

Topic V0D

Recursive Functions

Readings: (Section 2.8)

Non-Leaf Procedures

Procedures that call other procedures

- For nested calls, callee needs to:
 - Save on the stack:
 - ra
 - callee-saved registers
 - Restore values from the stack before returning

Calling Itself

```
int fact(int n){  
    if (n < 1)  
        return 1;  
    else  
        return (n * fact(n - 1));  
}
```

Step-by-step creation of the
assembly code for fact

```
int fact(int n){  
    if (n < 1)  
        return 1;  
    else  
        return (n * fact(n - 1));  
}
```

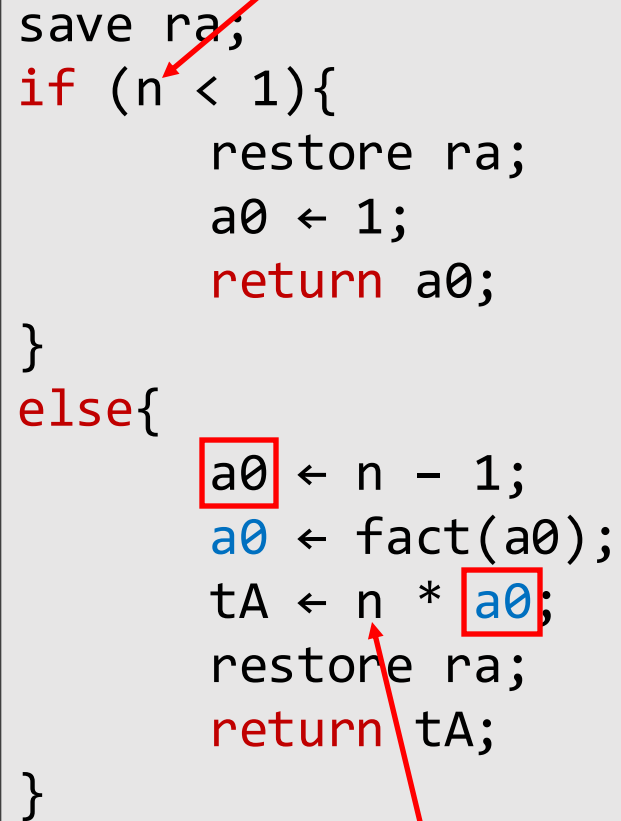
```
save ra;  
if (n < 1){  
    restore ra;  
    return 1;  
}  
else{  
    tA ← n * fact(n - 1);  
    restore ra;  
    return tA;  
}
```

```
save ra;  
if (n < 1){  
    restore ra;  
    a0 ← 1;  
    return a0;  
}  
else{  
    tA ← n * fact(n - 1);  
    restore ra;  
    return tA;  
}
```

```
save ra;  
if (n < 1){  
    restore ra;  
    a0 ← 1;  
    return a0;  
}  
else{  
    a0 ← n - 1;  
    tA ← n * fact(a0);  
    restore ra;  
    return tA;  
}
```

```
save ra;
if (n < 1){
    restore ra;
    a0 ← 1;
    return a0;
}
else{
    a0 ← n - 1;
    tA ← n * fact(a0);
    restore ra;
    return tA;
}
```

```
save ra;
if (n < 1){
    restore ra;
    a0 ← 1;
    return a0;
}
else{
    a0 ← n - 1;
    a0 ← fact(a0);
    tA ← n * a0;
    restore ra;
    return tA;
}
```



```
save ra, a0;
if (n < 1){
    restore ra, a0;
    a0 ← 1;
    return a0;
}
else{
    a0 ← n - 1;
    a0 ← fact(a0);
    restore ra, a0;
    tA ← n * a0;
    return tA;
}
```

This a0 is n

How to solve this?

```
save ra, a0;
if (n < 1){
    restore ra, a0;
    a0 ← 1;
    return a0;
}
else{
    a0 ← n - 1;
    a0 ← fact(a0);
    restore ra, a0;
    tA ← n * a0;
    return tA;
}
```

This a0 is fact(n-1)

```
save ra, a0;
if (a0 < 1){
    restore ra, a0;
    a0 ← 1;
    return a0;
}
else{
    a0 ← a0 - 1;
    a0 ← fact(a0);
    restore ra, a0;
    tA ← a0 * a0;
    return tA;
}
```

n-1 n

What is wrong with this?

This multiplication is n*n
It should be n*fact(n-1)

```
save ra, a0;
if (a0 < 1){
    restore ra, a0;
    a0 ← 1;
    return a0;
}
else{
    a0 ← a0 - 1;
    a0 ← fact(a0);
    t0 ← a0;
    restore ra, a0;
    a0 ← a0 * t0;
    return a0;
}
```

This is n This is fact(n-1)
This a0 is n

```

save ra, a0;
if (a0 < 1){
    restore ra, a0;
    a0 ← 1;
    return a0;
}
else{
    a0 ← a0 - 1;
    a0 ← fact(a0);
    t0 ← a0;
    restore ra, a0;
    a0 ← a0 * t0;
    return a0;
}

```

```

sp ← sp - 8;
M[sp + 4] ← ra;
M[sp] ← a0;
if (a0 < 1){
    restore ra, a0;
    a0 ← 1;
    return a0;
}
else{
    a0 ← a0 - 1;
    a0 ← fact(a0);
    t0 ← a0;
    a0 ← M[sp];
    ra ← M[sp + 4];
    sp ← sp + 8;
    a0 ← a0 * t0;
    return a0;
}

```

```

sp ← sp - 8;
M[sp + 4] ← ra;
M[sp] ← a0;
if (a0 < 1){
ra ← M[sp + 4];
a0 ← M[sp];
    sp ← sp + 8;
    a0 ← 1;
    return a0;
}
else{
    a0 ← a0 - 1;
    a0 ← fact(a0);
    t0 ← a0;
    a0 ← M[sp];
    ra ← M[sp + 4];
    sp ← sp + 8;
    a0 ← a0 * t0;
    return a0;
}

```


Recursive Procedure

```
sp ← sp - 8;  
M[sp + 4] ← ra;  
M[sp] ← a0;  
if (a0 < 1){  
    sp ← sp + 8;  
    a0 ← 1;  
    return a0;  
}  
else{  
    a0 ← a0 - 1;  
    a0 ← fact(a0);  
    t0 ← a0;  
    a0 ← M[sp];  
    ra ← M[sp + 4];  
    sp ← sp + 8;  
    a0 ← a0 * t0;  
    return a0;  
}
```

```
int fact(int n){  
    if (n < 1)  
        return 1;  
    else  
        return (n * fact(n - 1));  
}
```

Recursive Procedure

```
sp ← sp - 8;
M[sp + 4] ← ra;
M[sp] ← a0;
if (a0 < 1){
    sp ← sp + 8;
    a0 ← 1;
    return a0;
}
else{
    a0 ← a0 - 1;
    a0 ← fact(a0);
    t0 ← a0;
    a0 ← M[sp];
    ra ← M[sp + 4];
    sp ← sp + 8;
    a0 ← a0 * t0;
    return a0;
}
```

fact:

```
    addi    sp, sp, -8      # make room in stack for 2 more items
    sw      ra, 4(sp)      # save the return address
    sw      a0, 0(sp)      # save the argument n
    li      t1, 1          # t1 <- 1
    bge     a0, t1, L1     # if n ≥ 1, go to L1
    addi     a0, zero, 1    # return 1
    addi     sp, sp, 8      # pop two items from the stack
    jalr     zero, ra, 0    # return to the instruction after jal
L1: addi     a0, a0, -1     # subtract 1 from the argument
    jal      ra, fact      # jump to fact and save position to ra
    mv      t0, a0         # copy the return value; don't overwrite
    lw      a0, 0(sp)      # just returned from jal, restore n
    lw      ra, 4(sp)      # restore the return address
    addi     sp, sp, 8      # pop two items from the stack
    mul      a0, a0, t0     # return n * fact(n - 1)
    jalr     zero, ra, 0    # return to the caller
```

```
int fact(int n){
    if (n < 1)
        return 1;
    else
        return (n * fact(n - 1));
}
```

```

int fact(int n){
    save ra;
    if save ra:
        if (sp ← sp - 8;
            M[sp + 4] ← ra;
            M[sp] ← a0;
            if (a0 < 1){
                sp ← sp + 8;
                a0 ← 1;
                return a0;
            }
            else{
                a0 ← a0 - 1;
                a0 ← fact(a0);
                t0 ← a0;
                a0 ← M[sp];
                ra ← M[sp + 4];
                sp ← sp + 8;
                a0 ← a0 * t0;
                return a0;
            }
        }
    }
}

```

fact:

```

    addi    sp, sp, -8
    sw      ra, 4(sp)
    sw      a0, 0(sp)
    li      t1, 1
    bge     a0, t1, L1
    addi    a0, zero, 1
    addi    sp, sp, 8
    jalr    zero, ra, 0
L1: addi    a0, a0, -1
    jal     ra, fact
    mv      t0, a0
    lw      a0, 0(sp)
    lw      ra, 4(sp)
    addi    sp, sp, 8
    mul     a0, a0, t0
    jalr    zero, ra, 0

```

Recap

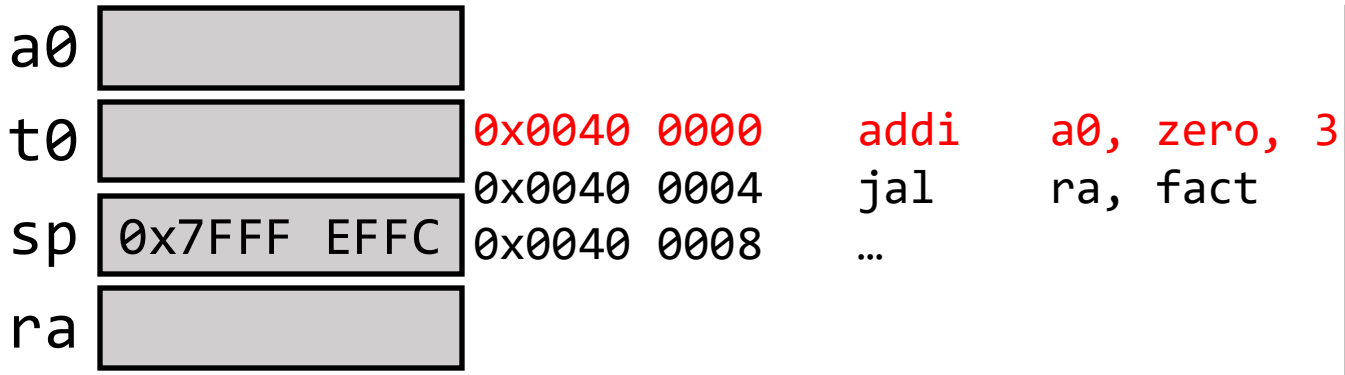
The simulation will not track t1 because it is only stores the value 1 for these instructions:

```
li          t1, 1          # t1 <- 1
bge         a0, t1, L1      # if n ≥ 1, go to L1
```

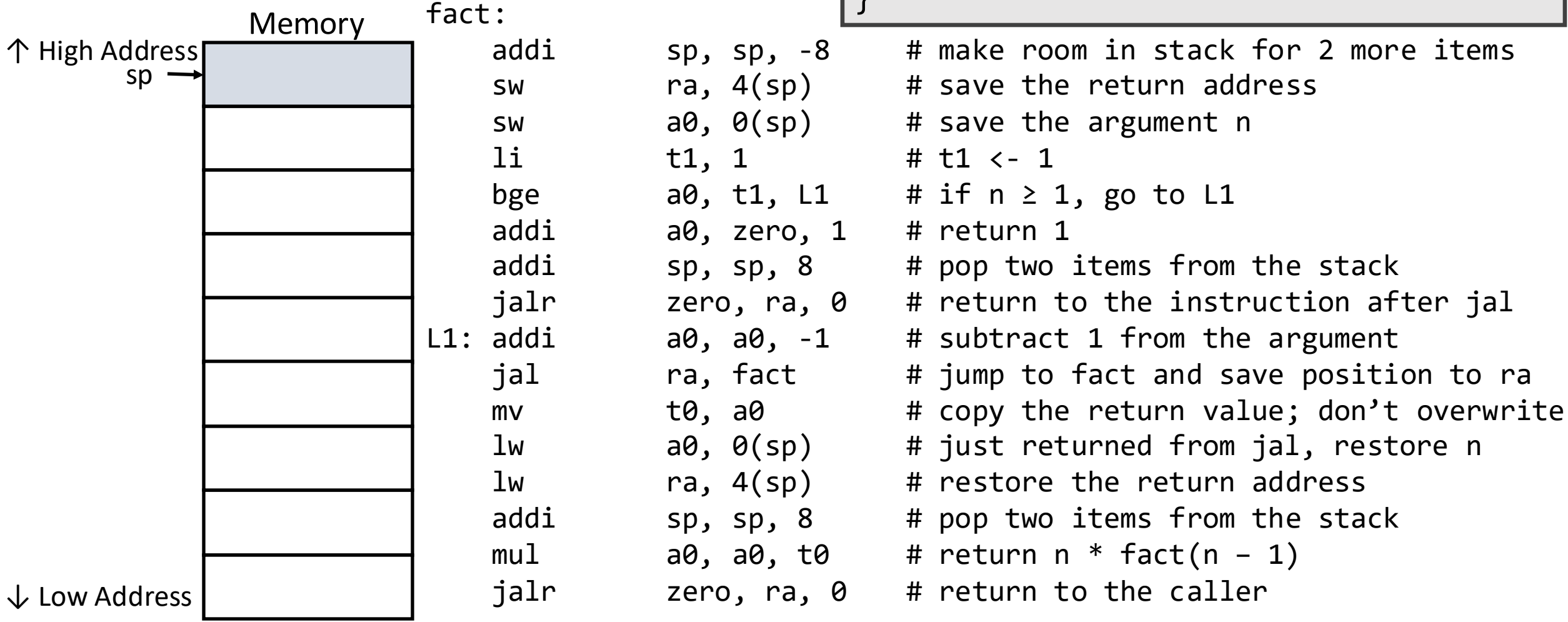
$n \geq 1 \equiv n > 0$ if n is an integer

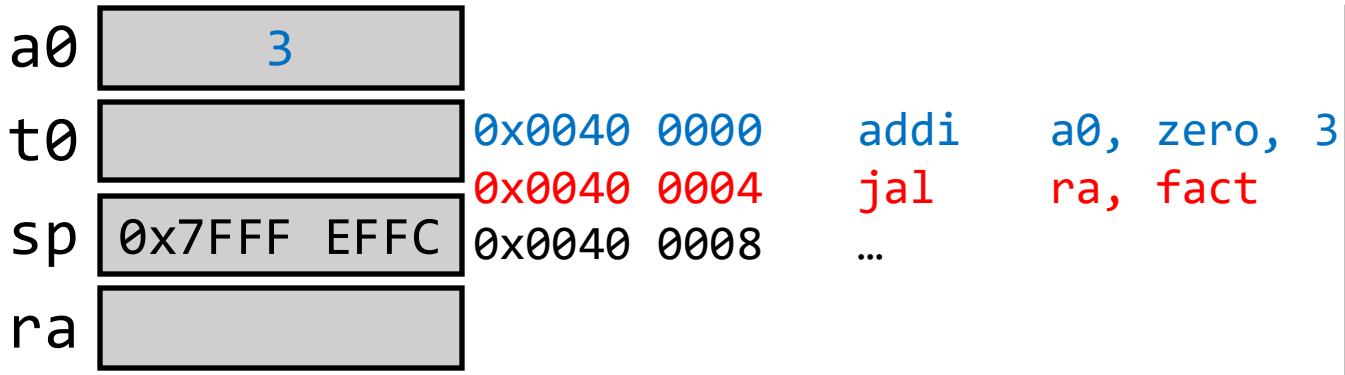
The two instructions above could be replaced with:

```
blt         zero, a0, L1    # if n > 0, go to L1
```

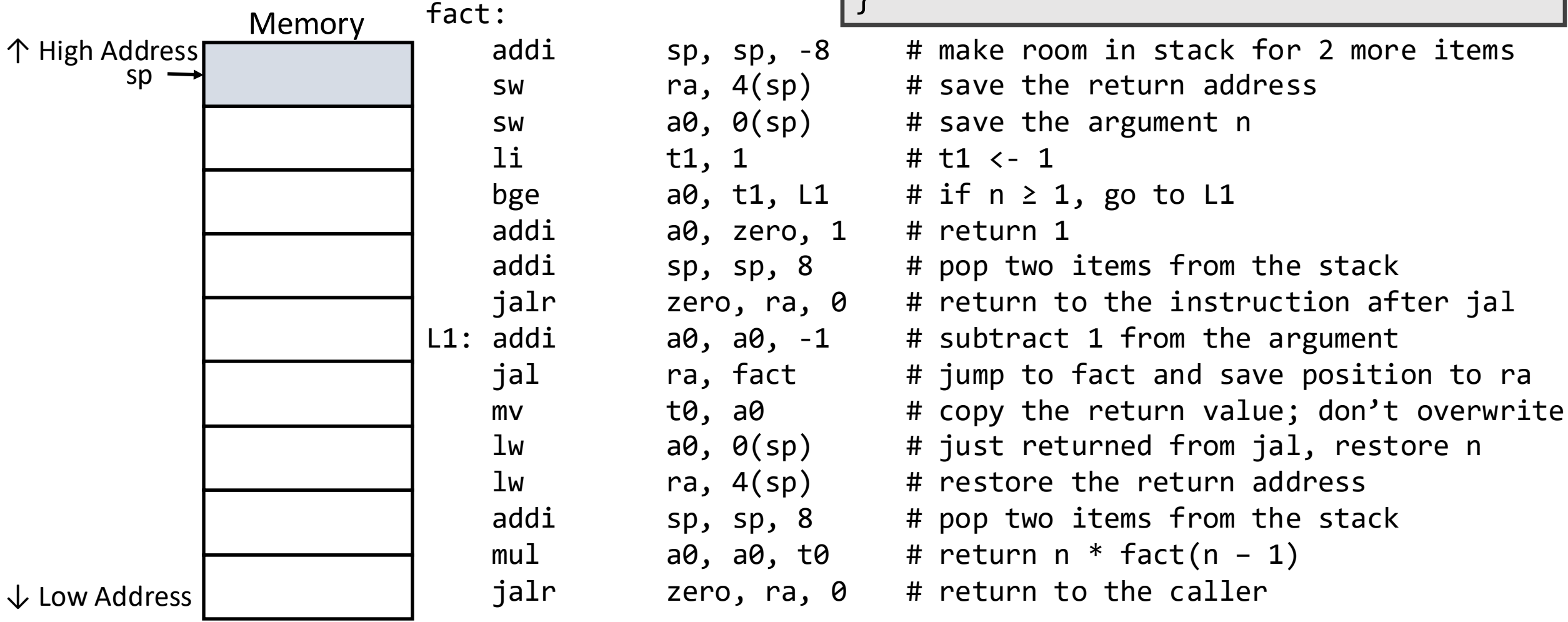


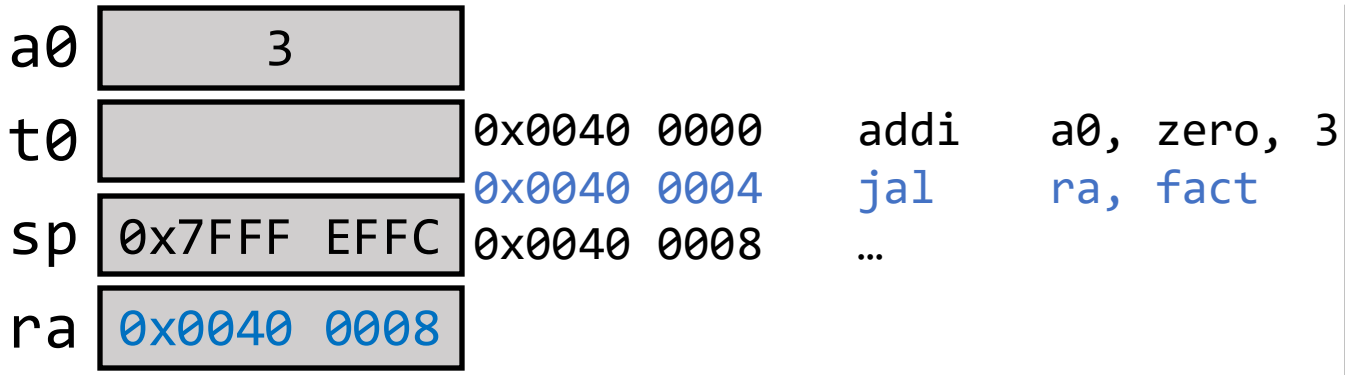
```
int fact(int n){
    if (n < 1)
        return 1;
    else
        return (n * fact(n - 1));
}
```



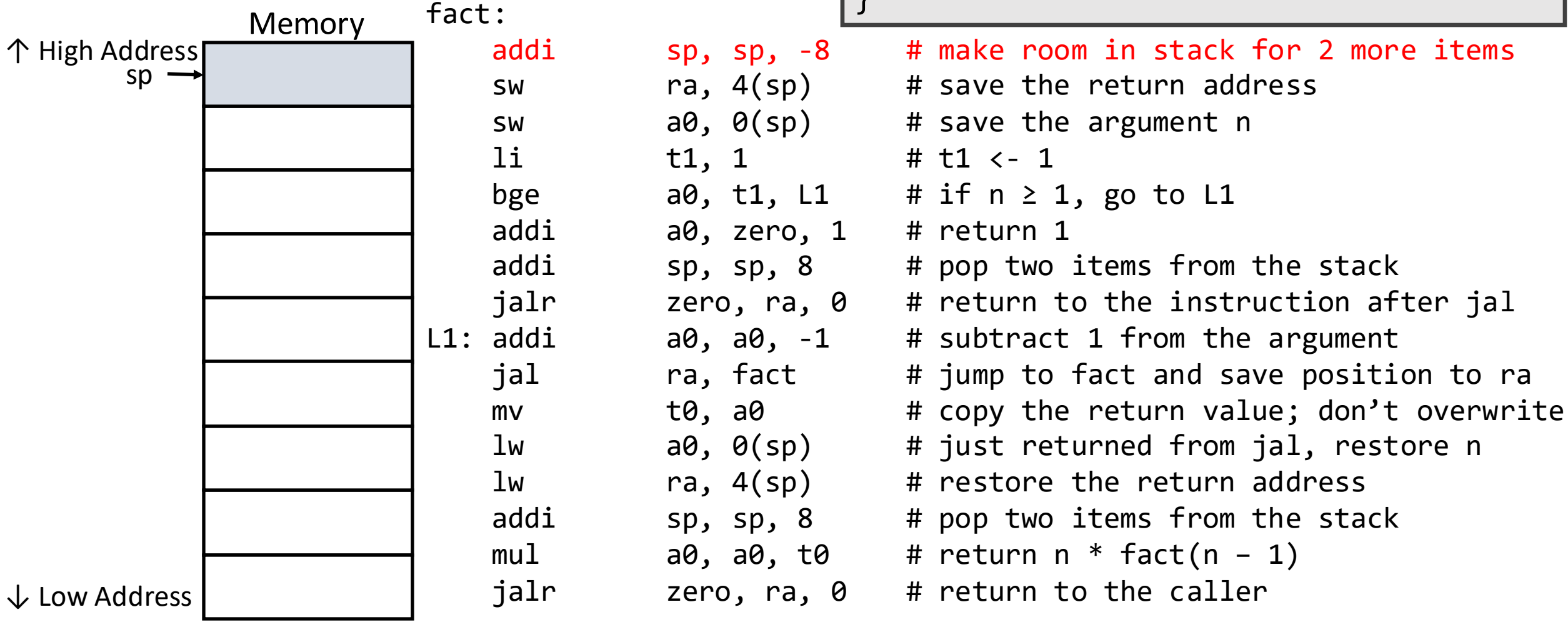


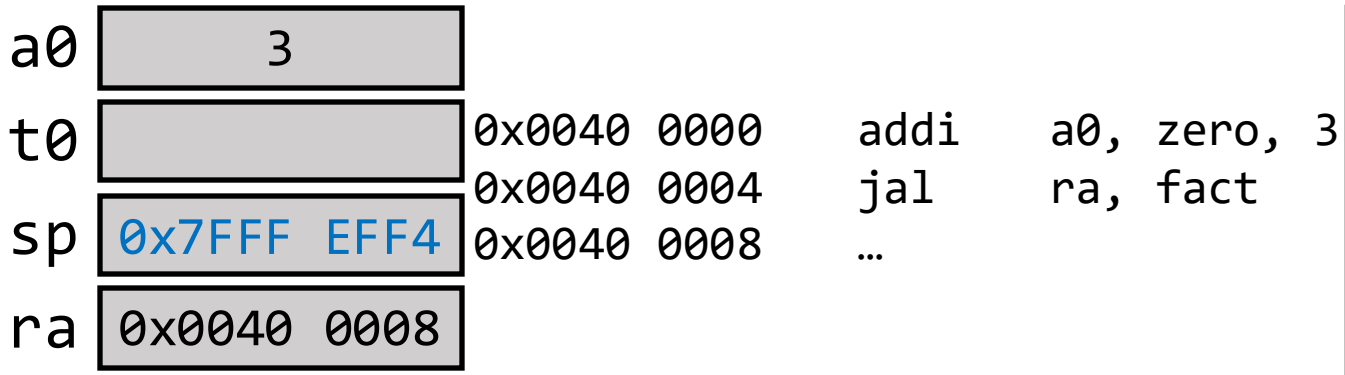
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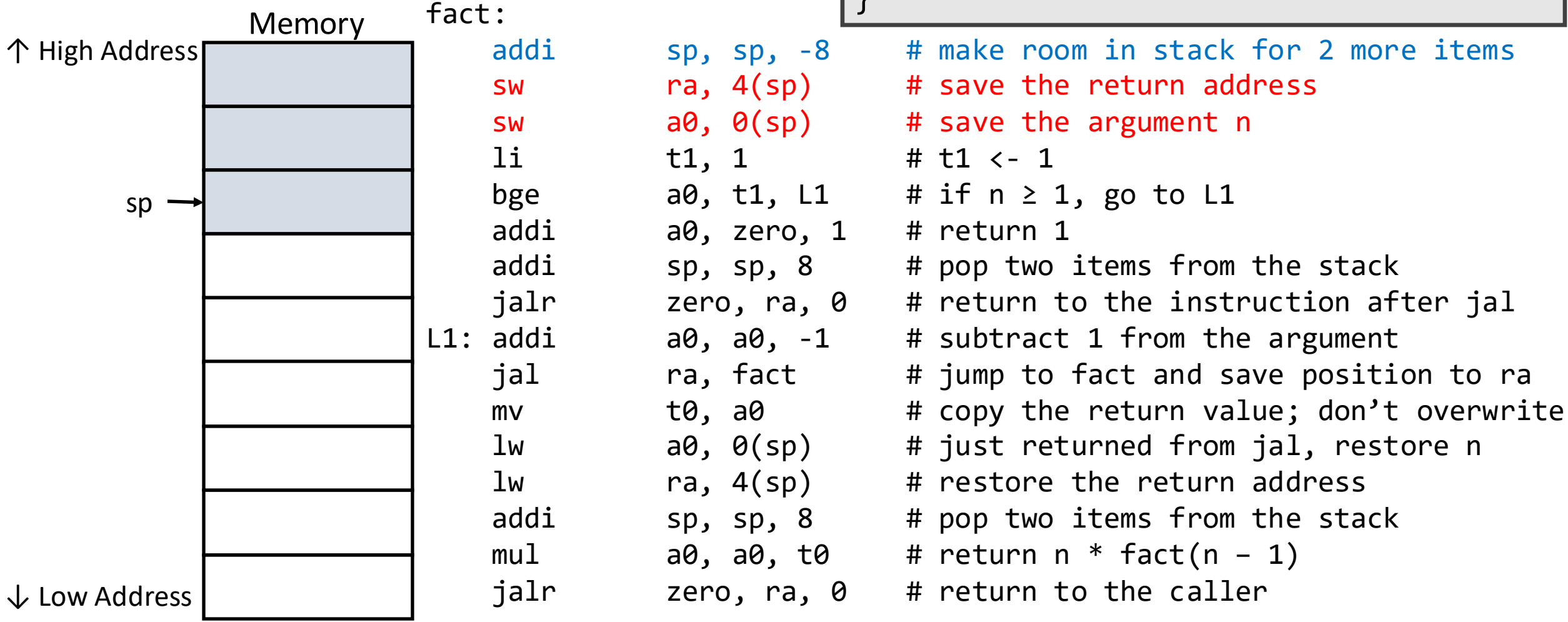


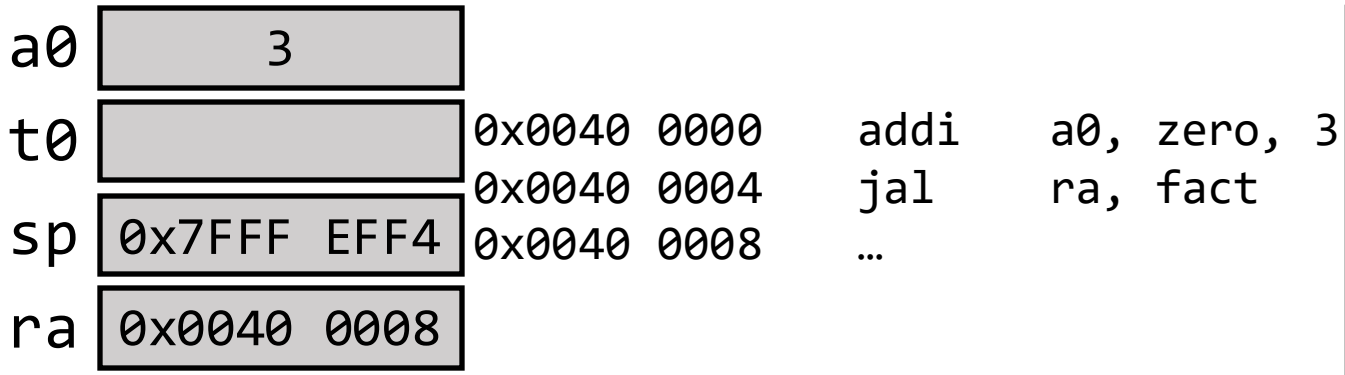
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        return 1;
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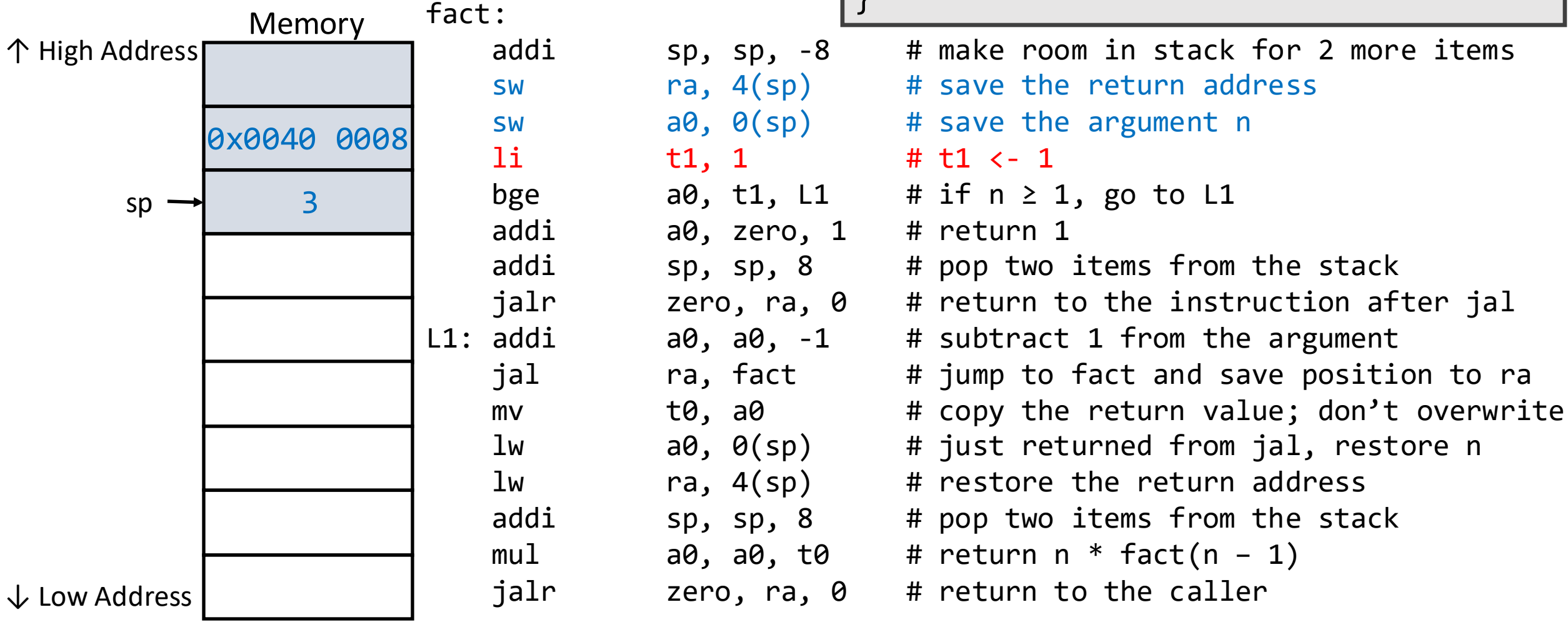


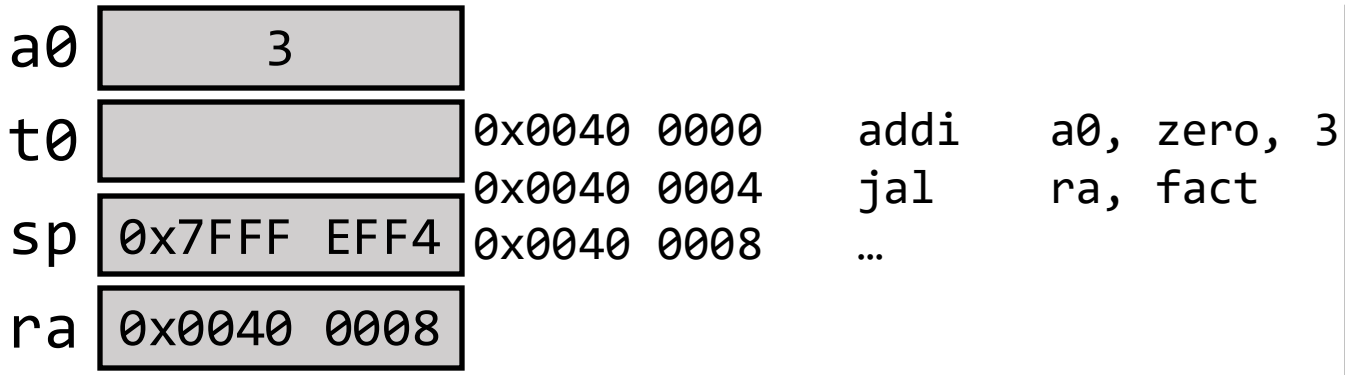
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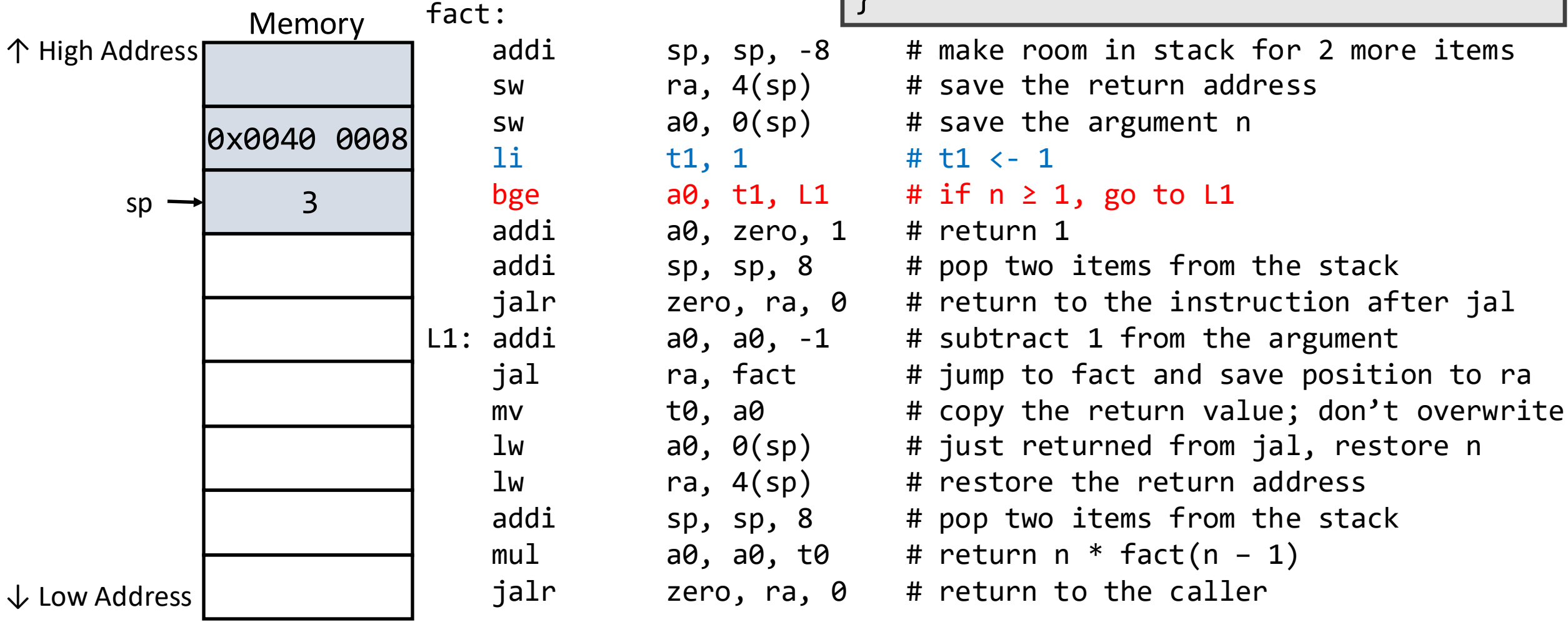


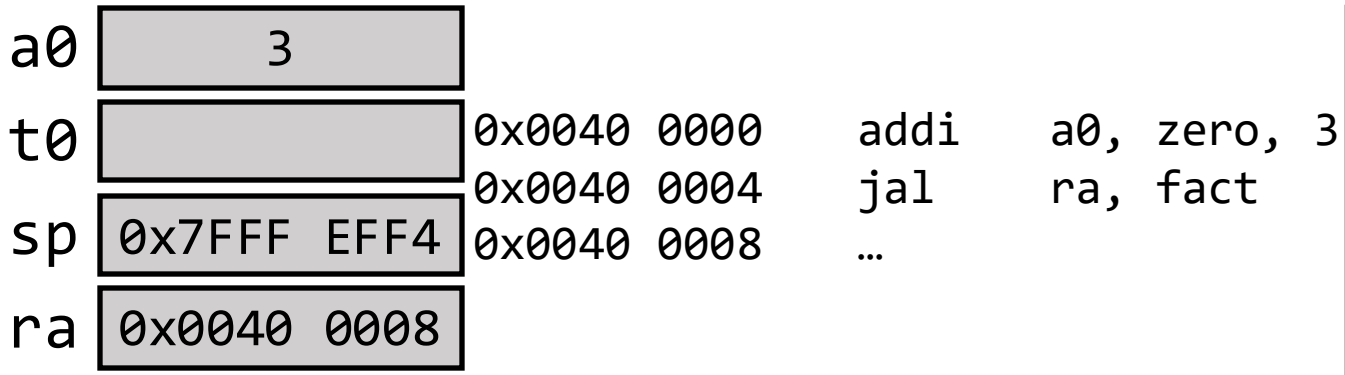
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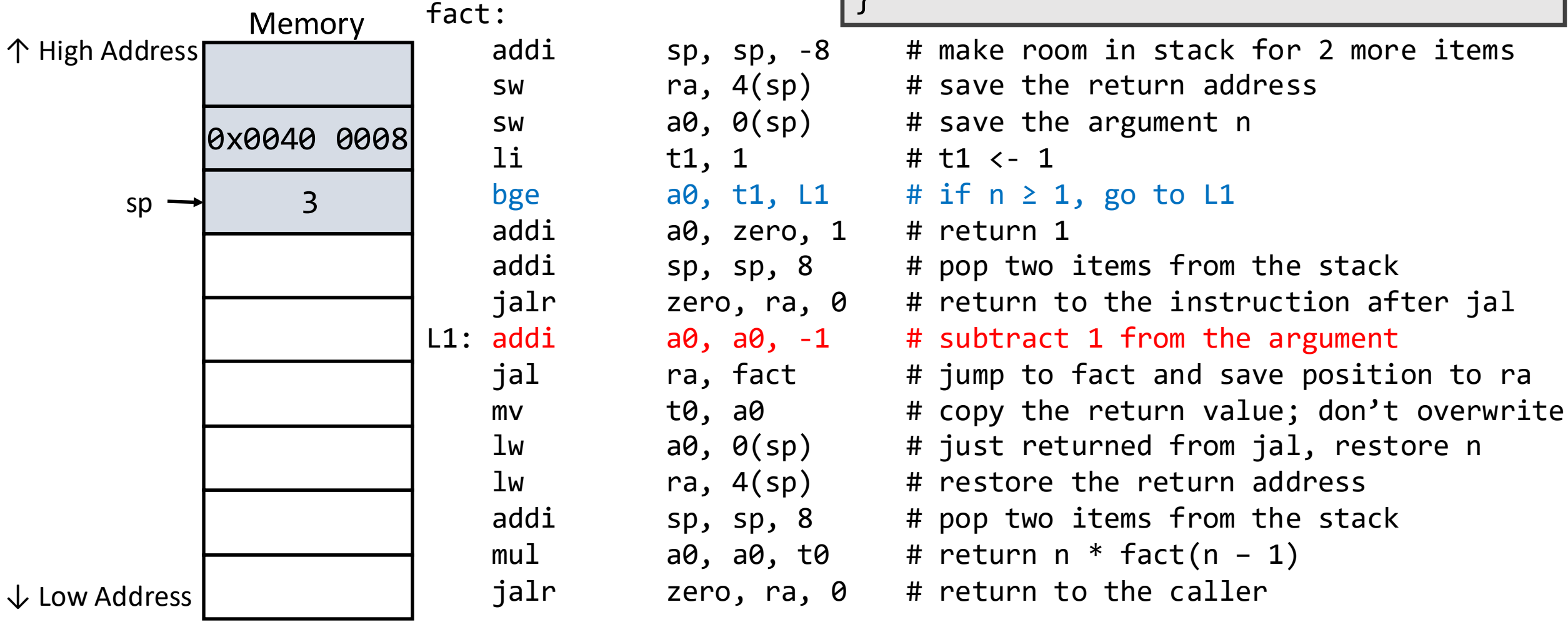


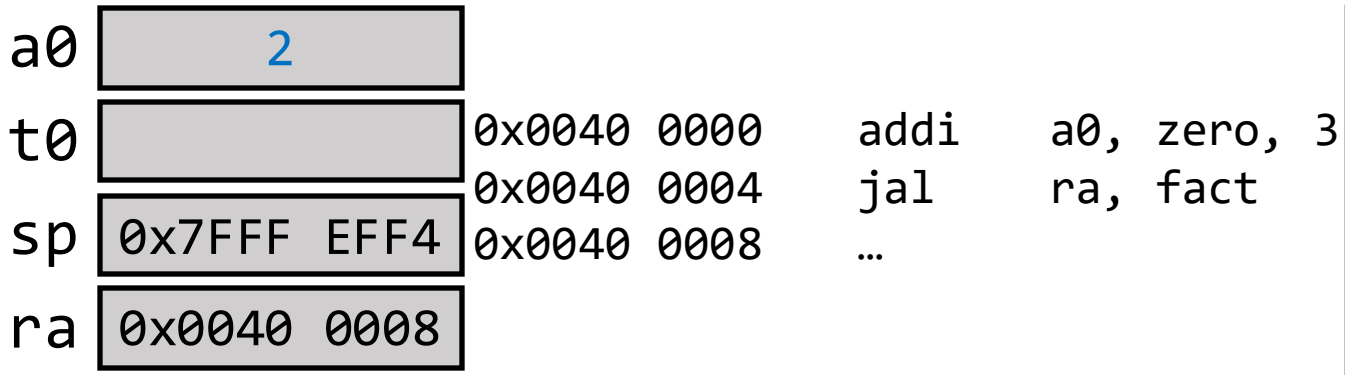
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int fact(int n){
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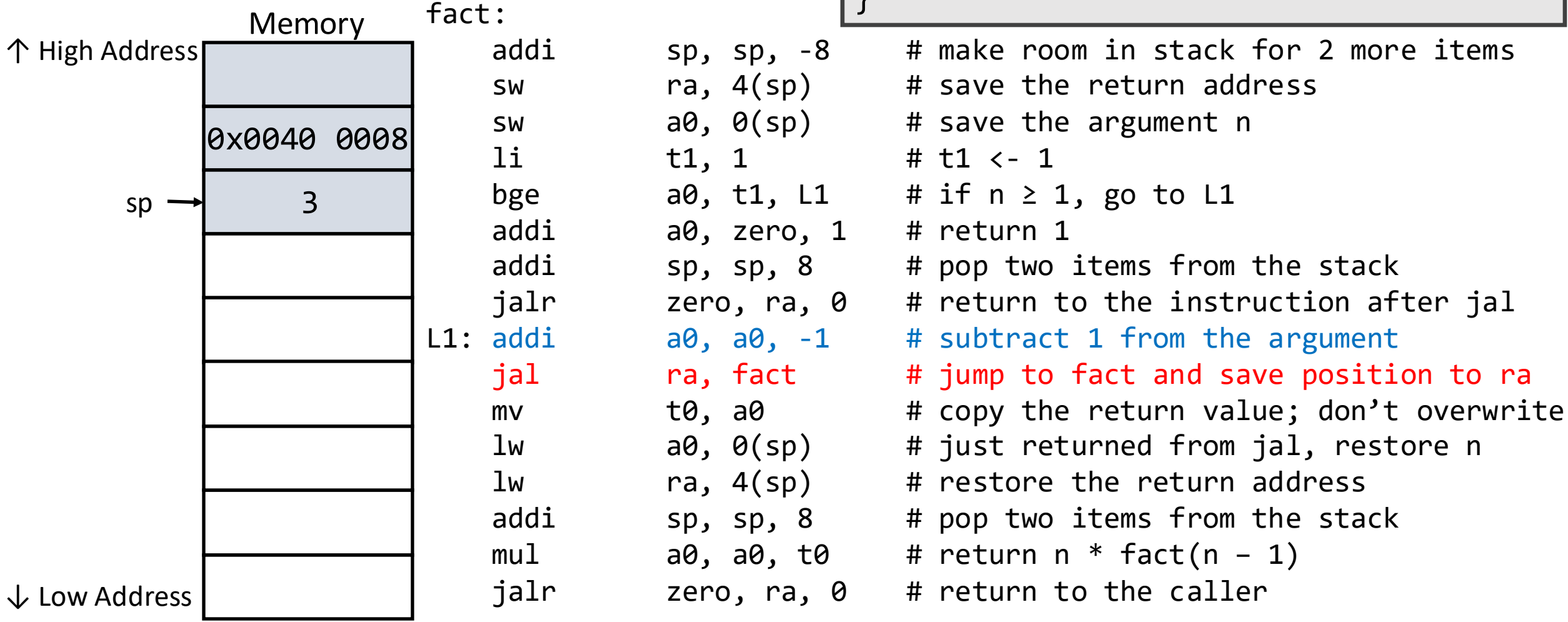


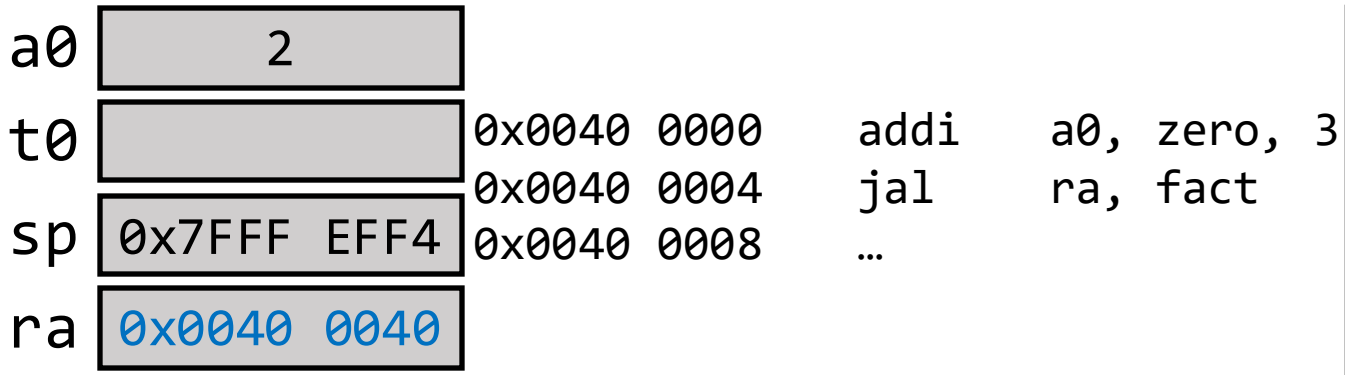
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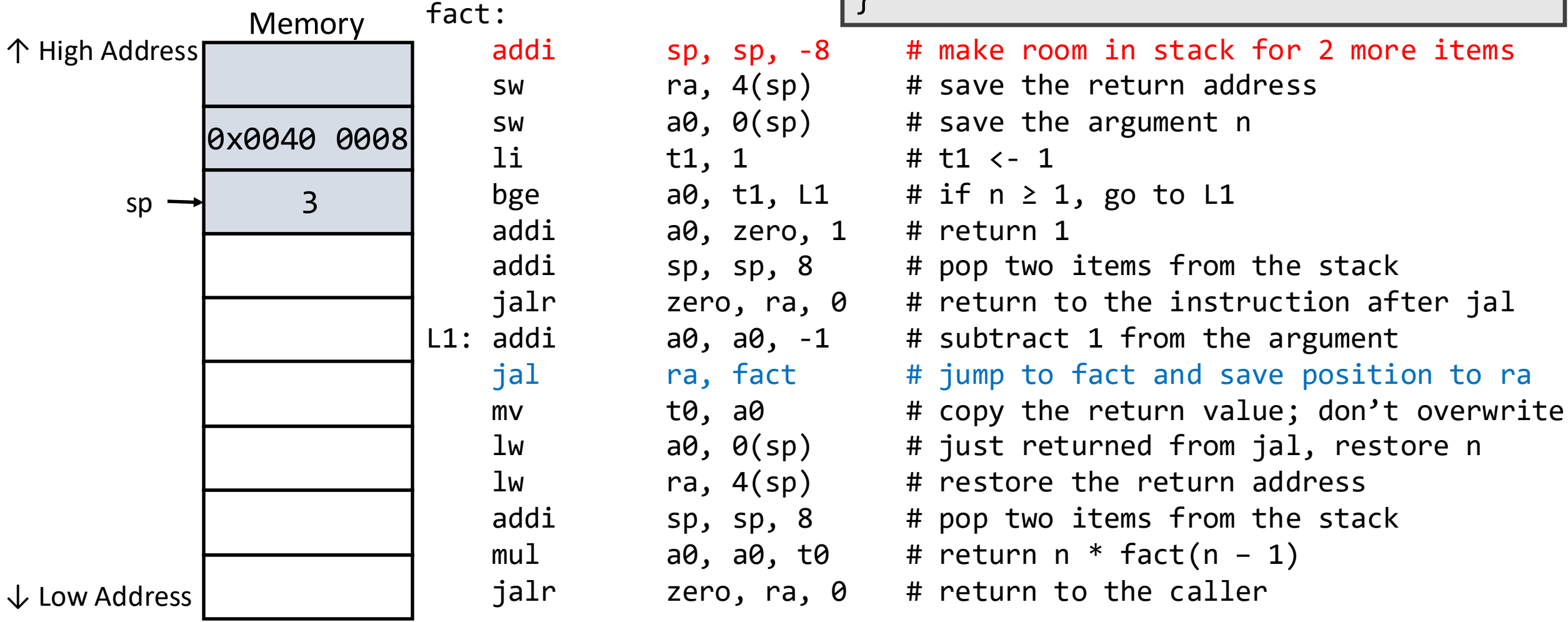


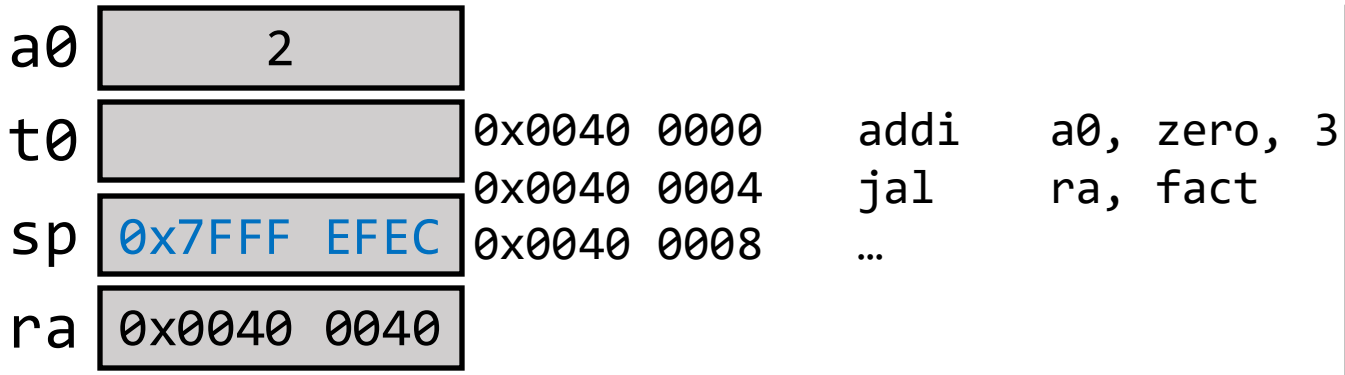
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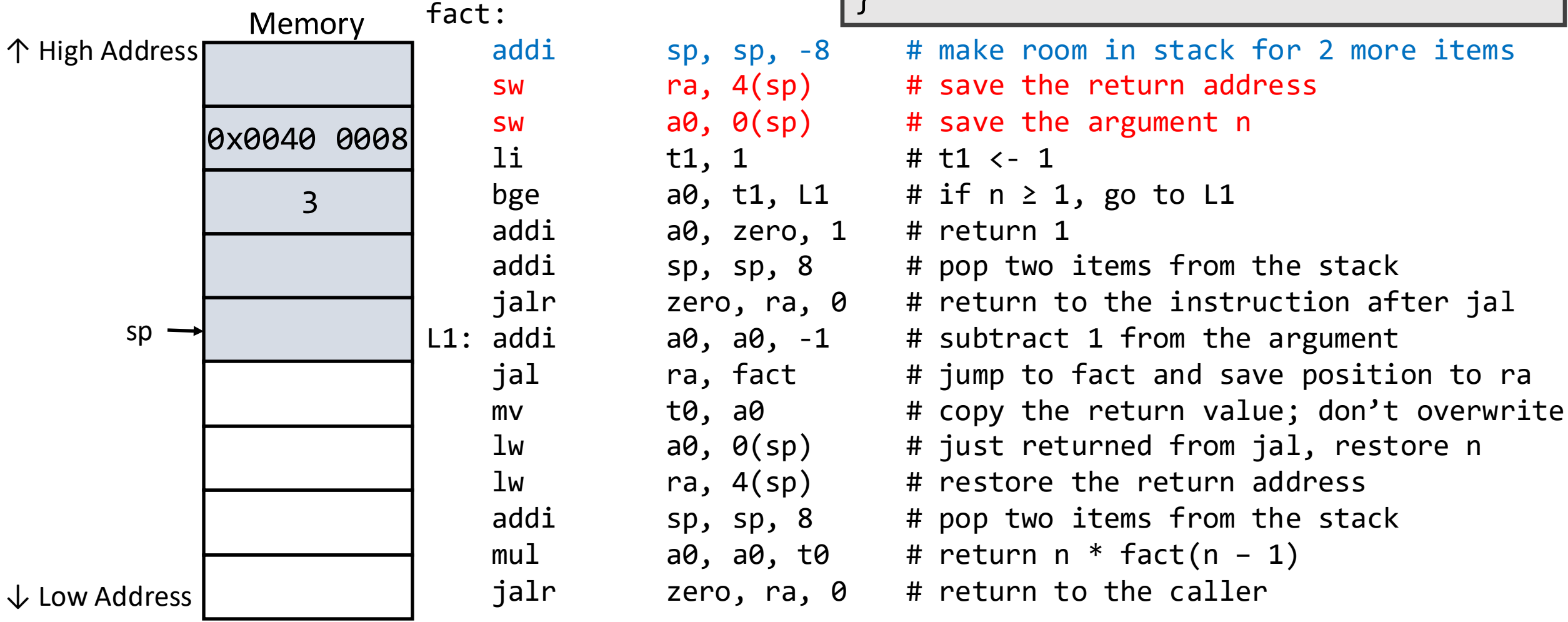


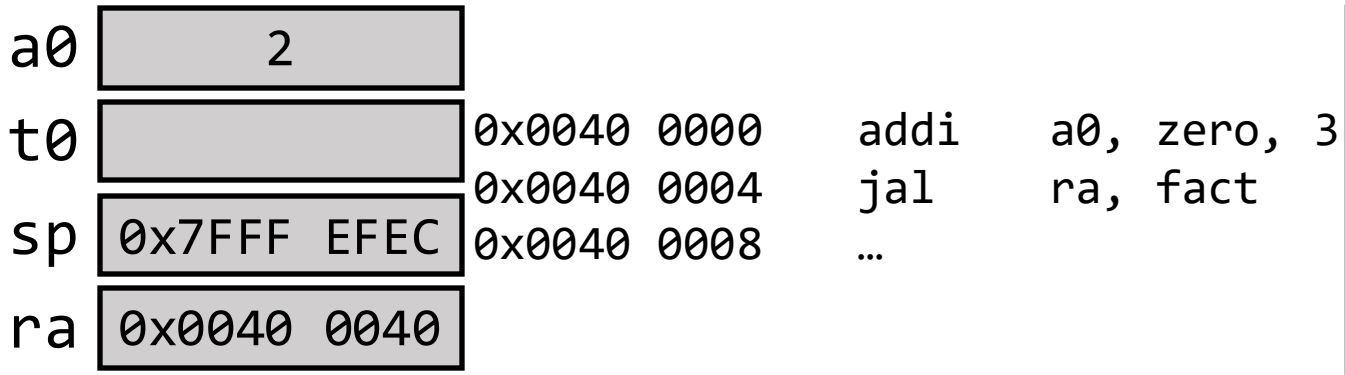
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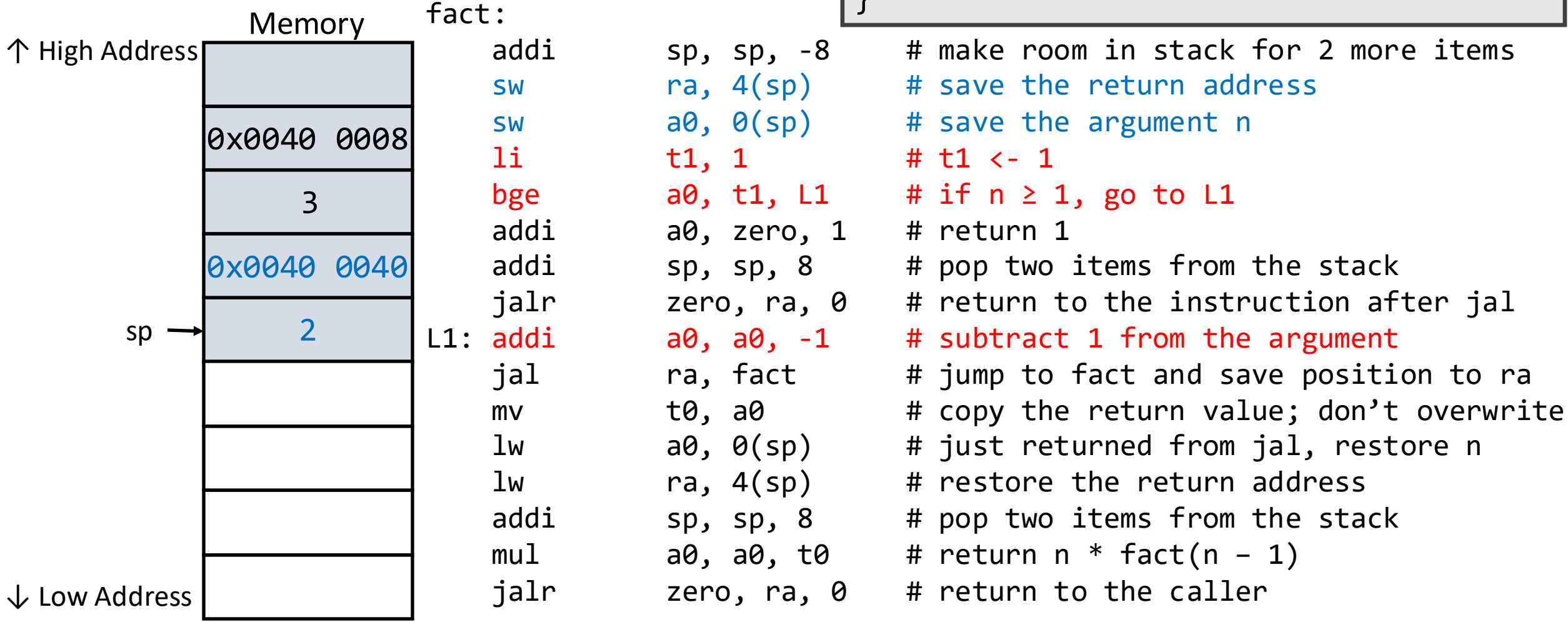


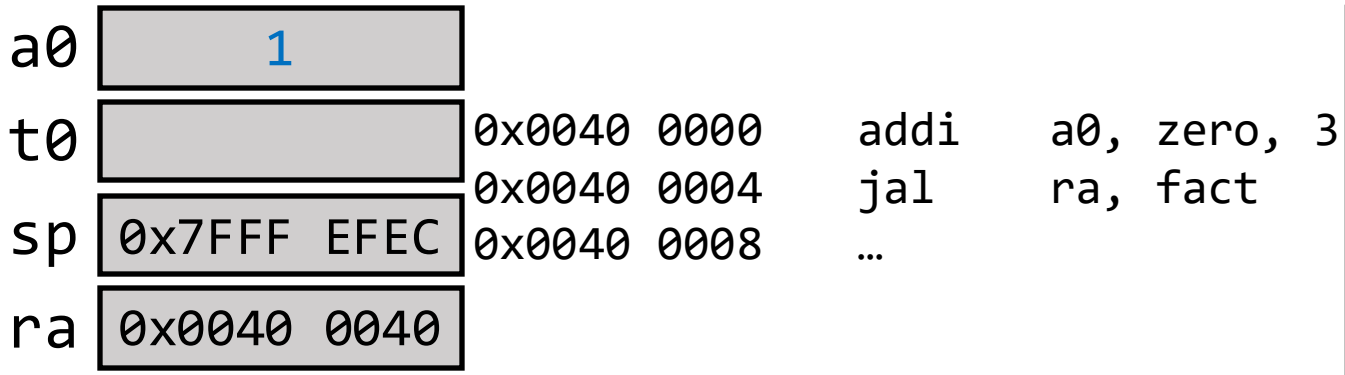
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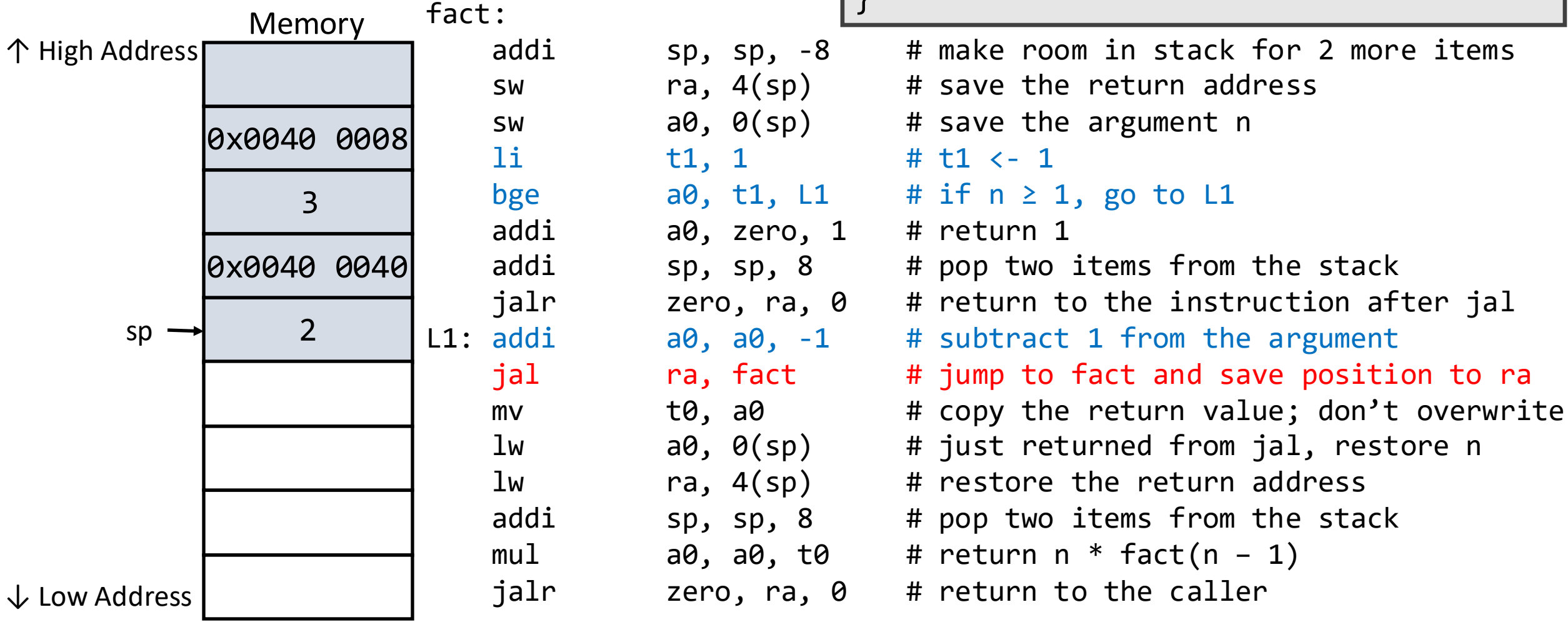


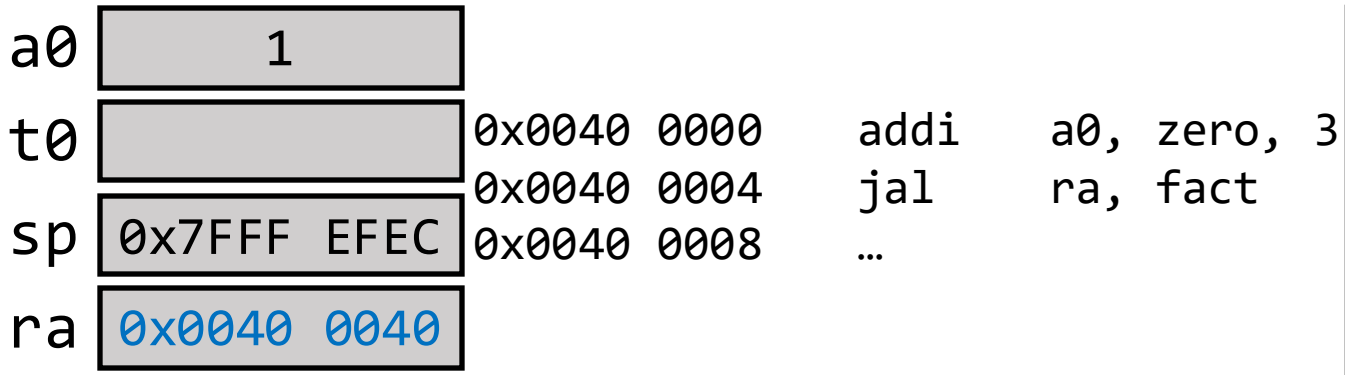
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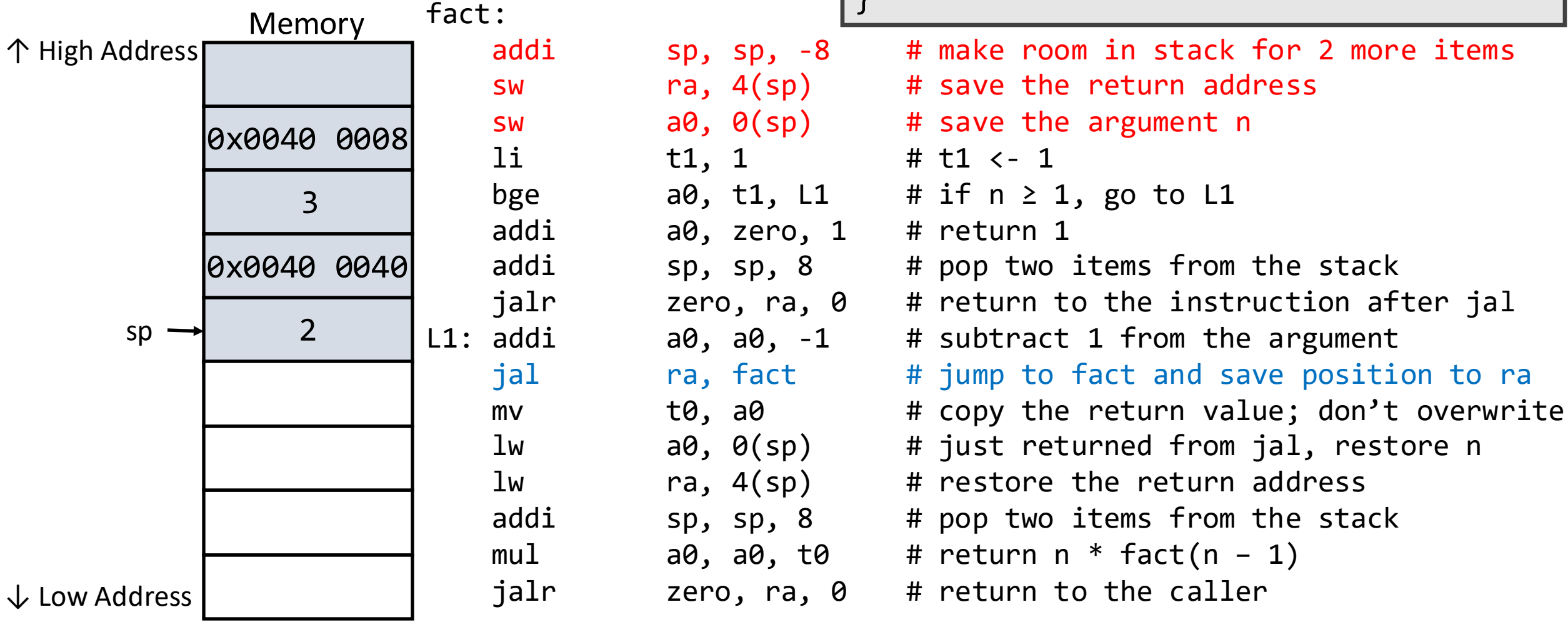


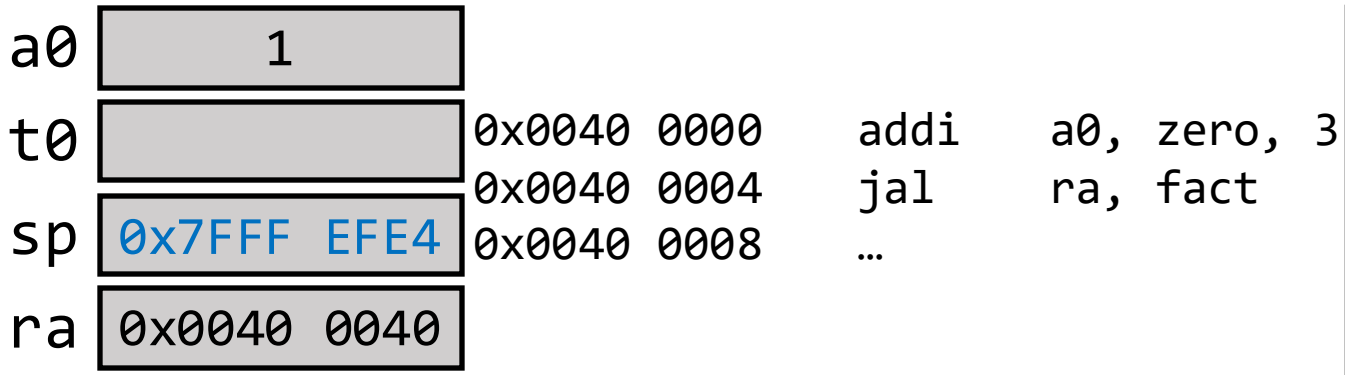
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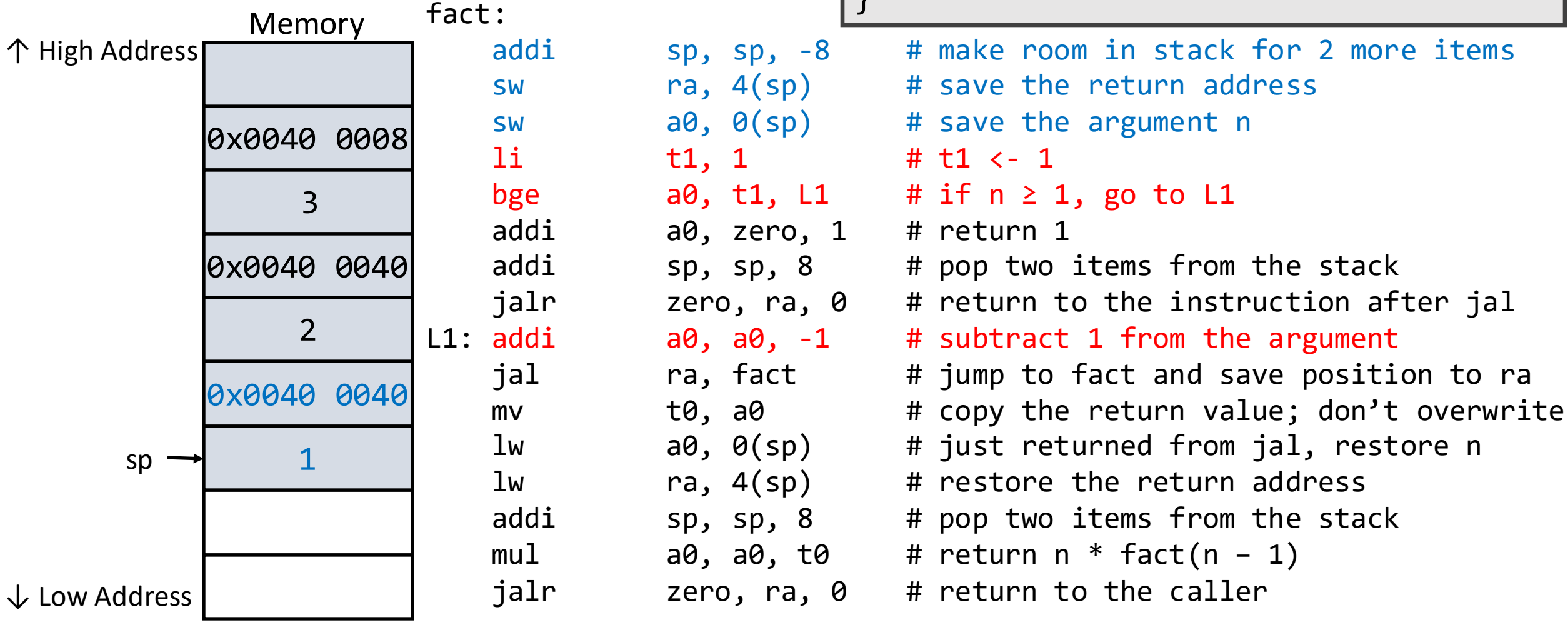


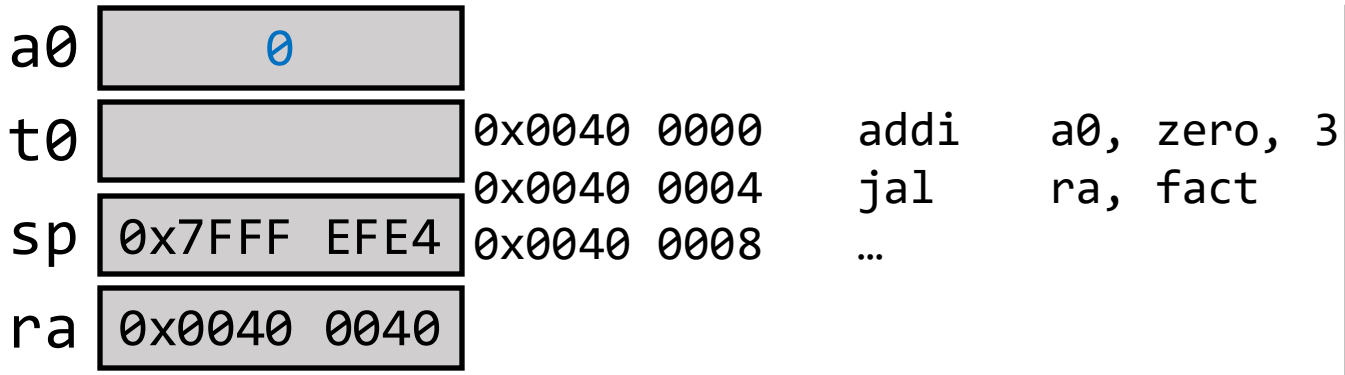
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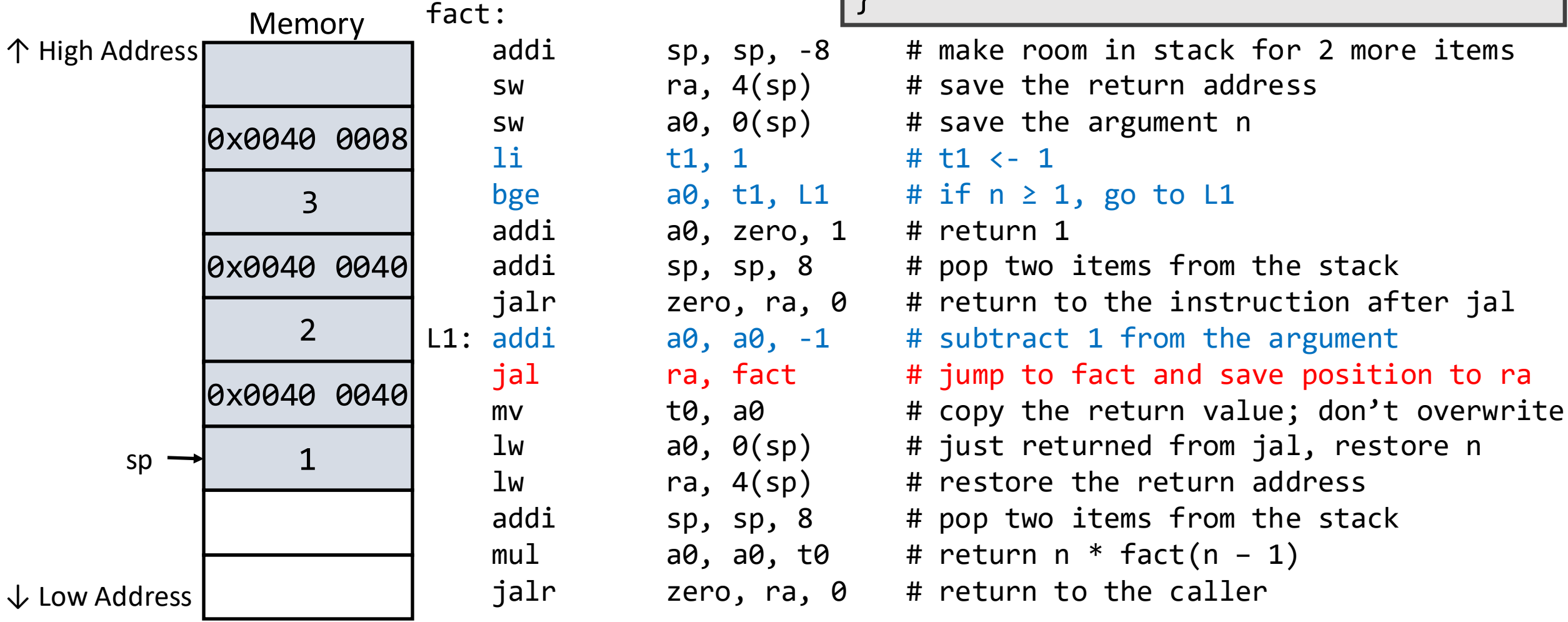


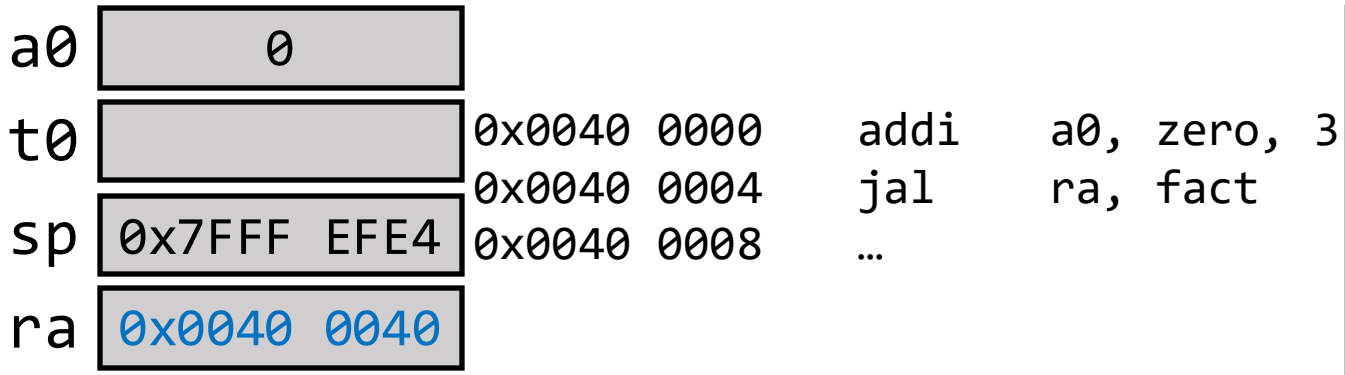
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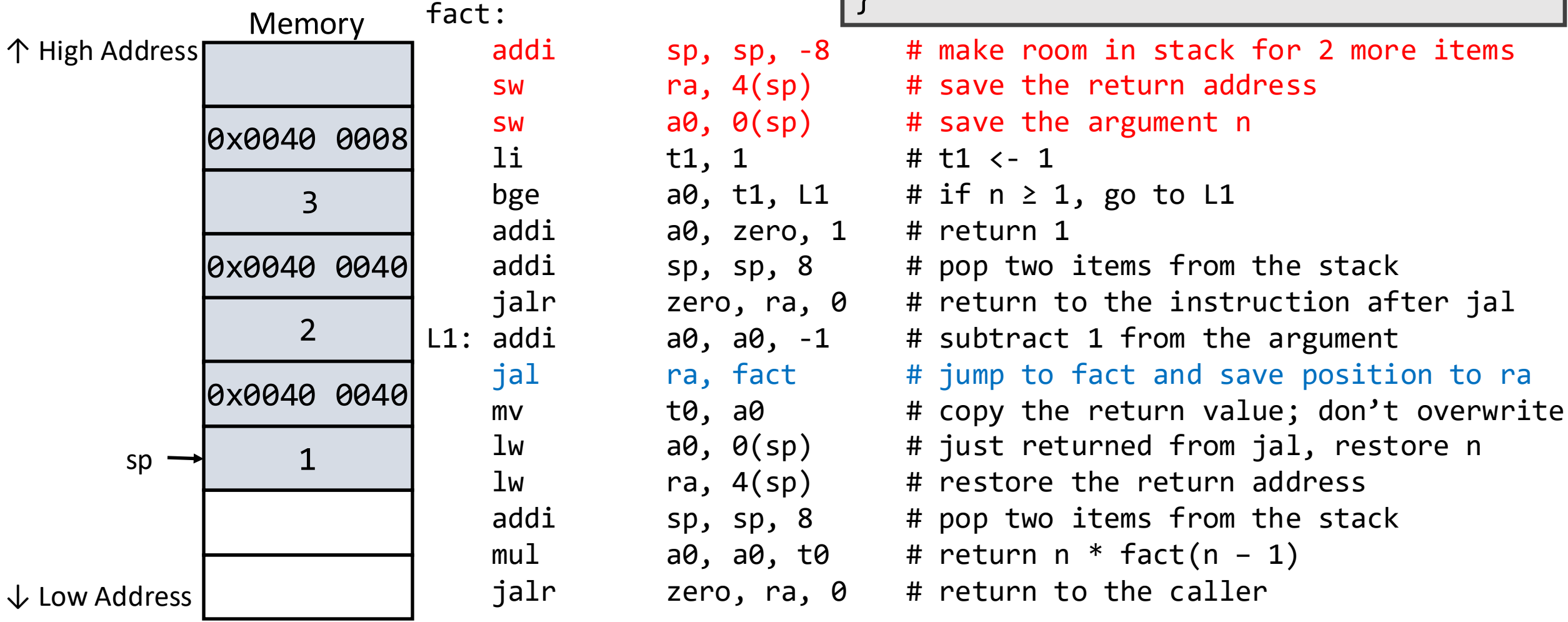


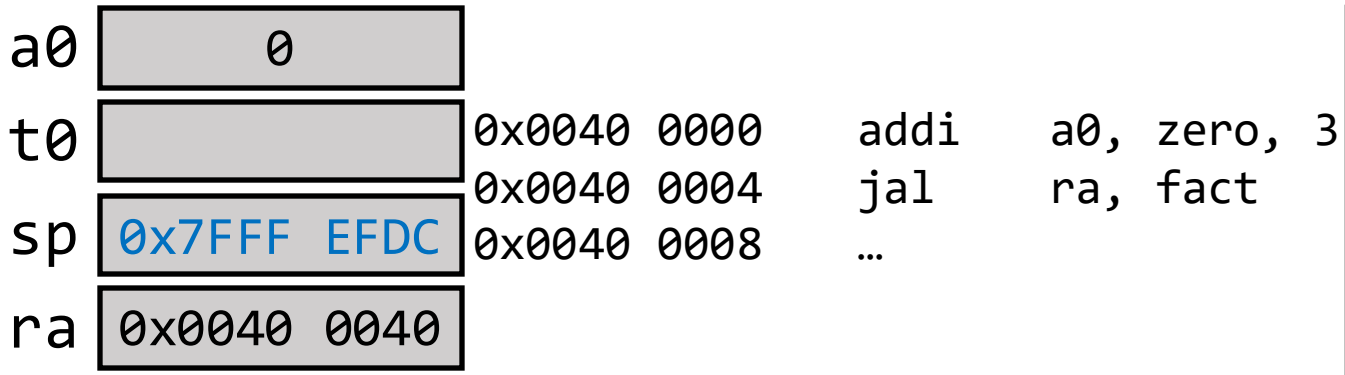
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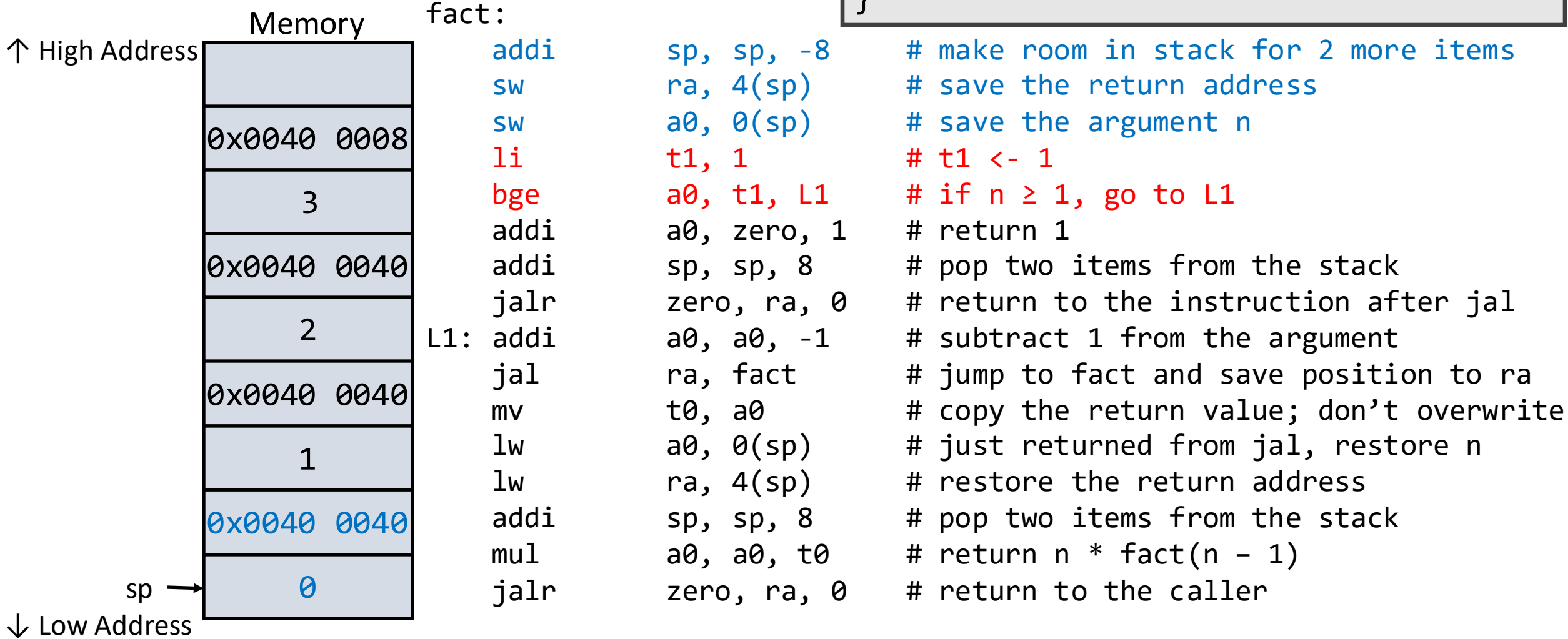


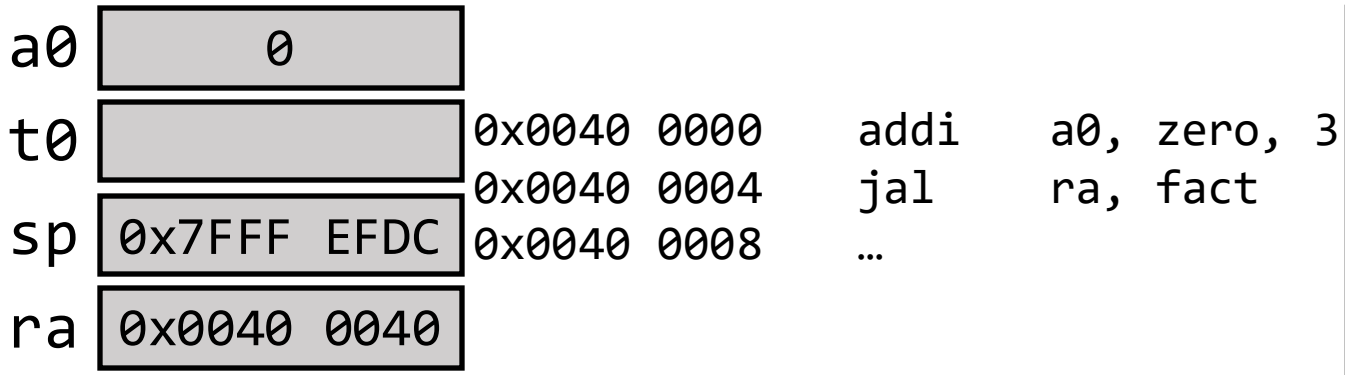
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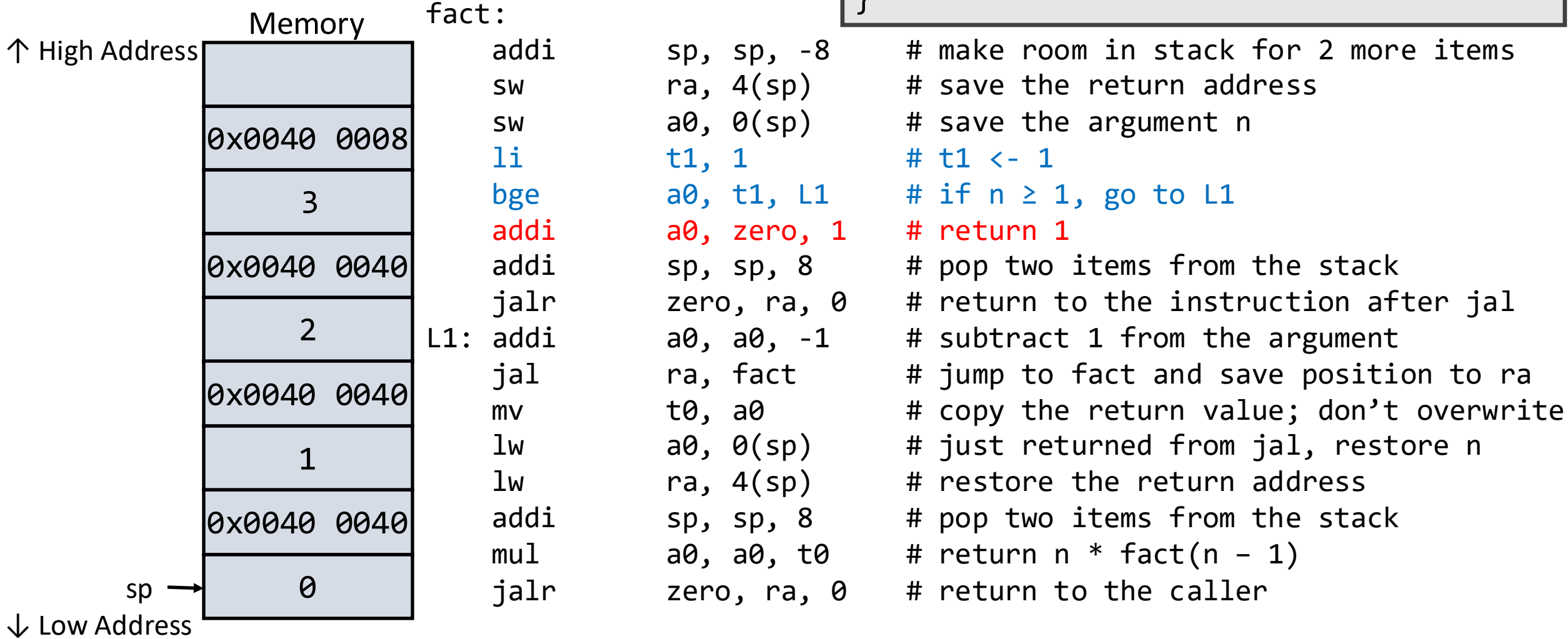


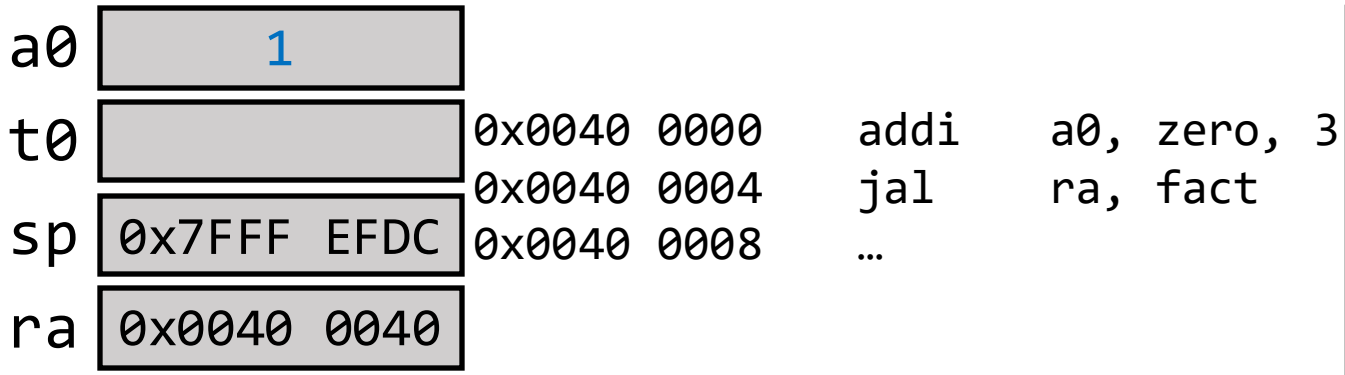
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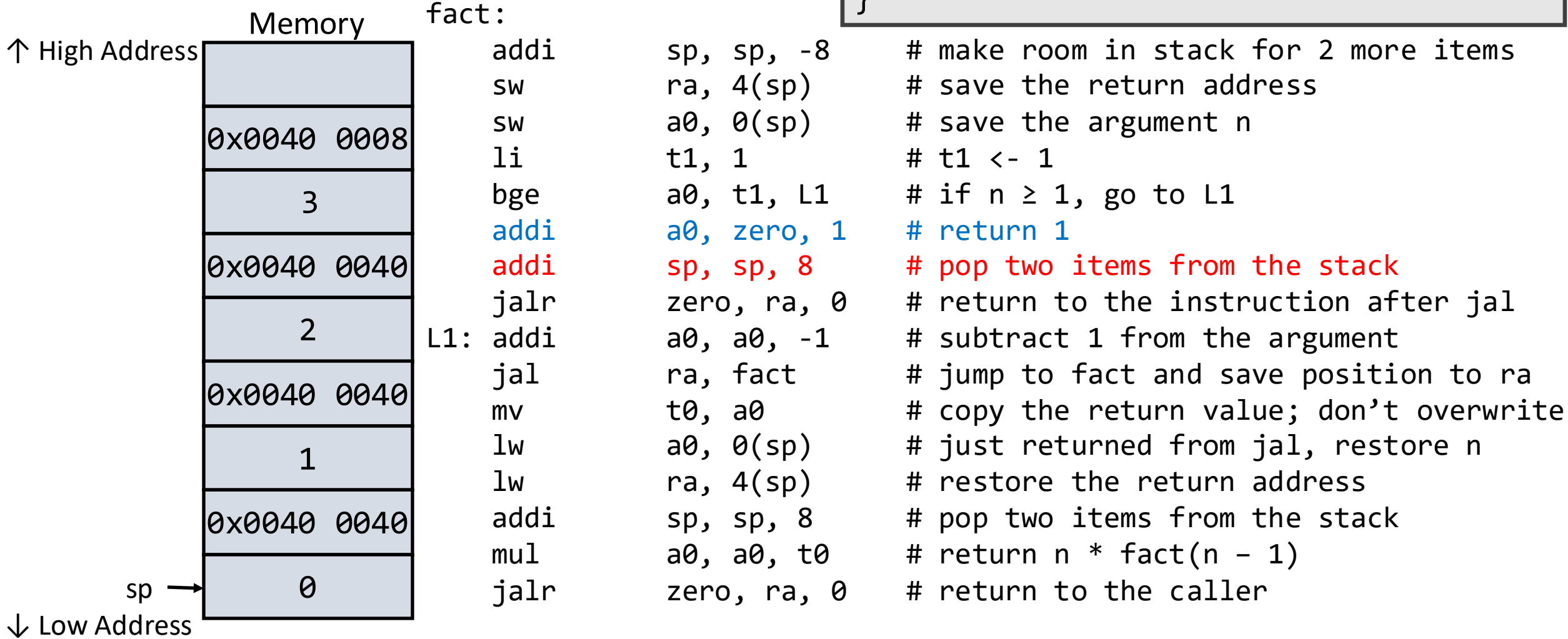


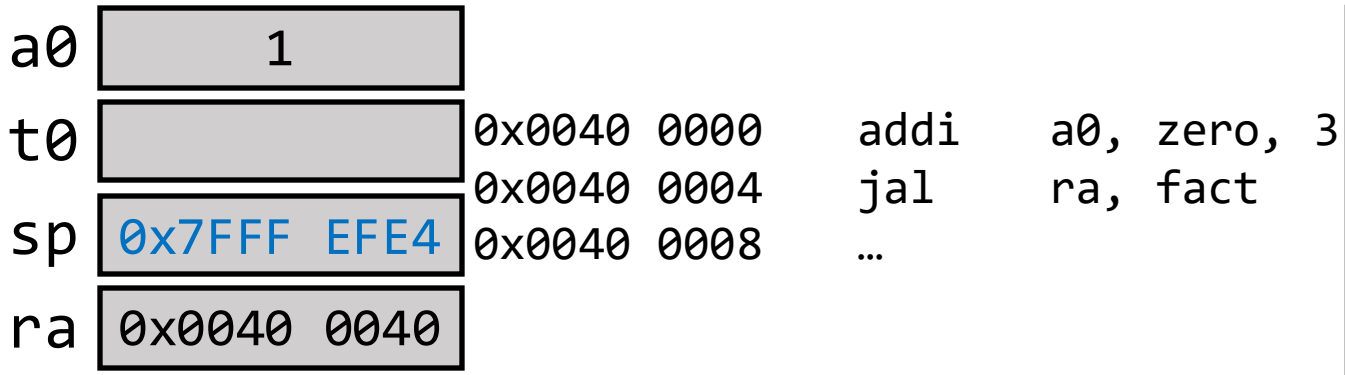
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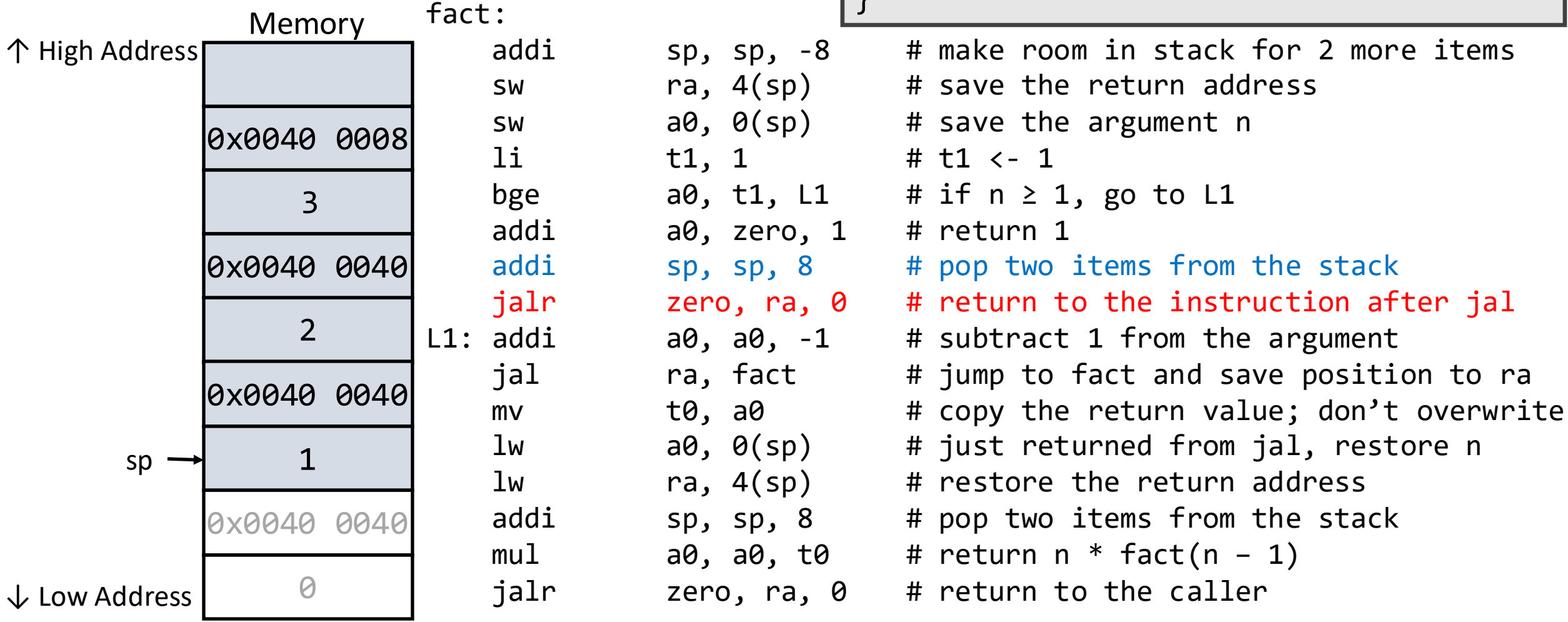


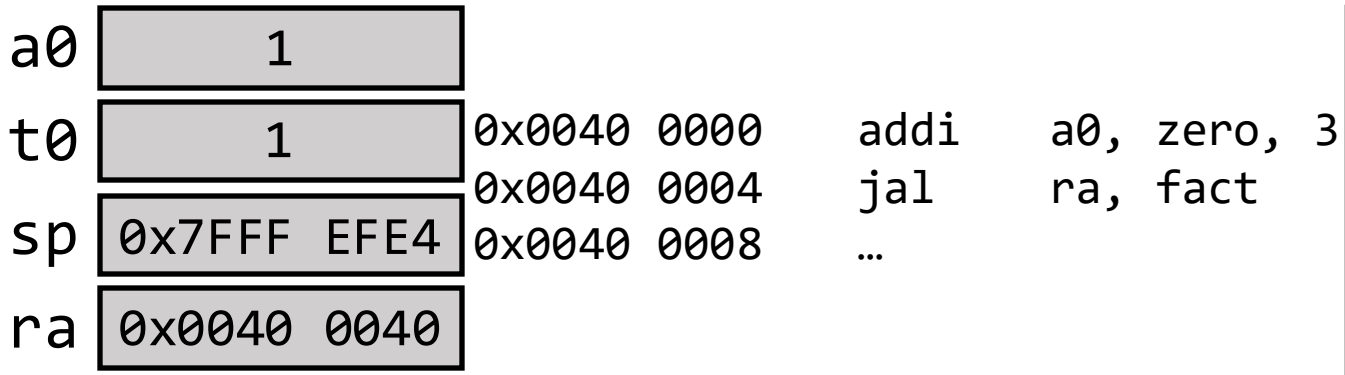
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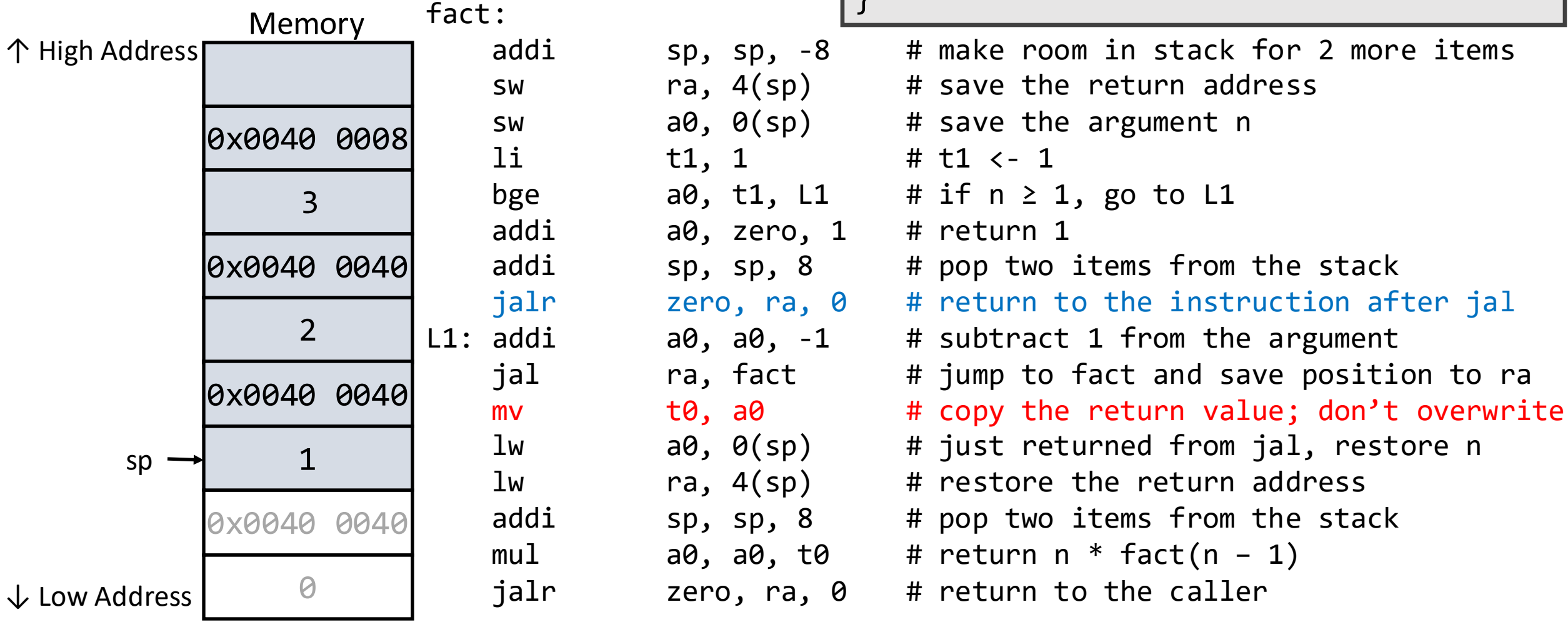


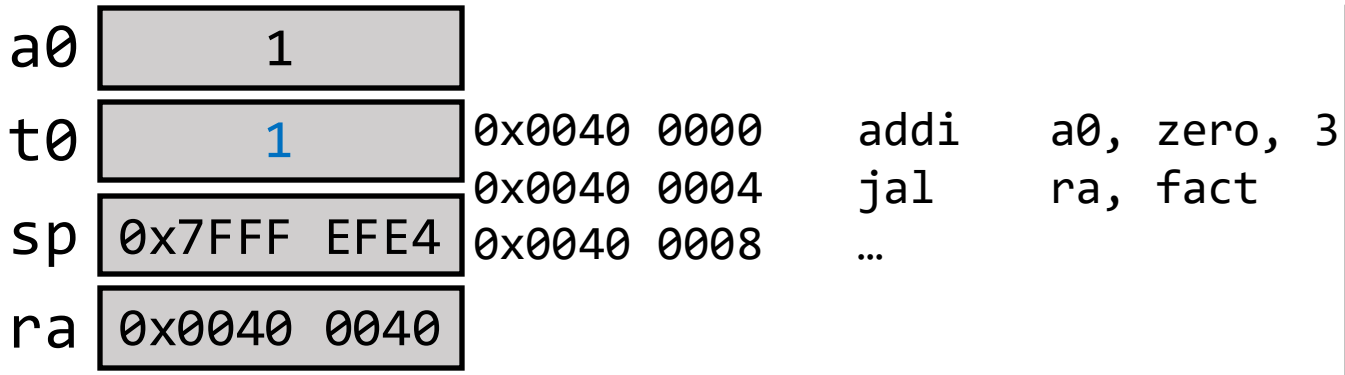
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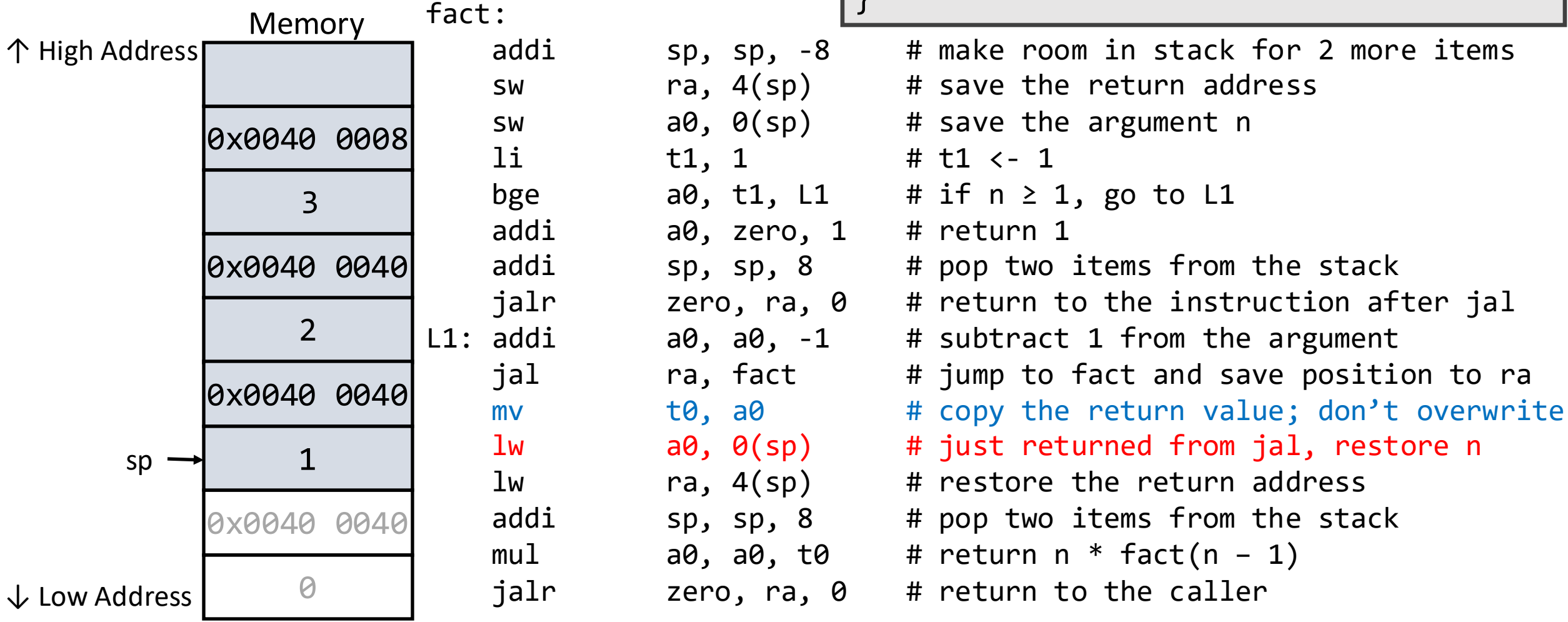


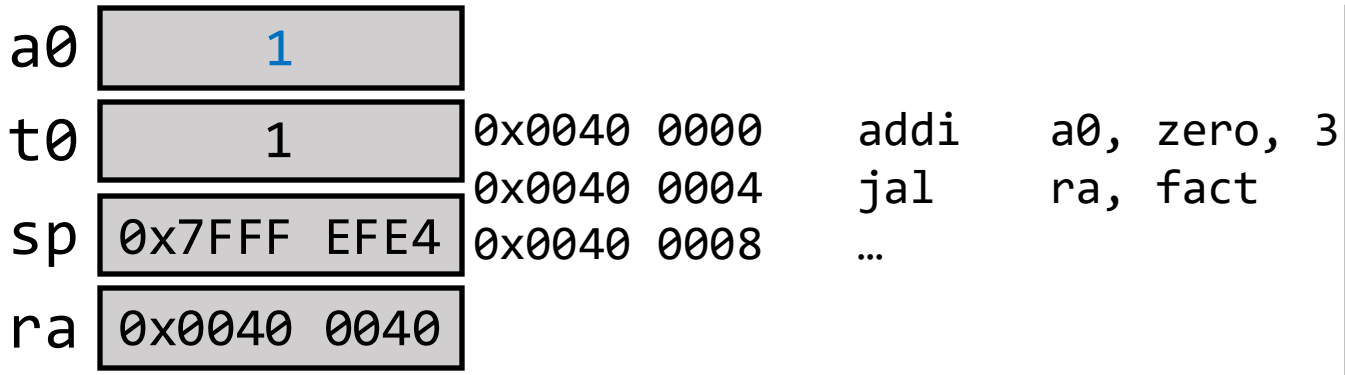
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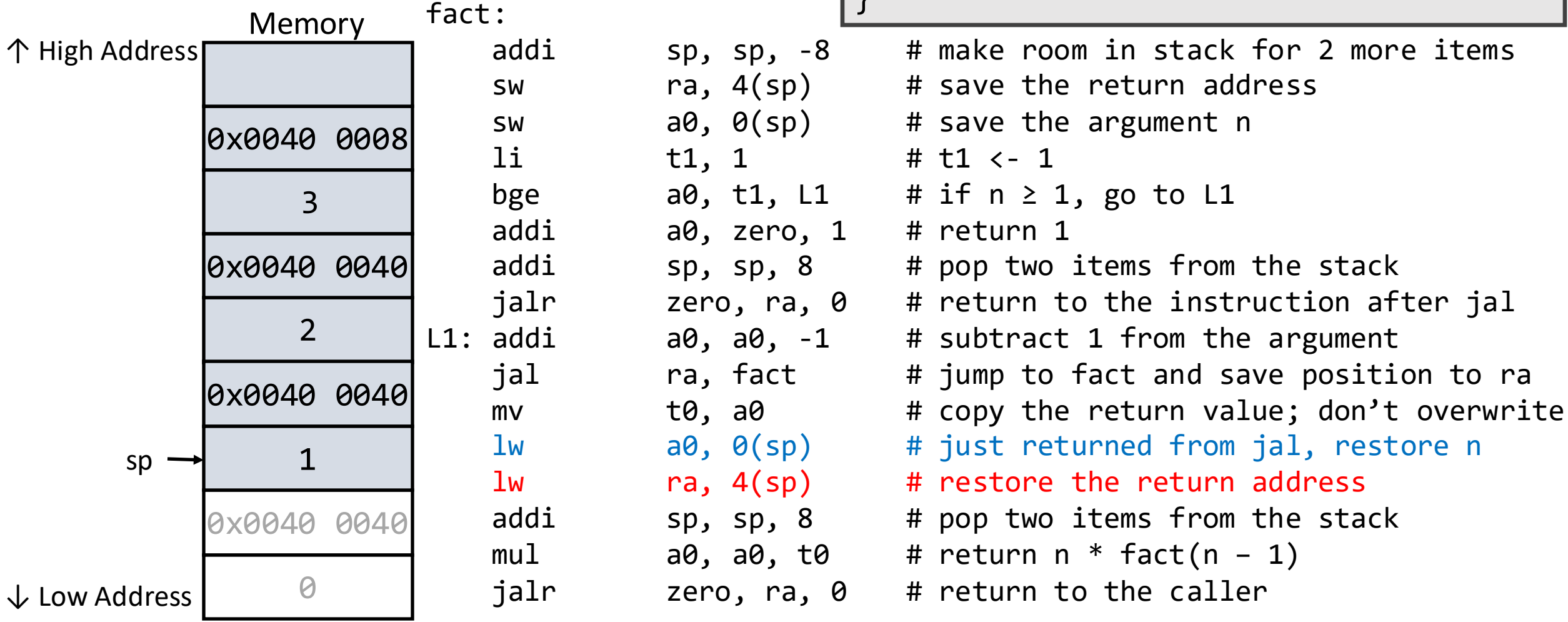


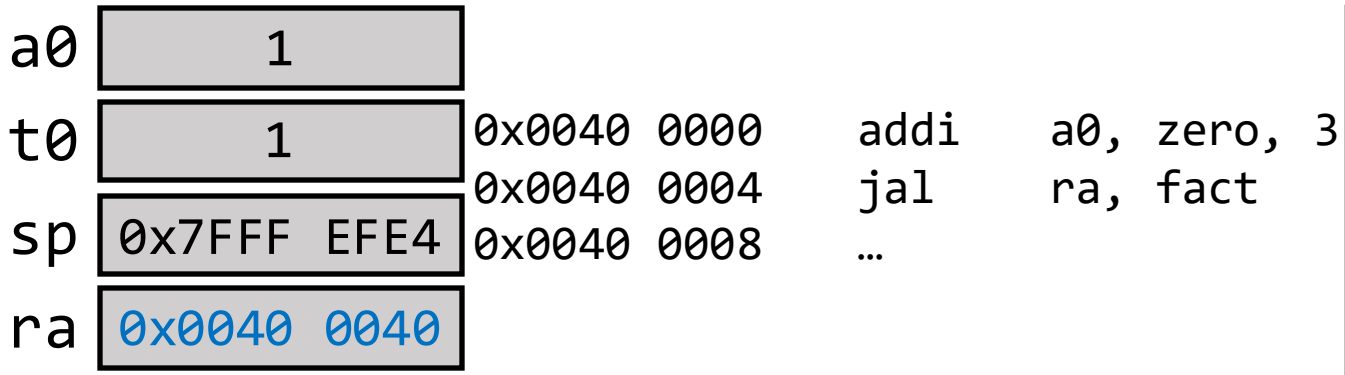
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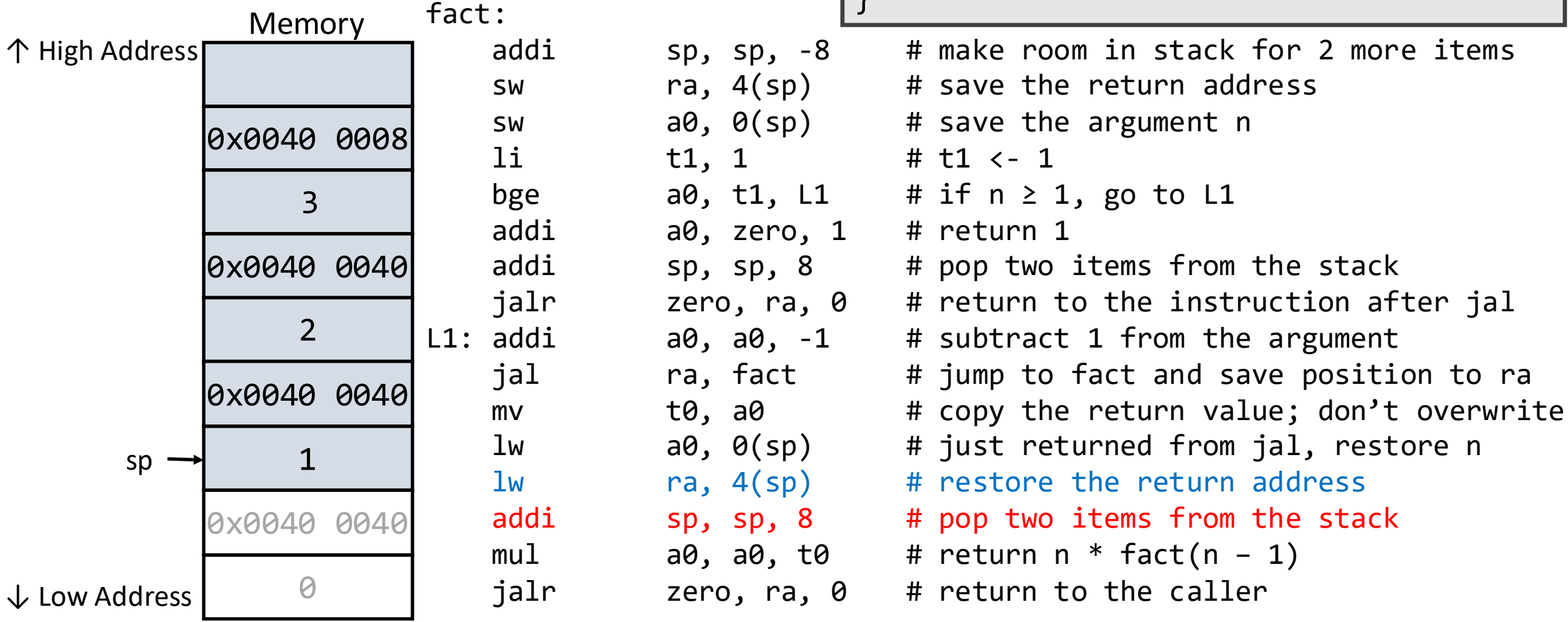


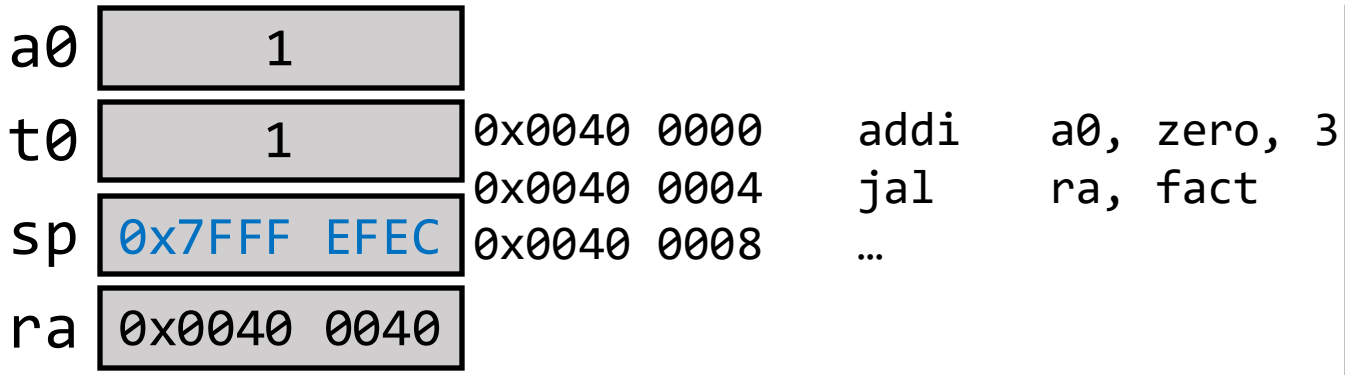
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    else
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}
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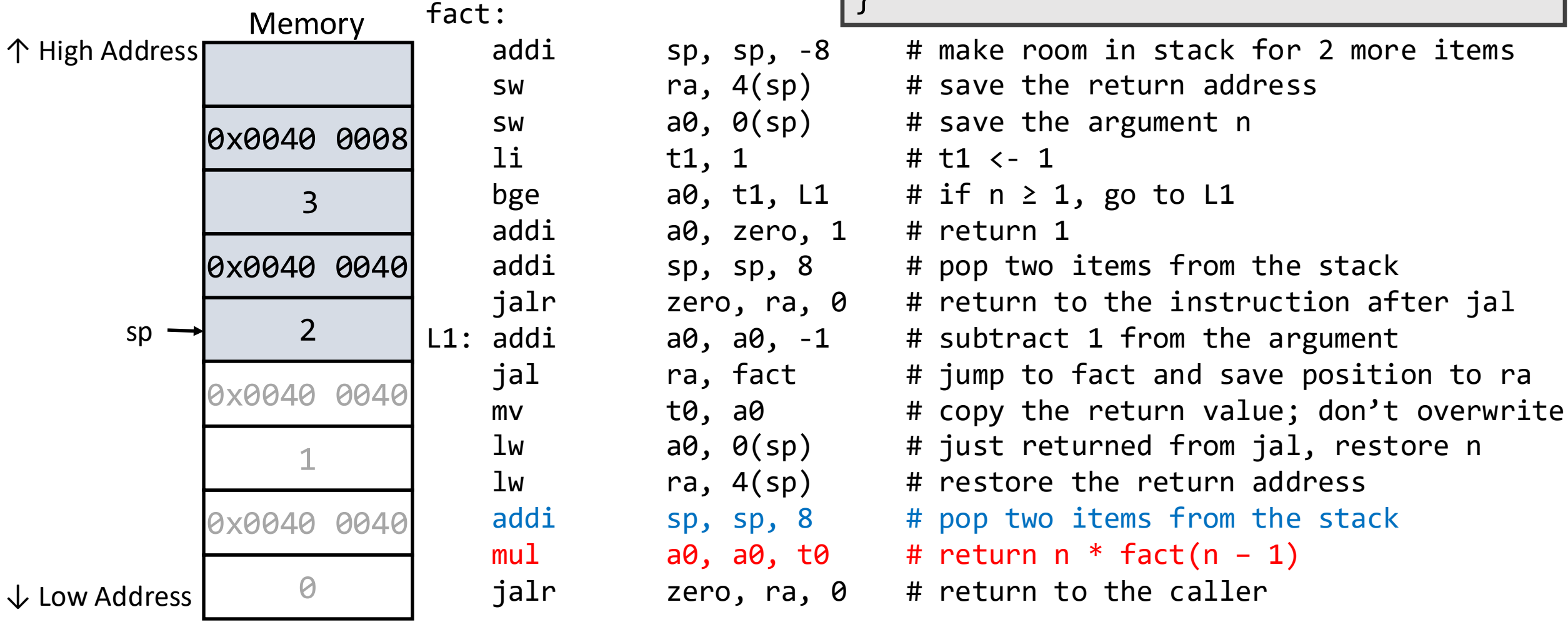


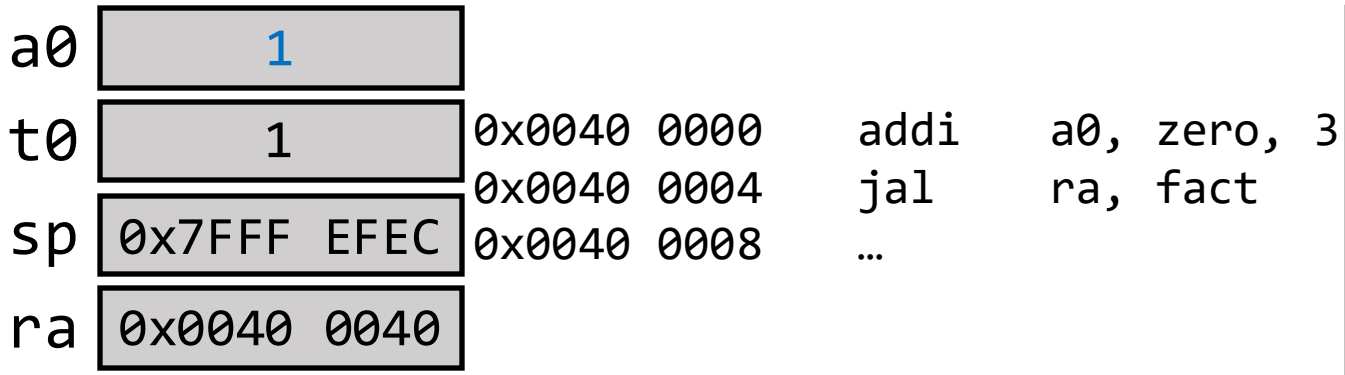
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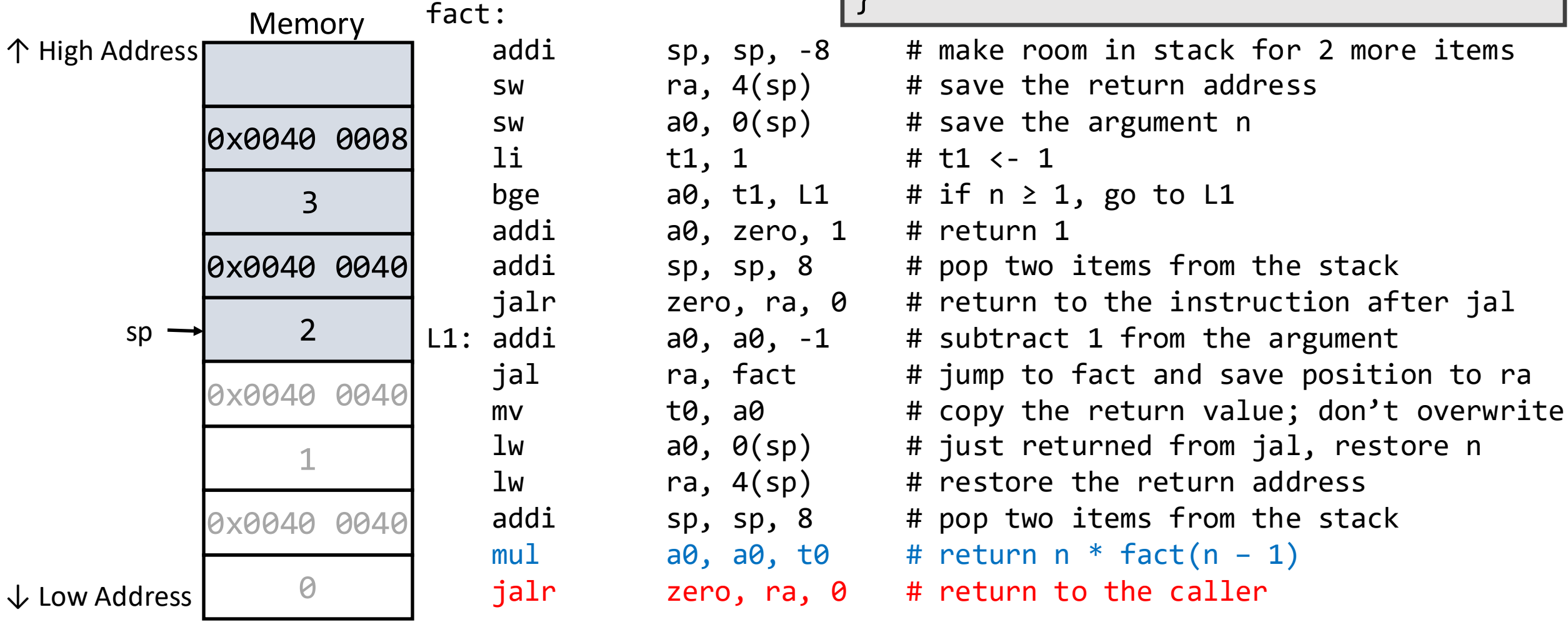


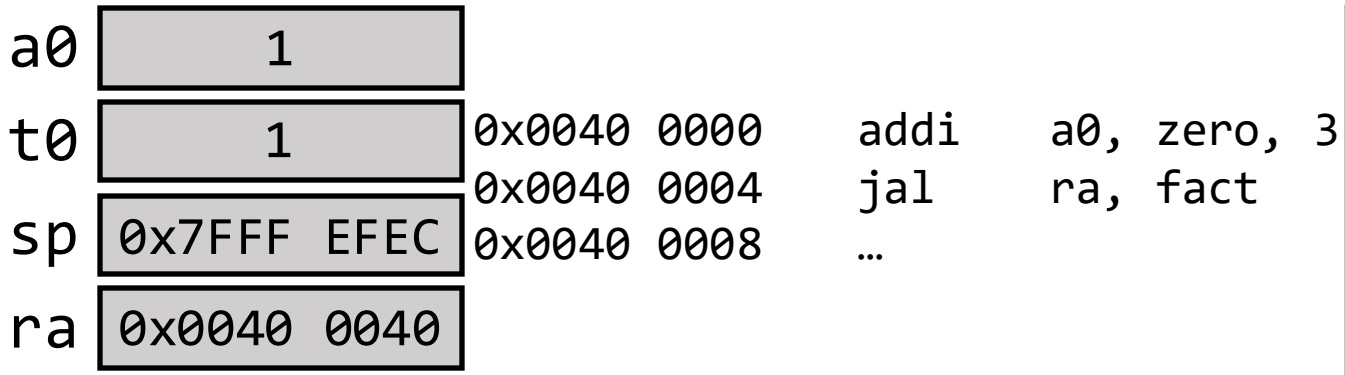
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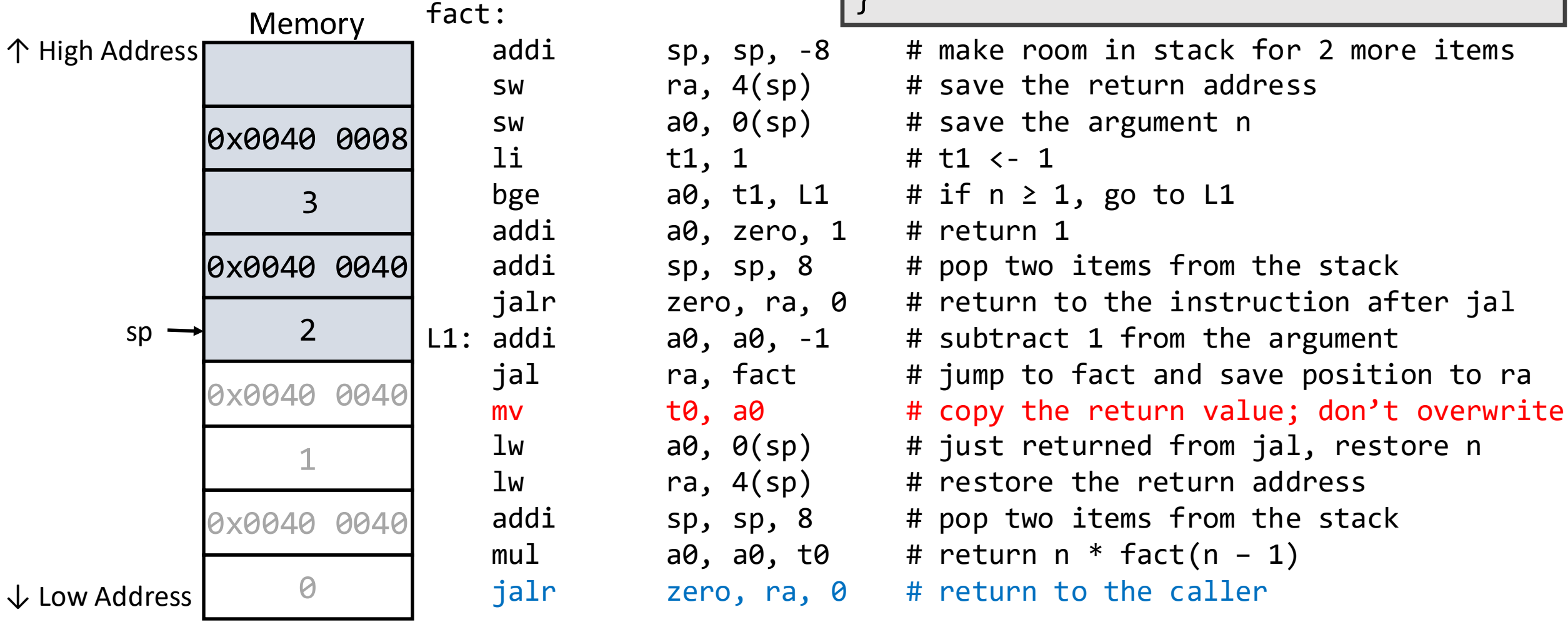


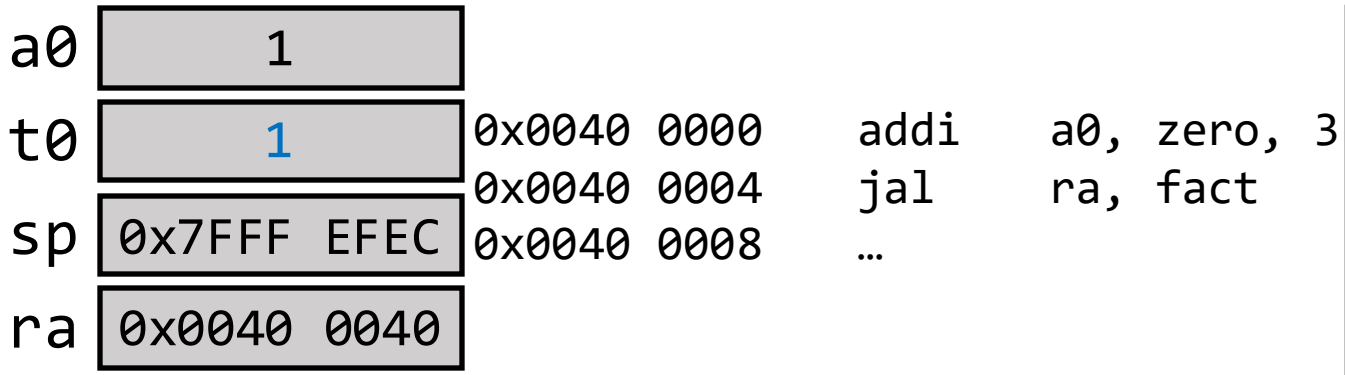
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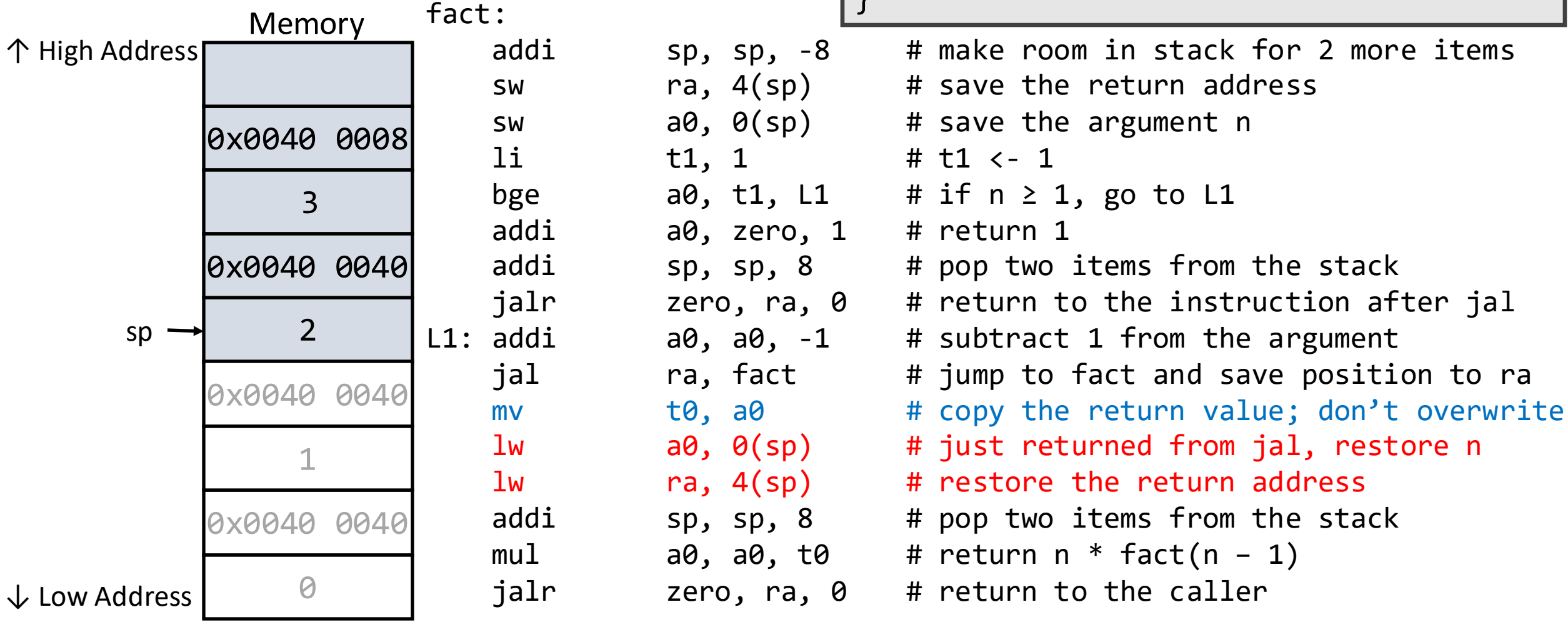


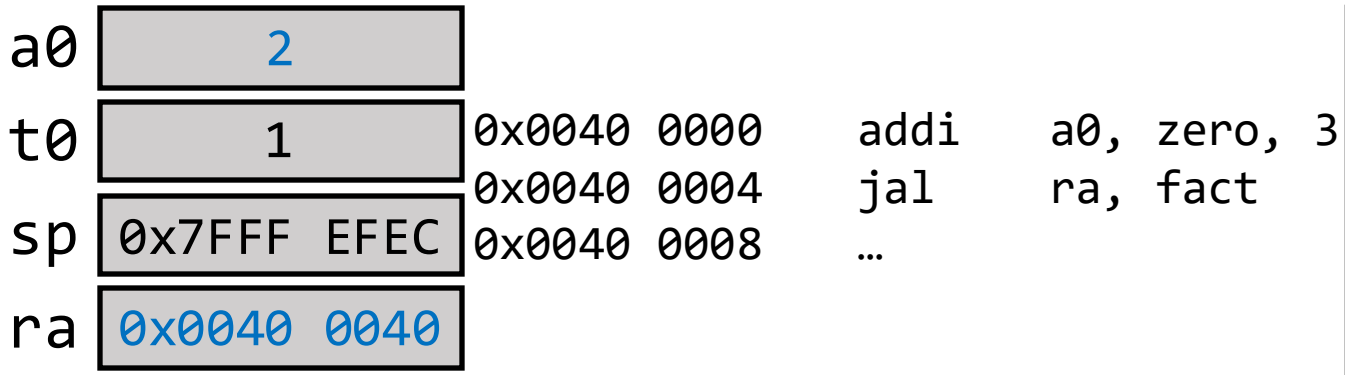
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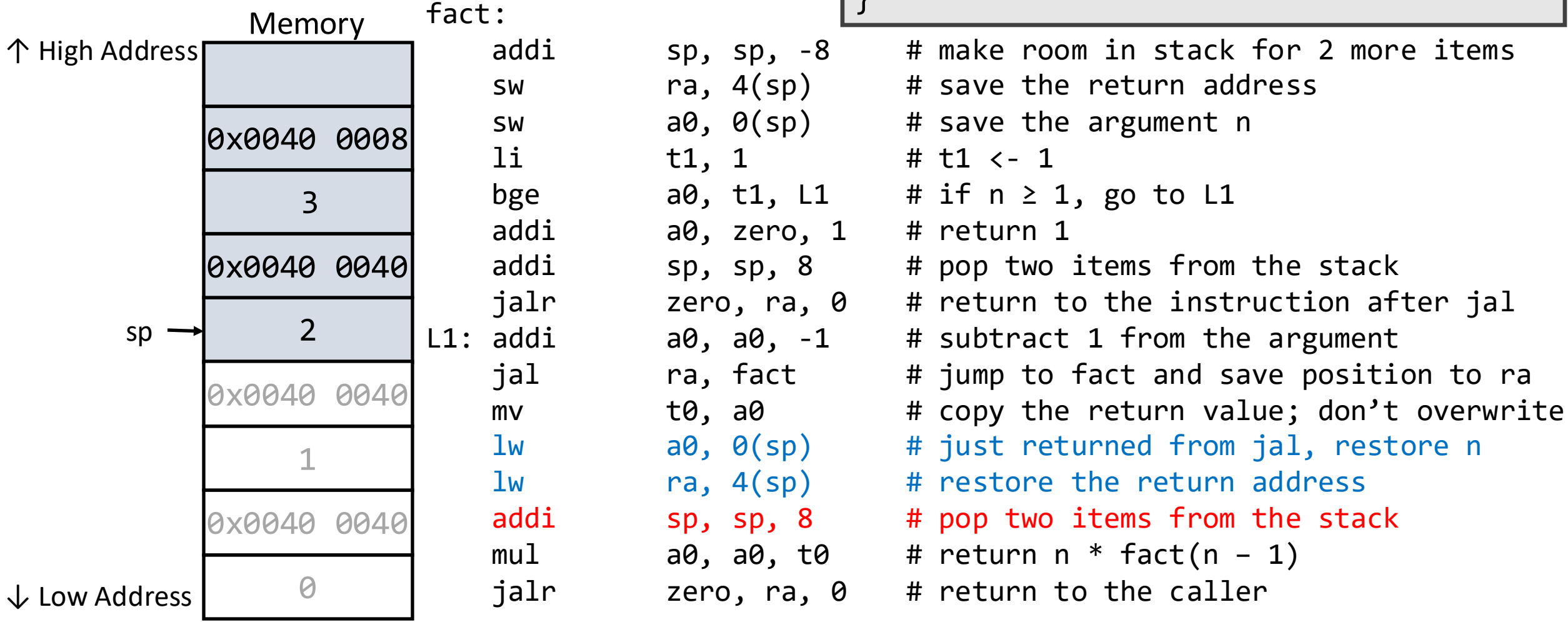


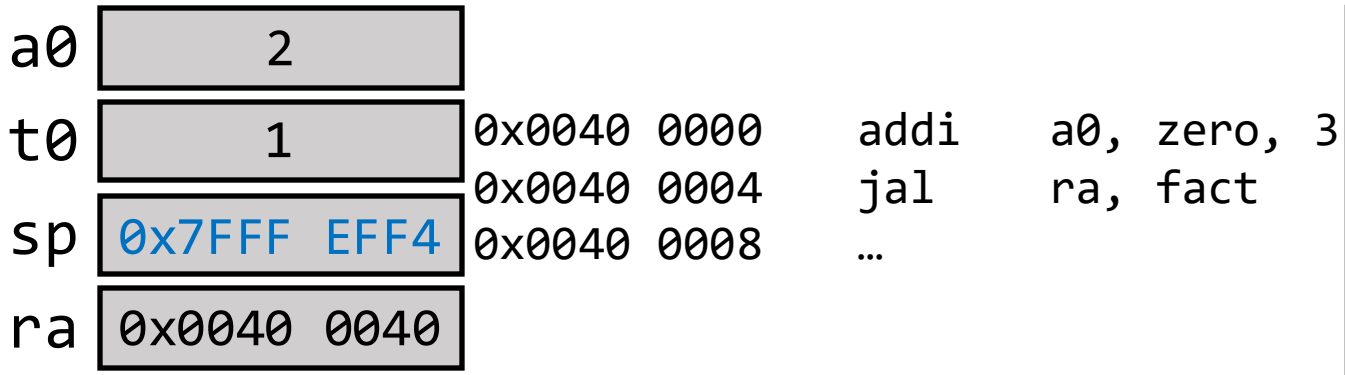
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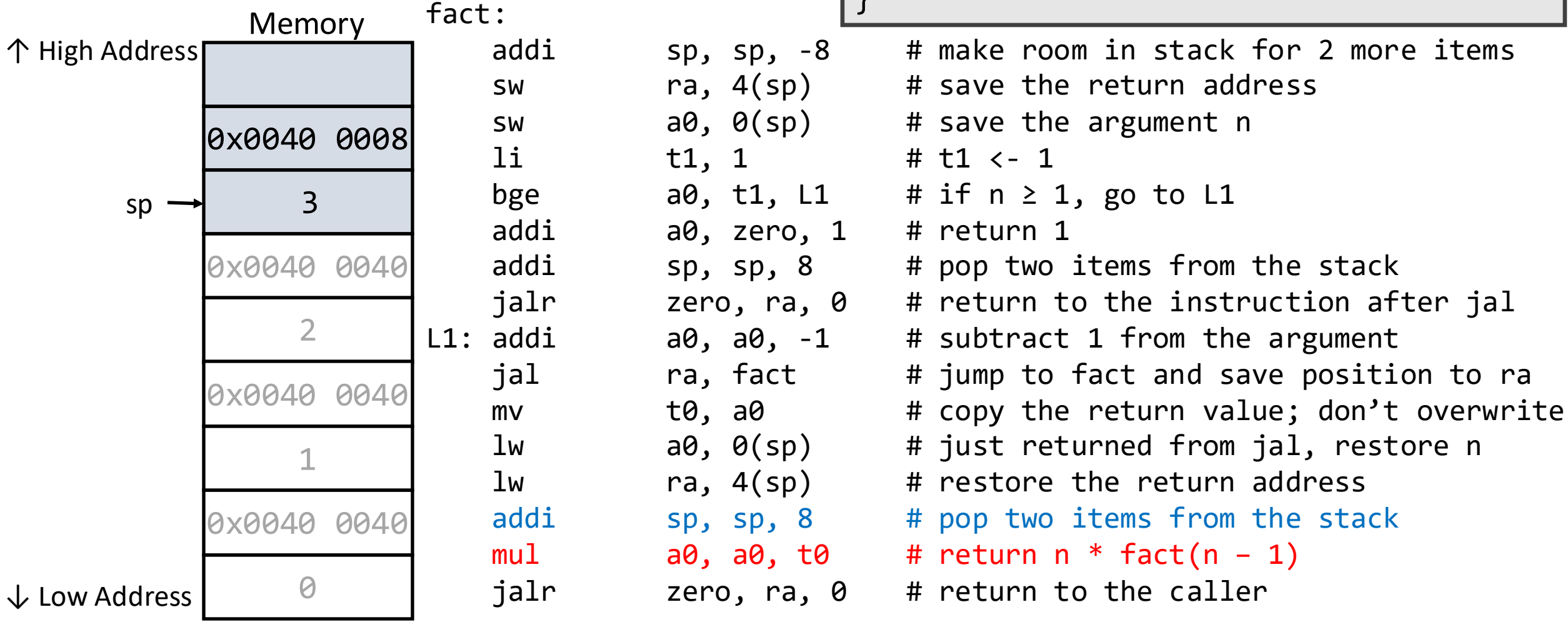


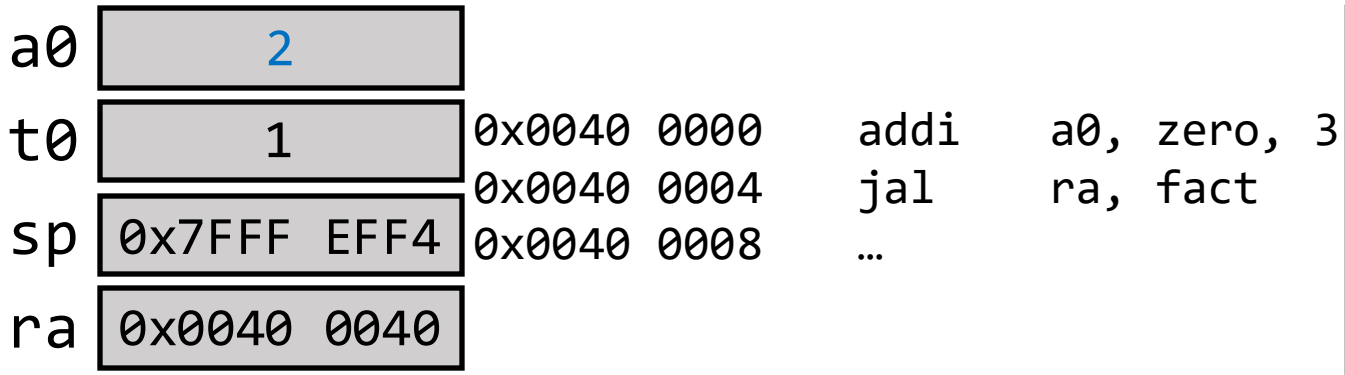
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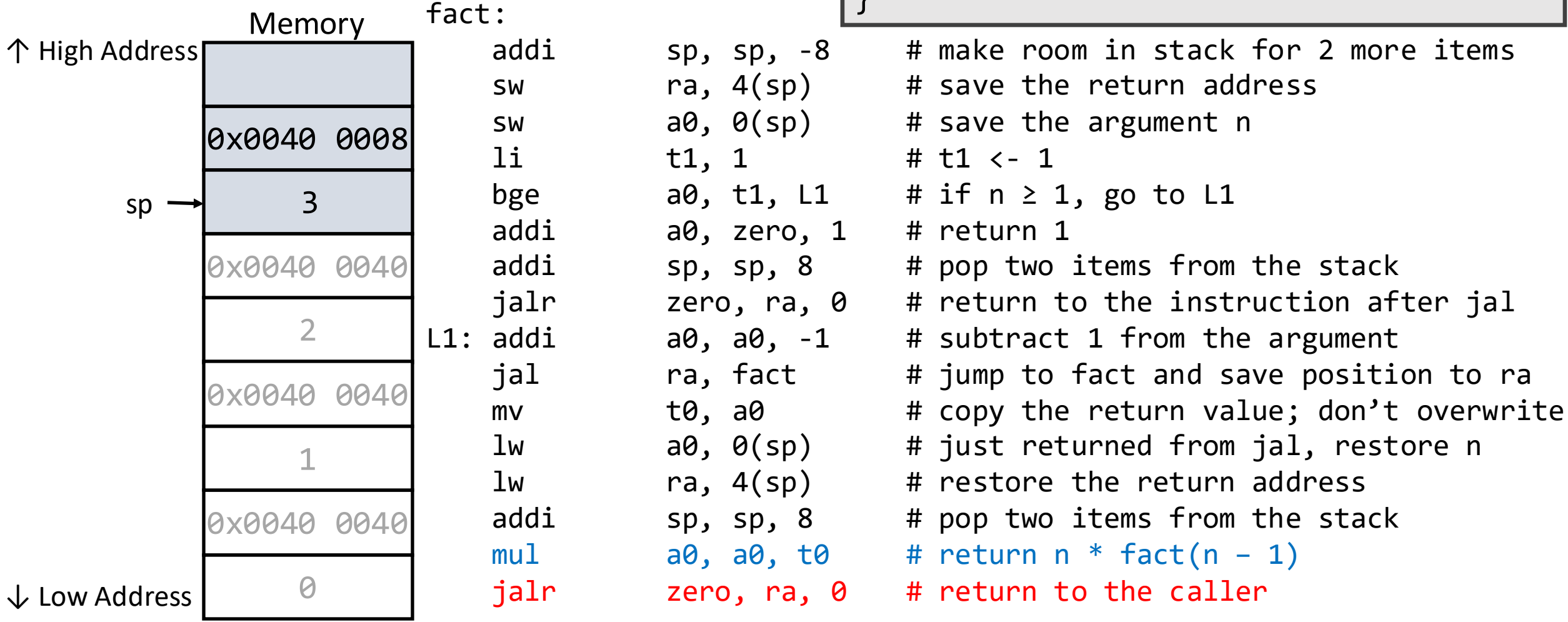


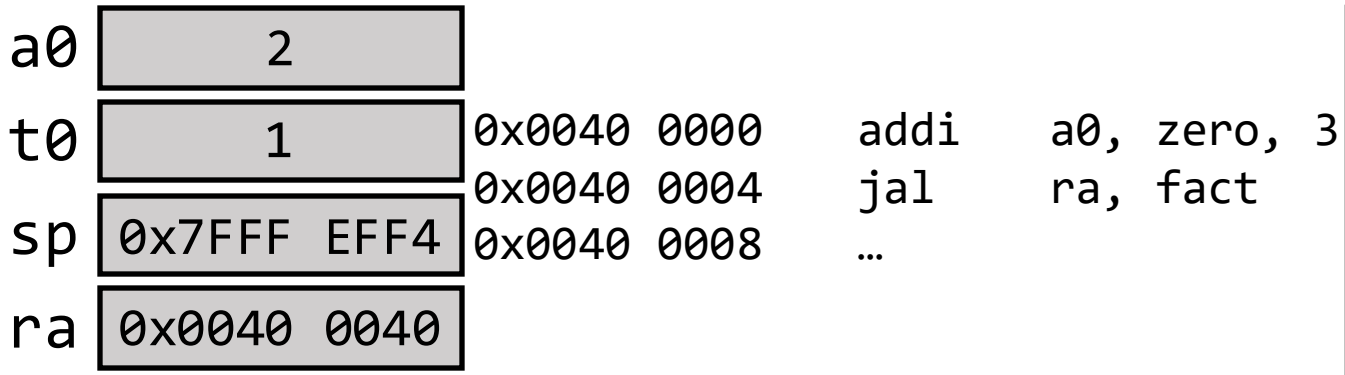
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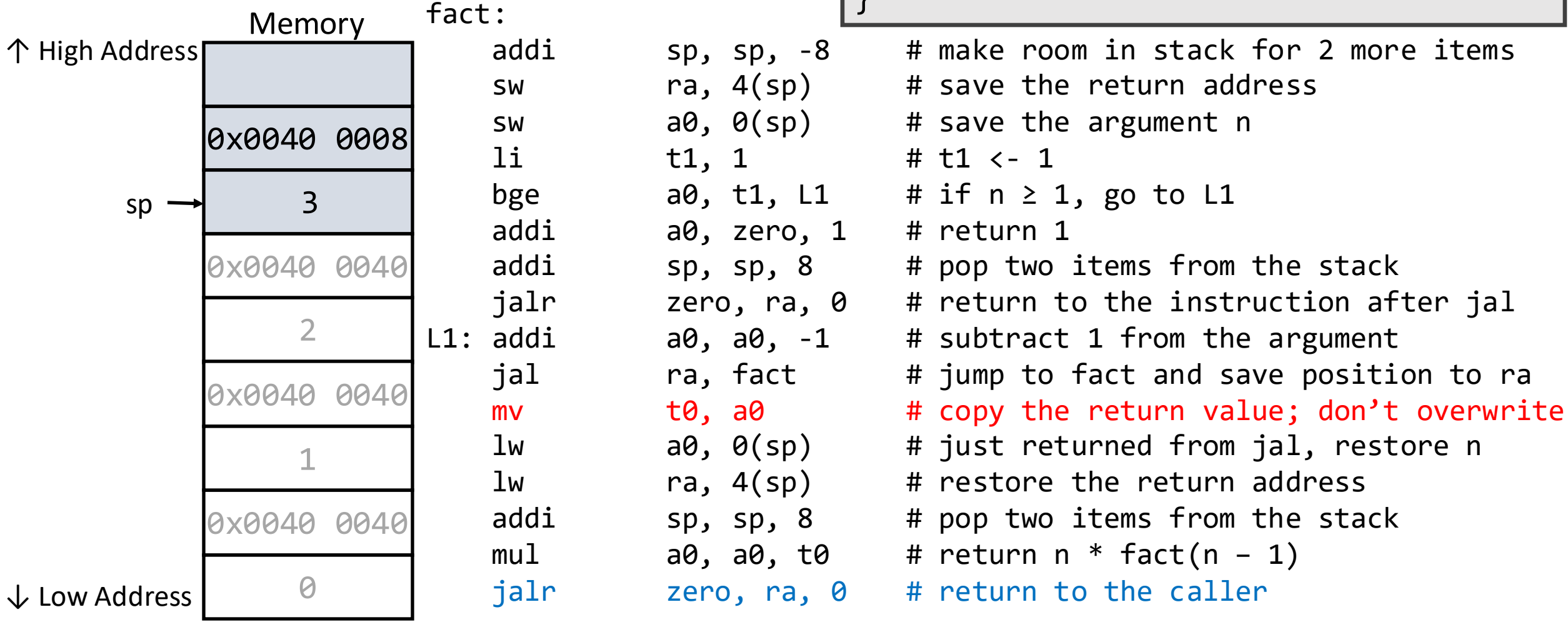


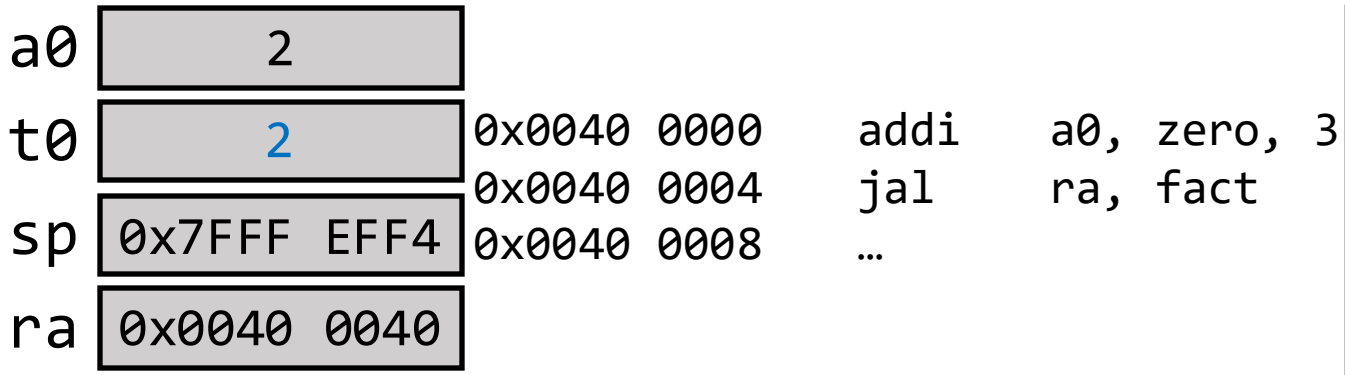
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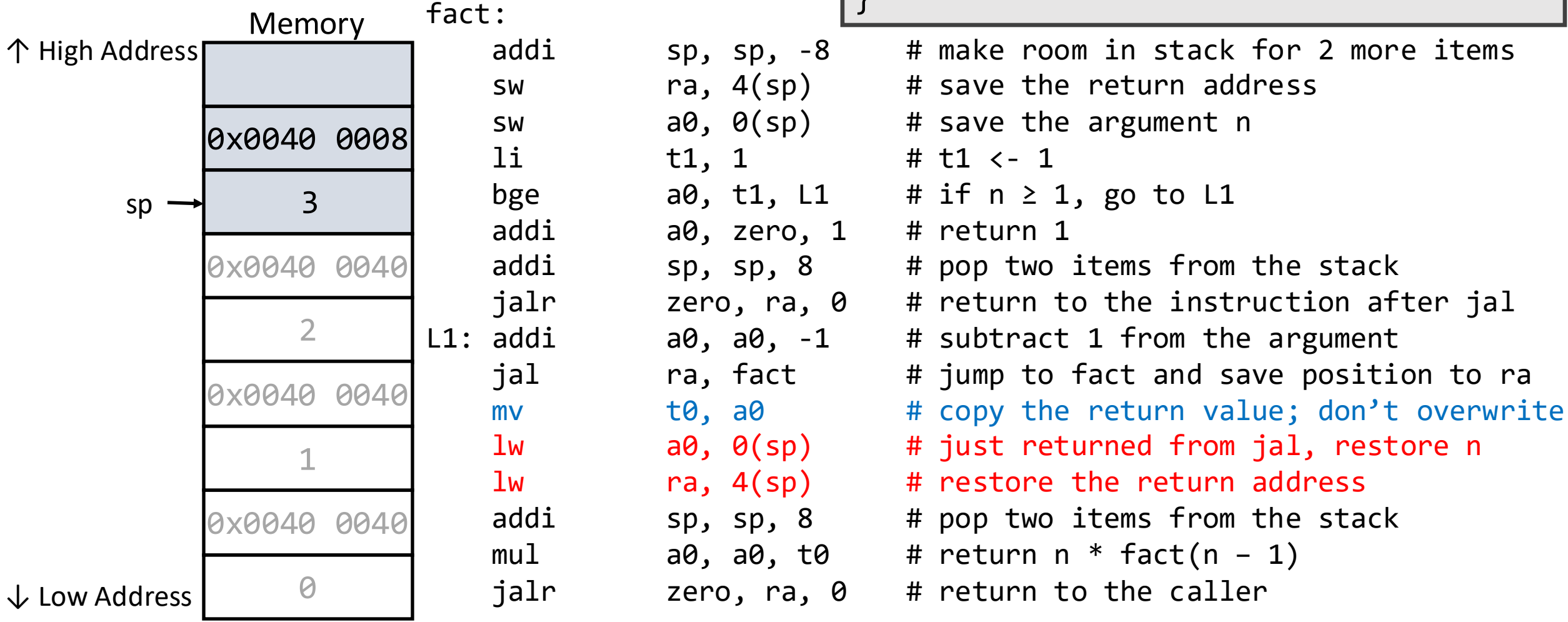


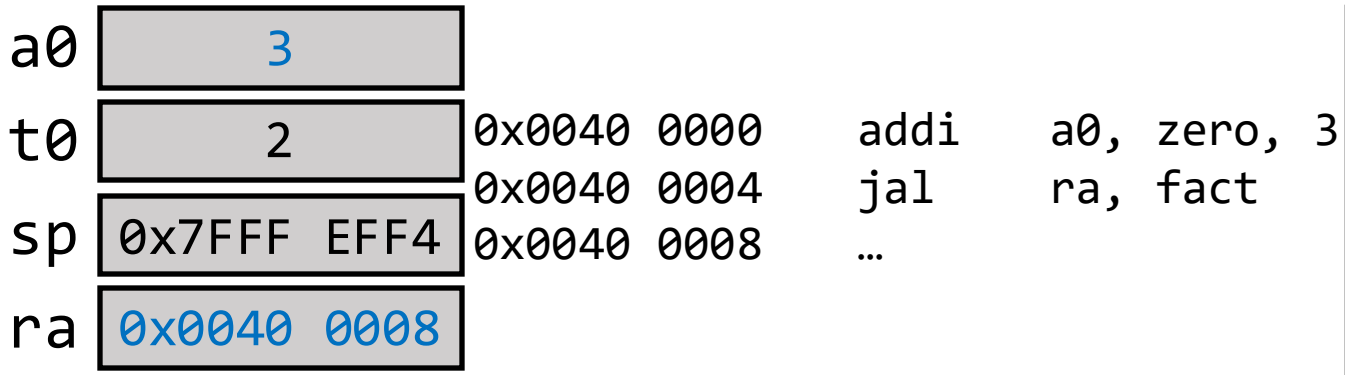
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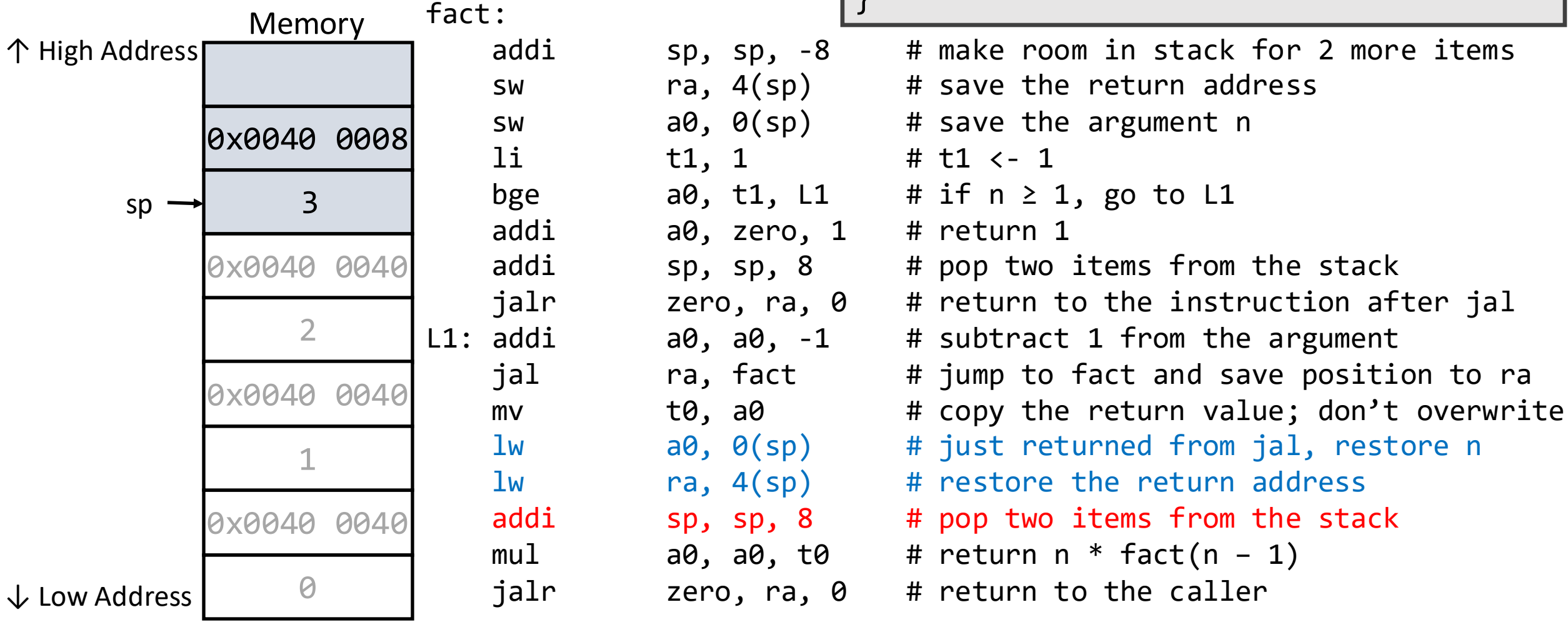


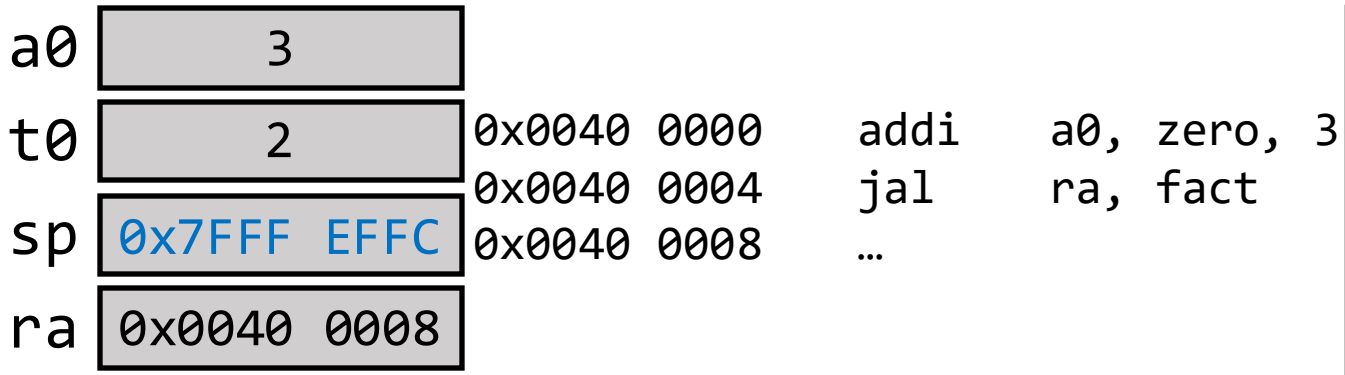
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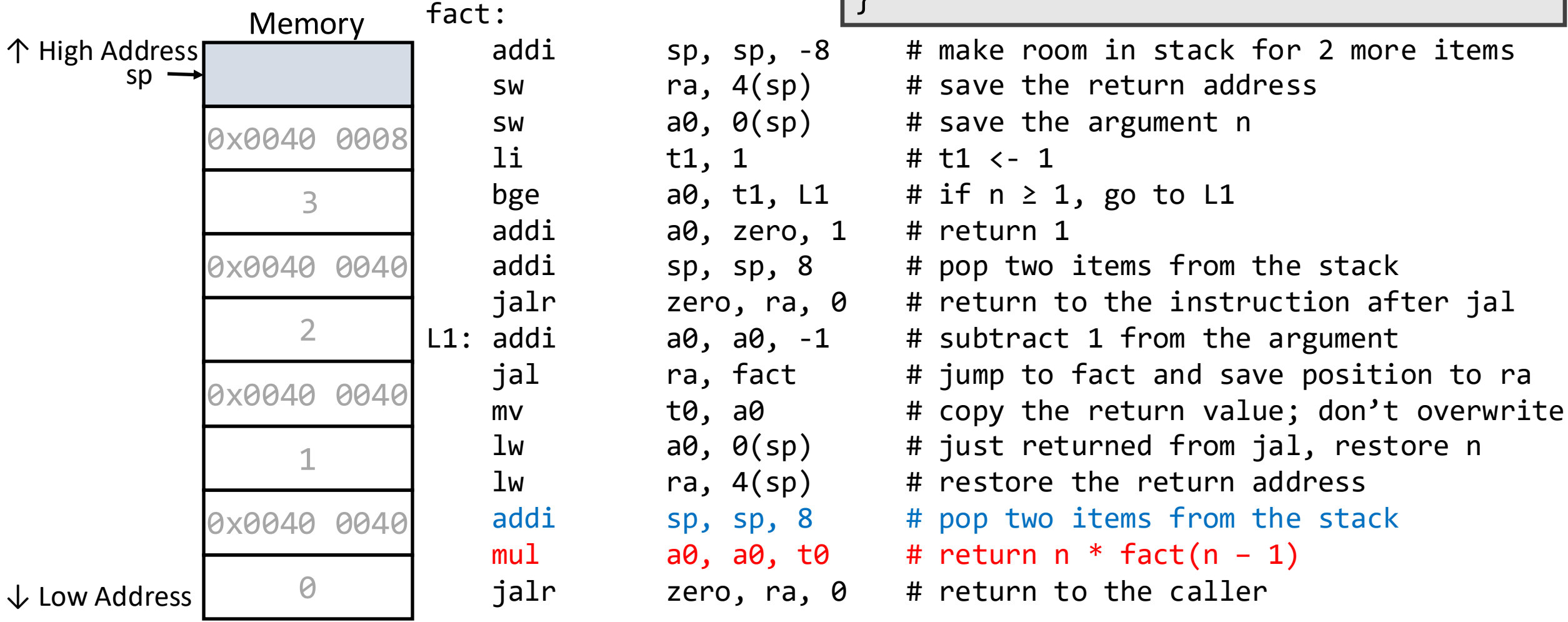


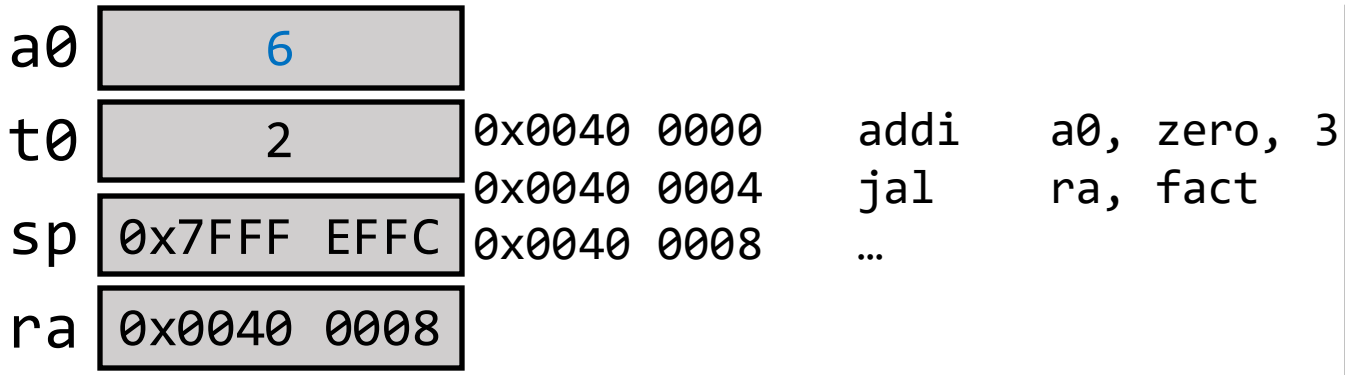
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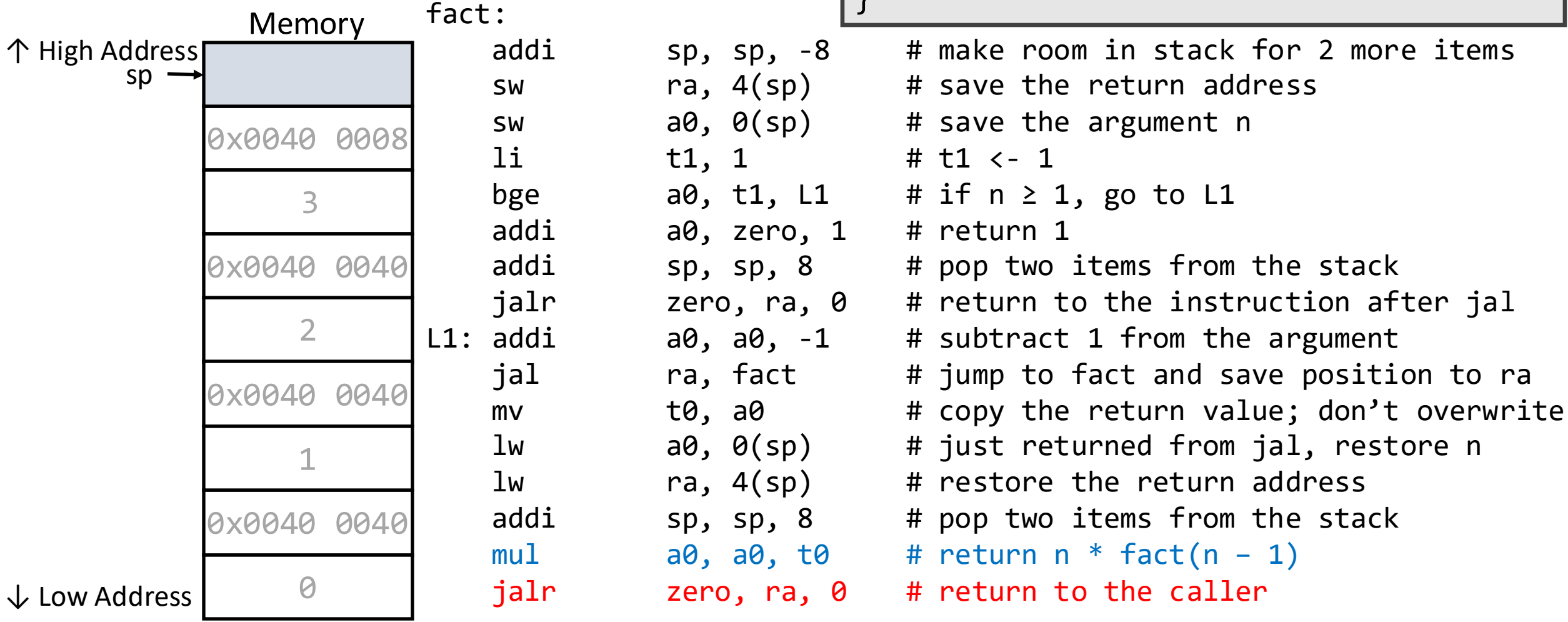


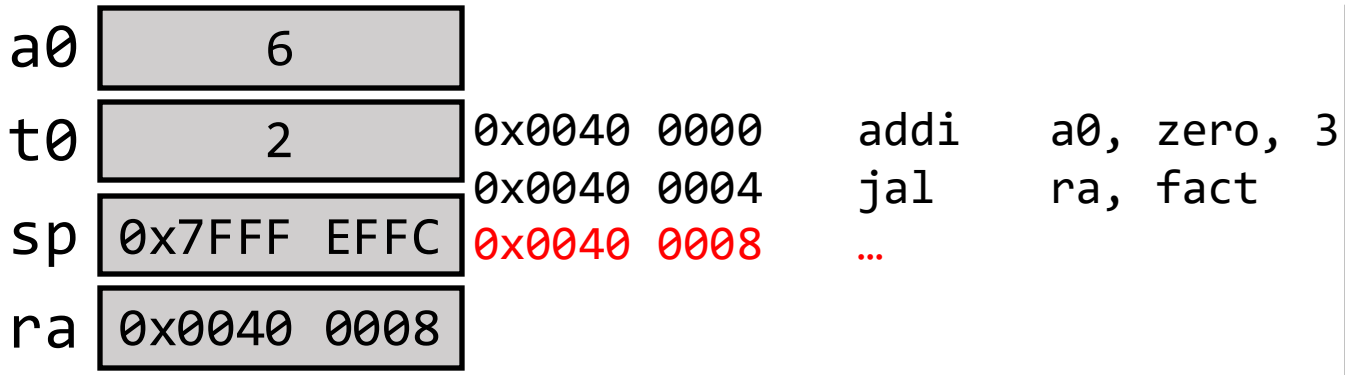
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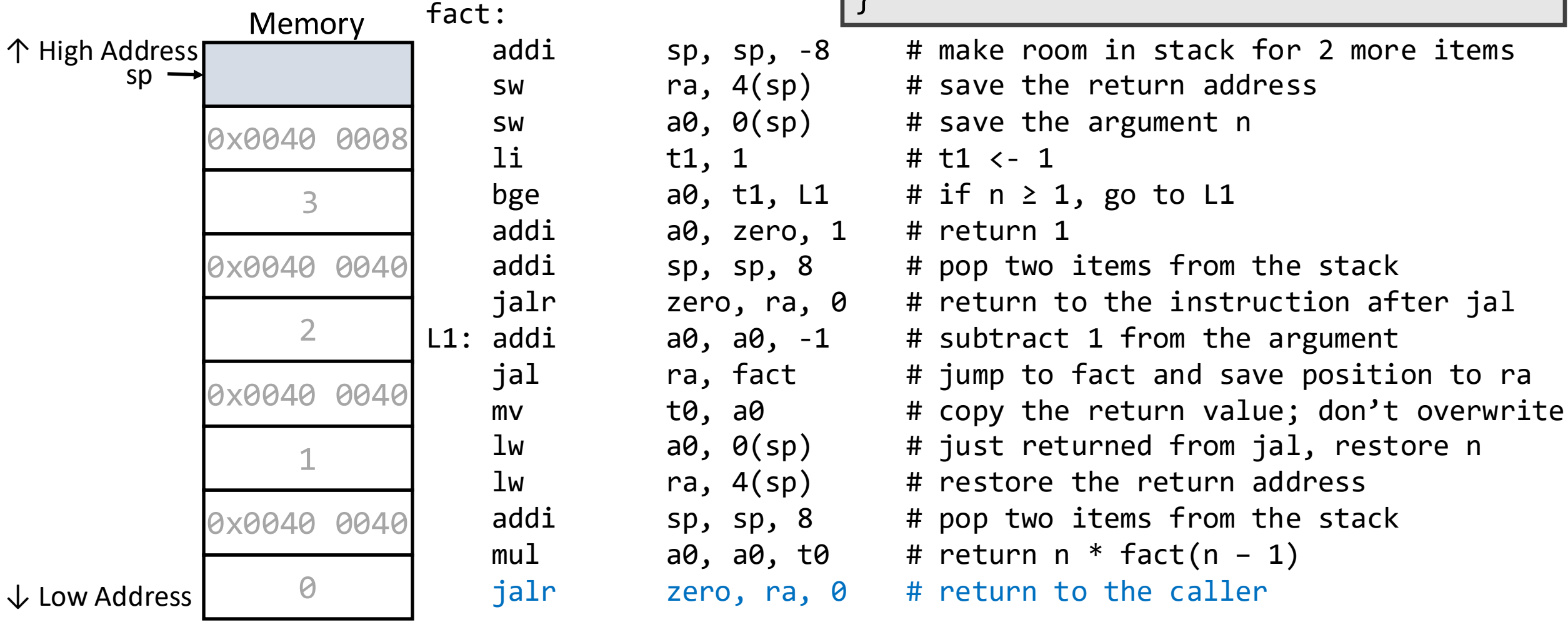


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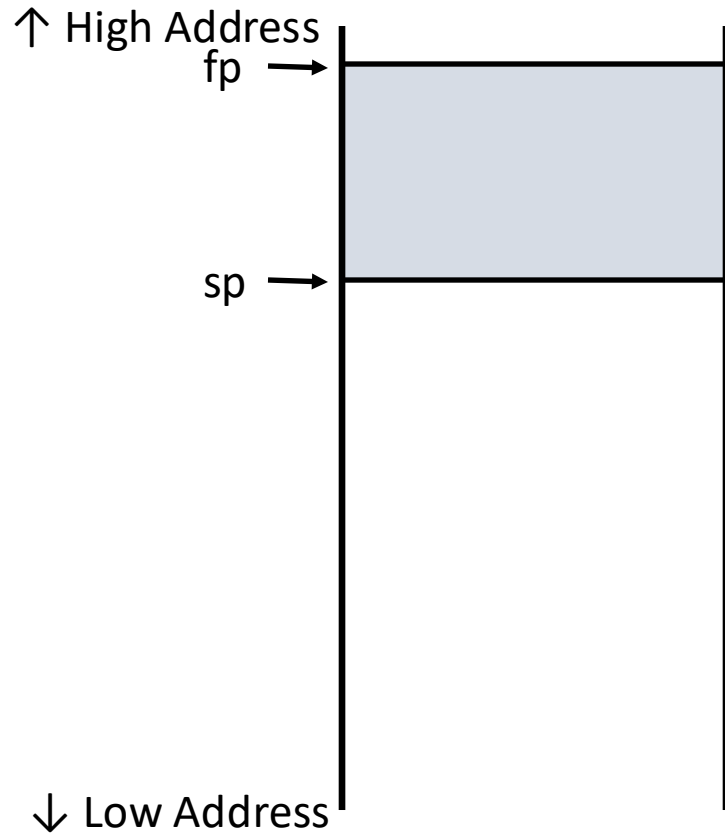




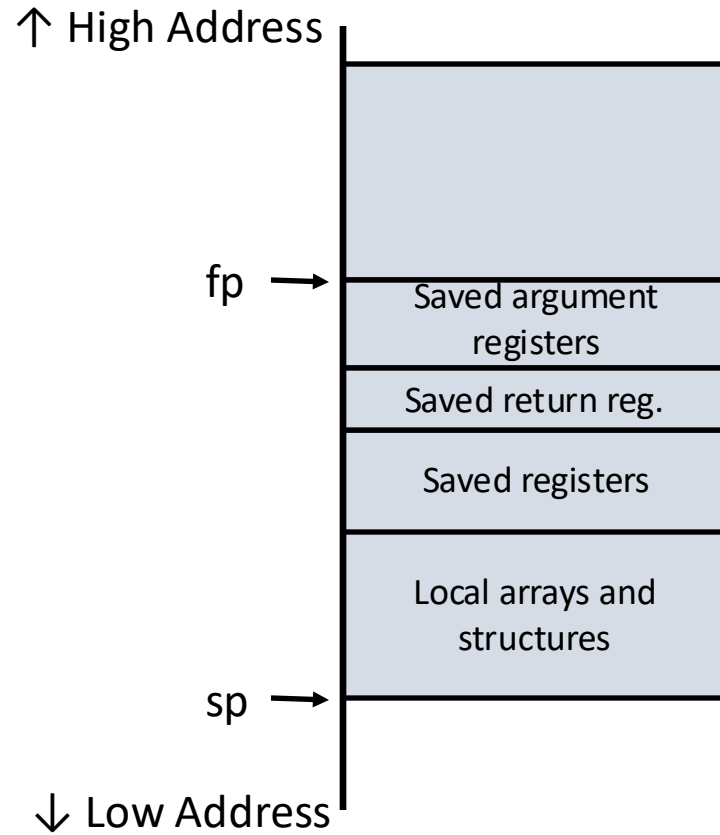
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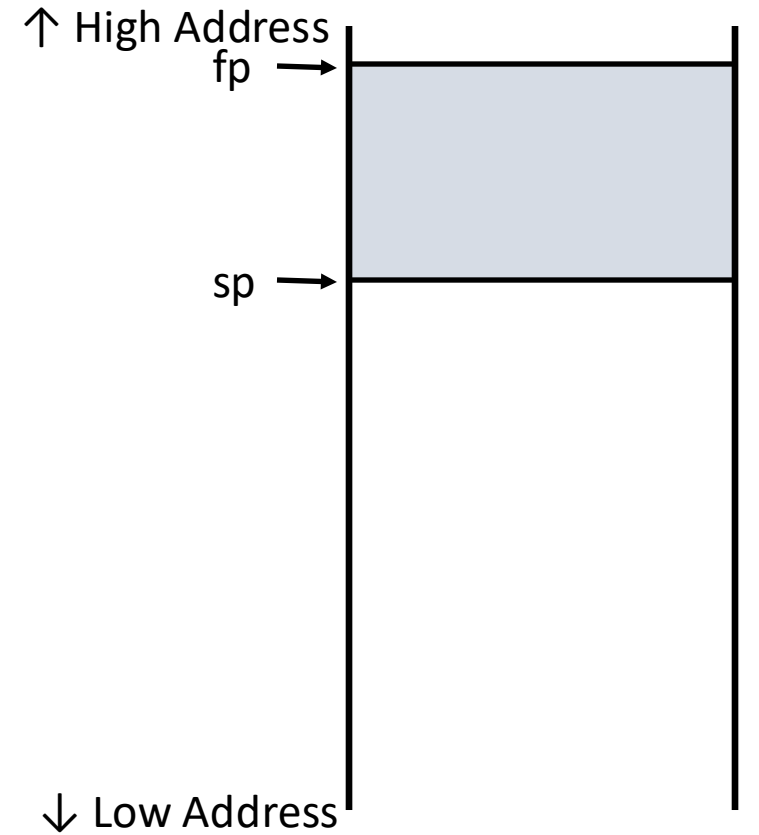
Other Data Stored in the Stack



Before procedure call



During procedure call



After procedure call

Recap

