1) a) A reasonable sample space

$\Omega = \{(1,1),(1,2),(1,3)(1,4),$
$(2,1),(2,2),(2,3),(2,4),$
$(3,1),(3,2),(3,3),(3,4),$
$(4,1),(4,2),(4,3),(4,4)\}$

The probabilities $P(\{\omega\})$ for each possible outcome is from sample space Ω is $1/16$.

$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

b) The state space S_X of X is:

$\{1,2,3,4\}$

c)

min(1,1) = 1	min(2,1) = 1	min(3,1) = 1	min(4,1) = 1
min(1,2) = 1	min(2,2) = 2	min(3,2) = 2	min(4,2) = 2
min(1,3) = 1	min(2,3) = 2	min(3,3) = 3	min(4,3) = 3
min(1,4) = 1	min(2,4) = 2	min(3,4) = 3	min(4,4) = 4

$$\text{pmf of } X = P_X(x) = \begin{cases} 7/16 & \text{if } x = 1 \\ 5/16 & \text{if } x = 2 \\ 3/16 & \text{if } x = 3 \\ 1/16 & \text{if } x = 4 \\ \emptyset & \text{otherwise} \end{cases}$$

d)

$$F_X(x) = \begin{cases} 0 & x < 0 \\ 7/16 & 0 \leq x \leq 1 \\ 12/16 & 1 \leq x < 2 \\ 15/16 & 2 \leq x < 3 \\ 1 & 3 \leq x < 4 \end{cases}$$

2) a) P(only one typo) = P(Student A does one typo) + P(Student B does one typo)

$= \left(\frac{1}{3}\right)\left(e^{-1} \times \frac{1}{1}\right) + \left(\frac{2}{3}\right)\left(e^{-10} \times \frac{10}{1}\right)$

$= \boxed{.1229}$ or ~.123

b) $\left(\frac{1}{3}\right)\left(e^{-1} \times \frac{1^0}{0!}\right) + \left(\frac{2}{3}\right)\left(e^{-10} \times \frac{10^0}{0!}\right)$ = P(Student A, no typos) + P (Student B, no typos)

$= .12262 + 3.0266 \times 10^{-5} = .12265$, or ~.1227

$\Rightarrow \dfrac{\left(\frac{2}{3}\right)\left(e^{-10} \times \frac{10^0}{0!}\right)}{(.1227)} = \dfrac{P(\text{Student B made it} \mid \text{No typo})}{P(\text{No typos})}$

$\boxed{= .000246671 \text{ or } \sim .0002467}$

3) a) - X is a binomial random variable that has parameters:

$n = 600 \quad p = 1/6$

- The state space $S_x = \{0, 1, 2, 3, 4, \dots 600\}$

b) $P(X \le 100) = \sum_{x=0}^{100} P(X = x)$

$= \sum_{x=0}^{100} \binom{600}{x} \left(\frac{1}{6}\right)^x \left(\frac{6}{6} - \frac{1}{6}\right)^{600-x}$

$= \boxed{.5266}$

c) The probability in 3b can be approximated by the value 1/2 using CLT (Central Limit Theorem)

$$\frac{\sqrt{n}\,(X - E(X))}{\sigma} \sim N(0,1)$$

$$P(X \le 100) = P\left(\frac{\sqrt{n}\,(x-100)}{\sigma} \le 0\right)$$

$$= \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}} e^{\frac{-t^2}{2}}\, dt$$

$$= \underline{1/2}$$

$n \cdot p = 600 \cdot \left(\frac{1}{6}\right) = 100$

$\sigma = \sqrt{100\left(\frac{5}{6}\right)} = 9.128$

$\frac{\sqrt{n}\,X}{\sigma} \sim N(0,1)$

4) a) $\int_0^2 c(4t - 2t^2) = 1$

$\left[c\left(\frac{4t^2}{2} - \frac{2t^3}{3}\right)\right]_0^2 = 1$

$= c\left(\frac{4(2)^2}{2} - \frac{2(2)^3}{3}\right) = 1$

$= c\left(8 - \frac{16}{3}\right) = 1$

$c\left(\frac{8}{3}\right) = 1$

$\boxed{c = 3/8}$

b) $\int_0^x c(4t - 2t^2)\, dt$

$= c\left[\frac{4t^2}{2} - \frac{2t^3}{3}\right]_0^x$

$= c\left[2t^2 - \frac{2t^3}{3}\right]_0^x$

$\frac{3}{8}\left(2x^2 - \frac{2x^3}{3}\right)$

$\frac{3}{8}\left(\frac{6x^2 - 2x^3}{3}\right)$

$= \frac{1}{8}(6x^2 - 2x^3)$

$= \frac{2x^2}{8}(3 - x)$

$$F_x(x) = \begin{cases} \frac{1}{4}x^2(3-x) & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

4c) $\cdot$ $P(X=1)$ is always going to equal zero

$\cdot$ $P(X \geq 1) = 1 - P(X \leq 1)$

$= 1 - F_X(1)$

$= 1 - \left(\left(\frac{1}{4}\right)(3-1)\right)$

$= 1 - \frac{2}{4}$

$= 1/2$

4d) $E(x) = \int_0^2 t\left(\frac{3}{8}(4t - 2t^2)\right)dt$

$= \frac{3}{8}\left[\frac{4t^3}{3} - \frac{2t^4}{4}\right]_0^2$

$= \frac{3}{8}\left[\frac{4(2)^3}{3} - \frac{2(2)^4}{4}\right]$

$= \frac{3}{8}\left[\frac{32}{3} - \frac{32}{4}\right]$

$= \frac{3}{8}\left[\frac{32}{3} - 8\right]$

$= \frac{3}{8}\left[\frac{8}{3}\right]$

$= 1$

$E(x^2) = \int_0^2 t^2\left[\frac{3}{8}(4t - 2t^2)\right]dt$

$= \frac{3}{8}\int_0^2 (4t^3 - 2t^4)\,dt$

$= \frac{3}{8}\left[\frac{4(2)^4}{4} - \frac{2(2)^5}{5}\right]_0^2$

$= \frac{3}{8}\left[16 - \frac{64}{5}\right]$

$\frac{48}{8} - \left(\frac{3}{8}\right)\left(\frac{64}{5}\right)$

$= \frac{48}{8} - \frac{24}{5}$

$= 6 - \frac{24}{5} = \frac{6}{5}$

$E(x^2) - (E(x))^2 = Var(x)$

$= \frac{6}{5} - (1)^2$

$= \boxed{1/5}$