4) Let there exist a function $c: 2^R \to \mathbb{R}$

where $(E_n)_{n \in \mathbb{N}}$ is a sequence of pairwise disjoint sets

Case 1) If a value of E_n is equal to infinite (E.g. $E_i = \infty$)

then it follows that $\bigcup E_n \supseteq E_i$ is infinte too.

Thus, $\sum_{n \in \mathbb{N}} c(E_n) = c(\bigcup_{k \in \mathbb{N}} E_k)$

Case 2) If any infinite number of values in E_n is nonempty.

because they are disjoints, this implies $\bigcup E_n$ is infinite

and $\sum_{n \in \mathbb{N}} c(E_n) \Rightarrow = c(\bigcup_{n \in \mathbb{N}} E_n)$

Case 3) If a finite number of E_n are nonempty $\Longrightarrow$ it is

established that the union will also be finite since E_n is finite

Thus, $\sum_{n \in \mathbb{N}} c(E_n) = c(\bigcup_{n \in \mathbb{N}} E_n)$

Let E be a finite set and $x \in \mathbb{R}$. Then, we may conclude

$E + x = \{ e + x \mid e \in E \}$ is a finite set. It then

follows that $c(E) = |E| = |x + E|$

Let E be infinite. This implies $x + E$ is infinite for any

$x \in \mathbb{R}$ and thus $c(E) = \infty = c(E + x)$

$= c(E)$ for all $x \in \mathbb{R}$

Thus, c is countably additive and translation invariant $\boxtimes$

6. Let there exist $A = [0,1] \setminus \mathbb{Q}$

$\mathbb{Q} \cap [0,1]$ is a countable set, as implied by Q being countable. This implies $m^*([0,1] \cap \mathbb{Q}) = 0$

Based on Proposition 31 we can conclude this (above).

As a result, we can conclude the following:

$m^*([0,1]) = 1 \leq m^*(\mathbb{Q} \cap [0,1]) + m^*(A)$

This can be simplified to $m^*(A) \leq m^*([0,1]) = 1$

Thus, $m^*(A) = 1$ $\boxtimes$

9) Let there exist $m^*(A \cup B) \leq m^*(A) + m^*(B)$.

Because $m^*(A) = 0$, this can be simplified to:

$$m^*(A \cup B) \leq m^*(B)$$

For any set $B \subseteq R$ implies $B \subset (A \cup B)$

Thus, $m^*(B) \leq m^*(A \cup B)$ and thus we may conclude $m^*(A \cup B) = m^*(B)$ $\boxtimes$