Two Way SS

4.7)

Chemical	Bolt 1	2	3	4	5	$y_{i\cdot}$
1	73	68	74	71	67	353
2	73	67	75	72	70	357
3	75	68	78	73	68	362
4	73	71	75	75	69	363
$y_{j\cdot}$	294	274	302	291	274	1435

RBD (random)

$\sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij}^2 - \frac{y_{\cdot\cdot}^2}{bt}$ SS Total

$SS_{Total} = [73^2 + 68^2 + 74^2 + 71^2 + 67^2 + 73^2 \ldots + 69^2] - \frac{(1435)^2}{20} = 191.75$ SS Total

SS_R $\frac{\sum_{i=1}^{t} y_{i\cdot}^2}{b} - \frac{y_{\cdot\cdot}^2}{bt}$

$SS_{Treatment} = \left[\frac{1}{5}[353^2 + 357^2 + 362^2 + 363^2]\right] - \frac{(1435)^2}{20} = 12.95$ SS R

5 ← despite 4 values; ↑ 4×5

SS_B $\frac{\sum_{j=1}^{b} y_{j\cdot}^2}{t} - \frac{y_{\cdot\cdot}^2}{bt}$

$SS_{Blocks} = \left[\frac{1}{4}[294^2 + 274^2 + 302 + 291 + 274]\right] = \frac{1435^2}{20} = 157$ SS B

4 ← Despite 5 values

$SS_{Total} = SS_R + SS_B + SS_E$

$191.75 = 12.95 + 157 + 21.8$ $\quad SS_E = 21.8$

Source	df	SS	$MS = \frac{SS}{df}$	F	p-val
Treatment	3	12.95	4.317	2.376	.1211
Blocks	4	157	39.25	21.606	
Error	12	21.8	1.817		
Total	19	191.75			

As shown by the F value being 2.376, the model is not significant relative to the noise. Theres a 12.11% chance the F-value will occur due to noise. Theres no difference between chemical types.

4.13) a)

Oil	Truck 1	2	3	4	5	
1	.5	.634	.487	.329	.512	2.462
2	.535	.675	.520	.435	.540	2.705
3	.513	.595	.488	.4	.510	2.506
	1.548	1.904	1.495	1.164	1.562	7.673

SS_{Total}: $[.5^2 + .634^2 + .487^2 \ldots + .4^2 + .510^2] - \frac{(7.673^2)}{15} = .103$

SS_R: $\left[\left(\frac{1}{5}\right)[2.462^2 + 2.705^2 + 2.506^2)\right] - \frac{(7.673)^2}{15} = .0067$

SS_B: $\left[\left(\frac{1}{3}\right)[1.548^2 + 1.904^2 + 1.495^2 + 1.164^2 + 1.562^2)\right] - \frac{(7.673)^2}{15} = .092$

$.103 = .0067 + .092 + S_E$, $S_E = .0043$

Source	df	SS	MS	F	P-value
Block	4	.092	.023	6.85	.0223
Treatment	2	.0067	.00335		
Error	8	.0048	.000527		
Total	14	.103			

There is a significant difference between lubrication oils & trucks, as shown by the p-value being $p = .0223$, which is less than $\alpha = .05$.

b)

$LSD = t_{.025,8}\sqrt{.000527\left(\frac{1}{3} + \frac{1}{5}\right)}$
$= .038688$

T1 vs T2 = $|.4924 - .541| = .0486 > .038688$
T1 vs T3 = $|.4924 - .5012| = .0088 < .03688$
T2 vs T3 = $|.541 - .5012| = .0398 > .03688$

Pairs 1 and 2 & 2 and 3 are significantly different from eachother.
Pair 1 and 3 are not different from eachother.

4.19) Parameters τ_i and β_j for CRBD

$$Y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij}$$

$i = 1, 2, 3, \ldots a$ for $\hat{\tau}_i = \bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot}$

$j = 1, 2, 3 \ldots b$ for $\hat{\beta}_j = \bar{y}_{\cdot j} - \bar{y}_{\cdot\cdot}$

$\hat{\mu} = \bar{y}_{\cdot\cdot}$

Using the textbook equations (4.17 & 4.18) the estimation for the model parameters are as follows:

$\hat{\mu} = 35/20 = 1.75$

$\hat{\tau}_1 = -23/20 = -1.15$

$\hat{\tau}_2 = -7/20 = -.35$

$\hat{\tau}_3 = 13/20 = .65$

$\hat{\tau}_4 = 17/20 = .85$

$\hat{\beta}_1 = 35/20 = 1.75$

$\hat{\beta}_2 = -65/20 = -3.25$

$\hat{\beta}_3 = 75/20 = 3.75$

$\hat{\beta}_4 = 20/20 = 1$

$\hat{\beta}_5 = -65/20 = -3.25$

★ Most calculations solved through calculator. ★

4.44)

Source	df	SS	MS	F	P
(R) Treatment	6	1317.43	219.57	10.42	.002
Block	6	394.10	65.68	3.12	.070
Error	8	168.58	21.07		
Total	20	1880.1			

$$SS_R = \sum_{i=1}^{t}\left(\left(\frac{1}{b}\right)Y_{i\cdot}^{2}\right) - \frac{Y_{\cdot\cdot}^{2}}{N} = 1317.43$$

$$SS_B = \sum_{i=1}^{t}\left(\left(\frac{1}{t}\right)Y_{j\cdot}^{2}\right) - \frac{Y_{\cdot\cdot}^{2}}{N} = 394.10$$

$$SS_{Total} = \sum\sum\left(Y_{ij}^{2} - \frac{Y_{\cdot\cdot}^{2}}{N}\right) = 1880.1$$

$$SS_E = 1880.1 - 1317.43 - 394.10 = 168.58$$

The F-value for treatment is less than the value found in the anova table (10.42), thus at least one treatment is causing a difference.

The F value for blocking is greater than the value found in the anova table (3.12), thus blocking is <u>not</u> causing a difference.

4.44)

TABLE

	1	2	3	4	5	6	7	
2%	114				120		117	351
4%	126	120				119		365
6%		137	117				134	388
8%	141		129	149				419
10%		145		150	143			438
12%			120		118	125		363
14%				136		130	127	393
	381	402	366	435	381	374	378	2717

4.51) BIBD , $a=8, r=8, k=4, b=16$

$$\lambda = \frac{r(k-1)}{t-1} \Rightarrow \frac{r(k-1)}{a-1} = \frac{8(3)}{7} = \frac{24}{7}$$

not whole number, cannot exist

Lecture ↑ Textbook ↑