1) a) $P(S_1 = 0 \mid S_5 = 0, S_0 = 1)$

$$= \frac{P(S_1 = 0, S_5 = 0, S_0 = 1)}{P(S_5 = 0, S_0 = 1)} = \frac{P(S_5 - S_1 = 0, S_1 - S_0 = 1, S_0 = 1)}{P(S_5 - S_0 = -1, S_0 = 1)}$$

$$= \frac{P(S_5 - S_1 = 0) \cdot P(S_0 = 1) \cdot P(S_1 - S_0 = 1)}{P(S_5 - S_0 = -1) P(S_0 = 1)} = \frac{P(S_5 - S_1 = 0) \cdot P(S_1 - S_0 = 1)}{P(S_5 - S_0 = -1)}$$

$$P(S_5 - S_1 = 0) = \binom{4}{2}(.3)^2(.7)^2 = 6(.3)^2(.7)^2$$

$$= .2646$$

$$P(S_1 - S_0 = 1) = P(X_1 = 1) = .7 = q$$

$$\binom{5}{2}(.3)^2(.7)^2 = P(S_5 - S_0 = -1) = 10(.3)^2(.7)^2$$

$$= .3087$$

$$\frac{P(S_5 - S_1 = 0) \cdot P(S_1 - S_0 = 1)}{P(S_5 - S_0 = -1)}$$

$$= \frac{(.2646)(.7)}{(.3087)} = \boxed{.6}$$

b) $P(S_5 = 0 \mid S_3 = 2)$ $S_0 = 1$, $q = .7$, $p = .3$

$P(S_5 = 0 \mid S_3 = 2, S_0 = 1)$

$$= \frac{P(S_5 = 0, S_3 = 2, S_0 = 1)}{P(S_3 = 2, S_0 = 1)} = \frac{P(S_5 - S_3 = -2, S_3 - S_0 = 1, S_0 = 1)}{P(S_3 - S_0 = 1, S_0 = 1)}$$

$$= \frac{P(S_5 - S_3 = -2) \cdot P(S_0 = 1) \cdot P(S_3 - S_0 = 1)}{P(S_3 - S_0 = 1) P(S_0 = 1)} = P(S_5 - S_3 = -2)$$

$$= \binom{2}{0}(.3)^0(.7)^2 = \boxed{.49}$$

1) c) $P(M_{10} \geq 4, S_{10} \geq 4) = P(S_{10} \geq 4) = P(S_{10} - S_0 \geq 3)$

$= P(S_{10} - S_0 = 4, 6, 8, 10)$

$= P(S_{10} - S_0 = 4) + P(S_{10} - S_0 = 6)$

$+ P(S_{10} - S_0 = 8) + P(S_{10} - S_0 = 10)$

- $P(S_{10} - S_0 = 4) = \binom{10}{7}(.3)^7(.7)^3$

$= 120 \cdot (.3)^7(.7)^3 = \underline{.0090016}$

- $P(S_{10} - S_0 = 6) = \binom{10}{8}(.3)^8(.7)^2$

$= (45)(.3)^8(.7)^2 = \underline{.0014670}$

- $P(S_{10} - S_0 = 8) = \binom{10}{9}(.3)^9(.7)^1$

$= 10 \cdot (.3)^9(.7)^1 = \underline{.0001378}$

- $P(S_{10} - S_0 = 10) = \binom{10}{10}(.3)^{10}(.7)^0$

$= (.3)^{10} \cdot 1 = \underline{.0000059}$

$.0090016 + .0014670 + .0001378 + .0000059$

$= \underline{.0106123}$

2) a) $P(S_2=0, S_4=0, S_5=-1 \mid S_0=0) = \dfrac{P(S_2=0, S_4=0, S_5=-1, S_0=0)}{P(S_0=0)}$

$= \dfrac{P(S_5-S_4=-1, S_4-S_2=0, S_2-S_0=0, S_0=0)}{P(S_0=0)}$

$= \dfrac{P(S_5-S_4=-1)\cdot P(S_4-S_2=0)\cdot P(S_2-S_0=0)\cdot P(S_0=0)}{P(S_0=0)}$

$= P(S_5-S_4=-1)\cdot P(S_4-S_2=0)\cdot P(S_2-S_0=0)$

- $P(S_5-S_4=-1) = \binom{1}{0}(.4)^0(.6)^1 = \underline{\underline{.6}}$
- $P(S_4-S_2=0) = \binom{2}{1}(.4)^1(.6)^1 = \underline{\underline{.48}}$
- $P(S_2-S_0=0) = \binom{2}{1}(.4)(.6) = \underline{\underline{.48}}$

$= P(S_5-S_4=-1)\cdot P(S_4-S_2=0)\cdot P(S_2-S_0=0)$

$= .6 \times .48 \times .48 = \underline{\underline{.13824}}$

b) $P(\{S_4=4\} \cup \{S_4=-2\}) = P(S_4=4 \mid S_0=0) + P(S_4=-2 \mid S_0=0)$

$= P(S_4-S_0=4) + P(S_4-S_0=-2)$

- $P(S_4-S_0=4) = \binom{4}{4}(.4)^4(.6)^0 = \underline{\underline{.0256}}$
- $P(S_4-S_0=-2) = \binom{4}{1}(.4)^1(.6)^3 = \underline{\underline{.3456}}$

$= P(S_4-S_0=4) + P(S_4-S_0=-2)$

$.0256 + .3456 = \underline{\underline{.3712}}$

2 c) $P(M_{17} \leq -5, S_7 = -5)$

$$\{M_{17} \leq -5\} \supseteq \{S_7 = -5\}$$

$$P(M_{17} \leq -5, S_7 = -5) = P(S_7 = -5)$$

$$= P(S_7 = -5 \mid S_0 = 0)$$

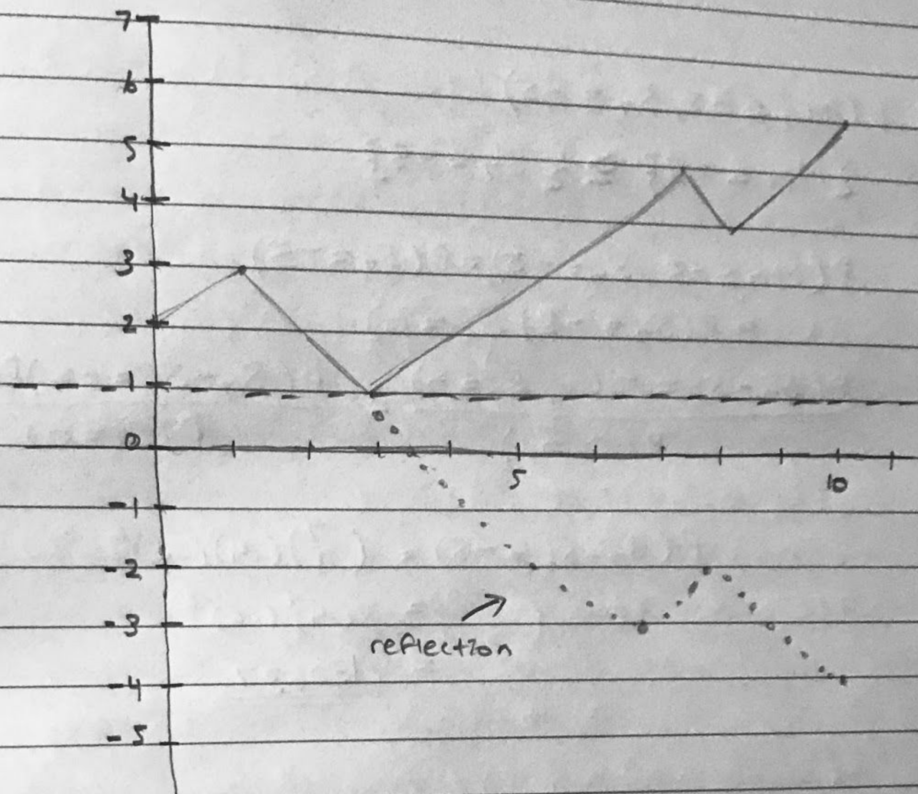
$$\frac{P(S_7 - S_0 = -6, S_0 = 0)}{P(S_0 = 1)} = \frac{P(S_7 - S_0 = -6)P(S_0 = 0)}{P(S_0 = 0)}$$

$$P(S_7 - S_0 = -5) = \binom{7}{1}(.4)^1(.6)^6$$

$$= 7(.4)^1(.6)^6$$

$$= \underline{\underline{.130637}}$$

3)



We can see that $S_3 = 2$ to $S_{10} = -4$ is a reflection of $S_3 = 2$ to $S_{10} = 6$. The number of paths $S_0 = 2$ to $S_{10} = 6$ is equivalent to the number of paths $S_0 = 2$ to $S_{10} = -4$.

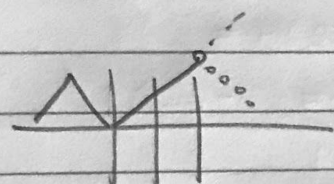
Assuming u is up-moves and d is down-moves,

$$r + s = 10 \quad \Big| \quad r + r + s - s = 4$$
$$r - s = 6 \quad \Big| \quad r = 2$$

$$\binom{10}{2} = \underline{\underline{45}}$$

4) $P(M_8 = 6) = P(M_8 \geq 6) - P(M_8 \geq 7)$

$$P(M_8 \geq 6) = P(M_8 \geq 6, S_8 \geq 6) + P(M_8 \geq 6, S < 6)$$
$$= P(S_8 \geq 6) + P(S_8 \geq 7)$$



$S_8 \geq 6 \Rightarrow M_8 \geq 6$

$$P(M_8 \geq 7) = P(S_8 \geq 7) + P(S_8 > 8)$$
$$P(M_8 \geq 6) = P(S_8 \geq 6) + P(S_8 \geq 7)$$
$$- P(S_8 \geq 7) - P(S_8 \geq 8)$$
$$P(S_8 \geq 6) - P(S_8 \geq 8) = P(S_8 = 6) + P(S_8 = 7)$$
$$= \binom{8}{\frac{1}{2}(8+0)} \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)$$
$$= \underline{\underline{.03125}}$$