1) Suppose f and g are continuous functions on $[a,b]$.
$f-g$ is continuous and $f-g=0$. Suppose there exists
some $c \in [a,b]$ such that $(f-g)(c) \neq 0$.
By continuity there exists $\delta > 0$ such that whenever
$x \in (c-\delta, c+\delta)$ there is $(f-g)(x) \in (0, 2(f-g)(c))$;
$f-g$ is nonzero on $(c-\delta, c+\delta)$ where $\delta(2) > 0$.
Thus, $f-g$ is identically zero so $f=g$ which is not
true for E measure zero

2)

true for E measure zero

3) Suppose is the measurable domain of f and we let $X = \{x_i\}_{i=1}^{n}$ be the finite collection of discontinuities. Because $|X|$ is finite, they can be ordered from smallest to largest $(x_1, \dots x_n)$.
We can then define $1 \leq i \leq n+1$ and $A_i = A \cap (x_{i-1}, x_i)$ where $x_0 = -\infty$ and $x_{n+1} = \infty$. Each A_i is measurable, and

$$\{x \in A : f(x) > c\} = \left(\bigcup_{i=1}^{n+1} \{x \in A_i : f(x) > c\}\right) \cup \{x \in X : f(x) > c\}$$

For each c, each set in the indexed union is measurable since f is measurable on each A_i and each subset of X is either empty or contains no more than n points and is therefore measurable as well.

9. Assume E_0 is measurable as well as $|f_n(x) - f_m(x)| < \frac{1}{n}$.
Using Cauchy's theorem, $E_0 = \{x \in E : |f_n(x) - f_m(x)| < \frac{1}{k}\}$
Because this is a σ algebra, E_0 is measurable.

12) Let f be a bounded measureable function on E. By approximation, the sequences exist such that $\Phi_n(x) \leq f \leq \Psi_n(x)$ and $\Psi_n(x) - \Phi_n(x) < \frac{1}{n}$ for each $n \in N$. using uniform continuity. For some $\epsilon > 0$, there exists $n \geq N$ such that $|\Phi_n(x) - f| < \frac{1}{n}$. We may take a max $\{\Phi_n(x)\}$.

For every non negative integer n, we may define

12) $A_n: R \to R$ by $A_n(t) = k \cdot g^{-n}$ for every t such that $k \leq 2^n \cdot t < k+1$ for some integer k. Thus, $|2^n A_n(t)$ is the integer part of $2^n t$. We can let $\Phi_n = A_n(t)$ and $\Psi_n = -A_n(t)$. Φ_n and Ψ_n are step functions where $\Phi_n \leq f \leq \Psi_n$ and $\Psi_n - \Phi_n \leq 2^{-n}$ for every n. The sequence (Φ_n) is nondecreasing and (Ψ) is non increasing.

14) Suppose f is measurable and $|f|$ is also measurable. Let us consider $A_n = \{x \in E : |f(x)| > n\}$, $n \in \mathbb{N}$. A_n is decreasing and $\bigcap_{n=1}^{\infty} A_n = \{x \in E : |f(x)| = \infty\}$. As $m(A_1) \leq m(E) < \infty$ we may apply monotone convergence thm and get that

$$\lim_{n \to \infty} m(A_n) = m(\{x \in E \mid |f(x)| = \infty\}) = 0 \quad \text{and}$$

$$m(\{x \in E \mid f(x) \mid = -\infty\}) = 0 \quad \text{because } f$$

is of finite measure. Thus, if we allow $\varepsilon > 0$, there exists an $n_\varepsilon \in \mathbb{N}$ such that $m(A_{n_\varepsilon}) < \varepsilon/2$.

Thus, $F = E / A_{n_\varepsilon}$ then $m(E \setminus A_{n_\varepsilon}) - m(F) < \varepsilon/2$