HW 17

11.3) 1. Equivalent classes of R:

$[1] = \{1\}$

$[2] = [3] = \{2, 3\}$

$[4] = [5] = [6] = \{4, 5, 6\}$

2. Reflexive → $(a,a)\ (b,b)\ (c,c)$
(d,d)

Symmetric → $(d,a)\ (c,b)\ (d,e)$

Transitive → $(a,d)(d,e) \in R$
$\Rightarrow (a,e) \in R$
$(e,d)(d,a) \in R$
$\Rightarrow (e,a) \in R$

$R = \{ (a,a)\ (b,b)\ (c,c)\ (d,d)$
$(e,e)\ (a,d)\ (b,c)\ (e,d)$
$(d,a)\ (c,b)\ (d,e)$
$(a,e)\ (e,a) \}$

6. $\{\{a\}, \{b\}, \{c\}\}$ no others
$\{\{a\}, \{b,c\}\}$ $b \equiv c$, $c \equiv b$
$\{\{b\}, \{a,c\}\}$ $a \equiv c$, $c \equiv a$
$\{\{c\}, \{a,b\}\}$ $a \equiv b$, $b \equiv a$
$\{\{a,b,c\}\}$ $a \equiv b$, $b \equiv a$, $a \equiv c$
$c \equiv a$, $b \equiv c$, $c \equiv b$

9. It is reflexive, for any $x \in \mathbb{Z}$, $4 \mid (x + 3x)$, thus xRx.
Suppose xRy. Then, $4 \mid (x+3y)$. So, $x + 3y = 4k$ for some int k. We can multiply this by 3 to get $3x + 9y = 12k$, which can be rewritten as: $y + 3x = 4(3k - 2y)$. So, $4 \mid (y+3x)$, and thus yRx, proving R is symmetric.
Suppose xRy and yRz. Then, $4 \mid (x+3y)$ and $4 \mid (y + 3z)$, so $x + 3y = 4k$ and $y + 3z = 4q$ for some integers k and q. Adding these equations with create $x + 4y + 3z = 4k + 4q$, which can be rewritten as $x + 3z = 4k + 4q - 4y = 4(k+q-y)$
Because of this, we know $4 \mid (x+3z)$, thus xRz, and R is transitive
Since R is reflective, symmetric, and transitive, so its an equivalence relation

$[0] = \{x \in \mathbb{Z} : 4 \mid x\} = \{-4, 0, 4, 8, 12 \ldots\}$
$[1] = \{x \in \mathbb{Z} : 4 \mid (x+3)\} = \{-3, 1, 5, 9, 13 \ldots\}$
$[2] = \{x \in \mathbb{Z} : 4 \mid (x+6)\} = \{-2, 2, 6, 10, 14, 18\}$
$[3] = \{x \in \mathbb{Z} : 4 \mid (x+9)\} = \{-1, 3, 7, 11, 15, 19\}$

Statement is (False)

12. Proof by counterexample.

Let $A = \{a, b, c\}$, $R = \{(a,a)(b,b)(c,c)(c,a)(a,c)\}$,

and $S = \{(a,a)(b,b)(c,c)(a,b)(b,a)\}$.

$R \cup S = \{(a,a)(b,b)(c,c)(a,c)(c,a)(a,b)(b,a)\}$

is not equivalence relation since does have transitive properties such as $c(R \cup S)a \cap a(R \cup S)b \not\Rightarrow c(R \cup S)b$

Thus the union of 2 equivalence relations aren't necessarily an equivalence relation. $\boxtimes$