Topic V01

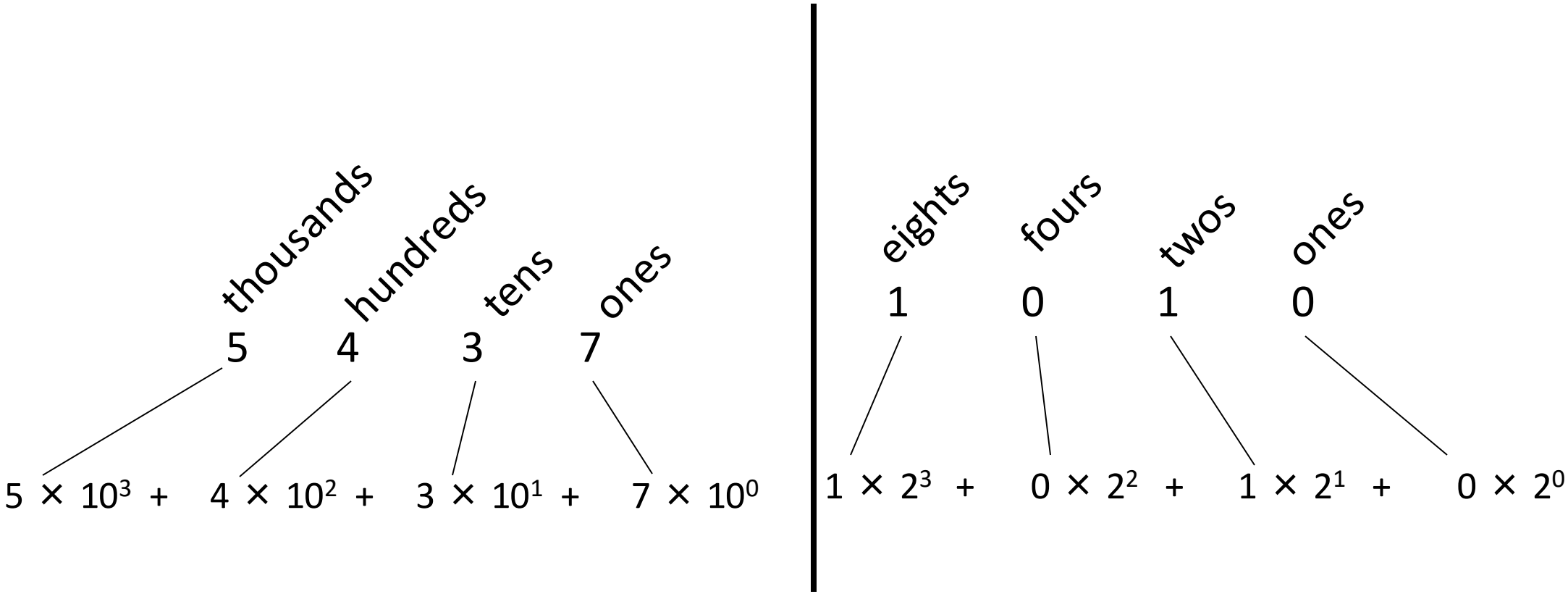
Binary Representation

Readings: (Sections 2.4)

Decimal Numbers

5437

Positional Numbering System



Decimal

Binary

Unsigned Binaries (example)

Largest unsigned number in four bits?

Binary Representation	Decimal Value
1 1 1 1	8 + 4 + 2 + 1 = 15
	$2^4 - 1 = 15$

In Decimal notation we know that:

$9999 = 10000 - 1 = 10^4 - 1$

In binary we have:

$1111_2 = 10000_2 - 1 = 2^4 - 1 = 16 - 1$

Unsigned Binaries (example)

Largest unsigned number in four bits?

Binary Representation	Decimal Value
1 1 1 1	8 + 4 + 2 + 1 = 15
	$2^4 - 1 = 15$

Largest unsigned number in k bits?

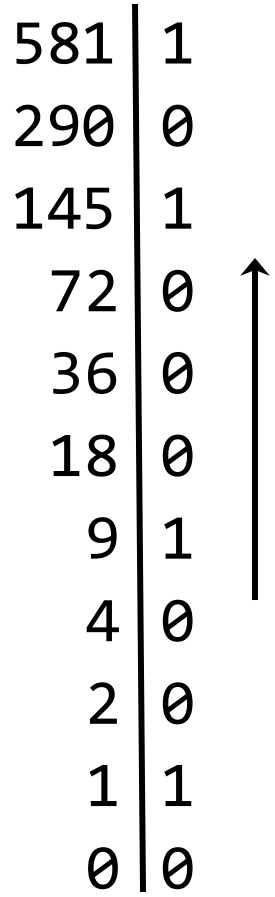
Binary Representation	Decimal Value
1 1 … 1	$2^k - 1$

Some Powers of Two to Memorize

Name	Exponent	Value

Decimal to Binary Conversion

Method of Repeated Division:



581	1
290	0
145	1
72	0
36	0
18	0
9	1
4	0
2	0
1	1
0	0

581 = 01001000101

Decimal to Binary Conversion

Method of Repeated Subtraction:

k	2^k
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

10	9	8	7	6	5	4	3	2	1	0
0	1	0	0	1	0	0	0	1	0	1

581
-512

69
-64

5
-4

1
-1

0

581 = 01001000101

Review of logic Operators

The complement of a binary number:

$$\begin{array}{rcccc} x = & 1 & 0 & 0 & 1 \\ + \quad \overline{x} = & 0 & 1 & 1 & 0 \\ \hline x + \overline{x} = & 1 & 1 & 1 & 1 \end{array}$$

By convention:

$$x + \overline{x} = -1$$

$$1 \quad 1 \quad 1 \quad 1 = -1$$

2-Complement Notation

By convention:

$$-1 = 1\ \ 1\ \ 1\ \ 1$$

Some algebra:

$$x + \overline{x} = -1$$

$$x + \overline{x} + 1 = 0$$

$$\overline{x} + 1 = -x$$

$$-x = \overline{x} + 1$$

Example:

$$6 = 0\ 1\ 1\ 0_2$$

$$-6 = ?$$

$$\begin{array}{r} \quad\quad\quad\quad 1 \\ \overline{6} = 1\ 0\ 0\ 1_2 \\ + \quad\quad\quad\quad 1_2 \\ \hline -6 = 1\ 0\ 1\ 0_2 \end{array}$$

A leading 1 indicates a negative number.

Given the binary number:

1010

How do you know if it is 10_{10} or -6_{10}?

You don't!

You need to be told if this is:

An unsigned representation

Or a 2-complement representation

Meaning Depends on Context

April, May and June

Theater play: *April, May and June* are three sisters confronting their late mother's affair.

Meaning Depends on Context

Given the binary representation:

01000001

What does it represent?

You don't know!

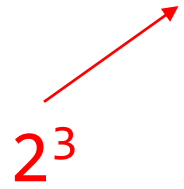
It could be:

- $+65_{10}$
- The ASCII character 'A'
- Something else

Range of 2-Complement Notation

Largest positive number in 2-complement in four bits?

Binary Representation	Decimal Value
0 1 1 1	$4 + 2 + 1 = 7$
	$2^3 - 1 = 7$
	$2^{4-1} - 1 = 7$



Largest positive number in 2-complement in k bits?

Binary Representation	Decimal Value
0 1 ... 1	$2^{k-1} - 1$

Range of 2-Complement Notation

Most negative number in 2-complement in four bits?

Binary Representation	Decimal Value
1 0 0 0	$-2^3 = -8$
	$-2^{4-1} = -8$

$$
\begin{array}{rl}
 & \ 1\ \ 1\ \ 1 \\
1\ \ 0\ \ 0\ \ 0 & =\ 0\ \ 1\ \ 1\ \ 1 \\
+ & \ 1 \\
\hline
 & \ 1\ \ 0\ \ 0\ \ 0 = 8_{10}
\end{array}
$$

Range of 2-Complement Notation

Most negative number in 2-complement in four bits?

Binary Representation	Decimal Value
1 0 0 0	$-2^3 = -8$
	$-2^{4-1} = -8$

Most negative number in 2-complement in k bits?

Binary Representation	Decimal Value
1 0 ··· 0	-2^{k-1}

Range of 2-Complement Notation

2-complement in k bits:

Largest positive number:

Binary Representation	Decimal Value
0 1 ⋯ 1	$2^{k-1} - 1$

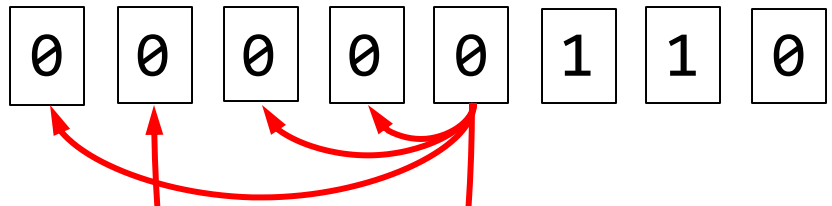
Most negative number:

Binary Representation	Decimal Value
1 0 ⋯ 0	-2^{k-1}

Sign Extension

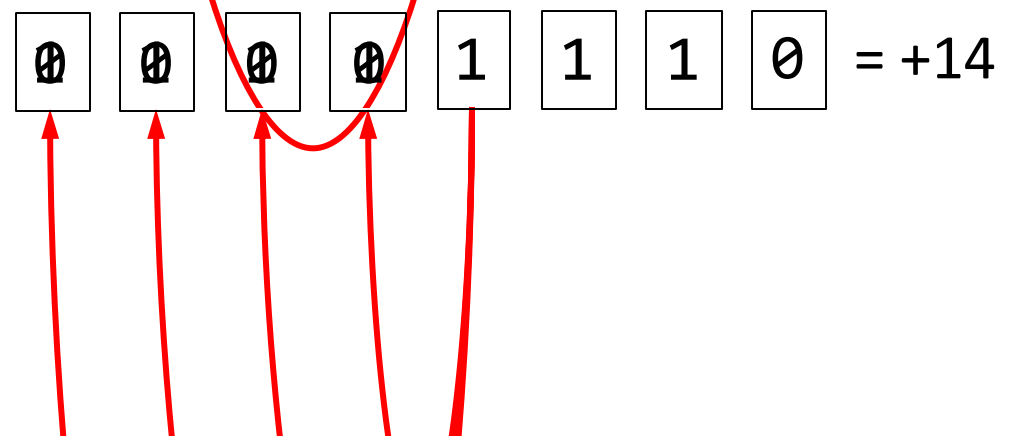
Number +6 represented in four digits: 0 1 1 0

How to represent +6
in eight digits?



Number -2 represented in four digits: 1 1 1 0

How to represent -2
in eight digits?



Sign Extension in RISC-V

In RISC-V a word has 32 bits

Sometimes a 12-bit (or a 20-bit) value has to be stored into a word

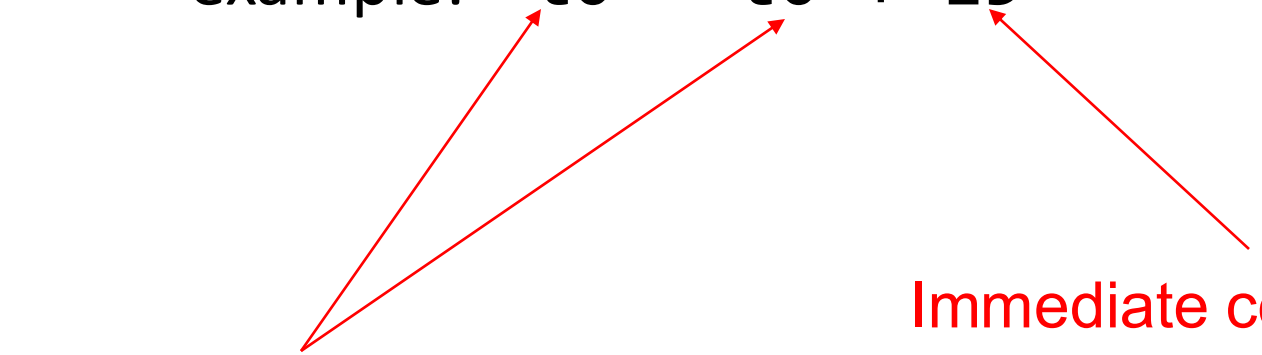
Sign Extension in RISC-V

Instructions with Immediate value

Add Immediate

`addi`: extend immediate value

example: `t0 ← t0 + 15`



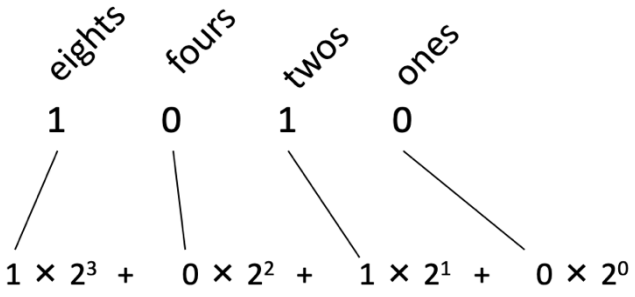
Registers are 32 bits

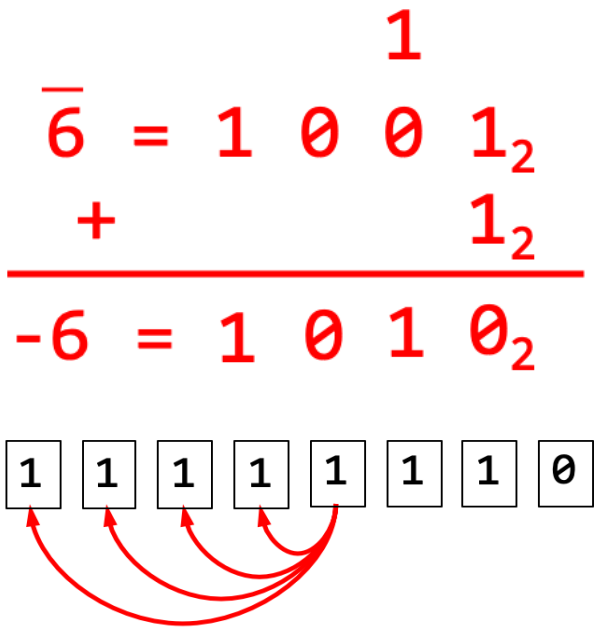
Immediate constants are 12 bits

Constant is sign-extended to 32 bits

P-H pp. 90-94

What have we learned?





$2^k - 1$

-2^{k-1}

$2^{k-1} - 1$

581	1
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April, May and June

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2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

581
-512
69
-64
5
-4
1
-1
0