PSTAT 120B - Quiz 1 (Shravan) Shenoy

1.)
$$X = -n \log \omega$$

 $U_i = -\log Y_i$

$$\Rightarrow \left| U_{\xi} \sim E_{xp} (1) \right|$$

$$\Rightarrow \left| E(U_{\xi}) = 1 \right|$$

$$\Rightarrow |Var(U_t) = 1|$$

$$\Rightarrow |m_U(t) = \frac{1}{1-t}|$$

$$m_{U}(t) = \frac{1}{1-t}$$

b)
$$X = -n \log \omega$$

$$= \sum_{i=1}^{n} U_i$$

$$M_{\dot{x}}(t) = \mathbb{E}(e^{xt})$$

$$= \mathbb{E}\left(e^{\frac{1}{2}\cdot\frac{\sqrt{2}}{2}} \cdot v_{\epsilon}\right)$$

$$= \prod_{i=1}^{n} \mathbb{E}\left(e^{\mathbf{U}_{i} \cdot \mathbf{t}}\right)$$

$$M_{x}(t) = \frac{1}{(1-t)^n}$$

Gamma Distribution has a similar M&F with the same Functional form

C) MGF from previous problem (16) shows properties of gamma distribution, in which

Because of this; we can find the density to be:

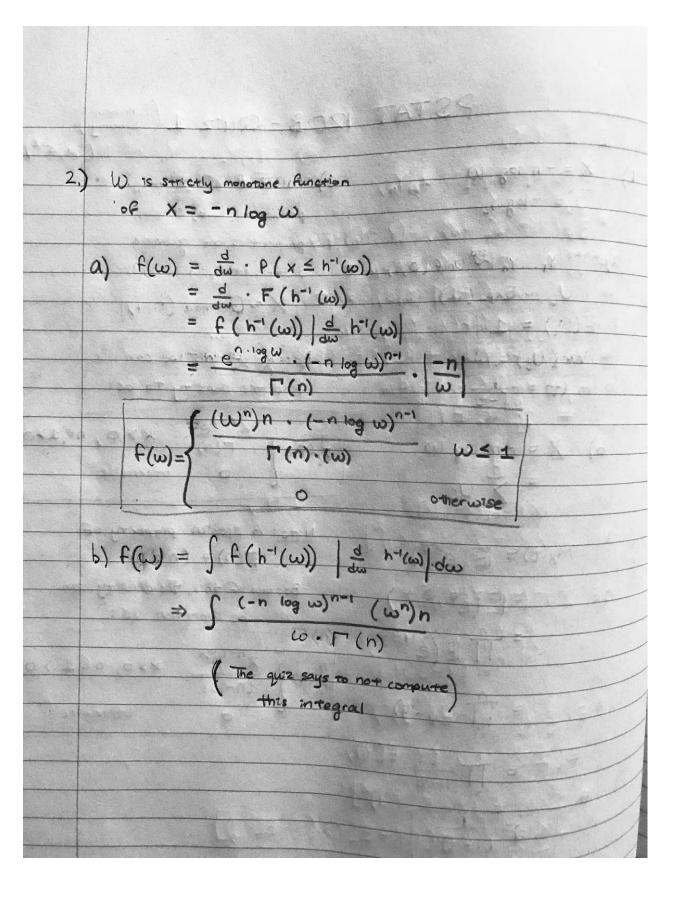
$$f_{\chi}(x) = \begin{cases} \frac{x^{n-1}e^{x}}{\Gamma(n)} & x > 0, n \neq 1 > 0 \\ 0 & \text{otherwise} \end{cases}$$

Note: A special property of

Because of this, the density can

also be written as:

$$f_{x}(x) \begin{cases} \frac{x^{n-1}e^{-x}}{(n-1)!} & x>0, \ n \leqslant 1 > 0 \end{cases}$$



3.)
$$-\log \omega = \overline{U}$$
 $\overline{U} \sim N(\mu, \frac{5^2}{n})$
 $50 - \log \omega \sim N(\mu, \frac{5^2}{n})$
Aln $\omega = \frac{1}{n} \sum_{i=1}^{n} \ln Y_i$
 $\sin \alpha = \omega = \left(\frac{1}{1} \cdot Y_i\right)^{1/n}$
For personal reference π

a)
$$n \cdot -|n w| = -\frac{n}{2} \ln \gamma$$
;
 $n \cdot +|n w| = Gamma(n, 1)$
 $-|n w| = Gamma(n, 1/n)$

$$E[-\ln \omega] = \frac{1}{n} \times n = 1$$

$$Var[-\ln \omega] = \left(\frac{1}{n}\right)^{2} \times n = \frac{1}{n}$$
Using CLT $\left(\frac{X - M}{S}\right)$:

Now compare

$$N(1,\frac{1}{n})$$
 and $N(\mu,\frac{\sigma^2}{n})$

$$\mu = 1, \sigma^2 = 1$$

we already: - In w ~ N (1, 1)

So, we can now do:

We know:

Tinus, 重(0) ⇒ 1/2