My Chosen Topic: Penalty!

Introduction:

"The origin of this project is the need to estimate expected detector count rates. Assume a radioactive source emitting radiation of some kind (particles or gamma rays) isotropically, i.e. with no preferred direction. Assume further a detector with given dimensions placed in some distance from the source. We would be interested in knowing what fraction of the emitted radiation would impinge on the detector surface. "

This problem can be transferred into the world of macroscopic objects. As an example, we chose a football pitch. The main goal of this final python project is to write a python code that performs the Monte Carlo simulation of a (high) number of penalties and determine the fraction of kicks where the ball ends up in the goal.

Since it is a simulation that I aim to reappear the reality when we shooting penalty in football patch as possible, some reasonable assumptions and parameters of objects need to be applied. spherical coordinate is used for calculation, important assumptions and strategies includes:

- I first start simulate the ball as only a point mass in an ideal way and then to a more realistic model as with finite diameter football.
- I first start simulate the goal area without goal posts and crossbar, then in a more realistic model as goal area with goal posts and crossbar
- In reality, the penalty direction of ball cannot go underground, i.e., $0° \leq \theta \leq 90°$
- In the more realistic model gravity is added compare to ideal case of without gravity
- A goalkeeper placed randomly in front of goal area and compare to no goalkeeper
- Look for penalty when φ in forward angles only and all direction angles
- Look for goals and own goals when φ angle in all directions

Essential parameters from the football field:

Some parameters that will be used in the simulation, are listed in the followings.

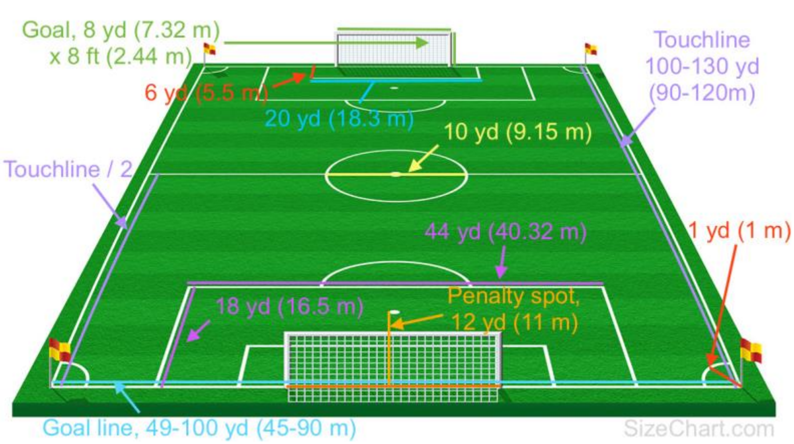

Figure 1 Football field illustration diagram [1]

Refer to Figure 1 (captured from online reference [1]), the width of goal area, the height of goal area, the distance between penalty spot and goal area, the distance of all touchline of the field, is illustrated. Also, from reference [2], the diameter of a football, the thickness of goal posts or crossbar, average speed of football penalty shot by professional football player, and our human body size are all considered in certain aspects.

Parameter	w	h	$d_{penalty}$	$d_{touchline}$	$d_{football}$	tk_{pb}	v_{avg}
Value	7.32 m	2.44 m	11 m	90 m	0.22 m	0.12 m	31.29 m/s

Where w is the width of goal area (including the goal posts), h is the height of goal area (including the crossbar), $d_{penalty}$ is the distance between penalty spot and the goal area line (as shown Figure 1), $d_{touchline}$ is the distance touchline the football field (from that we can later get the distance of penalty spot and own goal area line), $d_{football}$ is the finite diameter of a football, tk_{pb} is the thickness of goal posts and crossbar, and v_{avg} is the average speed of football penalty shot by professional football player.

Refer to Figure 2 (captured from reference [3]), an illustration of human body size was shown. In the ideal situation, the height and the width (stretch distance) are used as parameter, which is 1.88(+0.393) m and 1.964 m, respectively.

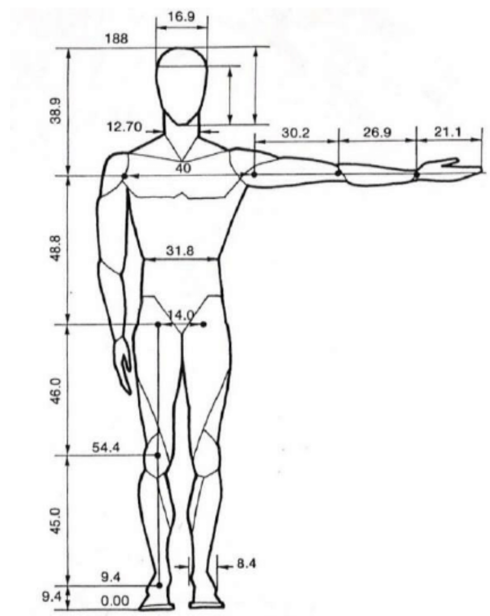

Figure 2 Human body size illustration [3]

Also, later when I simulate the situation when gravity involved, the constant of gravitational acceleration $g \approx 9.81\ m/\ s^2$ (directly use the constant from numpy which is more accurate and more than 2 decimals) is also included for calculation.

The Method and Math's Model for this Monte Carlo Simulation:

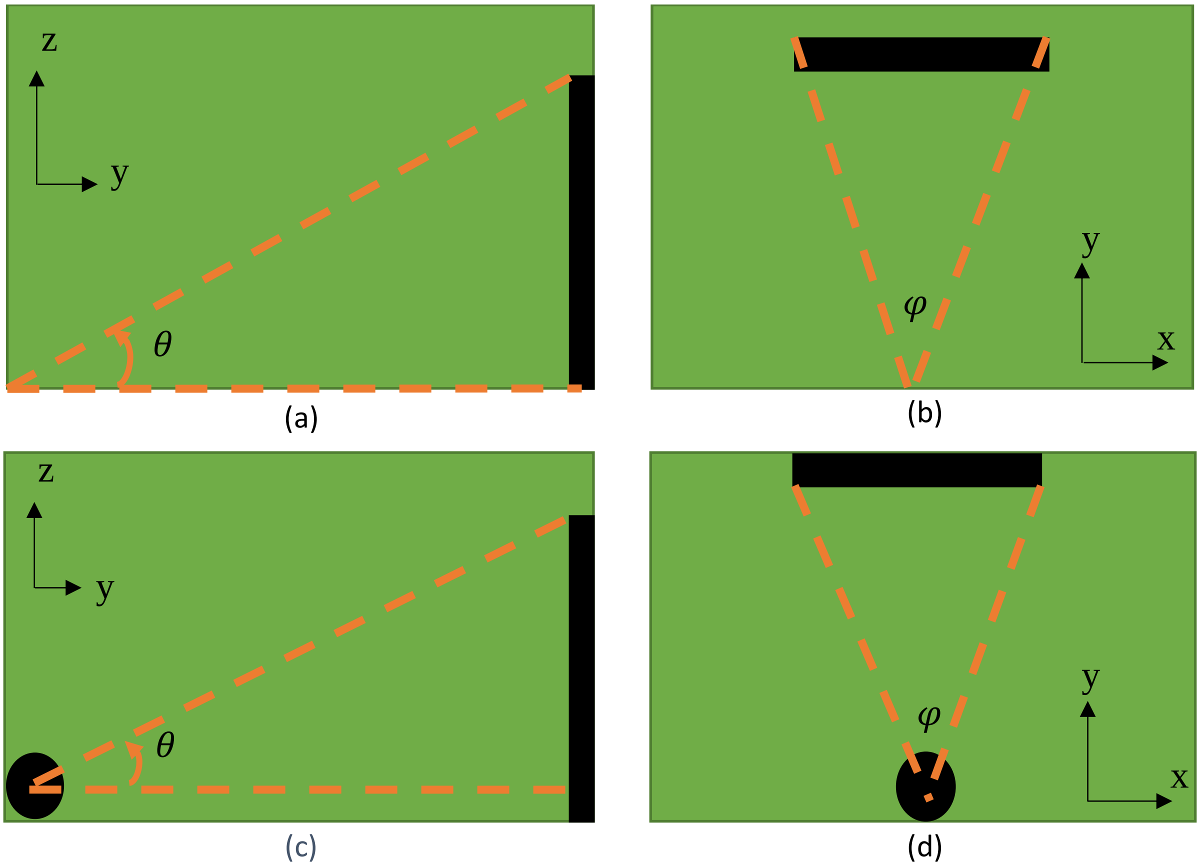

Figure 3 Illustration of theta and phi angle in spherical coordinate system

§ Refer to Figure 3, the core concept and idea of the method or strategy that I used for the MC simulation can be solely represented by these four subfigures.

My choice of coordinate system for the description of the football of penalty, it is the intuitively spherical coordinate system. The rectangle shape in Figure 3 represents the goal area, to be clarify. The diagram of Figure 3 (a) and (c) shows the projection in z-y plane, (b) and (d) shows the projection in x-y plane. Furthermore, the diagram of Figure 3 (a) and (b) shows the ideal situation of penalty with point mass ball (without diameter), while the (c) and (d) shows a more realistic situation of the football with a finite diameter ($d_{football} = 0.22m$).
In the latter case, the way I consider that is still treat it with a point, but this time would be the center of mass (sphere) under the assumption of mass is evenly distributed, thus just the center of the circle in the 2D projection. Then, when doing the calculation in simulation just also consider the radius with the original center of mass, in a similar logic.

Ideal point mass vs. finite diameter ball:

For the point mass "ball"	For the football with finite diameter
Directly use the relations of reaching boundary of goal area	Need to $\pm$ the radius of football as the boundary reaching the boundary of the goal area

§ The way is that, by the geometry relations, one can determined the θ and φ that can reach the goal area, and the main idea of my MC simulation code is to perform a large amount of number (I have tried 10 thousand, 100 thousand, and I have simulated 1 million events eventually) of randomly pick θ and φ these 2 sets of angles that satisfy the geometric relations in the football field.

- The geometrical relations

The geometrical relations are can mostly be found by trigonometric function. When the ball was shot out in $\varphi = 90°$, the distance $d_{penalty}$ and height of goal area h are known by that we can determine θ in this situation by:

$$\theta = \arctan\left(\frac{h}{d_{penalty}}\right)$$

On the other hand, the φ can be determined by half goal area width and distance $d_{penalty}$:

$$\varphi = 2\arctan\left(\frac{w/2}{d_{penalty}}\right)$$

At first, I naively think that would be done for the relations to describe θ and φ, but in fact the θ is a function of φ, i.e., $\theta = \theta(\Delta\varphi)$. When the $\varphi \neq 90°$, then the $d_{penalty}$ change a bit, so that I actually need to express θ changes as φ changes, where $\Delta\varphi = |\varphi - 90°|$.

$$\theta = \theta(\Delta\varphi) = \arctan\left(\frac{h}{d_{penalty}/\cos(\Delta\varphi)}\right)$$

- Possible range of θ and φ

About the possible range of θ and φ. For φ, I divide it into two categories, the forward angles case, i.e., $0 \leq \varphi \leq 180°$, and the all-directional angles case, i.e., $0 \leq \varphi < 360°$. With the restriction of reality, the ball can only go above the ground which means $0 \leq \theta \leq 90°$.
It is necessary to mention that, the diagram of Figure 3 illustrate the ideal case that without gravity, i.e., the ball go-on in a straight line. In the later time, the gravity is considered.

§ That is, use the analytical geometric relations to restrict θ and φ and then randomly generate them and MC simulation.
§ On the other hand, the other intuitive way is to use python ODE numerical solver to describe the trajectory and by the equation of motion under gravity or so.

§ In the python code, I here am trying to only using random generator method to determine those two angles that obey the geometric relations each time, but without using ODE numerical solver in python, things will work out also as we can visualize it but in a different level of difficulty, that can ultimately and in order to approach the reality.

- Include the interaction with goal posts and crossbar

The thickness of both goal posts and crossbar are the same as listed in the previous section. The way I simulate it is mainly divided the posts or bar into several sectors that each have a movement of length of 0.01 m, from inner area moving towards outer area. Which means that I start it from the place where if the center of the ball reaches the point that will lead to the boundary of the ball to touch the posts or the crossbar, and then begin with that points, and moving horizontally (for posts from inner side towards outer side) or vertically (from inner side towards outer side) and until the center of the ball reach the half thickness of the posts or crossbar, then from the point of half thickness of posts or bar to outer boundary of post or bar, there will not be a goal. But from the half thickness of post or bar to the inner area where the boundary of the ball start touching the posts or bar, there will be different possibilities that the football interaction with the posts or bar that can be goal or no goal.

§ Mainly I simulate the interaction of football with goal posts and crossbar, based on the assumption of when the ball hit different area of the posts or bar with different possibilities. If it is more closer to the inner side, the higher the possibility that the ball can go inside the goal area, and the boundary that I set here is from the inner side as when the boundary of the ball touches the posts or bar, and with 1 cm as a unit to divide the posts (separate it vertically or move it horizontal direction) or bar (separate it horizontally or move it vertical direction) area, until the center of the ball reaches the center of the posts or bar, then intuitively since the ball hit the outer half area of posts or bar, and it will not goal, if and only if the ball hit the half inner area of posts or bar that can have certain probability according to its distance in the unit of 1cm to the inner boundary that might have some chance to goal.

- Interactions with goal posts and crossbar:

If it hit the outer half area	If it hit the inner area of posts or bar
It will not be a goal. Since hitting the outer half of posts or bar cannot reflect into the inner direction	To check in which 1cm unit divided area it is in, the closer it is to the inner boundary, the higher of the chance it will be a goal.

In my model of 1cm units, recall from the parameter sections, the thickness of posts or bar is 0.12 m and the radius of the ball is $0.11m$. So that starting from the point of center of the ball of ball's boundary touches the posts or bar's inner boundary of each sector horizontally (for posts) or vertically (for bar) with 1cm, besides the boundary of direct goal when inside to inner boundary and reach the center of posts or bar, then I got 16 possible areas of each.

From the point of center of posts or bar with in between 0% to 100% in 18 areas, but except the 0% and 100% for the 2 boundary, the remaining area is randomly decided in my python code using random.randint function by these 18 total sectors, and we can only check for the 16 area in between:

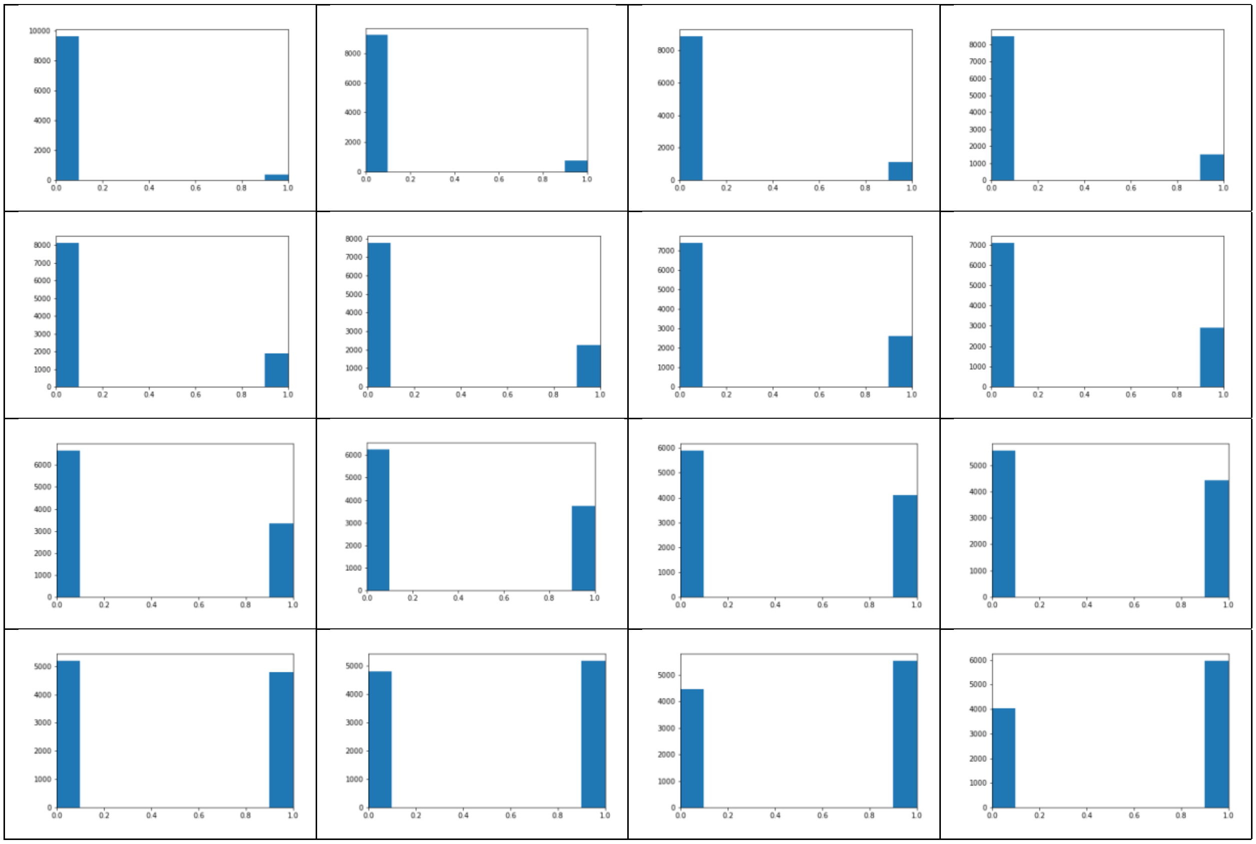

Figure 4 Test result of 16 different possibility distribution for 10 thousand events for the posts or bar inner half area (read it from top-left sub-figure to right then next row left to right until the last row bottom right sub-figure)

Refer to Figure 4, value 1.0 represents goals and 0.0 represents no goal. The purpose here is just to check the distribution of random generator that I use for further interaction sections at different half inner area of goal posts or crossbar. To see it from the left upper corner and then look for the right-hand side, and then next row and them continue to right hand side and go on, as we see that the possibility from the outer position to the more inner position, that it seems a reasonable possibility distribution, that means the validity is acceptable to accomplish the mission for possibility and distance based goal-posts and crossbar with football interaction.

- Including gravity

Here I am not using the traditional numerical solving ODE of equation of motion, but using the relations that the θ can reach under the analytical visualization and then again determine each time's $\theta\ and\ \varphi$. And the gravity affects mainly the θ angle, not the φ, but similarly the $\theta = \theta(\Delta\varphi)$, as I discussed previously that is a function of those 2 sets of angles, $\theta\ and\ \varphi$.

By the equation of motion of projectile motion, one can have the relation:

$$\tan\theta \ = \frac{v^2 \pm \sqrt{v^4 - g(gx^2 + 2yv^2)}}{gx}$$

Where x and y are the position of the plane, in my case it should be $x \to y$ and $y \to z$ instead. Then we have velocity of football as mentioned, and g as gravitational acceleration constant, so that the theta again can be determined with certain range, actually we have 2 sets of range, so this would be got two possible ranges. The first one is similar to the one with straight going, but here since we included gravity, the upper limit will be higher, that including more areas in the range, and the second area which is very close to vertical direction but it can reach somehow in some scenarios.

§ Here after consider the effect of gravity I can still update the range that obey the geometrical relations and projectile motion that leads to the range even larger. And then use the same way as MC simulation to determine the 2 angles and calculation.

For the drag force case using relations like

$$F_D = -\frac{1}{2}c\rho A v\mathbf{v}$$

One can also update the θ range and perform MC simulation by this model that I used, but I will not include this pat in the report.

The simulation results under different assumptions:

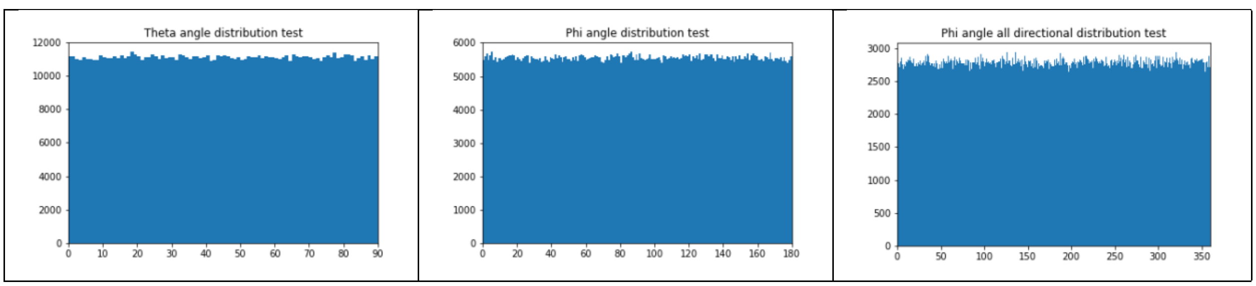

Figure 5 Angles distribution test in spherical coordinates

Using random function in python, I first test the distribution of $0° \leq \theta \leq 90°$ and the forward angles $0° \leq \varphi \leq 180°$ and the all directions of $0° \leq \varphi \leq 360°$, as shown in Figure 5, as it is valid for the random angles generation as it spread overall range evenly mostly.

Using the method and model that I previously introduce. The followings are the simulation results.

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- For Ideal point mass ball.

Assume φ in forwards angles only:	For φ in all directional angles:
The fraction of goals without consideration of theta as function of phi in 1000000 isotropic events is: 28476. $[0.028476 \approx 2.8476\%]$	The fraction of goals without consideration of theta as function of phi in 1000000 isotropic events is: 14292 [0.014292] $[0.014292 \approx 1.4292\%]$
The fraction of goals with theta as function of phi consideration in 1000000 isotropic events is: 27979 [0.027979] $[0.027979 \approx 2.7979\%]$	The fraction of goals with the consideration of theta as function of phi in 1000000 isotropic events is: 14079 [0.014079] $[0.014079 \approx 1.4079\%]$

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As shown above, I test the error of consider or not of the relations that θ as a function of $\Delta\varphi$

- Football with diameter and Goal area with posts and bar

Assume φ in forwards angles only:	For φ in all directional angles:
The fraction of goals with theta as function of phi consideration in 1000000 isotropic events with posts and bar is: 25415 $[0.025415 \approx 2.5415\%]$	The fraction of goals with theta as function of phi consideration in 1000000 isotropic events with posts and bar is: 10768 $[0.010768 \approx 1.0768\%]$

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- Own goal (when φ in all direction) and keeper save

Without posts and bar	With posts and bar	
Own goal when 1000000 isotropically shoot in any phi direction: 286 $[0.000286 \approx 0.0286\%]$	Own goals in 1000000 isotropic events with posts and bar is: 274 $[0.000274 \approx 0.0274\%]$	Keeper saving in 1000000 isotropic events with posts and bar is: 111 $[0.000111 \approx 0.0111\%]$

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- Goals are with goal posts and crossbar and with gravity (1 million events)

Goal	Own goal	Keeper save
Goals when 1000000 isotropically shoot in any phi direction: 27609 $[0.027609 \approx 2.7609\%]$	Own goals when 1000000 isotropically shoot in any phi direction: 4447 $[0.004447 \approx 0.04447\%]$	Keeper save when 1000000 isotropically shoot in any phi direction: 291 $[0.000291 \approx 0.0291\%]$

Summary and discussion

I shall summarize the goals, own goal and save result by these different categories.
Notice that is the simulation percentage result got from 1 million events by the method and model that I introduce above.

Table S1			
Goals (Ideal point mass without goal posts and crossbar)			
φ in forwards angles only		For φ in all directional angles	
Without $\theta(\Delta\varphi)$	Consider $\theta(\Delta\varphi)$	Without $\theta(\Delta\varphi)$	Consider $\theta(\Delta\varphi)$
2.8476%	2.7979%	1.4292%	1.4079%

As we seen above, the error caused by not considering $\theta(\Delta\varphi)$ is not really high, but frankly it affects at least from the second decimal, would be around at least 0.01~0.02 for its deviation. One can roughly find the ratio of rectangular goal area to the one fourth of sphere than it shows the validity of the simulation, not exactly but close with error due to roughly approximation.

Table S2		
Goals (Finite diameter football with goal posts and crossbar)		
Straight line motion		Motion with gravity
φ in forwards angles only	For φ in all directional angles	
2.5415%	1.0768%	2.7609%

As we expected, for forward φ only and all φ direction simulation can have a nearly double of its value is pretty imaginable. And when we include gravity, one can imagine that the projectile motion that can include more of the range in θ degree that can leads to more of the possible events to have a goal, compare to only straight-line motion, like in our microscopic world of particle which is nearly not affected by gravity.
From Table S1 and S2, it also shows the variation when we include a more realistic factor, such as goal posts and crossbar, or diameter of the ball, that would get lower goals due to those effect. For gravity it changes from straight-line to projectile thus can increase some possible θ as we see that can be more goals from this more range that can possibly goals.

<table>
<tr><td colspan="3">Table S3</td></tr>
<tr><td colspan="3">Own Goals (obviously φ in all directional angles)</td></tr>
<tr><td>Without goal posts and crossbar</td><td colspan="2">Consider goal posts and crossbar</td></tr>
<tr><td colspan="2">Straight line motion</td><td>Motion with gravity</td></tr>
<tr><td>0.0286%</td><td>0.0274%</td><td>0.04447%</td></tr>
</table>

Apparently from that far away of distance, that would be a relatively small value. And the trend of own goal is also as expected from the physics model that I am in, reasonably that the effect of gravity added is also shown.

<table>
<tr><td colspan="3">Table S4</td></tr>
<tr><td colspan="3">Keeper save (with goal posts and crossbar)</td></tr>
<tr><td></td><td>Straight line motion</td><td>Motion with gravity</td></tr>
<tr><td>Compare to total events</td><td>0.0111%</td><td>0.0291%</td></tr>
<tr><td>Compare to goal events</td><td>1.030%</td><td>1.054%</td></tr>
</table>

The goalkeeper randomly places in the goal area, one would be only considered if the ball would initially count as a goal (direct goal or smashing posts or bar and goal), also with the human size as set in parameter, I got that it is relatively low rate that the save event occurred compare to the one million total events. A more reasonable way would be compared to the actual goal events, then as shown in Table S4.

Lastly, it is fun to find another way rather the more familiar numerical ODE solver method, and as challenging as to explore many different processes towards the way of MC simulation.

Reference

[1]: Soccer Field (Football Pitch) Size. Available from: https://www.sizechart.com/sports/football-soccer/field-pitch-size/index.html [accessed 4 Nov, 2021]

[2]: Experts Corner: The Science Behind Elite Penalty Kicks. Available from: https://www.neurotrackerx.com/post/science-penalty-kicks [accessed 4 Nov, 2021]

[3]: Modeling and Simulation of a Passive Lower-Body Mechanism for Rehabilitation - Scientific Figure on ResearchGate. Available from: https://www.researchgate.net/figure/Dimensions-of-average-male-human-being-23_fig1_283532449 [accessed 11 Nov, 2021]