Asymptotic Normalization Coefficient of ^{27}P $\rightarrow$ ^{26}Si + p and Radius of ^{27}P Halo*

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The asymptotic normalization coefficient of the virtual decay $^{27}P \rightarrow {}^{26}Si + p$ is extracted to be 1840 ± 240 fm^{-1} from the peripheral $^{26}Mg(d,p)^{27}Mg$ reaction using charge symmetry of mirror pair, for the first time. It is then used to derive the rms radius of the valence proton in the ground state of ^{27}P. We obtain the rms radius $\langle r^2\rangle^{1/2} = 4.57 \pm 0.36$ fm, significantly larger than the matter radius of ^{27}P. The probability of the valence proton outside the matter radius of ^{27}P is found to be 73%. The present work supports the conclusion that the ^{27}P ground state has a proton halo structure.

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A considerable effort has been made to investigate the exotic property of halo nuclei since this phenomenon was discovered by Tanihata *et al.*[1] To date, a number of light unstable nuclei close to the dripline have been identified as halo nuclei.[2] Various approaches were employed to study the structure of these exotic neutron-rich or proton-rich nuclei, which include the measurement of reaction cross sections, fragment momentum distribution, quadrupole moment and Coulomb dissociation. Recently, the asymptotic normalization coefficient (ANC) approach has successfully been utilized to extract the rms radii of ^{8}B ground state,[3,4] ^{11}Be, ^{12}B and ^{13}C excited states[4,5] and ^{9}C ground state.[2]

The proton drip line nucleus ^{27}P has a very small proton binding energy ($S_p = 0.861$ MeV),[6] of which the proton halo structure has been proposed within the framework of shell model,[7] relativistic mean field[8−10] and cluster-core model.[11] In 1998, Navin *et al.* measured the de-excitation γ-rays in coincidence with the momentum distribution of projectile residues and demonstrated a proton halo character in the ground states of ^{26,27,28}P.[12] Recently, Fang *et al.* measured the reaction cross sections (σ_R) for some proton rich nuclei ($N = 11 - 15$ isotones) on ^{12}C target;[13] Zhang *et al.* measured the reaction cross sections for 44 nuclei with $A < 30$ on ^{12}C target;[14] Liu *et al.* measured the reaction cross sections of ^{27,28}P on ^{28}Si target.[15] These three works all observe an abnormally large σ_R value for ^{27}P and show that ^{27}P is a halo nucleus.

In the present work, we reanalyse the existing angular distribution of ^{26}Mg$(d,p)^{27}$Mg$_{\rm g.s.}$[16] and derive the neutron ANC for the virtual decay ^{27}Mg $\rightarrow$ ^{26}Mg + n, based on distorted wave Born approximation (DWBA) analysis. The ANC for ^{27}P $\rightarrow$ ^{26}Si + p is then extracted using charge symmetry of mirror pair, and utilized to deduce the rms radius of the valence proton in ^{27}P ground state. The probability of the valence proton outside the matter radius of ^{27}P and the contribution of the asymptotic part of the wavefunction to rms radius have also been calculated.

The spins and parities of ^{26}Mg and ^{27}Mg (ground state) are 0^+ and $1/2^+$, respectively. The ^{26}Mg$(d,p)^{27}$Mg$_{g.s.}$ cross section only includes one contribution from neutron transfer to $2s_{1/2}$ orbit in ^{27}Mg.[16] In the case of peripheral transfer reaction, the ANC can be extracted by the normalization of DWBA calculation to the experimental differential cross section,

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm exp} = C_d^2 (C_{0,1/2}^{^{27}\rm Mg})^2 R_{0,1/2}, \tag{1}$$

where

$$R_{0,1/2} = \frac{\sigma_{DW}}{b_d^2 (b_{0,1/2}^{^{27}\rm Mg})^2}, \tag{2}$$

$\left(\frac{d\sigma}{d\Omega}\right)_{\rm exp}$ and σ_{DW} are the measured and DWBA differential cross sections, respectively; $C_{0,1/2}^{^{27}\rm Mg}$, C_d and $b_{0,1/2}^{^{27}\rm Mg}$, b_d represent nuclear and corresponding single particle ANCs for virtual decays ^{27}Mg $\rightarrow$ ^{26}Mg + n and $d \rightarrow p + n$, respectively. $R_{0,1/2}$ is model independent in the case of peripheral process, therefore the ANC is insensitive to the geometric parameters (radius r_0 and diffuseness a) of bound state potential.

In the present calculations, the zero-range distorted wave Born approximation (DWBA) code DWUCK4[17] is employed to calculate the angular distributions. The optical potential parameters of entrance channel are obtained by fitting the elastic scattering angular distribution,[16] those of exit channel are taken from Refs. [18–20], respectively, as listed in Table 1. To verify that the transfer reaction is peripheral, the ANCs for ^{27}Mg $\rightarrow$ ^{26}Mg + n and the spectroscopic factors are computed by changing the geometric parameters of Woods–Saxon potential for single particle bound state, using one set of the optical potential

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parameters, as shown in Fig. 1. One can see that the spectroscopic factors vary ±40% around the average, while the ANCs vary less than ±2%, indicating that the ^{26}Mg$(d,p)^{27}$Mg$_{g.s.}$ reaction at present energy is dominated by peripheral process. In the DWBA analysis, C_d^2 is taken to be 0.76 fm^{-1} from Ref. [21]. The experimental data are manually taken from the figure on the ^{26}Mg$(d,p)^{27}$Mg$_{g.s.}$ angular distribution in Ref. [16], as shown in Fig. 2, together with the normalized angular distributions for four sets of optical potential parameters listed in Table 1. Thus, a manual uncertainty of 5% is assigned in the present calculations. Each set of optical potential corresponds to one ANC, the average value of four neutron ANCs is derived to be 44.0 ± 5.3 fm^{-1}, its error results from the manual uncertainty (5%) and the deviation from different optical potentials (11%).

Table 1. Optical potential parameters used in the DWBA calculation, D represents those of entrance channel, $P1 - P4$ represent four sets of optical potential parameters of exit channel, where V and W are in MeV, R and a in fm.

Set	V_r	r_{0r}	a_r	W_s	r_{0s}	a_s	V_{so}	r_{0so}	a_{so}	r_{0c}
D	95.2	1.049	0.836	24.14	1.399	0.619	7.0	0.9	0.6	1.3
$P1$	49.14	1.25	0.65	9.0	1.25	0.47	7.5	1.25	0.65	1.25
$P2$	53.2	1.17	0.75	9.15	1.32	0.59	6.2	1.01	0.75	1.36
$P3$	49.14	1.174	0.736	8.06	1.19	0.562	5.29	1.06	0.546	1.17
$P4$	52.6	1.27	0.66	6.6	1.2	0.66	6.1	1.27	0.66	1.25

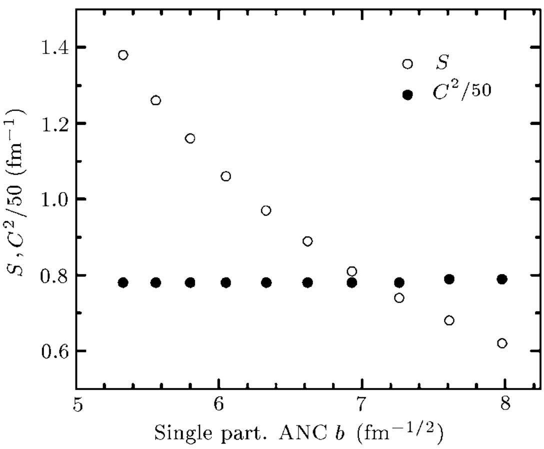

Fig. 1. Dependence of the spectroscopic factor (S) and the square of ANC (C^2) on the single particle ANC (b).

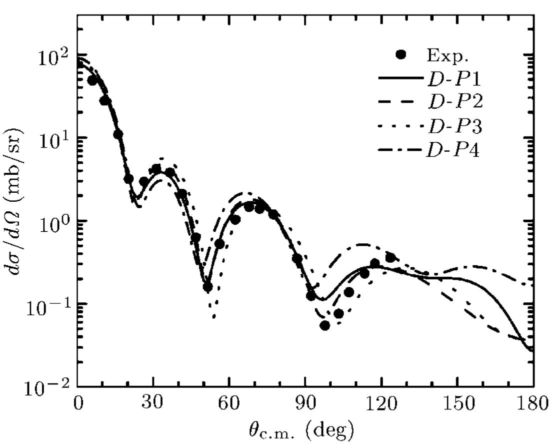

Fig. 2. Experimental angular distribution of ^{26}Mg$(d,p)^{27}$Mg$_{\rm g.s.}$ at $E_d = 12$ MeV, together with the DWBA calculations.

Recently, the relationship of the ANCs for mirror pairs has been established.[22,23] The ground states of ^{27}Mg and ^{27}P are mirror pair, the proton ANC of ^{27}P $\to$ ^{26}Si + p can be obtained through

$$(C_{0,1/2}^{^{27}\rm P})^2 = R(C_{0,1/2}^{^{27}\rm Mg})^2, \tag{3}$$

where R is the ratio between the squares of the proton and neutron ANCs, which can be computed with

$$R = \left|\frac{F_l(ik_pR_N)}{k_pR_Nj_l(ik_nR_N)}\right|^2, \tag{4}$$

where F_l and j_l are the regular Coulomb wavefunction and spherical Bessel function, respectively; R_N is the nuclear interaction radius between ^{26}Si and proton; k_p and k_n can be determined by the proton and neutron binding energies ε_p and ε_n via $k = (2\mu\varepsilon/\hbar^2)^{1/2}$. R is nearly a constant (44.2) when R_N varies from 2.5 to 5.0 fm.

On the other hand, if we assume that the single particle spectroscopic factors S_p and S_n are equal for mirror pair, the ratio R can also be obtained based on the relationship of the single particle spectroscopic factor and ANC, $C_{p(n)} = \sqrt{S_{p(n)}}b_{p(n)}$,

$$R = \frac{(b_{0,1/2}^{^{27}\rm P})^2}{(b_{0,1/2}^{^{27}\rm Mg})^2}. \tag{5}$$

This ratio of 39.5 is derived from the single particle wavefunctions calculated with the optical potential model. It should be noted that one must use the same radius r_0 and diffuseness a, the same spin–orbit interaction, and the depth adjusted to reproduce the neutron or proton binding energy in ^{27}Mg or ^{27}P.

Consequently, the average of the above two ratios (44.2 and 39.5) is derived to be 41.9 ± 2.4. Combining Eqs. (3)–(5), the square of the ANC for ^{27}P $\to$ ^{26}Si + p is then extracted to be 1840 ± 240 fm^{-1}, its error results from the uncertainties of the neutron ANC (12%) and the ANC^2 ratio R (6%).

The rms radius of the valence proton in $2s_{1/2}$ orbit is defined by the overlap integral for ^{27}P $\to$ ^{26}Si + p,

$$\langle r^2\rangle^{1/2} = \left(\int_0^\infty r^4[I_{^{26}{\rm Si},p}^{27{\rm P}}(r)]^2dr\right)^{1/2}. \tag{6}$$

The overlap integral inside the nucleus is difficult to compute since it covers the many-body functions. At large distances, $r > R_N$, it has asymptotic behaviour,

$$I^{^{27}\mathrm{P}}_{^{26}\mathrm{Si},p}(r) \to C^{^{27}\mathrm{P}}_{0,1/2}\frac{W(2k_B r)}{r}, \qquad (7)$$

where $W(2k_B r)$ is the Whittaker function, k_B is the wave number of the bound state. In the single particle approach, $I^{^{27}\mathrm{P}}_{^{26}\mathrm{Si},p}(r)$ can be approximated by the product of two factors, the single particle spectroscopic factor $S^{^{27}\mathrm{P}}_{^{26}\mathrm{Si},p}$ and the single particle wavefunction $\phi(r)$ calculated by solving the Schrödinger equation,

$$I^{^{27}\mathrm{P}}_{^{26}\mathrm{Si},p}(r) = (S^{^{27}\mathrm{P}}_{^{26}\mathrm{Si},p})^{1/2}\phi(r). \qquad (8)$$

Thus, the rms radius can be separated into the contributions of interior and asymptotic regions,

$$\langle r^2\rangle^{1/2} = \Big[S^{^{27}\mathrm{P}}_{^{26}\mathrm{Si},p}\int_0^{R_N} r^4\phi^2(r)dr + (C^{^{27}\mathrm{P}}_{0,1/2})^2 \cdot \int_{R_N}^{\infty} r^2 W^2(2k_B r)dr\Big]^{1/2}. \qquad (9)$$

The first term in this equation is somehow model dependent, while the second one describing the asymptotic part is model independent, which presents major contribution to the rms radius. Therefore, the uncertainty introduced by various parameters of the bound state potential is small, anyway it is still included into the total uncertainty in our calculation.

The rms radius of the valence proton has been computed with typical Woods-Saxon potential. The depth of the potential is adjusted to reproduce the proton binding energy. The geometric parameters, the radius r_0 and diffuseness a, are varied on a grid of 25 points for $r_0 = 1.15 - 1.35$ fm and $a = 0.55 - 0.75$ fm with a step of 0.05 fm. The results are listed in Table 2, the average value of them is extracted to be 4.57 ± 0.36 fm. The uncertainty results from the error of the proton ANC (6.6%) and the deviation of rms radii derived from different geometric parameters (4.4%). Our result is in agreement with the ones (4.46 fm, 4.47 fm, 4.19 fm and 4.26 fm) deduced from relativistic mean field calculations with different force parameters.[10] The ^{27}P matter radii of 3.22 ± 0.23 fm and 3.02 ± 0.16 fm are obtained from the measurement of reaction cross sections.[13,14] Thus, the rms radius of the valence proton is significantly larger than the ^{27}P matter radius, indicating that ^{27}P has a proton halo.

The contribution from the asymptotic part of the wavefunction can be calculated by

$$D_\lambda(R_N) = \Big[\frac{\int_{R_N}^{\infty} r^{2\lambda}\phi^2(r)dr}{\int_0^{\infty} r^{2\lambda}\phi^2(r)dr}\Big]^{1/\lambda}, \qquad (10)$$

where $\lambda = 1, 2$, D_1 denotes the probability of the valence proton outside the radius R_N; D_2 gives the contribution of the asymptotic part to the rms radius. For $R_N = 5$ fm, we find $D_{1(2)} = 0.26$ (0.75). Thus, the asymptotic part contributes 75%. In the case of $R_N = 3.2$ fm, almost as the same as the ^{27}P matter radius, D_1 is found to be 0.73, namely, the probability of the valence proton outside the ^{27}P matter radius is 73%. Based on the definition of proton halo[3], the ^{27}P ground state has a proton halo structure.

Table 2. The rms radius of the valence proton calculated with different geometric parameters of optical potential, $r_0 = 1.15 - 1.35$ fm, $a = 0.55 - 0.75$ fm with the step of 0.05 fm.

	a (fm)				
r_0 (fm)	0.55	0.60	0.65	0.70	0.75
1.15	4.10	4.21	4.32	4.39	4.56
1.20	4.21	4.32	4.44	4.56	4.69
1.25	4.32	4.44	4.55	4.68	4.81
1.30	4.44	4.56	4.68	4.81	4.95
1.35	4.57	4.69	4.82	4.96	5.09

In summary, we reanalyse the $^{26}\mathrm{Mg}(d,p)^{27}\mathrm{Mg}_{\mathrm{g.s.}}$ angular distribution and derive the neutron ANC of $^{27}\mathrm{Mg} \to {}^{26}\mathrm{Mg} + n$ via DWBA analysis. The ANC of $^{27}\mathrm{P} \to {}^{26}\mathrm{Si} + p$ is then extracted based on charge symmetry. We then compute rms radius of the valence proton, the probability of the valence proton outside the ^{27}P matter radius, and the contribution from the asymptotic part. Our result indicates that the ^{27}P ground state has a proton halo structure and presents an independent examination to the existing results.

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