11.3   11.4

13, 14, 15,   2, 6

HW 18

11.3) 13) Suppose R is an equiv. relation on a finite set A, and every eq. has same cardinality m. Express $|R|$ in terms of $|A|$ and m

$= m\,|A|$.

14) • Let $x \in A$,

If R is reflexive then xRx.

Let $x_1 = x$.

$\therefore xRx_1, x_1Rx \Rightarrow xSy$, Therefore R is reflexive.

• Let xSy, then there exists natural numbers n such that $xRx_1, x_1Rx_2, x_3Rx_4 \ldots x_{n-1}Rx_n$ and x_nRy. If R is symmetric then $x_1Rx, x_2Rx_4 \ldots x_nRx_{n-1}$ and yRx_n. Then, $yRx_n, x_nRx_{n-1} \ldots x_2Rx_2, x_3Rx_4, x_1Rx$

$\Rightarrow ySx$

$xSy \Rightarrow ySx$, thus S is Symmetric.

• Let xSy and ySz therefore $\exists$ nat. number m & n such that xRx_1, $x_1Rx_2 \ldots x_{n-1}Rx_n$ and x_nRy

$yRy_1, yRy_2 \ldots y_{m-1}Ry_m$ and y_mRz.

$\therefore xR'x_1, x_1R'x_2 \ldots x_nR'y$

So $xR'y \Rightarrow (x,y) \in R'$

$\therefore (x,y) \in S \Rightarrow (x,y) \in R'$

$\Rightarrow S \subseteq R_1$

Hence, S is unique smallest eq. on A containing R

15.) $R_1 = \{(a,a)(b,b)(c,c)(d,d)(a,b)(b,a)\}$

$R_2 = \{(a,a)(b,b)(c,c)(d,d)(a,c)(c,a)\}$

$R_3 = \{(a,a)(b,b)(c,c)(d,d)(a,d)(d,a)\}$

$R_4 = \{(a,a)(b,b)(c,c)(d,d)(b,c)(c,b)\}$

$R_5 = \{(a,a)(b,b)(c,c)(d,d)(b,d)(b,d)\}$

$R_6 = \{(a,a)(b,b)(c,c)(d,d)(a,b)(b,a)(a,c)(c,a)(b,c)(c,b)\}$

$R_7 = \{(a,a)(b,b)(c,c)(d,d)(c,d)(d,c)\}$

$\vdots$

$R_{15} = \{(a,a)(b,b)(c,c)(d,d)(a,b)(b,a)(a,c)(c,a)(b,c)(c,b) \ldots (d,a)(a,d)\}$

$= 15$ cases.

11.4) 2) $\{a,b,c\}$

$\{\{a,b\},\{c\}\}$

$\{\{a,c\},\{b\}\}$

$\{\{a\}\{b,c\}\}$

$\{\{a\}\{b\}\{c\}\}$

6) $P = \{\{0\},\{-1,1\}\{-2,2\}\{-3,3\}\ldots\}$

The equivalence relation of equivalence classes of P is:

$R = \{x \sim y \text{ iff } |x| = |y|\}$.