Radius of ^{9}C from the Asymptotic Normalization Coefficient *

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The asymptotic normalization coefficient (ANC) of $^9C = {}^8B+p$ deduced from $^8Li(d,p)^9Li$ reaction is used to obtain the root-mean-square (rms) radius of the loosely bound proton in the ^9C ground state. We obtain $\langle r^2\rangle^{1/2} = 3.61$ fm for the valence proton, which is significantly larger than the matter radius of ^9C. The probability of the valence proton outside the matter radius of ^9C is greater than 60%. The present work supports the conclusion that ^9C has a proton halo structure.

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After the discovery of halo nuclei which exhibit an extended valence nucleon distribution surrounding a nuclear core, there has been a considerable effort to study the nature of halo nuclei and their reactions.[1] Over the past 20 years, a number of light neutron (or proton) rich nuclei closed to the drip-line have been identified as the halo nuclei, such as the one-neutron halo nuclei ^{11}Be[2] and ^{19}C,[3] the two-neutron halo nuclei ^{6}He,[4] ^{11}Li,[5] ^{14}Be,[6] and ^{17}B,[7] the proton halo nuclei ^{8}B,[8,9] ^{17}F,[10] and ^{17}Ne,[11] the neutron–proton halo for the second excited state of ^{6}Li,[12] and the one-neutron halo structure for the excited states of ^{12}B[13,14] and ^{13}C.[14] To date, various methods have been used to study this new type of nuclear structure. The radii of such nuclei, in particular, can be deduced from the measurement of total reaction cross section and of the angular distribution of elastic scattering. Recently, a method, ANC, developed for the indirect measurement of direct radiative capture reaction was applied to extracting the rms radius of the ^{8}B ground state,[9] and ^{12}B and ^{13}C excited states[14] successfully.

The proton drip line nucleus ^{9}C has a very small one-proton separation energy ($S_p = 1.298\,\mathrm{MeV}$) and the valence proton is expected to penetrate substantially beyond the range of the nuclear force. Different theoretical calculations suggested that ^{9}C may be proton halo nucleus.[15,16] Two experiments have been carried out by measuring the interaction cross sections of ^{9}C with different targets to establish its halo nature,[17,18] the results showed that the radii of ^{9}C is not anomalously large, and the conclusion of ^{9}C being a proton halo is still ambiguous. Therefore, it is necessary to study the halo nature of ^{9}C with an independent approach, and ANC is a practicable method to extract the rms radius of ^{9}C.

In the case of the ^{9}C ground state, the rms radius of the valence proton is defined approximately by the overlap function,

$$\langle r^2\rangle^{1/2} = \Big(\int_0^\infty r^4[I^B_{A,p}(r)]^2dr\Big)^{1/2}, \tag{1}$$

where $I^B_{Ap}(r)$ is the radial part of the overlap function for the ^{9}C ground state, the valence proton and the ^{8}B core. In the single-particle approach, the overlap function is approximated by the product of two factors, the spectroscopic factor $S^B_{A,p}$ for the virtual process $B \to A + p$ and the single particle radial wavefunction of the ground state,

$$I^B_{A,p}(r) = (S^B_{A,p})^{1/2}\Phi(r). \tag{2}$$

The single particle radial wavefunction $\Phi(r)$ can be calculated by solving the Schrödinger equation with the distorted-wave Born approximation (DWBA) and an optical potential model. Usually, the calculated $\Phi(r)$ is model-dependent and effected by the geometrical parameters of optical potential, namely, the radius parameter r_0 and the diffuseness a. At a large distance, $\Phi(r)$ has the asymptotic behaviour,

$$\Phi(r) = b^B_{A,p}\frac{W_{\eta_B,l_B+1/2}(2k_Br)}{r}, \tag{3}$$

where $b^B_{A,p}$ is called the single particle ANC, which relates to the nuclear ANC and the single-particle spectroscopic factor by

$$C^B_{A,p} = b^B_{A,p}(S^B_{A,p})^{1/2}. \tag{4}$$

Therefore, the rms radius of the valence proton can be separated into the contributions of interior and exterior region,

$$\langle r^2\rangle^{1/2} = \Big[S^B_{A,p}\int_0^{R_N} r^4\Phi^2(r)dr + (C^B_{A,p})^2$$

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$$\cdot \int_{R_N}^{\infty} r^2 W^2_{\eta,l+1/2}(2k_B r)dr\Big]^{1/2}. \qquad (5)$$

The second term in the equation is model-independent and denotes the asymptotic part, which gives major contribution to the rms radius. The first term is somehow model-dependent, but its contribution is relatively small due to the r^4 dependence in the integral function, the difference induced by various potential parameters can be regarded as uncertainty of the calculation result. Consequently, the radius of valence proton in ^{9}C can be extracted by the measurement of the ANC for $^9\mathrm{C} \to {}^8\mathrm{B} + n$.

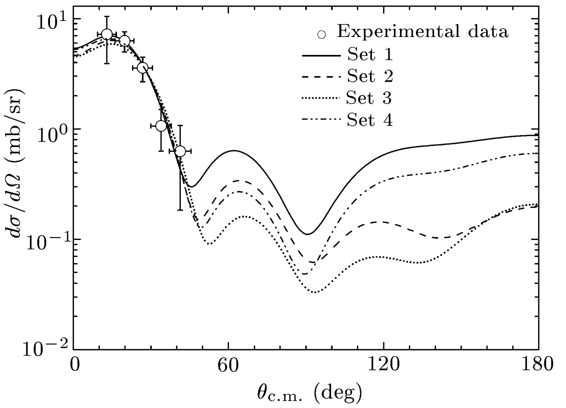

Fig. 1. Measured angular distribution of $^8\mathrm{Li}(d,p)^9\mathrm{Li}_{g.s.}$ at $E_{\mathrm{c.m.}} = 7.8\,\mathrm{MeV}$, together with the DWBA calculations for four sets of optical potential parameters.

Very recently, the relationship of the ANCs for the mirror systems has been established.[19,20] The ground states of ^{9}C and ^{9}Li are mirror nuclei, the valence proton (neutron) in ^{9}C (^{9}Li) is mostly in the $1p_{3/2}$ orbit.[21,22] The relation between the ANC for $^9\mathrm{C} \to {}^8\mathrm{B} + p$ and that for $^9\mathrm{Li} \to {}^8\mathrm{Li} + n$ can be expressed as

$$(C^{^9\mathrm{C}}_{^8\mathrm{B},p})^2 = (C^{^9\mathrm{Li}}_{^8\mathrm{Li},p})^2 (b^{^9\mathrm{C}}_{^8\mathrm{B},p})^2/(b^{^9\mathrm{Li}}_{^8\mathrm{Li},p})^2. \qquad (6)$$

The ratio of the proton and neutron single particle ANCs for ^{9}C and ^{9}Li is extracted to be $(b^{^9\mathrm{C}}_{^8\mathrm{B},p})^2/(b^{^9\mathrm{Li}}_{^8\mathrm{Li},p})^2 = 0.83$ from the single particle wavefunctions calculated with optical potential models. It should be noted that, in the calculations of the single particle wavefunctions, one must use the same r_0 and a, the same spin-orbit interaction for Woods-Saxon potentials, and the depths are adjusted to reproduce the neutron and proton binding energies of ^{9}Li and ^{9}C, respectively. Thus, the ANC for $^9\mathrm{C} \to {}^8\mathrm{B} + p$ can be deduced from the $^8\mathrm{Li}(d,p)^9\mathrm{Li}$ transfer reaction.

The measurement of $^8\mathrm{Li}(d,p)^9\mathrm{Li}$ angular distribution is performed using the secondary beam facility GIRAFFE[23,24] built at the HI-13 tandem accelerator of China Institute of Atomic Energy. The 44 MeV ^{7}Li primary beam from the tandem accelerator impinged on a deuterium gas cell at 1.6 atm pressure to produce the ^{8}Li ions through the $^2\mathrm{H}(^7\mathrm{Li},{}^8\mathrm{Li})^1\mathrm{H}$ reaction. The front and rear windows of the gas cell are 1.9-mg/cm^2-thick Havar foils. Using the magnetic separation and focusing with a dipole and a quadrupole doublet, the 39 MeV secondary ^{8}Li beam is delivered. After the collimation using two 3-mm apertures, the ^{8}Li beam is directed onto a deuterated polyethylene $(\mathrm{CD}_2)_n$ foil in thickness 1.5 mg/cm^2 to study the $^2\mathrm{H}(^8\mathrm{Li},\ {}^9\mathrm{Li})^1\mathrm{H}$ reaction. One can find the detail of the experiment in Refs. [25,26]. Here, we just give the angular distribution in the c.m. frame for the $^8\mathrm{Li}(d,p)^9\mathrm{Li}$ (ground state) reaction, as shown in Fig. 1.

The finite-range distorted wave Born approximation (DWBA) code PTOLEMY[27] is utilized to compute the angular distributions. All the optical potential parameters of the entrance channel are taken from Ref. [28], those of the exit channel from Refs. [28] and [29], respectively. If the reaction is dominated by a peripheral process, the differential cross section can be expressed as

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{exp}} = \frac{C_d^2}{b_d^2}\frac{(C^{^9\mathrm{Li}}_{^8\mathrm{Li},p})^2}{(b^{^9\mathrm{Li}}_{^8\mathrm{Li},p})^2}\sigma(\theta), \qquad (7)$$

where $\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{exp}}$ and $\sigma(\theta)$ denote the measured and DWBA differential cross sections, respectively. C_d^2 and b_d are the nuclear ANC and the single particle ANC for the virtual decay $d \to p + n$. In the DWBA calculations, C_d is taken to be $0.872\,\mathrm{fm}^{-1/2}$ from Ref. [30]. Figure 1 presents the normalized angular distributions for four sets of optical potential parameters, each curve corresponds to an ANC, $(C^{^9\mathrm{Li}}_{^8\mathrm{Li},p})^2$, their average value is $1.33 \pm 0.27\,\mathrm{fm}^{-1}$. $(C^{^9\mathrm{C}}_{^8\mathrm{B},p})^2$ is then derived to be $1.10 \pm 0.23\,\mathrm{fm}^{-1}$ by substituting the above values of $(C^{^9\mathrm{Li}}_{^8\mathrm{Li},p})^2$ and $(b^{^9\mathrm{C}}_{^8\mathrm{B},p})^2/(b^{^9\mathrm{Li}}_{^8\mathrm{Li},p})^2$ into Eq. (6). The error is mainly caused by statistics (16%) and the deviation from optical potentials used in the DWBA calculation (11%). This value is in agreement with the other experimental results[22,31] within the error bar. With this ANC, the rms radius of the valence proton in the ^{9}C ground state can be calculated from Eq. (5).

Table 1 lists the rms radius of the valence proton in The ^{9}C ground state calculated with typical Woods–Saxon potentials. The depths of the potential are adjusted to reproduce the proton binding energy, we vary the potential geometrical parameters r_0 and a of the potential on a grid of 81 points for $r_0 = 1.0$–1.4 fm and $a = 0.4$–0.8 fm with steps of 0.05 fm, respectively. The mean value of the calculated rms radius is 3.61 fm, with a standard deviation of only 0.03 fm, which can be negligible in comparison with the experimental ANC error. The maximum uncertainty arises from different potential parameters is less than 3%, this indicates the stability of the above method.

Table 1. The rms radius of the valence proton calculated with various potential parameters, r_0 = 1.0–1.4 fm and a = 0.4–0.8 fm in steps of 0.05 fm.

r_0	a (fm)								
(fm)	0.4	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80
1.00	3.52	3.53	3.55	3.58	3.58	3.59	3.60	3.61	3.61
1.05	3.53	3.55	3.57	3.58	3.59	3.60	3.61	3.61	3.61
1.10	3.55	3.57	3.58	3.59	3.60	3.61	3.61	3.62	3.62
1.15	3.57	3.58	3.59	3.60	3.61	3.62	3.62	3.62	3.62
1.20	3.59	3.60	3.61	3.61	3.62	3.62	3.62	3.62	3.62
1.25	3.60	3.61	3.62	3.62	3.63	3.63	3.63	3.63	3.62
1.30	3.62	3.62	3.63	3.63	3.63	3.63	3.63	3.63	3.62
1.35	3.63	3.63	3.64	3.64	3.64	3.64	3.63	3.63	3.62
1.40	3.64	3.64	3.64	3.64	3.64	3.64	3.64	3.63	3.62

The matter radius of ^{9}C can be computed by

$$r_m^2(A) = \frac{A-1}{A}\Big(r_c^2 + \frac{1}{A}r_p^2,\Big), \qquad (8)$$

where r_p and r_c are the rms radii of valence proton and the core nuclei, respectively. The ^{8}B rms radius extracted from reaction cross section measurements at high energy being 2.50 ± 0.04 fm,[32] the matter radius of ^{9}C is then deduced to be 2.62 ± 0.55 fm. This result is in agreement with those obtained from the measurement of interaction cross section (2.42 fm)[17] and the relativistic mean field calculation (2.45 fm).[16] The density distributions of the valence proton and ^{8}B core in ^{9}C are shown in Fig. 2, from which one can see that the density distribution of valence proton has a long tail expanding to the outside of the core nuclei. It is the typical behaviour of a halo nucleus.

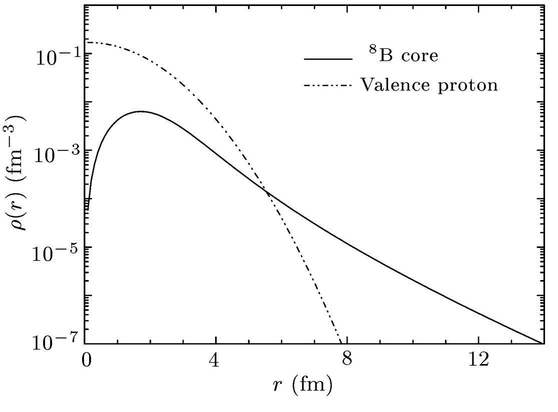

Fig. 2. The density distributions of valence proton and ^{8}B core in ^{9}C.

The contribution from the outer part of the wavefunction can be estimated by

$$D_\lambda(R_N) = \Big[\frac{\int_{R_N}^{\infty} r^{2\lambda}\Phi^2(r)dr}{\int_0^{\infty} r^{2\lambda}\Phi^2(r)dr}\Big]^{1/\lambda} \qquad (9)$$

with $\lambda = 1, 2$.[14] Here D_1 represents the probability of the valence proton outside the radius R_N, and D_2 gives the contribution of the asymptotic part to the rms radius of valence proton. For $R_N = 4$ fm, D_1 and D_2 are found to be 0.22 and 0.76, respectively. In the case of $R_N = 2.6$ fm, almost as the same as the rms radius of ^{9}C, the probability to find the valence proton outside the ^{9}C matter radius is 62%. According to the definition of proton halo,[9] we can thus conclude that the ground state of ^{9}C has a proton halo structure.

In summary, we have deduced the nuclear ANC of the ^{9}C ground state from the angular distribution of ^{8}Li$(d,p)^9$Li reaction, and then obtained the rms radius of the valence proton in ^{9}C to be 3.61 fm and the matter radius of ^{9}C to be 2.62 fm. The probability of the valence proton outside the matter radius of ^{9}C is greater than 60%, the asymptotic part of the wavefunction contributes about 76% to the rms radius of valence proton. The present result supports the conclusion that ^{9}C has a proton-halo structure.

References

[1] Jensen A S et al 2004 *Rev. Mod. Phys.* **76** 215
[2] Fukuda M et al 1991 *Phys. Lett.* B **268** 339
[3] Bazin D et al 1995 *Phys. Rev. Lett.* **74** 3569
[4] Jänecke J et al 1996 *Phys. Rev.* C **54** 1070
[5] Tanihata T et al 1985 *Phys. Rev. Lett.* **55** 2676
[6] Labiche M et al 2001 *Phys. Rev. Lett.* **86** 600
[7] Suzuki T et al 1999 *Nucl. Phys.* A **658** 313
[8] Guimarães V et al 2000 *Phys. Rev. Lett.* **84** 1862
[9] Carstoiu F et al 2001 *Phys. Rev.* C **63** 054310
[10] Lewis R and Hayes C A 1999 *Phys. Rev.* C **59** 1211
[11] Zhukov M V and Thompson I J 1995 *Phys. Rev.* C **52** 3505
[12] Li Z H et al 2002 *Phys. Lett.* B **527** 50
[13] Lin C J et al 2001 *Chin. Phys. Lett.* **18** 1183
[14] Liu Z H et al 2001 *Phys. Rev.* C **64** 034312
[15] Varga K, Suzuki Y and Tanihata I 1995 *Phys. Rev.* C **52** 3013
[16] Ren Z Z et al 1996 *Nucl. Phys.* A **605** 75
[17] Ozawa A et al 1996 *Nucl. Phys.* A **608** 63
[18] Wang Q J et al 2001 *High Energy Phys. Nucl. Phys.* **25** 1165 (in Chinese)
[19] Trache L et al 2003 *Phys. Rev.* C **67** 062801(R)
[20] Timofeyuk N K et al 2003 *Phys. Rev. Lett.* **91** 232501
[21] Mohr P 2003 *Phys. Rev.* C **67** 065802
[22] Beaumel D et al 2001 *Phys. Lett.* B **514** 226
[23] Bai X X et al 1995 *Nucl. Phys.* A **588** 273c
[24] Liu W P et al 2003 *Nucl. Instrum. Methods Phys. Res.* B **204** 62
[25] Li Z H et al 2005 *Phys. Rev.* C **71** 052801(R)
[26] Guo B et al 2005 *Nucl. Phys.* A (submitted)
[27] Macfarlane M H and Pieper S C *Argonne National Laboratory Report* (unpublished)
[28] Perey C M and Perey F G 1976 *At. Data Nucl. Data Tables* **17** 1
[29] Watson B A et al 1969 *Phys. Rev.* **182** 977
[30] Blokhintsev L D, Borbely I and Dolinskii E I 1977 *Sov. J. Part. Nucl.* **8** 485
[31] Trache L et al 2002 *Phys. Rev. C* **66** 035801
[32] Al-Khalili J S and Tostevin J A 1996 *Phys. Rev. Lett.* **76** 3903