State Estimation Using Different Estimators

The aim of this exercise is to learn how to use different estimators to solve a bivariate normal state estimation problem.

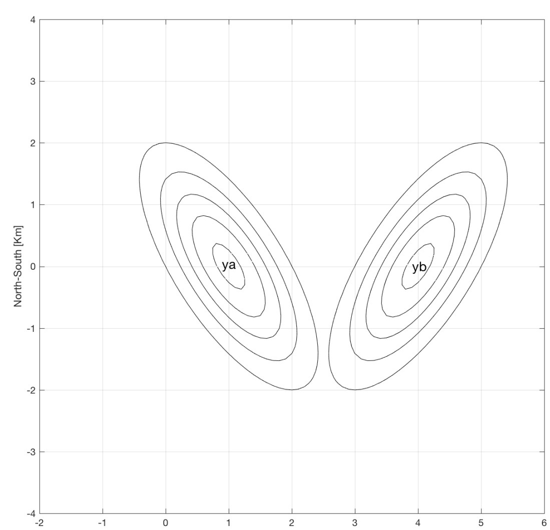
Remember the example of finding your car at the DIA parking lot.

- The state $\mathbf{x}$ is the location of your car : $\mathbf{x} = \begin{bmatrix} x_{ew} \\ x_{ns} \end{bmatrix}$
- There are two GPS devices locating your car at: $\mathbf{y}^a = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\mathbf{y}^b = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$
- GPS location errors from each device are correlated:

$$\boldsymbol{\epsilon}^a \sim \mathcal{N}(\mathbf{0}, \mathbf{R}^a) \quad \mathbf{R}^a = \begin{bmatrix} 0.6 & -0.6 \\ -0.6 & 1.2 \end{bmatrix}$$

$$\boldsymbol{\epsilon}^b \sim \mathcal{N}(\mathbf{0}, \mathbf{R}^b) \quad \mathbf{R}^b = \begin{bmatrix} 0.6 & 0.6 \\ 0.6 & 1.2 \end{bmatrix}$$

Note that $\boldsymbol{\epsilon}^a$ and $\boldsymbol{\epsilon}^b$ are uncorrelated.



(Locations are given in terms of km for Problem 3)

1. Maximum Likelihood Estimator (MLE)

   - Show the derivation of the maximum-likelihood estimator (MLE), starting from the log-likelihood function. Compute your car location by using MLE, and graphically show the location on the map.

   - Change the GPS location observation errors to be uncorrelated, i.e., $\mathbf{R}^a = \begin{bmatrix} 0.6 & 0 \\ 0 & 1.2 \end{bmatrix}$ and $\mathbf{R}^b = \begin{bmatrix} 0.6 & 0 \\ 0 & 1.2 \end{bmatrix}$, and recompute the MLE of your car location.

     Discuss the difference from the previous case.

2. Maximum A Posterior Estimator (MAP)

   - Let's suppose that there is prior information about the car location given as

$$\mathbf{x} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1.2 & 0.6 \\ 0.6 & 0.6 \end{bmatrix} \right)$$

     and compute your car location by using the MAP estimator. Use the original GPS location observation errors. Graphically show the location on the map, and discuss the difference from Problem 1 (first case) if any.

   - Describe how the inference may change if the Bayesian approach is adopted.

3. Suppose that the car is moving in the south-east direction at the speed of 7.2 km/hr because it is being towed. Track the car location by using the Kalman filter. Use the prior car location information given in Problem 2, and the original GPS location observation errors. You may assume that there is no (process) error associated with a dynamical model for the motion of car. Two GPS devices report the following locations every 1 minute:

$$[\mathbf{y}_0^a \ \mathbf{y}_1^a \ \mathbf{y}_2^a \ \mathbf{y}_3^a \ \mathbf{y}_4^a \ \mathbf{y}_5^a \ \mathbf{y}_6^a] = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & -0.1 & -0.2 & -0.3 & -0.4 & -0.5 & -0.5 \end{bmatrix}$$

$$[\mathbf{y}_0^b \ \mathbf{y}_1^b \ \mathbf{y}_2^b \ \mathbf{y}_3^b \ \mathbf{y}_4^b \ \mathbf{y}_5^b \ \mathbf{y}_6^b] = \begin{bmatrix} 4 & 4 & 4 & 4 & 4 & 4 & 4 \\ 0 & -0.1 & -0.2 & -0.3 & -0.4 & -0.5 & -0.5 \end{bmatrix}$$

   What is the probability of finding your car at $x_{ew} = 2$ and $x_{ns} = -2$ in 6 minutes into the search?

4. Show the equivalence of the Bayesian approach and variational approach to the univariate normal state estimation problem by equations.

   (*Submit your derivations, codes, plots, and brief discussion via Canvas.*)