Instruction Class	Frequency	# of clock cycles
CF	20%	3
ALU	30%	1
L/S	20%	10
FP	30%	5

Table 1: Frequency and latency per instruction class.

Question 1 (15 points): You are an intern with the Edmonton Compiler Group (ECG). Your current task is to assist the team in an optimization that aims to improve the performance of an important client application. In discussions with your team, it is decided that the instructions executed by this application can be divided into four classes: Control Flow (CF) instructions, Integer ALU instructions, Load/Store (L/S) instructions, and Floating Point (FP) instructions. You are tasked with performing experiments to determine the relative frequency of these instructions during the execution of the application and the average number of clock cycles that an instruction of each class takes to execute. After performing experiments, you present your team with the results in Table 1.

Upon looking into this data, one of the members of the compiler team, who is responsible for the memory hierarchy in the system, is very interested in what you found. Two days later she comes to you and tells you that she has implemented an optimization in the order in which the data in the application is accessed. She tells you "We still execute the same number of load/store instructions, but now they do not take as much time. The application is now 1.3 times faster than before!"

a. (**5 points**) Based on what she told you, you know that the number of instructions executed has not changed. You also know that she used the same machine with the same clock frequency as before. What is the new average number of clock cycles to execute an L/S instruction after her optimization?

$$
\begin{aligned}
\text{Time}_{\text{Base}} &= \frac{\text{NumInstr} \times \text{CPI}_{\text{Base}}}{\text{Freq}} \\
\text{CPI}_{\text{Base}} &= 0.2 \times 3 + 0.3 \times 1 + 0.2 \times 10 + 0.3 \times 5 = 4.4 \frac{\text{Clocks}}{\text{Instruction}} \\
\text{Time}_{\text{New}} &= \frac{\text{NumInstr} \times \text{CPI}_{\text{New}}}{\text{Freq}} \\
\text{CPI}_{\text{New}} &= 0.2 \times 3 + 0.3 \times 1 + 0.2 \times \text{L/S}_{\text{New}} + 0.3 \times 5 = 2.4 + 0.2 \times \text{L/S}_{\text{New}} \frac{\text{Clocks}}{\text{Instruction}}
\end{aligned}
$$

$$
\begin{aligned}
\text{Speedup} &= \frac{\text{Time}_{\text{Base}}}{\text{Time}_{\text{New}}} = 1.3 \\
&= \frac{\frac{\text{NumInstr} \times \text{CPI}_{\text{Base}}}{\text{Freq}}}{\frac{\text{NumInstr} \times \text{CPI}_{\text{New}}}{\text{Freq}}} = \frac{\text{CPI}_{\text{Base}}}{\text{CPI}_{\text{New}}} = 1.3 \\
&= \frac{4.4}{2.4 + 0.2 \times \text{L/S}_{\text{New}}} = 1.3 \\
1.3 \times (2.4 + 0.2 \times \text{L/S}_{\text{New}}) &= 4.4 \\
0.26 \times \text{L/S}_{\text{New}} &= 4.4 - 3.12 \\
\text{L/S}_{\text{New}} &= \frac{1.76}{0.352} = 4.9 \text{ clock cycles}
\end{aligned}
$$

b. (**5 points**) The next day, she tells you "I have a few more tricks up my sleeve, I will reduce the average number of clock cycles for L/S instructions even further and make this application faster." What is the theoretical maximum speedup over the original version of the application that she can obtain by reducing the average time needed to execute an L/S instruction?

The maximum occurs when the average time for L/S instructions is reduced to zero.

$$
\begin{aligned}
\text{Time}_{\text{Base}} &= \frac{\text{NumInstr} \times \text{CPI}_{\text{Base}}}{\text{Freq}} \\
\text{CPI}_{\text{Base}} &= 0.2 \times 3 + 0.3 \times 1 + 0.2 \times 10 + 0.3 \times 5 = 4.4 \frac{\text{Clocks}}{\text{Instruction}} \\
\text{Time}_{\text{New}} &= \frac{\text{NumInstr} \times \text{CPI}_{\text{New}}}{\text{Freq}} \\
\text{CPI}_{\text{NewMax}} &= 0.2 \times 3 + 0.3 \times 1 + 0.2 \times 0 + 0.3 \times 5 = 2.4 \frac{\text{Clocks}}{\text{Instruction}} \\
\text{MaxSpeedup} &= \frac{4.4}{2.4} = 1.83
\end{aligned}
$$

c. (**5 points**) If the original program, before any of the modifications, executed in 11 seconds and the processor's clock frequency is 4GHz (1 GHz = 10^9 Hz), how many L/S instructions does the original program execute?

$$
\begin{aligned}
\text{Time}_{\text{Base}} &= \frac{\text{NumInstr} \times \text{CPI}_{\text{Base}}}{\text{Freq}} \\
11 &= \frac{\text{NumInstr} \times 4.4}{4 \times 10^9} \\
\text{NumInstr} &= \frac{11 \times 4 \times 10^9}{4.4} = 10 \times 10^9 \text{ instructions} \\
\text{Num L/S} &= 0.2 \times \text{NumInstr} = 0.2 \times 10 \times 10^9 = 2 \times 10^9 \text{ L/S instructions}
\end{aligned}
$$