Math 8 HW 8

1. Suppose $n \in \mathbb{Z}$. If n^2 is even, then n is even.

$\Rightarrow$ Suppose $n \in \mathbb{Z}$. If n is odd, then n^2 is odd.

Let $n = 2k+1$, an odd integer.

$$n^2 = (2k+1)^2 = 4k^2 + 4k + 1$$
$$= 2(2k^2 + 2k) + 1$$
$$= 2(a) + 1 \quad \text{where } a = 2k^2+2k \text{ is integer}$$

Because $n^2 = 2a+1$ will be odd,

n is not even $\boxtimes$

3. Suppose $a, b \in \mathbb{Z}$. If $a^2(b^2-2b)$ is odd, then a & b are odd

$\Rightarrow$ If a or b are even, then $a^2(b^2-2b)$ is even.

<u>Case 1</u> Suppose a is even. Let $a = 2k$

$$a^2(b^2-2b) = (2k)^2(b^2-2b)$$
$$= 4k^2(b^2-2b)$$
$$= 2(2k^2(b^2-2b))$$

$2(2k^2(b^2-2b))$ will be even for any integer values k and b.

<u>Case 2</u>. Suppose b is even. Let $b = 2k$

$$a^2(b^2-2b) = a^2(4k^2-4k)$$
$$= 2(a^2(2k^2-2k))$$

$2(a^2(2k^2-2k))$ will be even for any integer values a and k.

Because $a^2(b^2-2b)$ is even, in any cause in which a or b are even, then a & b are not odd $\boxtimes$

7. Suppose $a, b \in \mathbb{Z}$. ~~If ab and a+b are even, then both a & b are even.~~

$\Rightarrow$ If both a & b are not even, then ab and a+b are not even

Case 1: Let a be even & b be odd integers

$a = 2k \quad b = 2d+1$

$ab = 2k(2d+1)$ is even.

$a+b = 2k + 2d + 1$

$a+b = 2(k+d)+1$ is odd

Case 2: Let a be odd & b be even integers

$a = 2d+1 \quad b = 2k$

$ab = 2k(2d+1)$ is even

$a+b = 2d+1 + 2k$

$a+b = 2(k+d)+1$ is odd

Case 3: Let a be odd & b be odd integers

$a = 2k+1 \quad b = 2d+1$

$ab = (2k+1)(2d+1)$

$= 4kd + 2k + 2d + 1$

$= 2(2kd + k + d) + 1$ is odd

$a+b = 2k+1+2d+1$

$= 2(k+d)+2$ is even

Thus, if both a & b are not even, then ab and a+b aren't even

12. Suppose $a \in \mathbb{Z}$. If a^2 is not divisible by 4, then a is odd.

$\Rightarrow$ If a is not odd, then a^2 is divisible by 4.

Case 1: Let a be an even integer

$$a = 2k$$

$$a^2 = (2k)^2$$

$$= 4k^2 \quad \leftarrow \text{ is divisible by 4.}$$

$$4k^2/4 = k^2$$

Thus, if a is not odd, then a^2 is divisible by 4,

thus if a^2 is not divisible by 4, then a is odd.