16-822: Geometry-based Methods in Vision (F18)	Lecture #0 September 20, 2018

LaTeX instructions

Lecturer: Martial Hebert

TA: Xiaofang Wang & Xinshuo Weng *Scribes: Scribe-1 & Scribe-2*[1]

1 Recap

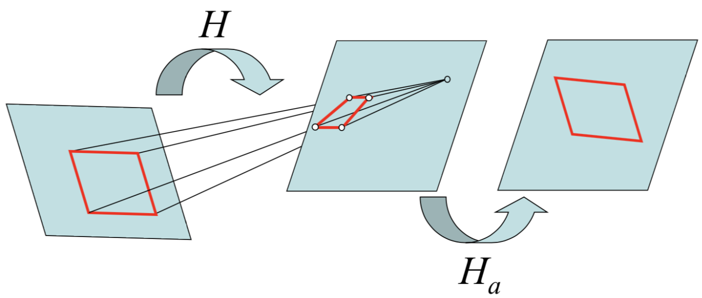

Figure 1: Affine planar rectification

Given a projective version of the world scene, as shown in Fig. 1, we may find a homography H_a that preserves the parallelism and thus recovers affine properties as in the original plane. So $H_a H$ is an affine transformation. Actually, H_a can be computed if we know the projection of the line at infinity. In this case, we may denote $l_\infty = \begin{bmatrix} l_1 & l_2 & l_3 \end{bmatrix}^\top$. H_a can be expressed as

$$H_a = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 & l_2 & l_3 \end{bmatrix}$$

2 Metric rectification

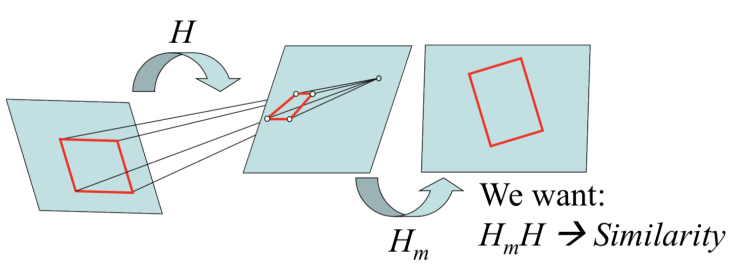

Figure 2: Metric planar rectification

Similarly, for metric rectification, the euclidean properties of the original plane can be recovered if we find a homography H_m, which preserves the angle between two lines on the plane. We first

[1] Your name, and Andrew-ID

denote two lines $l = \begin{bmatrix} a & b & c \end{bmatrix}^\top$ and $l' = \begin{bmatrix} a' & b' & c' \end{bmatrix}^\top$. The angle between these two lines is

$$\cos\theta = \frac{aa' + bb'}{\sqrt{a^2+b^2}\sqrt{a'^2+b'^2}}$$

Given $aa' + bb' = l'^\top C^* l$ and $C^* = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, we may derive that

$$\cos\theta = \frac{l'^\top C^* l}{\sqrt{l^\top C^* l}\sqrt{l'^\top C^* l'}}$$

If l' and l are orthogonal, we have

$$l'^\top C^* l = 0$$

In Fig. 2, assume we have a line l_0 in the original plane and its homography transformation l. After metric rectification, the line became l' and right angle preserved. We get the following equation

$$l' = H_m^{-\top} H^{-\top} l_0 = H^{-\top} l$$

$$l$$