HW # 9

8. Suppose $a, b, c, \in \mathbb{Z}$. If $a^2+b^2=c^2$, then a or b is even.

$\Rightarrow$ a & b are both odd, and $a^2+b^2=c^2$

$$a = 2k+1 \ , \ b = 2d+1$$

$$a^2+b^2 = (2k+1)^2 + (2d+1)^2 = c^2$$

$$4k^2+4k+1+4d^2+4d+1 = c^2$$

$$4(k^2+k+d^2+d)+2 = c^2$$

It is contradictory to think the square of c will be in the form $4p$ or $4p+1$

9. Suppose $a, b \in \mathbb{R}$. If a is rational and ab is irrational, b is irrational.

$\Rightarrow$ b is rational & a is rational, ab is irrational

$$a = \frac{c}{d} \ , \ b = \frac{e}{f}$$

$$ab = \left(\frac{c}{d}\right)\left(\frac{e}{f}\right) = \frac{ce}{df}$$

$\frac{ce}{df}$ must be rational, which contradicts ab being irrational ⊠

10. There exists no integers a and b for which $21a+30b=1$.

$\Rightarrow$ a & b in $\mathbb{Z}$ such that $21a+30b=1$

$$3 \mid 21a \ \ \& \ \ 3 \mid 30b \Rightarrow 3 \mid (21a+30b)$$

There exists a value k in $\mathbb{Z}$ such that

$3k = 21a+30b$. But since $21a+30b=1$ and $1/3 = k$ is not in $\mathbb{Z}$, this contradicts $k \in \mathbb{Z}$.

The statement there exists a & b for which $21a+30b=1$ is false

Sorry for sloppy work, had to evacuate from fire