Math 8 HW 10

3. Given an integer a, then a^3+a^2+a is even iff a is even

$\Rightarrow$ If a is odd, then a^3+a^2+a is odd

Let $a=2k+1$

$a^3+a^2+a = (2k+1)^3+(2k+1)^2+(2k+1)$

$= 8k^3+16k^2+12k+2+1$

$= 2(4k^2+8k^2+6k+1)+1$

Because a^3+a^2+a can be written in this form, it is odd.

Thus, a^3+a^2+a is even if a is even ⊠

12. There exists a positive real number x for which $x^2<\sqrt{x}$

Let x be rational & $x=\frac{1}{k^2}$, where k is a pos integer.

$x^2=\frac{1}{k^4}$, $\sqrt{x}=\frac{1}{k}$

$x^2-\sqrt{x}=\frac{1}{k^4}-\frac{1}{k}=\frac{1-k^3}{k^4}$

Since $k\in\mathbb{N}$, $1-k^3<0$ and $\frac{1-k^3}{k^4}<0$

So, $x^2-\sqrt{x}<0$ for some irrational number

thus, $x^2<\sqrt{x}$ for some positive real number ⊠

14. Suppose $a\in\mathbb{Z}$. Then $a^2|a$ iff $a\in\{-1,0,1\}$

14 $a=ka^2 \Rightarrow -a+ka^2=0$

$a(-1+ka)=0$ (thus $a=0$ & $k=\frac{1}{a}$, so $k\in\mathbb{Z}$)

So, a is either 1 or -1

Let $a\in\{-1,0,1\}$

if $a=-1$, $-1|(-1)^2=$

$a=0$, $a^2=0$, $a^2|a$

$a=1$, $1|1$

Thus, $a^2|a$ iff $a\ \{-1,0,-1\}$ ⊠

20. There exists an $n \in \mathbb{N}$ for which $11 \mid (2^n - 1)$

Let $n \in \mathbb{N}$ s.t. $\frac{2^n - 1}{11} = k$ $\times$

For all values of $n \in \mathbb{N}$, $11 \nmid (2^n - 1)$

Let $n \in \mathbb{N}$, s.t. $n = k = 10$

$$\frac{2^n - 1}{11} \Rightarrow \frac{2^{10} - 1}{11} = \frac{1024 - 1}{11}$$

$$= \frac{1023}{11} = 93$$

Because $11 \mid 2^n - 1$ when $n = 10$, we know the statement for all $n \in \mathbb{N}$, $11 \nmid (2^n - 1)$ is false. Thus, there exists an $n \in \mathbb{N}$ for which $11 \mid 2^n - 1$ is proved true $\boxtimes$