The BTL Model

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1 The Standard BTL Model $-$

Base on the standard BTL model:

$$p(i \succ j|u) = \frac{ui}{ui + uj}$$

$$p(i \prec j|u) = \frac{uj}{uj + ui}$$

Suppose that there are K underlying aspects. for each pair, $p(< w, v >) = p^k(w \succ v)$. then the probability of generating a session observation d is defined as:

$$p(< w, v > |V, U) = \frac{\sum_k u_k w_k}{\sum_k u_k w_k + \sum_k u_k v_k} \tag{1}$$

The likelihood function can be written as follows. The model parameters are denoted as $\Theta = \{v \in V, u \in U)\}$

$$L(\Theta) = p(D|\Theta) = \Pi_{d \in D} \Pi_{w \in W^d, v \in L^d} \Pi_{k=1}^{K} \big[\frac{u_k w_k}{u_k w_k + u_k v_k} \big] \tag{2}$$

Thus, the log likelihood is:

$$l(\Theta) = log\ L(\Theta) \tag{3}$$

$$= log\ \Pi_{d \in D} \Pi_{w \in W^d, v \in L^d} \Pi_{k=1}^{K} \big[\frac{u_k w_k}{u_k w_k + u_k v_k} \big] \tag{4}$$

$$= \sum_{d \in D} log\ \Pi_{w \in W^d, v \in L^d} \Pi_{k=1}^{K} \big[\frac{u_k w_k}{u_k w_k + u_k v_k} \big]$$

we want to choose $\{U\ and\ V\}$ to maximize $l(\Theta)$ by stochastic gradient ascent(SGA).

1.1 For u

$$u_j := u_j + \alpha \frac{\partial \, l(\Theta)}{\partial \, u_j}$$

$$\frac{\partial \, l(\Theta)}{\partial \, u_j} = \sum_{d \in D} log \, \Pi_{w \in W^d, v \in L^d} \left[\frac{uwv}{(uw + uv)^2} \right] \tag{5}$$

1.2 For v

$$v_j := v_j + \alpha \frac{\partial \, l(\Theta)}{\partial \, v_j}$$

$$\frac{\partial \, l(\Theta)}{\partial \, v_j} = \sum_{d \in D} log \, \Pi_{w \in W^d, v \in L^d} \left[\frac{u^2 v - u^2 w}{(uw + uv)^2} \right] \tag{6}$$