

1) Suppose $(x, y) \in R \circ R$. Then there exists $(x, a), (a, y) \in R$ such that $(a, y) \circ (x, a) \in R \circ R$ and $(a, y) \circ (x, a) = (x, y)$. Since R is transitive and (x, a) and $(a, y) \in R \Rightarrow (x, y) \in R$. Therefore, $R \circ R \subseteq R$ \square

2) (1) Suppose $f: A \rightarrow B$ and $g: B \rightarrow C$. Let $A = \{a\}$,

$B = \{a, b\}$, and $C = \{b\}$ (Counterexample)

$f = \{(a, a)\}$ is injective, not surjective

$g = \{(a, b), (b, b)\}$ is not injective since $g(a) = g(b)$

Hence, if $g \circ f$ is injective, ~~both~~ f, g are not ^{both} injective

(2) Suppose $f: A \rightarrow B$ and $g: B \rightarrow C$. Let $A = \{a\}$,

$B = \{a, b\}$ and $C = \{b\}$

$f = \{(a, a)\}$ is not surjective as nothing maps to $b \in B$.

$g = \{(a, b), (b, b)\}$

Hence since f is not surjective, then $f \circ g$ are ^{both} not surjective

(3) Suppose $f: A \rightarrow B$ and $g: B \rightarrow C$. Let $A = \{a\}$,

$B = \{a, b\}$, and $C = \{b\}$

injective $\Rightarrow f = \{(a, a)\}$

surjective $\Rightarrow g = \{(a, b), (b, b)\}$

$g \circ f: A \rightarrow C$ implies must be bijection

$g \circ f = \{(a, b)\}$ is a bijection

Hence if $g \circ f$ is not bijective, ~~both~~ f, g are not both bijective