Math 8 - HW # 1 Shravan Shenoy

1) Prove: A set A is infinite IFF its power set is infinite $\left(\begin{array}{c}|A| = \infty \\ \text{iff} \\ |P(A)| = \infty\end{array}\right)$

- Let an infinite set $\{a_1, a_2, a_3 \ldots\} \subseteq A$. Then $\{a_i\} \in P(A)$ for any $i \in \mathbb{N}$. Because $|\mathbb{N}| = \infty$, then $|P(A)| = \infty$.
- If $|P(A)| = \infty$ and $|A| = n$ such that n is finite, then $|P(A)| = 2^n$ would be finite instead of ∞. This means towards contradiction, $|A| = \infty$ if $|P(A)| = \infty$.

2) Prove: If $|A| = m$, and $|B| = n$, then $|A \cup B| \leq m+n$, $|A \cap B| \leq \min(m,n)$

- If $A = \{a_1, a_2, a_3 \ldots a_m\}$
  $B = \{b_1, b_2, b_3 \ldots b_n\}$, and $(A \cup B) = \{x \mid x \in A \text{ or } x \in B\}$
  $= \{\underbrace{a_1, a_2, a_m}_{m}, \underbrace{b_1, b_2, b_n}_{n}\}$.
- Thus, $|A \cup B| \leq m + n$.
- If $|A \cap B| = |\{x \mid x \in A \text{ and } x \in B\}|$,
  where every element of $A \subseteq B$ or vice-versa, then $|A \cap B| \leq \min(m,n)$. ← (also if there exists $a_i = b_i$)

1.4

4) $P(\{R, Q\})$
$= \boxed{\emptyset, \{R\}, \{Q\}, \{R, Q\}}$

14) $|P(P(A))|$ $= \boxed{2^{2^m}}$ not sure?

$A = \{1, 2\}$
$P(A) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$
$P(P(A)) = \{\{1\}, \{2\}, \{1,2\}, \{2,1\}\}$

15) $|P(A \times B)|$ $(|A| \cdot |B|)$
$= \boxed{2^{m \cdot n}}$

since $|P(A)| = 2^{|A|}$ and $|A| = m$ AND $|P(B)| = 2^{|B|}$ $|B| = n$

20) $|\{X \subseteq P(A) : |X| \leq 1\}|$

Since $|X| \leq 1$, $|X \subseteq P(A)| \leq 1$ because it can only contain $\emptyset$ or a singular element, despite that element potentially being any real number.

1.5

$A = \{4, 3, 6, 7, 1, 9\}$
$B = \{5, 6, 8, 4\}$
$C = \{5, 8, 4\}$

1a. $A \cup B = \boxed{\{4, 3, 1, 6, 7, 5, 8, 9\}}$

b. $A \cap B = \boxed{\{4, 6\}}$

c. $A - B = \boxed{\{3, 7, 1, 9\}}$

$A = \{0, 1\}$, $B = \{1, 2\}$

3a. $(A \times B) \cap (B \times B)$
$\{0,1\}, \{0,2\}, \{1,1\}, \{1,2\} \cap \{1,1\}\{1,2\}, \{2,1\}, \{2,2\}$
$= \boxed{\{1,1\}, \{1,2\}}$

g. $P(A) - P(B)$
$P(A) = \{\emptyset, \{0\}, \{1\}, \{0,1\}\}$
$P(B) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$
$= \boxed{\{\{0\}, \{0,1\}\}}$