The BTL Model

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1 The Standard BTL Model –

Base on the standard BTL model:

$$p(i \succ j|u) = \frac{ui}{ui + uj}$$
$$p(i \prec j|u) = \frac{uj}{uj + ui}$$

Suppose that there are K underlying aspects. for each pair, $p(< w, v >) = p^k(w \succ v)$. then the probability of generating a session observation d is defined as:

$$p(< w, l > |V, U) = \Pi_{k=1}^{K}[\frac{u_k w_k}{u_k w_k + u_k v_k}] \tag{1}$$

The likelihood function can be written as follows. The model parameters are denoted as $\Theta = \{v \in V, u \in U)\}$

$$L(\Theta) = p(D|\Theta) = \Pi_{d \in D}\Pi_{w \in W^d, v \in L^d}\Pi_{k=1}^{K}[\frac{u_k w_k}{u_k w_k + u_k v_k}] \tag{2}$$

Thus, the log likelihood is:

$$\begin{aligned} l(\Theta) &= log\ L(\Theta) && (3)\\ &= log\ \Pi_{d \in D}\Pi_{w \in W^d, v \in L^d}\Pi_{k=1}^{K}[\frac{u_k w_k}{u_k w_k + u_k v_k}] && (4)\\ &= \sum_{d \in D} log\ \Pi_{w \in W^d, v \in L^d}\Pi_{k=1}^{K}[\frac{u_k w_k}{u_k w_k + u_k v_k}] \end{aligned}$$

afterwards,