

# The BTL Model

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## 1 The Standard BTL Model –

Base on the standard BTL model:

$$\begin{aligned} p(i \succ j|u) &= \frac{u_i}{u_i + u_j} \\ p(i \prec j|u) &= \frac{u_j}{u_j + u_i} \end{aligned}$$

Suppose that there are  $K$  underlying aspects. for each pair,  $p(< w, v >) = p^k(w \succ v)$ . then the probability of generating a session observation  $d$  is defined as:

$$p(< w, l > |V, U) = \prod_{k=1}^K \left[ \frac{u_k w_k}{u_k w_k + u_k v_k} \right] \quad (1)$$

The likelihood function can be written as follows. The model parameters are denoted as  $\Theta = \{v \in V, u \in U\}$

$$p(D|\Theta) = \prod_{d \in D} \prod_{w \in W^d, v \in L^d} \prod_{k=1}^K \left[ \frac{u_k w_k}{u_k w_k + u_k v_k} \right] \quad (2)$$