RL homework 3

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Start date: *7th March 2018*

Due date: *21st March 2018, 11:55 pm*

How to Submit

When you have completed the exercises and everything has finsihed running, click on 'File' in the menu-bar and then 'Download .ipynb'. This file must be submitted to Moodle named as **studentnumber_RL_hw3.ipynb** before the deadline above.

Also send a **sharable link** to the notebook at the following email: ucl.coursework.submit@gmail.com. You can also make it sharable via link to everyone, up to you.

Please compile all results and all answers to the understanding questions into a PDF. Name convention: **studentnumber_RL_hw3.pdf**. Do not include any of the code (we will use the notebook for that).

Page limit: 10 pg

Context

In this assignment, we will investigate the properties of 3 distinct reinforcement learning algorithms:

- Online Q-learning
- Experience Replay
- Dyna-Q

We will consider two different dimensions:

- Tabular vs Function Approximation
- Stationary vs Non-Stationary environments

Background reading

- Sutton and Barto (2018), Chapters 8

The Assignment

Objectives

You will use Python to implement several reinforcement learning algorithms **[50 pts]**.

You will then run these algorithms on a few problems, to understand their properties.

Finally you will answer a few question about the performance of these algorithms in the various problems **[50pts]**.

Setup

In [0]:

```
import matplotlib.pyplot as plt
import numpy as np
from collections import namedtuple

np.set_printoptions(precision=3, suppress=1)
plt.style.use('seaborn-notebook')
```

Grid worlds

Tabular Grid-World

Simple tabular grid world.

You can visualize the grid worlds we will train our agents on, by running the cells below. `S` indicates the start state and `G` indicates the goal. The agent has four possible actions: up, right, down, and left. Rewards are: `-5` for bumping into a wall, `+10` for reaching the goal, and `0` otherwise. The episode ends when the agent reaches the goal, and otherwise continues. The discount, on continuing steps, is $\gamma = 0.9$.

We will use three distinct GridWorlds:

- `Grid` tabular grid world withh a goal in the top right of the grid
- `AltGrid` tabular grid world withh a goal in the bottom left of the grid
- `FeatureGrid` a grid world with a non tabular representation of states, the features are such to allow some degree of state aliasing

In [0]:

```
#@title Grid
class Grid(object):

  def __init__(self, discount=0.9):
    # -1: wall
    # 0: empty, episode continues
    # other: number indicates reward, episode will terminate
    self._layout = np.array([
      [-1, -1, -1, -1, -1, -1, -1, -1, -1, -1],
      [-1,  0,  0,  0,  0,  0,  0,  0,  0, -1],
      [-1,  0,  0,  0, -1, -1,  0,  0, 10, -1],
      [-1,  0,  0,  0, -1, -1,  0,  0,  0, -1],
      [-1,  0,  0,  0, -1, -1,  0,  0,  0, -1],
      [-1,  0,  0,  0,  0,  0,  0,  0,  0, -1],
      [-1,  0,  0,  0,  0,  0,  0,  0,  0, -1],
      [-1,  0,  0,  0,  0,  0,  0,  0,  0, -1],
      [-1, -1, -1, -1, -1, -1, -1, -1, -1, -1]
    ])
    self._start_state = (2, 2)
    self._goal_state = (8, 2)
    self._state = self._start_state
    self._number_of_states = np.prod(np.shape(self._layout))
    self._discount = discount

  @property
  def number_of_states(self):
      return self._number_of_states

  def plot_grid(self):
    plt.figure(figsize=(3, 3))
    plt.imshow(self._layout > -1, interpolation="nearest", cmap = 'pink')
    ax = plt.gca()
    ax.grid(0)
    plt.xticks([])
    plt.yticks([])
    plt.title("The grid")
    plt.text(
        self._start_state[0], self._start_state[1],
        r"$\mathbf{S}$", ha='center', va='center')
    plt.text(
        self._goal_state[0], self._goal_state[1],
        r"$\mathbf{G}$", ha='center', va='center')
    h, w = self._layout.shape
    for y in range(h-1):
      plt.plot([-0.5, w-0.5], [y+0.5, y+0.5], '-k', lw=2)
    for x in range(w-1):
      plt.plot([x+0.5, x+0.5], [-0.5, h-0.5], '-k', lw=2)


  def get_obs(self):
    y, x = self._state
    return y*self._layout.shape[1] + x

  def int_to_state(self, int_obs):
    x = int_obs % self._layout.shape[1]
    y = int_obs // self._layout.shape[1]
    return y, x

  def step(self, action):
```

```
    y, x = self._state

    if action == 0:  # up
      new_state = (y - 1, x)
    elif action == 1:  # right
      new_state = (y, x + 1)
    elif action == 2:  # down
      new_state = (y + 1, x)
    elif action == 3:  # left
      new_state = (y, x - 1)
    else:
      raise ValueError("Invalid action: {} is not 0, 1, 2, or 3.".format(action

    new_y, new_x = new_state
    if self._layout[new_y, new_x] == -1:  # wall
      reward = -5.
      discount = self._discount
      new_state = (y, x)
    elif self._layout[new_y, new_x] == 0:  # empty cell
      reward = 0.
      discount = self._discount
    else:  # a goal
      reward = self._layout[new_y, new_x]
      discount = 0.
      new_state = self._start_state

    self._state = new_state
    return reward, discount, self.get_obs()
```

In [0]:

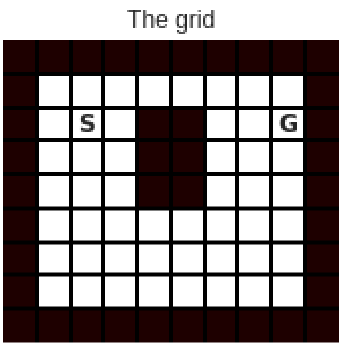
```
#@title AltGrid
class AltGrid(Grid):

    def __init__(self, discount=0.9):
      # -1: wall
      # 0: empty, episode continues
      # other: number indicates reward, episode will terminate
      self._layout = np.array([
        [-1, -1, -1, -1, -1, -1, -1, -1, -1, -1],
        [-1,  0,  0,  0,  0,  0,  0,  0,  0, -1],
        [-1,  0,  0,  0, -1, -1,  0,  0,  0, -1],
        [-1,  0,  0,  0, -1, -1,  0,  0,  0, -1],
        [-1,  0,  0,  0, -1, -1,  0,  0,  0, -1],
        [-1,  0,  0,  0,  0,  0,  0,  0,  0, -1],
        [-1,  0,  0,  0,  0,  0,  0,  0,  0, -1],
        [-1,  0, 10,  0,  0,  0,  0,  0,  0, -1],
        [-1, -1, -1, -1, -1, -1, -1, -1, -1, -1]
      ])
      self._start_state = (2, 2)
      self._goal_state = (2, 7)
      self._state = self._start_state
      self._number_of_states = np.prod(np.shape(self._layout))
      self._discount = discount
```

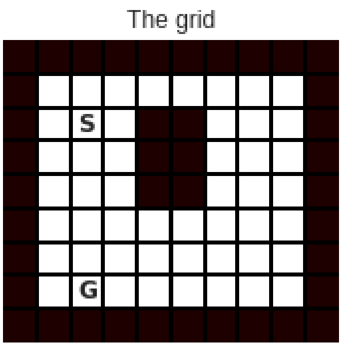
In [0]:

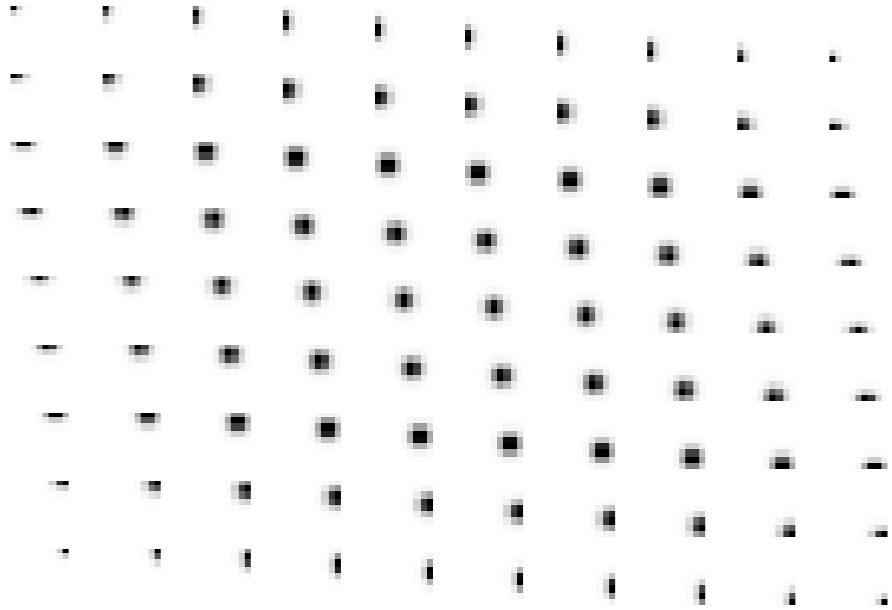
```
#@title FeatureGrid
class FeatureGrid(Grid):

  def get_obs(self):
    return self.state_to_features(self._state)

  def state_to_features(self, state):
    y, x = state
    x /= float(self._layout.shape[1] - 1)
    y /= float(self._layout.shape[0] - 1)
    markers = np.arange(0.1, 1.0, 0.1)
    features = np.array([np.exp(-40*((x - m)**2+(y - n)**2))
                         for m in markers
                         for n in markers] + [1.])
    return features / np.sum(features**2)

  def int_to_features(self, int_state):
    return self.state_to_features(self.int_to_state(int_state))

  @property
  def number_of_features(self):
      return len(self.get_obs())
```

In [5]:

```
# Instantiate the two tabular environments
grid = Grid()
alt_grid = AltGrid()

# Plot tabular environments
grid.plot_grid()
alt_grid.plot_grid()

# Instantiate the non tabular version of the environment.
feat_grid = FeatureGrid()

# Plot the features of each state
shape = feat_grid._layout.shape
f, axes = plt.subplots(shape[0], shape[1])
for state_idx, ax in enumerate(axes.flatten()):
  ax.imshow(np.reshape((feat_grid.int_to_features(state_idx)[:-1]),(9,9)), inte
  ax.set_xticks([])
  ax.set_yticks([])
```







In [6]:

```
1 grid.get_obs()
```

Out[6]:

22

In [7]:

```
1 grid._layout.size
```

Out[7]:

90

Helper functions

In [0]:

```
def run_experiment(env, agent, number_of_steps):
    mean_reward = 0.
    try:
      action = agent.initial_action()
    except AttributeError:
      action = 0
    for i in range(number_of_steps):
      reward, discount, next_state = env.step(action)
      action = agent.step(reward, discount, next_state)
      mean_reward += (reward - mean_reward)/(i + 1.)

    return mean_reward

map_from_action_to_subplot = lambda a: (2, 6, 8, 4)[a]
map_from_action_to_name = lambda a: ("up", "right", "down", "left")[a]

def plot_values(values, colormap='pink', vmin=-1, vmax=10):
  plt.imshow(values, interpolation="nearest", cmap=colormap, vmin=vmin, vmax=vma
  plt.yticks([])
  plt.xticks([])
  plt.colorbar(ticks=[vmin, vmax])

def plot_state_value(action_values):
  q = action_values
  fig = plt.figure(figsize=(4, 4))
  vmin = np.min(action_values)
  vmax = np.max(action_values)
  v = 0.9 * np.max(q, axis=-1) + 0.1 * np.mean(q, axis=-1)
  plot_values(v, colormap='summer', vmin=vmin, vmax=vmax)
  plt.title("$v(s)$")

def plot_action_values(action_values):
  q = action_values
  fig = plt.figure(figsize=(8, 8))
  fig.subplots_adjust(wspace=0.3, hspace=0.3)
  vmin = np.min(action_values)
  vmax = np.max(action_values)
  dif = vmax - vmin
  for a in [0, 1, 2, 3]:
    plt.subplot(3, 3, map_from_action_to_subplot(a))

    plot_values(q[..., a], vmin=vmin - 0.05*dif, vmax=vmax + 0.05*dif)
    action_name = map_from_action_to_name(a)
    plt.title(r"$q(s, \mathrm{" + action_name + r"})$")

  plt.subplot(3, 3, 5)
  v = 0.9 * np.max(q, axis=-1) + 0.1 * np.mean(q, axis=-1)
  plot_values(v, colormap='summer', vmin=vmin, vmax=vmax)
  plt.title("$v(s)$")

def random_policy(q):
  return np.random.randint(4)

def plot_greedy_policy(grid, q):
  action_names = [r"$\uparrow$",r"$\rightarrow$", r"$\downarrow$", r"$\leftarro
  greedy_actions = np.argmax(q, axis=2)
  grid.plot_grid()
  plt.hold('on')
  for i in range(9):
```

```
    for j in range(10):
      action_name = action_names[greedy_actions[i,j]]
      plt.text(j, i, action_name, ha='center', va='center')
```

Part 1: Implement Agents

Each agent, should implement a step function:

`__init__(self, number_of_actions, number_of_states, initial_observation):`

The constructor will provide the agent the number of actions, number of states, and the initial observation. You can get the initial observation by first instatiating an environment, using `grid = Grid()`, and then calling `grid.get_obs()`.

All agents should be in pure Python - so you cannot use TensorFlow to, e.g., compute gradients. Using `numpy` is fine.

`step(self, reward, discount, next_observation, ...):`

where `...` indicates there could be other inputs (discussed below). The step should update the internal values, and return a new action to take.

When the discount is zero ($\text{discount} = \gamma = 0$), then the `next_observation` will be the initial observation of the next episode. One shouldn't bootstrap on the value of this state, which can simply be guaranteed when using "$\gamma \cdot v(\text{next\_observation})$" (for whatever definition of v is appropriate) in the update, because $\gamma = 0$. So, the end of an episode can be seamlessly handled with the same step function.

`q_values():`

Tabular agents implement a function `q_values()` returning a matrix of Q values of shape: (`number_of_states`, `number_of_actions`)

`q_values(state):`

Agents with Linear function approximation implement a method `q_values(state)` returning an array of Q values of shape: (`number_of_actions`)

A note on the initial action

Normally, you would also have to implement a method that gives the initial action, based on the initial state. As in the previous assignment you can use the action `0` (which corresponds to `up`) as initial action, so that otherwise we do not have to worry about this. Note that this initial action is only executed once, and the beginning of the first episode---not at the beginning of each episode.

Q-learning and it's variants needs to remember the last action in order to update its value when they see the next state. In the `__init__`, make sure you set the initial action to zero, e.g.,

```
def __init__(...):
  (...)
  self._action = 0
  (...)
```

Part 1: Implement Agents

We are going to implement 5 agent:

- Online Tabular Q-learning
- Tabular Experience Replay
- Tabular Dyna-Q (with a Tabular model)
- Experience Replay with linear function approximation
- Dyna-Q with linear function approximation (with a linear model)

1.1 Tabular Model

[5 pts] Implement a trainable tabular Model of the environment.

The Model should implement:

- a *next_state* method, taking a state and action and returning the next state in the environment.
- a *reward* method, taking a state and action and returning the immediate reward associated to execution that action in that state.
- a *discount* method, taking a state and action and returning the discount associated to execution that action in that state.
- a *transition* method, taking a state and an action and returning both the next state and the reward associated to that transition.
- a *update* method, taking a full transition *(state, action, reward, next_state)* and updating the model (in its reward, discount and next_state component)

Given that the environment is deterministic and tabular the model will basically reduce to a simple lookup table.

In [0]:

```
class TabularModel(object):

  def __init__(self, number_of_states, number_of_actions):
    self.number_of_states = number_of_states
    self.number_of_actions = number_of_actions
    self._store = {}
    # self._store[(state, action)] = [reward, discount, next_action]

  def next_state(self, s, a):
    try:
      return self._store[(s,a)][2]
    except KeyError:
      pass

  def reward(self, s, a):
    try:
      return self._store[(s,a)][0]
    except KeyError:
      pass

  def discount(self, s, a):
    try:
      return self._store[(s,a)][1]
    except KeyError:
      pass

  def transition(self, state, action):
    return (
        self.reward(state, action),
        self.discount(state, action),
        self.next_state(state, action))

  def update(self, state, action, reward, discount, next_state):
      self._store[(state,action)] = [reward, discount, next_state]


```

1.2 Linear Model

[5 pts] Implement a trainable linear model of the environment.

The Model should implement:

- a *next_state* method, taking a state and action and returning the predicted next state in the environment.
- a *reward* method, taking a state and action and returning the predicted immediate reward associated to execution that action in that state.
- a *discount* method, taking a state and action and returning the predicted discount associated to execution that action in that state.
- a *transition* method, taking a state and an action and returning both the next state and the reward associated to that transition.
- a *update* method, taking a full transition *(state, action, reward, next_state)* and updating the model (in its reward, discount and next_state component)

For each selected action, the predicted reward, discount and next state will all be a linear function of the state.

- $s' = T_a s$

- r' = $R_a s$
- g' = $G_a s$

Where T_a is a matrix of shape $(\text{number\_of\_features}, \text{number\_of\_features})$, R_a and G_a are vectors of shape $(\text{number\_of\_features},)$

The parameters of all these linear transformations must be trained by gradient descent. Write down the update to the parameters of the models and implement the update in the model below.

$$T_a = T_a + \alpha(s' - T_a \cdot s) \cdot s^T$$
$$R_a = R_a + \alpha(r' - R_a \cdot s) \cdot s^T$$
$$G_a = G_a + \alpha(g' - G_a \cdot s) \cdot s^T$$

In [0]:

```
class LinearModel(object):

  def __init__(self, number_of_features, number_of_actions):
    self._number_of_features = number_of_features
    self._number_of_actions = number_of_actions
    self._T = np.zeros((number_of_actions, number_of_features, number_of_feature
    self._R = np.zeros((number_of_actions, number_of_features))
    self._G = np.zeros((number_of_actions, number_of_features))

  def next_state(self, s, a):
    return np.dot(self._T[a, :, :], s)

  def reward(self, s, a):
    return np.dot(self._R[a, :], s)

  def discount(self, s, a):
    return np.dot(self._G[a, :], s)

  def transition(self, state, action):
    return (
        self.reward(state, action),
        self.discount(state, action),
        self.next_state(state, action))

  def update(self, state, action, reward, discount, next_state, step_size=0.1):

    r, g, s = self.transition(state, action)

    self._R[action, :] += step_size *(reward - r) * state
    self._G[action, :] += step_size *(discount - g) * state
    self._T[action,:,:] += step_size * np.dot((next_state - s).reshape(-1,1), st
```

1.3 Experience Replay

[10 pts] Implement an agent that uses **Experience Replay** to learn action values, at each step:

- select actions randomly
- accumulate all observed transitions *(s, a, r, s')* in the environment in a *replay buffer*,
- apply an online Q-learning
- apply multiple Q-learning updates based on transitions sampled (uniformly) from the *replay buffer* (in addition to the online updates).

Initialize $Q(s, a)$ and $\mathrm{Model}(s, a)$ for all s ∈ S and a ∈ A(s)

Loop forever:

1. $S \leftarrow$ current (nonterminal) state
2. $A \leftarrow \mathrm{random\_action}(S)$
3. Take action A; observe resultant reward R, discount γ, and state, S'
4. $Q(S,A) \leftarrow Q(S,A) + \alpha(R + \gamma \max_a Q(S', a) - Q(S,A))$
5. $\mathrm{ReplayBuffer.append\_transition}(S, A, R, \gamma, S')$
6. Loop repeat n times:

   A. $S, A, R, \gamma, S' \leftarrow \mathrm{ReplayBuffer.sample\_transition}()$
   B. $Q(S,A) \leftarrow Q(S,A) + \alpha(R + \gamma \max_a Q(S', a) - Q(S,A))$

In [0]:

```
class ExperienceQ(object):

 def __init__(
     self, number_of_states, number_of_actions, initial_state,
     behaviour_policy, num_offline_updates=0, step_size=0.1):

   self._number_of_states = number_of_states
   self._number_of_actions = number_of_actions
   self._state = initial_state
   self._behaviour_policy = behaviour_policy
   self._num_offline_updates = num_offline_updates
   self._step_size = step_size
   self._action = 0
   self._replay_buffer = []
   self._q = np.zeros((number_of_states, number_of_actions))

 @property
 def q_values(self):
   return self._q

 def step(self, reward, discount, next_state):
   s = self._state
   a = self._action
   r = reward
   g = discount
   self._state = next_state
   self._q[s,a] += self._step_size * (r + g * np.max(self._q[next_state,:]) - s
   self._replay_buffer.append([s, a, r, g, next_state])

   for n in range(self._num_offline_updates):
     length = len(self._replay_buffer)
     which = np.random.randint(length)
     s_, a_, r_, g_, next_s_ = self._replay_buffer[which]
     self._q[s_, a_] += self._step_size * (r_ + g_ * np.max(self._q[next_s_,:]

   self._action = self._behaviour_policy(s)

   return self._action
```

1.4 Dyna-Q

[10 pts] Implement an agent that uses **Dyna-Q** to learn action values.

- select actions randomly
- accumulate all observed transitions *(s, a, r, s')* in the environment in a *replay buffer*,
- apply an online Q-learning to Q-value
- apply an update to the *model* based on the latest transition
- apply multiple Q-learning updates based on transitions *(s, a, model.reward(s), model.next_state(s))* for some previous state and action pair *(s, a)*.

Initialize $Q(s, a)$ and $\text{Model}(s, a)$ for all s ∈ S and a ∈ A(s)

Loop forever:

1. $S \leftarrow$ current (nonterminal) state
2. $A \leftarrow \text{random\_action}(S)$
3. Take action A; observe resultant reward R, discount γ, and state, S'
4. $Q(S, A) \leftarrow Q(S, A) + \alpha(R + \gamma \max_a Q(S', a) - Q(S, A))$
5. $\text{ReplayBuffer.append\_transition}(S, A)$
6. $\text{Model.update}(S, A, R, \gamma, S')$
7. Loop repeat n times:

    A. $S, A \leftarrow \text{ReplayBuffer.sample\_transition}()$
    B. $R, \gamma, S' \leftarrow \text{Model.transition}(S, A)$
    C. $Q(S, A) \leftarrow Q(S, A) + \alpha(R + \gamma \max_a Q(S', a) - Q(S, A))$

In [0]:

```
class DynaQ(object):

  def __init__(
      self, number_of_states, number_of_actions, initial_state,
      behaviour_policy, num_offline_updates=0, step_size=0.1):

    self._number_of_states = number_of_states
    self._number_of_actions = number_of_actions
    self._state = initial_state
    self._behaviour_policy = behaviour_policy
    self._num_offline_updates = num_offline_updates
    self._step_size = step_size
    self._action = 0
    self._replay_buffer = []
    self._q = np.zeros((number_of_states, number_of_actions))
    self._model = TabularModel(number_of_states, number_of_actions)

  @property
  def q_values(self):
    return self._q

  def step(self, reward, discount, next_state):
    s = self._state
    a = self._action
    r = reward
    g = discount
    self._state = next_state
    self._q[s,a] += self._step_size * (r + g * np.max(self._q[next_state,:]) - s
    self._replay_buffer.append([s, a, r, g, next_state])
    self._model.update(s,a,r,g,next_state)

    for n in range(self._num_offline_updates):
      length = len(self._replay_buffer)
      which = np.random.randint(length)
      s_, a_, _, _, _ = self._replay_buffer[which]
      r_,g_, next_s_ = self._model.transition(s_, a_)
      self._q[s_, a_] += self._step_size * (r_ + g_ * np.max(self._q[next_s_,:]

    self._action = self._behaviour_policy(s)

    return self._action
```

1.5 Experience Replay with Linear Function Approximation

[10 pts] Implement an agent that uses **Experience Replay** to learn action values as a linear function approximation over a given set of features.

Training: To make sure of the experience in an online fashion, we will learn this linear model via gradient descent. Write down the update to the parameters of the value function and implement the update in the agent below.

· Consider a linear approximation for the action value function with $\theta_a^T \phi(s) \approx q(s,a)$ \

· Learning uses an observed transition $[\phi(s), a, r, \phi(s')]$

$$\delta \leftarrow max_{a'} r + \gamma \theta_{a'}^T \phi(s') - \theta_a^T \phi(s)$$

$$\theta_a \leftarrow \theta_a + \alpha\delta\phi(s)$$

In [0]:

```
class FeatureExperienceQ(ExperienceQ):

  def __init__(
      self, number_of_features, number_of_actions, *args, **kwargs):
    super(FeatureExperienceQ, self).__init__(
        number_of_actions=number_of_actions, *args, **kwargs)

    self.theta = np.zeros((number_of_actions, number_of_features))

  def q(self, state):
    return np.dot(self.theta, state)

  def step(self, reward, discount, next_state):
    s = self._state
    a = self._action
    r = reward
    g = discount
    next_s = next_state

    delta = r + g*np.max(np.dot(self.theta, next_s)) - np.dot(self.theta[a,:], s
    self.theta[a,:] += self._step_size * delta * s
    self._replay_buffer.append([s,a,r,g,next_s])

    for n in range(self._num_offline_updates):
      length = len(self._replay_buffer)
      which = np.random.randint(length)
      s_, a_, r_, g_, next_s_ = self._replay_buffer[which]
      delta_ =  r_ + g_*np.max(np.dot(self.theta, next_s_)) - np.dot(self.theta
      self.theta[a_,:] += self._step_size * delta_ * s_

    self._action = self._behaviour_policy(s)
    self._state = next_state
    return self._action
```

1.6 Dyna-Q with Linear Function Approximation

[10 pts] Implement an agent that uses **Dyna-Q** that uses a linear function approximation to represent the value functions and a learnt linear model of the environment (represent and learn both the **transition model**(action conditioned) and the **reward model** as linear transformations of the given set of features).

- select actions randomly
- accumulate all observed transitions *(s, a, r, s')* in the environment in a *replay buffer*,
- apply an online Q-learning to Q-value
- apply an update to the *model* based on the latest transition, use a step_size of 0.01
- apply multiple Q-learning updates based on transitions *(s, a, model.reward(s), model.next_state(s))* for some previous state and action pair *(s, a)*.

Initialize $Q(s, a)$ and $\text{Model}(s, a)$ for all s ∈ S and a ∈ A(s)

Loop forever:

1. $S \leftarrow$ current (nonterminal) state
2. $A \leftarrow \text{random\_action}(S)$
3. Take action A; observe resultant reward R, discount γ, and state, S'

4. $Q(S, A) \leftarrow Q(S, A) + \alpha(R + \gamma \max_a Q(S', a) - Q(S, A))$
5. $\text{ReplayBuffer.append\_transition}(S, A)$
6. $\text{Model.update}(S, A, R, \gamma, S')$
7. Loop repeat n times:

   A. $S, A \leftarrow \text{ReplayBuffer.sample\_transition}()$
   B. $R, \gamma, S' \leftarrow \text{Model.transition}(S, A)$
   C. $Q(S, A) \leftarrow Q(S, A) + \alpha(R + \gamma \max_a Q(S', a) - Q(S, A))$

In [0]:

```
class FeatureDynaQ(DynaQ):

  def __init__(self, number_of_features, number_of_actions, *args, **kwargs):
    super(FeatureDynaQ, self).__init__(
        number_of_actions=number_of_actions, *args, **kwargs)
    self._number_of_actions = number_of_actions
    self._number_of_features = number_of_features
    self.theta = np.zeros((number_of_actions, number_of_features))
    self._model = LinearModel(self._number_of_features, number_of_actions)

  def q(self, state):
    return np.dot(self.theta, state)

  def step(self, reward, discount, next_state):
    s = self._state
    a = self._action
    r = reward
    g = discount
    next_s = next_state

    delta = r + g*np.max(self.q(next_s)) - np.dot(self.theta[a,:], s.reshape(-1,
    self.theta[a,:] += self._step_size * delta * s
    self._replay_buffer.append([s,a])
    self._model.update(s,a,r,g,next_s)

    for n in range(self._num_offline_updates):
      length = len(self._replay_buffer)
      which = np.random.randint(length)
      s_, a_ = self._replay_buffer[which]
      r_,g_,next_s_ = self._model.transition(s_,a_)
      delta_ =  r_ + g_*np.max(self.q(next_s_)) - np.dot(self.theta[a_,:], s_)
      self.theta[a_,:] += self._step_size * delta_ * s_

    self._action = self._behaviour_policy(s)
    self._state = next_state
    return self._action


```

Assignment 2: Analyse Results

2.1 Tabular Learning

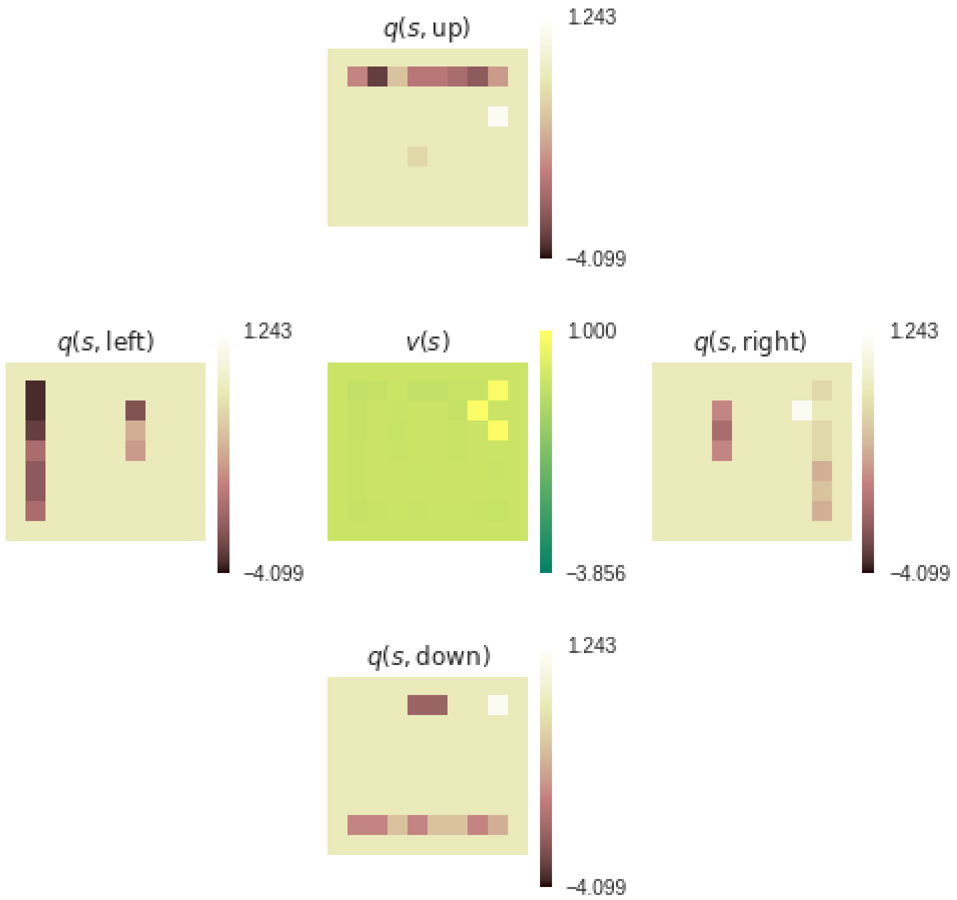
2.1.1 Data Efficiency

Online Q-learning

- number_of_steps = $1e3$ and num_offline_updates = 0

In [15]:

```
grid = Grid()
agent = ExperienceQ(
  grid._layout.size, 4, grid.get_obs(),
  random_policy, num_offline_updates=0, step_size=0.1)
run_experiment(grid, agent, int(1e3))
q = agent.q_values.reshape(grid._layout.shape + (4,))
plot_action_values(q)
```

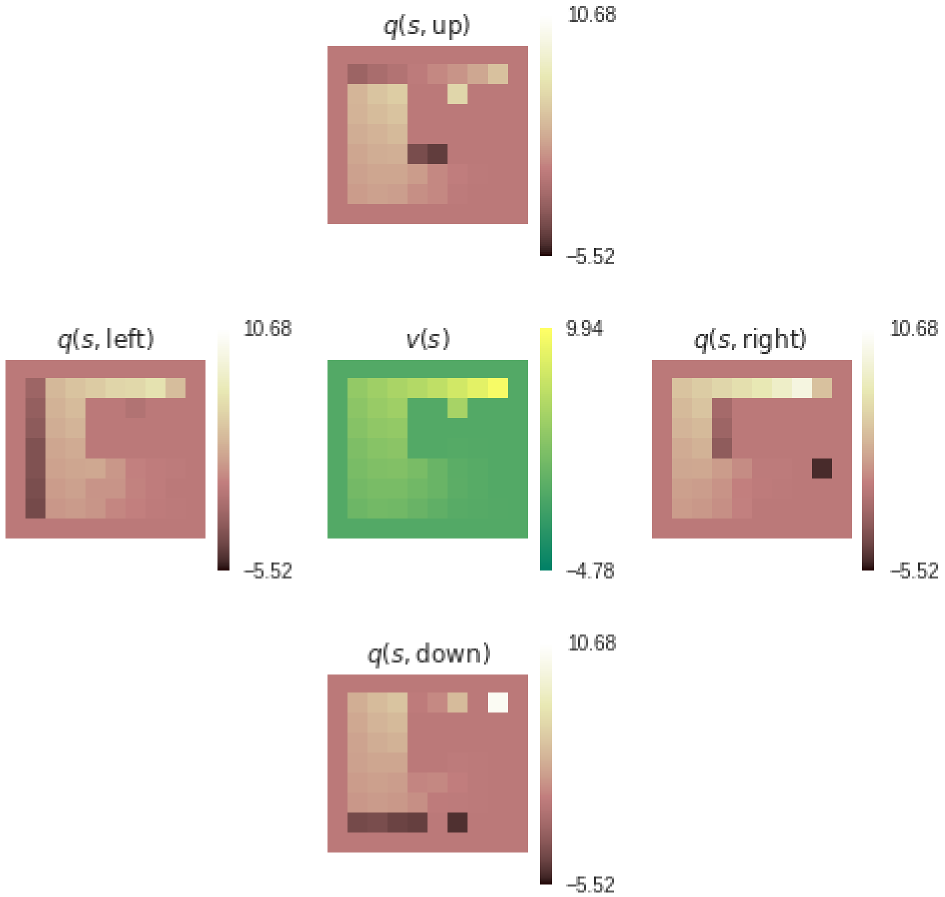


Experience Replay

- number_of_steps = $1e3$ and num_offline_updates = 0

In [16]:

```
grid = Grid()
agent = ExperienceQ(
  grid._layout.size, 4, grid.get_obs(),
  random_policy, num_offline_updates=30, step_size=0.1)
run_experiment(grid, agent, int(1e3))
q = agent.q_values.reshape(grid._layout.shape + (4,))
plot_action_values(q)
```

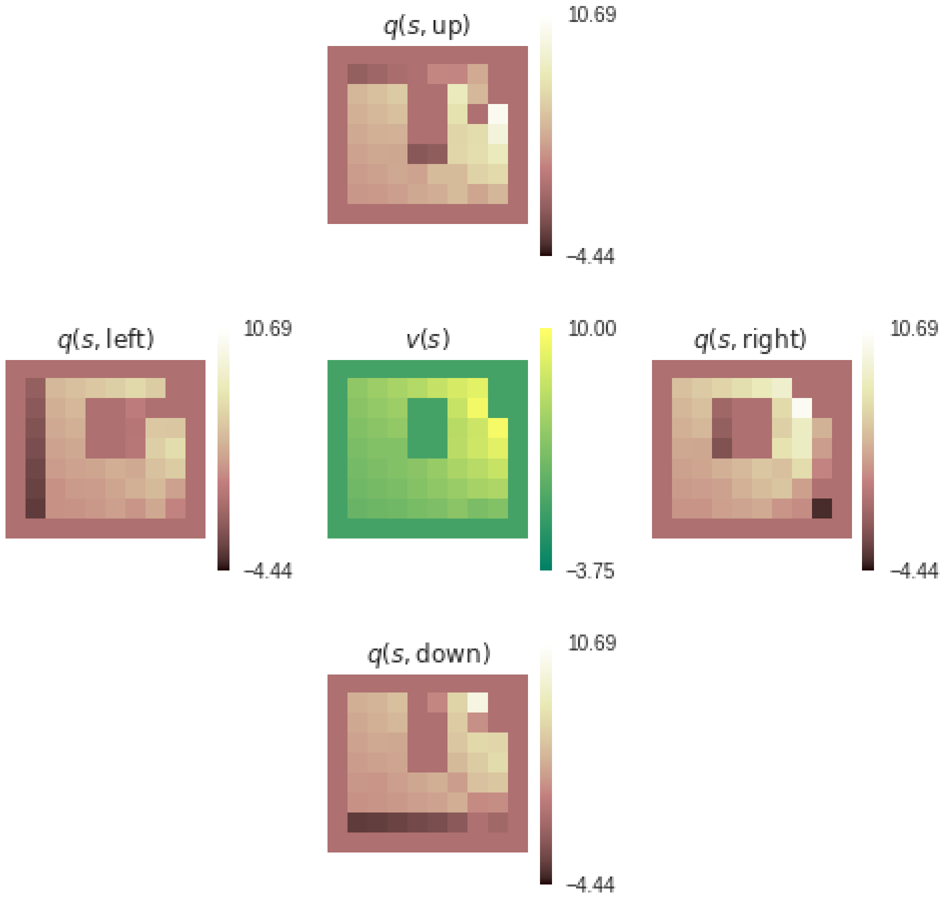


DynaQ

- number_of_steps = $1e3$ and num_offline_updates = 30

In [17]:

```
grid = Grid()
agent = DynaQ(
  grid._layout.size, 4, grid.get_obs(),
  random_policy, num_offline_updates=30, step_size=0.1)
run_experiment(grid, agent, int(1e3))
q = agent.q_values.reshape(grid._layout.shape + (4,))
plot_action_values(q)
```



2.1.2 Computational Cost

What if sampling from the environment is cheap and I don't care about data efficiency but only care about the number of updates to the model?

How do Online Q-learning, ExperienceReplay and Dyna-Q compare if I apply the same number of total updates?

Online Q-learning

- $\text{number\_of\_steps} = 3e4$ and $\text{num\_offline\_updates} = 0$

In [18]:

```
grid = Grid()
agent = ExperienceQ(
  grid._layout.size, 4, grid.get_obs(),
  random_policy, num_offline_updates=0, step_size=0.1)
run_experiment(grid, agent, int(3e4))
q = agent.q_values.reshape(grid._layout.shape + (4,))
plot_action_values(q)
plot_greedy_policy(grid, q)
```

```
/usr/local/lib/python3.6/dist-packages/ipykernel_launcher.py:58: Matpl
otlibDeprecationWarning: pyplot.hold is deprecated.
    Future behavior will be consistent with the long-time default:
    plot commands add elements without first clearing the
    Axes and/or Figure.
/usr/local/lib/python3.6/dist-packages/matplotlib/__init__.py:805: Mat
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rom your matplotlibrc and/or style files.
  mplDeprecation)
/usr/local/lib/python3.6/dist-packages/matplotlib/rcsetup.py:155: Matp
lotlibDeprecationWarning: axes.hold is deprecated, will be removed in
3.0
  mplDeprecation)
```
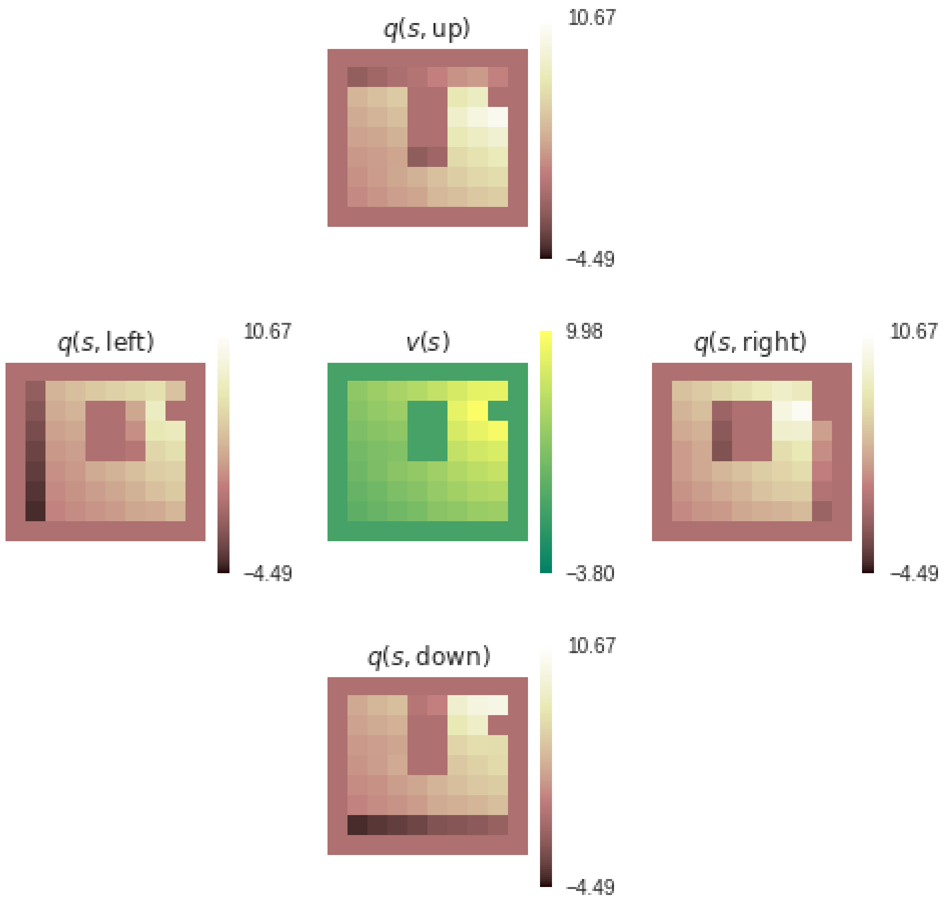
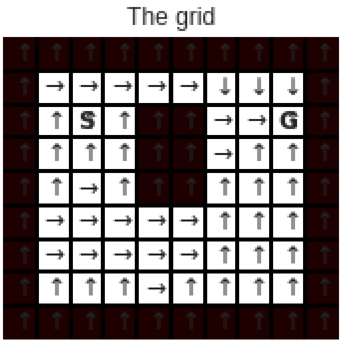





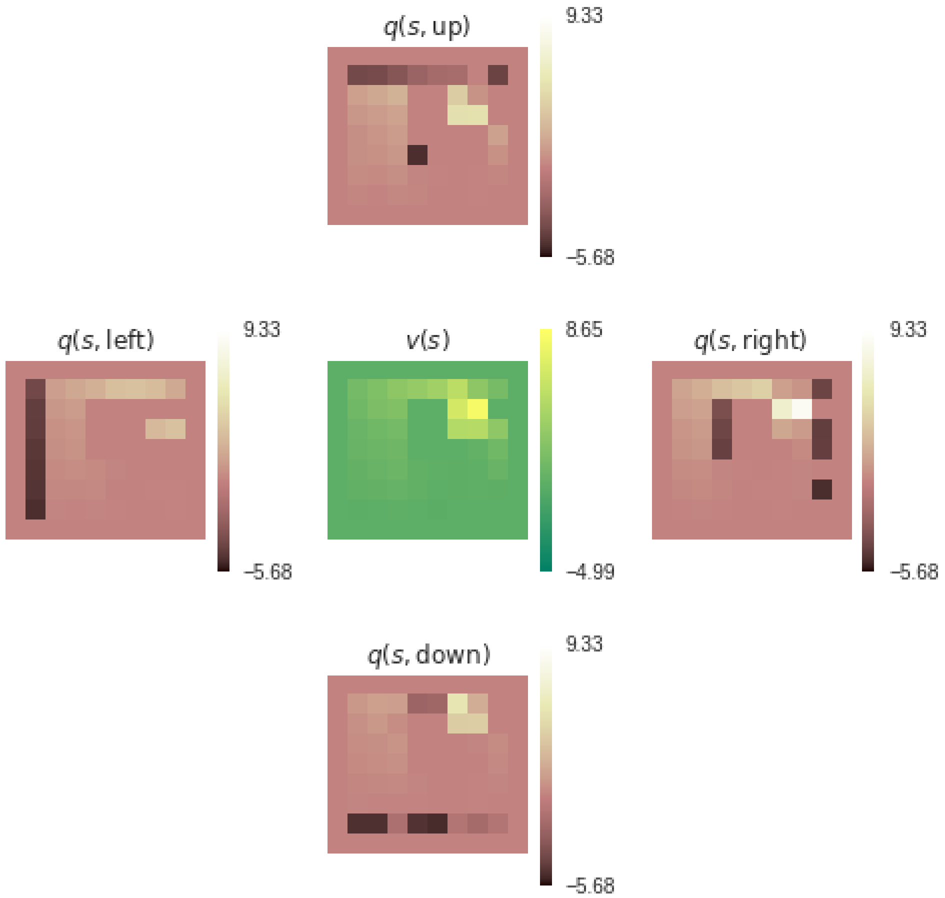
ExperienceReplay

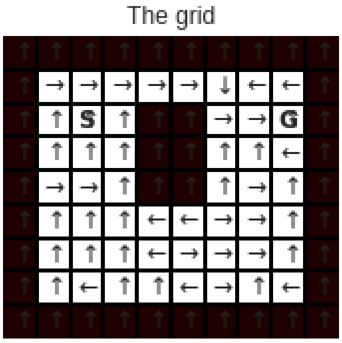
- number_of_steps = $1e3$ and num_offline_updates = 30

In [19]:

```
grid = Grid()
agent = ExperienceQ(
  grid._layout.size, 4, grid.get_obs(),
  random_policy, num_offline_updates=30, step_size=0.1)
run_experiment(grid, agent, int(1e3))
q = agent.q_values.reshape(grid._layout.shape + (4,))
plot_action_values(q)
plot_greedy_policy(grid, q)
```

```
/usr/local/lib/python3.6/dist-packages/ipykernel_launcher.py:58: Matpl
otlibDeprecationWarning: pyplot.hold is deprecated.
    Future behavior will be consistent with the long-time default:
    plot commands add elements without first clearing the
    Axes and/or Figure.
/usr/local/lib/python3.6/dist-packages/matplotlib/__init__.py:805: Mat
plotlibDeprecationWarning: axes.hold is deprecated. Please remove it f
rom your matplotlibrc and/or style files.
  mplDeprecation)
/usr/local/lib/python3.6/dist-packages/matplotlib/rcsetup.py:155: Matp
lotlibDeprecationWarning: axes.hold is deprecated, will be removed in
3.0
  mplDeprecation)
```





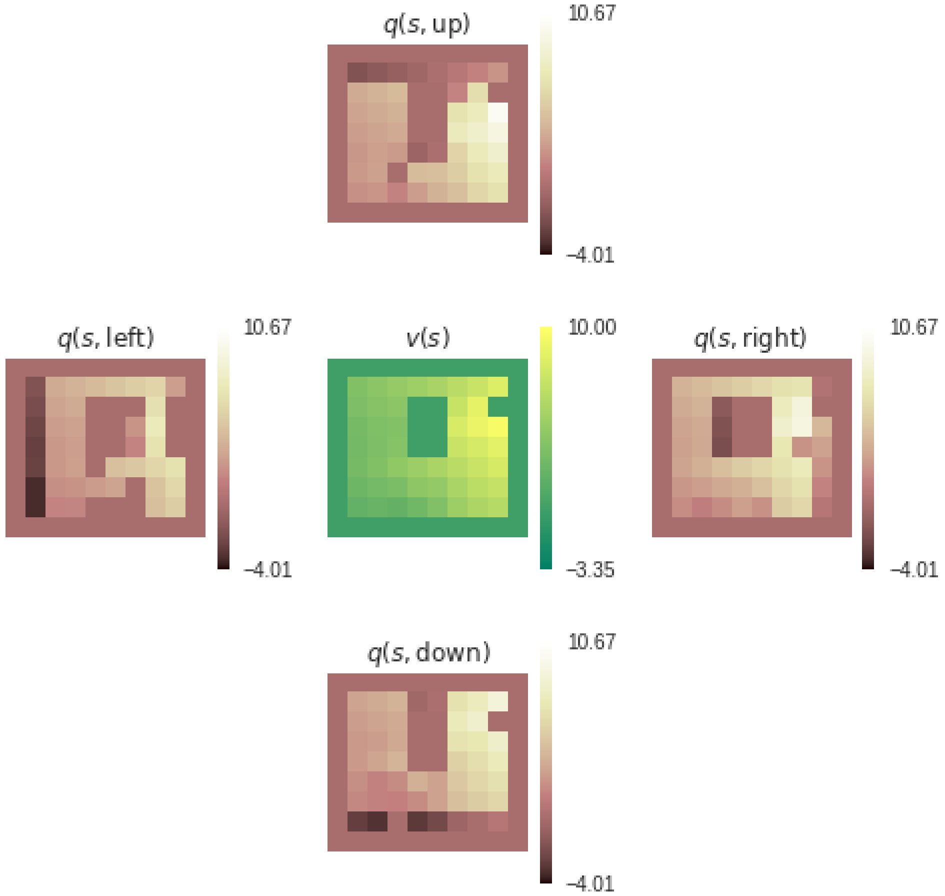
DynaQ

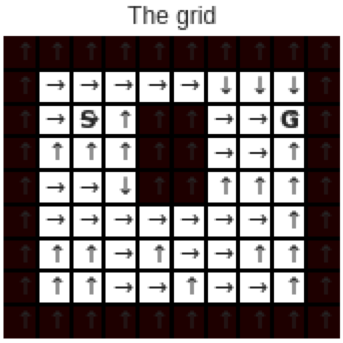
- number_of_steps = $1e3$ and num_offline_updates = 30

In [20]:

```
grid = Grid()
agent = DynaQ(
  grid._layout.size, 4, grid.get_obs(),
  random_policy, num_offline_updates=30, step_size=0.1)
run_experiment(grid, agent, int(1e3))
q = agent.q_values.reshape(grid._layout.shape + (4,))
plot_action_values(q)
plot_greedy_policy(grid, q)
```

```
/usr/local/lib/python3.6/dist-packages/ipykernel_launcher.py:58: Matpl
otlibDeprecationWarning: pyplot.hold is deprecated.
    Future behavior will be consistent with the long-time default:
    plot commands add elements without first clearing the
    Axes and/or Figure.
/usr/local/lib/python3.6/dist-packages/matplotlib/__init__.py:805: Mat
plotlibDeprecationWarning: axes.hold is deprecated. Please remove it f
rom your matplotlibrc and/or style files.
  mplDeprecation)
/usr/local/lib/python3.6/dist-packages/matplotlib/rcsetup.py:155: Matp
lotlibDeprecationWarning: axes.hold is deprecated, will be removed in
3.0
  mplDeprecation)
```





2.3 Linear function approximation

We will now consider the FeatureGrid domain.

And evaluate Q-learning, Experience Replay and DynaQ, in the context of linear function approximation.

All experiments are run for $\text{number\_of\_steps} = 1e5$

Online Q-learning with Linear Function Approximation

In [21]:

```
grid = FeatureGrid()

agent = FeatureExperienceQ(
  number_of_features=grid.number_of_features, number_of_actions=4,
  number_of_states=grid._layout.size, initial_state=grid.get_obs(),
  num_offline_updates=0, step_size=0.01, behaviour_policy=random_policy)
run_experiment(grid, agent, int(1e5))
q = np.reshape(
    np.array([agent.q(grid.int_to_features(i)) for i in range(grid.number_of_sta
    [grid._layout.shape[0], grid._layout.shape[1], 4])
plot_action_values(q)
plot_greedy_policy(grid, q)
```

```
/usr/local/lib/python3.6/dist-packages/ipykernel_launcher.py:58: Matpl
otlibDeprecationWarning: pyplot.hold is deprecated.
    Future behavior will be consistent with the long-time default:
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  mplDeprecation)
/usr/local/lib/python3.6/dist-packages/matplotlib/rcsetup.py:155: Matp
lotlibDeprecationWarning: axes.hold is deprecated, will be removed in
3.0
  mplDeprecation)
```

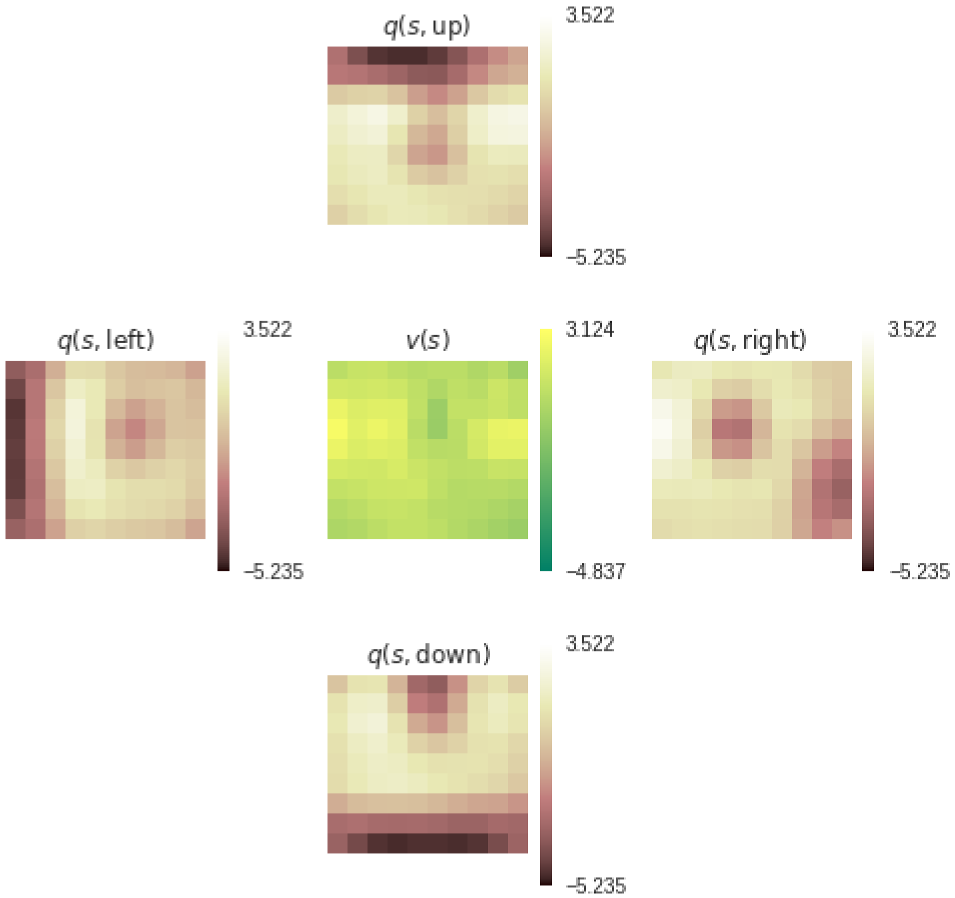

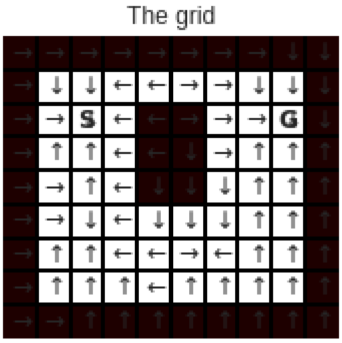

Experience Replay with Linear Function Approximation

In [22]:

```
grid = FeatureGrid()

agent = FeatureExperienceQ(
  number_of_features=grid.number_of_features, number_of_actions=4,
  number_of_states=grid._layout.size, initial_state=grid.get_obs(),
  num_offline_updates=10, step_size=0.01, behaviour_policy=random_policy)
run_experiment(grid, agent, int(1e5))
q = np.reshape(
    np.array([agent.q(grid.int_to_features(i)) for i in range(grid.number_of_sta
    [grid._layout.shape[0], grid._layout.shape[1], 4])
plot_action_values(q)
plot_greedy_policy(grid, q)
```

```
/usr/local/lib/python3.6/dist-packages/ipykernel_launcher.py:58: Matpl
otlibDeprecationWarning: pyplot.hold is deprecated.
    Future behavior will be consistent with the long-time default:
    plot commands add elements without first clearing the
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/usr/local/lib/python3.6/dist-packages/matplotlib/__init__.py:805: Mat
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  mplDeprecation)
/usr/local/lib/python3.6/dist-packages/matplotlib/rcsetup.py:155: Matp
lotlibDeprecationWarning: axes.hold is deprecated, will be removed in
3.0
  mplDeprecation)
```

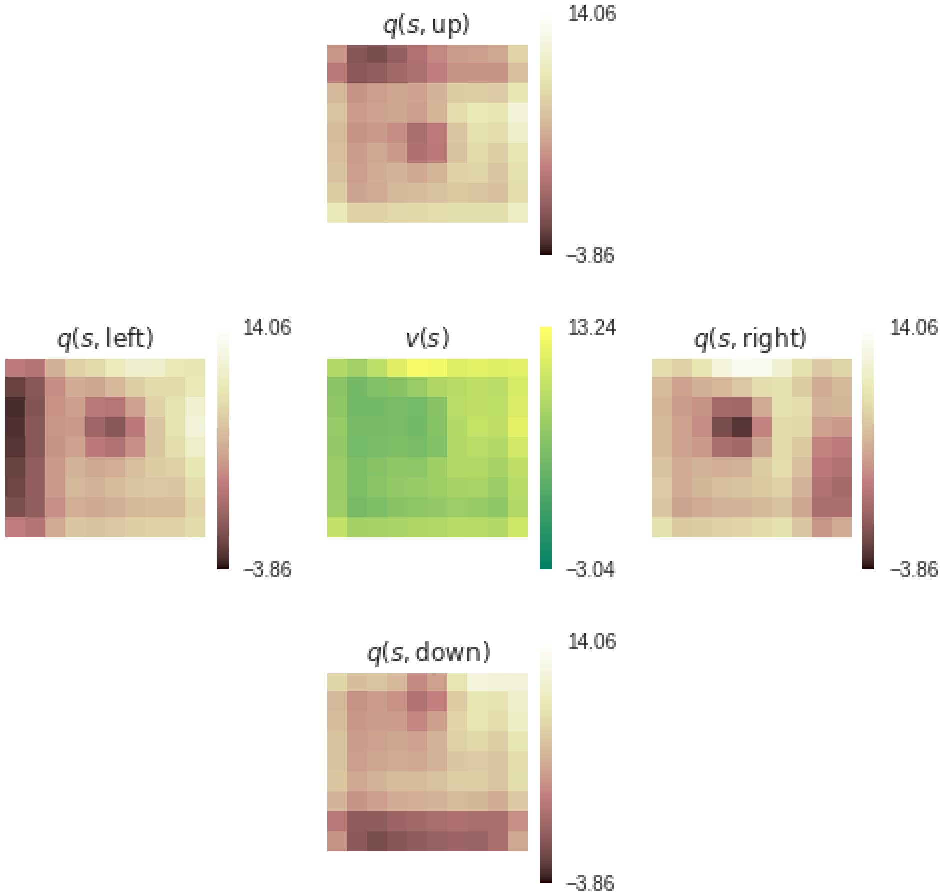

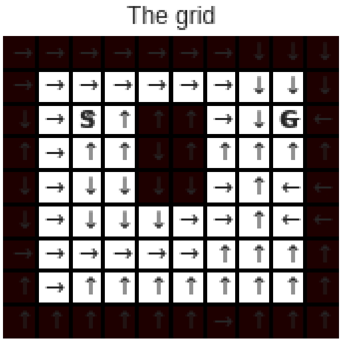

DynaQ with Linear Function Approximation

In [23]:

```
grid = FeatureGrid()

agent = FeatureDynaQ(
  number_of_features=grid.number_of_features,
  number_of_actions=4,
  number_of_states=grid._layout.size,
  initial_state=grid.get_obs(),
  num_offline_updates=10,
  step_size=0.01,
  behaviour_policy=random_policy)

run_experiment(grid, agent, int(1e5))
q = np.reshape(
    np.array([agent.q(grid.int_to_features(i)) for i in range(grid.number_of_sta
    [grid._layout.shape[0], grid._layout.shape[1], 4])
plot_action_values(q)
plot_greedy_policy(grid, q)
```

```
/usr/local/lib/python3.6/dist-packages/ipykernel_launcher.py:58: Matpl
otlibDeprecationWarning: pyplot.hold is deprecated.
    Future behavior will be consistent with the long-time default:
    plot commands add elements without first clearing the
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  mplDeprecation)
/usr/local/lib/python3.6/dist-packages/matplotlib/rcsetup.py:155: Matp
lotlibDeprecationWarning: axes.hold is deprecated, will be removed in
3.0
  mplDeprecation)
```

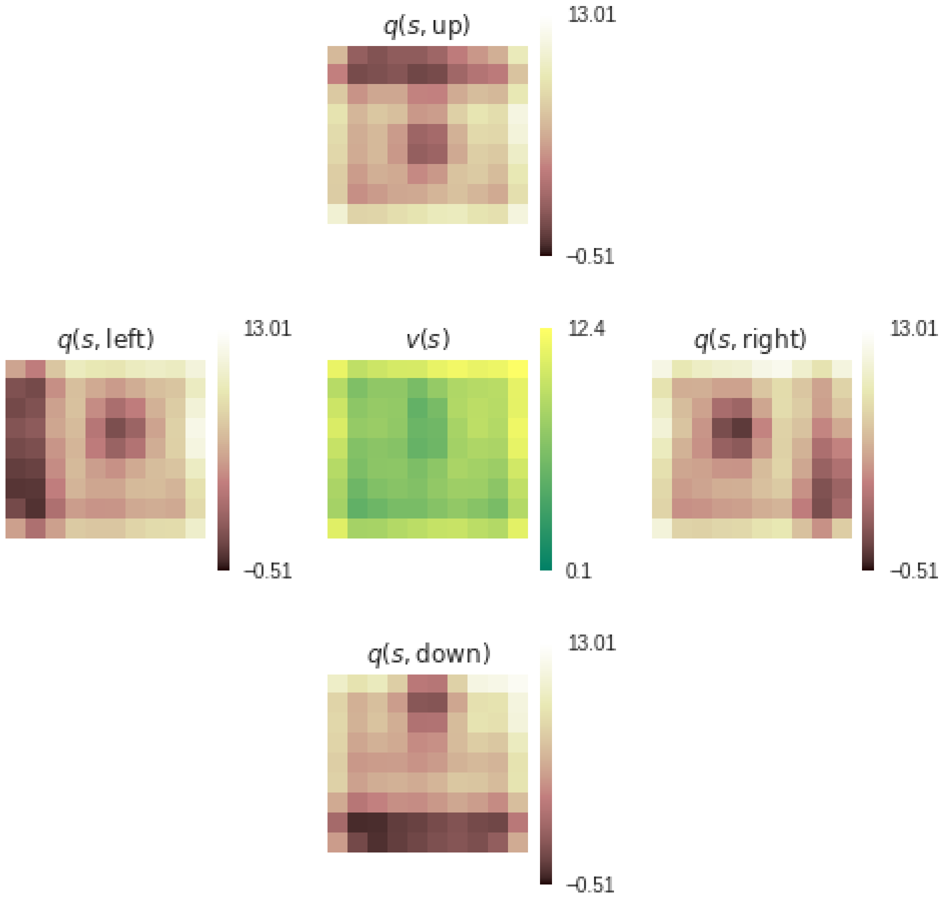

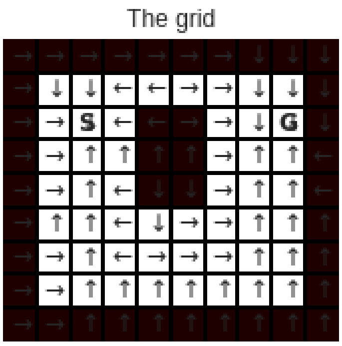

2.4 Non stationary Environments

We now consider a non-stationary setting where after `pretrain_steps` in the environment, the goal is moved to a new location (from the top-right of the grid to the bottom-left).

The agent is allowed to continue training for a (shorter) amount of time in this new setting, and then we evaluate the value estimates.

In [0]:

```
pretrain_steps = 2e4
new_env_steps = pretrain_steps / 30
```

Online Q-learning

In [25]:

```
# Train on first environment
grid = Grid()
agent = ExperienceQ(
  grid._layout.size, 4, grid.get_obs(),
  random_policy, num_offline_updates=0, step_size=0.1)
run_experiment(grid, agent, int(pretrain_steps))
q = agent.q_values.reshape(grid._layout.shape + (4,))
plot_state_value(q)
plot_greedy_policy(grid, q)

# Change goal location
alt_grid = AltGrid()
run_experiment(alt_grid, agent, int(new_env_steps))
alt_q = agent.q_values.reshape(alt_grid._layout.shape + (4,))
plot_state_value(alt_q)
plot_greedy_policy(alt_grid, alt_q)
```

```
/usr/local/lib/python3.6/dist-packages/ipykernel_launcher.py:58: Matpl
otlibDeprecationWarning: pyplot.hold is deprecated.
    Future behavior will be consistent with the long-time default:
    plot commands add elements without first clearing the
    Axes and/or Figure.
/usr/local/lib/python3.6/dist-packages/matplotlib/__init__.py:805: Mat
plotlibDeprecationWarning: axes.hold is deprecated. Please remove it f
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  mplDeprecation)
/usr/local/lib/python3.6/dist-packages/matplotlib/rcsetup.py:155: Matp
lotlibDeprecationWarning: axes.hold is deprecated, will be removed in
3.0
  mplDeprecation)
```
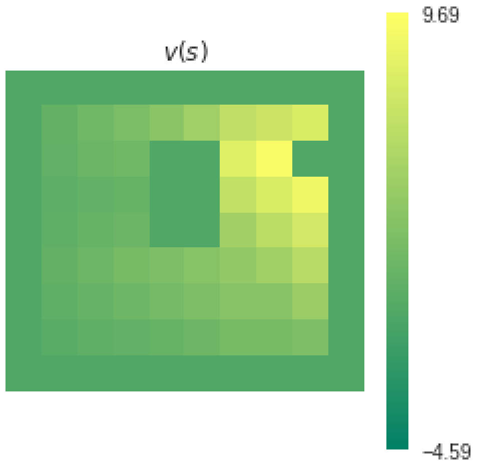
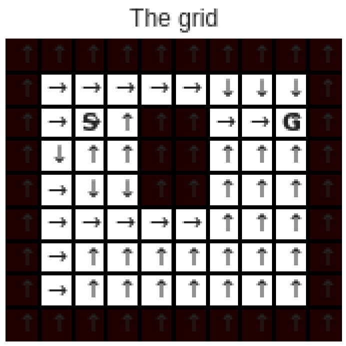
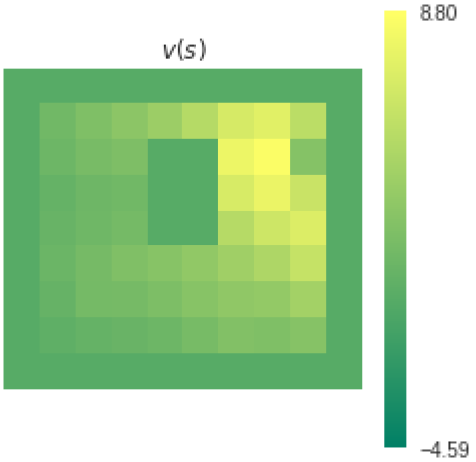
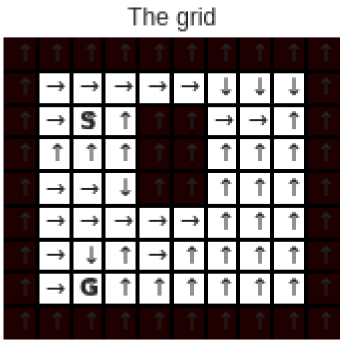







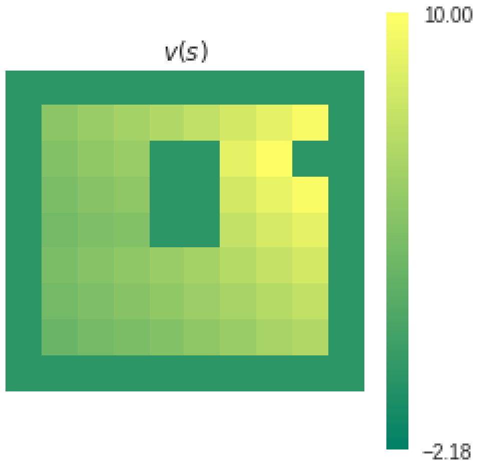


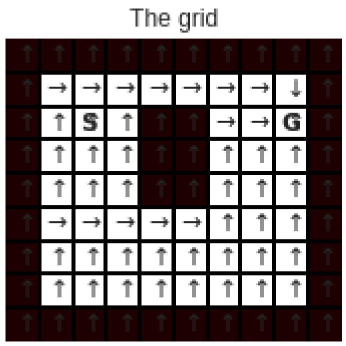
Experience Replay

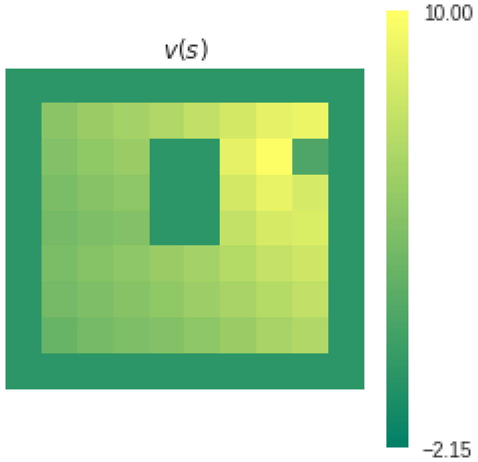
In [26]:

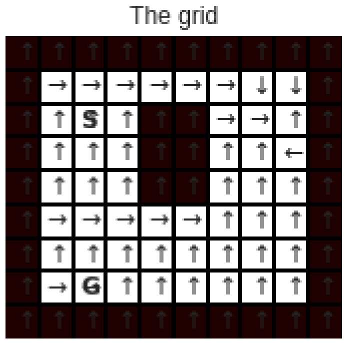
```
# Train on first environment
grid = Grid()
agent = ExperienceQ(
  grid._layout.size, 4, grid.get_obs(),
  random_policy, num_offline_updates=30, step_size=0.1)
run_experiment(grid, agent, int(pretrain_steps))
q = agent.q_values.reshape(grid._layout.shape + (4,))
plot_state_value(q)
plot_greedy_policy(grid, q)

# Change goal location
alt_grid = AltGrid()
run_experiment(alt_grid, agent, int(new_env_steps))
alt_q = agent.q_values.reshape(alt_grid._layout.shape + (4,))
plot_state_value(alt_q)
plot_greedy_policy(alt_grid, alt_q)
```

```
/usr/local/lib/python3.6/dist-packages/ipykernel_launcher.py:58: Matpl
otlibDeprecationWarning: pyplot.hold is deprecated.
    Future behavior will be consistent with the long-time default:
    plot commands add elements without first clearing the
    Axes and/or Figure.
/usr/local/lib/python3.6/dist-packages/matplotlib/__init__.py:805: Mat
plotlibDeprecationWarning: axes.hold is deprecated. Please remove it f
rom your matplotlibrc and/or style files.
  mplDeprecation)
/usr/local/lib/python3.6/dist-packages/matplotlib/rcsetup.py:155: Matp
lotlibDeprecationWarning: axes.hold is deprecated, will be removed in
3.0
  mplDeprecation)
```







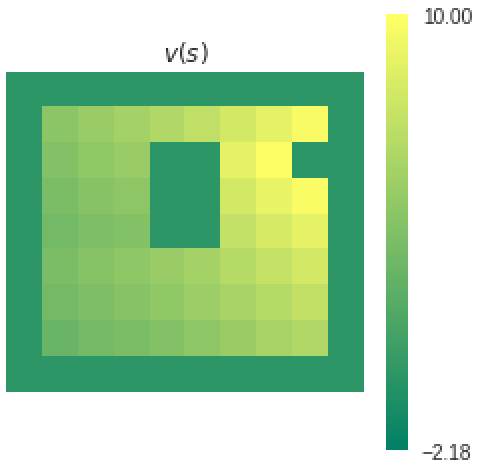


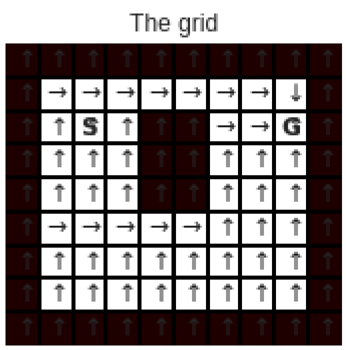
Dyna

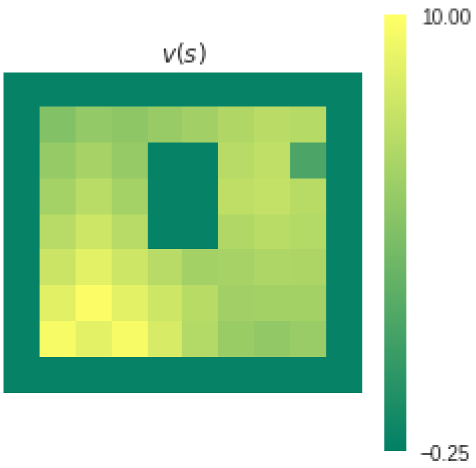
In [27]:

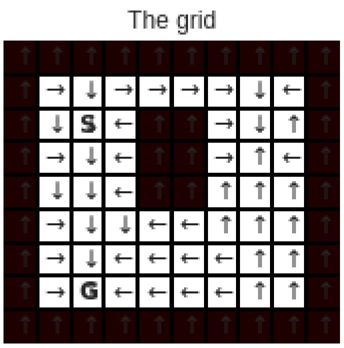
```
# Train on first environment
grid = Grid()
agent = DynaQ(
  grid._layout.size, 4, grid.get_obs(),
  random_policy, num_offline_updates=30, step_size=0.1)
run_experiment(grid, agent, int(pretrain_steps))
q = agent.q_values.reshape(grid._layout.shape + (4,))
plot_state_value(q)
plot_greedy_policy(grid, q)

# Change goal location
alt_grid = AltGrid()
run_experiment(alt_grid, agent, int(new_env_steps))
alt_q = agent.q_values.reshape(alt_grid._layout.shape + (4,))
plot_state_value(alt_q)
plot_greedy_policy(alt_grid, alt_q)
```

```
/usr/local/lib/python3.6/dist-packages/ipykernel_launcher.py:58: Matpl
otlibDeprecationWarning: pyplot.hold is deprecated.
    Future behavior will be consistent with the long-time default:
    plot commands add elements without first clearing the
    Axes and/or Figure.
/usr/local/lib/python3.6/dist-packages/matplotlib/__init__.py:805: Mat
plotlibDeprecationWarning: axes.hold is deprecated. Please remove it f
rom your matplotlibrc and/or style files.
  mplDeprecation)
/usr/local/lib/python3.6/dist-packages/matplotlib/rcsetup.py:155: Matp
lotlibDeprecationWarning: axes.hold is deprecated, will be removed in
3.0
  mplDeprecation)
```









Questions

Basic Tabular Learning

[5 pts] Why is the ExperienceReplay agent so much more data efficient than online Q-learning?

Answer: In ExperienceReplay, the agent stores previous experience and learn it multiple times during "offline learning". While in online Q-learning, the agents just learns from the outer environment, therefore do not converge quickly compared with ExperienceReplay.

[5 pts] If we run the experiments for the same number of updates, rather than the same number of steps in the environment, which among online Q-learning and Experience Replay performs better? Why?

Answer: online Q-learning will perform better than ExperienceReplay. With the same number of updates, online Q-learning agent learnings everything from the outer environment (3e4 steps from outer evnironment) while ExperienceReplay does not learn as much from outer environment as online Q-learing (1e3 form outer environment). With the total number of updates, ExperienceReplay agents updates primarily from the past experiences rather than outer environment, hence resulting not acquiring enough knowledge of the outer environment compared with online Q-learning, therefore the perforance is worse than online Q-learning.

[5 pts] Which among online Q-learning and Dyna-Q is more data efficient? why?

Answer: Dyna-Q is more data efficient. The agent for Dyna-Q integrates planning, model-learning and direct RL in parallel. The agent learns from previous experience as in ExperienceReplay, but also updates the previous experience

when the outer environment changes with the tabular table. Hence taking more updates then online-Q learning, resulting being less data efficient than Dyna-Q.

[5 pts] If we run the experiments for the same number of updates, rather than the same number of steps in the environment, which among online Q-learning and Dyna-Q performs better? Why?

Answer: online Q-learning will perform better than Dyna-Q. With the same number of updates, online Q-learning agent learnings everything from the outer environment (3e4 steps from outer evnironment) while Dyna-Q does not learn as much from outer environment as online Q-learing (1e3 form outer environment). With the total number of updates, Dyna-Q agents updates primarily from the past experiences rather than outer environment, hence resulting not acquiring enough knowledge of the outer environment compared with online Q-learning, therefore the perforance is worse than online Q-learning.

Linear function approximation

[5 pts] The value estimates with function approximation are considerably more blurry than in the tabular setting despite more training steps and interactions with the environment, why is this the case?

Answer: In this case, Radial basis features are used, resulting more smooth features. Also, gradient descent for linear approximation is used, and the number of weights is less than number os states, hence when a single state is updated, it generalise to affect the value of other states.

[5 pts] Inspect the policies derived by training agents with linear function approximation on `FeatureGrid` (as shown by `plot_greedy_policy`). How does this compare to the optimal policy? Are there any inconsistencies you can spot? What is the reason of these?

Answer: We compare two Dyna-Q agents (with and without linear function approximation) to find influence by linear funciton approximation. We note that policy obtained without linear function approxiamtion is better than the one with linear function approximation. Policy with linear approximation can hardly find the optimal policy. We spot policy inconsistency for lienar approximation, which means the policy has not converged. Reason for this: we need more information and iterations to train it in order to make it converge.

Learning in a non stationary environment

Consider now the tabular but non-stationary setting of section 2.4.

After an initial pretraining phase, the goal location is moved to a new location, where the agent is allowed to train for some (shorter) time.

[10 pts] Compare the value estimates of online Q-learning and Experience Replay, after training also on the new goal location, explain what you see.

Answer: The value estimates of online Q-learning is slightly higher than those of ExperienceReplay. And the learning outcome visualisation has slightly higher values near or at left cornor (new goal), as ExperienceReplay agent learns more from previous experience. After changing the goal, online Q-learning agent performs better as it learns new knowledge from the new outer environment, while ExperienceReplay agent learns from previous experience, which might not be as useful in an non-stationary environment after changing the goal.

[10 pts] Compare the value estimates of online Q-learning and Dyna-Q, after training also on the new goal location, explain what you see.

Answer: The value estimates of Dyna-Q is slightly higher than those of online Q-learning. And the learning outcome visualisation has much higher values near left cornor (new goal), as the visualisation

is much more brighter. If we change the goal, Dyna-Q agent combines experience learning and model learning, and therefore resulting more efficient learning compared with online Q-learning in an non-stationary environment. And the Dyna-Q agent performs much better than online Q-learning.

Back up your observations with visualizations of the value/policy.

In [0]:

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1
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