Linear Algebra
HW3 Cosine Transform

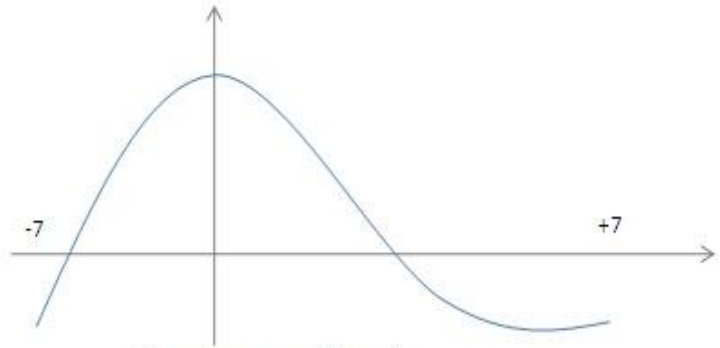
吳思霖

Outline

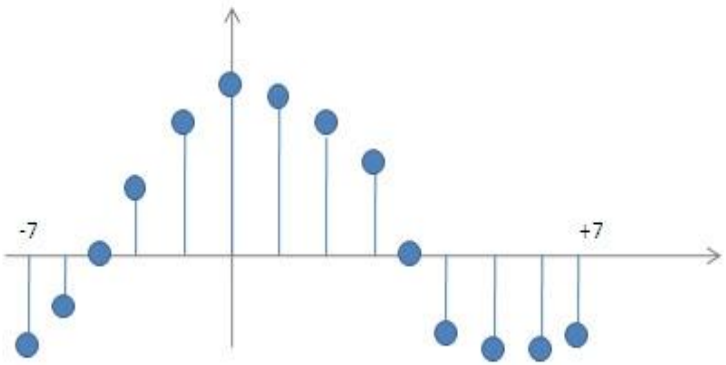
- What is signal
- Fourier Transform
- Cosine Transform
- HW3
  - Input&output
  - Code&util function
  - Rules

What is signal

- 傳遞有關一些現象的行為或屬性的資訊的函數
- Example
  - f(t) : 音訊
  - f(x, y) : 圖片
- Type
  - Continous
  - Discrete
- In this homework, we use
  - discrete signal



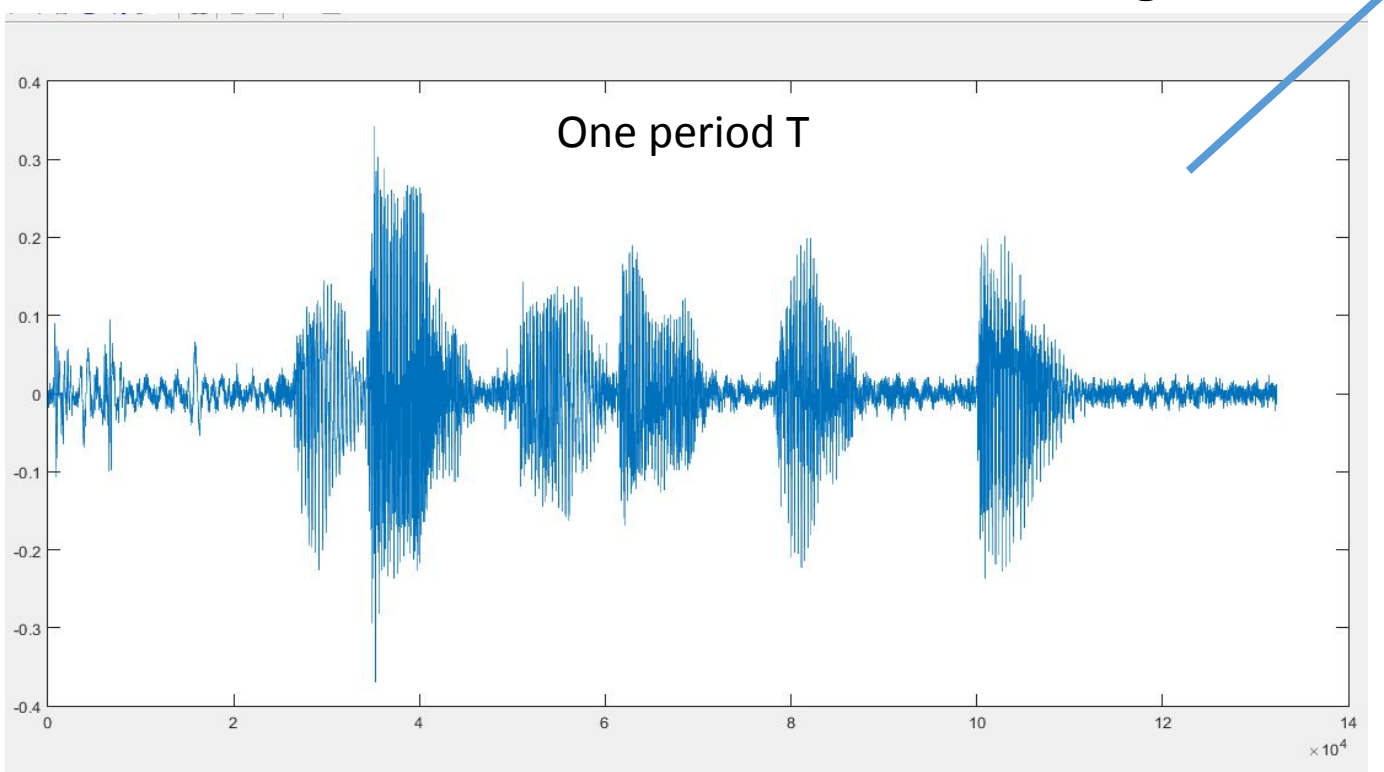
Continuous Signal
(takes values *in the set* [-7,7])



Discrete Signal
(takes values *at the integers* {-7,-6...0...6,7})

Basis

- How to use basis in signal analysis
- Given a speech signal
  - Can we find basis to describe this signal ?



$$\mathbf{x} = [x_0, x_1, \dots, x_{N-1}]$$

$$\mathbf{B} = \{\mathbf{b}_0, \mathbf{b}_1, \dots, \mathbf{b}_{N-1}\}$$

$$[\mathbf{x}]_{\mathbf{B}} = [a_0, a_1, \dots, a_{N-1}] = \mathbf{a}$$

$$\mathbf{x} = \sum_{k=0}^{N-1} a_k \mathbf{b}_k$$

exists **b** that is easy to analysis?
(basis is also a signal)

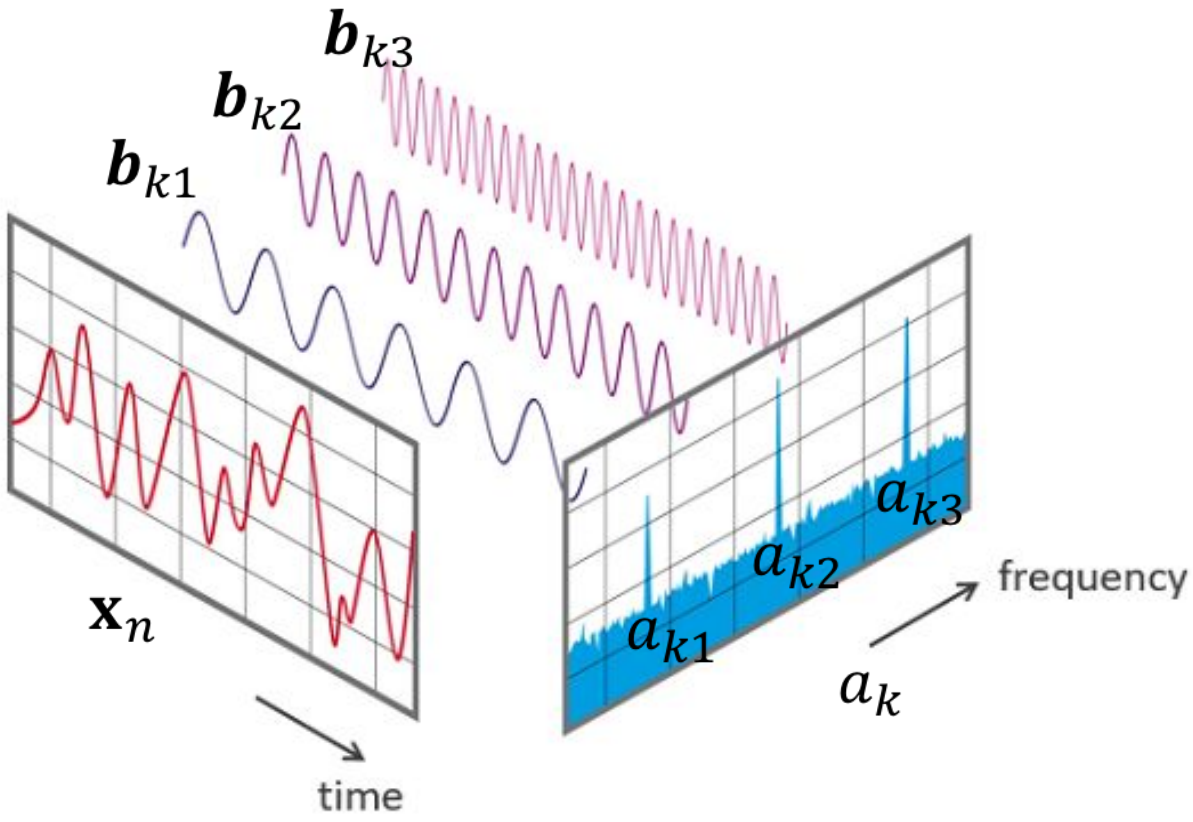
Joseph Fourier

Any periodic signal can be represented as a sum of sinusoids.

Fourier Transform

假設x主要由三個basis vector組成
→三個頻率的cosine signal組成

$$\mathbf{x} = a_{k1}\mathbf{b}_{k1} + a_{k2}\mathbf{b}_{k2} + a_{k3}\mathbf{b}_{k3} + \cdots$$
$$0 \le k1, k2, k3 \le N - 1$$



x_t 時間上的強度

$Fourier\ Transform$

a_k 頻率上的強度

Cosine Transform

- Fourier Transform includes complex number computation
  - We use cosine transform instead
- Cosine Transform Formula
  - Given a discrete signal $x = [\, x_0, x_1, \dots, x_n, \dots, x_{N-1}]$ with N length
  - Basis Matrix:
- $\mathbf{B} = \{\mathbf{b}_0, \mathbf{b}_1, \dots, \mathbf{b}_{N-1}\}$

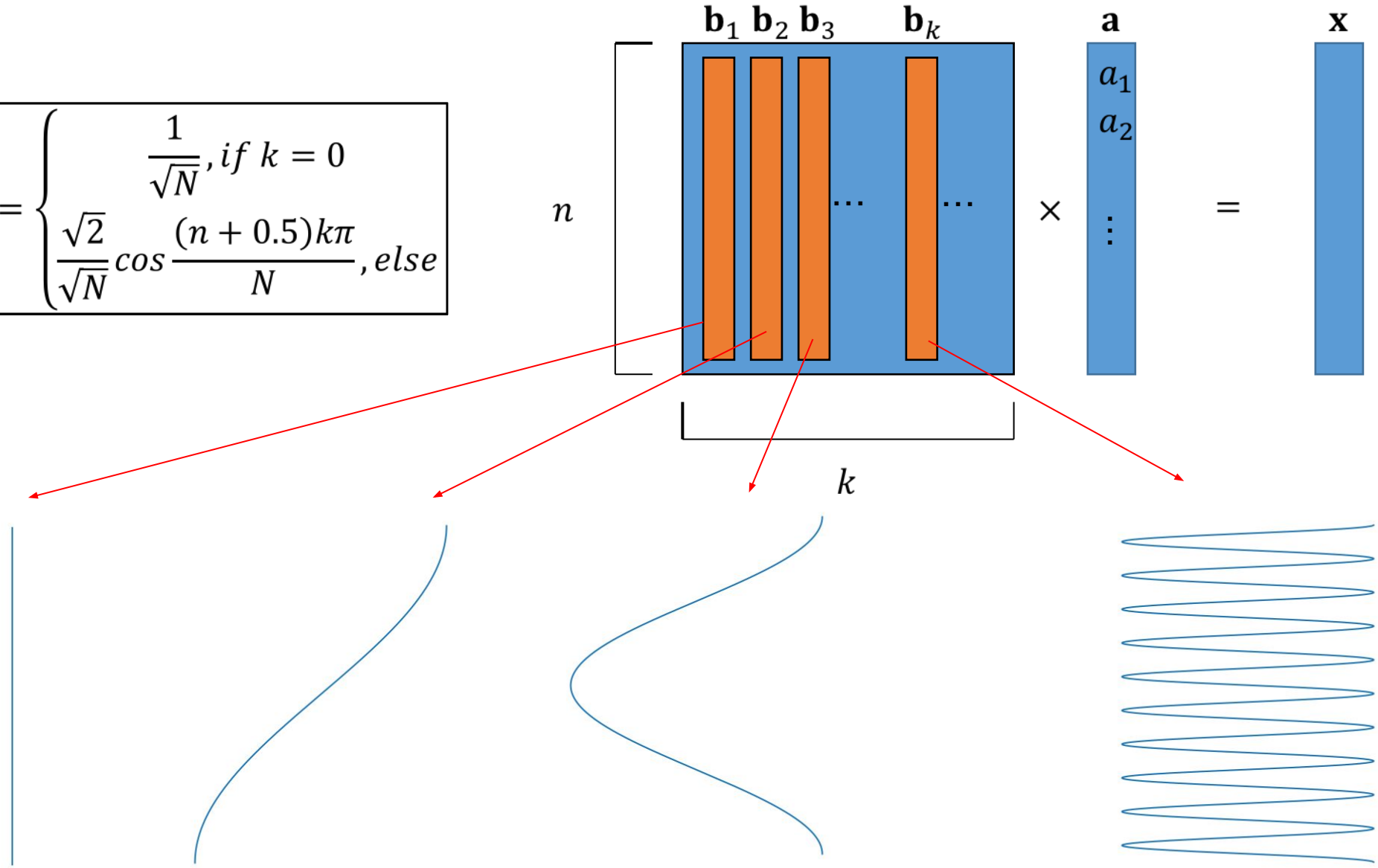
$$\mathbf{b}_{k,n} = \begin{cases} \frac{1}{\sqrt{N}}, if\ k = 0 \\ \frac{\sqrt{2}}{\sqrt{N}} cos \frac{(n+0.5)k\pi}{N}, else \end{cases}$$

How to get **a**

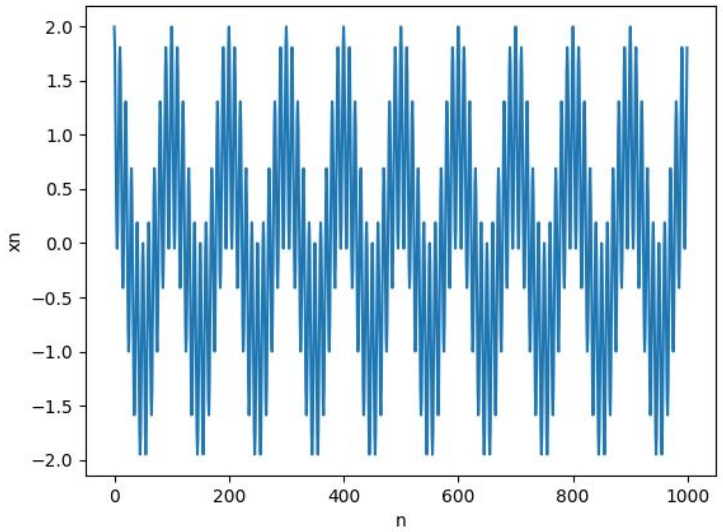
$$\mathbf{x} = \mathbf{Ba}$$
$$\mathbf{a} = \mathbf{B}^{-1}\mathbf{x}$$

n: 時間上的index
k: 頻率上的index

$$\mathbf{b}_{k,n} = \begin{cases} \frac{1}{\sqrt{N}}, if\ k = 0 \\ \frac{\sqrt{2}}{\sqrt{N}} cos \frac{(n+0.5)k\pi}{N}, else \end{cases}$$

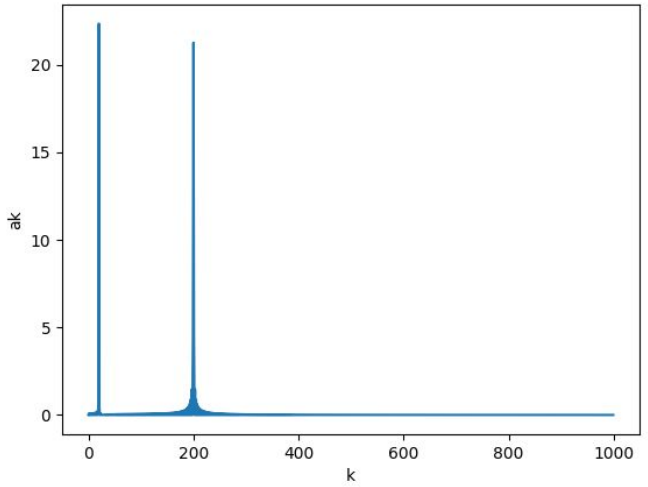


If we want to get the low frequency signal of a mixed signal…



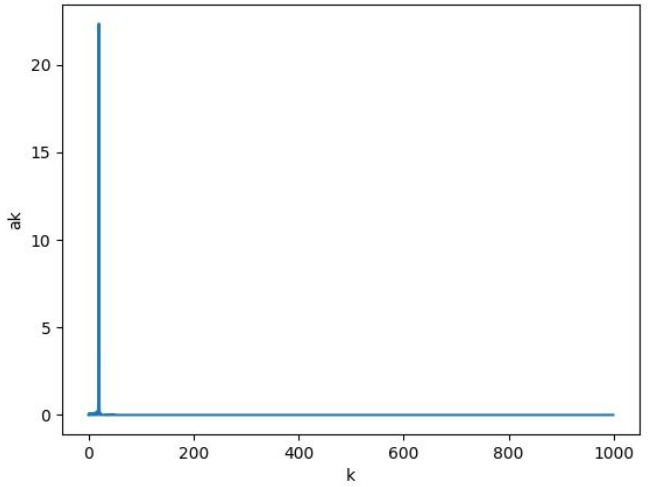
Cosine transform

$$\mathbf{a} = \mathbf{B}^{-1}\mathbf{x}$$



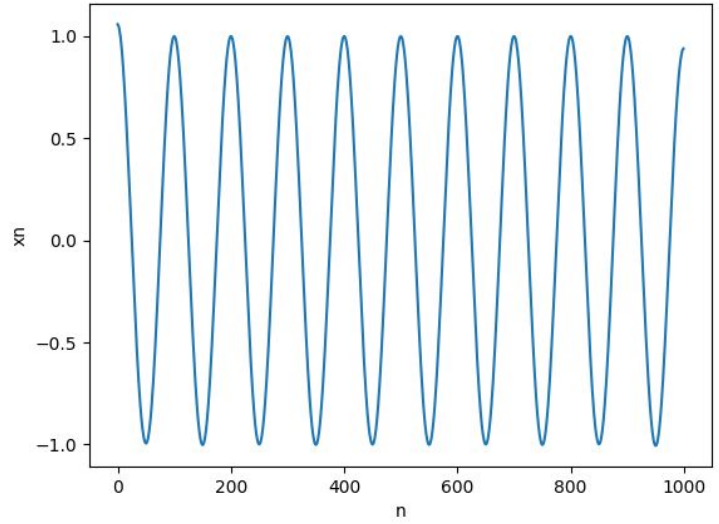
x

mask [1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 …]



Inverse cosine transform

$$\mathbf{x} = \mathbf{a}^{\mathbf{T}}\mathbf{B}$$



Application

- Filter
  - Human voice
    - Man: 85-180Hz
    - Woman: 165-255Hz
  - Remove high frequency noise from speech signal

Input & Output

- $x = \sum_{i=1}^{5} Cosine(2\pi f_i)$
  - $f_1 < f_2 < f_3 < f_4 < f_5$
- Input data
  - Please download the file of <student_id>.txt
  - Total 1000 lines, one value per line.
- Output f_1 and f_3 signal to following file
  - f_1 : <student_id>_f1.txt (ex: b01901118_f1.txt)
  - f_3 : <student_id>_f3.txt
  - Use numpy.savetxt to output the answer signal
  - Same format as input: 1000 lines
- Output the picture of **a** of input signal
  - <student_id>_freq.png
  - Use util function
- You can use test.txt for testing.
  - Only two cosine signal with different freq are mixed.

```
 1 2.0000000000000000e+00
 2 1.807043722803219010e+00
 3 1.301131695689425438e+00
 4 6.732702563537411589e-01
 5 1.595661667536837358e-01
 6 -4.894348370484646882e-02
 7 1.207594915133041180e-01
 8 5.958100580910720145e-01
 9 1.185323674418810924e+00
10 1.653344919876962527e+00
11 1.809016994374947451e+00
12 1.579530237150736927e+00
13 1.037985621796358338e+00
14 3.755301115537416079e-01
15 -1.715930046262574837e-01
16 -4.122147477075268629e-01
17 -2.731901993959511277e-01
18 1.727366797267690379e-01
19 7.347962859400198887e-01
20 1.771415470596251480e+00
21 1.309016994374947451e+00
22 1.057706881539801413e+00
23 4.963983089606727184e-01
24 -1.836837608106426378e-01
25 -7.462264748456348684e-01
26 -9.999999999999998890e-01
27 -8.718075139042609223e-01
28 -4.343502279392522092e-01
29 1.216356797892221842e-01
30 5.603271072100921568e-01
31 6.909830056250526598e-01
32 4.408924416902699206e-01
33 -1.167622971901245421e-01
34 -7.907706684766655503e-01
```

Code & Data link

- Link
- Code
  - hw3.py
- Data
  - <student_id>.txt
  - test.txt

HW3 Code

```
def CosineTrans(x, B):
    # TODO
    # implement cosine transform
    return

def InvCosineTrans(a, B):
    # TODO
    # implement inverse cosine transform
    return

def gen_basis(N):
    # TODO
    return

if __name__ == '__main__':

    signal_path = sys.argv[1]
```

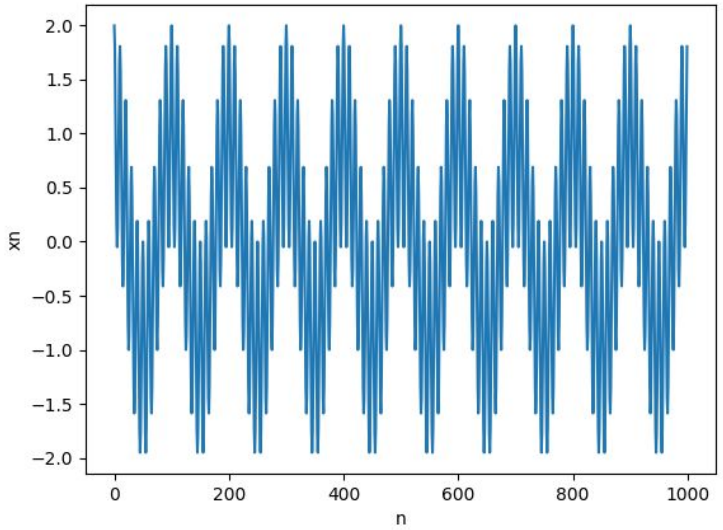
Run the code

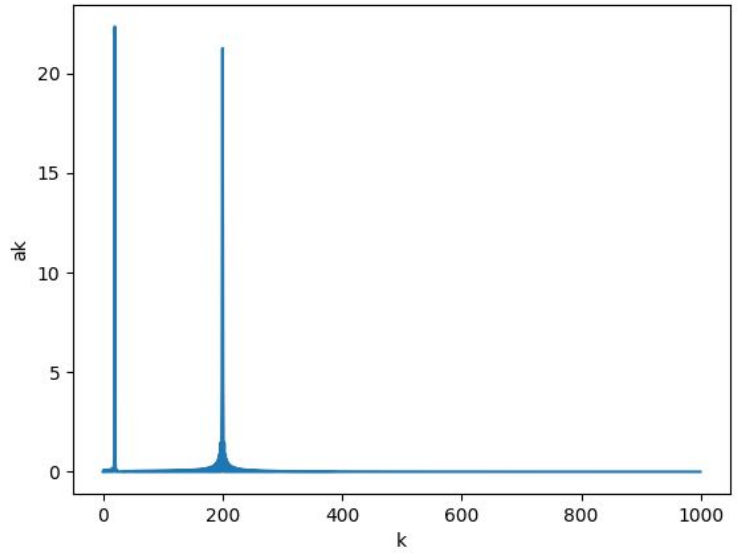
- python hw3.py <input_signal_txt>

- Your code should generate 2 txt files and 1 png file.
  - <student_id>_f1.txt
  - <student_id>_f3.txt
  - <student_id>_freq.png
  - These three files should be in the same folder with hw3.py

Some util function

```
 9
10 def plot_wave(x, path = './wave.png'):
11     # util function
12     plt.gcf().clear()
13     plt.plot(x)
14     plt.xlabel('n')
15     plt.ylabel('xn')
16     plt.savefig(path)
17
18 def plot_ak(a, path = './freq.png'):
19     # util function
20     plt.gcf().clear()
21
22     # Only plot the mag of a
23     a = np.abs(a)
24     plt.plot(a)
25     plt.xlabel('k')
26     plt.ylabel('ak')
27     plt.savefig(path)
```





Scoring

1. Plot the figure of a_k. (2%)
2. Output correct f_1 signal (2%)
3. Output correct f_3 signal (2%)

Submit

- Code you download

  |-- hw3.py

- Code you submit should be put in a folder and compressed in a zip file

  r07922072_hw3.zip

  |-- ./r07922072_hw3

  |-- hw3.py

  |-- r07922072_f1.txt

  |-- r07922072_f3.txt

  |-- r07922072_freq.png

Standard Rules

- 不要抄作業，不要交別人的答案，作弊一律0分計算
- 上傳 zip 檔案到 CEIBA
- 注意繳交的資料夾學號開頭英文用**小寫**

- **DEADLINE: 2018/11/15(四) 23:59 (GMT+8:00)**
- **遲交每過一天：分數×0.8 (per day)**
- **格式、檔案、各種奇怪的錯誤讓我無法改作業：分數×0.8**

Code Rules

- You can‘t
  - Use cosine transform formula in appendix to generate seperated signal
    - 只准使用inverse matrix的方法產生指定的三個檔案
    - 可以實作，但僅限於檢查inverse matrix的方法是否正確
  - import scipy
    - Or other cosine transform package

Appendix - Cosine Transform Formula

- Cosine transform

$$a_k = s_k * 2 \sum_{n=0}^{N-1} x_n \cos\left(\frac{\pi}{N} k \left(n + \frac{1}{2}\right)\right)$$

$$if\ k = 0, s_0 = \sqrt{\frac{1}{4N}}$$

$$else, s_i = \sqrt{\frac{1}{2N}}$$

- Inverse Cosine transform

$$x_n = \frac{1}{\sqrt{N}} a_0 + \frac{\sqrt{2}}{\sqrt{N}} \sum_{k=1}^{N-1} a_k \cos\left(\frac{\pi}{N} k \left(n + \frac{1}{2}\right)\right)$$