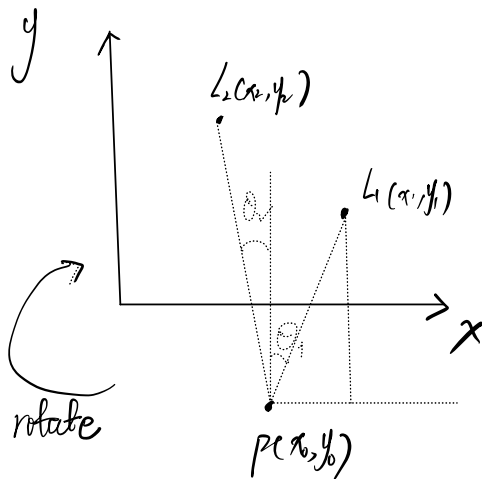


1. (a)



on map, we can observe that:

$$\frac{y_i - y_0}{x_i - x_0} = \cot(\theta_i)$$

$$\frac{y_i - y_0}{x_i - x_0} = \tan\left(\frac{\pi}{2} - \theta_i\right) \quad (x_i \neq x_0)$$

We have $\begin{cases} y_0 = m_i x_0 + b_i \\ y_i = m_i x_i + b_i \end{cases} \rightarrow \frac{y_i - y_0}{x_i - x_0} = m_i \quad (x_i \neq x_0)$

$$\Rightarrow m_i = \frac{y_0 - y_i}{x_0 - x_i} = \tan\left(\frac{\pi}{2} - \theta_i\right)$$

then $y_0 = \frac{(y_0 - y_i)x_0}{x_0 - x_i} + b_i$

$$b_i = \frac{x_0 y_0 - x_i y_0 - x_0 y_i + x_i y_i}{x_0 - x_i} = \frac{x_0 y_i - x_i y_0}{x_0 - x_i}$$

$$\left. \begin{aligned} m_i &= \frac{y_0 - y_i}{x_0 - x_i} \Rightarrow m_i x_i - y_i = m_i x_0 - y_0 \\ b_i &= \frac{x_0 y_i - x_i y_0}{x_0 - x_i} \Rightarrow b_i x_i = b_i x_0 - y_i x_0 + x_i y_0 \end{aligned} \right\} \Rightarrow \begin{bmatrix} m_i x_i - y_i \\ b_i x_i \end{bmatrix} = \begin{bmatrix} m_i & -1 \\ b_i & x_i \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

(b) $m_i = T_i$

We have: $m_i x_i - y_i = m_i x_0 - y_0$

$T_i x_i - y_i = m_i x_0 - y_0$

$$\begin{bmatrix} T_1 x_1 - y_1 \\ T_2 x_2 - y_2 \end{bmatrix} = \begin{bmatrix} T_1 & -1 \\ T_2 & -1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \Rightarrow \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} T_1 & -1 \\ T_2 & -1 \end{bmatrix}^{-1} \begin{bmatrix} T_1 x_1 - y_1 \\ T_2 x_2 - y_2 \end{bmatrix}$$

(T₁ ≠ T₂)