Determination of the astrophysical ^{26}Si$(p,\gamma)^{27}$P reaction rate from the asymptotic normalization coefficients of ^{27}Mg $\rightarrow$ ^{26}Mg + n

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The squares of neutron asymptotic normalization coefficient (ANC) for ^{27}Mg $\rightarrow$ ^{26}Mg + n are extracted to be 44.0 ± 5.3, 3.40 ± 0.32 and 0.90 ± 0.08 fm^{-1} from the angular distributions of the ^{26}Mg$(d,p)^{27}$Mg reaction leading to the ground, first, and second excited states of ^{27}Mg, respectively, based on distorted wave Born approximation (DWBA) analysis. According to charge symmetry of mirror nuclei, the square of proton ANC for ^{27}P $\rightarrow$ ^{26}Si + p is determined to be 1840 ± 240 fm^{-1} and then utilized to calculate the astrophysical S-factor and rate for the direct capture into the ^{27}P ground state. In addition, the proton widths for the first and second excited states in ^{27}P are derived to be $1.30 \pm 0.12 \times 10^{-8}$ and $1.79 \pm 0.15 \times 10^{-5}$ MeV from the neutron ANCs and used to compute the contribution of the resonant captures. Furthermore, we have also presented the total astrophysical ^{26}Si$(p,\gamma)^{27}$P reaction rate.

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The 1.809 MeV γ-ray from the decay of ^{26}Al is an excellent observable for some astrophysical events, such as novae [1] and x-ray burst [2]. The nucleosynthesis of ^{26}Al is dominated by the reaction chain ^{24}Mg$(p,\gamma)^{25}$Al$(\beta^+)^{25}$Mg$(p,\gamma)^{26}$Al, however, it is complicated due to the presence of a short-lived ($T_{1/2} = 6.34$ s) isomer. This dominant chain can be bypassed through ^{25}Al$(p,\gamma)^{26}$Si$(p,\gamma)^{27}$P, and thus ^{26}Si$(p,\gamma)^{27}$P is thought to be one of the relevant reactions in the production of ^{26}Al. It has been supposed that the thermal equilibrium between the ground and isomeric states of ^{26}Al may be reached at high temperature ($T_9 \approx 0.4$) novae [3]. The isomeric state of ^{26}Al can also be produced via the reaction chain ^{25}Al$(p,\gamma)^{26}$Si$(\beta^+)^{26m}$Al. Therefore, ^{26}Si depletion via its (p,γ) reaction is of importance for determining the quantity of ^{26gs}Al produced by the thermal equilibrium because ^{26m}Al would not be fed by ^{26}Si β decay. Wiescher *et al*. estimated the ^{26}Si$(p,\gamma)^{27}$P reaction rate based on the structure of better known mirror nucleus [4]. Herndl *et al*. performed detailed shell model calculations, and presented the proton widths of 1.7×10^{-9} and 1.36×10^{-5} MeV, and the gamma widths of 1.36×10^{-9} and 3.3×10^{-10} MeV for the first and second excited states of ^{27}P, respectively [5]. Rauscher *et al*. calculated this reaction based on statistical Hauser-Feshbach theory [6]. Caggiano *et al*. measured the ground state mass excess and the first excited state in ^{27}P, and found the location of the first resonance through the ^{28}Si(^{7}Li,^{8}He)^{27}P reaction [7]. Recently, the gamma width for the first excited state was measured to be $3.60 \pm 0.54 \times 10^{-10}$ MeV through Coulomb dissociation method by Togano *et al*. [8], and renewed to be $3.7 \pm 2.2 \times 10^{-9}$ MeV very recently [9]. To date, there has not yet been any experimental information concerning direct capture rate of the ^{26}Si$(p,\gamma)^{27}$P reaction. Here we reanalyze the existing ^{26}Mg$(d,p)^{27}$Mg angular distributions [10], and then deduce the neutron ANCs of ^{27}Mg $\rightarrow$ ^{26}Mg + n. The proton ANC of ^{27}P $\rightarrow$ ^{26}Si + p is extracted using charge symmetry and employed to obtain the astrophysical ^{26}Si$(p,\gamma)^{27}$P S-factor and rate for the direct capture into the ^{27}P ground state. We have also derived the proton widths from the neutron ANCs and then computed the contribution of resonant captures into the first and second excited states of ^{27}P.

For a peripheral transfer reaction, the ANC can be extracted by comparison of the experimental angular distribution with DWBA calculation. In the present calculation, the zero-range DWBA code DWUCK4 [11] is used to fit the experimental data. The optical potential parameters of entrance channel are obtained by fitting the angular distribution of deuteron elastic scattering on ^{26}Mg at $E_d = 12$ MeV [10], those of the exit channel are taken from Refs. [12–14], respectively, as listed in Table I. Figure 1 shows the normalized angular distributions for four sets of optical potentials, together with the experimental data. The extracted three neutron ANCs for the ground, first and second excited states of ^{27}Mg are listed in Table II. The error results from the deviation of different optical potentials and experimental uncertainty. To verify that the transfer reaction is peripheral, the ANCs for ^{27}Mg $\rightarrow$ ^{26}Mg + n and the spectroscopic factors are computed by changing the geometry parameters of Woods-Saxon potential for single particle bound state. The spectroscopic factors vary more than ±30% around the average, while the ANCs vary less than ±2%, indicating that the ^{26}Mg$(d,p)^{27}$Mg reactions leading to the ground, first and second excited states of ^{27}Mg are all dominated by peripheral process.

Recently, the relationship of the ANCs for mirror pairs has been established [15,16], and applied to the study of the ^{8}B$(p,\gamma)^{9}$C reaction [17]. ^{27}Mg and ^{27}P are mirror pair, thus the proton ANC of ^{27}P $\rightarrow$ ^{26}Si + p is written as $(C_{lj}^{^{27}\mathrm{P}})^2 = R(C_{lj}^{^{27}\mathrm{Mg}})^2$, where R is computed according to [15]

$$R = \left|\frac{F_l(ik_pR_N)}{k_pR_N\, j_l(ik_nR_N)}\right|^2, \tag{1}$$

where F_l and j_l are regular Coulomb wave function and spherical Bessel function, and R_N radius of nuclear interior. k_p and k_n can be determined by the proton and neutron separation energies ε_p and ε_n via $k = (2\mu\varepsilon/\hbar^2)^{1/2}$. The ratio R is nearly

TABLE I. Optical potential parameters used in DWBA calculation, D and P denote entrance and exit channels respectively, where V and W are in MeV, r and a in fm.

Set	D	$P1$	$P2$	$P3$	$P4$
V_r	95.2	57.9-0.55E_p	58.3-0.32E_p	49.14	52.6
r_{0r}	1.049	1.25	1.17	1.174	1.27
a_r	0.836	0.65	0.75	0.736	0.66
W_s	24.14	9.0	13.13-0.25E_p	8.06	6.6
r_{0s}	1.399	1.25	1.32	1.19	1.2
a_s	0.619	0.47	0.59	0.562	0.66
V_{so}	7.0	7.5	6.2	5.29	6.1
r_{0so}	0.9	1.25	1.01	1.06	1.27
a_{so}	0.6	0.65	0.75	0.546	0.66
r_{0c}	1.3	1.25	1.36	1.17	1.25

a constant (44.2) when R_N runs from 2.5 to 5.0 fm in present calculation.

On the other hand, if we assume that the single particle spectroscopic factors S_p and S_n are equal for mirror pair, the ratio R can also be obtained by

$$R = \left(b_{lj}^{^{27}\mathrm{P}}\right)^2 / \left(b_{lj}^{^{27}\mathrm{Mg}}\right)^2. \tag{2}$$

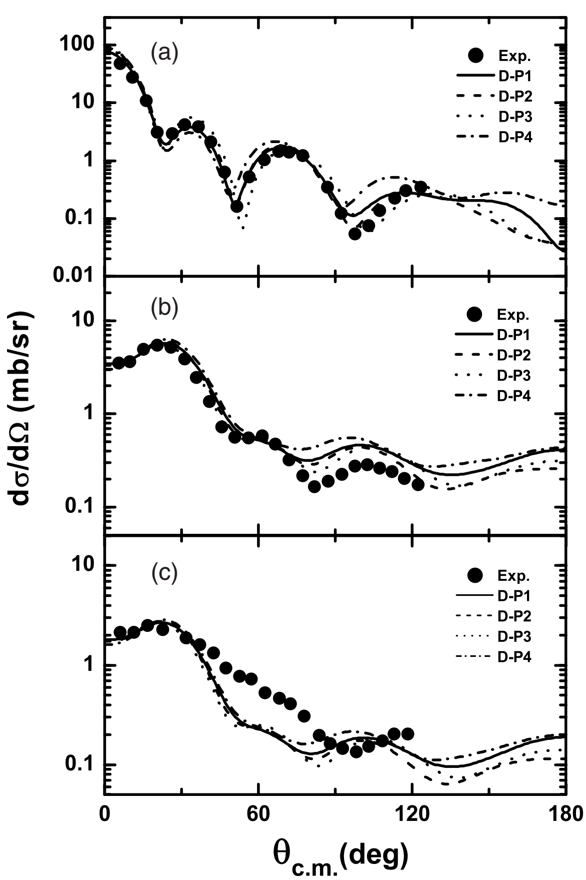

FIG. 1. Experimental ^{26}Mg$(d, p)^{27}$Mg angular distributions at $E_d = 12$ MeV, together with the DWBA calculations using four sets of optical potentials. (a)–(c) correspond to the ground, first, and second excited states of ^{27}Mg, respectively.

TABLE II. The squares of ANC for $^{27}\mathrm{Mg} \rightarrow {}^{26}\mathrm{Mg} + n$. n, l, and j are the number of nodes excluding the origin and infinity for the radial function, the orbital and total angular momenta of the transferred neutron in ^{27}Mg, respectively.

Reaction	E_x (MeV)	J_π	nlj	ANC^2 (fm^{-1})
	0	$1/2^+$	$1s_{1/2}$	44.0 ± 5.3
^{26}Mg$(d, p)^{27}$Mg	0.985	$3/2^+$	$0d_{3/2}$	3.40 ± 0.32
	1.698	$5/2^+$	$0d_{3/2}$	0.90 ± 0.08

R is derived to be 39.5 from the single particle wave functions in the same manner as described in Ref. [17].

The average of above two ratios is 41.9 ± 2.4, the error arises from their systematic deviation. The squared ANC for $^{27}\mathrm{P} \rightarrow {}^{26}\mathrm{Si} + p$ is then extracted to be 1840 ± 240 fm^{-1}, its error results from the uncertainties of $(C_{lj}^{^{27}\mathrm{Mg}})^2$ and R. These results are listed in Table III.

The relationship between the width Γ_p of a proton resonance and the neutron ANC of its mirror bound analog has also been found [15]. Γ_p can be expressed as $\Gamma_p = R^{\mathrm{res}}(C_{lj}^{^{27}\mathrm{Mg}})^2$, where R^{res} is given by

$$R^{\mathrm{res}} = \frac{\hbar^2 k_p}{\mu}\left|\frac{F_l(k_p R_N)}{k_p R_N j_l(i k_n R_N)}\right|^2. \tag{3}$$

Similarly, under the assumption of $S_p = S_n$ for mirror pair, R^{res} can also be approximated by

$$R^{\mathrm{res}} = \Gamma_p^{\mathrm{s.p.}} / \left(b_{lj}^{^{27}\mathrm{Mg}}\right)^2, \tag{4}$$

where $\Gamma_p^{\mathrm{s.p.}}$ denotes the single particle width, which can be computed from the scattering phase shifts in a Woods-Saxon potential with the depth determined by reproducing the resonance energy. The Γ_p for the first and second excited states of ^{27}P are then deduced. The resonance strength $\omega\gamma$ is given by $\omega\gamma = \frac{2J+1}{(2J_1+1)(2J_2+1)} \frac{\Gamma_p \Gamma_\gamma}{\Gamma_{\mathrm{tot}}}$, where J, J_1, and J_2 are the spins of ^{27}P, proton and ^{26}Si, respectively. All the results are listed in Table IV. The Γ_p and $\omega\gamma$ of the first resonance in the present work is significantly larger than those in Refs. [5,7]. The Γ_p of the second resonance is approximately in agreement with

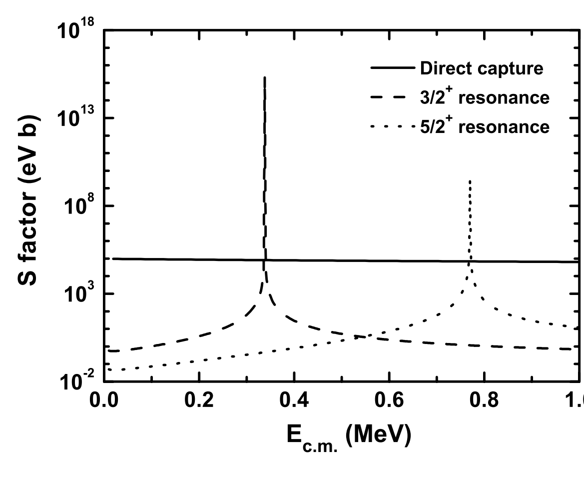

FIG. 2. Astrophysical S-factors as a function of $E_{\mathrm{c.m.}}$ for the direct and resonant captures.

TABLE III. The parameters used in the calculation, the square of ANCs for virtual decay $^{27}P \rightarrow {}^{26}Si + p$, and astrophysical S-factor of the direct capture. l represent the angular momentum of incident wave.

Reaction	E_x (MeV)	J_π	R		ANC^2 (fm^{-1})	l	S_0 (eV b)	
$^{26}Si(p, \gamma)^{27}P$	0.0	$1/2^+$	44.2	39.5	1840 ± 240	p	$8.7 \pm 1.1 \times 10^4$	3.6×10^4 [5]

that given in Ref. [5], while twice larger than that in Ref. [7]. However, the corresponding resonance strengths in these three works agree with each other because $\Gamma_p \gg \Gamma_\gamma$, resulting in $\frac{\Gamma_p \Gamma_\gamma}{\Gamma_{tot}} \approx \Gamma_\gamma$.

For the $^{26}Si(p, \gamma)^{27}P$ reaction, the direct capture into the ground state of ^{27}P is believed to be dominated by $E1$ transition from incoming p wave to bound s state. Since the direct capture at astrophysical energies is a peripheral process, the absolute normalization of its cross section and astrophysical S-factor is entirely defined by the ANC of $^{27}P \rightarrow {}^{26}Si + p$. The reaction rate for the direct capture (cm^3 mole^{-1} s^{-1}) can be parametrized by [18]

$$N_A \langle \sigma v \rangle_{dc} = 3.7313 \times 10^{10} \mu^{-1/2} T_9^{-3/2} \times \int_0^\infty \sigma_{dc}(E) E \exp(-11.605 E/T_9) dE, \quad (5)$$

where T_9 and $\sigma_{dc}(E)$ are the temperature in unit of 10^9 K and cross section in barn, respectively.

The cross section of resonant capture is given by the Breit-Wigner formula. For the gamma transition of the first excited state, the mixing ratio $E2/M1$ is estimated to be 0.033 using simple single particle model [19]. For the one of the second excited state, only pure $E2$ transition is taken into account due to selection rule. Since the incoming angular momentum and multipolarity of the direct and resonant capture γ-radiation are different, the interference effects between direct and resonant captures are negligible [20]. Shown in Figure 2 is the energy dependence of astrophysical S-factors for the direct capture into the ground state and resonant captures into the first and second excited states of ^{27}P. The S_0 listed in Table III is the average of S-factors for the direct capture over the energy range of 0.1–0.3 MeV, which is about 2.4 times larger than that from shell-model calculation [5].

For isolated and narrow resonance, the reaction rate (cm^3 mole^{-1} s^{-1}) is approximated by [18]

$$N_A \langle \sigma v \rangle_{res} = 1.54 \times 10^{11} (\mu T_9)^{-3/2} \omega\gamma \times \exp(-11.605 E_r/T_9), \quad (6)$$

where both resonance strength $\omega\gamma$ and resonance energy E_r are in units of MeV. The resonance energies are determined with $E_r = E_x - Q$, using the newly measured excitation energies of 1.199 MeV and 1.631 MeV [7] and the latest Q value of 0.861 MeV [21]. The reaction rates for the direct capture and two resonant captures are shown in Fig. 3. It can be seen that the resonant capture into the first excited state of ^{27}P accounts for almost total of reaction rate in a wide range of temperature ($T_9 = 0.1 - 1$).

The total rates are listed in Table V, together with the previous results [4–7]. They are orders of magnitude larger than the estimates [4], where the level ($3/2^+$, $E_x =$ 1.199 MeV) was neglected, however, it has a strong effect on total rate. They accord with Hauser-Feshbach predictions [6] within a factor of 5, the deviation may be caused by the relatively low level density of the compound nucleus ^{27}P. They are roughly in agreement with those in Refs. [5,7], the deviation mainly results from the difference of E_r and $\omega\gamma$ used in these three works.

In summary, we reanalyze the existing $^{26}Mg(d, p)^{27}Mg$ angular distributions for the ground, first, and second excited states of ^{27}Mg, and then extract the neutron ANCs for $^{27}Mg \rightarrow {}^{26}Mg + n$ through DWBA analysis. By using charge symmetry, the proton ANC of $^{27}P \rightarrow {}^{26}Si + p$ is derived and utilized to calculate the astrophysical $^{26}Si(p, \gamma)^{27}P$ S-factor and rate of

TABLE IV. The parameters used in the calculation, the proton and gamma widths and resonance strengths.

Reaction	$^{26}Si(p, \gamma)^{27}P$	
E_x (MeV)	1.199	1.631
E_r (MeV)	0.338	0.770
J^π	$3/2^+$	$5/2^+$
Γ_γ (MeV)	3.43×10^{-9}[7]	3.3×10^{-10}[7]
R^{res}	3.81×10^{-9}	1.99×10^{-5}
(MeV · fm)	3.64×10^{-9}	1.86×10^{-5}
	$1.27 \pm 0.12 \times 10^{-8}$	$1.74 \pm 0.16 \times 10^{-5}$
Γ_p (MeV)	1.7×10^{-9}[5]	1.36×10^{-5}[5]
	3.5×10^{-9}[7]	7.5×10^{-6}[7]
	$5.4 \pm 0.1 \times 10^{-9}$	9.9×10^{-10}
$\omega\gamma$ (MeV)	1.51×10^{-9}[5]	9.9×10^{-10}[5]
	3.5×10^{-9}[7]	9.9×10^{-10}[7]

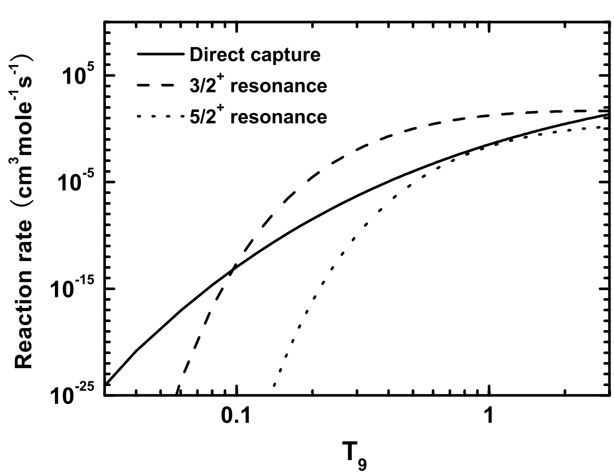

FIG. 3. The temperature dependence of reaction rates for the direct and resonant captures.

TABLE V. The total reaction rate $N_A < \sigma v >$ (cm^3 $mole^{-1}$ s^{-1}) as a function of temperature, together with the previous results.

T_9	Present work	Ref. [4]	Ref. [5]	Ref. [6]	Ref. [7]
0.1	3.63×10^{-13}	1.68×10^{-13}	6.19×10^{-13}	2.50×10^{-9}	2.67×10^{-13}
0.2	2.96×10^{-5}	5.34×10^{-9}	2.37×10^{-5}	7.50×10^{-5}	2.98×10^{-5}
0.3	1.11×10^{-2}	7.92×10^{-7}	6.30×10^{-3}	4.28×10^{-3}	1.16×10^{-2}
0.4	1.90×10^{-1}	1.85×10^{-5}	9.04×10^{-2}	4.29×10^{-2}	2.02×10^{-1}
0.5	9.64×10^{-1}	1.85×10^{-4}	4.14×10^{-1}	1.97×10^{-1}	1.04×10^{0}
0.6	2.71×10^{0}	1.16×10^{-3}	1.09×10^{0}	5.92×10^{-1}	2.95×10^{0}
0.7	5.48×10^{0}	5.10×10^{-3}	2.09×10^{0}	1.36×10^{0}	5.99×10^{0}
0.8	9.04×10^{0}	1.70×10^{-2}	3.32×10^{0}	2.62×10^{0}	9.92×10^{0}
0.9	1.31×10^{1}	4.53×10^{-2}	4.67×10^{0}	4.45×10^{0}	1.44×10^{1}
1.0	1.73×10^{1}	1.02×10^{-1}	6.04×10^{0}	6.88×10^{0}	1.90×10^{1}
1.5	3.54×10^{1}	1.46×10^{0}	1.17×10^{1}	2.76×10^{1}	3.90×10^{1}
2.0	4.68×10^{1}	7.15×10^{0}	1.57×10^{1}	5.88×10^{1}	5.06×10^{1}

the direct capture into the ground state of ^{27}P. The proton widths are also deduced from the experimental neutron ANCs and used to compute the contribution of resonant captures into the first and second excited states. In addition, the total reaction rates are presented as a function of temperature. It can be seen that the $^{26}\mathrm{Si}(p,\gamma)^{27}\mathrm{P}$ reaction is dominated by the resonant capture into the first excited state of ^{27}P in a wide range of temperature ($T_9 = 0.1 - 1$).

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[1] J. José, A. Coc, and M. Hernanz, Astrophys. J. **520**, 347 (1999).
[2] O. Koike, M. Hashimoto, K. Arai, and S. Wanajo, Astron. Astrophys. **342**, 464 (1999).
[3] A. Coc, M.-G. Porquet, and F. Nowacki, Phys. Rev. C **61**, 015801 (1999).
[4] M. Wiescher, J. Görres, F.-K. Thielemann, and H. Ritter, Astron. Astrophys. **160**, 56 (1986).
[5] H. Herndl, J. Görres, M. Wiescher, B. A. Brown, and L. Van Wormer, Phys. Rev. C **52**, 1078 (1995).
[6] T. Rauscher, F.-K. Thielemann, and K. L. Kratz, Phys. Rev. C **56**, 1613 (1997).
[7] J. A. Caggiano, D. Bazin, W. Benenson, B. Davids, R. Ibbotson, H. Scheit, B. M. Sherrill, M. Steiner, J. Yurkon, A. F. Zeller *et al.*, Phys. Rev. C **64**, 025802 (2001).
[8] Y. Togano, T. Gomi, T. Motobayashi, Y. Ando, N. Aoi, H. Baba, K. Demichi, Z. Elekes, N. Fukuda, Z. Fülöp *et al.*, Nucl. Phys. **A758**, 182c (2005).
[9] Y. Togano, T. Gomi, T. Motobayashi, Y. Ando, N. Aoi, H. Baba, K. Demichi, Z. Elekes, N. Fukuda, Z. Fülöp *et al.*, Proceedings of the International Symposium on Origin of Matter and Evolution of Galaxies, Tokyo, 2005.
[10] F. Meurders and A. V. D. Steld, Nucl. Phys. **A230**, 317 (1974).
[11] P. D. Kunz, Computer code DWUCK4 (unpublished).
[12] F. G. Perey, Phys. Rev. **131**, 745 (1963).
[13] F. D. Becchetti and G. W. Greenlees, Phys. Rev. **182**, 1190 (1969).
[14] C. M. Perey and F. G. Perey, At. Data Nucl. Data Tables **17**, 1 (1976).
[15] N. K. Timofeyuk, R. C. Johnson, and A. M. Mukhamedzhanov, Phys. Rev. Lett. **91**, 232501 (2003).
[16] L. Trache, A. Azhari, F. Carstoiu, H. L. Clark, C. A. Gagliardi, Y.-W. Lui, A. M. Mukhamedzhanov, X. Tang, N. Timofeyuk, and R. E. Tribble, Phys. Rev. C **67**, 062801(R) (2003).
[17] B. Guo, Z. H. Li, W. P. Liu, X. X. Bai, G. Lian, S. Q. Yan, B. X. Wang, S. Zeng, J. Su, and Y. Lu, Nucl. Phys. **A761**, 162 (2005).
[18] C. Angulo, M. Arnould, M. Rayet, P. Descouvemont, D. Baye, C. Leclercq-Willain, A. Coc, S. Barhoumi, P. Aguer, C. Rolfs *et al.*, Nucl. Phys. **A656**, 3 (1999).
[19] A. Bohr and B. R. Mottelson, *Nuclear Structure* (World Scientific, Singapore, 1988).
[20] L. Buchmann, J. M. D'Auria, and P. Mccorquodale, Astrophys. J. **324**, 953 (1988).
[21] G. Audi, A. H. Wapstra, and C. Thibault, Nucl. Phys. **A729**, 337 (2003).