2\. i) For each real number $a \neq 0$, $a^2 > 0$.

Let $a = 3$, $a^2 = 9$, $9 > 0$

Let $a = 5$, $a^2 = 25$, $25 > 0$

Let $a = 12$, $a^2 = 144$, $144 > 0$

Let $a = -2$, $a^2 = 4$, $4 > 0$

Let $a = 1$, $a^2 = 1$, $1 > 0$

For each real number $a \neq 0$, $a^2 > 0$ ⊠

ii) For each positive number a, its multiplicative inverse is positive

Let $a = 3$, $3 \times \frac{1}{3} = 1$, $1 > 0$

Let $a = 1$, $1 \times \frac{1}{1} = 1$, $1 > 0$

Let $a = 100$, $100 \times \frac{1}{100} = 1$, $1 > 0$

Because $1 > 0$, 1 is positive, thus for each positive number a, its multiplicative inverse is positive ⊠

iii) If $a > b$, then let

$a = 8$, $b = 3$. If $c > 0$, Let $c = 5$

$ac > bc$

$8(5) > 3(5)$

$40 > 15$

$ac > bc$ ✓

Let $a = 8$, $b = 3$. If $c < 0$, Let $c = -5$

$ac < bc$

$8(-5) < 3(-5)$

$-40 < -15$

$ac < bc$ ✓

4) i) If $ab = 0$, either $a = 0$ or $b = 0$

Let $a \neq 0$ and $a^{-1} = \frac{1}{a}$.

Let $ab = 0$

$$a^{-1} \cdot (ab) = 0 \cdot a^{-1}$$

$$(a^{-1}a)\, b = 0$$

$$\Rightarrow 1 \cdot b = 0$$

$$b = 0$$

The same can be argued for $a = 0$ assuming the variables are switched

ii) $(a-b)(a+b)$

$= a^2 + ba - ba - b^2$

$= a^2 - b^2$

If $a^2 = b^2$, then $a^2 - b^2 = 0$

$\Rightarrow (a-b)(a+b) = 0$

$a+b = 0$ or $a - b = 0$

Either $a = b$ or $a = -b$ $\boxtimes$

iii) Let c be a positive real number, let $E = \{x \in \mathbb{R} \mid x^2 < c\}$

For $0 = 0^2$, $0^2 \in \mathbb{R}$, $0 <$ any positive number

$\Rightarrow E$ is non-empty.

$x_0 = \sup E$; this implies $x \in E$, $x_0 < x$

Thus, $x_0 = E$ & $x_0^2 = c$

$x^2 = c \Rightarrow x^2 - c = 0$

$\Rightarrow x^2 - (\sqrt{c})^2 = 0$

$\Rightarrow (x - \sqrt{c})(x + \sqrt{c}) = 0$

$x = \sqrt{c}, -\sqrt{c}$

$c > 0$, $x = \sqrt{c} \Rightarrow x > 0$

6) Let $-E := \{-x : x \in E\}$

Let a be a lower bound for E. $\Rightarrow$

$x > a, \forall x \in E \Rightarrow -x < -a, \forall x \in E$.

$\Rightarrow y < -a, \forall y \in -E$

Because of the completeness axiom $-E$ has supremum, s

Let $-s$ be infimum of E.

Since $y < b, \forall y \in -E \Rightarrow x > -b, \forall x \in E$.

Let c exist such that $x > c, \forall x \in E$

$\Rightarrow -x < -c, \forall -x \in E \Rightarrow y < -c, \forall y \in -E$

$\Rightarrow b < -c \Rightarrow -b > c$ (true because b is supremum of the set $-E$.)

Thus, $-b$ is infinum of E $\boxtimes$