AI Planning and Decision Making 2019
National University of Singapore CS4246/CS5446
Prof Lee Wee Sun

Homework 1

Please write the following on your homework:

- Name
- Collaborators (write none if no collaborators)
- Source, if you obtained the solution through research, e.g. through the web.

While you may collaborate, you *must write up the solution yourself*. While it is okay for the solution ideas to come from discussion, it is considered as plagiarism if the solution write-up is highly similar to your collaborator's write-up or to other sources.
You solution should be submitted to IVLE workbin. Scanned handwritten solutions are acceptable but must be legible.
Late Policy: A late penalty of 20% per day will be imposed for the written assignment (no submission accepted after 5 late days) unless prior permission is obtained. Late submissions will not be accepted for the programming assignment.

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1. **Classical Planning**

   (a) Consider the 8 puzzle with the Slide schema.

7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

Goal State

$$\begin{aligned}&Action(Slide(t, s_1, s_2),\\ &\quad \textsc{Precond:} On(t, s_1) \land Tile(t) \land Blank(s_2) \land Adjacent(s_1, s_2)\\ &\quad \textsc{Effect:} On(t, s_2) \land Blank(s_1) \land \neg On(t, s_1) \land \neg Blank(s_2))\end{aligned}$$

Consider (i) ignoring $Blank(s_2)$ in the precondition as a heuristic and (ii) ignoring $Blank(s_2) \land Adjacent(s_1, s_2)$ in the precondition as a heuristic. Which of (i) or (ii) will result in fewer nodes being explored when used with the A^* algorithm?

Solution: Both (i) and (ii) results in relaxation of the problem, so are admissible heuristics. As (ii) is a further relaxation of (i), the heuristic (i) dominates heuristic (ii), hence (i) will result in fewer nodes being expanded when used with A^*.

$Init(At(C_1, SFO) \wedge At(C_2, JFK) \wedge At(P_1, SFO) \wedge At(P_2, JFK)$
$\wedge\ Cargo(C_1) \wedge Cargo(C_2) \wedge Plane(P_1) \wedge Plane(P_2)$
$\wedge\ Airport(JFK) \wedge Airport(SFO))$
$Goal(At(C_1, JFK) \wedge At(C_2, SFO))$
$Action(Load(c, p, a),$
PRECOND: $At(c, a) \wedge At(p, a) \wedge Cargo(c) \wedge Plane(p) \wedge Airport(a)$
EFFECT: $\neg At(c, a) \wedge In(c, p))$
$Action(Unload(c, p, a),$
PRECOND: $In(c, p) \wedge At(p, a) \wedge Cargo(c) \wedge Plane(p) \wedge Airport(a)$
EFFECT: $At(c, a) \wedge \neg In(c, p))$
$Action(Fly(p, from, to),$
PRECOND: $At(p, from) \wedge Plane(p) \wedge Airport(from) \wedge Airport(to)$
EFFECT: $\neg At(p, from) \wedge At(p, to))$

(b) Consider the Air Cargo problem. Describe how to modify the problem so that each plane can only carry one cargo.

Solution: Introduce a fluent $Full(p)$. Add $Full(p)$ to the add list of $Load(c, p, c)$ and to the delete list of $Unload(c, p, a)$.

(c) In the Air Cargo problem, write the successor state axiom for the fluent $At(P_1, SFO)$.

Solution: $At(P_1, SFO)^{t+1} \Leftrightarrow Fly(P_1, JFK, SFO)^t \vee (At(P_1, SFO)^t \wedge \neg Fly(P_1, SFO, JFK)^t)$.

2. **Decision Theory**

(a) Bob is risk adverse but rational. His utilities for A, B, and C are $U(A) = 0$, $U(B) = 100$ and $U(C) = 40$. He is given a choice between C and a lottery $[0.4, A; 0.6, B]$. Which would he choose and why?

Solution: Bob is rational, hence would maximize his expected utility. He would choose the lottery which has expected utility of 60 instead of C which has utility of 40.

(b) Alice's utility function for money is $U(x) = x^2$. Argue that Alice is risk seeking. (Hint: $U(x)$ is a strictly convex function. Jensen's inequality may be useful here.)

Solution: Jensen's inequality states that for a strictly convex function $U(x)$, $pU(x_1) + (1-p)U(x_2) > U(px_1 + (1-p)x_2)$ for $p \in (0, 1)$. Hence, Alice will always prefer a lottery $[p, x_1; (1-p), x_2]$ to the expected monetary value of $px_1, +(1-p)x_2$.

(c) Cathy prefers A to B but prefers lottery $C = [0.2, A; 0.8, B]$ to lottery $D = [0.3, A; 0.7, B]$. Argue that there is no utility function that satisfies Cathy's preferences.

Solution: Cathy prefers A to B, so $U(A) > U(B)$. Cathy also prefers C to D, so

$$\begin{aligned} & 0.2U(A) + 0.8U(B) > 0.3U(A) + 0.7U(B) \\ \Leftrightarrow\ & 0.1U(A) < 0.1U(B) \\ \Leftrightarrow\ & U(A) < U(B) \end{aligned}$$

giving a contradiction.