Math 8 HW 19

1) $\forall a, b \in \mathbb{N} : aRb \leftrightarrow a \mid b$

- aRa since $a \mid a$, and thus R is reflexive
- Let $a \mid b$ and $b \mid c$.

  This implies $b = aq$ and $c = bk$ for some q & $k \in \mathbb{Z}$.

  Now $c = (aq)k$

  $c = aqr$

  $\Rightarrow a \mid c$, thus R is transitive.
- Let $a \mid b$ and $a = b$, since $a \leq b$ and $b \leq a$, R is not symmetric, and so
- R is a partial ordering but not total order.

2) If A is a set and R is a total order, but not a well ordering of A, then there exists an infinite sequence $\{a_n \mid n \in \mathbb{N}\}$ s.t. $\forall n \in \mathbb{N}, a_n \in A, a_{n+1} R a_n$

- Since A is not well ordered, there exists a nonempty subset S of A that doesn't contain at least one element.
- Since S is not empty, elements are to be observed, such as $a_1 \in S$. But because S does not have a least element in it, a_1 is not the least element of S; Thus, $\exists$ a_2 in S such that $a_2 R a_1$ $(a_2 \neq a_1)$.
- This will result in an infinite sequence of distinct elements $\{a_n \mid n \in \mathbb{N}\}$ such that $a_{n+1} R a_n$ for all $n \in \mathbb{N}$.