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Determination of astrophysical ^{11}C(p, γ)^{12}N reaction rate from the asymptotic normalization coefficients of ^{12}B $\rightarrow$ ^{11}B + n

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Abstract
The squares of the neutron asymptotic normalization coefficient (ANC) for the virtual decay ^{12}B $\rightarrow$ ^{11}B + n are extracted to be 1.20 $\pm$ 0.26, 0.354 $\pm$ 0.107 and 1.98 $\pm$ 0.35 fm^{-1} from the angular distributions of the ^{11}B(d, p)^{12}B reaction leading to the ground, first and second excited states of ^{12}B respectively, using the Johnson–Soper approach. According to charge symmetry of strong interaction, the square of proton ANC of virtual decay ^{12}N $\rightarrow$ ^{11}C + p is determined to be 1.63 $\pm$ 0.35 fm^{-1} and then utilized to calculate the astrophysical S-factor and the rate of the direct capture contribution in the ^{11}C(p, γ)^{12}N reaction. The astrophysical S-factor at zero energy for the direct capture, $S(0)$, is derived to be 0.088 $\pm$ 0.019 keV b. An evaluated $S(0)$ of 0.092 $\pm$ 0.009 keV b is then given by using the present and pre-existing experimental results. In addition, the proton widths of the first and second excited states of ^{12}N are derived to be 0.91 $\pm$ 0.29 and 99 $\pm$ 20 keV from the neutron ANCs of ^{12}B and used to compute the contribution from the first two resonances of ^{12}N, respectively. We have also calculated the contribution from the interference effect between the direct capture and the second resonance.

1. Introduction

Nuclear capture reactions, such as (p, γ) and (α, γ), play a crucial role in the evolution of stars. The reaction chains ^{7}Be(α, γ)^{11}C(p, γ)^{12}N(β^+)^{12}C and ^{7}Be(p, γ)^{8}B(α, p)^{11}C(p, γ)^{12}N(β^+)^{12}C are considered as two of the possible alternative paths to the 3α process for transforming the nuclei in the pp chains to the CNO nuclei in the peculiar astrophysical sites where the densities and temperatures are so high that the proton- and α-capture reactions become faster than the competing β-decays [1]. The ^{11}C(p, γ)^{12}N reaction may play an important role in the evolution of massive stars with very low metallicities, and thus has increasingly attracted both theoretical and experimental studies [1–8].

At the temperature of astrophysical relevance, the ^{11}C(p, γ)^{12}N reaction is believed to be dominated the direct capture into the ground state and the resonant captures via the first (2^+, $E_x = 0.960$ MeV) and second (2^-, $E_x = 1.191$ MeV) excited states of ^{12}N. In 1989, Wiescher *et al* extracted the gamma width Γ_γ of 2^+ state to be 2.59 meV from the lifetime of the first excited state of ^{12}B. Because no experimental data on the gamma width were available for the higher excited state, a Γ_γ of 2.0 meV for 2^- state was suggested according to the systematic rule of $E1$ matrix elements in light nuclei [1]. In 1990, Descouvement *et al* presented a Γ_γ of 140 meV for 2^- state based on a microscopic cluster model calculation [2]. In 1999, a repeated calculation yielded an updated value of 68 meV [4]. In 2003, Timofeyuk *et al* calculated the astrophysical S-factor at zero energy $S(0)$ for the direct capture to be 0.149 keV b using a microscopic approach. A separate estimation gave $S(0) = 0.111^{+0.025}_{-0.020}$ keV b based on the ratio of mirror ANCs [6]. On the experimental side, it is very difficult to directly measure this reaction at energies of astrophysical interest because of very small cross section and low intensity of the available ^{11}C beam at present. Some indirect methods have been applied to study this reaction [3, 5, 7, 8]. In 1995, Lefebvre *et al* derived $\Gamma_\gamma = 6.0^{+7.0}_{-3.5}$ meV for 2^- state through the Coulomb dissociation of ^{12}N on ^{208}Pb [3], while a similar measurement by Minemura *et al* yielded $\Gamma_\gamma = 13.0 \pm 0.5$ meV [5]. In 2003, Tang *et al* measured the ^{14}N(^{11}C,^{12}N)^{13}C angular distribution, derived the asymptotic normalization coefficient (ANC) of virtual decay $^{12}\mathrm{N} \to {}^{11}\mathrm{C} + \mathrm{p}$ to be 1.73 ± 0.25 fm^{-1}, and then calculated the direct capture S-factor and rate for the ^{11}C(p, γ)^{12}N reaction [7]. Recently, Liu *et al* extracted the ANC of 2.86 ± 0.91 fm^{-1} for $^{12}\mathrm{N} \to {}^{11}\mathrm{C} + \mathrm{p}$ via the measurement of the ^{2}H(^{11}C,^{12}N)n angular distribution, and then computed the S-factor and rate of the direct capture [8].

Here we reanalyse the existing angular distributions of the ^{11}B(d, p)^{12}B reaction [9–11] and then deduce the neutron ANCs for $^{12}\mathrm{B} \to {}^{11}\mathrm{B} + \mathrm{n}$ through Johnson–Soper adiabatic approximation [12]. The proton ANC of $^{12}\mathrm{N} \to {}^{11}\mathrm{C} + \mathrm{p}$ is then extracted from charge symmetry of mirror nuclei and utilized to obtain the ^{11}C(p, γ)^{12}N astrophysical S-factors and rates of the direct capture into the ground state of ^{12}N. We have also derived the proton widths from the neutron ANCs of ^{12}B and then computed the contribution of the captures via 2^+ and 2^- resonances, as well as the interference effect between the direct capture and the broad 2^- resonance.

2. The neutron ANCs of $^{12}\mathrm{B} \to {}^{11}\mathrm{B} + \mathrm{n}$

For a peripheral transfer reaction, the ANC can be extracted by comparison of the experimental angular distribution with theoretical calculation,

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\exp} = \sum_{j_i j_f} \left(C^{d}_{l_i j_i}\right)^2 \left(C^{^{12}\mathrm{B}}_{l_f j_f}\right)^2 R_{l_i j_i l_f j_f}, \tag{1}$$

where

$$R_{l_i j_i l_f j_f} = \frac{\sigma^{\mathrm{th}}_{l_i j_i l_f j_f}}{\left(b^{d}_{l_i j_i}\right)^2 \left(b^{^{12}\mathrm{B}}_{l_f j_f}\right)^2}, \tag{2}$$

$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\exp}$ and $\sigma^{\mathrm{th}}_{l_i j_i l_f j_f}$ are the measured and theoretical differential cross sections, respectively. $C^{^{12}\mathrm{B}}_{l_f j_f}$, $C^{d}_{l_i j_i}$ and $b^{^{12}\mathrm{B}}_{l_f j_f}$, $b^{d}_{l_i j_i}$ represent nuclear and corresponding single-particle ANCs for virtual decays $^{12}\mathrm{B} \to {}^{11}\mathrm{B} + \mathrm{n}$ and $\mathrm{d} \to \mathrm{p} + \mathrm{n}$, respectively. l_i, j_i and l_f, j_f denote the orbital and total angular momenta of the transferred neutron in initial and final nuclei d and ^{12}B, respectively. $R_{l_i j_i l_f j_f}$ is model independent in the case of peripheral transfer reaction; therefore,

Table 1. Optical potential parameters used in the calculation, where V and W are in MeV, r and a in fm.

Set No.	1		2		3	
Channel	Entrance	Exit	Entrance	Exit	Entrance	Exit
V_r	92.8	51.6	104.9	60.0	94.6	53.5
r_{0r}	1.15	1.15	1.14	1.14	1.12	1.12
a_r	0.72	0.69	0.61	0.57	0.71	0.68
W	2.65	1.29			1.97	1.04
r_w	1.72	1.14			1.72	1.12
a_w	0.42	0.69			0.42	0.68
W_s	13.2	7.9	13.9	8.14	15.4	8.49
r_{0s}	1.14	1.14	1.14	1.14	1.31	1.31
a_s	0.69	0.69	0.54	0.5	0.56	0.52
V_{so}	5.9	5.9	5.5	5.5	5.49	5.49
r_{0so}	0.8	0.8	1.14	1.14	0.89	0.89
a_{so}	0.63	0.63	0.57	0.57	0.57	0.59
r_{0c}	1.29	1.29	1.14	1.14	1.58	1.58

the extraction of ANC is insensitive to the geometry parameters (radius r_0 and diffuseness a) of the bound state potential.

In the present calculation, the code PTOLEMY [13] is used to calculate the angular distributions. In order to include the breakup effects of deuteron in the entrance channel, the transfer cross sections are calculated within the Johnson–Soper adiabatic approximation to the neutron, proton and target three-body system [12]. The adiabatic distorting potential governing the centre-of-mass motion of the deuteron is well described by the sum of the neutron- and proton-target optical potentials [12]. In the present calculation, the optical potentials of nucleon target are taken from [14–16], respectively. The optical potential parameters for ^{11}B(d, p)^{12}B$_{\mathrm{g.s.}}$ are listed in table 1. For entrance channel, the parameters of the excited states of ^{12}B are the same as those of the ground state. For exit channel, the parameters (V_r, W, W_s and V_{so}) of the excited states may slightly differ from those of the ground state, which depends on the energy of outgoing protons. Each set of an optical potential corresponds to one ANC. Figures 1, 2 and 3 show the normalized angular distributions of the ^{11}B(d, p)^{12}B reaction leading to the ground, first and second excited states of ^{12}B with three sets of optical potential parameters, together with the experimental data [9–11]. To verify if the transfer reaction is peripheral, the ANCs for virtual decay $^{12}\mathrm{B} \to {}^{11}\mathrm{B} + \mathrm{n}$ and the spectroscopic factors are computed by changing the geometry parameters of the Woods–Saxon potential for the single-particle bound state, using one set of the optical potential parameters, as shown in figure 4. One can see that the spectroscopic factors vary significantly, while the ANCs are nearly a constant, indicating that the ^{11}B(d, p)^{12}B reactions leading to the first three states of ^{12}B are all dominated by peripheral process at present energy.

The spins and parities of ^{11}B and ^{12}B (ground state) are $3/2^-$ and 1^+, respectively. Therefore, the ^{11}B(d, p)^{12}B$_{\mathrm{g.s.}}$ cross section includes two contributions from the neutron transfers to $\mathrm{p}_{3/2}$ and $\mathrm{p}_{1/2}$ orbits in ^{12}B. The ratio of $\mathrm{p}_{3/2} : \mathrm{p}_{1/2}$ $\left[\left(C^{^{12}\mathrm{B}}_{\mathrm{p}_{3/2}}\right)^2 / \left(C^{^{12}\mathrm{B}}_{\mathrm{p}_{1/2}}\right)^2\right]$ is respectively derived to be 0.165 ± 0.007 and 0.24 based on the shell model calculations [6, 7]. Very recently, Timofeyuk *et al* extracted this ratio to be 0.34 and 0.39 with two nucleon–nucleon potentials in a microscopic cluster model [17]. C_d^2 is taken to be 0.76 fm^{-1} from [18]. The ANCs for the virtual decay $^{12}\mathrm{B}_{\mathrm{g.s.}} \to {}^{11}\mathrm{B} + \mathrm{n}$ are then extracted using the differential cross sections at several forward angles ($\theta_{\mathrm{c.m.}} < 20^\circ$ and listed in table 2. For the total ANCs derived from these three

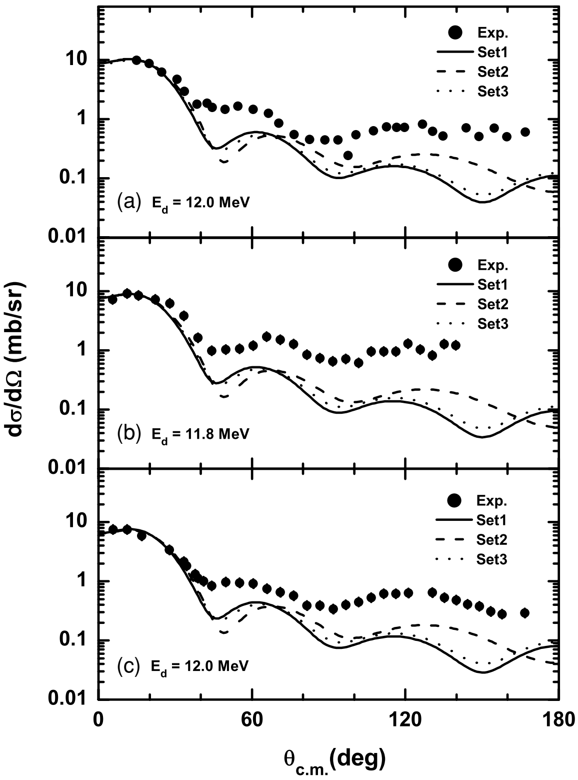

Figure 1. Angular distributions of $^{11}B(d, p)^{12}B_{g.s.}$. The experimental data are from [9–11], respectively.

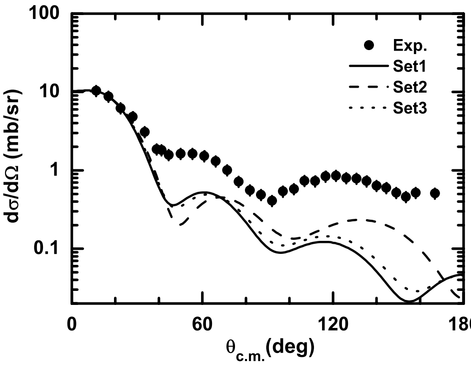

Figure 2. Angular distribution of $^{11}B(d, p)^{12}B$ leading to the first excited state.

experiments, the error is caused by the statistics, the deviation of optical potentials, as well as the uncertainty of $p_{3/2} : p_{1/2}$ (<1% for different values 0.165, 0.24, 0.34 and 0.39). $\left(C^{^{12}B}_{p_{3/2}}\right)^2$ and $\left(C^{^{12}B}_{p_{1/2}}\right)^2$ listed in tables 2 and 3 are obtained according to the average of 0.34 and 0.39 from [17]. In addition to above three uncertainties, the systematic deviation between three

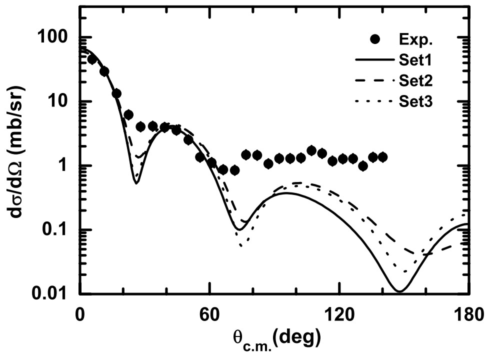

Figure 3. Angular distribution of $^{11}B(d, p)^{12}B$ leading to the second excited state.

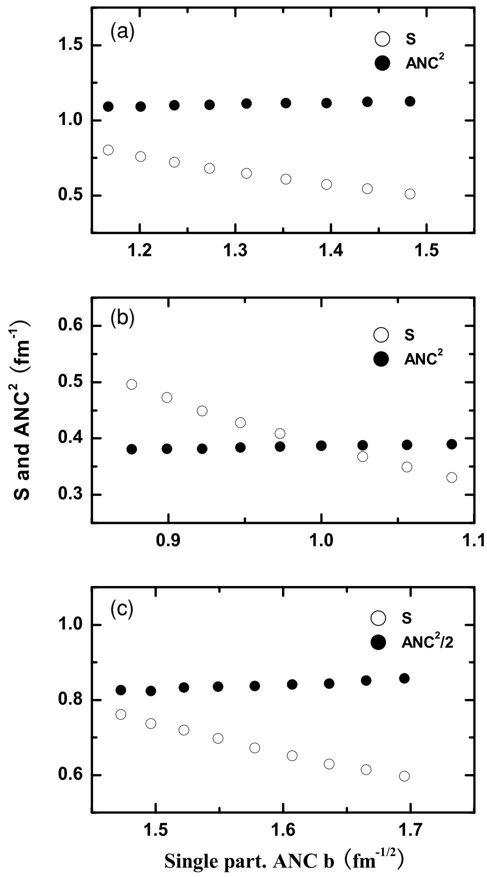

Figure 4. Dependence of the spectroscopic factor (S) and the square of ANC (C^2) on the single-particle ANC (b). (a)–(c) represent the ground, first and second excited states of ^{12}B.

experiments is included in total error of the averaged ANCs. Similarly, the neutron ANCs for the first and second excited states of ^{12}B are also deduced, as listed in table 3.

Table 2. The ANCs of the virtual decay $^{12}B_{g.s.} \rightarrow\ ^{11}B + n$ derived from three experiments [9–11].

Experiment	ANC^2 (fm^{-1})		
	$p_{1/2}$	$p_{3/2}$	Total
1	1.01 ± 0.20	0.369 ± 0.072	1.38 ± 0.27
2	0.890 ± 0.180	0.325 ± 0.066	1.22 ± 0.25
3	0.744 ± 0.190	0.271 ± 0.069	1.02 ± 0.26
Average	0.881 ± 0.189	0.322 ± 0.069	1.20 ± 0.26

Table 3. Summary of the ANCs for the ground, first and second excited states of ^{12}B. n, l and j are the number of nodes excluding the origin and infinity for the radial function, the orbital and total angular momenta of the transferred neutron in ^{12}B, respectively.

Reaction	E_x (MeV)	J^π	nlj	ANC^2 (fm^{-1})
	0	1^+	$0p_{3/2}$	0.322 ± 0.069
			$0p_{1/2}$	0.881 ± 0.189
$^{11}B(d, p)^{12}B$			Total	1.20 ± 0.26
	0.953	2^+	$0p_{1/2}$	0.354 ± 0.107
	1.674	2^-	$1s_{1/2}$	1.98 ± 0.35

3. The proton ANC and widths for ^{12}N

Recently, the relationship of the ANCs for mirror pairs has been established [19], and utilized to study the $^8B(p, \gamma)^9C$ and $^{26}Si(p, \gamma)^{27}P$ reactions [20, 21]. The ground states of ^{12}B and ^{12}N are mirror pair, and charge symmetry implies that the spectroscopic amplitudes for the proton single-particle orbitals entering the ^{12}N wavefunction are nearly the same as those of the neutron single-particle orbitals in ^{12}B. Although the weak matrix elements connecting the ground states of ^{12}B and ^{12}N to the ^{12}C ground state break mirror symmetry [22, 23], the calculations using a microscopic approach based on the solution of the inhomogeneous differential equation [6] and a microscopic cluster model [17] demonstrate that the spectroscopic factors for the two nuclei are very similar, with differences of 2–4%. Therefore, the proton ANC of $^{12}N \rightarrow {}^{11}C + p$ can be related to the neutron ANC of $^{12}B \rightarrow {}^{11}B + n$ through

$$\left(C^{^{12}\mathrm{N}}_{l_f j_f}\right)^2 = R\left(C^{^{12}\mathrm{B}}_{l_f j_f}\right)^2, \tag{3}$$

where R can be computed with [19]

$$R = \left|\frac{F_l(ik_pR_N)}{k_pR_N j_l(ik_nR_N)}\right|^2, \tag{4}$$

where F_l and j_l are the regular Coulomb wavefunction and the spherical Bessel function, and R_N is the radius of nuclear interior. The wave number k_p (k_n) can be determined by the proton (neutron) separation energy ε_p (ε_n) via $k = (2\mu\varepsilon/\hbar^2)^{1/2}$. The ratio R is nearly a constant (1.38) when R_N varies from 2.5 to 5.0 fm in the present calculation.

On the other hand, if we assume that the single-particle spectroscopic factors S_p and S_n are equal for mirror pair, the ANC ratio R can also be obtained based on the relationship of

Table 4. The parameters used in the calculation, the square of ANC for virtual decay $^{12}\mathrm{N} \to {}^{11}\mathrm{C} + \mathrm{p}$, and the astrophysical S-factor of the direct capture at zero energy. l represents the angular momentum of the incident wave.

Reaction	E_x (MeV)	J^π		R	ANC^2 (fm^{-1})	l	S_0 (keV b)
^{11}C(p, γ)^{12}N	0.0	1^+	1.38	1.33(2)	1.63 ± 0.35	0	0.088 ± 0.019

Table 5. The parameters used in the calculation, the proton widths for the first and second excited states of ^{12}N.

Reaction	E_x (MeV)	J^π	R^{res} (keV fm)		Γ_p (keV)		
^{11}C(p, γ)^{12}N	0.960	2^+	2.78	2.38(5)	0.91 ± 0.29	<20 [24]	5.5 [3]
	1.191	2^-	55.1 [25]	44.0(4) [25]	99 ± 20	118 ± 14 [24]	109 [26]

the single-particle spectroscopic factor and ANC, $C_{p(n)} = \sqrt{S_{p(n)}} b_{p(n)}$,

$$R = \frac{\left(b_{l_f j_f}^{^{12}\mathrm{N}}\right)^2}{\left(b_{l_f j_f}^{^{12}\mathrm{B}}\right)^2}. \tag{5}$$

The single-particle ANCs $b_{l_f j_f}^{^{12}\mathrm{N}}$ and $b_{l_f j_f}^{^{12}\mathrm{B}}$ can be derived from the single-particle wavefunctions calculated with the optical potential model. The ratio R is then extracted to be 1.33 ± 0.02 from equation (5). It should be noted that one must use the same geometry parameters r_0 and a, the same spin–orbit interaction for Woods–Saxon potentials, and the depth adjusted to reproduce the neutron or proton separation energy in ^{12}B or ^{12}N.

The average of above two ratios is 1.36 ± 0.03, its error arises from the systematic deviation of these two ANC ratios. Combining equations (3)–(5), the squared ANCs for $^{12}\mathrm{N} \to {}^{11}\mathrm{C} + \mathrm{p}$ are then found to be $\left(C_{\mathrm{p}_{1/2}}^{^{12}\mathrm{N}}\right)^2 = 1.19 \pm 0.26\ \mathrm{fm}^{-1}$, $\left(C_{\mathrm{p}_{3/2}}^{^{12}\mathrm{N}}\right)^2 = 0.436 \pm 0.094\ \mathrm{fm}^{-1}$, and the total ANC is $\left(C_{\mathrm{p_{eff}}}^{^{12}\mathrm{N}}\right)^2 = \left(C_{\mathrm{p}_{1/2}}^{^{12}\mathrm{N}}\right)^2 + \left(C_{\mathrm{p}_{3/2}}^{^{12}\mathrm{N}}\right)^2 = 1.63 \pm 0.35\ \mathrm{fm}^{-1}$. The error results from the uncertainties of the neutron ANC $\left(C_{l_f j_f}^{^{12}\mathrm{B}}\right)^2$ and the ANC ratio R. The results are listed in table 4.

The relationship between the width Γ_p of a narrow proton resonance and the neutron ANC of its mirror bound analogue has also been found [19]. The Γ_p of 2^+ resonance in ^{12}N can be expressed as

$$\Gamma_p = R^{\mathrm{res}} \left(C_{l_f j_f}^{^{12}\mathrm{B}}\right)^2, \tag{6}$$

where R^{res} is given by [19]

$$R^{\mathrm{res}} = \frac{\hbar^2 k_p}{\mu} \left| \frac{F_l(k_p R_N)}{k_p R_N j_l(i k_n R_N)} \right|^2. \tag{7}$$

Similarly, under the assumption of $S_p = S_n$ for mirror pair, R^{res} can also be approximated by

$$R^{\mathrm{res}} = \Gamma_p^{\mathrm{sp}} \Big/ \left(b_{l_f j_f}^{^{12}\mathrm{B}}\right)^2, \tag{8}$$

where Γ_p^{sp} denotes the single-particle width, which can be computed from the scattering phase shift in a Woods–Saxon potential with the depth determined by reproducing the resonance energy. The R^{res} are derived to be 2.78 and 2.38 ± 0.05 keV fm according to equations (7) and (8), respectively. The Γ_p for 2^+ resonance is then deduced, as listed in table 5. The present result is by order of magnitude smaller than the upper limit presented in [24] and by a factor of

6 smaller than that given in [3]. For the broad 2^- resonance, the R^{res} from more exact model calculations [25] are used to deduce the Γ_p together with the present neutron ANC, as listed in table 5. It can be seen that the present result is in agreement with those from [24] and most recent measurement of ^{11}C + p resonance scattering [26].

4. The astrophysical *S*-factor and reaction rate of ^{11}C(p, γ)^{12}N

At the temperatures of astrophysical interest, the direct capture for the ^{11}C(p, γ)^{12}N reaction is believed to be dominated by the $E1$ transition from incoming s wave to the bound p state. The direct capture cross section can be expressed as

$$\sigma_{\text{dc}} = \lambda \left| \left\langle I^{^{12}\text{N}}_{^{11}\text{C}p}(\vec{r}) | \hat{O}(\vec{r}) | \psi^{(+)}_{\vec{k}}(\vec{r}) \right\rangle \right|^2, \tag{9}$$

where λ is a kinematic factor, $I^{^{12}\text{N}}_{^{11}\text{C}p}(\vec{r})$ is the overlap function for $^{12}\text{N} \to {}^{11}\text{C} + \text{p}$, which includes radial and angular dependences. $\hat{O}(\vec{r})$ denotes the electromagnetic transition operator, and $\psi^{(+)}_{\vec{k}}(\vec{r})$ is the scattering wavefunction in the initial state. For a peripheral capture reaction at low energies, $\psi^{(+)}_{\vec{k}}(\vec{r})$ may be approximated by Coulomb wavefunction, and the radial overlap function $I^{^{12}\text{N}}_{^{11}\text{C}p}(r)$ has the asymptotic form,

$$I^{^{12}\text{N}}_{^{11}\text{C}p}(r) = C^{^{12}\text{N}}_{^{11}\text{C}p} W_{-\eta, l+1/2}(2k_B r)/r, \qquad r \geqslant R_N, \tag{10}$$

where $C^{^{12}\text{N}}_{^{11}\text{C}p}$ stands for the ANC of $^{12}\text{N} \to {}^{11}\text{C} + \text{p}$. R_N denotes the nuclear interaction radius of proton and ^{11}C, which is empirically taken to be $R_N = 1.25(A^{1/3} + 1)$ fm ≈ 4.0 fm in the present calculation. $W_{-\eta, l+1/2}(2k_B r)$ represents the Whittaker function, k_B the wave number for the bound state. Therefore, the direct capture cross section only depends on the square of the ANC.

The astrophysical *S*-factor is related to the cross section by

$$S(E) = E\sigma(E)\exp(E_G/E)^{1/2}, \tag{11}$$

where the Gamow energy $E_G = 0.978 Z_1^2 Z_2^2 \mu$ MeV, μ is the reduced mass of the system. According to the square of experimental ANC (1.63 ± 0.35 fm^{-1}), the energy dependence of the *S*-factor for the direct capture contribution in ^{11}C(p, γ)^{12}N is extracted, as shown in figure 5. The direct capture *S*-factor $S(0)$ at zero energy is found to be 0.088 ± 0.019 keV b. Figure 6 displays the comparison of the present $S(0)$ factor with other studies. One can see that the $S(0)$ factor in this work is in agreement with those from [6–8] within the uncertainties. The weighted average value of these five experimental results is 0.092 ± 0.009 keV b, as indicated by two dashed lines in figure 6.

The *S*-factors for the captures via the 2^+ and 2^- resonances are calculated with the Breit–Wigner formula for a single-level resonance,

$$S_{\text{res}}(E) = \pi \frac{(\hbar c)^2}{2\mu c^2} \frac{(2J+1)}{(2J_1+1)(2J_2+1)} \frac{\Gamma_p(E)\Gamma_\gamma(E)}{(E-E_R)^2 + (\Gamma_{\text{tot}}/2)^2} \exp\left(\frac{E_G}{E}\right)^{1/2}, \tag{12}$$

where E_R represents the resonance energy; J, J_1 and J_2 the spins of ^{12}N, proton and ^{11}C, respectively; Γ_p, Γ_γ and Γ_{tot} are the proton, gamma and total widths. The energy dependence of these widths is given by [27]

$$\Gamma_p(E) = \Gamma_p(E_R) \frac{\exp[-(E_G/E)^{1/2}]}{\exp[-(E_G/E_R)^{1/2}]}, \tag{13}$$

and

$$\Gamma_\gamma(E) = \Gamma_\gamma(E_R) \frac{(Q+E)^{2l+1}}{(Q+E_R)^{2l+1}}, \tag{14}$$

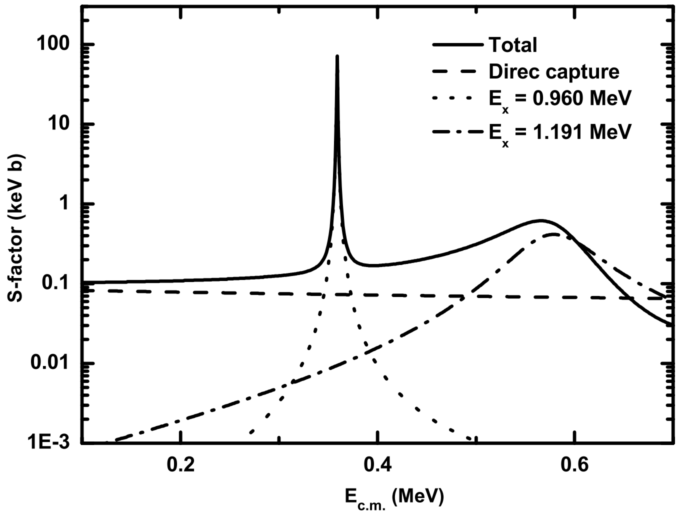

Figure 5. The astrophysical S-factor of the $^{11}C(p, \gamma)^{12}N$ reaction as a function of $E_{c.m.}$.

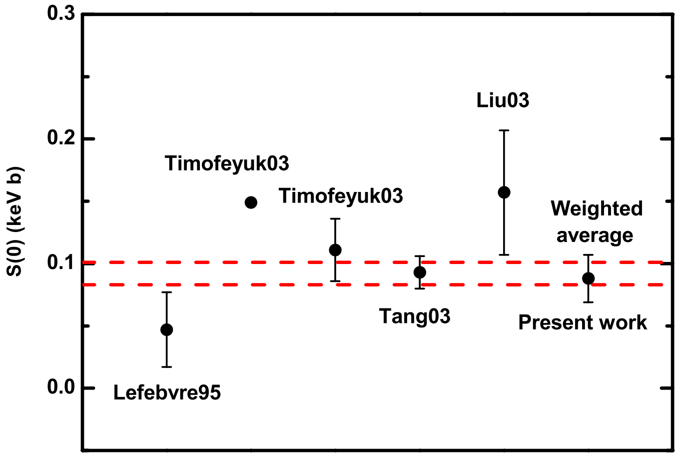

Figure 6. Direct capture S-factor at zero energy from theoretical and experimental works. The two dashed lines represent the weighted average value of the present and pre-existing experimental results.

where Q is the reaction Q value of $^{11}C(p, \gamma)^{12}N$, and l represents the multipolarity of the gamma transition. The present proton widths (0.91 ± 0.29 keV for 2^+ resonance, 99 ± 20 keV for 2^- resonance) are used in the present calculation, together with the corresponding gamma widths (2.59 meV for 2^+ resonance, 13.0 ± 0.5 meV for 2^- resonance) taken from [1, 5]. The S-factors of 2^+ and 2^- resonances are then derived, as demonstrated in figure 5. The present S-factor at resonant energy ($E_R = 0.359$ MeV) is by a factor of 6 larger than those in [7, 8] for 2^+ resonance, because the present Γ_p is six times less than that (5.5 keV) used in [7, 8] as mentioned above.

Since the incoming angular momentum (s wave) and the multipolarity ($E1$) of the direct capture and 2^- resonance γ-radiation are identical, they can interfere with each other. In this

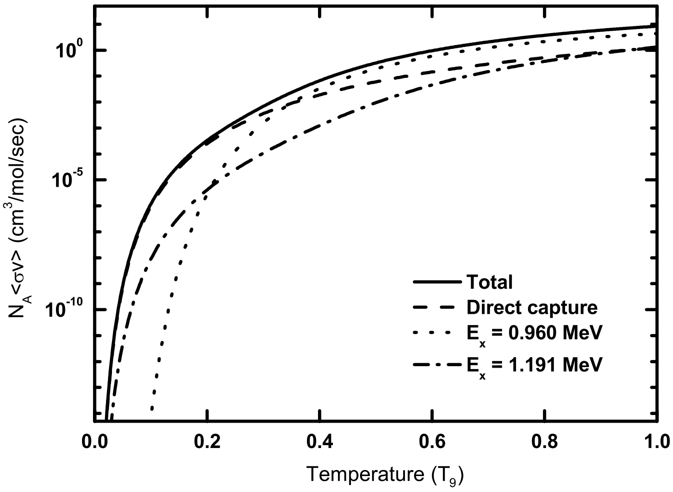

Figure 7. Temperature dependence of the $^{11}C(p, \gamma)^{12}N$ reaction rates, T_9 is the temperature in unit of 10^9 K.

case, the total S-factor is calculated with [27, 28]

$$S_{\mathrm{tot}}(E) = S_{\mathrm{dc}}(E) + S_{\mathrm{res}}(E) \pm 2[S_{\mathrm{dc}}(E)S_{\mathrm{res}}(E)]^{1/2}\cos(\delta), \tag{15}$$

where δ is the resonance phase shift, which can be given by

$$\delta = \arctan\left[\frac{\Gamma_p(E)}{2(E - E_R)}\right]. \tag{16}$$

Generally, the sign of the interference in equation (15) has to be determined experimentally. However, it is possible to infer this sign sometimes. Recently, Tang *et al* found constructive interference below the resonance and destructive one above it using an R-matrix method [7]. Based on this interference pattern, the present total S-factor is then obtained, as shown in figure 5.

The astrophysical $^{11}C(p, \gamma)^{12}N$ reaction rates for the direct capture, the broad 2^- resonance and their inference effect are calculated with

$$N_A\langle\sigma v\rangle_{\mathrm{nr}} = N_A\left(\frac{8}{\pi\mu}\right)^{1/2}\frac{1}{(kT)^{3/2}}\int_0^\infty S(E)\exp\left[-\left(\frac{E_G}{E}\right)^{1/2} - \frac{E}{kT}\right]\mathrm{d}E, \tag{17}$$

where N_A is Avogadro's number. For the first narrow resonance, the reaction rate can be approximated by

$$N_A\langle\sigma v\rangle_{\mathrm{res}} = N_A\left(\frac{2\pi}{\mu kT}\right)^{3/2}\hbar^2\omega\gamma\exp(-E_R/kT), \tag{18}$$

where the resonance strength $\omega\gamma$ is given by

$$\omega\gamma = \frac{2J+1}{(2J_1+1)(2J_2+1)}\frac{\Gamma_p\Gamma_\gamma}{\Gamma_{\mathrm{tot}}}. \tag{19}$$

Figure 7 shows the reaction rates as a function of temperature for the direct and resonant captures, as well as the interference effect. It can be seen that the direct capture accounts for almost total of reaction rate at temperatures relevant to CNO chain ($T_9 = 0.02$) and novae ($T_9 = 0.3$). It should be mentioned that the reaction rate for the capture via the narrow 2^+

Table 6. The present total rate of the $^{11}C(p, \gamma)^{12}N$ reaction, $N_A\langle\sigma v\rangle$ (cm^3 $mole^{-1}$ s^{-1}), as a function of temperature, together with the previous result.

T_9	Present work	REACLIB [1]	Present/REACLIB
0.01	9.40×10^{-21}	4.94×10^{-22}	19.0
0.02	2.85×10^{-15}	1.48×10^{-16}	19.3
0.03	1.26×10^{-12}	6.45×10^{-14}	19.6
0.04	5.82×10^{-11}	2.94×10^{-12}	19.8
0.05	8.80×10^{-10}	4.41×10^{-11}	19.9
0.06	6.95×10^{-9}	3.47×10^{-10}	20.0
0.07	3.61×10^{-8}	1.79×10^{-9}	20.1
0.08	1.40×10^{-7}	6.93×10^{-9}	20.2
0.09	4.39×10^{-7}	2.17×10^{-8}	20.3
0.10	1.18×10^{-6}	5.79×10^{-8}	20.3
0.15	3.75×10^{-5}	1.83×10^{-6}	20.5
0.2	3.34×10^{-4}	1.87×10^{-5}	17.9
0.3	6.86×10^{-3}	1.86×10^{-3}	3.70
0.4	6.67×10^{-2}	3.55×10^{-2}	1.88
0.5	3.22×10^{-1}	2.02×10^{-1}	1.59
0.6	9.54×10^{-1}	6.17×10^{-1}	1.55
0.7	2.08×10^{0}	1.32×10^{0}	1.58
0.8	3.74×10^{0}	2.28×10^{0}	1.64
0.9	5.88×10^{0}	3.42×10^{0}	1.72
1.0	8.42×10^{0}	4.66×10^{0}	1.81

resonance does not depend on the Γ_p of the 2^+ state in ^{12}N since $\Gamma_\gamma \ll \Gamma_p$, while the rate for the capture via the broad 2^- resonance will be reduced by 1–19% in the temperature range of $T_9 = 0.01$–1.0 with the present Γ_p (99 ± 20 keV) as compared with that (118 ± 14 keV) used in the previous calculations [7, 8]. The present total reaction rates are listed table 6, together with REACLIB's compilation [1]. The new rates are by order of magnitude more than those in [1] in the temperature range of $T_9 < 0.2$ because the present direct capture rates are much larger than those in [1].

The reaction rate for the narrow 2^+ resonance is fitted as

$$N_A\langle\sigma v\rangle_{\rm res} = \exp\big[5.628\,16 - 4.166\,16 T_9^{-1} - 0.004\,303\,43 T_9^{-1/3} - 0.016\,4188 T_9^{1/3} - 0.002\,375\,95 T_9 + 0.000\,340\,253 T_9^{5/3} - 1.505\,06 \ln(T_9)\big], \tag{20}$$

while the summed rate for direct capture, broad 2^- resonance and the interference between them is fitted as

$$N_A\langle\sigma v\rangle_{\rm nr} = \exp\big[-8.219\,80 + 0.033\,0198 T_9^{-1} - 20.2445 T_9^{-1/3} + 32.9930 T_9^{1/3} - 3.04015 T_9 - 0.272\,922 T_9^{5/3} - 9.911\,23 \ln(T_9)\big]. \tag{21}$$

The fitting errors are less than 5% over the range from $T_9 = 0.01$ to $T_9 = 1$. The present rate can be used in the calculation of nuclear reaction network.

5. Summary and conclusion

In summary, we have reanalysed the existing $^{11}B(d, p)^{12}B$ angular distributions leading to the ground, first and second excited states of ^{12}B. The neutron ANCs of the virtual decay $^{12}B \to {}^{11}B + n$ are then extracted using Johnson–Soper adiabatic approximation. Based on charge symmetry, the proton ANC of ^{12}N $\rightarrow$ ^{11}C + p is derived and utilized to calculate the astrophysical ^{11}C(p, γ)^{12}N S-factor and rate of the direct capture into the ground state of ^{12}N. The proton widths of the first and second excited states of ^{12}N are also derived from the neutron ANCs of the first two excited states in ^{12}B and employed to calculate the contribution of the captures via the corresponding resonances in ^{12}N. The contribution from the interference between the direct capture and the second resonance is also computed. Our result shows that the direct capture dominates the ^{11}C(p, γ)^{12}N reaction at the temperature range below $T_9 = 0.35$. This work supports the conclusion demonstrated in [7, 8] and provides an independent examination to the existing results for the ^{11}C(p, γ)^{12}N reaction.

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