Math 8 HW # 7 (Shravan Shenoy)

1. If $x \in \mathbb{N}$ then $x + \frac{1-(-1)^x}{2}$ is even

$P =$ "$x \in N$"

$Q =$ "$x + \frac{1-(-1)^x}{2}$ is even"

Cases $\begin{cases} A_1 = \text{" } x \text{ is odd, } x + \frac{1-(-1)^x}{2} \mid 2 \text{ "} & (1) \\ A_2 = \text{" } x \text{ is even, } x + \frac{1-(-1)^x}{2} \mid 2 \text{ "} & (2) \end{cases}$ $x \in \mathbb{N} \Rightarrow A_1 \wedge A_2$ ✓

If x is a natural number, such as 1 (which is odd) or 2 (which is even), this implies $x + \frac{1-(-1)^x}{2}$ is even

Proof: Let x be a natural number

Case 1: suppose $x = 1$, which is a odd natural number. then, $1 + \frac{1-(-1)^1}{2}$ should be divisible by 2, thus shows if x is a natural number, such an equation will result in an even number. In this case, when x equals one, the equation will result in the number 2, which is divisible by 2, show that the equation results in an even number

Case 2: Suppose $x = 2$, which is an even natural number. Then, $2 + \frac{1-(-1)^2}{2}$ should be divisible by 2, thus showing if x is a natural number, such an equation will result in an even number. In this case, when x equals two, the equation will result in the number 2, which is divisible by 2, showing that the equation results in an even number.

Therefore for any natural number x, $x + \frac{1-(-1)^x}{2}$ is even ∎

2. If $y \in \mathbb{Z}$, then $2\cos\left(\frac{2\pi}{3}y\right) \in \mathbb{Z}$.

$P = y \in \mathbb{Z}$

$Q = 2\cos\left(\frac{2\pi}{3}y\right) \in \mathbb{Z}$

$$\text{Cases}\begin{cases} A_1 = \text{"}y = 3k \wedge 2\cos\left(\frac{2\pi}{3}(1)\right) \in \mathbb{Z}\text{"} \\ A_2 = \text{"}y = 3k+1 \wedge 2\cos\left(\frac{2\pi}{3}(0)\right) \in \mathbb{Z}\text{"} \\ A_3 = \text{"}y = 3k+2 \wedge 2\cos\left(\frac{2\pi}{3}(-1)\right) \in \mathbb{Z}\text{"} \end{cases}$$

$y \in \mathbb{Z} \Rightarrow A_1 \wedge A_2 \wedge A_3$ ✓

If y is an integer, then cases where y equals $3k, 3k+1, 3k+2$ is plugged in to the equation $2\cos\left(\frac{2\pi}{3}y\right)$ should result ~~in~~ a whole number that is within the range $(-\infty, \infty)$ so its an integer.

Proof: Let Y be an integer.

Case 1: $y = 3k$ where $k \in \mathbb{Z}$

$2\cos\left(\frac{2\pi}{3}(3k)\right) \Rightarrow 2\cos(2\pi \cdot k)$

$= 2$

$\boxed{2 \in \mathbb{Z}}$

Case 2: $y = 3k+1$ where $k \in \mathbb{Z}$

$2\cos\left(\frac{2\pi}{3}(3k+1)\right) = 2\cos\left(2\pi k + \frac{2\pi}{3}\right)$

Since $2\cos\left(\frac{2\pi}{3}\right) = 1$, then $\cos\left(\frac{2\pi}{3}(3k+1)\right) = 1$

$\boxed{1 \in \mathbb{Z}}$

Case 3: $y = 3k+2$ where $k \in \mathbb{Z}$

$2\cos\left(\frac{2\pi}{3}(3k+2)\right) = 2\cos\left(2\pi k + \frac{4\pi}{3}\right)$

Since $2\cos\left(\frac{4\pi}{3}\right) = -1$, then $2\cos\left(2\pi k + \frac{4\pi}{3}\right) = -1$

$\boxed{-1 \in \mathbb{Z}}$

Therefore for any integer y, $2\cos\left(\frac{2\pi}{3}y\right)$ results in an integer ⊠