

$$1. (a) \begin{cases} x_{k+1} = Ax_k \\ y_k = g(x_k) + Dw_k \end{cases} \quad \text{system} \quad A = I_3$$

$$\hat{x}_{k+1|k} = A \hat{x}_{k|k}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_k (y_{k+1} - g(\hat{x}_{k+1|k})) \quad C_{k+1} = \frac{\partial g}{\partial x} \bigg|_{\hat{x}_{k+1|k}} = \begin{bmatrix} \frac{\hat{x}_k - x_1}{\sqrt{(\hat{x}_k - x_1)^2 + (\hat{x}_k - y_1)^2}} & \frac{\hat{x}_k - y_1}{\sqrt{(\hat{x}_k - x_1)^2 + (\hat{x}_k - y_1)^2}} \\ \frac{\hat{x}_k - x_2}{\sqrt{(\hat{x}_k - x_2)^2 + (\hat{x}_k - y_2)^2}} & \frac{\hat{x}_k - y_2}{\sqrt{(\hat{x}_k - x_2)^2 + (\hat{x}_k - y_2)^2}} \\ \frac{\hat{x}_k - x_3}{\sqrt{(\hat{x}_k - x_3)^2 + (\hat{x}_k - y_3)^2}} & \frac{\hat{x}_k - y_3}{\sqrt{(\hat{x}_k - x_3)^2 + (\hat{x}_k - y_3)^2}} \end{bmatrix}$$

$$K_k = P_{k+1|k} C_{k+1}^T (C_{k+1} P_{k+1|k} C_{k+1}^T + R_{k+1})^{-1}$$

$$P_{k+1|k} = A_k P_{k|k} A_k^T + Q_k$$

$$P_{k+1|k+1} = P_{k+1|k} - K_k C_{k+1} P_{k+1|k}$$