

The BTL Model

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1 The Standard BTL Model –

Base on the standard BTL model:

$$\begin{aligned} p(i \succ j|u) &= \frac{u_i}{u_i + u_j} \\ p(i \prec j|u) &= \frac{u_j}{u_j + u_i} \end{aligned}$$

Suppose that there are K underlying aspects. for each pair, $p(< w, v >) = p^k(w \succ v)$. then the probability of generating a session observation d is defined as:

$$p(< w, v > | V, U) = \prod_{k=1}^K \left[\frac{u_k w_k}{u_k w_k + u_k v_k} \right] \quad (1)$$

The likelihood function can be written as follows. The model parameters are denoted as $\Theta = \{v \in V, u \in U\}$

$$L(\Theta) = p(D|\Theta) = \prod_{d \in D} \prod_{w \in W^d, v \in L^d} \prod_{k=1}^K \left[\frac{u_k w_k}{u_k w_k + u_k v_k} \right] \quad (2)$$

Thus, the log likelihood is:

$$l(\Theta) = \log L(\Theta) \quad (3)$$

$$\begin{aligned} &= \log \prod_{d \in D} \prod_{w \in W^d, v \in L^d} \prod_{k=1}^K \left[\frac{u_k w_k}{u_k w_k + u_k v_k} \right] \\ &= \sum_{d \in D} \log \prod_{w \in W^d, v \in L^d} \prod_{k=1}^K \left[\frac{u_k w_k}{u_k w_k + u_k v_k} \right] \end{aligned} \quad (4)$$

we want to choose $\{U \text{ and } V\}$ to maximize $l(\Theta)$ by stochastic gradient ascent(SGA).

1.1 For \mathbf{u}

$$u_j := u_j + \alpha \frac{\partial l(\Theta)}{\partial u_j}$$

$$\frac{\partial l(\Theta)}{\partial u_j} = \sum_{d \in D} \log \Pi_{w \in W^d, v \in L^d} \left[\frac{uwv}{(uw + uv)^2} \right] \quad (5)$$

1.2 For \mathbf{v}

$$v_j := v_j + \alpha \frac{\partial l(\Theta)}{\partial v_j}$$

$$\frac{\partial l(\Theta)}{\partial v_j} = \sum_{d \in D} \log \Pi_{w \in W^d, v \in L^d} \left[\frac{u^2v - u^2w}{(uw + uv)^2} \right] \quad (6)$$