

# Math 8 HW # 15

25. <sup>Prove</sup> Fibonacci Sequence,  $F_1 + F_2 \dots F_n = F_{n+2} - 1$

When  $n=1$ ,  $F_1 = F_{1+2} - 1 = F_3 - 1 = 2 - 1 = 1$ .

Assume  $k \geq 1$  and  $F_1 + F_2 \dots F_k = F_{k+2} - 1$ .

$$\begin{aligned} \text{If } F_1 + F_2 + F_3 + F_4 + \dots F_k + F_{k+1} &= \\ (F_1 + F_2 + F_3 + F_4 + \dots F_k) + F_{k+1} &= \\ F_{k+2} - 1 + F_{k+1} &= (F_{k+1} + F_{k+2}) - 1 \\ &= F_{k+3} - 1 \end{aligned}$$

Thus by induction  $F_1 + F_2 + \dots F_n = F_{n+2} - 1$   $\square$

27. <sup>Prove</sup> Fibonacci Sequence,  $F_1 + F_3 + F_5 + \dots F_{2n-1} = F_{2n}$

$$\text{If } n \geq 1, \sum_{i=1}^n F_{2i-1} = F_{2n}$$

$$\text{Then, } \sum_{i=1}^{n+1} F_{2i-1} = F_{2n+1} + \sum_{i=1}^n F_{2i-1}$$

$$= F_{2n+1} + F_{2n}$$

$$= F_{2n+2}$$

$$= F_{2(n+1)}$$

Thus by induction,  $F_1 + F_3 + F_5 + \dots F_{2n-1} = F_{2n}$   $\square$

1. Let  $A = \{0, 1, 2, 3, 4, 5\}$   $R$  expresses  $>$  on  $A$

$$R = \{(5,4)(5,3)(5,2)(5,1)(5,0)(4,3)(4,2)(4,1)(4,0)(3,2)(3,1)(3,0)(2,1)(2,0)(1,0)\}$$

2. Let  $A = \{1, 2, 3, 4, 5, 6\}$   $R$  expresses  $!$  on  $A$

$$R = \{(6,6)(5,5)(4,4)(3,3)(2,2)(1,1)(6,5)(5,4)(4,3)(3,2)(2,1)(6,4)(4,2)(2,6)(6,3)(3,6)(3,1)(1,3)(1,6)(6,1)(1,5)(5,1)(5,6)(6,5)\}$$

3. Let  $A = \{0, 1, 2, 3, 4, 5\}$

$$R = \{(5,5)(5,4)(5,3)(5,2)(5,1)(5,0)(4,4)(4,3)(4,2)(4,1)(4,0)(3,3)(3,2)(3,1)(3,0)(2,2)(2,1)(2,0)(1,1)(1,0)(0,0)\}$$