12.1) 7) $f = \{(x,y) \in \mathbb{Z} \times \mathbb{Z} : 3x + y = 4\}$

- This function is from $\mathbb{Z}$ to $\mathbb{Z}$ since $3x+y=4$ if and only if $y = 4-3x$, as $f: \mathbb{Z} \to \mathbb{Z}$ defined as $f(x) = 4-3x$

9) $f = \{(x^2, x) : x \in \mathbb{R}\}$

- No. If f contains the coords $(4,2)$ and $(4,-2)$, then 4 will occur as the first coordinate of more than 1 element of f.

10) $f = \{(x^3, x) \mid x \in \mathbb{R}\}$

- Yes, because for $f = \{(x^3, x) \mid x \in \mathbb{R}\}$ $\Rightarrow f = \{(x, x^{1/3}) \mid x \in \mathbb{R}\}$. For any x, $x^{1/3}$ is unique.

12.2) 6) $f(m,n) = 3n - 4m$

- Let $f(m1, n1) = f(m2, n2)$

$3n1 - 4m1 = 3n2 - 4m2$

$3(n1 - n2) = 4(m1 - m2)$

Let $n1 = 5, n2 = 1, m1 = 3, m2 = 0$

Thus $(3,5)$ and $(0,1)$, $f(m,n)$ is not injective

- For any $3n-4m$, we must find (m,n). Any number k can be written in the form $3n-4m$, Thus is it surjective.

8) $f(m,n) = (m+n, 2m+n)$

- Suppose $f(m,n) = f(a,b)$ for some $a,b,m,n \in \mathbb{Z}$. Then $(m+n, 2m+n) = (a+b, 2a+b)$. So $m+n = a+b$ and $2m+n = 2a+b$. $2m+n-(m+n) = 2a+b-(a+b) \Rightarrow m = a$. But $m+n = a+b$. So $a+n = a+b$, subtracting a from both sides, $n = b$. Then $(m,n) = (a,b)$, so f is injective.

- Suppose $(x,y) \in \mathbb{Z} \times \mathbb{Z}$. $(m+n, 2m+n) = (x,y)$. We need $x = m+n$ and $y = 2m+n$. Then $m = x-n$, $y = 2x-2n+n$, $y = 2x-n$, $n = 2x-y$, $-2x+y+x = m = y-x$

$f(y-x, 2x-y) = (y-x+2x-y,\ 2y-2x+2x-y) = (x,y)$.

Thus it is surjective.