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Math 8 HW # 5

2.6) 7.) $P \Rightarrow Q \equiv (P \wedge \sim Q) \Rightarrow (Q \wedge (\sim Q))$

P	Q	$P \wedge \sim Q$	$Q \wedge \sim Q$	$P \wedge \sim Q \Rightarrow Q \wedge (\sim Q)$	$P \Rightarrow Q$
T	T	F	F	T	T
T	F	T	F	F	F
F	T	F	F	T	T
F	F	F	F	T	T

Since the circled columns are identical, the statements are equivalent

8.) $\sim P \Leftrightarrow Q \equiv (P \Rightarrow \sim Q) \wedge (\sim Q \Rightarrow P)$

P	Q	$P \Rightarrow \sim Q$	$\sim Q \Rightarrow P$	$P \Rightarrow \sim Q \wedge \sim Q \Rightarrow P$	$\sim P \Leftrightarrow Q$
T	T	F	F	F	F
T	F	T	T	T	T
F	T	T	T	T	T
F	F	T	F	F	F

Since the circled columns are identical, the statements are equivalent

10) $(P \Rightarrow Q) \vee R$ and $\sim((P \wedge \sim Q) \wedge \sim R))$

$\hookrightarrow ((\sim P) \vee Q) \vee R$

P	Q	R	$P \Rightarrow Q$	$(P \Rightarrow Q) \vee R$	$\sim((P \wedge \sim Q) \wedge \sim R$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

Since the circled columns are identical, the statements are equivalent

11) $(\sim P) \wedge (P \Rightarrow Q)$ and $\sim (Q \Rightarrow P)$

P	Q	$P \Rightarrow Q$	$(\sim P) \wedge (P \Rightarrow Q)$	$\sim (Q \Rightarrow P)$
T	T	T	F	F
T	F	F	F	F
F	T	T	T	T
F	F	T	T	F

Since the circled columns are NOT identical, the statements are NOT equivalent

12. $\sim(P \Rightarrow Q)$ and $P \wedge \sim Q$

P	Q	$\sim(P \Rightarrow Q)$	$P \wedge \sim Q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	F	F

Because the circled columns are identical, the two statements are equivalent.

2.7) 1.) $\forall x \in \mathbb{R}, x^2 > 0$

False → For all real numbers, x^2 is greater than zero

3) $\exists a \in \mathbb{R}, \forall x \in \mathbb{R}, ax = x$

True → There exists real numbers where $ax = x$ for all real numbers

4) $\forall X \in P(\mathbb{N}), X \subseteq \mathbb{R}$

True. → For any X in the power set of natural numbers, X is a subset of the real numbers

5) $\forall n \in \mathbb{N}, \exists X \in P(\mathbb{N}), |X| < n$

True → For every natural number there exists subsets X of natural numbers in which $|X| < n$

7) $\forall X \subseteq \mathbb{N}, \exists n \in \mathbb{Z}, |X| = n$

False → For all subsets X of $\mathbb{N}$, there exists integer n in which $|X| = n$.