

$H_0$ : The effects of the 5 ingredients will have no significant difference

$H_1: \beta_1 \neq \beta_2 = \beta_3$

$H_2$

$H_3$

4.24)

	1	2	3	4	5	
1	A=8	B=7	D=1	C=7	E=3	26
2	C=11	E=2	A=7	D=3	B=8	31
3	B=4	A=9	C=10	E=1	D=5	29
4	D=6	C=8	E=6	B=6	A=10	36
5	E=4	D=2	B=3	A=8	C=8	25
	33	28	27	25	34	147

$$CF = \frac{147^2}{25} = 864.36$$

$$SS_{Total} = (8^2 + 7^2 + 1^2 + 7^2 + \dots + 8^2) - 864.36$$

$$= 1071 - 864.36 = 206.64$$

$$SS_{Rows} = \frac{1}{5} [26^2 + 31^2 + 29^2 + 36^2 + 25^2] - 864.36$$

$$= 879.8 - 864.36 = 15.44$$

$$SS_{Columns} = \frac{1}{5} [33^2 + 28^2 + 27^2 + 25^2 + 34^2] - 864.36$$

$$= \frac{1}{5} (4383) - 864.36 = 12.24$$

$$SS_{Treatment} = \frac{1}{5} [42^2 + 28^2 + 44^2 + 17^2 + 16^2] - 864.36$$

$$= \frac{1}{5} (5029) - 864.36 = 141.44$$

$$SS_{Error} = 206.64 - 15.44 - 12.24 - 141.44 = 37.52$$

Source	df	SS	MS	F
Rows	4	15.44	3.86	1.2345
Columns	4	12.24	3.06	0.9787
Treatment	4	141.44	35.36	11.309
Error	12	37.52	3.1267	
Total	24	206.64		

$$F_{4,6}(.05) = 4.5337$$

$$H_1: \alpha_1 \dots = \alpha_t$$

$$H_2: \beta_1 \dots = \beta_t$$

$$H_3: \tau_1 \dots = \tau_t$$

Since  $11.309 > 4.5337$ , the variation due to treatments has a significant effect on the yield. The variation due to rows and columns is not significant to the yield, thus we reject the null hypothesis for rows and columns.

4.25)

	1	2	3	4	
1	C=10	D=14	A=7	C=8	39
2	B=7	C=18	D=11	A=8	44
3	A=5	B=10	C=11	D=9	35
4	D=10	A=10	B=12	C=14	46
	32	52	41	39	164

$$CF = \frac{(164)^2}{16} = 1681$$

$$SS_{\text{Total}} = (10^2 + 14^2 + 7^2 + 8^2 + \dots + 14^2) - 1681$$

$$1834 - 1681 = 153$$

$$SS_{\text{Rows}} = \frac{1}{4} ((39)^2 + (44)^2 + (35)^2 + (46)^2) - 1681$$

$$189.5 - 1681 = 18.5$$

$$SS_{\text{Col.}} = \frac{1}{4} (32^2 + 52^2 + 41^2 + 39^2) - 1681$$

$$= 1732.5 - 1681 = 51.5$$

$$SS_R = \frac{1}{4} (30^2 + 37^2 + 53^2 + 44^2) - 1681$$

$$(treatment) = 1753.5 - 1681 = 72.5$$

$$SS_{\text{Error}} = 153 - 72.5 - 18.5 - 51.5 = 10.5$$

$$H_1: \alpha_1, \dots = \alpha_t$$

$$H_2: \beta_1, \dots = \beta_t$$

$$H_3: \tau_1, \dots = \tau_t$$

Source	df	SS	MS	F
Treatment	3	72.5	24.167	13.80
Row	3	18.5	6.167	3.52
Column	3	51.5	17.167	9.80
Error	6	10.5	1.75	
Total	15	153		

$$F_{3,6,0.05} = 4.76$$

We fail to reject the null hypothesis for Row data.

Because  $13.80 > 4.76$ , we reject the null hypothesis. Thus, at least two assembly methods are significantly different (due to treatment) being significant diff

Because  $9.80 > 4.76$ , we reject the null hypothesis as the assembly methods are different (due to column data) being significant different



$$\begin{aligned}
 H_1: \alpha_1 &= \alpha_2 = \dots = \alpha_t \\
 H_2: \beta_1 &= \beta_2 = \dots = \beta_t \\
 H_3: \tau_1 &= \tau_2 = \tau_3 = \dots = \tau_t \\
 H_4: &\text{Replication Effects}
 \end{aligned}$$

2 Replicates

4.38)

	1	2	3	4	
1	C=11	B=10	D=14	A=8	43
2	B=8	C=12	A=10	D=12	42
3	A=9	D=11	B=7	C=15	42
4	D=9	A=8	C=18	B=6	41
	37	41	49	41	168

$$2(4^2) = 32$$

$$CF = \frac{(168)^2}{32} = 3444.5$$

$$\begin{aligned}
 SS_{\text{Total}} &: [10^2 + 7^2 + 5^2 + \dots + 8^2 + 12^2 + 15^2 + 6^2] - 3444.5 \\
 &= 3748 - 3444.5 = \boxed{303.5}
 \end{aligned}$$

$$\begin{aligned}
 SS_{\text{Rows}} &: \left[ \frac{1}{8} (82^2 + 86^2 + 77^2 + 87^2) \right] - 3444.5 \\
 &= \boxed{7.75}
 \end{aligned}$$

$$\begin{aligned}
 SS_{\text{Cols}} &: \left[ \frac{1}{8} (69^2 + 93^2 + 90^2 + 80^2) \right] - 3444.5 \\
 &= \boxed{44.25}
 \end{aligned}$$

$$\begin{aligned}
 SS_{\text{Treatment}} &: \left[ \frac{1}{8} (65^2 + 68^2 + 109^2 + 90^2) \right] - 3444.5 \\
 &= \boxed{159.25}
 \end{aligned}$$

$$\begin{aligned}
 SS_{\text{Replicates}} &: \left[ \frac{1}{4^2} (164^2 + 168^2) \right] - 3444.5 = \boxed{.5}
 \end{aligned}$$

$$\begin{aligned}
 SS_{\text{Error}} &: 303.5 - 7.75 - 44.25 - 159.25 - .5 = \boxed{91.5}
 \end{aligned}$$

Source	df	SS	MS	F
Rows	3	7.75	2.58	.59
Cols	3	44.25	14.75	3.38
Treatment	3	159.25	53.08	12.15
Replicates	1	.5	.5	.11
Error	21	91.75	4.37	
Total	31	303.5		

$$F_{.05, 3, 21} = 3.07$$

$F_2$  &  $F_3 > 3.07$ , thus we fail to reject null hypotheses  $H_1$ ,  $H_4$ ,  $H_2$  &  $H_3$ .

Replication & order of assembly have no effect. Operator & Assembly method have an effect on assembly time.