16-822: Geometry-based Methods in Vision (F18) Lecture #5 September 20, 2018

Calibration and PnP Solutions

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1 Recap

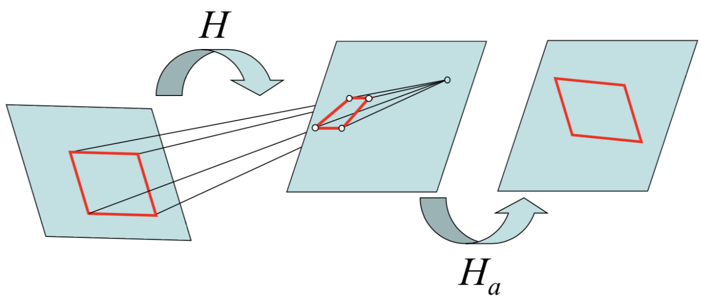

Figure 1: Affine planar rectification

Given a projective version of the world scene, as shown in Fig. 1, we may find a homography H_a that preserves the parallelism and thus recovers affine properties as in the original plane. So H_aH is an affine transformation. Actually, H_a can be computed if we know the projection of the line at infinity. In this case, we may denote $l_\infty = \begin{bmatrix} l_1 & l_2 & l_3 \end{bmatrix}^\mathsf{T}$. H_a can be expressed as

$$H_a = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 & l_2 & l_3 \end{bmatrix}$$

2 Metric rectification

2.1 For 2D plane

For metric rectification, the euclidean properties of the original plane can be recovered if we find a homography H_m, which preserves the angle between two lines on the plane. We first denote two lines $l = \begin{bmatrix} a & b & c \end{bmatrix}^\mathsf{T}$ and $l' = \begin{bmatrix} a' & b' & c' \end{bmatrix}^\mathsf{T}$. The angle between these two lines is

$$\cos\theta = \frac{aa' + bb'}{\sqrt{a^2+b^2}\sqrt{a'^2+b'^2}}$$

Given $aa' + bb' = l'TC^*l$ and $C^* = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, we may derive that

$$\cos\theta = \frac{l'TC^*l}{\sqrt{l^\mathsf{T}C^*l}\sqrt{l'TC^*l'}}$$

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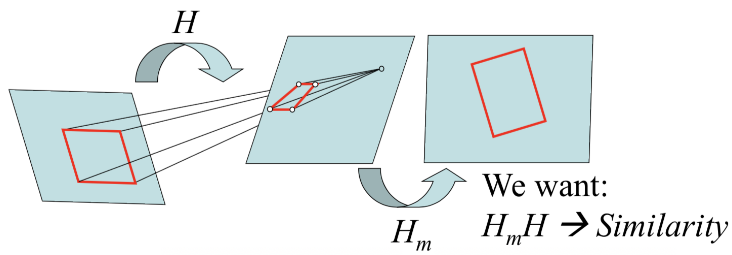

Figure 2: Metric planar rectification

If l' and l are orthogonal, we have

$$l'TC^*l = 0$$

In Fig. 2, assume we have a line l_0 in the original plane and its homography transformation l. After metric rectification, the line became l' and right angle preserved. We get the following equation

$$l' = H_m^{-\mathsf{T}} H^{-\mathsf{T}} l_0 = H^{-\mathsf{T}} l \tag{1}$$

$$l_0^{i\mathsf{T}} C^* l_0^j = l_0^{i\mathsf{T}} H_m^{-1} C^* H_m^{-\mathsf{T}} l_0^j = 0 \tag{2}$$

To solve for H_m, we may need at least 4 pairs of lines.

2.2 For 3D space

Assume that we have two perpendicular planes π_1 and π_2 in 3D space and π_∞ at infinity. Ω_∞ is an absolute conic on π_∞. The dual of the absolute conic Ω_∞ is called the absolute dual quadric, denoted Ω_∞^*

$$\Omega_\infty^* = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

Similarly, the properties are the same as in the plane. The angle between planes is

$$\cos\theta = \frac{\pi_1^{\mathsf{T}} \Omega_\infty^* \pi_2}{\sqrt{\pi_1^{\mathsf{T}} \Omega_\infty^* \pi_1} \sqrt{\pi_2^{\mathsf{T}} \Omega_\infty^* \pi_2}} \tag{4}$$

Note that C_∞^* is invariant if and only if H is a similarity transformation.

3 Calibration

3.1 Calibration form 3D points (PnP)

For the camera matrix, $M = K[R\ t]$. We want to solve this intrinsic matrix K from calibration.

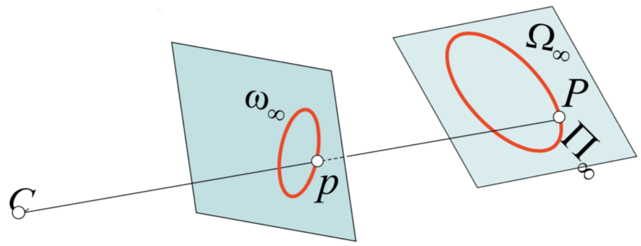

Figure 3: Calibration from plane homographies: The image of the absolute conic

Given a plane at infinity Π_∞ as in Fig. 3, the absolute conic Ω_∞ is the conic of equation

$$x^2 + y^2 + z^2 = 0,\ \ w = 0 \tag{5}$$

where $P = \begin{bmatrix} x & y & z & 0 \end{bmatrix}^\mathsf{T}$. The point p at the image plane can be expressed as

$$\begin{aligned} p = k\begin{bmatrix} R & t \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix} &= KR\begin{bmatrix} x \\ y \\ z \end{bmatrix} \\ \implies \begin{bmatrix} x \\ y \\ z \end{bmatrix} &= R^\mathsf{T} K^{-1} p \end{aligned} \tag{6}$$

From Eqns. 5 and 6, we have

$$\begin{aligned} \begin{bmatrix} x & y & z \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} &= 0 \\ p^\mathsf{T} K^{-\mathsf{T}} R R^\mathsf{T} K^{-1} p &= 0 \\ p^\mathsf{T} K^{-\mathsf{T}} K^{-1} p &= 0 \\ p^\mathsf{T} \omega_\infty p &= 0 \end{aligned} \tag{7}$$

Therefore, ω_∞ can be represented by the matrix $K^{-\mathsf{T}} K^{-1}$.

3.2 Outline of the calibration solution

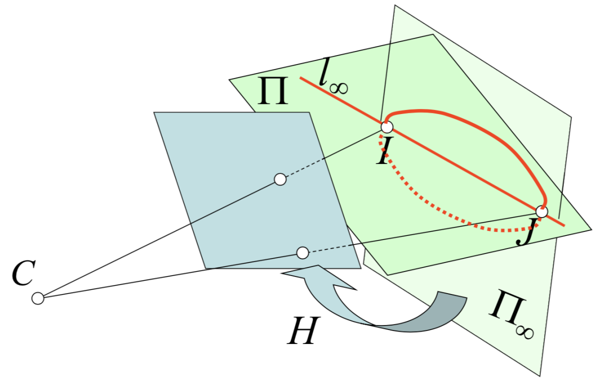

Figure 4: Point I, J at plane Π_∞

Now we assume that a plane Π intersects the conic Ω_∞ at point $I, J = \begin{bmatrix} x & y & 0 \end{bmatrix}^\top$. Since $x^2 + y^2 = 0$ does not have a real solution. I, J can be expressed as

$$I = \begin{bmatrix} 1 \\ i \\ 0 \end{bmatrix} \quad J = \begin{bmatrix} i \\ -1 \\ 0 \end{bmatrix} \tag{8}$$

Since $I \in \Omega_\infty$ and $HI \in \omega_\infty$, we know

$$I^\top H^\top \omega_\infty HI = 0$$
$$J^\top H^\top \omega_\infty HI = 0$$

Thus we have 2 equations per plane. Note that ω_∞ has 5 dof since a 3×3 matrix gives 6 dof while it is also up to scale.

4 PnP solutions

Suppose we have 3 world coordinates of point i: $p_i^w = \begin{bmatrix} x_i^w & y_i^w & z_i^w \end{bmatrix}^\top$, and 3 coordinates of point i with respect to camera coordinate system: $p_i^c = \begin{bmatrix} x_i^c & y_i^c & z_i^c \end{bmatrix}^\top$.

Now let $M = K \begin{bmatrix} R & t \end{bmatrix}$, in this case K is a known variable. Since we have $p = MP$, $M = \begin{bmatrix} \mathbf{m}_1^\top \\ \mathbf{m}_2^\top \\ \mathbf{m}_3^\top \end{bmatrix}$,

$$x = \frac{\mathbf{m}_1^\top P}{\mathbf{m}_3^\top P}, \quad y = \frac{\mathbf{m}_2^\top P}{\mathbf{m}_3^\top P}$$

$$\Rightarrow \begin{cases} P_i^\top \mathbf{m}_1 - xP_i^\top \mathbf{m}_3 = 0 \\ P_i^\top \mathbf{m}_2 - yP_i^\top \mathbf{m}_3 = 0 \end{cases} \tag{9}$$

Suppose we have N sets of Eqn. 9, we may write in the matrix form

$$\begin{bmatrix} P_i^\top & 0 & -xP_i^\top \\ 0 & P_i^\top & -yP_i^\top \\ & \vdots & \end{bmatrix} \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} = 0 \tag{10}$$