32) i) Assume $A = \mathrm{int}(A) = \bigcup \{ G : G \subset A, G \text{ is open} \}$

The set of A is open since the arbitrary union of open sets is also open.

Assume A is open set. For all $x \in A$, a positive real number r_a exists such that $x \in (x - r_a, x + r_a) \subset A$

$\Rightarrow x$ is an interior pt of set A $\Rightarrow x \in \mathrm{int}(A)$

$\Rightarrow A \subseteq \mathrm{int}(A) \Rightarrow A = \mathrm{int}(A)$ $\boxtimes$

ii) Assume E is dense in R $\Rightarrow Cl(E) = R$

$\Rightarrow R \setminus Cl(E) = \phi$

$\Rightarrow \mathrm{int}(R \setminus Cl(E)) = \phi$

$\Leftrightarrow \mathrm{int}(R \setminus E) = \phi$ $\boxtimes$

35) Any open set is the complement of a closed set

A collection $\mathcal{B}$ of borel sets is the smallest σ-algebra that contains all open sets.

Let Σ be any σ-algebra containing all closed sets

$\mathcal{B} \subset \Sigma$ by definition of Borel set $\Rightarrow$ $\mathcal{B}$ is the smallest

Borel set is the smallest σ-algebra that contains the closed sets. $\boxtimes$

36) Let R be the set of all real numbers.

Let $\varphi = B \cdot B$ be a base set. Because finite union of countable operations are countable & countable union of countable operations are countable

Let $[a, b]$, $a, b \in R$ and $a < b$

$$[a,b] = \cap_{n=1}^{\infty} [a - \frac{1}{n}, b + \frac{1}{n}]$$

For each $n \in N$, $[a - \frac{1}{n}, b + \frac{1}{n}]$ is an open set

$\Rightarrow$ $[a,b]$ is a countable intersection of open intervals

$\Rightarrow$ This is a borel set $\boxtimes$