1) Suppose R is symmetric. This means $(x,y) \in R \Rightarrow (y,x) \in R \Rightarrow (x,y) \in R^{-1}$

Since $(x,y) \in R^{-1} \Rightarrow (y,x) \in R \Rightarrow (x,y) \in R$,

then $R^{-1} \subseteq R$, so $R^{-1} = R$

Therefore, R is symmetric iff $R^{-1} = R$ $\blacksquare$

2) Suppose $R \subseteq A \times B$ and $S \subseteq B \times C$ are relations.

Let $(x,y) \in (R \circ S)^{-1}$ in which $x \in C$, $R \in A$

It is important to note that here, $(R \circ S)^{-1} \subseteq C \times A$

$(x,y) \in (R \circ S)^{-1} \Leftrightarrow (y,x) \in (R \circ S)$

$\Leftrightarrow \exists\, k \in B$ such that $(y,k) \in R$ & $(k,x) \in S$

$\Leftrightarrow (k,y) \in R^{-1}$ and $(x,k) \in S^{-1}$

$\Leftrightarrow (x,y) \in S^{-1} \circ R^{-1}$. Thus, $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$

Therefore if $R \subseteq A \times B$ and $S \subseteq B \times C$, then $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$ $\boxtimes$

3) Suppose $R \subseteq A \times B$. Let $(x,y) \in R$ such that $x \in A$, $y \in B$

Then, $(y,x) \in R^{-1}$ and $(x,y) \in (R^{-1})^{-1}$

Thus, $(x,y) \in R \Leftrightarrow (y,x) \in (R^{-1})^{-1}$

Therefore, $(R^{-1})^{-1} = R$ $\boxtimes$

4) Suppose $R \subseteq A \times B$ is a relation.

Case 1) Let $(x,y) \in R \circ R^{-1}$ s.t. x & $y \in A$

Thus, $\exists\, k \in B$ s.t $(x,k) \in R$ & $(k,y) \in R^{-1}$

This implies $(k,x) \in R^{-1}$ and $(y,k) \in R$.

$\Rightarrow (y,k) \in R$ and $(k,x) \in R^{-1}$.

$\Rightarrow (y,x) \in R \circ R^{-1}$,

$(x,y) \in R \circ R^{-1} \Rightarrow (y,x) \in R \circ R^{-1}$

Thus, $R \circ R^{-1}$ is symmetric $\boxtimes$

Case 2) Let $(x,y) \in R^{-1} \circ R$ s.t. $x, y \in R$

Thus, $\exists\, k \in A$ s.t. $(x,k) \in R^{-1}$ & $(k,y) \in R$

This implies $(k,x) \in R$ and $(y,k) \in R^{-1}$

$\Rightarrow (y,k) \in R^{-1}$ and $(k,x) \in R$

$\Rightarrow (y,x) \in R^{-1} \circ R$

$(x,y) \in R^{-1} \circ R \Rightarrow (y,x) \in R^{-1} \circ R$

Thus, $R^{-1} \circ R$ is symmetric $\boxtimes$