HW 5

5.5) a)

Source	SS	df	MS	F	p-val (b)
A	50	1	50	50	< .001
B	80	2	40	40	< .001
AB	30	2	15	15	< .001
Error	12	12	1		
Total	172	17			

b) At $\alpha = .05$, there is a significant difference between A, B, and AB (interaction between A & B) as shown above

c) True, the pure estimate of the std. dev. of sample observations is 1.

5.8) a)

		.15	.18	.20	.25	
3	.20	74	79	82	99	Avg
3-1		64	68	88	104	81.58
=2		60	73	92	96	
	.25	92	98	99	104	Avg
		86	104	108	110	97.58
		88	88	95	99	
	.30	99	104	108	114	Avg
		98	99	110	111	103.83
		102	95	99	107	

4 $\quad$ 4-1 = 3

$$SS_A = \frac{1}{bn}\sum_{i=1}^{a} y_{i..}^2 - \frac{y_{...}^2}{abn} = \boxed{2125.11}$$

$$SS_B = \frac{1}{an}\sum_{j=1}^{b} y_{.j.}^2 - \frac{y_{...}^2}{abn} = \boxed{3160.5}$$

$$SS_{AB} = \left(\frac{1}{n}\sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij.}^2 - \frac{y_{...}^2}{abn}\right) - SS_A - SS_B = 5842.67 - 2125.11 - 3160.5$$

$$= \boxed{557.06}$$

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}^2 - \frac{y_{...}^2}{abn} = \boxed{6532}$$

$$SS_E = SS_T - SS_A - SS_B - SS_{AB} = 6532 - 2125.11 - 3160.5 - 557.06$$

$$= \boxed{689.33}$$

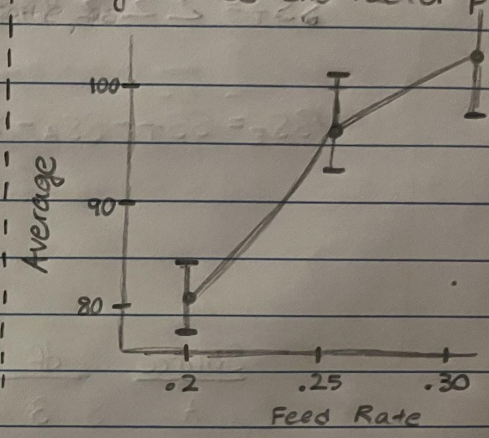
Source	df	SS	MS	F	P-val
A	3	2125.11	708.37	24.66	<.0001
B	2	3160.5	1580.25	55.02	<.0001
AB	6	557.06	92.84	3.23	.018
Error	24	689.33	28.72		
Total	35	6532.00			

The depth of cut and feed rate, as well as their interactions, are all significant. The model is significant as well.

5.8 c) The values for averages at each feed rate, as shown in part a, is:

	Feed Rates		Avg
1)	.20	→	81.58
2)	.25	→	97.58
3)	.30	→	103.83

The generated one factor plot:



d) The p-values, as shown in the ANOVA table in part a, are as follows:

Source	P-val
A	$< .0001$
B	$< .0001$
AB	.018

5.9) μ_1 = mean surface finish for .20 in/min

μ_2 = mean surface finish for .25 in/min

$$\bar{y}_{1..} - \bar{y}_{2..} - t_{\alpha/2,\, ab(n-1)}\sqrt{\frac{2MSE}{n}} \leq \mu_1 - \mu_2 \leq \bar{y}_{1..} - \bar{y}_{2..} + t_{\alpha/2,\, ab(n-1)}\sqrt{\frac{2MSE}{n}}$$

95% CI $(\mu_1 - \mu_2)$ $\Rightarrow (81.5833 - 97.5833) \pm (2.064)\sqrt{\frac{2(28.722)}{3}}$

$\Rightarrow -16 \pm (9.032)$

$= \boxed{(-25.032, -6.968)}$

5.28)

Formulation	# of Washings = 1		# of Washings = 2		
	Booster: Yes	Booster: No	Booster: Yes	Booster: No	
New	(11) 6,5	6,5 (11)	(5) 3,2	4,1 (5)	32
Original	(19) 10,9	11,11 (22)	(19) 10,9	9,10 (19)	79
	30	33	24	24	Total 111

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n} y_{ijkl}^2 - \frac{y_{....}^2}{abcn} = \boxed{166.94}$$

$$SS_A = \frac{1}{bcn}\sum_{i=1}^{a} y_{i...}^2 - \frac{y_{....}^2}{abcn} = \boxed{138.06}$$

$$SS_B = \frac{1}{acn}\sum_{j=1}^{b} y_{.j..}^2 - \frac{y_{....}^2}{abcn} = \boxed{14.06}$$

$$SS_C = \frac{1}{abn}\sum_{k=1}^{c} y_{..k.}^2 - \frac{y_{....}^2}{abcn} = \boxed{0.56}$$

$$SS_{AB} = \frac{1}{cn}\sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij..}^2 - \frac{y_{....}^2}{abcn} - SS_A - SS_B = \boxed{5.06}$$

$$SS_{AC} = \frac{1}{bn}\sum_{i=1}^{a}\sum_{k=1}^{c} y_{i.k.}^2 - \frac{y_{....}^2}{abcn} - SS_A - SS_C = \boxed{.56}$$

$$SS_{BC} = \frac{1}{an}\sum_{j=1}^{b}\sum_{k=1}^{c} y_{.jk.}^2 - \frac{y_{....}^2}{abcn} - SS_B - SS_C = \boxed{.56}$$

$$SS_{ABC} = \frac{1}{n}\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} y_{ijk.}^2 - \frac{y_{....}^2}{abcn} - SS_A - SS_B - SS_C = \boxed{.56}$$

$$SS_E = SS_T - SS_{\text{subtotals}} = 166.94 - 159.44 = \boxed{7.5}$$

Source	df	SS	MS	F	P-val	
A	1	138.06	138.06	147.27	<.0001	✗
B	1	14.06	14.06	15	.0047	✗
C	1	.56	.56	.6	.4609	
AB	1	5.06	5.06	5.4	.0486	✗
AC	1	.56	.56	.6	.4609	
BC	1	.56	.56	.6	.4609	
ABC	1	.56	.56	.6	.4609	
Error	8	7.5	.94			
Total	15	166.94				

✗ = significant

Formulation, number of washings, & interaction between them are significant, A, B, and AB are significant terms (at $\alpha = .05$)

118.667 − 10 − 12.167 − 96.333 = .1666

96.333 / 96.333 = 1

11 − 6 − 2 − 1 = 2

10/6 = 1.667

5.41)

Source	df	SS	MS	F	P-value
Factor A	2	.1666	.0833	.05	.952
Factor B	1	96.333	96.333	57.80	< .001
Interaction	2	12.167	6.0833	3.65	.092
Error	6	10	1.667	–	
Total	11	118.667			

a) .1666

b) 2

c) 1

d) 1.667

e) Upper bound is .1

f) 2 + 1 = 3 levels

g) 1 + 1 = 2 levels

h) 2 replicates

i) No (False) since the p-value is .092, > .05

j) An estimate of the std dev of the response variable is 1.29