27) Let a set H be open if $H = H^\circ$

Let $\mathbb{Q}^\circ$ = empty set ($\mathbb{Q}$ is not equal to $\mathbb{Q}^\circ$, not open set)

A set H is closed if $cl(H) = H$.

However, $cl(\mathbb{Q}) = \mathbb{R}$ ($cl(\mathbb{Q}) \neq \mathbb{Q}$, the set of rational numbers is not a closed set either.

The set of rational numbers is neither open or closed $\boxtimes$

30) i) Suppose x_0 is a accumulation point E'. Given $\varepsilon > 0$, there exists $x \in E'$ with $|x - x_0| < \frac{\varepsilon}{2}$

$$|x' - x_0| = |x' + x - x - x_0| \leq |x' - x| + |x - x_0|$$

$$< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$$

As such, x_0 is a limit point of E and contained by E'

$\Rightarrow$ E' is closed as it contains all its limit points $\boxtimes$

ii) If $x \in E$ then $x \in \bar{E}$ because $E \subseteq \bar{E}$

If $x \in E'$ then $x \in \bar{E}$ because $\bar{E}$ contains all its limit pts.

$\Rightarrow E \cup E' \subseteq E' \Rightarrow \bar{E} \subseteq E \cup E'$

Let $x \in \bar{E}$ ($x \in E$ or $x \in \bar{E} - E$)

$\therefore$ $x \in \bar{E}$ & $x \notin E$, x is a limit

$\Rightarrow x \in E'$

$x \in E \Rightarrow x \in E$

$\bar{E} \subseteq E \cup E'$. Thus, $\bar{E} = E \cup \bar{E}$ $\boxtimes$

31) Assuming F is an uncountable set, there exists a limit point of F in R. Let this point be x. This is because

$$F \subseteq \bigcup_{i=1}^{\infty} (r_i, r_i) = R \text{ such that } r_i > 0 \ \& \ r_i \to \infty \text{ as } i \to \infty$$

Since F is uncountable, there exists an interval (r_a, r_b) that contains infinitely many points of F, x_k.
Apart from this, we know F is countable (countable unions of countable sets are countable.

x_k in (r_a, r_b) has a limit point

$\exists\ x_n \subseteq F$ such that $x_k \to x$ as $k \to \infty \Rightarrow x \in F$ since F is closed

Because $x_n \to x$, x is not an isolated point of E $\boxtimes$