Astrophysical Rates for ^{12}N$(p,\gamma)^{13}$O Direct Capture Reaction *

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The proton capture on the unstable nuclei plays a very important role in nucleonsynthesis. The $^{12}N(p,\gamma)^{13}O$ reaction rates at the energies of astrophysical interest are estimated with the spectroscopic factor and asymptotic normalization coefficient methods. The present results show that the $^{12}N(p,\gamma)^{13}O$ reaction may play an important role in x-ray bursts.

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Unstable nuclei play a very important role in the nucleonsynthesis. If there are not unstable nuclei but stable nuclei, most elements will have not been synthesized due to the lack of reaction routes. The proton- and α-capture reactions on light nuclei close to the proton drip line are very important in the evolution of super massive stars. The studies on the energy release and the explosion mechanism of the high-mass, low-metallicity stars (Population Ⅲ) are mainly dependent on their reaction rates. Therefore, the determination of their reaction rates is of great importance, particularly at higher burning temperatures.

The ^{12}N unstable nucleus, which is produced via the ^{9}C$(\alpha,p)^{12}$N and ^{11}C$(p,\gamma)^{12}$N reactions, may capture a proton and form ^{13}O via the ^{12}N$(p,\gamma)^{13}$O reaction, if the capture rate is sufficiently large as compared with the β^+-decay. The ^{12}N$(p,\gamma)^{13}$O reaction is believed to play a role in the nucleonsynthesis of Population Ⅲ stars.[1] Up to date, although the ^{12}N$(p,\gamma)^{13}$O reaction has been discussed by many researchers,[1-4] the uncertainty of the ^{12}N$(p,\gamma)^{13}$O reaction rates is still exist. Thus, it is necessary to determine the ^{12}N$(p,\gamma)^{13}$O reaction rates at the temperatures of astrophysical interest with an independent method.

The proton separation energy of ^{13}O is very low (1.51 MeV). The direct capture of ^{12}N$(p,\gamma)^{13}$O reaction is believed to be dominated by the $E1$ transition from incoming s-wave to bound p state, the contribution of d-wave is negligible at the energies of astrophysical interests. According to the traditional direct capture model,[5] the cross section of proton direct capture to the ground state of ^{13}O with the orbit and total angular momenta l_f and j_f can be expressed as

$$\sigma_{DC} = \frac{16\pi}{9}k_\gamma^3\frac{e_{\rm eff}^2}{k^2}\frac{1}{\hbar v}\frac{(2I_3+1)}{(2I_1+1)(2I_2+1)}S_{l_f,j_f} \times \Big|\int_0^\infty r^2\psi_{l_i}(kr)\phi_{l_f}(\kappa_B r)dr\Big|^2, \quad (1)$$

where $k_\gamma = \epsilon_\gamma/\hbar c$ is the wave number of the emitted γ-ray (of energy ϵ_γ); $e_{\rm eff} = eN/(A+1)$ is the proton effective charge for the E1 transition in the potential produced by a target nucleus with mass number A and neutron number N; v corresponds to the relative velocity between ^{12}N and proton; $k = \sqrt{2\mu E_{\rm cm}}/\hbar$ is the wave number of the incident proton; I_1, I_2, and I_3 are the spins of proton, ^{12}N, and ^{13}O, respectively; S_{l_f,j_f} is the spectroscopic factor of the configuration ^{13}O $\to$ ^{12}N + p; $\psi_{l_i}(r)$ is the optical model scattering wave function of the colliding proton and ^{12}N; and $\varphi_{l_f}(r)$ is the radial wave function of the bound state $p+^{12}$N in ^{13}O, which can be calculated by solving the respective Schrödinger equation with the optical potential model. If the spectroscopic factor S_{l_f,j_f} is known, the ^{12}N$(p,\gamma)^{13}$O cross section can then be calculated by Eq. (1).

The integrand of the E1 transition matrix element based on a single-particle model at 0.5 MeV is shown in Fig. 1, One can see that the contribution to the ^{12}N$(p,\gamma)^{13}$O direct capture reaction at small r is not significant. In this case, the ^{12}N$(p,\gamma)^{13}$O direct capture reaction is dominated by the peripheral process. The cross section of peripheral proton capture is not sensitive to the optical potential parameters and can be calculated with the asymptotic normalization coefficient (ANC) method and Eq. (1) can be replaced by

$$\sigma_{DC} = \frac{16\pi}{9}k_\gamma^3\frac{e_{\rm eff}^2}{k^2}\frac{1}{\hbar v}\frac{(2I_3+1)}{(2I_1+1)(2I_2+1)}C_{l_f,j_f}^2 \times \Big|\int_{R_N}^\infty r\psi_{l_i}(kr)W(2\kappa_B r)dr\Big|^2, \quad (2)$$

where C_{l_f,j_f}^2 is the squared proton ANC for ^{13}O ground state; $W(2\kappa_B r)$ is the Whittaker hypergeometric function; k_B is the bound state wave number for the last proton in ^{13}O; and R_N is the interaction radius between proton and ^{12}N, which can be calcu-

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lated by

$$R_N = 1.25(A^{1/3} + 1.0). \tag{3}$$

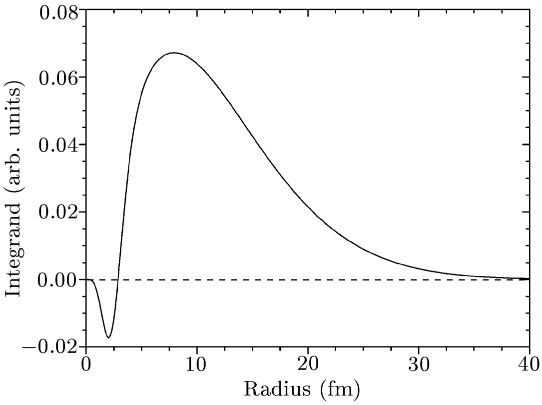

Fig. 1. Integrand of the E1 transition matrix element base on a single-particle model at 0.5 MeV.

The spectroscopic factor describes the overlap between the initial and final states in the reaction channels and yields important information about single-particle orbitals in many nuclei. It is also an important ingredient for calculation of direct transfer reaction cross sections in the distorted wave Born approximation (DWBA) and capture reaction cross sections in the direct capture (DC) model. Thus, great efforts have been expended in the studies of spectroscopic factors theoretically and experimentally.[6−9] The valence proton can occupy the $1p_{1/2}$ or $1p_{3/2}$ states in ^{13}O. The spectroscopic factors $S_{1,3/2} = 0.086$ and $S_{1,1/2} = 0.537$ are obtained, based on the calculations of the shell model code OXBASH.[10] These values are in good agreement with the recent experimental data of the one proton removal cross section.[11] The square of nuclear ANC is deduced to be $3.02\,\mathrm{fm}^{-1}$ based on the optical model with the standard geometry parameters. Figure 2 shows the cross section of the $^{12}\mathrm{N}(p,\gamma)^{13}\mathrm{O}$ direct capture reaction calculated with Eq. (1) and (2), respectively. The two results agree with each other within 2%, which demonstrates the identical result from the two methods in the calculations of the $^{12}\mathrm{N}(p,\gamma)^{13}\mathrm{O}$ direct capture reaction.

For the astrophysically important class of charged-particle-induced fusion reactions, the cross section of the fusion reaction drops nearly exponentially with decreasing energy due to the tunnelling effect through the Coulomb barrier. The astrophysical S-factor is often used to extrapolate the reaction data to lower energies in the Gamow window. The S-factor is defined as

$$S(E) = E\sigma(E)\exp(E_G/E)^{1/2}, \tag{4}$$

where the Gamow energy $E_G = 0.978 Z_1^2 Z_2^2\,\mu\,\mathrm{MeV}$, $Z_{1,2}$ denote the atomic number of ^{12}N and proton, μ is the reduced mass of the system. Figure 3 shows the S-factors for the $^{12}\mathrm{N}(p,\gamma)^{13}\mathrm{O}$ direct capture reaction as functions of energies with the above-mentioned two methods. The present S-factor at zero energy is 0.31 keV b, which is about two orders of magnitude less than the only theoretical one (4.0×10^{-2} MeV b) in Ref. [1]. Further experimental works are necessary to determine the cross section and S-factor of $^{12}\mathrm{N}(p,\gamma)^{13}\mathrm{O}$.

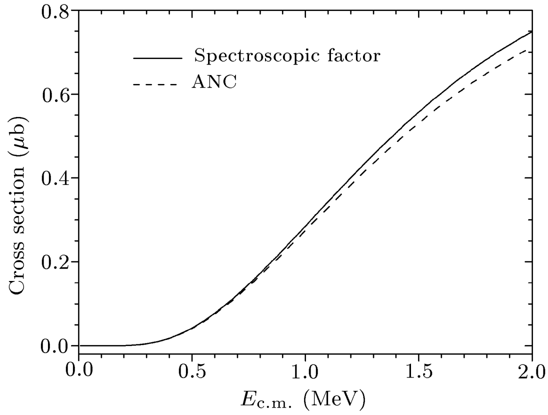

Fig. 2. Cross section of the $^{12}\mathrm{N}(p,\gamma)^{13}\mathrm{O}$ direct capture reaction calculated with Eqs. (1) and (2), respectively.

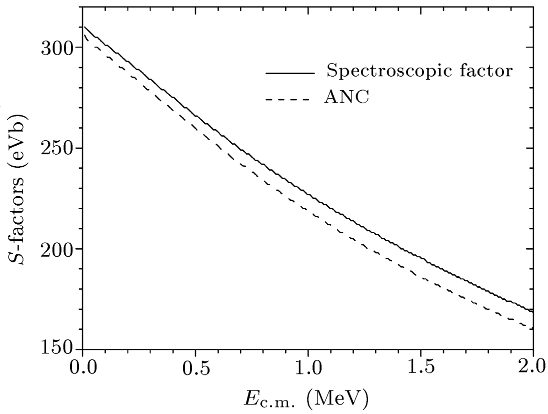

Fig. 3. Astrophysical S-factors for the $^{12}\mathrm{N}(p,\gamma)^{13}\mathrm{O}$ direct capture reaction.

The astrophysical reaction rate of the $^{12}\mathrm{N}(p,\gamma)^{13}\mathrm{O}$ direct capture can be calculated by

$$N_A\langle\sigma v\rangle = N_A\Big(\frac{8}{\pi\mu}\Big)^{1/2}\frac{1}{(kT)^{3/2}} \times\int_0^\infty S(E)\exp\Big[-\Big(\frac{E_G}{E}\Big)^{1/2}-\frac{E}{kT}\Big]dE, \tag{5}$$

where N_A is Avogadro's constant. The average value of the two S-factors in Fig. 3 is used in the present calculations.

Figure 4 shows the reaction rates as a function of temperature T_9 (in units of GK), the rates are fitted with an expression used in the astrophysical reaction rate library REACLIB,[13]

$$N_A\langle\sigma v\rangle = \exp[15.1767 + 0.0037T_9^{-1} - 15.5324T_9^{-1/3} + 0.4554T_9^{1/3} - 0.0318T_9 - 0.0115T_9^{5/3} - 0.9821\ln(T_9)]. \quad (6)$$

The overall fitting errors are less than 2% in the range from $T_9 = 0.01$ to $T_9 = 10$. The present reaction rates of ^{12}N$(p,\gamma)^{13}$O can be used in the nuclear reaction network calculations.

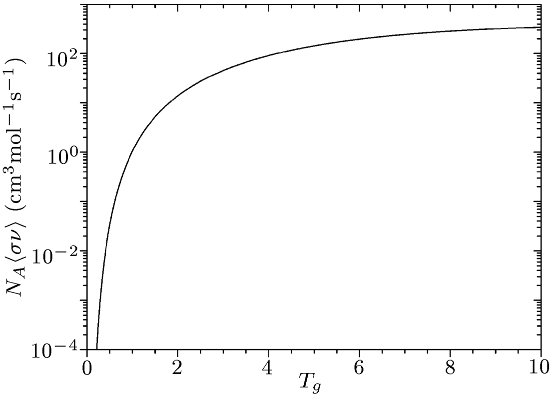

Fig. 4. Rates of the ^{12}N$(p,\gamma)^{13}$O direct capture reaction as a function of temperature.

The competition between the ^{12}N$(p,\gamma)^{13}$O direct capture reaction and ^{12}N β^+-decay depends on the density, temperature and mass fraction of proton in stars. The solid line in Fig. 5 shows the temperature and density boundary at which the ^{12}N$(p,\gamma)^{13}$O direct capture reaction and the competing β^+-decay are of equal strength assuming the hydrogen mass fraction of $X_H = 0.77$. The typical temperature and density conditions for x-ray bursts, novae, main sequence stars and the hydrogen shell of AGB-stars taken from Ref. [12] are also shown in this figure. In the region above the solid curve the proton capture reaction dominates, while below the solid line the ^{12}N nuclei are exhausted by the β^+-decay. Compared with the typical temperature and density conditions for x-ray bursts, novae, main sequence stars and the hydrogen shell of AGB-stars, one can find that the ^{12}N$(p,\gamma)^{13}$O direct capture reaction may play an important role in x-ray bursts.

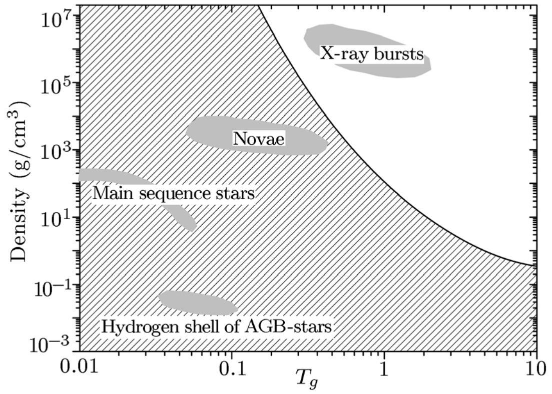

Fig. 5. Temperature and density boundary at which the ^{12}N$(p,\gamma)^{13}$O direct capture reaction and the competing β^+-decay are of equal strength assuming the hydrogen mass fraction of $X_H = 0.77$. The typical temperature and density conditions for x-ray bursts, novae, Main Sequence stars and the Hydrogen shell of AGB-stars are taken from Ref. [12].

In summary, using the spectroscopic factor and ANC of ^{13}O ground state, the astrophysical rates of the ^{12}N$(p,\gamma)^{13}$O direct capture reaction have been estimated at the energies of astrophysical interests. The present results show that the ^{12}N$(p,\gamma)^{13}$O reaction may play an important role in x-ray bursts.

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