HW 13

There exists numbers $x, y \in \mathbb{Z}$ such that $x^2 + 4y = 2$

$\forall\ x, y \in \mathbb{Z}$, such that $x^2 + 4y \neq 2$

Let x be $0, \in \mathbb{Z}$. Let y be $1, \in \mathbb{Z}$.

$$x^2 + 4y \neq 2$$

$$0^2 + 4(1) = 4 \neq 2$$

Thus, by contradiction, $\forall\ x, y \in \mathbb{Z}$ s.t. $x^2 + 4y \neq 2$ ⊠

1) Prove that $1+2+3+4+\dots n = \frac{n^2+n}{2}$ for every integer $n \in \mathbb{N}$

Show S_k implies S_{k+1}

Let $k \geq 1, \in \mathbb{N}, k = n$

$$(1+2+3+4\dots k) + (k+1) = \frac{k^2+k+2(k+1)}{2}$$

$$= \frac{k^2+2k+1+k+1}{2}$$

$$= \frac{(k+1)^2+(k+1)}{2}$$

Therefore, $1+2+3\dots k + (k+1) = \frac{(k+1)^2+(k+1)}{2}$

and $1+2+3\dots n = \frac{n^2+n}{2}$ for every int. $n \in \mathbb{N}$ ⊠

2) Prove that $1^2+2^2+3^2\dots n^2 = \frac{n(n+1)(2n+1)}{6}$ for every pos int. n.

let $n = k+1$. $= 1^2+2^2\dots k^2+(k+1)^2$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= \frac{(k+1)(k(2k+1) + 6(k+1)^2}{6}$$

$$= \frac{(k+1)(2k^2+7k+6)}{6}$$

$$= \frac{(k+1)[(k+1)+1][2(k+1)+1]}{6}$$

Thus its true for $n = k+1$. By mathematical induction for every integer $n \in \mathbb{N}$: $1^2+2^2+3^2\dots n^2 = \frac{n(n+1)(2n+1)}{6}$ ⊠

4) If $n \in \mathbb{N}$, then $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + 4 \cdot 5 + \ldots n(n+1) = \frac{n(n+1)(n+2)}{3}$.

Let $n = k+1$.

$$= 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots k(k+1) + (k+1)(k+2)$$

$$= [1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 \ldots k(k+1)] + (k+1)(k+2)$$

$$= \frac{k(k+1)(k+2)}{3} + (k+1)(k+2)$$

$$= \frac{k(k+1)(k+2) + 3(k+1)(k+2)}{3}$$

$$= \frac{(k+1)(k+2)(k+3)}{3}$$

Thus, It is true if $n = k+1$. Therefore by mathematical induction for every integer $n \in \mathbb{N}$,

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 \cdots n(n+1) = \frac{n(n+1)(n+2)}{3} \quad \boxtimes$$