# Texural Features Corresponding to Texural Properties

MOSES AMADASUN, MEMBER IEEE, AND ROBERT KING

*Abstract* —**Textural features corresponding to visual properties of texture are highly desirable for two main reasons. They will not only be optimum in terms of feature selection, but will also be applicable to all kinds of textures. Five properties of texture, namely: coarseness, contrast, busyness, complexity and texture strength, were given conceptual definitions in terms of spatial changes in intensity. These conceptual definitions were then approximated in computational forms. In comparison with human perceptual measurements, the computational measures had good correspondences in the rank ordering of ten natural textures. The extent to which the measures approximate visual perception was investigated in the form of texture similarity measurements. The results obtained were also encouraging, though not as good as in the ranking ordering of the textures. The differences may be due to the complex mechanism of human usage of multiple cues. Improved classification results were obtained using the developed features compared with two existing texture analysis techniques.**

## I. Introduction

Texture is an important item of information that humans use in analyzing a scene. It is particularly useful in the analysis of natural environments, as most natural scenes consist of textured surfaces. Literally, texture refers to the arrangement of the basic constituents of a material. In a digital image, texture is depicted by spatial interrelationships between, and/or spatial arrangement of the image pixels. Visually, these spatial interrelationships, or arrangement of image pixels, are seen as changes in the intensity patterns, or gray tones. Thus in automatic analysis, information about texture has to be derived from the gray tones of the image pixels.

A number of texture analysis methods have been proposed, some of which [1]–[6] are frequently referred to in the correspondence. Haralick [7] categorized the various proposals into three groups: the statistical techniques, the structural methods, and the statistical–structural approaches. A major disadvantage of almost all of these approaches is that they do not have general applicability—they cannot be applied to different classes of textures with reasonable success. For instance while the statistical techniques are generally good for microtextures and are poor performers on macrotextures, the reverse is the case for the structural techniques. Another disadvantage of some of the existing methods is the computational cost involved, either in terms of memory requirement, computation time or implementational complexity.

The human perception mechanism, in comparison, seems to work well for almost all types of textures. The properties that humans use to discriminate between different textural patterns include coarseness, contast, complexity, busyness (fineness), shape, directionality, and texture strength. Therefore for general applicability of developed texture measures, and also for improved performance in automatic texture classification, it is relevant that measures reflect or represent to some extent some of the aforementioned textural properties. Tamura *et al.* [8] did some work in this direction, but used already developed features, only modifying a given feature or combining some features in one way or another to have a close relationship to a specific property. However the extraction of their features may be computationally expensive, as diverse analysis techniques are involved in their derivation.

Some other investigations carried out in the study of human perception of textures are reported by Julesz [9]–[11]. However in these investigations, the aim was not the development of texture measures, but rather the study of the extent to which one can just perceive differences in artificially generated textures when all familiar cues are removed. He concluded that the discrimination of textures depends mostly on the difference in second-order statistics.

In the present approach, an attempt is made to develop completely new computational measures corresponding to some textural properties, so as to ensure general applicability, while at the same time minimize the cost of computation. Five perceptual attributes of texture, namely: coarseness, contrast, busyness, complexity, and strength of texture, were approximated in computational forms. The computational form for each property was derived by expressing a perceptual description of the property in terms of spatial changes in intensity and/or dynamic range of intensity, as the degree to which a given texture possesses any particular property is considerably dependent on these two factors. In a digital image, information about spatial changes in intensity can be obtained by looking at the difference between the gray tone of each image pixel and the gray tones of its surrounding neighbors. Therefore central to the development of the reported features is the computation of a one-dimensional (1-D) matrix for an image, in which the $i$th entry is a summation of the differences between the gray level of all pixels with gray level $i$, and the average gray level of their surrounding neighbors. The computational measures are derived from this matrix.

A description of the matrix, which shall be referred to as the neighborhood gray-tone difference matrix (NGTDM) follows in Section II. In Section III, a discussion of the five textural properties, together with their conceptualized relationships to changes in gray tones, and their computational expressions derived from the matrix, is presented. The approximations of the texture measures to textural properties are experimentally evaluated in Section IV, while in Section V the features are employed in agricultural land-use classification. A discussion of the features in relation to the size of neighborhood used in their computation is given in Section VI, while the Conclusion follows in Section VII.

## II. Neighborhood Gray-Tone Difference Matrix (NGTDM)

This is a column matrix formed as follows.

Let $f(k,l)$ be the gray tone of any pixel at $(k,l)$ having gray tone value $i$. Then we find the average gray-tone over a neighborhood centered at, but excluding $(k,l)$

$$\bar{A}_i = \bar{A}(k,l) = \frac{1}{W-1}\left[\sum_{m=-d}^{d}\sum_{n=-d}^{d} f(k+m,l+n)\right] \qquad (m,n)\neq(0,0), \quad (1)$$

where $d$ specifies the neighborhood size and $W=(2d+1)^2$.

Then the $i$th entry in the NGTDM is

$$s(i) = \begin{cases} \sum\left|i-\bar{A}_i\right|, & \text{for } i\in N_i \text{ if } N_i\neq 0,\\ 0, & \text{otherwise} \end{cases} \qquad (2)$$

where $\{N_i\}$ is the set of all pixels having gray tone $i$ (except in the peripheral regions of width $d$).

### A. Illustration

Consider the $5\times 5$ sample image shown in Fig. 1(a). Specifying a distance, $d=1$, results in a $3\times 3$ neighborhood. This neighborhood can only be centered on pixels within the indicated square.

Manuscript received September 28, 1987; revised April 13, 1989.

The authors are with the Electrical Engineering Department, Imperial College, Exhibition Road, London, SW7 2BT, England.

IEEE Log Number 8929126.

0018-9472/89/0900-1264$01.00 

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 4 | 3 | 1 |
| 3 | 4 | 0 | 1 | 1 |
| 5 | 4 | 2 | 2 | 2 |
| 2 | 1 | 1 | 4 | 4 |
| 0 | 2 | 2 | 5 | 1 |

| i | $s(i)$ |
|---|---|
| 0 | 2.750 |
| 1 | 4.125 |
| 2 | 0.250 |
| 3 | 0.000 |
| 4 | 4.875 |

Fig. 1. (a) Sample image. (b) NGTDM for sample image.

The other pixels are considered as being in the periphery of the image.

There are two pixels within the indicated square with gray tone = 2. Thus for this image

$$s(2)=\left|2-\frac{17}{8}\right|+\left|2-\frac{15}{8}\right|=0.250.$$

In similar fashion, we have $s(0)=2.750$; $s(1)=4.125$; $s(4)=4.875$; and $s(3)$ is necessarily zero. The NGTDM for this sample image is as shown in Fig. 1(b).

## III. Textural Properties and Their Computational Approximations

### A. Coarseness

This is the most fundamental property of texture, and in a narrow sense, it is used to imply texture. In a coarse texture, the primitives or basic patterns making up the texture are large. As a result, such a texture tends to possess a high degree of local uniformity in intensity, even over a fairly large area. In other words, the spatial rate of change in intensity is slight. Therefore the intensities of neighboring pixels would tend to be similar; thus there would be small differences between the gray tones of pixels and the average gray tones of their neighborhoods. Hence the summation of such differences computed over all image pixels would give an indication of the level of spatial rate of change in intensity, and thereby (in an inverse manner) show the level of coarseness of the texture.

This summation is the same as adding up the entries in the NGTDM. However in the summation each entry is weighted by the probability of occurrence of the corresponding intensity value.

We therefore define a computational measure for coarseness as

$$f_{\cos}=\left[\epsilon+\sum_{i=0}^{G_h} p_i s(i)\right]^{-1} \tag{3}$$

where $G_h$ is the highest gray-tone value present in the image and $\epsilon$ is a small number to prevent $f_{\cos}$ becoming infinite.

For an $N\times N$ image, $p_i$ is the probabiity of occurrence of gray-tone value $i$, and is given by

$$p_i=N_i/n^2, \qquad \text{where } n=N-2d.$$

The $f_{\cos}$ is the reciprocal of normalized sum of the deviations of pixel intensities from their neighborhood average intensities. Large values represent areas where gray-tone differences are small, i.e., coarse texture.

### B. Contrast

Perceptually, an image is said to have a high level of contrast if areas of different intensity levels are clearly visible. Thus high contrast means that the intensity difference between neighboring regions is large. This is usually the case when the dynamic range of gray scale is large or when it is stretched.

Also the spatial frequency of the changes in intensity (i.e., the amount of local intensity variations) will affect the contrast of an image. For instance, a small checkerboard will appear to have a higher contrast than a coarse checkerboard for the same gray scale range.

Taking these two factors into consideration, we propose the following computational form for this property

$$f_{\text{con}}=\left[\frac{1}{N_g(N_g-1)}\sum_{i=0}^{G_h}\sum_{j=0}^{G_h} p_i p_j (i-j)^2\right]\left[\frac{1}{n^2}\sum_{i=0}^{G_h} s(i)\right] \tag{4}$$

where $N_g$ is the total number of different gray levels present in the image.

$$N_g=\sum_{i=0}^{G_h} Q_i, \qquad \text{where } Q_i=\begin{cases}1, & \text{if } p_i\neq 0\\ 0, & \text{otherwise.}\end{cases}$$

The $f_{\text{con}}$ is a product of two terms. The first quantity is the average weighted squared difference between the different gray-tone values taken in pairs, and is used to reflect the dynamic range of gray scale; the weighting factor is a product of the probabilities of the two gray-tone values under consideration. The second term is the average difference between pixel gray tones and the average gray tone of their neighborhoods; this quantity increases with the amount of local variation in intensity.

### C. Busyness

A busy texture is one in which there are rapid changes of intensity from one pixel to its neighbor; that is the spatial frequency of intensity changes is very high. If these changes are very small in magnitude, they may not be visually noticeable and a high level of local uniformity in intensity may be perceived. On the other hand, if the spatial frequency of changes in intensity is very low, a high degree of local uniformity may still be perceived, even if the magnitude of the changes is large. While the spatial frequency of intensity changes reflects the level of busyness, the magnitude of these changes depends upon the dynamic range of gray scale, and thus relates to contrast.

Therefore a suppression of the contrast aspect from the information about spatial rate of change in intensity may indicate the degree of busyness of a texture. The following computational measure is proposed:

$$f_{\text{bus}}=\left[\sum_{i=0}^{G_h} p_i s(i)\right]\Big/\left[\sum_{i=0}^{G_h}\sum_{j=0}^{G_h} ip_i - jp_j\right], \qquad p_i\neq 0,\ p_j\neq 0. \tag{5}$$

The numerator is essentially a measure of the spatial rate of change in intensity, while the denominator is a summation of the magnitude of differences between the different gray-tone values. Each value is weighted by its probability of occurrence. The denominator results in the suppression of the effect of contrast variations. Hence the expression would tend to emphasize the frequency of spatial changes in intensity values, i.e., busyness.

### D. Complexity

Complexity refers to the visual information content of a texture. A texture is considered complex if the information content is high. This occurs when there are many patches or primitives present in the texture, and more so when the primitives have different average intensities. (Complexity could also be related to the shape of the individual primitives, but this is not used in this definition.) Again a texture with a large number of sharp edges and/or lines may be considered as complex. All these depend upon the spatial period of pattern repetition and on the dynamic range of gray scale. Thus complexity is in part correlated with busyness and contrast.

Generally textures in which the spatial rate of change in intensity is slight tend to have few different values of gray tones, but with a high probability of each value occurring. Consequently in these textures, there may not be many patches having different average intensity levels, but the patches would be large. Also the resulting high level of local uniformity in intensity will produce few edges. Thus a texture in which there are very rapid spatial changes in intensity is more likely to be complex than a texture that has a high degree of local uniformity in intensity. However rapid spatial changes from one intensity level to another would generally result in the presence of a large number of different levels of intensity (gray-tone values), but with a low probability of each individual value occurring. Therefore the sizes of primitives and/or probabilities of occurrence of gray-tone values tend to have inverse relationship with complexity.

A proposed measure for complexity is as follows:

$$f_{\mathrm{com}} = \sum_{i=0}^{G_h} \sum_{j=0}^{G_h} \left\{ (|i-j|) / \left( n^2 (p_i + p_j) \right) \right\} \left\{ p_i s(i) + p_j s(j) \right\}, \quad p_i \neq 0, p_j \neq 0. \quad (6)$$

The $f_{\mathrm{com}}$ is a sum of normalized differences between intensity values taken in pairs. These differences are weighted by the sum of the (probability-weighted) entries in the NGTDM corresponding to the two intensity values under consideration. The normalizing factor, $n^2(p_i + p_j)$, which would take high values for coarse textures and small values for busy or fine textures, is used to represent the inverse relationship between complexity and the sizes of primitives and/or the probabilities of intensity values. The absolute differences between the different intensity values are used to convey the influence of contrast variations on complexity, while the weighting factor reflects the rapidity or otherwise with which spatial changes in tonal values occur. High values of $f_{\mathrm{com}}$ should indicate a high degree of information content.

### E. Texture Strength

The term texture strength is a difficult concept to define concisely. However a texture is generally referred to as strong when the primitives that comprise it are easily definable and clearly visible. Such textures generally tend to look attractive, as they present a high degree of visual feel. But the ease with which distinctions can be made between the component primitives of a texture depends to a considerable extent upon the sizes of the primitives and the differences between their average intensities. For instance it may be possible to distinguish between large primitives (coarse texture) even with small differences between their average intensities. However for such distinctions to be made between small primitives (e.g., in busy textures), there must be wide differences between their intensities. Thus, in part, texture strength may be correlated with coarseness and contrast. We therefore propose the following computational approximation for this property

$$f_{\mathrm{str}} = \left[ \sum_{i=0}^{G_h} \sum_{j=0}^{G_h} (p_i + p_j)(i-j)^2 \right] \Bigg/ \left[ \epsilon + \sum_{i=0}^{G_h} s(i) \right], \quad p_i \neq 0, p_j \neq 0. \quad (7)$$

This expression involves two terms. The numerator is a factor stressing the differences between intensity levels, and therefore may reflect intensity differences between adjacent primitives, particularly as the intensities are weighted by the sum of their probabilities of occurrence; these probabilities would tend to be high for large primitives.

Also the denominator can convey information about the size of texture primitives, as it is essentially a sum of the difference between a pixel gray tone and the average gray tone in its neighborhood over all pixels. Its value would be small for coarse textures (since neighboring pixels generally have similar intensities in such textures), and high for busy or fine textures.

The expression would therefore tend to emphasize the boldness or distinctiveness of the primitives. Hence a high value of $f_{\mathrm{str}}$ would correspond to a strong texture.

## IV. Correspondence of Features with Textural Properties

Two sets of experiments were performed to determine the extent to which the texture features correspond to the properties, and therefore to human perception of textures. In each set of experiments, human subjects performed perceptual measurements on a set of natural textures, and the computer also performed corresponding tasks using the developed features. The experiments were carried out with the following aims:

- to investigate the degree to which each of the five textural features relates to each of the five textural properties, and consequently to determine whether the theoretically conceptualized textural property-textural feature relationship agrees with the practical case;
- to investigate the extent to which the features relate to each other, and also how the properties are correlated with one another;
- to investigate the extent, if any, to which certain combinations of the features can indicate similarity between different textural patterns, and therefore to determine the extent to which the features approximate human perception of textures.

In all, 88 subjects performed the experiments: 48 men and 40 women. Ten natural textures taken from Brodatz's album [12] were used in the experiments. The natural textures were: crushed rose quartz (D98); depressed cork (D4); straw matting (D55); herringbone weave (D16); beach pebbles and sand (D27); grass lawn (D9); beach pebbles (D23); oriental glass fiber cloth (D79); pigskin (D92); unborn calf fur (D93). They are shown in Fig. 2. From henceforth, the textures will simply be referred to as quartz, cork, matting, weave, beach, grass, pebbles, fiber, pigskin, and calf. A part of each of the original picture from the album was photographed on a 35-mm negative film and digitized into a $384 \times 384$ digital image with 256 gray levels. There was no other operation performed on the digital images (e.g., gray-scale contraction or histogram flattening).

However since a comparison was to be made between the results of the perceptual measurements and those produced by the texture features, it was only natural that both processes used the same pictures. Thus in the perceptual measurements, the original pictures from the album were not used, but rather the printed copies of the digital versions. In this regard, the digital pictures were displayed on a monitor and photographed.

### A. Ranking Experiments

This was the first set of experiments. Subjects were told to rank the ten textures using each of the five properties—coarseness, contrast, busyness, complexity, and texture strength. Prior to performing the experiment, the subject was given a brief explanation of the concept of texture and each of the five textural properties.

In the case of the computer, each of the $384 \times 384$ digital images was divided into sixteen subimages, each of size $96 \times 96$. It is reasonable to assume that a subimage of this size is large enough to capture the desired textural properties satisfactorily. The five features were computed for each of the subimages and

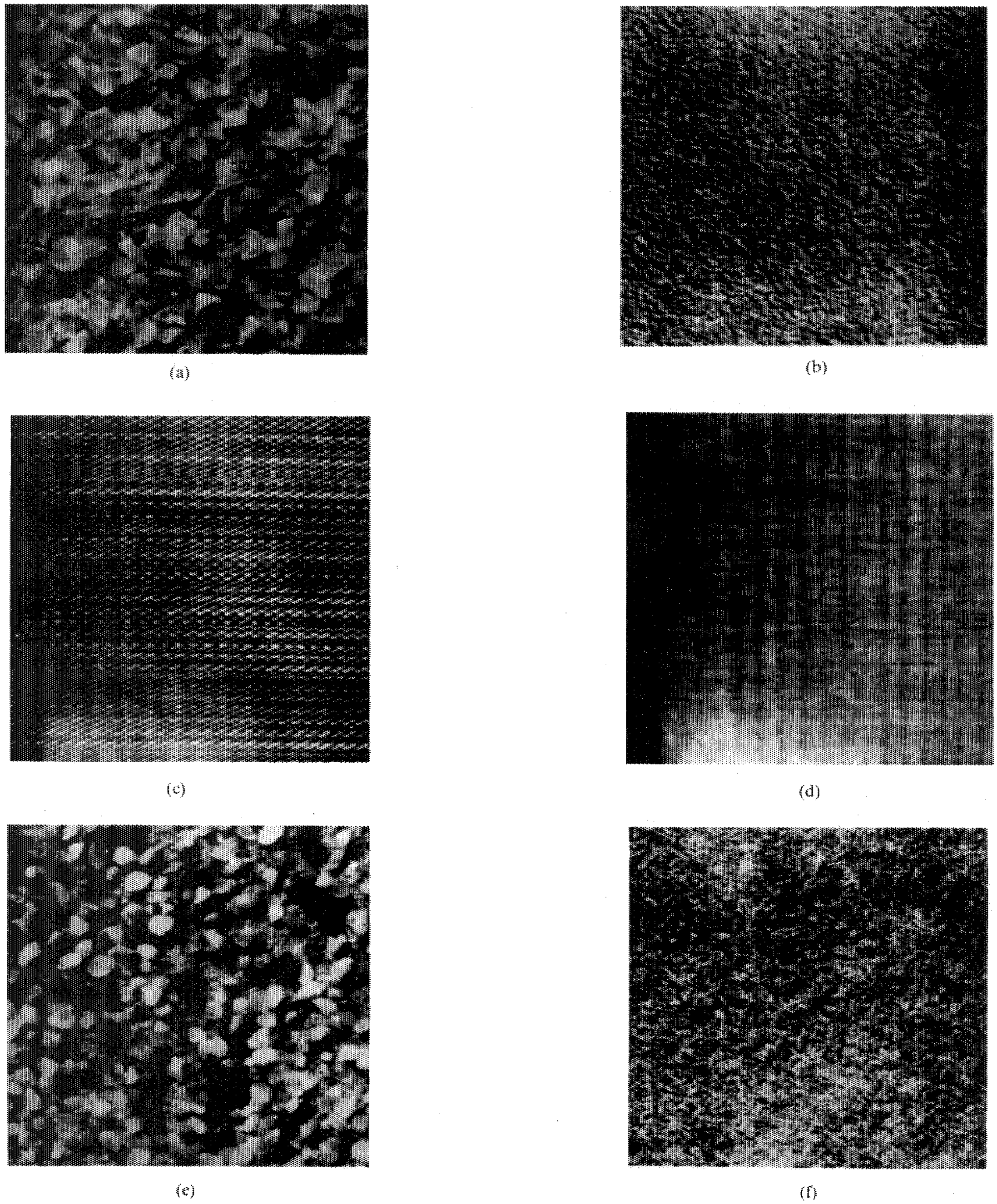


Fig. 2. Natural textures used in experiments. (a) Crushed rose quartz. (b) Depressed cork. (c) Straw matting. (d) Herringbone weave. (e) Beach pebbles and sand. (f) Grass lawn. (g) Beach pebbles. (h) Oriental glass fiber cloth. (i) Pigskin. (j) Unborn calf fur.

the average over the sixteen was determined. Two distances, $d=1$ and $d=2$, were used in feature computation, corresponding to neighborhood sizes of $3\times3$ and $5\times5$ respectively. These average values of features were used to rank the textures. The texture having the highest average value for a given feature was given a rank of 1 with respect to that feature, and the one with the least value a rank of 10. In the computation of the features $f_{\cos}$ and $f_{str}$, the value of $\epsilon$ was put at $10^{-7}$ The rankings are presented in Table I.

### B. Comparison of Human and Feature Rankings

In order to make a comparison between the rankings produced by humans and those by features, it was first of all necessary to determine a representative human ranking for each texture prop-

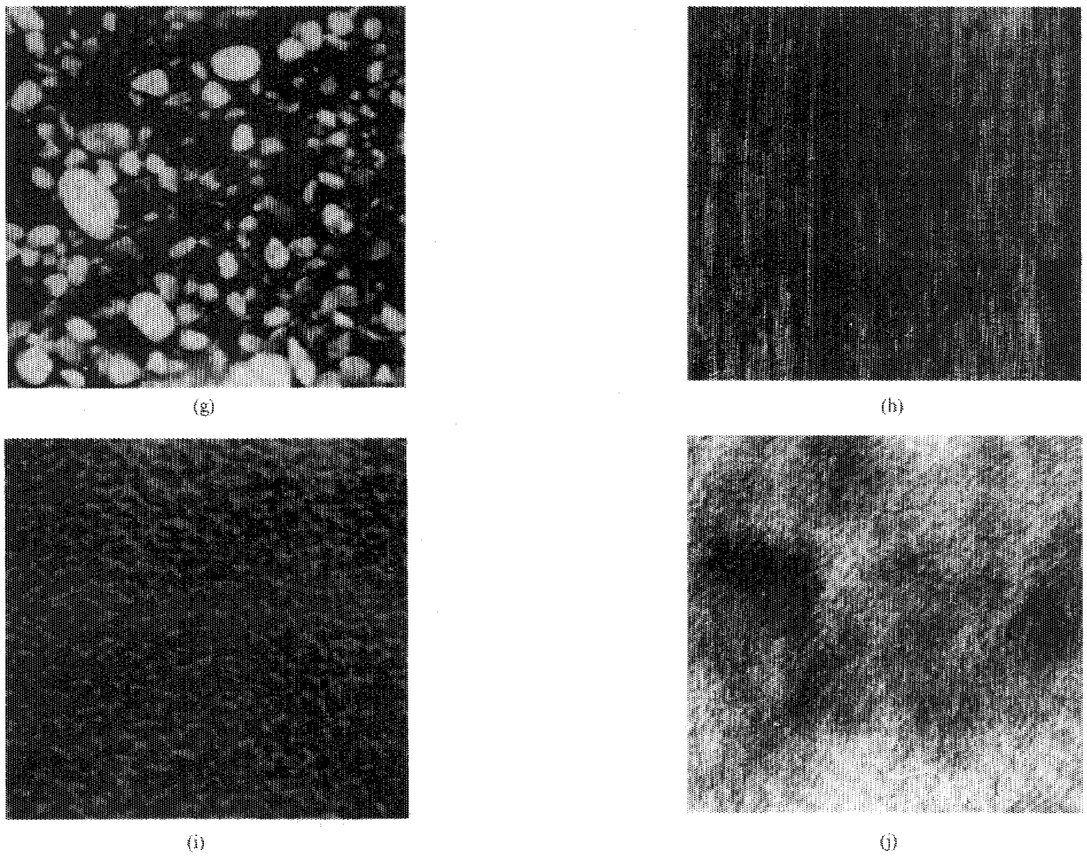


Fig. 2. (*Continued*).

TABLE I
RANKING OF TEXTURES USING FEATURES COMPUTED AT TWO DISTANCES: $d = 1$ AND $d = 2$

| Ranks | Features | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $f_{\cos}$ | | $f_{\mathrm{con}}$ | | $f_{\mathrm{bus}}$ | | $f_{\mathrm{com}}$ | | $f_{\mathrm{str}}$ | |
| $(k)$ | $d=1$ | $d=2$ | $d=1$ | $d=2$ | $d=1$ | $d=2$ | $d=1$ | $d=2$ | $d=1$ | $d=2$ |
| 1 | G | E | G | G | C | H | B | B | A | A |
| 2 | E | G | B | B | H | C | F | F | G | E |
| 3 | A | A | F | F | D | B | C | C | E | G |
| 4 | I | I | E | C | B | D | G | H | F | F |
| 5 | J | J | C | E | I | F | I | I | B | B |
| 6 | F | F | I | H | F | I | E | G | I | I |
| 7 | C | B | H | I | J | J | H | E | C | J |
| 8 | B | D | A | A | G | G | A | A | J | C |
| 9 | H | C | D | D | E | E | J | J | H | H |
| 10 | D | H | J | J | A | A | D | D | D | D |

erty from the rankings produced by the 88 subjects. The psychometric method of rank order, discussed in [13], was adapted to determine these representative, or composite, rankings.

The technique involves the computation of a quantity called the sum of rank values. Assuming that $N$ objects are ranked, the sum of rank values for the $j$th object is given

$$Z_j = \sum_{k=1}^{N} f_{jk} R_k \tag{8}$$

where $f_{jk}$ is the frequency of giving the rank $k$ $(k = 1, 2, \cdots, N)$ to the $j$th object. This is the same as the number of subjects that give the $j$th object the rank $k$.

The frequencies of ranks for the ten textures are presented in Tables XII–XVI in the Appendix.

$R_k$ is a series made up of rank values. These values are in exact reverse order to the rank $k$. $R_k$ is related to $k$ by

$$R_k = N - k + 1. \tag{9}$$

The sums of rank values are then used to obtain the representative ranking. The object (and in the present case the texture) whose sum of rank values is highest is assigned a rank of 1, that with the second highest a rank of 2, and so on. The resulting representative human rankings for the five textural properties are shown in Table II.

The comparison of human and feature rankings involved the determination of the degree of correspondences between them. In

TABLE II
REPRESENTATIVE HUMAN RANKINGS OF TEXTURES USING TEXTURAL PROPERTIES

| Ranks ($k$) | Representative Ranking: Coarseness | Contrast | Busyness | Complexity | Texture Strength |
|---|---|---|---|---|---|
| 1 | G | G | D | C | G |
| 2 | A | E | H | F | E |
| 3 | E | C | F | H | C |
| 4 | I | H | B | I | A |
| 5 | C | F | C | G | H |
| 6 | F | I | J | E | I |
| 7 | B | B | I | B | F |
| 8 | H | J | E | D | B |
| 9 | J | A | G | J | D |
| 10 | D | D | A | A | J |

TABLE III
COEFFICIENTS OF RANK CORRELATIONS BETWEEN HUMAN AND FEATURE RANKINGS[a]

| Textural Features | Textural Properties: Coarseness | Contrast | Busyness | Complexity | Texture Strength |
|---|---|---|---|---|---|
| $f_{cos}$ | 0.856 | 0.442 | −0.927 | −0.152 | 0.612 |
| $f_{con}$ | 0.527 | 0.685 | −0.176 | 0.467 | 0.515 |
| $f_{bus}$ | −0.600 | −0.018 | 0.782 | 0.552 | −0.272 |
| $f_{com}$ | 0.321 | 0.503 | −0.006 | 0.600 | 0.261 |
| $f_{str}$ | 0.879 | 0.321 | −0.794 | −0.139 | 0.600 |

[a]Features computed at $d=1$.

TABLE IV
COEFFICIENTS OF RANK CORRELATIONS BETWEEN HUMAN AND FEATURE RANKINGS[a]

| Textural Features | Textural Properties: Coarseness | Contrast | Busyness | Complexity | Texture Strength |
|---|---|---|---|---|---|
| $f_{cos}$ | 0.721 | 0.224 | −0.842 | −0.382 | 0.418 |
| $f_{con}$ | 0.455 | 0.697 | −0.079 | 0.539 | 0.515 |
| $f_{bus}$ | −0.624 | 0.018 | 0.830 | 0.564 | −0.297 |
| $f_{com}$ | 0.091 | 0.406 | 0.236 | 0.685 | 0.127 |
| $f_{str}$ | 0.806 | 0.248 | −0.794 | −0.248 | 0.503 |

[a]Features computed at $d=2$.

TABLE V
COEFFICIENTS OF RANK CORRELATIONS BETWEEN FEATURE RANKINGS[a]

| | $f_{str}$ | $f_{com}$ | $f_{bus}$ | $f_{con}$ |
|---|---|---|---|---|
| $f_{cos}$ | 0.806 | 0.079 | −0.830 | 0.152 |
| $f_{con}$ | 0.552 | 0.867 | −0.079 | |
| $f_{bus}$ | −0.782 | 0.176 | | |
| $f_{com}$ | 0.370 | | | |

[a]Features computed at $d=1$.

TABLE VI
COEFFICIENTS OF RANK CORRELATIONS BETWEEN FEATURE RANKINGS[a]

| | $f_{str}$ | $f_{com}$ | $f_{bus}$ | $f_{con}$ |
|---|---|---|---|---|
| $f_{cos}$ | 0.830 | −0.345 | −0.939 | 0.079 |
| $f_{con}$ | 0.345 | 0.745 | 0.164 | |
| $f_{bus}$ | −0.794 | 0.539 | | |
| $f_{com}$ | 0.042 | | | |

[a]Features computed at $d=2$.

this regard, the well-known Spearman's coefficient of rank correlation was used. This coefficient is given as

$$r_s = 1 - \frac{6D}{N^3 - N} \tag{10}$$

where $D$, called the summed squared difference, is given by

$$D = \sum_{j=1}^{N} \left(k_{mj} - k_{nj}\right)^2 \tag{11}$$

and $k_{mj}$ and $k_{nj}$ are the ranks given to the $j$th object in the $m$th and $n$th ranking, respectively. $N$ is the number of objects ranked; in the present case, $N=10$.

The value of $r_s$ is between $-1$ and 1. The value $-1$ corresponds to complete disagreement between the two rankings, and the value 1 indicates complete agreement. Equation [10] assumes that, as in the present case, there are no ties in ranks, i.e., no two or more objects are given the same rank in any of the rankings. A more complex expression exists for situations where there are ties (see [14] for details).

TABLE VII
COEFFICIENTS OF RANK CORRELATIONS BETWEEN REPRESENTATIVE HUMAN RANKINGS

| | Texture Strength | Complexity | Busyness | Contrast |
|---|---|---|---|---|
| Coarseness | 0.842 | 0.091 | −0.855 | 0.539 |
| Contrast | 0.782 | 0.661 | −0.261 | |
| Busyness | −0.588 | 0.309 | | |
| Complexity | 0.345 | | | |

TABLE VIII
REPRESENTATIVE HUMAN SIMILARITY ASSIGNMENTS

| Reference Texture | Most Similar Texture | Second Most Similar Texture |
|---|---|---|
| A | E | G |
| B | F | I |
| C | H | D |
| D | H | C |
| E | G | A |
| F | B | I |
| G | E | A |
| H | C | D |
| I | F | B and F |
| J | F | B |

TABLE IX
COMPUTER SIMILARITY ASSIGNMENTS USING MAXIMUM LIKELIHOOD CRITERION

| | Feature Combination[a] | | | | | | | | | | Feature Combination[b] | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Reference Texture | $f_{cos}$<br>$f_{con}$<br>$f_{bus}$ | | $f_{cos}$<br>$f_{con}$<br>$f_{com}$ | | $f_{bus}$<br>$f_{com}$<br>$f_{str}$ | | $f_{cos}$<br>$f_{con}$<br>$f_{bus}$<br>$f_{com}$ | | $f_{con}$<br>$f_{bus}$<br>$f_{com}$<br>$f_{str}$ | | $f_{cos}$<br>$f_{con}$<br>$f_{bus}$ | | $f_{cos}$<br>$f_{con}$<br>$f_{com}$ | | $f_{bus}$<br>$f_{com}$<br>$f_{str}$ | | $f_{cos}$<br>$f_{con}$<br>$f_{bus}$<br>$f_{com}$ | | $f_{con}$<br>$f_{bus}$<br>$f_{com}$<br>$f_{str}$ | |
| A | **E** | **G** | **E** | **G** | G | E | **E** | **G** | G | E | **E** | **G** | **E** | **G** | G | E | **E** | **G** | G | E |
| B | **F** | **I** | **F** | C | **F** | **I** | **F** | **I** | **F** | **I** | **F** | C | **F** | C | **F** | H | **F** | C | **F** | C |
| C | I | F | B | F | **H** | G | **H** | F | G | H | F | B | **B** | F | **H** | G | F | B | G | F |
| D | J | H | J | **C** | J | **C** | J | **C** | J | G | J | **C** | J | B | J | **C** | J | **C** | J | **C** |
| E | **G** | **A** | **G** | **A** | **G** | **A** | **G** | **A** | **G** | **A** | **G** | **A** | **G** | **A** | **G** | **A** | **G** | **A** | **G** | **A** |
| F | **B** | **I** | **B** | C | **B** | **I** | **B** | **I** | **B** | **I** | **B** | C | **B** | C | **B** | C | **B** | C | **B** | C |
| G | **E** | **A** | **E** | **A** | **E** | J | **E** | **A** | **E** | **A** | **E** | **A** | **E** | **A** | **E** | J | **E** | **A** | **E** | **A** |
| H | **C** | **D** | **C** | B | **C** | G | **C** | F | **C** | G | **C** | B | **C** | **D** | **C** | G | **C** | B | **C** | G |
| I | C | J | C | F | C | H | C | D | C | F | J | D | J | D | **F** | G | D | F | **F** | G |
| J | D | H | D | **B** | D | C | D | C | C | G | D | F | D | **B** | D | F | D | C | D | G |
| Number of Agreements | **6** | **6** | **6** | **5** | **6** | **4** | **7** | **6** | **5** | **4** | **6** | **4** | **7** | **5** | **7** | **2** | **6** | **4** | **6** | **3** |

[a] $d=1$.
[b] $d=2$.

TABLE X
COMPUTER SIMILARITY ASSIGNMENTS USING EUCLIDEAN DISTANCE CRITERION

| | Feature Combination[a] | | | | | | | | | | Feature Combination[b] | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| References Texture | $f_{con}$<br>$f_{con}$<br>$f_{bus}$ | | $f_{cos}$<br>$f_{con}$<br>$f_{com}$ | | $f_{bus}$<br>$f_{com}$<br>$f_{str}$ | | $f_{cos}$<br>$f_{con}$<br>$f_{bus}$<br>$f_{com}$ | | $f_{con}$<br>$f_{bus}$<br>$f_{com}$<br>$f_{str}$ | | $f_{cos}$<br>$f_{con}$<br>$f_{bus}$ | | $f_{cos}$<br>$f_{con}$<br>$f_{com}$ | | $f_{bus}$<br>$f_{com}$<br>$f_{str}$ | | $f_{cos}$<br>$f_{con}$<br>$f_{bus}$<br>$f_{com}$ | | $f_{con}$<br>$f_{bus}$<br>$f_{com}$<br>$f_{str}$ | |
| A | **E** | **G** | **E** | I | **E** | **G** | **E** | **G** | **E** | **G** | **E** | **G** | **E** | **G** | **E** | **G** | **E** | **G** | **E** | **G** |
| B | **F** | C | **F** | C | **F** | C | **F** | C | **F** | C | **F** | C | **F** | C | **F** | C | **F** | C | **F** | C |
| C | **H** | B | I | H | **H** | I | **H** | I | **H** | I | **H** | D | **H** | F | **H** | I | **H** | B | **H** | B |
| D | J | I | J | H | J | H | J | H | J | H | J | I | J | I | J | I | J | I | J | I |
| E | A | G | **G** | **A** | A | G | A | G | A | G | A | G | A | G | A | G | A | G | A | G |
| F | **B** | C | **B** | C | **B** | C | **B** | C | **B** | C | **B** | **I** | **B** | C | **B** | C | **B** | C | **B** | C |
| G | **E** | **A** | **E** | **A** | **E** | **A** | **E** | **A** | **E** | **A** | **E** | **A** | **E** | **A** | **E** | **A** | **E** | **A** | **E** | **A** |
| H | **C** | I | I | C | **C** | I | **C** | I | **C** | I | **C** | I | I | C | **C** | I | **C** | I | **C** | I |
| I | H | D | H | C | C | H | H | C | H | C | D | J | H | C | J | D | J | D | J | H |
| J | D | I | D | H | D | I | D | I | D | I | D | I | D | I | D | I | D | I | D | I |
| Number of Agreements | **6** | **2** | **5** | **2** | **6** | **2** | **6** | **2** | **6** | **2** | **6** | **3** | **5** | **2** | **6** | **2** | **6** | **2** | **6** | **2** |

[a] $d=1$.
[b] $d=2$.

Using (10) and (11), the coefficients of rank correlation were determined for the following:

- between each feature ranking and the representative human ranking for each textural property,
- between each feature ranking and every other feature ranking,
- between the representative ranking for each property and the representative ranking for every other property.

The results are presented in Tables III–VII.

The results in Tables III and IV show that each feature is more correlated with the appropriate texture property than the other properties, except for the feature $f_{str}$. There is a stronger correlation of this feature with coarseness than texture strength. A strong correlation also exists between the features $f_{\cos}$ and $f_{\mathrm{str}}$, and between $f_{\cos}$ and texture strength. It is very likely that the two features, and perhaps the two properties as well (as they are also very correlated), convey essentially the same information about a texture.

There is also a strong correlation between the features $f_{\mathrm{con}}$ and $f_{\mathrm{com}}$, and between the properties of contrast and complexity, though not as strong as that between $f_{\cos}$ and $f_{\mathrm{str}}$, and coarseness and texture strength, respectively. The feature $f_{\mathrm{bus}}$ is shown to be the most independent feature.

### C. Measurement of Texture Similarity

In this experiment, subjects were told to find a most similar, and a second most similar, texture to each of the ten textures; similarity need not be reciprocal. For instance if $B$ was considered to be most similar to $A$, this did not necessarily mean that $A$ was most similar to $B$; $C$ might be more similar to $B$ than $A$. The number of subjects that considered a given texture as the most similar, or second most similar, to a reference texture constitutes the frequency of assignment of the given texture as the most similar or second most similar one to the reference texture. These frequencies were used to obtain representative human similarity assignments. The texture that had the highest frequency as being the most similar to a reference texture was considered to be the representative most similar texture. The same applied for the second most similar case. The human representative similarity assignments obtained are given in Table VIII. The frequencies of similarity assignments are shown in Table XVIII in the Appendix.

For the automatic case, five different combinations of features were used, and two distance criteria were employed to measure similarity. The first criterion finds for each texture the one having the maximum likelihood from among the other nine or eight textures. This corresponds to finding, from among the other textures, the one with the minimum (squared) Mahalanobis distance to the mean of the reference texture. This is the so-called minimum error-rate classifier described in [15]. The second distance criterion is a normalized Euclidean distance.

The similarity assignments for the two distance criteria and for the five combinations of features are shown in Tables IX and X. Under each feature combination, there are two columns, the one on the left being for the most similar assignment, and the one on the right for the second most similar. A letter in bold type corresponds to an agreement with the representative human similarity assignment. The total number of such agreements is written under each column.

The results show that, for the most-similar assignment category, there is agreement between the human and computer similarity assignments for at least half of the number of textures for the two distance criteria. For the second most similar assignments, the results are not as good. Overall however, the results are very encouraging. They indicate that the features, to some extent, approximate visual perception.

## V. Application of Features in Agricultural Land-Use Classification

This experiment was carried out to assess the performance of the features in an actual classification task. A black-and-white aerial picture of an agricultural area was obtained from the Ministry of Agriculture in Cambridge, England. They also provided a corresponding "ground truth information." A part of this picture is shown in Fig. 3. The area consists of five major agricultural land-use categories, namely: wheat fields, potato fields, spring barley, young coniferous forest, and coniferous field under planting.

The picture was re-photographed and then digitized into a $1024 \times 1024$ image. Twenty subimages, each of size $54 \times 54$, were obtained for each of the five agricultural categories, giving a total of 100 subimages or class samples. In the classification, the technique of training on the data was employed, in this case leaving out four samples for each category at a time and training the classifier on the remaining sixteen; after which the twenty samples left out (four for each category) were presented to the classifier to identify. Thus in all, there were five runs of training and identification for a given set of features, and five different sets of features were used. The minimum error-rate classifier was also employed in these classifications.

Fig. 3. Part of aerial picture of agricultural terrain.

For the purpose of comparison of performance, the classification procedures were repeated in the same fashion, but using a set of four features, each from methods in [1], [4]. For the first method, the four features were angular second moment (ASM), contrast (CON), entropy (ENT) and correlation (COR). For the second technique, the features were angular second moment (asm), contrast (con), entropy (ent) and mean (MN). The first three features have been abbreviated in lower case letters in order to distinguish them from feautres of the same name in the other technique. The eight features were computed using intersample spacing of 1.

The classification results are shown in Table XI. Overall, the table shows that better classification results were obtained using the features developed and presented in this correspondence. Moreover the computation of the features is less expensive, as only one matrix is involved, compared to the other two techniques that require four matrices in their feature computation. Furthermore with regard to memory requirement, the method described here has considerable advantage.

## VI. Texture Features and Neighborhood Size Specifier, $d$

The features are derived from a matrix in which the entries are essentially measures of the spatial rate of change in intensity. A large difference between the gray tone of a pixel and the average gray tone of its surrounding neighbors over a small neighborhood would indicate a greater spatial rate of change in intensity than for the same difference over a large neighborhood. In other words higher sensitivity to spatial changes in intensity would be derived using small values of $d$ than for large $d$. Thus it is expected that features derived from the NGTDM computed at small values of $d$ should perform better than those obtained from

TABLE XI
CLASSIFICATION RESULTS FOR AGRICULTURAL LAND-USE CATEGORIES

| Features | Number of Correctly Classified Samples Per Category | | | | | Total Number of Correctly Classified Samples | Accuracy (percent) |
|---|---|---|---|---|---|---|---|
| | WH | POT | WB | YCF | CFP[a] | | |
| $f_{bus}$, $f_{com}$, $f_{str}$ | 15 | 11 | 16 | 19 | 20 | 81 | 81 |
| $f_{cos}$, $f_{con}$, $f_{bus}$ | 16 | 13 | 17 | 19 | 17 | 82 | 82 |
| $f_{cos}$, $f_{con}$, $f_{bus}$, $f_{str}$ | 16 | 12 | 17 | 20 | 18 | 83 | 83 |
| $f_{cos}$, $f_{bus}$, $f_{com}$, $f_{str}$ | 17 | 14 | 17 | 20 | 18 | 86 | 86 |
| $f_{cos}$, $f_{con}$, $f_{bus}$, $f_{com}$ | 16 | 13 | 17 | 20 | 18 | 84 | 84 |
| ASM, CON, ENT, COR | 17 | 11 | 17 | 19 | 19 | 83 | 83 |
| asm, con, ent, MN | 19 | 13 | 19 | 12 | 19 | 82 | 82 |

[a] WH is wheat; POT is potato; WB is winter barley; YCF is young coniferous forest; CFP is coniferous field under planting.

TABLE XII
FREQUENCY OF RANKS FOR TEXTURES USING COARSENESS

| Ranks ($k$) | Frequency of Ranks for Each Texture A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 40 | | | | 1 | | 47 | | | |
| 2 | 26 | | | 1 | 24 | | 36 | | 1 | |
| 3 | 18 | 1 | 2 | | 57 | | 4 | 1 | 5 | |
| 4 | 2 | 2 | 14 | 3 | 2 | 5 | | 3 | 50 | 7 |
| 5 | 2 | 5 | 26 | 1 | 1 | 27 | | 7 | 11 | 7 |
| 6 | | 24 | 12 | 1 | | 22 | | 18 | 5 | 5 |
| 7 | | 26 | 14 | 2 | 1 | 22 | | 21 | 3 | |
| 8 | | 21 | 10 | 3 | | 8 | | 26 | 6 | 13 |
| 9 | | 7 | 8 | 15 | | 3 | 1 | 11 | 4 | 39 |
| 10 | | 2 | 2 | 62 | | 1 | | 1 | 3 | 17 |

TABLE XIII
FREQUENCY OF RANKS FOR TEXTURES USING CONTRAST

| Ranks ($k$) | Frequency of Ranks for Each Texture A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | 1 | 11 | 2 | 6 | 2 | 58 | 5 | | 3 |
| 2 | | 4 | 10 | 1 | 41 | 5 | 11 | 9 | 3 | 4 |
| 3 | 8 | 5 | 21 | | 11 | 15 | 1 | 18 | 8 | 1 |
| 4 | 7 | 5 | 15 | | 6 | 12 | 3 | 20 | 15 | 5 |
| 5 | 3 | 6 | 8 | 4 | 6 | 21 | 2 | 20 | 11 | 7 |
| 6 | 3 | 10 | 11 | 4 | 8 | 20 | 4 | 7 | 13 | 8 |
| 7 | 12 | 27 | 3 | 2 | 5 | 7 | 2 | 6 | 10 | 14 |
| 8 | 10 | 23 | 5 | 3 | 2 | 5 | 2 | 2 | 13 | 23 |
| 9 | 26 | 6 | 2 | 28 | 1 | 1 | 3 | 1 | 4 | 16 |
| 10 | 19 | 1 | 2 | 44 | 2 | | 2 | | 11 | 7 |

TABLE XIV
FREQUENCY OF RANKS FOR TEXTURES USING BUSYNESS

| Ranks ($k$) | Frequency of Ranks for Each Texture A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | 2 | 10 | 39 | 4 | 9 | 3 | 9 | 2 | 10 |
| 2 | | 10 | 9 | 6 | 2 | 15 | 5 | 20 | 2 | 19 |
| 3 | | 16 | 13 | 9 | 3 | 15 | 2 | 15 | 7 | 8 |
| 4 | 2 | 21 | 11 | 6 | 1 | 11 | 1 | 15 | 11 | 9 |
| 5 | 1 | 18 | 6 | 2 | 4 | 20 | 2 | 12 | 16 | 7 |
| 6 | 2 | 9 | 21 | 6 | 7 | 14 | 4 | 4 | 16 | 5 |
| 7 | 8 | 4 | 10 | 4 | 4 | 4 | 7 | 10 | 25 | 12 |
| 8 | 12 | 6 | 5 | 2 | 39 | | 15 | 3 | 1 | 5 |
| 9 | 19 | | 3 | 5 | 21 | | 29 | | 2 | 9 |
| 10 | 44 | 2 | | 9 | 3 | | 20 | | 6 | 4 |

large $d$. In fact an indication of this trend is shown by results obtained from the ranking and texture similarity measurements, where the levels of correspondence between perceptual and computer measurements are higher for features computed using $d=1$ than for $d=2$.

Moreover a smaller amount of computation is involved in deriving the NGTDM when small values of $d$ are used. Hence it would be more advantageous to use small values of $d$ in the computation of these features.

## VII. CONCLUSION

In this study, an attempt has been made to develop measures that correspond to some textural properties, and therefore to visual perception of textures. Five basic properties of texture, namely: coarseness, contrast, busyness, complexity, and texture strength, were conceptually defined or expressed in terms of spatial changes in intensity. The conceptual expressions were then put into computational forms. In this approach, a 1-D matrix, called a neighborhood gray-tone difference matrix (NGTDM), was computed for a given image, and from this matrix the features were derived.

The measures were used in two experiments that also involved perceptual measurements by human subjects. One experiment involved the ranking, by humans, of a set of natural textures according to the degree to which they possessed a given textural property; the computer performed a similar task using the features. The second experiment was the measurement of texture similarity, both by humans and by the computer, the latter using certain combinations of the features.

TABLE XV
FREQUENCY OF RANKS FOR TEXTURES USING COMPLEXITY

| Ranks | Frequency of Ranks for Each Texture | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| ($k$) | A | B | C | D | E | F | G | H | I | J |
| 1 | | 7 | 23 | 7 | 3 | 14 | 18 | 4 | 5 | 7 |
| 2 | 1 | 10 | 9 | 6 | 14 | 11 | 8 | 19 | 8 | 2 |
| 3 | 4 | 10 | 15 | 9 | 7 | 7 | 8 | 12 | 11 | 5 |
| 4 | 9 | 8 | 7 | 5 | 14 | 15 | 4 | 10 | 9 | 7 |
| 5 | 6 | 5 | 8 | 7 | 7 | 15 | 8 | 9 | 21 | 2 |
| 6 | 12 | 6 | 10 | 6 | 6 | 8 | 7 | 15 | 8 | 10 |
| 7 | 9 | 16 | 7 | 4 | 9 | 5 | 4 | 11 | 13 | 10 |
| 8 | 19 | 9 | 4 | 6 | 13 | 10 | 9 | 6 | 5 | 7 |
| 9 | 16 | 11 | 3 | 7 | 11 | 3 | 11 | 2 | 2 | 22 |
| 10 | 12 | 6 | 2 | 31 | 4 | | 11 | | 6 | 16 |

TABLE XVI
FREQUENCY OF RANKS FOR TEXTURES USING TEXTURE STRENGTH

| Ranks | Frequency of Ranks for Each Texture | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| ($k$) | A | B | C | D | E | F | G | H | I | J |
| 1 | 8 | | 23 | 1 | 6 | | 48 | | 2 | |
| 2 | 8 | | 10 | 3 | 37 | 1 | 16 | 10 | 3 | |
| 3 | 33 | 2 | 11 | 5 | 14 | 1 | 9 | 7 | 5 | 1 |
| 4 | 10 | 6 | 18 | | 12 | 8 | 5 | 11 | 10 | 8 |
| 5 | 11 | 11 | 5 | 6 | 6 | 15 | 1 | 21 | 9 | 3 |
| 6 | 8 | 6 | 4 | 13 | 5 | 16 | | 12 | 21 | 3 |
| 7 | 5 | 25 | 6 | 8 | 4 | 14 | 3 | 6 | 14 | 3 |
| 8 | 4 | 20 | 7 | 9 | 4 | 14 | 1 | 10 | 10 | 9 |
| 9 | 1 | 15 | | 21 | | 15 | 5 | 9 | 3 | 19 |
| 10 | | 3 | 4 | 22 | | 4 | | 2 | 11 | 42 |

TABLE XVII
FREQUENCY OF TEXTURE ASSIGNMENTS TO REFERENCE TEXTURE[a]

| Reference Texture | A | | B | | C | | D | | E | | F | | G | | H | | I | | J | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | | | 2 | 2 | | | 1 | | 50 | 32 | 2 | 3 | 26 | 46 | | | 6 | 3 | 1 | 2 |
| B | | | | | | 5 | | 2 | 2 | | 58 | 25 | | 1 | | 2 | 27 | 42 | 1 | 11 |
| C | 1 | | 4 | 9 | | | 10 | 56 | | | | 4 | 1 | | 66 | 8 | 6 | 11 | | |
| D | 1 | 1 | 2 | 6 | 12 | 41 | | | | | 1 | 1 | | 1 | 54 | 23 | 3 | 4 | 15 | 11 |
| E | 14 | 64 | 1 | 1 | | | | | | | 2 | 7 | 71 | 14 | | | | 1 | | 1 |
| F | | | 46 | 36 | 1 | 4 | | | | | | | | | | | 37 | 39 | 4 | 10 |
| G | 3 | 73 | 2 | 1 | | 1 | | 2 | 81 | 3 | | 5 | | | | 1 | 1 | 2 | 1 | |
| H | | | 3 | 9 | 44 | 30 | 32 | 37 | | | 3 | 5 | | | | | | 2 | 6 | 5 |
| I | 1 | 1 | 38 | 36 | 2 | 4 | 1 | 4 | 2 | 1 | 43 | 36 | | | 1 | | | | | 6 |
| J | | | 26 | 28 | 2 | | 19 | 8 | | | 36 | 26 | | 2 | 4 | 20 | 1 | 4 | | |

[a]The first column under each heading is for "most similar;" the second column under each heading represents "second-most similar."

With respect to ranking, very successful results were obtained. The results show not only that there are very high levels of correspondences between computational and perceptual measurements, but also that each feature relates more to the appropriate textural property, except for the feature $f_{str}$. This feature is found to be slightly more correlated to coarseness than to texture strength for the textures used in the experiments. In any case the two properties, coarseness and texture strength, are also very correlated with each other.

For the experiments designed to indicate similarity between different textural patterns, the results were also encouraging. The most similar pattern was correctly identified by the computer for at least five of the ten test textures for each of the five feature combinations used. The results for the second most similar textures varied more widely. Only in two to six cases did the computer results agree with the representative human similarity assignments. However, it should be realized that the representative human similarity assignments were derived using a vote-type count (i.e., a simple majority), and not taking into consideration the variations between the assignments of the individual subjects. If the variations are taken into account, then the results obtained could be considered as very successful, especially if one also considers the results of similar experiments in [8]. It is also probable that the differences between human and computer similarity assignments arise from the fact that the mechanism of human usage of multiple cues may be much more complex than the maximum likelihood and Euclidean distance criteria used by machine.

The results also show better correlation between human and computer measurements using features computed at $d = 1$ than for $d - 2$. This may be due to the fact that higher sensitivity to spatial changes in intensity is obtained at the smaller distance. Moreover the feature computation at $d = 1$ involved a smaller amount of computation. Hence for any texture, small values of $d$ are recommended for the computation of these features.

Finally slightly improved results in terms of classification accuracy were obtained using the developed features compared with two existing approaches—the methods of cooccurrence matrices and spatial gray-level difference. Other advantages of the technique described in this paper are that it is computationally less expensive, and the memory requirement is very small.

## APPENDIX

### A. Frequency of Ranks

In Tables XII–XVI, the frequencies of ranks for the ten textures are presented. These are the same as the number of subjects who gave a particular rank to each of the textures. A blank in the table indicates zero frequency.

### B. Frequency of Similarity Assignments

The frequency of assignment of a given texture (i.e., the number of subjects who considered a given texture as most similar to, or second most similar to, a reference texture) is shown in Table XVII. The frequencies are in two columns for each texture. The first column is for the assignment as a most similar texture to the reference one, while the second column is for the assignment as the second most similar one. For instance, 58 subjects considered texture $F$ to be most similar to texture $B$, while 25 subjects considered it as the second most similar. Again, a blank indicates zero assignment.

## ACKNOWLEDGMENT

The authors are grateful to the referees for their helpful comments leading to improvements of the manuscript. Thanks also go to Mr. Alan Blakeman of the Ministry of Agriculture, Cambridge, England, for providing the aerial photograph used for the agricultural land-use classification.

## References


[1] R. M. Haralick, K. Shanmugam, and I. Dinstein, "Textural features for image classification," *IEEE Trans. Syst. Man Cybern.*, vol. SMC-3, pp. 610–662, Nov. 1973.

[2] M. Galloway, "Texture analysis using gray-level run lengths," *Computer Graphics Image Processing*, vol. 4, pp. 172–199, 1974.

[3] A. Rosenfeld and M. Thurston, "Edge and curve detection for visual scene analysis," *IEEE Trans. Comput.*, vol. C-20, pp. 562–569, May 1971.

[4] J. S. Weszka, C. R. Dyer, and A. Rosenfeld, "A comparative study of texture measures for terrain classification," *IEEE Trans. Syst. Man Cybern.*, vol. SMC-6, pp. 269–285, Apr. 1976.

[5] P. C. Chen and T. Pavlidis, "Segmentation by texture using correlation," *IEEE Trans. Pattern Anal. Machine Intell.*, vol. PAMI-5, pp. 64–69, 1983.

[6] B. H. McCormick and S. N. Jayaramurthy, "Time series model for texture synthesis," *J. Computing Inform. Sci.*, vol. 3, pp. 329–343, Dec. 1974.

[7] R. M. Haralick, "Statistical and structural approaches to texture," *Proc. IEEE*, vol. 67, 1979 pp. 786–804.

[8] H. Tamura, S. Mori, and T. Yamawaki, "Textural features corresponding to visual perception," *IEEE Trans. Syst. Man Cybern.*, vol. SMC-8, pp. 460–473, June 1978.

[9] B. Julesz, "Texture and visual perception," *IRE Trans. Info. Theory*, vol. IT-8, pp. 84–92, Feb. 1962.

[10] ____, "Texture and visual perception," *Scientific American*, vol. 212, pp. 38–54, Feb. 1965.

[11] B. Julesz, E. N. Gilbert, L. A. Shepp, and H. L. Frisch, "Inability of humans to discriminate between visual textures that agree in second-order statistics—Revisited," *Perception*, vol. 2, pp. 391–405, 1973.

[12] P. Brodatz, *Textures: A Photographic Album for Artists and Designers*. New York: Dover, 1966.

[13] J. P. Guildford, *Psychometric Methods*, ch. 8. New York: McGraw-Hill, 1954.

[14] S. Siegel, *Nonparametric Statistics for the Behavioural Sciences*, ch. 9. New York: McGraw-Hill, 1956.

[15] R. O. Duda and P. E. Hart, *Pattern Classification and Scene Analysis*. New York: Wiley, 1973, pp. 16–31.


# Point Pattern Matching Using Centroid Bounding

PAUL M. GRIFFIN AND CHRISTOS ALEXOPOULOS

***Abstract*—Given two point patterns $A$ and $B$ in the plane, where $|A| = |B| = N$, an algorithm is presented to find a feasible matching and registration between the point patterns. The matching results are invariant to rotation, translation, noise, and scaling.**

## I. Introduction

Point pattern matching has found many applications in the field of pattern recognition and computer vision. Examples include pattern matching, automated visual inspection of flat objects and the registration of images. References to past approaches to point pattern matching are given in [2]. More recent work is cited in [3] and [4].

This correspondence describes a simple method for matching of two point patterns $A$ and $B$ in the plane, where the number of points in both patterns are the same. The method works by computing the two pattern controids and aligning them, and then constructing a bipartite graph of feasible point assignments. A feasible point pattern matching is then found by determining the maximal cardinality matching of the bipartite graph. The matching results are invariant to translation, rotation, noise and scaling. A computer vision application is also given.

In the next section, definitions that are useful in this correspondence are given. The point pattern matching algorithm is discussed in Section III. Section IV gives experimental results of the performance of the algorithm. Conclusions are discussed in Section V.

Manuscript received April 23, 1988; revised April 23, 1989.

The authors are with School of Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0205.

IEEE Log Number 8929621.

## II. Definitions

A point pattern is a nonempty finite set of points in the plane. The location of each point $p_i$ is described by a location vector $\langle x_i, y_i \rangle$. Given two point patterns $A$ and $B$, a matching $M$ is a one-to-one mapping of the points of $B$ onto the points of $A$. A registration $R$ is an affine transformation composed of translation, rotation, and scaling. A registration acts on a point $p_i$ in the plane by the following:

$$R(p_i) = s\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix} \tag{1}$$

where $s$ is the scale factor, $t_x$ and $t_y$ are the translation offsets in the $x$ and $y$ direction, and $\theta$ is the rotation angle. The point pattern matching problem may be stated as follows: given a template point pattern $A$ and a sample point pattern $B$, find a feasible matching of the points of $B$ onto the points of $A$, and the corresponding registration.

It is assumed that $A$ is a known point pattern specified in a database, and that $B$ is an observed point pattern. Due to inaccuracies in any observation system, each matched point $p_i' \in B$ is subject to a location error $l_i$:

$$l_i = d(R(M(p_i')), p_i)$$

where $d$ is the Euclidean distance between point $p_i' \in B$ and the point $p_i \in A$ that corresponds to it after the matching and registration. A point pattern where each point is subject to location error is referred to as *noisy*. It is assumed that the location error of each point is bounded from the previous by $r$. Circles with radius $r$ will be referred to as *noisy circles*.

The *centroid* of a point pattern $A$ (denoted by $c(A)$) with points $p_1, \cdots, p_N$ is given by

$$c(A) = \left\langle \frac{1}{N}\sum_{i=1}^{N} x_i, \frac{1}{N}\sum_{i=1}^{N} y_i \right\rangle$$

where $\langle x_i, y_i \rangle$ is the location of point $p_i$. The smallest enclosing circle for a set of $N$ points is the circle with the smallest area that contains all $N$ points.

## III. Algorithm

An algorithm is presented that determines a feasible matching between $A$ and $B$ as well as the corresponding registration. The algorithm first determines the scale factor $s$ using the smallest enclosing circles, matches the point pattern centroids and then determines the rotation angle $\theta$ that corresponds to a feasible match.

The first step of the algorithm is to compute the smallest enclosing circle for both $A$ and $B$. An $O(N \log N)$ algorithm for determining the smallest enclosing circle using the Voronoi diagram appears in [5]. Once the smallest enclosing circles have been found, the scale factor $s$ may be determined by the ratio of the radii of the two circles. That is

$$s = \frac{R(A)}{R(B)}. \tag{2}$$

Point pattern $B$ is then scaled to $A$ using $s$.

The translation offsets $t_x$ and $t_y$ are determined by matching the point pattern centroids. Each centroid $c(A)$ and $c(B)$ is calculated, and the difference between them is the translation offset. Once the translation offset is determined, each point $p_i' \in B$ is translated by the offset amount so that the centroid of each point pattern overlaps.

The next step of the algorithm is the computation of the Euclidean distances from the centroid of $A$, $c(A)$, to the points $p_i \in A$ and $p_i' \in B$. The distances are denoted by $d(p_i)$ and