2b) Suppose we let $k=1$, and $m^*(A \cap E) = m^*(A \cap E)$, this holds true.

Let us assume that for some n, $m^* A \cap \bigcup_{k=1} E_k = \sum_{k=1}^{n} n^*(A \cap E_k)$ so we may prove for $n+1$ disjoint measurable sets, since E_{n+1} is measurable.

$$
\begin{aligned}
m^*\left(A \cap \bigcup_{k=1}^{n+1} E_k\right) &= m^*\left(A \cap \bigcup_{k=1}^{n+1} E_k\right) \cap E_{n+1} + m^*\left(A \cap \left(\bigcup_{k=1}^{n+1} E_k\right) \cap E_{n+1}^c\right) \\
&= m^*(A \cap E_{n+1}) + m^*\left(A \cap \left(\bigcup_{k=1}^{n} E_k\right)\right) \\
&= m^*(A \cap E_{n+1}) + \sum_{k=1}^{n} m^*(A \cap E_k) \\
&= \sum_{k=1}^{n+1} m^*(A \cap E_k)
\end{aligned}
$$

Which remains true for $n+1$. With $n \to \infty$, this will result in the following:

$$m^*\left(A \cap \bigcup_{k=1}^{\infty} E_k\right) = \sum_{k=1}^{\infty} m^*(A \cap E_k).$$

28) Suppose $\{E_k\}_{k=1}^{\infty}$ is a disjoint collection of measurable sets. Let us define $A_n = \bigcup_{k=1}^{n} E_k$ and that $\{A_k\}_{k=1}^{\infty}$ is ascending and $\bigcup_{k=1}^{\infty} A_n = \bigcup_{k=1}^{\infty} E_n$.

The continuity of measure implies that $m\left(\bigcup_{k=1}^{\infty} A_k\right) = m\left(\bigcup_{k=1}^{\infty} E_k\right) = \lim_{n\to\infty} m(A_n)$

By finite additivity, $\lim_{n\to\infty} m(A_n) = \lim_{n\to\infty} \sum_{k=1}^{n} m(E_k) = \lim_{n\to\infty} \sum_{k=1}^{n} m(A_k)$

Thus, $m\left(\bigcup_{k=1}^{\infty} E_k\right) = \sum_{k=1}^{\infty} m(E_k)$ ☒

Thus, $m\left(\bigcup_{k=1} E_k\right) = \sum_{k=1} m(E_k)$ $\boxtimes$

39) Let F_i be the result of the ith iteration of removing all $\alpha/3^i$ wide intervals. We can let $F_0 = [0,1]$, $F_1 = [0, 1/2 - \frac{\alpha}{6}] \cup [\frac{1}{2} + \frac{\alpha}{6}$
This will result in $F = \bigcap_{k=1}^{\infty} F_k$.
Every F_i will have a 2^i disjoint closed intervals of equal length
This means each closed interval is less than $\frac{1}{2}^i$ wide.
Suppose $(a,b) \subseteq F$ with $a<b$. This means $(a,b) \leq F_i$ for all i by definition of $\cap$. However this can't be true because $(b-a) > \frac{1}{2^k}$ for some value of k. There is no interval of that length and thus there can be no nonempty open intervals in F. This in turn makes the complement of F dence.

Thus, $[0,1] \setminus F$ dense in $[0,1]$ $\boxtimes$