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Determination of Astrophysical ^{13}N(p,γ)^{14}O S-factors from the Asymptotic Normalization Coefficient of ^{14}C $\to$ ^{13}C + n *

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The angular distribution of the $^{13}C(d,p)^{14}C$ reaction is reanalysed using the Johnson–Soper approach. The squared asymptotic normalization coefficient (ANC) of virtual decay $^{14}C \to {}^{13}C + n$ is then derived to be $21.4 \pm 5.0\,fm^{-1}$. The squared ANC and spectroscopic factor (SF) of $^{14}O \to {}^{13}N + p$ are extracted to be $30.4 \pm 7.1\,fm^{-1}$ and 1.94 ± 0.45, respectively. The astrophysical S-factors and reaction rates of $^{13}N(p,\gamma)^{14}O$ are determined from the ANC of $^{14}O \to {}^{13}N + p$ using the R-matrix approach.

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Nuclear capture reactions, such as (p,γ) and (α,γ), play an important role in evolution of stars. ^{13}N$(p,\gamma)^{14}$O is one of the key reactions which trigger the onset of the hot CNO cycle. The hot CNO cycle is

$$^{12}\mathrm{C}(p,\gamma)^{13}\mathrm{N}(p,\gamma)^{14}\mathrm{O}(\beta^+)^{14}\mathrm{N}(p,\gamma)^{15}\mathrm{O}$$
$$(\beta^+)^{15}\mathrm{N}(p,\alpha)^{12}\mathrm{C},$$

which will be a crucial source of stellar energy generation when the proton capture on ^{13}N becomes faster than its competing β decay.[1] Thus, the precise determination of the ^{13}N$(p,\gamma)^{14}$O reaction rates is vital for determining the conditions of the transition from the cold CNO cycle to the hot one.

In the ^{13}N$(p,\gamma)^{14}$O reaction, the s-wave capture on the broad 1^- resonance at $E_r = 527.9\,$keV dominates the reaction rate at the energies of astrophysical relevance. Great effort has been expended in recent years to determine the parameters for the resonance, including the direct measurements using the radioactive ^{13}N beam,[2,3] particle transfer reactions,[4–7] and Coulomb dissociation of high-energy ^{14}O beam in the field of a heavy nucleus.[8–10] As a result, the resonance parameters are well determined. The contribution of the direct capture is significantly smaller than that of the tail of the resonance within the Gamow window. Since both direct and resonant captures proceed via s-waves and then decay by E1 transitions, there is an interference between the two components. Thus the capture reaction within the Gamow window can be enhanced by constructive interference or reduced by destructive interference. The direct component of the cross section has been calculated by several groups, either separately or as part of the calculation of the total cross section.[1,11–13] Since there are significant differences between the various calculations, the determination of the ^{13}N(p,γ) ^{14}O direct capture component with an independent approach is greatly needed. A practicable method is to extract the direct capture cross section of the ^{13}N$(p,\gamma)^{14}$O reaction using the direct capture model[14] and the spectroscopic factor (SF) or asymptotic normalization coefficient (ANC), which can be deduced from one proton transfer reaction. In 1993, Decrock *et al.*[15] extracted the SF for ^{14}O $\to$ ^{13}N + p from the ^{13}N$(d,n)^{14}$O cross section. In 2004, Tang *et al.*[16] derived the ANC for ^{14}O $\to$ ^{13}N + p from the ^{14}N(^{13}N, ^{14}O)^{13}C angular distribution. Recently, we have extracted the SF and ANC of ^{14}O $\to$ ^{13}N + p from the ^{13}N$(d,n)^{14}$O$_{\rm g.s.}$ angular distribution.[17] The S-factors of the direct capture for ^{13}N$(p,\gamma)^{14}$O in Ref. [15] are 30% smaller than those in Refs. [16,17]. In order to determine the direct contribution, King *et al.*[18] proposed the direct measurement of ^{13}N$(p,\gamma)^{14}$O using a DRAGON spectrometer at the TRIUMF Laboratory of Canada.

In this Letter, we reanalyse the existing angular distribution of the ^{13}C$(d,p)^{14}$C$_{\rm g.s.}$ reaction[19] and then deduce the neutron ANC for ^{14}C $\to$ ^{13}C + n through Johnson–Soper adiabatic approximation.[20] The proton ANC and SF of ^{14}O $\to$ ^{13}N + p are then extracted from charge symmetry of mirror nuclei and utilized to obtain the ^{13}N$(p,\gamma)^{14}$O astrophysical S-factors and rates of the direct capture into the ground state of ^{14}O.

For a peripheral transfer reaction, the ANC can be extracted by comparison of the experimental angular distribution with theoretical calculation,

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm exp} = \sum_{j_i j_f} (C^d_{l_i j_i})^2 (C^{^{14}\rm C}_{l_f j_f})^2 R_{l_i j_i l_f j_f}, \qquad (1)$$

where

$$R_{l_i j_i l_f j_f} = \frac{\sigma^{th}_{l_i j_i l_f j_f}}{(b^d_{l_i j_i})^2 (b^{^{14}\rm C}_{l_f j_f})^2}, \qquad (2)$$

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$(d\sigma/d\Omega)_{\rm exp}$ and $\sigma^{th}_{l_ij_il_fj_f}$ are the measured and theoretical differential cross sections. $C^{^{14}\rm C}_{l_fj_f}$, $C^{d}_{l_ij_i}$ and $b^{^{14}\rm C}_{l_fj_f}$, $b^{d}_{l_ij_i}$ represent the nuclear and the corresponding single particle ANCs for virtual decays ${}^{14}{\rm C} \to {}^{13}{\rm C} + n$ and $d \to p+n$, respectively; l_i, j_i and l_f, j_f denote the orbital and total angular momenta of the transferred neutron in the initial and final nuclei d and ^{14}C, respectively. $R_{l_ij_il_fj_f}$ is model independent in the case of peripheral transfer reaction, therefore the extraction of ANC is insensitive to the geometry parameters (radius r_0 and diffuseness a) of bound state potential.

In the present calculation, the code PTOLEMY[21] is used to calculate the angular distributions. In order to include the breakup effects of deuteron in the entrance channel, the transfer cross sections are calculated within the Johnson–Soper adiabatic approximation to the neutron, proton, and target three-body system.[20] The adiabatic distorting potential governing the centre-of-mass motion of the deuteron is well described by the sum of the neutron- and proton-target optical potentials.[20] In the present calculation, the optical potentials of the nucleon target are taken from Refs. [22–24], respectively, as listed in Table 1. Each set of optical potential corresponds to one ANC. Figure 1 shows the normalized angular distributions of the ${}^{13}{\rm C}(d,p){}^{14}{\rm C}$ reaction with three sets of optical potential, together with the experimental data. The spins and parities of ^{13}C and ^{14}C (ground state) are $1/2^-$ and 0^+, respectively. Therefore, the ${}^{13}{\rm C}(d,p){}^{14}{\rm C}_{\rm g.s.}$ cross section only includes one contribution from the neutron transfer to $1p_{1/2}$ orbit in ^{14}C. C_d^2 is taken to be $0.76\,{\rm fm}^{-1}$ from Ref. [25]. The squared ANC for ${}^{14}{\rm C}_{\rm g.s.} \to {}^{13}{\rm C} + n$ is then extracted to be $21.4 \pm 5.0\,{\rm fm}^{-1}$ using the differential cross sections at four forward angles. The error results from the statistics (9%) and the deviation of optical potentials (22%).

Table 1. Optical potential parameters used in the calculation, where V and W are taken in units of MeV, r and a in fm, the squared ANC in fm^{-1}.

Set No.	1		2		3	
Channel	Entrance	Exit	Entrance	Exit	Entrance	Exit
V_r	105.0	58.2	94.7	51.3	92.9	49.7
r_{0r}	1.14	1.13	1.13	1.13	1.15	1.15
a_r	0.61	0.57	0.71	0.68	0.72	0.69
W_s	14.1	11.2	15.4	8.2	13.2	7.3
r_{0s}	1.14	1.14	1.31	1.31	1.15	1.15
a_s	0.54	0.5	0.56	0.53	0.69	0.69
V_{so}	5.5	5.5	5.5	5.4	5.9	5.9
r_{0so}	1.14	1.13	0.91	0.91	0.83	0.83
a_{so}	0.57	0.57	0.57	0.59	0.63	0.63
r_{0c}	1.14	1.13	1.5	1.5	1.29	1.29
ANC^2	16.3 ± 2.4		22.6 ± 3.4		25.2 ± 3.8	

To verify if the transfer reaction is peripheral, the ANCs for virtual decay ${}^{14}{\rm C} \to {}^{13}{\rm C} + n$ and the SFs are computed by changing the geometry parameters of the Woods–Saxon potential for single particle bound state, using one set of the optical potential parameters, as shown in Fig. 2. One can see that the SFs vary significantly, while the ANCs are nearly a constant, indicating that the ${}^{13}{\rm C}(d,p){}^{14}{\rm C}$ reaction is dominated by peripheral process at present energy.

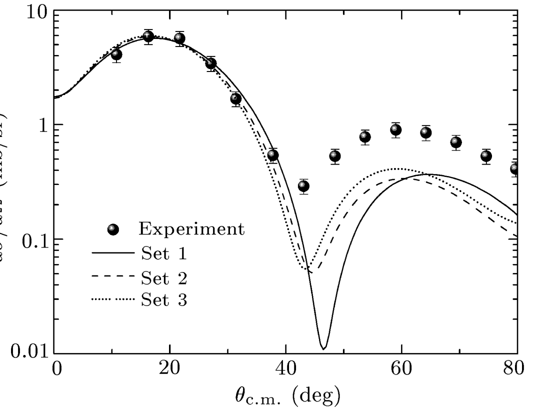

Fig. 1. Angular distributions of ${}^{13}{\rm C}(d,p){}^{14}{\rm C}_{\rm g.s.}$. The experimental data are from Ref. [19].

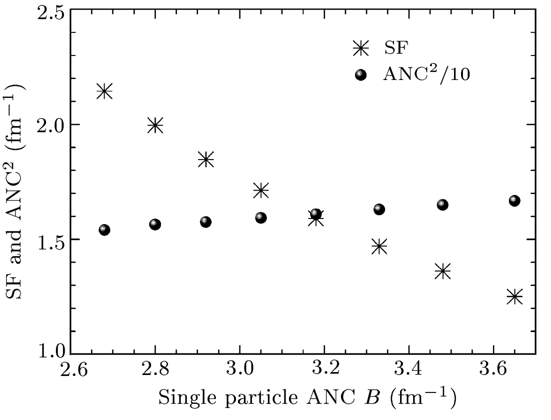

Fig. 2. Dependence of the spectroscopic factor (SF) and the squared ANC (ANC^2) on the single particle ANC (b).

Recently, the relationship of the ANCs for mirror pairs has been established[26] and utilized to study the ${}^{8}{\rm B}(p,\gamma){}^{9}{\rm C}$ and ${}^{26}{\rm Si}(p,\gamma){}^{27}{\rm P}$ reactions.[27,28] The ground states of ^{14}C and ^{14}O are a mirror pair, thus the proton ANC of ${}^{14}{\rm O} \to {}^{13}{\rm N} + p$ can be related to the neutron ANC of ${}^{14}{\rm C} \to {}^{13}{\rm C} + n$ by

$$(C^{^{14}\rm O}_{l_fj_f})^2 = R(C^{^{14}\rm C}_{l_fj_f})^2, \tag{3}$$

where R can be computed with[26]

$$R = \Big|\frac{F_l(ik_pR_N)}{k_pR_Nj_l(ik_nR_N)}\Big|^2, \tag{4}$$

where F_l and j_l are the regular Coulomb wave function and the spherical Bessel function, and R_N is the radius of nuclear interior. The wave number k_p (k_n) can be determined by the proton (neutron) separation energy ε_p (ε_n) via $k = (2\mu\varepsilon/\hbar^2)^{1/2}$. The ratio R

is nearly a constant (1.43) when R_N varies from 2.5 to 5.0 fm in the present calculation.

On the other hand, if we assume that the single particle spectroscopic factors S_p and S_n are equal for mirror pair, the ANC ratio R can also be obtained based on the relationship of the single particle spectroscopic factor and ANC, $C_{p(n)} = \sqrt{S_{p(n)}}b_{p(n)}$,

$$R = (b_{l_f j_f}^{^{14}\mathrm{O}})^2/(b_{l_f j_f}^{^{14}\mathrm{C}})^2. \tag{5}$$

The single particle ANCs $b_{l_f j_f}^{^{14}\mathrm{O}}$ and $b_{l_f j_f}^{^{14}\mathrm{C}}$ can be derived from the single particle wave functions calculated with the optical potential model. The ratio R is then extracted to be 1.41 from Eq. (5). It should be noted that one must use the same geometry parameters r_0 and a, the same spin-orbit interaction for the Woods–Saxon potentials, and the depth adjusted to reproduce the neutron or proton separation energy in ^{14}C or ^{14}O.

The average of the above two ratios is 1.42 ± 0.01, its error arises from the systematic deviation of both the ANC ratios. Combining Eqs. (3)–(5), the squared ANC and SF for $^{14}\mathrm{O} \to {}^{13}\mathrm{N} + p$ are found to be $30.4 \pm 7.1\,\mathrm{fm}^{-1}$ and 1.94 ± 0.45, respectively. The present results are by a factor of 2 larger than that (SF=0.9) in Ref. [15], while they are in good agreement with those ($\mathrm{ANC}^2 = 29.0 \pm 4.3\,\mathrm{fm}^{-1}$, $29.4 \pm 5.3\,\mathrm{fm}^{-1}$) in Refs. [16,17].

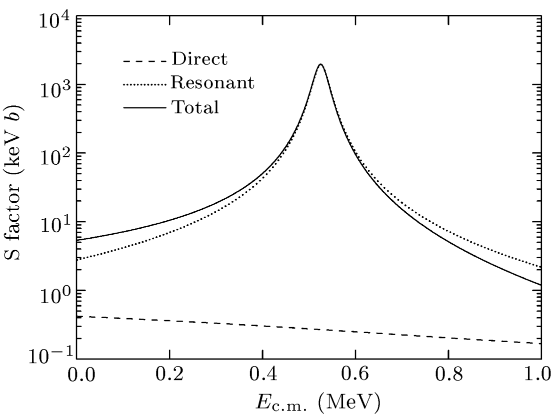

Fig. 3. Astrophysical S-factors as a function of $E_{\mathrm{c.m.}}$ for the $^{13}\mathrm{N}(p,\gamma)^{14}\mathrm{O}$ reaction.

The cross section for the direct and resonant captures in $^{13}\mathrm{N}(p,\gamma)^{14}\mathrm{O}$ can be calculated from the experimental ANC and resonance parameters using the R-matrix approach.[17,29] The astrophysical S-factor is related to the cross section by

$$S(E) = E\sigma(E)\exp(E_G/E)^{1/2}, \tag{6}$$

where the Gamow energy $E_G = 0.978 Z_1^2 Z_2^2\,\mu\,\mathrm{MeV}$, Z_1 and Z_2 are the charge numbers of proton and ^{13}N, μ is the reduced mass of the system. According to the experimental ANC ($30.4 \pm 7.1\,\mathrm{fm}^{-1}$) from the present work, and the resonance parameters ($E_R = 527.9 \pm 1.7\,\mathrm{keV}$, $\Gamma_{\mathrm{tot}}(E_R) = 37.3 \pm 0.9\,\mathrm{keV}$, and $\Gamma_\gamma(E_R) = 3.36 \pm 0.72\,\mathrm{eV}$) from Ref. [7], the S-factors for direct and resonant captures can then be derived, as demonstrated in Fig. 3.

Since the incoming angular momentum (s-wave) and the multipolarity ($E1$) of the direct and resonant capture γ-radiation are identical, there is an interference between the direct and the resonant captures. In this case, the total S-factor can be calculated by[30,31]

$$\begin{aligned} S_{\mathrm{tot}}(E) &= S_{dc}(E) + S_{\mathrm{res}}(E) \\ &\quad \pm 2[S_{dc}(E)S_{\mathrm{res}}(E)]^{1/2}\cos(\delta), \end{aligned} \tag{7}$$

where the resonance phase shift δ can be given by

$$\delta = \arctan\left[\frac{\Gamma_p(E)}{2(E-E_r)}\right]. \tag{8}$$

Generally, the sign of the interference in Eq. (7) has to be determined experimentally. However, it is possible sometimes to infer this sign. The interference between the resonant and direct capture contributions is constructive below the resonance energy and destructive above it, which has been observed from the isospin analogue $^{13}\mathrm{C}(p,\gamma)^{14}\mathrm{N}^*$ (2.31 MeV) reaction.[15] Recently, Tang *et al.*[16] deduced constructive interference below the resonance using an R-matrix method. Based on this interference pattern, the present total S-factor is then obtained, as shown in Fig. 3. One can see that the total S-factors at low energies are significantly enhanced due to constructive interference. Our updated total S-factors at low energies are about 40% higher than the previous ones in Ref. [15] and is in good agreement with those in Refs. [16,17].

Table 2. The present total reaction rate for $^{13}\mathrm{N}(p,\gamma)^{14}\mathrm{O}$, $N_A\langle\sigma v\rangle$ ($\mathrm{cm}^3\mathrm{mole}^{-1}\mathrm{s}^{-1}$), as a function of temperature, together with NACRE's compilation.

T_9	Present	NACRE	Present/NACRE
0.01	4.18×10^{-22}	2.01×10^{-22}	2.08
0.02	5.75×10^{-16}	2.78×10^{-16}	2.07
0.03	5.38×10^{-13}	2.63×10^{-13}	2.05
0.04	4.01×10^{-11}	1.99×10^{-11}	2.02
0.05	8.60×10^{-10}	4.34×10^{-10}	1.98
0.06	8.92×10^{-9}	4.58×10^{-9}	1.95
0.07	5.77×10^{-8}	3.02×10^{-8}	1.91
0.08	2.70×10^{-7}	1.44×10^{-7}	1.88
0.09	9.96×10^{-7}	5.43×10^{-7}	1.83
0.10	3.07×10^{-6}	1.71×10^{-6}	1.80
0.15	1.67×10^{-4}	1.03×10^{-4}	1.62
0.20	2.16×10^{-3}	1.45×10^{-3}	1.49
0.30	6.58×10^{-2}	4.66×10^{-2}	1.41
0.40	9.61×10^{-1}	7.65×10^{-1}	1.26
0.50	8.20×10^{0}	7.41×10^{0}	1.11
0.60	3.91×10^{1}	3.74×10^{1}	1.04
0.70	1.21×10^{2}	1.19×10^{2}	1.02
0.80	2.82×10^{2}	2.79×10^{2}	1.01
0.90	5.38×10^{2}	5.34×10^{2}	1.01
1.00	8.91×10^{2}	8.86×10^{2}	1.01

The astrophysical reaction rate of ^{13}N$(p,\gamma)^{14}$O is calculated with

$$N_A\langle\sigma v\rangle = N_A\Big(\frac{8}{\pi\mu}\Big)^{1/2}\frac{1}{(kT)^{3/2}}\int_0^\infty S(E) \times \exp\Big[-\Big(\frac{E_G}{E}\Big)^{1/2}-\frac{E}{kT}\Big]dE, \quad (9)$$

where N_A is the Avogadro constant. The updated rates are listed in Table 2, together with NACRE's compilation. The results from the two works agree with each other within a factor of 2 at low temperature of $T_9 < 0.1$ and are almost identical at higher temperature of $T_9 > 0.6$.

The present total reaction rates are fitted as

$$\begin{aligned} N_A\langle\sigma v\rangle = {} & \exp[79.7813 + 0.0558947T_9^{-1} \\ & - 19.4717T_9^{-1/3} - 105.969T_9^{1/3} \\ & + 97.0679T_9 - 46.0914T_9^{5/3} \\ & + 4.85749\ln(T_9)] + \exp[83.9653 \\ & - 0.329231T_9^{-1} - 20.1698T_9^{-1/3} \\ & - 105.974T_9^{1/3} + 71.8732T_9 \\ & - 22.7856T_9^{5/3} + 3.53815\ln(T_9)]. \end{aligned} \quad (10)$$

The fitting errors are less than 7% in the range from $T_9 = 0.01$ to $T_9 = 1$. The present rates can be used in the calculation of nuclear reaction network.

In summary, we have reanalysed the existing angular distribution of ^{13}C$(d,p)^{14}$C leading to the ground state of ^{14}C. The neutron ANC of $^{14}\mathrm{C_{g.s.}} \to {}^{13}\mathrm{C} + n$ is then extracted using the Johnson–Soper adiabatic approximation. Based on charge symmetry, the proton ANC and SF of $^{14}\mathrm{O_{g.s.}} \to {}^{13}\mathrm{N} + p$ are derived and utilized to calculate the astrophysical ^{13}N$(p,\gamma)^{14}$O S-factors and rates of the direct capture into the ground state of ^{14}O using the R-matrix method. This work supports the conclusion demonstrated in Ref. [16.17] and provides an independent examination to the existing results for the ^{13}N$(p,\gamma)^{14}$O reaction.

References

[1] Mathews G J and Dietrich F S 1984 *Astrophys. J.* **287** 969
[2] Decrock P et al 1991 *Phys. Rev. Lett.* **67** 808
[3] Delbar T et al 1993 *Phys. Rev.* C **48** 3088
[4] Chunpp T E et al 1985 *Phys. Rev.* C **31** 1023
[5] Fernandez P B et al 1989 *Phys. Rev.* C **40** 1887
[6] Smith M S et al 1993 *Phys. Rev.* C **47** 2740
[7] Magnus P V et al 1994 *Phys. Rev.* C **49** R1755
[8] Bauer G and Rebel H 1994 *J. Phys.* G **20** 1
[9] Motobayashi T et al 1991 *Phys. Lett.* B **624** 259
[10] Kiener J et al 1993 *Nucl. Phys.* A **552** 66
[11] Barker F C 1985 *Aust. J. Phys.* **38** 657
[12] Funck C and Langanke K 1987 *Nucl. Phys.* A **464** 90
[13] Descouvemont P and Baye D 1989 *Nucl. Phys.* A **500** 155
[14] Rolfs C 1973 *Nucl. Phys.* A **217** 29
[15] Decrock P et al 1993 *Phys. Rev.* C **48** 2057
[16] Tang X et al 2004 *Phys. Rev.* C **69** 055807
[17] Li Z H et al 2006 *Phys. Rev.* C **74** 035801
[18] King J et al 2006 *TRIUMF EEC Experiment Proposal* E805 (http://dragon.triumf.ca/experiments.html)
[19] Schiffer J P et al 1967 *Phys. Rev.* **164** 1274
[20] Wales G L and Johnson R C 1976 *Nucl. Phys.* A **274** 168
[21] Rhoades-Brown M, Macfarlane M H and Pieper S C 1980 *Phys. Rev.* C **21** 2417
Rhoades-Brown M, Macfarlane M H and Pieper S C 1980 *Phys. Rev.* C **21** 2436
[22] Watson B A, Singh P P and Segel R E 1969 *Phys. Rev.* **182** 977
[23] Koning A J and Delaroche J P 2003 *Nucl. Phys.* A **713** 231
[24] Varner R L et al 1991 *Phys. Rep.* **201** 57
[25] Blokhintsev L D, Borbely I and Dolinskii E I 1977 *Sov. J. Part. Nucl.* **8** 485
[26] Timofeyuk N K, Johnson R C and Mukhamedzhanov A M 2003 *Phys. Rev. Lett.* **91** 232501
[27] Guo B et al 2005 *Nucl. Phys.* A **761** 162
[28] Guo B et al 2006 *Phys. Rev.* C **73** 048801
[29] Barker F C and Kajino T 1991 *Aust. J. Phys.* **44** 369
[30] Rolfs C E and Rodney W S 1988 *Cauldrons in the Cosmos* (Chicago: The University of Chicago Press)
[31] Buchmann L, D'Auria J M and Mccorquodale P 1988 *Astrophys. J.* **324** 953