PSTAT HW 3

3.2) 1.) $3000\left(\frac{1-1.0025^{-60}}{.0025}\right)$

$= 3000(55.65)$

$= \boxed{\$166957.07}$ with $i^{(12)} = 3\%$

$3000\left(\frac{1-1.002534^{-60}}{.002534}\right)$

$= 3000(55.68)$

$= \boxed{166751.66}$ using $d = 3\%$

When $d=8\%$, the equivalent $i^{(12)}$ is greater than 3%, resulting in a smaller present value than the first case where $i^{(12)} = 3\%$

3.2) 3.) $150000 = P\left(\frac{1.05^{18}-1}{.05}\right) = P\left(\frac{1.4066}{.005}\right)$

5%

$\frac{150000}{28.1324} = P \Rightarrow \underline{5331.93}$

$PV = 5331.93\left(\frac{1.05^{10}-1}{.05}\right)$

$= 67064.44$

$PV = 67064.44(1.045)^8 = 95372.38$

$150000 - 95372.38$

$= 54687.61$

$54687.61 = P\left(\frac{1.045^8-1}{.045}\right)$

$P = \frac{54627.61}{9.38} = 5823.82$

$5823.82 - 5331.93 = \boxed{\$491.90 \text{ increase}}$

3.3) 3)

(a) $a_{\overline{n+1}|} - a_{\overline{n}|}$ measures the value of a payment of 1 made n+1 periods later.

b) $a_{\overline{n+1}|i} - a_{\overline{n}|i} = .177208656$

$\ddot{a}_{\overline{n+1}|i} - \ddot{a}_{\overline{n}|i} = .185248436$

$(1+i)[a_{\overline{n+1}|i} - a_{\overline{n}|i}]$

$= .185248436$

$\frac{.185248436}{.177208656} = 1.045369003$

$i = .045369003$

$a_{\overline{n}|i} = \frac{1-(1-i)^{-n}}{i} = .177208656$

$(\lambda^2(1+i)^{-(n+1)} - (1-(1+i)^{-n}) = .00803978$

$\frac{1}{1.045369003^n}(.0434) = .00803978$

$5.398 = 1.0454^n$

$\frac{\ln(5.398)}{\ln(1.0454)} = n = 37.97 \Rightarrow \boxed{38}$

3.3) 7) $\ddot{a}_{\overline{n}|} = 31.61667882$

$s_{\overline{n+1}|} = 64024.90944$

$64083.90944 = (1+i)^n (31.61667882)$

$(1+i)^n = 2025.004265$

$d = \frac{(1+i)^n - 1}{\ddot{s}_{\overline{n}|}} = \frac{2025.004265 - 1}{64023.90944}$

$i = .03264528 \Rightarrow 3.26\%$

$(1+.0326)^n = 2025.004265$

$n = \frac{\ln(2025.004265)}{\ln(1.0326)}$

$\boxed{n = 237}$

5) $54\ddot{a}_{\overline{\infty}|} = 1000$

$\ddot{a}_{\overline{\infty}|} = 18.5185 = \frac{1}{d}$

$d = \frac{1}{18.5185} = .054$

$\ddot{a}_{\overline{\infty}|} = 1 + a_{\overline{\infty}|} \Rightarrow 18.5185 = 1 + a_{\overline{\infty}|} = 17.5185$

$17.5185 = \frac{1}{i} \quad i = .05708$

$\ddot{s}_{\overline{22}|} = \frac{(1+.05708)^{22} - 1}{.054}$

$= \frac{2.392}{.054} = 44.288 \Rightarrow s_{\overline{22}|} = \frac{\ddot{s}_{\overline{22}|}}{1+i} = \frac{44.288}{140.057}$

$\Rightarrow \frac{44.28}{1.05708} = 41.896$

3.4) 3) a) $\frac{5000}{.07} = 71428.57$

$\frac{78428.57}{40000} = (1.07)^n$

$\frac{\ln(1.786)}{\ln(1.07)} = 8.57$

$+ 1$ yr

$9.57 \rightarrow 10$ yrs

$1980 + 10 \Rightarrow \boxed{1990}$

b) $40000(1.07)^9$

$= 73538.38$

$73538.38 - 71428.57$

$= \boxed{2109.79}$

3.5) 2) $\left(\frac{1}{i}\right)\left(1 - \frac{1}{(1+i)^n}\right) = 3\left(\frac{1}{i}\right)\left(\frac{1}{(1+i)^n}\right)$

$= 1 - \frac{1}{(1+i)^n} = \frac{3}{(1+i)^n}$

$1 = \frac{4}{(1+i)^n} \Rightarrow (1+i)^n = 4$

$1.1225^n = 4$

$\frac{\ln(4)}{\ln(1.1255)} = n \quad \boxed{n = 12}$

$\frac{\ln(4)}{\ln(1.06)} = n \Rightarrow n = 24$

$PV_{[1,n]} \Rightarrow \frac{1}{1.06}\left(1 - \frac{1}{1.06^{24}}\right) = (.9434)(.75302) = .7104$

$PV_{[n,\infty]} \Rightarrow 3\left(\frac{1}{1.06}\right)\left(\frac{1}{1.06^{24}}\right) = 3(.9434)(.24698) = .69899$

$\Rightarrow \frac{.69899}{(.698999 + .7104)} = \boxed{@\ i = 6\% \quad 49.6969\%}$

3.5) 3) $i = 7.8\%$, $PV = 21092.04$

$$21092.04 = A \times \left(\frac{1-(1.078)^{-12}}{.078/1.078} \right) \times 1.078^{-12}$$

$$A = \frac{21092.04}{5.333} = \boxed{\$6328.00}$$

3.11) 1) $$r = \frac{-\ln\left(1 - \frac{(.01)(200000)}{25000}\right)}{\ln(1.01)} \Rightarrow 8.3798$$

(8 level payments, 1 drop payment)

$\boxed{\text{9 payments total}}$

$$25000 \left(\frac{(1.01)^{8.3798} - 1}{.01} \right) (1.01)^{1-.3798}$$

$$25000(.37863)(1.00619)$$

$$\boxed{\text{drop payment} = \$9523.87}$$

3.11) 2) $$25000 + 25000 \left(\frac{(1.01)^{.3798} - 1}{1.01} \right) (1.01)^{-.3798}$$

$$25000 + 25000(.37863)(.99623)$$

$$= \boxed{\$34{,}429.58}$$