Draft of User's manual

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1 General description

Geometric nonlinearity plays an important role in the design of thin structures because of their large deflection and leads to complicated motions with typical nonlinear phenomena. Although the nonlinear response of such structures can be investigated experimentally, many efforts are devoted to predicting the nonlinear response by using finite element (FE) simulation. Consequently, a large number of expansion functions is needed for the discretization of the structure to obtain convergence through the Galerkin method. As a consequence, full-order nonlinear analysis of complex structures featuring important nonlinearity is computationally expensive, setting a clear limitation in terms of analysis and parametric design.

The purpose of this program is to generate a nonlinear reduced order model (ROM) from a full-order finite element model by applying the DNF approach in ABAQUS. The ROM can be used to compute nonlinear dynamics, e.g. time response, backbone curves, and frequency response functions. Most importantly, thanks to invariant manifold theory, the ROM is able to accurately capture the nonlinear properties and characteristics with only a small approximation error as compared to the full-order model.

One should install MATLAB and ABAQUS in order to launch the program. The program is able to generate the ROM from any mesh files in *.inp format which ABAQUS can read. The program is divided into two parts, one for the **Main_code.m** where to launch the full program and change input parameters, and the folder **SRC_DNF** including all the functions:

DNF_in_FE.m Main function with all equations of the DNF are programmed.
Compute_MK.m This is to compute the mass and stiffness matrix from the input meshes by launching ABAQUS.
Compute_PHI.m This is to extract eigenmodes and frequencies of the selected master modes from ABAQUS linear modal analysis.
generate_STEP_inp.m & generate_DNF_inp.m For generating *.inp files embedded the equations of the DNF approach that used in further ABAQUS analysis.
read_rpt.m Extracting ABAQUS field output data and transfer to vectors that MATLAB can read.
Compute_PHI.py&STEP_full.py&DNF_full.py Codes in python and can be called by MATLAB in order to launch ABAQUS simulation.

2 How to use the program

Please make sure the main code **Main_code.m** and the folder **SRC_DNF** including all functions are put under the same folder. Temporary files during the simulations will be saved in the folder named "temp_files_X" which is automatically generated.

Your mesh file should be in the *.inp format that can be read by ABAQUS. It should only include model information (*Part,*Assembly,*Material,*Boundary, etc.) but does not have any analysis part (*Step, etc.). The models can be in any type of element (while the user needs to know how many dofs of displacement and rotations for each node in advance) and can be in any shape.

By simply running **Main_code.m**, the program will compute all the nonlinear coefficients and tensors, the users do not need to provide any other input during the running. In the **Main_code.m**, the user can freely define path information, modal basis, etc, details can be found in the comments of the code, and some examples about the input are also provided.

It will take around a few minutes to complete the computation. Once the nonlinear coefficients (AH, BH) and tensors (a_ten, b_ten) are successfully computed. The reduced dynamics can thus be obtained, details can be found in Section 4. One can use numerical tools, for example, MANLAB [**?**], to obtain time response, backbone curves, frequency response functions, etc., of the reduced order model. The prediction of the nonlinear dynamics should be close to the results of the full-order model. A few test cases are given in the next section.

3 Examples

The backbone curve computed by the MANLAB are shown in the following. Corresponding codes for the MANLAB are also provided.

3.1 Examples 1: Clamped beam with 1:1 resonance

The first example is a straight clamped-clamped beam that is allowed to vibrate in the two bending directions, leading to different polarizations and consequently a 1:1 internal resonance. The parameters of the beam are: the length of the beam, $L = 1\ m$, the width and thickness are $0.03\ m$ and $0.0306\ m$ respectively, the density $\rho = 4400\ kg/m^3$, the Young modulus $E = 1.04e11$ Pa, and the Poisson's ratio $\nu = 0.3$. For the space discretization, 3D elements have been used. The master modes are selected as the first and second modes.

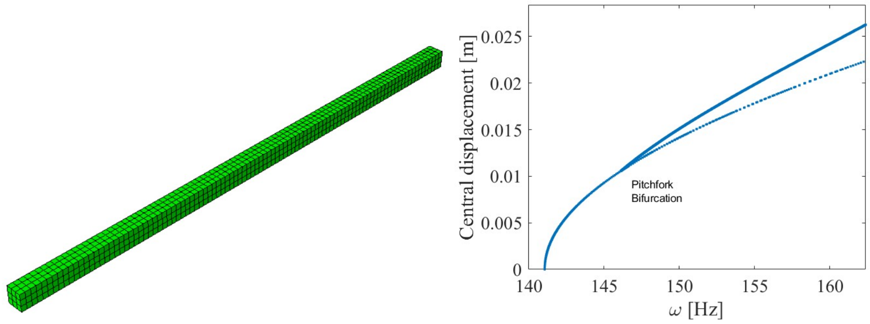

Figure 1: Left: The clamped beam discretized with 3D element. Right: Backbone curves of the first mode computed by the ROM.

3.2 Examples 2: Shell

The benchmark structure is an exhaust cover plate that fails at its center due to a nonlinear modal interaction. Figure 1 illustrates the finite element (FE) model approximating the perforated cover by a thin, smoothly curved plate, with curvature mapped through 3D surface measurement. The master modes are selected as the first and seventh modes.

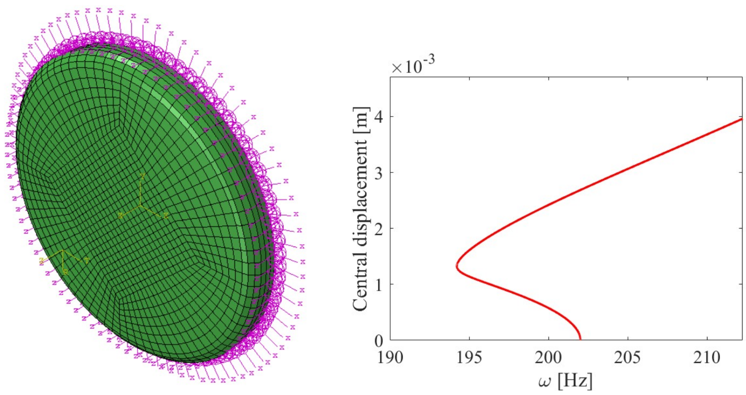

Figure 2: Left: The shell discretized with S4R element. Right: Backbone curves of the first mode computed by the ROM.

4 Description of the Direct normal form method

The Direct normal form (DNF) approach allows direct computation of the nonlinear mapping enabling to pass from the physical space (dofs of the FE mesh) to the invariant manifolds of the system that are tangent to their linear counterpart at the origin (Nonlinear Normal Modes in the sense of Shaw and Pierre [1, 2]). The method builds on earlier results where the normal form was computed from the problem expressed in the modal basis, such that develops a nonlinear mapping up to the third-order. The main advantage of the direct approach proposed in [3] is to bypass the step of eigenmode projection, as this can be out of reach in complex FE mesh with millions of dofs. Instead, the method uses a starting point of the physical space and the dofs of the FE mesh. The equations of motion in physical coordinates are given by:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} + \hat{\mathbf{\Gamma}}(\mathbf{q}) = \mathbf{F}, \tag{1}$$

where the internal force vector reads $\hat{\mathbf{\Gamma}}(\mathbf{q}) = \mathbf{G}(\mathbf{q}, \mathbf{q}) + \mathbf{H}(\mathbf{q}, \mathbf{q}, \mathbf{q})$. The detailed expressions of the quadratic and cubic polynomial terms, *i.e.* $\mathbf{G}(\mathbf{q}, \mathbf{q})$ and $\mathbf{H}(\mathbf{q}, \mathbf{q}, \mathbf{q})$, representing the nonlinear internal restoring force, are given by:

$$\mathbf{G}(\mathbf{q}, \mathbf{q}) = \sum_{r=1}^{N}\sum_{s=1}^{N} \mathbf{G}_{rs} q_r q_s, \tag{2}$$

$$\mathbf{H}(\mathbf{q}, \mathbf{q}, \mathbf{q}) = \sum_{r=1}^{N}\sum_{s=1}^{N}\sum_{t=1}^{N} \mathbf{H}_{rst} q_r q_s q_t, \tag{3}$$

where $\mathbf{G}_{rs}$ and $\mathbf{H}_{rst}$ are the N-dimensional vectors of coefficients G^p_{rs} and H^p_{rst}, for $p = 1, ..., N$. Using the matrix of eigenvectors, this problem can be rewritten in a modal basis using similar notations for the quadratic and cubic tensors of coefficients. The p-th modal equation thus writes:

$$\ddot{X}_p + 2\zeta_p\omega_p\dot{X}_p + \omega_p^2 X_p + \sum_{i=1}^{N}\sum_{j=i}^{N} g^p_{ij}X_iX_j + \sum_{i=1}^{N}\sum_{j=i}^{N}\sum_{l=j}^{N} h^p_{ijl}X_iX_jX_l = F_p, \tag{4}$$

and the relationships between $\mathbf{G}$ and $\mathbf{g}$, $\mathbf{H}$ and $\mathbf{h}$ reads:

$$\mathbf{g}_{ij} = \mathbf{P}_\phi^T\mathbf{G}(\boldsymbol{\phi_i}, \boldsymbol{\phi_j}), \tag{5a}$$

$$\mathbf{h}_{ijk} = \mathbf{P}_\phi\mathbf{H}(\boldsymbol{\phi_i}, \boldsymbol{\phi_j}, \boldsymbol{\phi_k}), \tag{5b}$$

where $\mathbf{P}_\phi$ is the matrix of eigenvectors $\boldsymbol{\phi_i}$.

Two versions of the DNF are presented in [3], a second- and a third-order development. Also, a method to take into account Rayleigh damping is proposed so that one can get reduced dynamics where the losses of the reduced dynamics do not neglect those of the slave modes. At present, the inclusion of Rayleigh damping in the form of $\mathbf{C} = \zeta_M\mathbf{M} + \zeta_K\mathbf{K}$ is only possible with second-order DNF. For that reason, the presentation retained here will focus on the case of second-order DNF including damping.

The nonlinear mapping up to second-order reads:

$$\mathbf{u} = \sum_{i}^{n}\boldsymbol{\phi}_iX_i + \sum_{i=1}^{n}\sum_{j=1}^{n}\bar{\boldsymbol{a}}_{ij}X_iX_j + \sum_{i=1}^{n}\sum_{j=1}^{n}\bar{\boldsymbol{b}}_{ij}\dot{X}_i\dot{X}_j + \sum_{i=1}^{n}\sum_{j=1}^{n}\bar{\boldsymbol{c}}_{ij}X_i\dot{X}_j, \tag{6}$$

where n is the number of master modes used to build the ROM, and X_i together with its velocity $Y_i = \dot{X}_i$ are the coordinates used to span the i-th invariant manifold. In Eq. (6), $\boldsymbol{\phi}_i$ is the eigenvector and $\bar{\boldsymbol{a}}_{ij}$, $\bar{\boldsymbol{b}}_{ij}$ and $\bar{\boldsymbol{c}}_{ij}$ are second-order tensors, the full expressions are expressed as:

$$\bar{\boldsymbol{a}}_{ij} =\frac{1}{2}(\bar{\boldsymbol{Z}}\boldsymbol{d}_{ij} + \bar{\boldsymbol{Z}}\boldsymbol{s}_{ij}), \tag{7a}$$

$$\bar{\boldsymbol{b}}_{ij} =\frac{1}{2\omega_i\omega_j}(\bar{\boldsymbol{Z}}\boldsymbol{d}_{ij} - \bar{\boldsymbol{Z}}\boldsymbol{s}_{ij}), \tag{7b}$$

$$\begin{aligned}\bar{\boldsymbol{c}}_{ij} =&(\zeta_M + 3\omega_i^2\zeta_K)\bar{\boldsymbol{b}}_{ij} - (2\zeta_K)\bar{\boldsymbol{a}}_{ij} + (-\zeta_M + 2\omega_i^2\zeta_K)(\bar{\boldsymbol{Z}}\boldsymbol{ss}_{ij} + \bar{\boldsymbol{Z}}\boldsymbol{dd}_{ij})\\ &+ (-\zeta_M + 2\omega_j^2\zeta_K)(\omega_i/\omega_j)(\bar{\boldsymbol{Z}}\boldsymbol{ss}_{ij} - \bar{\boldsymbol{Z}}\boldsymbol{dd}_{ij}),\end{aligned} \tag{7c}$$

where

$$\bar{\boldsymbol{Z}}\boldsymbol{ss}_{ij} = ((+\omega_i + \omega_j)^2\mathbf{M} - \mathbf{K})^{-1}\mathbf{M}\bar{\boldsymbol{Z}}\boldsymbol{s}_{ij}, \tag{8a}$$

$$\bar{\boldsymbol{Z}}\boldsymbol{dd}_{ij} = ((-\omega_i + \omega_j)^2\mathbf{M} - \mathbf{K})^{-1}\mathbf{M}\bar{\boldsymbol{Z}}\boldsymbol{d}_{ij}, \tag{8b}$$

and

$$\bar{\boldsymbol{Z}}\boldsymbol{s}_{ij} = ((+\omega_i + \omega_j)^2\mathbf{M} - \mathbf{K})^{-1}\mathbf{G}(\boldsymbol{\phi}_i, \boldsymbol{\phi}_j), \tag{9a}$$

$$\bar{\boldsymbol{Z}}\boldsymbol{d}_{ij} = ((-\omega_i + \omega_j)^2\mathbf{M} - \mathbf{K})^{-1}\mathbf{G}(\boldsymbol{\phi}_i, \boldsymbol{\phi}_j). \tag{9b}$$

By taking into account the Rayleigh damping with an assumption of small damping ratios on the master modes, the nonlinear dynamics reads, $\forall\, p = 1, ...m$:

$$\ddot{X}_p + (\zeta_M + \zeta_K\omega_p^2)\dot{X}_p + \omega_p^2X_p + \sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m}[(A^p_{ijk} + h^p_{ijk})X_iX_jX_k + B^p_{ijk}X_i\dot{X}_j\dot{X}_k + C^p_{ijk}X_iX_j\dot{X}_k] = F_p, \tag{10}$$

where the coefficients $A^p_{ijk}, B^p_{ijk}, C^p_{ijk}$ arise from the cancellation of non-resonant quadratic terms, their full expression reads [3]:

$$A^p_{ijk} = 2\phi_p^T\mathbf{G}(\boldsymbol{\phi}_i, \bar{\boldsymbol{a}}_{jk}), \tag{11a}$$

$$B^p_{ijk} = 2\phi_p^T\mathbf{G}(\boldsymbol{\phi}_i, \bar{\boldsymbol{b}}_{jk}), \tag{11b}$$

$$C^p_{ijk} = 2\phi_p^T\mathbf{G}(\boldsymbol{\phi}_i, \bar{\boldsymbol{c}}_{jk}). \tag{11c}$$

The reduced dynamics given in (10) is expressed on the $2m$ dimensional invariant manifold.

References

[1] S. W. Shaw and C. Pierre. Non-linear normal modes and invariant manifolds. *Journal of Sound and Vibration*, 150(1):170–173, 1991.

[2] S. W. Shaw and C. Pierre. Normal modes for non-linear vibratory systems. *Journal of Sound and Vibration*, 164(1):85–124, 1993.

[3] A. Vizzaccaro, Y. Shen, L. Salles, J. Blahos, and C. Touzé. Direct computation of nonlinear mapping via normal form for reduced-order models of finite element nonlinear structures. *Computer Methods in Applied Mechanics and Engineering*, 384:113957, 2021.