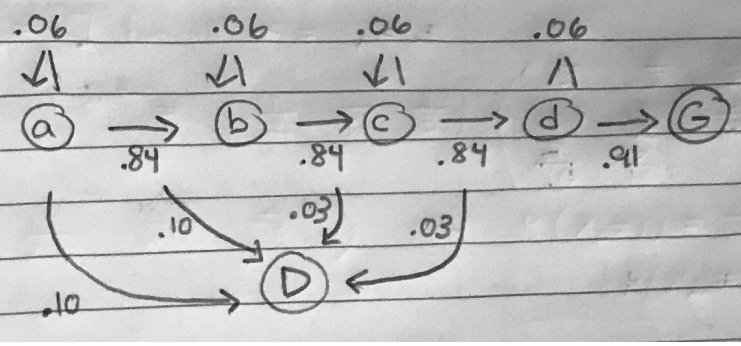
HW 6

1) Let: G = graduate (1) a = freshman (1st yr)
D = drop-out (2) b = sophomore
c = junior
d = senior

$S = \{ G, D, a, b, c, d \}$

$$P = \begin{array}{c} \\ (1)\ a \\ (2)\ b \\ (3)\ c \\ (4)\ d \\ (5)\ G \\ (6)\ D \end{array} \begin{array}{c} \begin{array}{cccccc} (1) & (2) & (3) & (4) & (5) & (6) \\ a & b & c & d & G & D \end{array} \\ \begin{pmatrix} .06 & .84 & 0 & 0 & 0 & .10 \\ 0 & .06 & .84 & 0 & 0 & .10 \\ 0 & 0 & .06 & .91 & 0 & .03 \\ 0 & 0 & 0 & .06 & .91 & .03 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$



2) $S = \{1, 2, 3\}$

(1) (2) (3)

$\alpha^T = (1/2, 1/2, 0)$

(1) $\hookrightarrow P =$

	(1)	(2)	(3)
(1)	0	1	0
(2)	1/3	0	2/3
(3)	1/5	1/10	7/10

(2) $P = \begin{pmatrix} 0 & 1 & 0 \\ 1/3 & 0 & 2/3 \\ 1/5 & 1/10 & 7/10 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1/3 & 0 & 2/3 \\ 1/5 & 1/10 & 7/10 \end{pmatrix} = \begin{pmatrix} 1/3 & 0 & 0 \\ 2/15 & 2/5 & 7/15 \\ 13/75 & 27/100 & 167/300 \end{pmatrix}$

a) $P[X_5 = 2 \mid X_4 = 1]$

$= P[X_1 = 2 \mid X_0 = 1] = P'_{1,2} = 1$

b) $P[X_1 = 1, X_2 = 2]$

$= P[X_2 = 2 \mid X_1 = 1] \times P[X_1 = 1]$

$= P'_{1,2} \times P[X_1 = 1] = 1 \times \frac{1}{6} = \frac{1}{6}$

$(1/2 \quad 1/2 \quad 0) \begin{pmatrix} 0 & 1 & 0 \\ 1/3 & 0 & 2/3 \\ 1/5 & 1/10 & 7/10 \end{pmatrix} = (1/6 \quad 1/2 \quad 1/3)$

$P[X_1 = 1] = 1/6, \; P[X_1 = 2] = 1/2, \; P[X_1 = 3] = 1/3$

c) $P[X_1 = 2 \mid X_2 = 1]$

$= \dfrac{P[X_2 = 1 \mid X_1 = 2] \cdot P[X_1 = 2]}{P[X_2 = 1]}$

$P[X_2 = 1 \mid X_1 = 2] = P^2_{2,1} = 1/3$

$P[X_1 = 2] = 1/2$

$P[X_1 = 3] = 1/3$

$P(X_2 = k) = \alpha^T P^2$

$(1/2 \quad 1/2 \quad 0) \begin{pmatrix} 0 & 1 & 0 \\ 1/3 & 0 & 2/3 \\ 1/5 & 1/10 & 7/10 \end{pmatrix}$

$= (7/30 \quad 1/5 \quad 17/30)$

(Check Next Page) $P[X_2 = 1] = \frac{7}{30}$ $P[X_2 = 2] = \frac{1}{5}$, $P[X_2 = 3] = \frac{17}{30}$

c) $P[X_2=1] = \frac{7}{30}$

$$\frac{P[X_2=1 \mid X_1=2] - P[X_1=2]}{P[X_2=1]}$$

DNT

$$= \left(\left(\tfrac{1}{3}\right)\left(\tfrac{1}{2}\right)\right) / (7/30)$$

$$= \underline{\underline{5/7}}$$

d) $P[X_5=1 \mid X_1=2, X_2=3, X_3=2]$

$$P[X_5=1 \mid X_1=2, X_2=3, X_3=2]$$
$$= P[X_5=1 \mid X_3=2]$$
$$= P[X_2=1 \mid X_0=2]$$
$$= P_{2,1}^2 = \underline{\underline{2/15}}$$

3) a) $P[X_2=k]$ for all $k=1,2,3$

$$= \alpha^T P^2 = \begin{pmatrix} \frac{1}{2} & \frac{1}{6} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} \frac{1}{5} & \frac{3}{5} & \frac{1}{5} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{3}{10} & \frac{7}{10} & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{5} & \frac{3}{5} & \frac{1}{5} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{3}{10} & \frac{7}{10} & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{19}{200} & \frac{167}{300} & \frac{209}{600} \end{pmatrix} \Rightarrow \boxed{P(X_2=1) = \frac{19}{200},\ P(X_2=2) = \frac{167}{300},\ P(X_2=3) = \frac{209}{600}}$$

b) $E[X_2] = \sum_{k=1}^{3} k\, P(X_2=k) = \frac{19}{200} \times 2\left(\frac{167}{300}\right) \times 3\left(\frac{209}{600}\right)$

$$= \frac{169}{75}$$

P1 → The dist of X_2 in (a) does depend on initial dist α. (Yes)
P2 → The dist of X_2 in (b) does depend on initial dist α. (Yes)
P1 → $\alpha^T P^2$ is used to calculate $P[X_2=k]$ for all $k=1,2,3$
P2 → We use the discrete distribution info involving $\alpha^T P^2$ to find $E[X^2]$