The BTL Model

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1 The BTL Model –

Suppose g is a $K-$dim vector, with one and only one component to be equal to 1, for each pair, $p(<w,v>|g_k=1) = p^k(w \succ v) \times \Pi_{k' \neq k} p^{k'}(w \succeq v)$. then the probability of generating a session observation d given the hidden aspect a is defined as:

$$p(d|g,\theta,V,U) = p(d|\Theta^t, g) = \Pi_{w \in W^d, v \in L^d, w \succ_R v} a_{r_i}^K \frac{w_k}{w_k + \theta^t v_k} \Pi_{k' \neq k} \frac{\theta^t w_{k'}}{v_{k'} + \theta^t w_{k'}} \tag{1}$$

where $w \succ_R v$ is the ranking pair in R relation existing in session d. e.g.$r1 \in R:$ buy v.s. click , $r2 \in R:$ buy v.s. add to cart ... And a is the given value.

Besides,

$$\begin{aligned} p(g|d,\Theta^t) &\propto p(g|u,\Theta^t)p(d|g,\Theta^t) \\ p(g_k=1|d,\Theta^t) &\propto u_k^t \tfrac{w_k^t}{w_k^t + \theta^t l_k^t} \prod_{k' \neq k} [\tfrac{\theta^t w_{k'}^t}{l_{k'}^t + \theta^t w_{k'}^t}] \end{aligned} \tag{2}$$

First, let's use $\gamma(d,k,\Theta^t)$ to denote the conditional probability $p(g_k=1|d,\Theta^t)$ given parameters in the $t-$th round, when the current session specific favorite aspect is $g_k=1$, defined as follows

$$\begin{aligned} \gamma(d,k,\Theta^t) &= \frac{p(d,g|\Theta^t)}{\Sigma_g p(d,g|\Theta^t)} = \frac{p(g|\Theta^t)p(d|\Theta^t,g)}{\Sigma_g p(g|\Theta^t)p(d|\Theta^t,g)} \\ &= \frac{u_k \Pi_{w \in W^d, v \in L^d, w \succ_R v} a_{r_i}^K \frac{w_k}{w_k+\theta^t v_k} \Pi_{k' \neq k} \frac{\theta^t w_{k'}}{v_{k'} + \theta^t w_{k'}}}{\Sigma_{k=1}^K u_k \Pi_{w \in W^d, v \in L^d, w \succ_R v} a_{r_i}^K \frac{w_k}{w_k+\theta^t v_k} \Pi_{k' \neq k} \frac{\theta^t w_{k'}}{v_{k'} + \theta^t w_{k'}}} \end{aligned} \tag{3}$$

Note that $\forall d, \Sigma_k \gamma(d,k,\Theta^t) = 1$.