1) $\frac{\sqrt{n}(\bar{Y}-p)}{\sqrt{\bar{Y}(1-\bar{Y})}} \xrightarrow{d} N(0,1)$

a) $Q(\theta) = P_\theta(T(y) \in R)$

$\rightarrow P_\theta\left(\frac{\sqrt{n}(\bar{Y}-\theta)}{\sqrt{\bar{Y}(1-\bar{Y})}} < C\right)$

$T(Y) \xrightarrow{d} N(0,1)$

$T(Y)$ is normal

Because of this, we can conclude

$\boxed{P_\theta\left(\frac{\sqrt{n}(\bar{Y}-\theta)}{\sqrt{\bar{Y}}(1-\bar{Y})} < C\right) \Rightarrow \Phi(C)}$

which is in terms of Gaussian CDF

b) $Q(\theta_0) = \alpha$ $\qquad \Phi(C) = \alpha$

$= \Phi(C)$

Because of this, we know that

$\Phi^{-1}(\alpha) = C$. We know from the HW that $\Phi^{-1}(\alpha) = -Z_\alpha$.

We can apply the same principle, so we can solve $-Z_\alpha = C$.

$\boxed{C = -Z_\alpha}$

c) $T(Y) \in R \Rightarrow \frac{\sqrt{n}(\bar{Y}-\theta)}{\sqrt{\bar{Y}(1-\bar{Y})}} < \Phi^{-1}(\alpha)$

From this, we know that:

$\Phi\left(\frac{\sqrt{n}(\bar{Y}-\theta)}{\sqrt{\bar{Y}(1-\bar{Y})}}\right) < \alpha$

As a result, the p value is:

$\boxed{\Phi\left(\frac{\sqrt{n}(\bar{Y}-\theta)}{\sqrt{\bar{Y}(1-\bar{Y})}}\right).}$

2) $Y_1 \dots Y_n \overset{iid}{\sim} \text{Bernoulli}(\theta)$

$n = 95$, $\bar{y} = 5/95$ $\qquad \text{eff}(\theta) = 1 - \frac{\theta}{1-\theta}$

a) $\begin{cases} H_0 : \theta = \theta_0 \\ H_A : \theta < \theta_0 \end{cases}$ $\rightarrow (\theta_0 = \theta)$

$1 - \frac{\theta}{1-\theta_0} \rightarrow 1 - \frac{\theta_0}{1-\theta_0}$

$1 - \frac{\theta_0}{1-\theta_0} = .9$

$.1 = \frac{\theta_0}{1-\theta_0}$

$.1(1-\theta_0) = \theta_0$

$.1 - .1\theta_0 = \theta_0$

$.1 = 1.1\theta_0$

$\theta_0 = \frac{.1}{1.1}$

$\boxed{\theta_0 = \frac{1}{11}}$

b) $\begin{cases} H_0 : \theta = \frac{1}{11} \\ H_A : \theta < \frac{1}{11} \end{cases}$

b) $T(Y) = \dfrac{\sqrt{n}(\bar{Y} - \theta)}{\sqrt{\bar{Y}(1-\bar{Y})}}$

$= \dfrac{\sqrt{95}\left(\frac{5}{95} - \theta\right)}{\sqrt{\frac{5}{95}\left(1 - \frac{5}{95}\right)}}$

$= \dfrac{9.746(.052 - .09)}{\sqrt{.052(.947)}}$

$\boxed{T(Y) = -1.668}$

c) $\alpha = .05 \qquad z_{.05} = 1.645$

$R = (-\infty, \Phi^{-1}(\alpha))$

- This is a lower tail test.
- We know from the problem that $\Phi^{-1}(\alpha) = -1.645$
- From the last problem, we also know that $T(Y) = -1.668$.
- $-1.668 < -1.645$ is true.

As a result, we should reject the null hypothesis.

3) a) Using the calculator from the link provided, we can solve for the p-value

$\Phi(-1.668) = .0476$

b) $(0, \hat{\theta}_u) = \left(0, \bar{Y} + Z_{\alpha}\sqrt{\frac{\bar{Y}(1-\bar{Y})}{n}}\right)$

$\Rightarrow (\text{eff}(\hat{\theta}_u), 1) = \left(1 - \frac{\hat{\theta}_u}{1-\hat{\theta}_u}, 1\right)$

$\Phi(-1.668) = .0476$

- 96% Confidence interval implies $\alpha = .04$

- Since $.04 < .0476$, we can expect to find the value .9 to be in the 96% CI, so the null hyp is not rejected