Forward: This is a short report presenting the figures requested in the various questions, and with short explanation of the procedures that I used to get them.

Question 1:

a proton moving in x-direction in a homogeneous magnetic field with a field axis in z-direction. The proton has an energy of 1 MeV and the field strength is 3 T.

1 Applying the condition that constraint the proton in the magnetic field, set all the given parameters, in SI units. Define the function for equation of motion, I then calculate with ordinary differential equation solver (solve_ivp) in python.

1.1 I first plot the motion in x-y plane for 1 μs, an obvious stiffness curve by default RK45 method, see Figure1(a). To improve it and choose Radau method (used followings), a smoother curve of motion in x-y plane for 1 μs obtained, see Figure1(b):

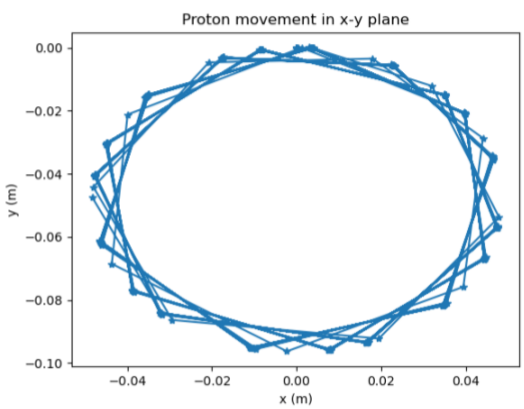

Figure 1(a)

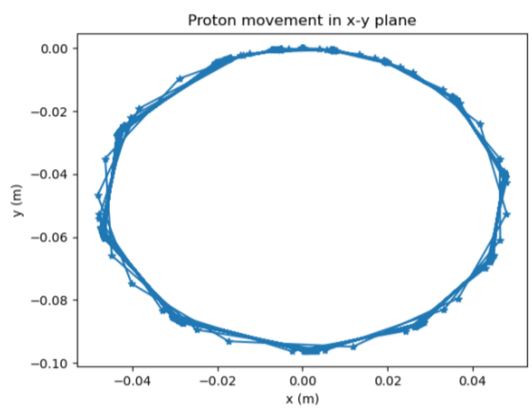

Figure 1(b)

1.2 As expected, the situation described in Question1, according to Lorentz force, by the cross-product relation, this would be only a 2D circular motion (in x-y plane). That means projection in x-z and y-z plane evolute in the same altitude, refer to figure 2(a) and 2(b):

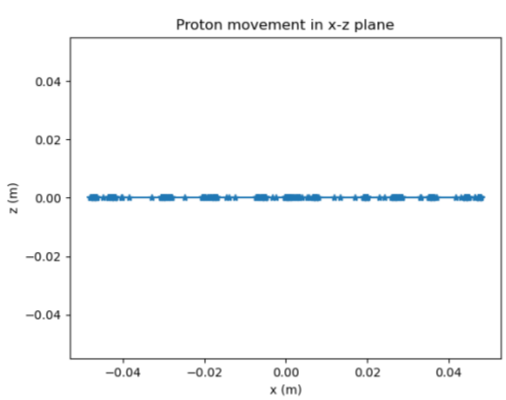

Figure 2(a)

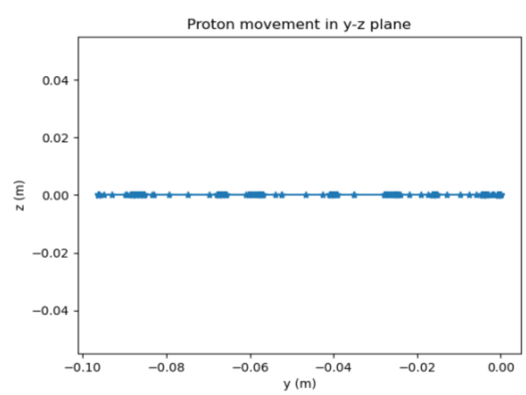

Figure 2(b)

For Question1.
Correction (Improvement): I update it to be smaller time steps, then given smoother plots.

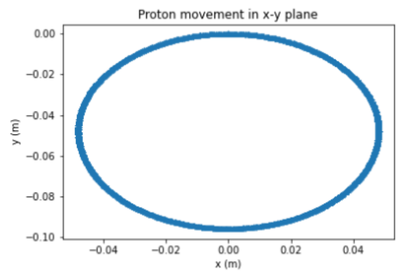

Figure 1

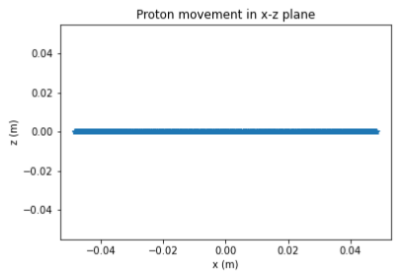

Figure 2(a)

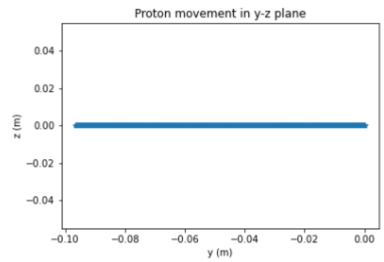

Figure 2(b)

Question2:

The same proton (energy 1 MeV) in the same magnetic field but this time the initial proton velocity has both an *x*- and a *z*-component.

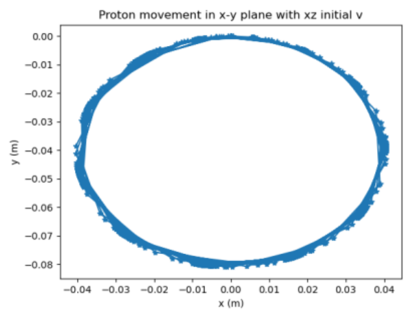

Figure 3(a)

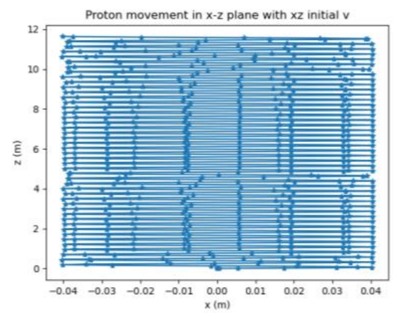

Figure 3(b)

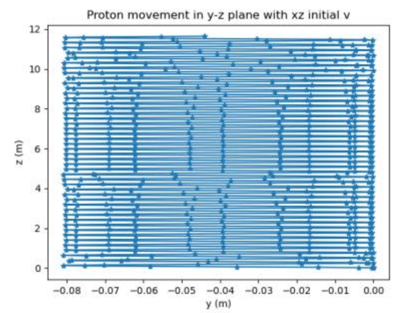

Figure 3(c)

2 Used the same function defined in question 1 to solve the ODE. Parameters are same except the initial condition of velocity changed, both x and z component exist.

2.1 I then assume the angle between x-z axis is 45 degree initially. Then apply to the ODE solver, and for 1 μs also, plot the motion of 2D projection in xy, xz, and yz plane, respectively (refer to Figure 3(a), 3(b), 3(c)).

2.2 As the calculation shows, since our initial velocity involved in 2 components, x and z, it leads to a 3D motion in upwards direction. It can be imagined similar to a circular motion of spiral along the z-axis.

For Question2.
Correction (Improvement): update it to smaller time steps, then given smoother plots.

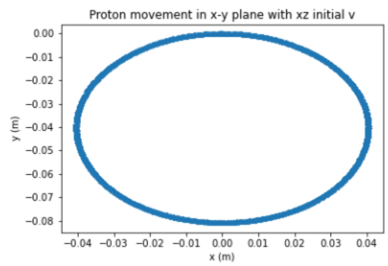

Figure 3(a)

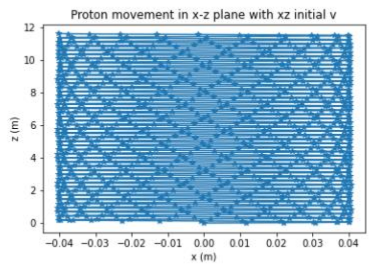

Figure 3(b)

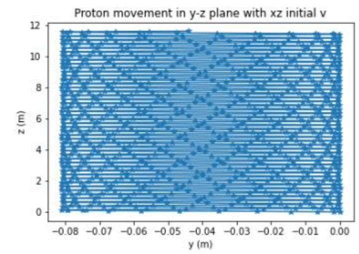

Figure 3(c)

Question3:

The magnetic field is now cylindrically symmetric around the z-axis.
If R is the distance from the z-axis, in m, the magnitude of the magnetic field is $9\mathrm{T}/R$.
The direction of the B-field at any point should be along a ring around z in the clockwise direction.
Launch a 1 MeV proton at R = 3 meters straight up (in the z direction).

For Question3.
Correction (Improvement): update the function of description of B field and the results of plots.

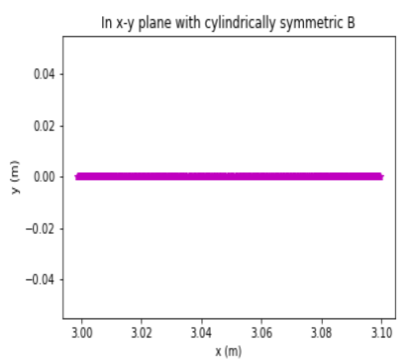

Figure 4(a)

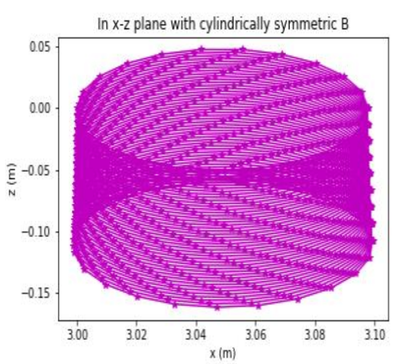

Figure 4(b)

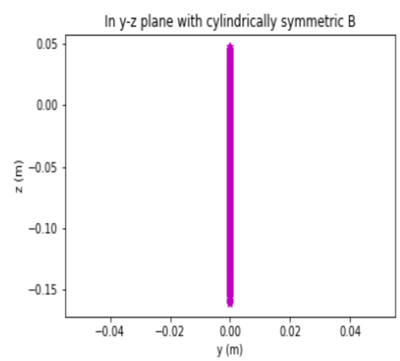

Figure 4(c)

3 Define a function B_field to satisfy the condition of cylindrically symmetric around the z-axis, i.e., evolution of direction and magnitude of magnetic field, and also define a function as always to describe the equation of motion by Lorentz force.
Then apply the ODE solver for 1 μs.

3.1 By the result of ODE solver, plot the motion in 2D projection in xy, xz, yz, refer to Figure 4(a), 4(b), 4(c).

3.2 ~~These 2D projection indicates that, the trajectory is a spiral of course, and it moves~~

~~circularly in x-z plane, with drifting mostly towards the direction of y-axis, not parallel to y-axis, but with some degree deflect in both x-y and y-z plane.~~

3.2 These 2D projection indicates that, the trajectory is a spiral of in the x-z plane, while the y direction changing is very tiny nearly nothing or neglectable, and it show a trend of drifting towards z direction, with oscillation and then go downwards (minus z direction) of the overall drift motion.

Question4:

Repeat question 2 in a 3D plot.

For Question4, continue with question 2.
Correction (Improvement): update it to smaller time steps, then given smoother plots in 3D projection.

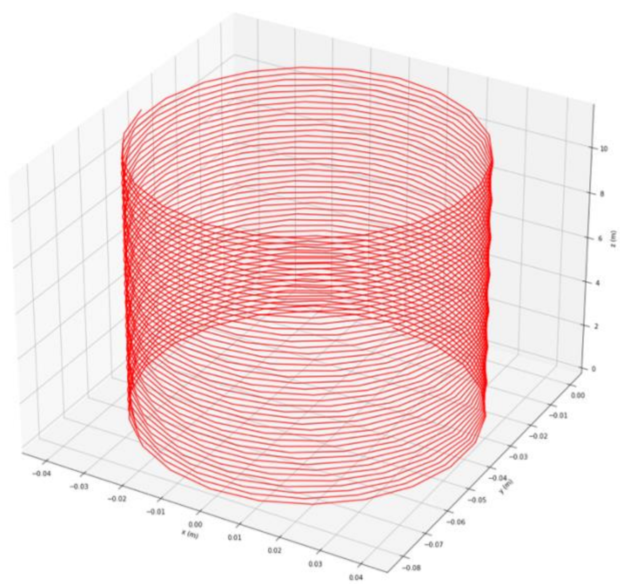

Figure 5

4 Used the ODE result for 1 μs in question 2, apply the 3D projection of the motion, in particularly, import the necessary 3D plot package. Then this would be a better visualization to see the motion of proton refer to Figure 5. As correspond to the result I discussed in question 2, a circular spiral trajectory, circular in x-y plane, with upward drift along z-direction.

Question5:
Repeat question 3 in a 3D plot.

For Question5.
Correction (Improvement): update the function of description of B field and the results of plot in 3D projection.

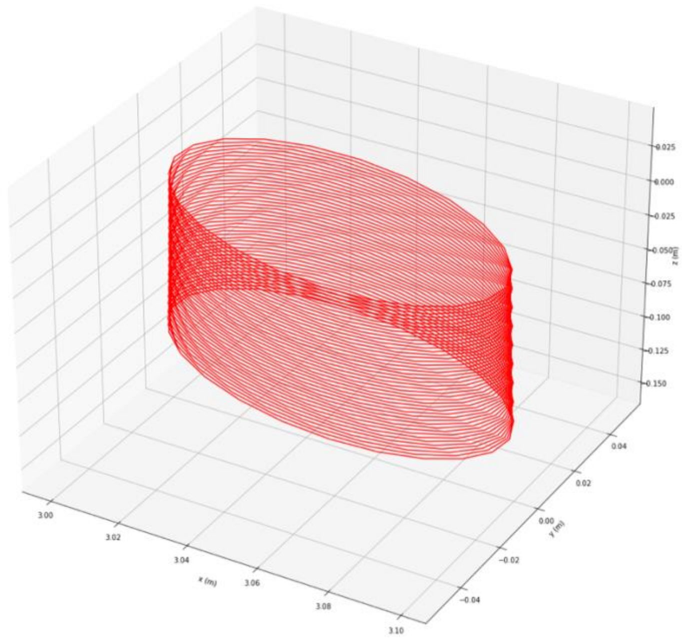

Figure 6

5 Used the ODE result for 1 μs in question 3, apply the 3D projection of the motion, in particularly, import the necessary 3D plot package. Also, a better visualization to see the motion of proton refer to Figure 6. As correspond to the result I discussed in question 3, a circular spiral moving caused by cylindrically symmetric magnetic field. Circular spiral motion in x-z plane, oscillation in x and z, and with overall drift tendency towards $-$z direction.

Question6:
Drift velocity in question 3.

6 Refer to Figure 4(c). As the tendency is to drift z-direction, in the perspective of z-axis, for 1 μs, read from the 2D projection data plot, estimate the difference of position in this direction, divide by the time period 1 μs, then the drift velocity approximates to:

$$v_D \approx 111\,864 \approx 10^5\ m/s\ \ (-z\ direction)$$