that $\emptyset \mapsto 0$ and $\forall X \in A$, $X = \{x_1, \ldots, x_k\} \mapsto p_{x_1} \cdot p_{x_2} \cdot p_{x_k}$. f is a function because $\emptyset \mapsto 0$ and every nonempty element of A is mapped to a product of primes defined as P_n is the nth prime. f is insective by FTA.

A	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$
$f(A)$	0	$P_1 = 2$	$P_2 = 3$	5	6
element of list	1	2	3	4	5

So, $|A| = \aleph_0 = |\mathbb{N}|$ $\boxtimes$

14.3) 1) For each $b \in B$ let a_b be an element of A for which $f(a_b) = b$. Suppose $U = \{a_b : b \in B\}$. Then the function $f: U \to B$ is bisective, by construction. Then since B is uncountable, so is U. Therefore U is an uncountable subset of A. So A is uncountable by Thm 14.9

2) Let C be the set of countable numbers such that $C = \{(x,y) : x, y \in \mathbb{R}\}$. Suppose $f(x) = (x, y_0)$; $\mathbb{R} \to C$ where y_0 is fixed, but arbitrary real numbers. Because this function is insective but not surjective, the uncountable set of $\mathbb{R}$ is a proper subset of C. $(\mathbb{R} \subset C)$

Thus, R is uncountable & C is uncountable $\boxtimes$

9) Suppose if $A \to B$ is an injective function. Let $n = |A| = |B|$.
Since A, B are finite & $A = \{a_1 \ldots a_n\}$, and $B = \{b_1, \ldots b_n\}$.
f is injective, so if $f(a_j) = f(a_k)$, then $a_j = a_k$. Then
$|\{f(a_1) \ldots f(a_n)\}| = n$. Then $|f(A)| = n = |B|$. Since
$f: A \to B$, $f(A) \subseteq B$. Then $|B| = |f(A)| \Rightarrow f(A) = B$.
Therefore, f is surjective $\boxtimes$

10) Let A and B be 2 finite sets with $|A| = |B| = n$ and
$f: A \to B$ is surjective, $f(A) = B$. Let $\{x_1, x_2 \ldots x_n\} = n$
be the set of ordered elements of A. Suppose f is not
injective $(f(x_i) = f(x_j))$ for $i \neq j$. $\therefore f$ is injective
If A & B are not finite then $A = \mathbb{R}$, $B = \mathbb{R}^+$. Then
$|A| = |B|$ and $f: A \to B$ define by $f(x) = x^2$ which is
surjective, but $f: A \mapsto$ is not injective $\boxtimes$