HW 2

3.4) $F_0 = 2.91$

N = 5 replicates × 3 factor levels = 15

degrees freedom

Factor: $3-1=2$

Total: $15-1=14$

Error: $14-2=12$

2 & 12 dg. freedom

$\Rightarrow \boxed{.05 < P < .1}$

3.7)

Source	DF	SS	MS	F	P
Factor	$\boxed{4}$	$\boxed{987.71}$	246.93	$\boxed{33.09}$	less than .0001
Error	25	186.53	$\boxed{7.46}$		
Total	29	1174.24			

dg: $29-25 = 4$

$SSF = SST - SSE$

$= 1174.24 - 186.53$

987.71

$MSE = \frac{SSE}{df_{SSE}}$

$= 186.53/25$

$= 7.46$

$F = \frac{MSF}{MSE}$

$= \frac{246.93}{7.46}$

$= 33.09$

p value < .0001

3.9) a) H_0: Mixing Techniques do not affect strength of the cement

H_1: Mixing techiques affect strength of cement

Upon construction of the ANOVA model, the F-value is 12.73 with $p = .0005$.

Thus, we reject the H_0 where mixing does not have an effect.

$\boxed{\text{Mixing techniques do affect strength of the cement}}$

3.9 c) $LSD = t_{\alpha/2, N-\alpha}\sqrt{\frac{2\,MSE}{n}}$

$= t_{.025, 16-4}\sqrt{\frac{2(12825.7)}{4}}$

$= 2.179\sqrt{6412.85}$

$= 174.497$

Treatments		
2 vs 4	$\lvert 3156.25 - 2666.25\rvert = 490 > 174.495$
2 vs 3	$\lvert 3156.25 - 2933.75\rvert = 222.5 > 174.495$
2 vs 1	$\lvert 3156.25 - 2971\rvert = 185.25 > 174.495$
1 vs 4	$\lvert 2971 - 2666.25\rvert = 304.75 > 174.495$
1 vs 3	$\lvert 2971 - 2933.75\rvert = 37.25 < 174.495$
3 vs 4	$\lvert 2933 - 2666.25\rvert = 267.5 > 174.495$

$\boxed{\text{All means are different apart from techniques 1 vs 3}}$

3.10 a)

$\bar{y}_{1.} - \bar{y}_{2.} = |2971 - 3156.25| = 185.25$

3.10) Tukey Test

$T_\alpha = q_\alpha(t, N-t)\sqrt{\frac{MS_E}{n}}$

$T_{.05} = q_{.05}(4, 12)\sqrt{\frac{12825.6875}{4}}$

$= 4.2\,(56.6252)$

$= 237.826$

$\bar{y}_{1.} - \bar{y}_{3.} = |2971 - 2933.75| = 37.25$

$\bar{y}_{1.} - \bar{y}_{4.} = |2971 - 2666.25| = 304.75$

$\bar{y}_{2.} - \bar{y}_{3.} = |3156.25 - 2933.75| = 222.25$

$\bar{y}_{2.} - \bar{y}_{4.} = |3156.25 - 2666.25| = 490.00$

$\bar{y}_{3.} - \bar{y}_{4.} = |2933.75 - 2666.25| = 267.50$

The Tukey test shows that techniques that were compared to the 4th technique were different compared to 1, 2, 3. Also, Tukey test shows the mean of T2 is not different to T1 & T3

3.20) a) H_0: All means of conductivity values are the same.

H_1: At least one mean of conductivity value is different

Source	DF	SS	MS	F	P
Factor	3	844.688	281.563	14.3	.0003
Error	12	236.25	19.688		
Total	15	1080.938			

The F_0 value is 14.3 and the p-value is .0003

Since $a=4$, and $n=4$, the rejection criteria is $F_{3,12,.05} = 3.49$

Since $14.3 > 3.49$, the null hypothesis is rejected.

The type of coating does have an effect on conductivity.

b) $\hat{\mu}: \frac{\sum_{i=1}^{4}\sum_{j=1}^{4} M_{ij}}{4\times4} = 137.9375$

$\hat{\tau}_1 = \bar{y}_1 - \hat{\mu} = 145 - 137.9375 = 7.0625$

$\hat{\tau}_2 = \bar{y}_2 - \hat{\mu} = 145.25 - 137.9375 = 7.3125$

$\hat{\tau}_3 = \bar{y}_3 - \hat{\mu} = 132.25 - 137.9375 = |-5.6875| = 5.6875$

$\hat{\tau}_4 = \bar{y}_4 - \hat{\mu} = 129.25 - 137.9375 = |-8.6875| = 8.6875$

3.20 c) 95% CI of Trt. 4 mean : $\mu_4 \pm t_{\alpha/2, N-a}\sqrt{\frac{MS_E}{n}} = 129.25 \pm t_{.025,12}\sqrt{\frac{19.6875}{4}}$

$= 129.25 \pm 4.834 = \boxed{(124.416, 134.084)}$

99% CI of mean difference T_1, T_4 : $(\mu_1 - \mu_4) \pm t_{\alpha/2, N-a}\sqrt{\frac{2\,MS_E}{n}}$

$= 145 - 129.25 \pm t_{.005,12}\sqrt{\frac{2 \times 19.6875}{4}}$

$= 15.75 \pm 9.58$

$= \boxed{(6.17, 25.33)}$

d) $LSD = t_{\alpha/2, N-a}\sqrt{\frac{2\,MS_E}{n}} = t_{.025,}\sqrt{\frac{2 \times 19.6875}{4}} = 2.1788 \times 3.1375 = 6.8359$

T_1 vs $T_2 \Rightarrow |145 - 145.25| = .250 < 6.8359$ ✗

T_1 vs $T_3 \Rightarrow |145 - 132.25| = 12.75 > 6.8359$

T_1 vs $T_4 \Rightarrow |145 - 129.25| = 15.75 > 6.8359$

T_2 vs $T_3 \Rightarrow |145.25 - 132.25| = 13 > 6.8359$

T_2 vs $T_4 \Rightarrow |145.25 - 129.25| = 16 > 6.8359$

T_3 vs $T_4 \Rightarrow |132.25 - 129.25| = 3 < 6.8359$ ✗

At $\alpha = .05$, all pairs apart from T_1 & T_2 and T_3 & T_4 are significantly different. This means pairs T_1 & T_2 and T_3 & T_4 are not different.

Anova Table Created

3.31 a)

Source	DF	SS	MS	F	P
Wafer Pos	3	16.2298	5.4066	8.29	.008
Error	8	5.2175	0.6522		
Total	11	21.4373			

	Variance Comp.	Error Term	MS
Wafer Pos		2	(2) + 3Q [1]
Error	.6522		(2)

R code: qf(.95, 3, 8, lower.tail=TRUE) = .1130552

Since 8.29 > .1130552, there is a difference.

b) $\hat{\sigma}_{\tau}^2 = \frac{MS_{\tau} - MS_E}{n}$

$\hat{\sigma}_{\tau}^2 = \frac{5.4066 - .6522}{3}$

$= \boxed{1.584}$

c) The calculated ANOVA table shows that

$\boxed{\hat{\sigma}^2 = .6522}$

3.32) a) Total Variability in Uniformity Response

$$\sigma_{\tau}^{2} + \hat{\sigma}^{2} = 1.58463 + 0.6523 = \underline{\underline{2.23693}}$$

b) $$\text{cor}(y_{ij}, y_{ik}) = \frac{\sigma_{\tau}^{2}}{\sigma^{2} + \sigma_{\tau}^{2}} = \frac{1.58463}{2.23693} = \underline{\underline{.7083}}$$

c) Since $\hat{\sigma}^{2} = .6523$, $$\frac{2.23693 - .6523}{2.23693} = .7083 \approx 70.83\%$$

$\boxed{\text{The reduction would roughly 70.83\%, which is a significant reduction.}}$