Math 8 HW 11

3. If $k \in \mathbb{Z}$, then $\{n \in \mathbb{Z} : n | k\} \subseteq \{n \in \mathbb{Z} : n | k^2\}$

Let $x \in \{n \in \mathbb{Z} : n | k\}$. In this case, we know that $x | k$. Because of this, there exists an integer y such that $k = xy$. Because of this, $k^2 = x^2y^2$, which is equivalent to $k^2 = x(xy^2)$, which demonstrates that $x | k^2$. As a result, $x | k^2$ implies $x \in \{n \in \mathbb{Z} : n | k^2\}$. Thus, $\{n \in \mathbb{Z} : n | k\} \subseteq \{n \in \mathbb{Z} : n | k^2\}$ ∎

8. If A, B, C are sets, then $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

- def of intersec.
- distrib. property
- def of union
- def. of intersection

$$A \cup (B \cap C) = \{x : (x \in A) \cup ((x \in B) \cap (x \in C))\}$$
$$= \{x : (x \in A \cup x \in B) \cap (x \in A \cup x \in C)\}$$
$$= \{x : (x \in A \cup B) \cap (x \in A \cup C)\}$$
$$\Rightarrow (A \cup B) \cap (A \cup C)$$

Hence, if A, B, C are sets, then $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ ∎

9. If A, B, C are sets, then $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

- def of intersec
- def of union
- dist. law
- def of inter.
- def of union

$$A \cap (B \cup C) = \{x : (x \in A) \cap (x \in B \cup C)\}$$
$$= \{x : (x \in A) \cap ((x \in B) \cup (x \in C))\}$$
$$= \{x : ((x \in A) \cap (x \in B)) \cup ((x \in A) \cap (x \in C))$$
$$= \{x : (x \in (A \cap B)) \cup (x \in (A \cap C))\}$$
$$= (A \cap B) \cup (A \cap C)$$

Hence, if A, B, C are sets, then $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ ∎

10. If A & B are sets in a universal set U, then $\overline{A \cap B} = \bar{A} \cup \bar{B}$

Let $x \in \overline{A \cap B}$.

$$= \quad x \notin (A \cap B)$$

$$= \quad (x \notin A) \cup (x \notin B)$$

$$= \quad (x \in \bar{A}) \cup (x \in B)$$

$$\Rightarrow \quad x \in (\bar{A} \cup \bar{B})$$

Thus, $\overline{A \cap B} = \bar{A} \cup \bar{B}$ $\blacksquare$