

动态规划 (Dynamic Programming)

本节内容

1. 递归+记忆化 \rightarrow 递推
2. 状态的定义: $\text{opt}[n]$, $\text{dp}[n]$, $\text{fib}[n]$
3. 状态转移方程: $\text{opt}[n] = \text{best_of}(\text{opt}[n-1], \text{opt}[n-2], \dots)$
4. 最优子结构

斐波那契数列

0, 1, 1, 2, 3, 5, 8, 13, 21, ...

递推公式: $F[n] = F[n-1] + F[n-2]$

$N \rightarrow R$
 MEMOIZATION
 +
 DYNAMIC PROGRAMMING
 $S \rightarrow R$

$$fib(n) = fib(n-1) + fib(n-2)$$

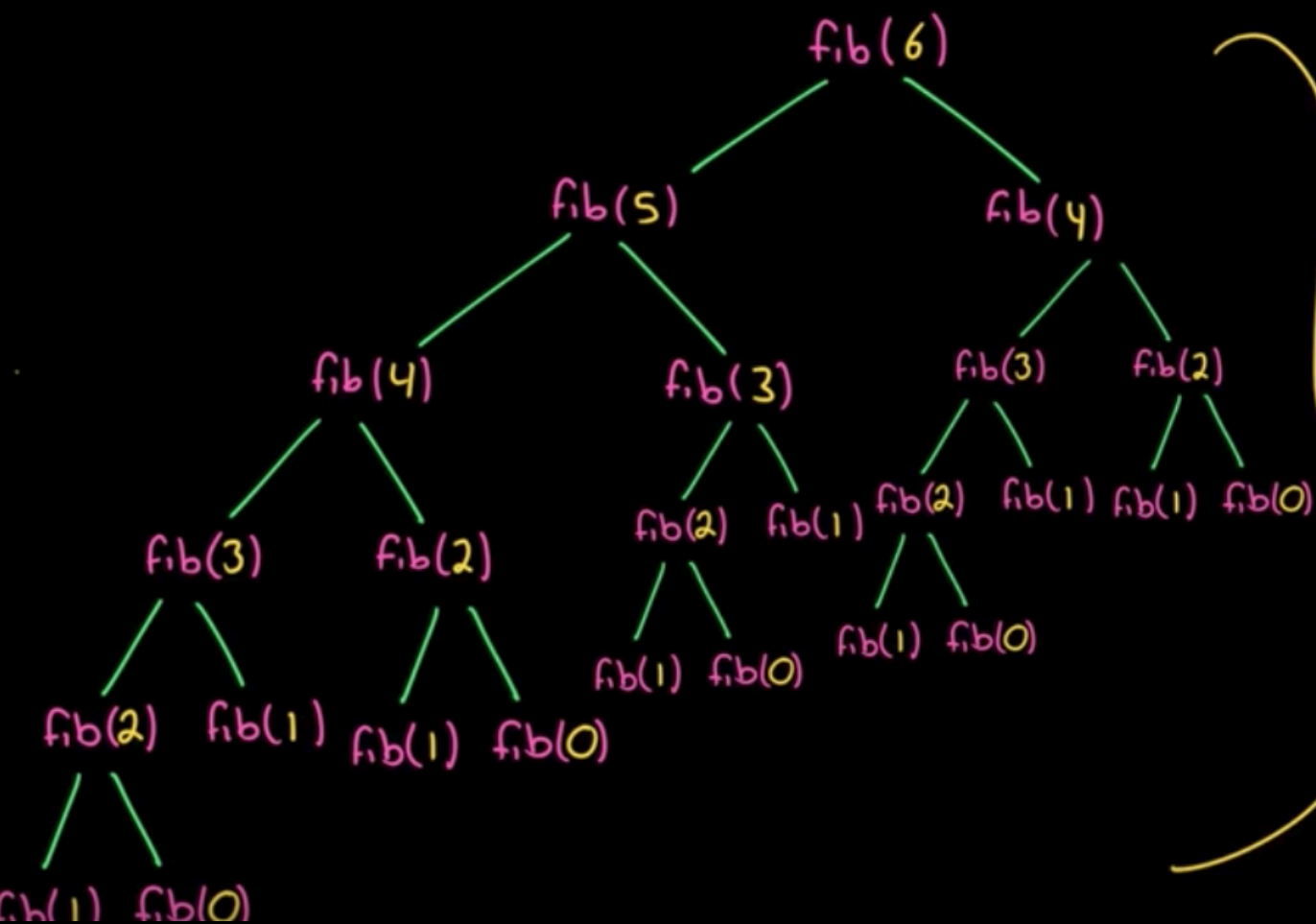
$$fib(0) = 0$$

$$fib(1) = 1$$

```

int fib(int n) {
    if (n <= 0) {
        return 0;
    } else if (n == 1) {
        return 1;
    } else {
        return fib(n-1) + fib(n-2);
    }
}

```



call tree has n levels

Level 1: 1 node

Level 2: 2

Level 3: 4

Level 4: 8

⋮

$$1 \times 2 \times 2 \times \dots \times 2 = O(2^n)$$

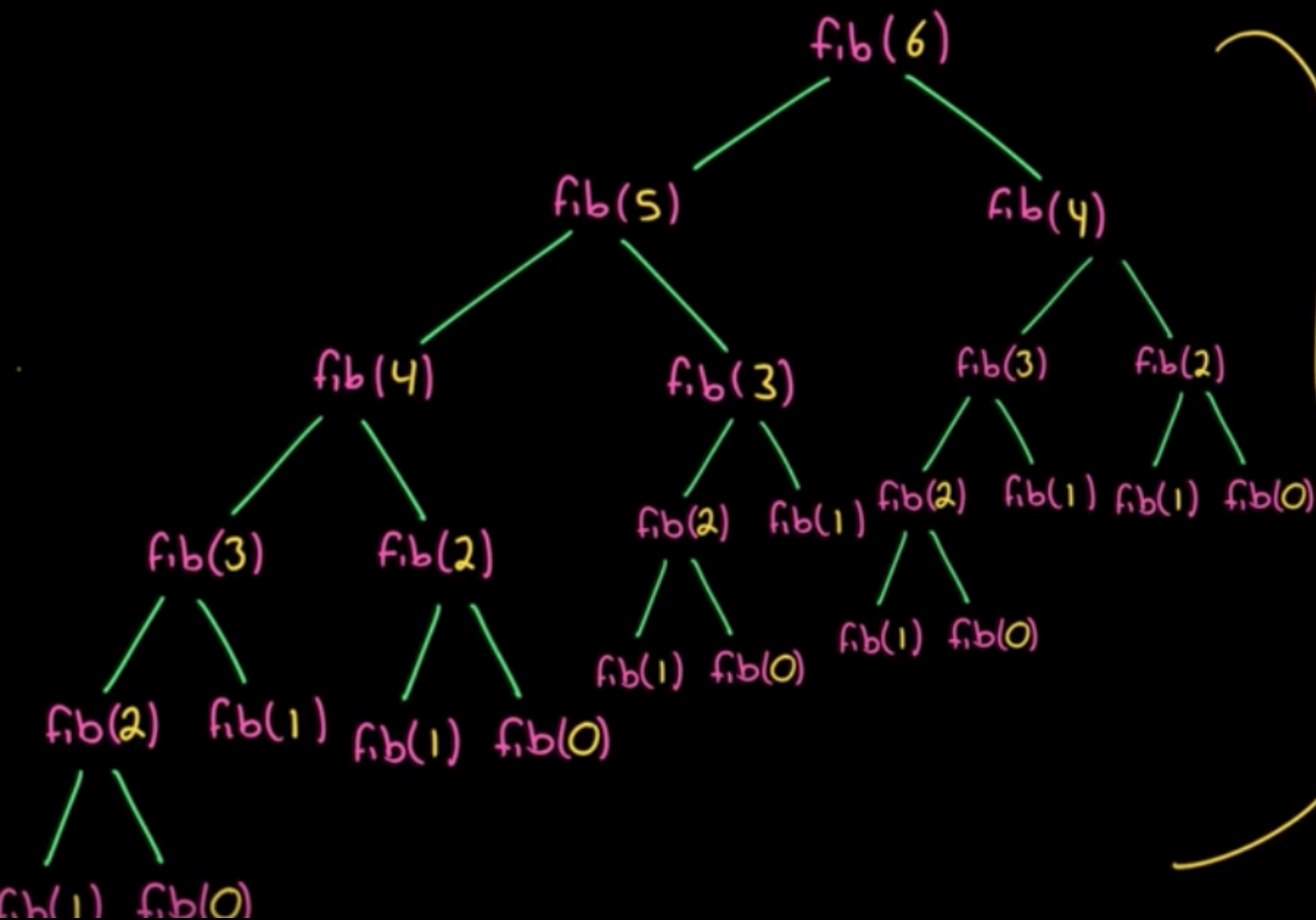
```
int fib(int n) {  
    return n <= 1 ? n : fib(n - 1) + fib(n - 2);  
}
```

$N \rightarrow R$
 MEMOIZATION
 +
 DYNAMIC PROGRAMMING
 $S \rightarrow R$

$$\text{fib}(n) = \text{fib}(n-1) + \text{fib}(n-2)$$

$$\text{fib}(0) = 0$$

$$\text{fib}(1) = 1$$



```

int fib(int n) {
    if (n <= 0) {
        return 0;
    } else if (n == 1) {
        return 1;
    } else {
        return fib(n-1) + fib(n-2);
    }
}

```

call tree has n levels

Level 1: 1 node

Level 2: 2

Level 3: 4

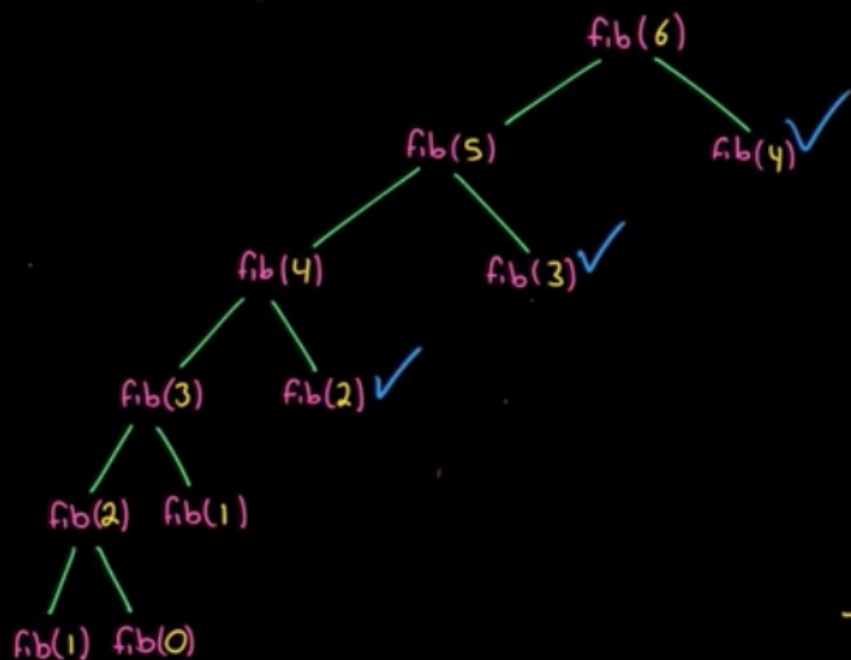
Level 4: 8

⋮

$$1 \times 2 \times 2 \times \dots \times 2 = O(2^n)$$

$N \rightarrow R$
 O MEMOIZATION
 $+$
 I DYNAMIC PROGRAMMING
 S R

$fib(n) = fib(n-1) + fib(n-2)$
 $fib(0) = 0$
 $fib(1) = 1$



$memo[2] = 1$
 $[3] = 2$
 $[4] = 3$
 $[5] = 5$

```

int fib(int n) {
    if (n <= 0) {
        return 0;
    } else if (n == 1) {
        return 1;
    } else {
        return fib(n-1) + fib(n-2);
    }
}
  
```

call tree has n levels
 Level 1: 1 node
 Level 2: 2
 Level 3: 4
 Level 4: 8
 \vdots

```

int fib(int n, int[] memo) {
    if (n <= 0) {
        return 0;
    } else if (n == 1) {
        return 1;
    } else if (memo[n]) {
        memo[n] = fib(n-1) + fib(n-2);
    }
    return memo[n];
}
  
```

MEMOIZATION

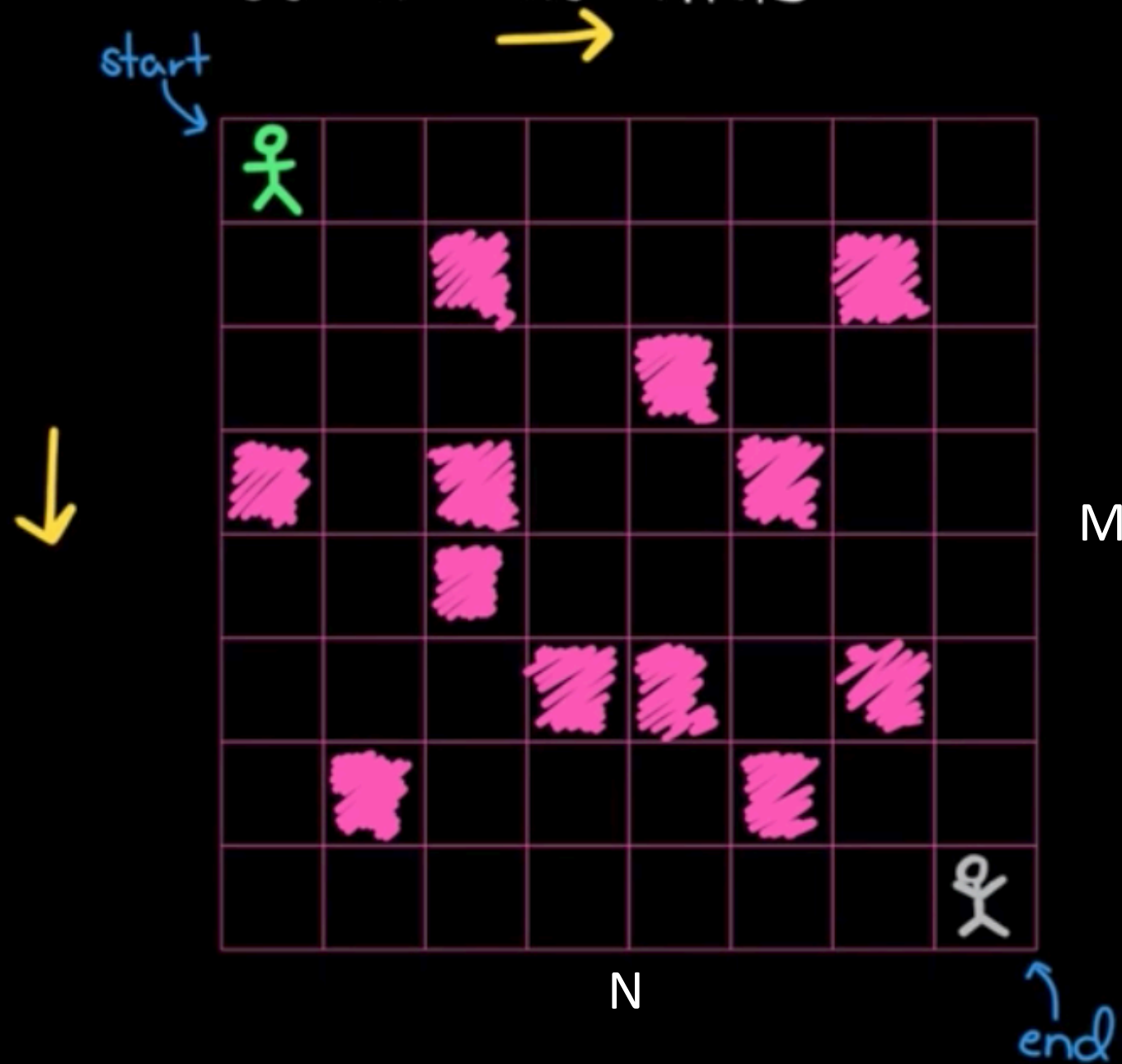
$\hookrightarrow O(n)$ time

递归 + 记忆化 ==> 递推

递推公式: $F[n] = F[n-1] + F[n-2]$

```
F[0] = 0, F[1] = 1;  
for (int i = 2; i <= n; ++i) {  
    F[i] = F[i - 1] + F[i - 2];  
}
```


COUNT THE PATHS



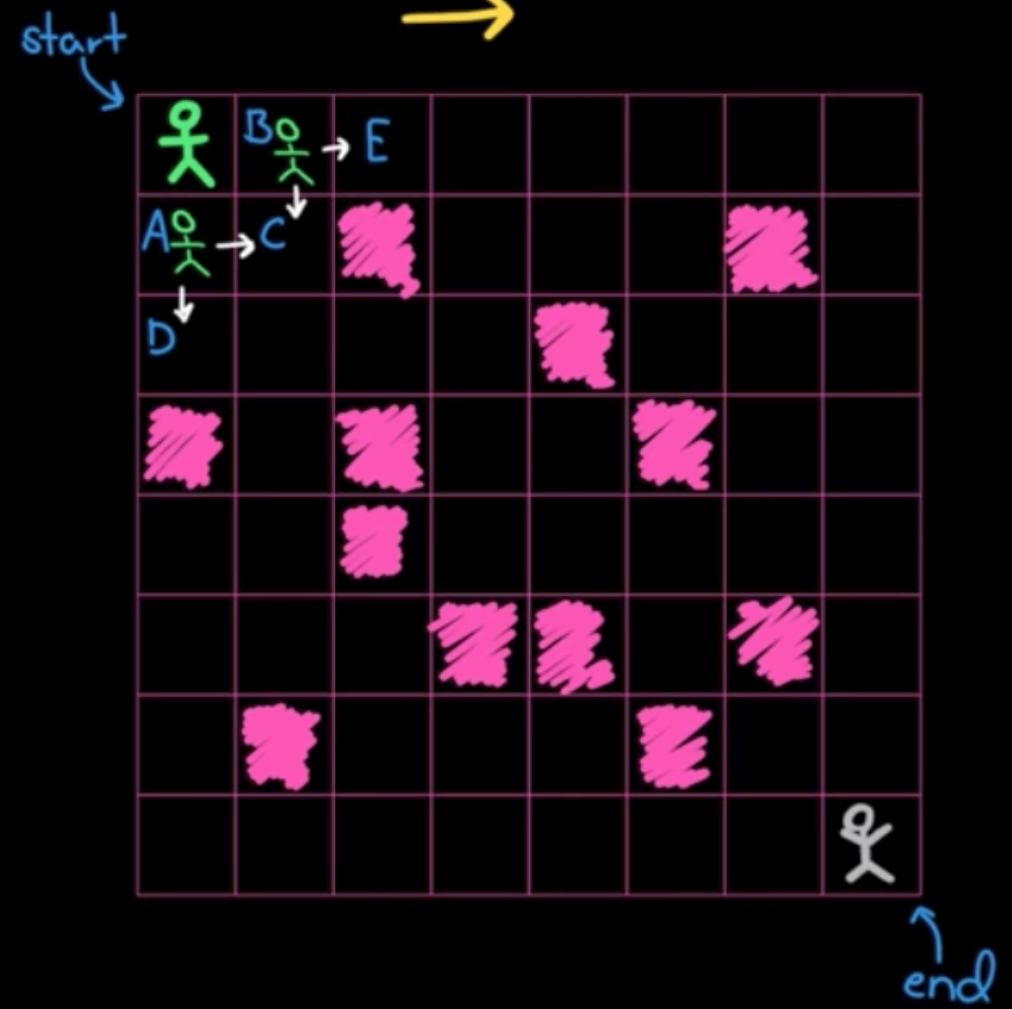
N → R
 MEMOIZATION
 +
 DYNAMIC PROGRAMMING
 S R

$$\begin{aligned}
 &\text{paths}(\text{start}, \text{end}) = \\
 &\underbrace{\text{paths}(\text{A}, \text{end})}_{\text{paths}(\text{D}, \text{end}) + \text{paths}(\text{C}, \text{end})} + \underbrace{\text{paths}(\text{B}, \text{end})}_{\text{paths}(\text{C}, \text{end}) + \text{paths}(\text{E}, \text{end})}
 \end{aligned}$$

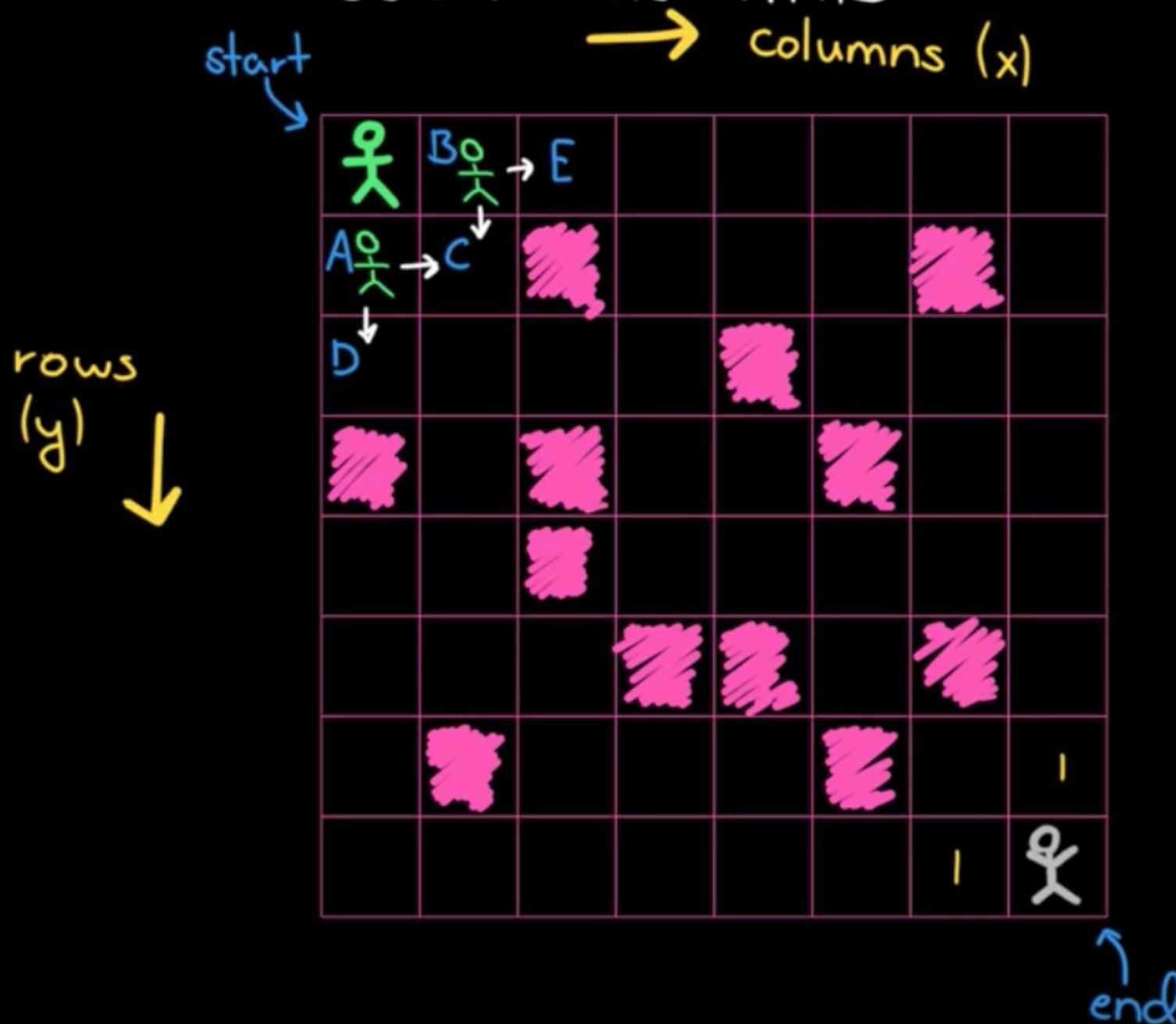
```

int countPaths(boolean[][] grid, int row, int col) {
  if (!validSquare(grid, row, col)) return 0;
  if (isAtEnd(grid, row, col)) return 1;
  return countPaths(grid, row+1, col) + countPaths(grid, row, col+1);
}
  
```

COUNT THE PATHS



COUNT THE PATHS



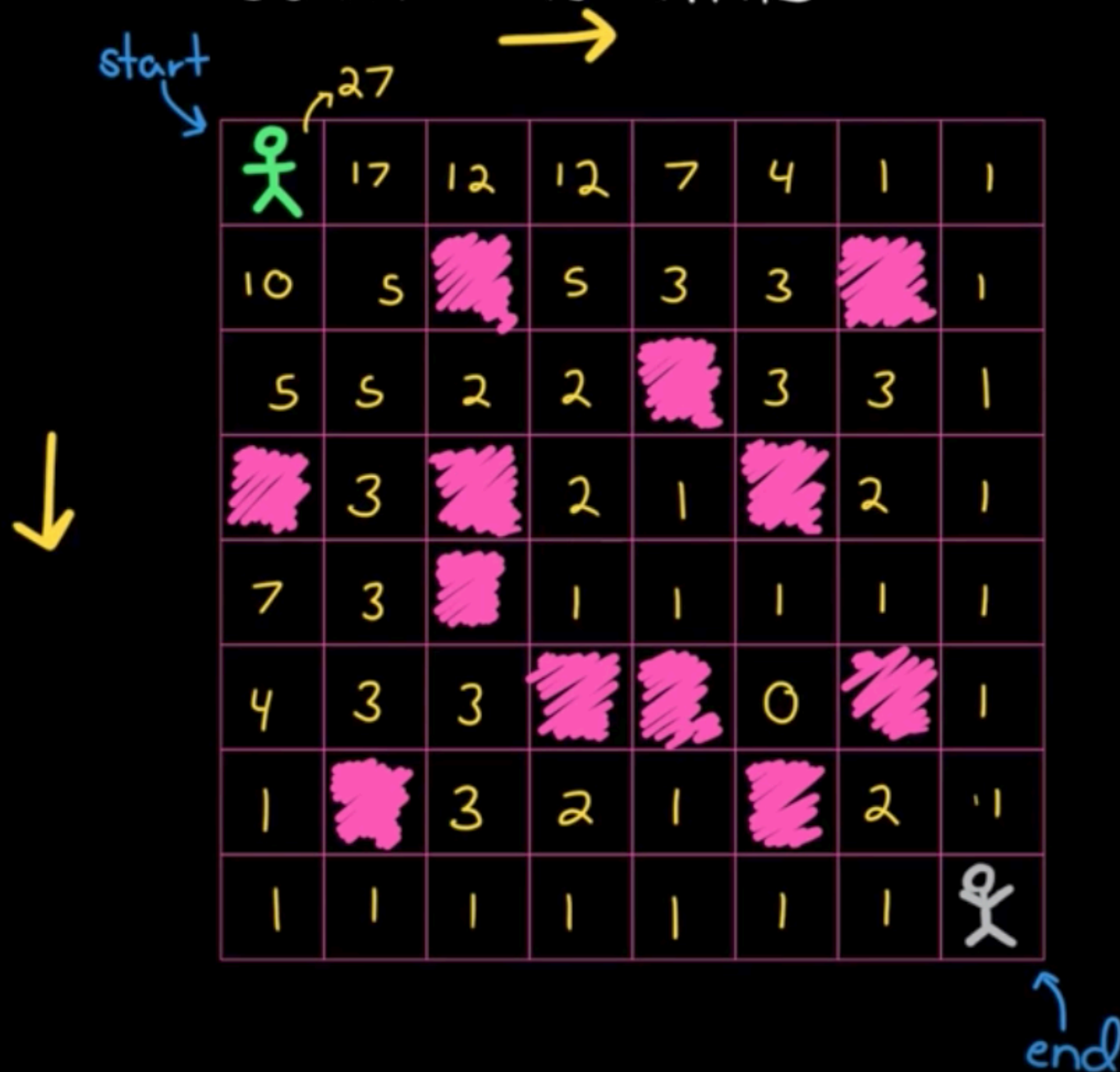
start

end

```
opt[i , j] = opt[i - 1, j] + opt[i, j - 1]
```

```
if a[i, j] = '空地':  
    opt[i , j] = opt[i - 1, j] + opt[i, j - 1]  
else: // 石头  
    opt[i , j] = 0
```


COUNT THE PATHS



动态规划 Dynamic Programming

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2. 状态的定义： $\text{opt}[n]$, $\text{dp}[n]$, $\text{fib}[n]$
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4. 最优子结构

动态规划 vs 回溯 vs 贪心算法

- 回溯（递归） — 重复计算
- 贪心算法 — 永远局部最优
- 动态规划 — 记录局部最优子结构 / 多种记录值

实战题目

1. <https://leetcode.com/problems/climbing-stairs/description/>
2. <https://leetcode.com/problems/triangle/description/>
3. <https://leetcode.com/problems/maximum-product-subarray/description/>
4. <https://leetcode.com/problems/best-time-to-buy-and-sell-stock/#/description>
5. <https://leetcode.com/problems/best-time-to-buy-and-sell-stock-ii/>
6. <https://leetcode.com/problems/best-time-to-buy-and-sell-stock-iii/>
7. <https://leetcode.com/problems/best-time-to-buy-and-sell-stock-iv/>
8. <https://leetcode.com/problems/best-time-to-buy-and-sell-stock-with-cooldown/>
9. <https://leetcode.com/problems/best-time-to-buy-and-sell-stock-with-transaction-fee/>
10. <https://leetcode.com/problems/longest-increasing-subsequence>
11. <https://leetcode.com/problems/coin-change/>
12. <https://leetcode.com/problems/edit-distance/>