ExplicitAndImplicitFeedback

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1 Original EM Derivation

1.1 E-step

1.2 M-step

the updates of the parameters $\{$u,v,$\theta\}$ $as follow:$

$$u_k = \frac{\Sigma_{u(d)=u}\gamma(d,k,\Theta^t)}{\Sigma_{s=1}^{K}\Sigma_{u(d)=u}\gamma(d,s,\Theta^t)} \tag{1}$$

2 Stochastic EM Derivation

2.1 E-step

$$\begin{aligned} p(g|d,\Theta^t) &\propto p(g|u,\Theta^t)p(d|g,\Theta^t) \\ p(g_k=1|d,\Theta^t) &\propto u_k^t \tfrac{w_k^t}{w_k^t+\theta^t l_k^t} \textstyle\prod_{k'\neq k}[\tfrac{\theta^t w_{k'}^t}{l_{k'}^t+\theta^t w_{k'}^t}] \end{aligned} \tag{2}$$

then simply add an S-step after the E-step,the value of g for each session d is

2.2 S-step

$$k \sim u_k^t \frac{w_k^t}{w_k^t+\theta^t l_k^t}\prod_{k'\neq k}[\frac{\theta^t w_{k'}^t}{l_{k'}^t+\theta^t w_{k'}^t}] \tag{3}$$

In the E-step of $t-$th EM round, compute the expectation $Q(\Theta^t) = E_G \ln p(D,G|\Theta)$
$\mathrm{E}_G \ln p(D,G|\Theta) = \Sigma_d \ln p(d,g|\Theta)$
$= \Sigma_d\{\ln u_k + \Sigma_{w\in W_d, v\in V_d}[\ln\frac{w_k}{w_k+\theta v_k} + \Sigma_{k'\neq k}\ln\frac{\theta w_{k'}}{v_{k'}+\theta w_{k'}}]\}$

2.3 M-step

In the M-step, for the : $\ln\frac{y}{x} \geq 1 - \frac{x}{y}$
we can derive a lower bound for the log-likelihood over the complete data, given the parameters learnt from previous round. hence, we obtain a minorization function of $\tilde{Q}(\Theta^t)$.
$\tilde{Q}(\Theta^t) = \Sigma_d \Sigma_{w \in W_d, v \in L_d} \{[\ln w_k + 1 - \ln(w_k^t + \theta^t v_k^t) - \frac{w_k + \theta v_k}{w_k^t + \theta^t v_k^t}] +$
$\Sigma_{k' \neq k}[\ln(\theta w_{k'}) + 1 - \ln(v_{k'}^t + \theta^t w_{k'}^t) - \frac{v_{k'} + \theta w_{k'}}{v_{k'}^t + \theta^t w_{k'}^t}]\}$(4)

$\tilde{Q}(\Theta^t)$ can be seperated for each item v. Considering only the $k-$th component v_k, $\tilde{Q}(v_k, \Theta^t)$ involves two terms, one of which is relevant to observations $d \in W(v)$ where v acts as skyline object, the other is relevant to observations $d \in L(v)$ where v acts as comparisons, $\tilde{Q}(v_k, \Theta^t) = \tilde{Q}^1(v_k, \Theta^t) + \tilde{Q}^2(v_k, \Theta^t)$. Removing all constants and irrelevant terms for v_k, we have the following minorizing function:

$\tilde{Q}^1(v_k, \Theta^t) = \Sigma_{d \in W(v)} |L_d| \ln v_k - v_k \Sigma_{d \in W(v)} \Sigma_{v' \in L_d} [\frac{1}{\alpha(v, v', k, \Theta^t)} + \Sigma_{k' \neq k} \frac{\theta^t}{\alpha(v', v, k, \Theta^t)}]$
$\tilde{Q}^2(v_k, \Theta^t) = -v_k \Sigma_{d \in L(v)} \Sigma_{v' \in W_d} [\frac{\theta^t}{\alpha(v', v, k, \Theta^t)} + \Sigma_{k' \neq k} \frac{1}{\alpha(v, v', k, \Theta^t)}]$
where:

$|L_d|$ is the number of objects being dominanted in d,
$\alpha(v, v', k, \Theta^t) = v_k^t + \theta^t {v'}_k^t$.
(1)By setting the partial derivative of $\frac{\partial \tilde{Q}(v_k, \Theta^t)}{\partial v_k} = 0$, we have:

$$\frac{1}{v_k} = \frac{\Sigma_{d \in W(v)} \Sigma_{v' \in L_d} [\frac{1}{\alpha(v, v', k, \Theta^t)} + \Sigma_{k' \neq k} \frac{\theta^t}{\alpha(v', v, k, \Theta^t)}]}{\Sigma_{d \in W(v)} |L_d|} + \frac{\Sigma_{d \in L(v)} \Sigma_{v' \in W_d} [\frac{\theta^t}{\alpha(v', v, k, \Theta^t)} + \Sigma_{k' \neq k} \frac{1}{\alpha(v, v', k, \Theta^t)}]}{\Sigma_{d \in W(v)} |L_d|} \tag{5}$$

(2)Fix u, v, update θ by:

Fix $v \in V$ and $u \in U$, rearranging Equ9, we have the solution for $\frac{\partial \tilde{Q}(\Theta^t)}{\partial \theta} = 0$ as:

$$\theta = \frac{(K-1)\Sigma_d |W_d||L_d|}{\Sigma_d \Sigma_{w,v} [\frac{v_k}{\alpha(w, v, k, \Theta^t)} + \Sigma_{k' \neq k} \frac{w_{k'}}{\alpha(v, w, k', \Theta^t)}]} \tag{6}$$