8) Let $n = k+1 \Rightarrow (k+1, k+2) \cap \mathbb{N} = \phi$

There exists $\lambda \in (k+1, k+2) \cap \mathbb{N}$

$\Rightarrow k+1 < \lambda < k+2$ s.t $\lambda \in \mathbb{N}$

$\Rightarrow k < \lambda - 1 < k+1$

λ is a natural number, and k is also a natural number $\Rightarrow (\lambda - 1)$ is a natural number

(by contradiction)

$\lambda - 1 \in (k, k+1) \cap \mathbb{N}$

$\Rightarrow (k+1, k+2) \cap \mathbb{N} = \phi$ $\boxtimes$

10) Let ~~k~~ $k \neq n$, such that it still belongs to $[n, n+1)$

Thus, $|k-n| \geq 1$

$\Rightarrow \quad -1 \geq k-n \geq 1$

$\Rightarrow \quad n-1 \geq k \geq n+1$

Because the only integers within $n-1$ and $n+1$ are $n-1$, n, and $n+1$

$(n-1)$ & $(n+1) \notin [n, n+1)$

$n \in [n, n+1)$

Since the only integer in $[n, n+1)$ is n, theres only one integer in $[n, n+1]$ $\boxtimes$

12) We know that $\mathbb{Q}$ is dense in $\mathbb{R}$

There exists a rational number k such that

$$\frac{a}{\sqrt{2}} < k < \frac{b}{\sqrt{2}} \quad \text{Thus,} \quad a < k\sqrt{2} < b$$

$$\text{Let } i = k\sqrt{2}.$$

There exists irrational number i between a & b, thus implying irrational numbers are dense in $\mathbb{R}$ $\boxtimes$

14) $(1+r)^n \geq 1+r*n$

Let $n=1$

$(1+r)^1 \geq 1+r*1$

$1+r \geq 1+r$

Assume $(1+r)^m \geq 1+r*m$ for some $m>1$

Consider $(1+r)^{m+1} \geq 1+r*(m+1)$

$(1+r)^m \times (1+r) \geq (1+r*m)*(1+r)$

$\Rightarrow 1+r+r*m+r*(m+1) = 1+r*m > 0$

For every n, there exists $(1+r)^n \geq 1+r*n$ $\boxtimes$