PSTAT 171 HW # 1

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1.3) 6) $2 + 2^2 + 2^3 \dots 2^n$

$2(2^0 + 2^1 \dots 2^{n-1})$

$= \frac{(1 - 2^{n-1})2}{-1}$

$= -2(1 - 2^{n-1})$

$= -2 + 2 \cdot 2^{n-1}$

$= \boxed{2^{n+1} - 2}$

1.4) 6) time = 0, simple interest = .05

a) $\frac{i}{1 + i(n-1)} = \frac{1}{23}$

$\Rightarrow \frac{.05}{1 + .05(n-1)} = \frac{1}{23}$

$\Rightarrow (1 + .05(n-1)) = 23(.05)$

$n - 1 = \frac{23(.05) - 1}{.05}$

$n = \frac{.15}{.05} + 1$

$n = 4 \quad [n-1, n]$

Thus, the period is: $[3, 4]$

b.) $i[4, 6]$ — $t = 6$, with gap being 2

$= \frac{2(.05)}{1 + (.05)(5-1)}$

$= \frac{.10}{1 + .20} = \frac{.1}{1.2}$

$= \boxed{8.33\%}$

1.5) 4) $A_k(t) = K \cdot a(t)$

$i = .05, \ t = 4$ (and 3)

$1000[1 + .05]^4 \leftarrow$ Fourth yr

$1000[1 + .05]^3 \leftarrow$ Third yr

$\Rightarrow a(4) - a(3)$

$\Rightarrow 1215.506 - 1157.625 = \boxed{\$57.88}$

8.) invest $2,500, 10% simple annual interest

a) Simple Interest: $2500(1 + (.10 \times 7))$ ← (2+5)

$2500(1 + .7)$

$= 2500(1.7)$

$= \$4250$

Compound Interest:

After 2 yrs:

$2500 + (2500 \times .10 \times 2)$

$= 3000$

Compounded:

$3000(1 + .7)^5$

$= \$4207.66$

Stick with simple Interest, $\$4250 > \$4207.66$

b.) Simple Interest: $2500(1 + .10 \times 10)$

$= 2500(2.0)$

$= \$5000$

Compound Interest: $2000(1 + .07)^8 = \$5154.56$

Move to compound interest account

5154.56 > 5000

1.5) 10) $\left(\frac{\text{Accum.}}{\text{Value}}\right) = PV(1+i)^t$

$t = \ln\left(\frac{AV}{PV}\right) / \ln(1+i)$

$t = \ln\left(\frac{5000}{4200}\right) / \ln(1+.04)$

$t = 4.445$ yrs

$6 - 4.445 = 1.55$

$AV = 5000(1+.055)^{1.55}$

$AV = \$5,433$

Withdraw \$1000 to get 4,433

$t = \ln\left(\frac{5000}{4433}\right) / \ln(1+.04)$

$t = 3.06$ yrs

$AV = 5000(1+.055)^{(4-3.06)}$

$= \boxed{\$5,257.17}$

1.6) 4) $K_i = 256$ (Here, K_i is $K \times i$)

$(K-236)(1+i) = K$

$K - 236 + K_i - 236i = K$

$-236 + 256 - 236i = 0$

$20 = 236i$

$i = .0847$

$K = \frac{256}{i}$ since $K = \frac{K \cdot i}{i}$

$K = 256/.0847 = \boxed{\$3020.80}$

1.7) 4) $Q(t) = K(1+i)^t$

$= 6000(1+.065)^2$

$= \$6805.35$

$(6805.35 - x)(1.065)^2 = 2x$

$(6805.35 - x)(1.065)^2 - 2x = 0$

$(6805.35)(1.065)^2 - x(1.065)^2 - 2x = 0$

$7718.78 - x(1.1342 + 2) = 0$

$7718.78 - x(3.1342) = 0$

$7718.78 = x$

3.1342

$\boxed{x = \$2462.75}$

8.) Project 1 $\left| (-)20,000 + \frac{8000}{(1+.06)^1} + \frac{15000}{(1+.06)^2} \right.$

$= \$897.12$

Project 2 $\left| (-)10000 + \frac{3000}{(1+.06)^1} + \frac{X}{(1+.06)^2} + \frac{14000}{(1+.06)^3} \right.$

$= 4584.86 - \frac{X}{(1.06)^2}$

$897.12 = 4584.86 - \frac{X}{(1.06)^2}$

$\frac{X}{(1.06)^2} = 3687.74$

$X = 3687.74 \cdot 1.06^2$

$X = 4143.54$

Project 1 $\left| (-)20000 + \frac{8000}{(1+.05)^2} + \frac{15000}{(1+.05)^2} \right.$

$= 1224.49$

Project 2 $\left| (-)10000 + \frac{3000}{(1+.05)^1} + \frac{4143.55}{(1+.05)^2} + \frac{14000}{(1+.05)^3} \right.$

$= 1192.55$

$1224.49 - 1192.55 = \boxed{31.94}$

1.8) 4) $300 \times .08 \times 3$

$= 72$ $(300+72)$

$520 - 372 = 148$

$148 = 520 \cdot 5 \cdot r$

$\frac{148}{520 \cdot 5} = r$

$r = 5.7\%$

$372 = X(1 - .057)$ $X = 394.45$

$394.49 - 372 = \boxed{\$22.49}$

1.9) 4) $480 + 52 = 532$

$532 = 480(1+i)^3$

$\sqrt[3]{\frac{532}{480}} - 1 = i$

$i = .03488$

$PV = \frac{1000}{(1+.3488)^2}$

$PV = 933.7266$

To find discount, do Principal − PV. So,

$1000 - 933.73$

$= \boxed{\$66.27}$

1.10) 3) $i^{(12)} = 12 \times .5\%$

$\boxed{i^{(12)} = 6.0\%}$

$i = (1 + .5\%)^{12} - 1$

$i = (1.05)^{12} - 1$

$i = .06167$

$\boxed{i = 6.167\%}$

$d^{(m)} = \frac{i^{(m)}}{1 + \frac{i^{(m)}}{m}} = \frac{.06}{1 + \frac{.06}{12}}$

$= .0597$

$d = 1 - \left[1 - \frac{.0597}{12}\right]^{12} \Rightarrow .05809 \Rightarrow \boxed{5.809\%}$

4) $i_{[0,2]} = \left[1 + \frac{.02}{12}\right]^{12} - 1$

$i_{[0,2]} = .0218$

$i_{[3,4]} = \left[1 - \frac{.03}{2}\right]^{\frac{1}{2}} - 1$

$= .03069$

$i_{[5,9]} = \left[1 + \frac{.042}{1}(2)\right]^{\frac{1}{2}} - 1$

$= .04115$

$i_{[10,12]} = \frac{1}{1-d} - 1$

$= \frac{1}{1 - .58} - 1$

$= .06157$

$2480(1.0218)^3(1.03069)^2(1.04115)^4(1.06157)^3$

$= \boxed{3932.32}$

1.11) 3.) A EIR = 5.2% (lowest)

B $EIR = (1 + .0044)^{12} - 1$

$= (1.0044)^{12} - 1$

$= 5.409\%$

C $EIR = e^{.0516} - 1$

$= .0529$

$= 5.29\%$

- B is the best choice.
- A has the lowest annual rate

1.12) 4.) $a(t) = e^{.03t + .002t^2}$

$\delta = \frac{a'(t)}{a(t)} = \frac{d}{dt}\left(\ln(a(t))\right)$

$\ln(a(t)) = .03t + .002t^2$

$\frac{d}{dt}\ln(a(t)) = .03 + .004t$

$\delta_2 = .03 + .004(2)$

$= .038$

$\delta_2 = 3.8\%$