

# Topic V01

Binary Representation

Readings: (Sections 2.4 )

# Decimal Numbers

5437

# Positional Numbering System

A diagram illustrating the decimal positional numbering system for the number 5437. The number is broken down into its constituent parts: 5 thousands, 4 hundreds, 3 tens, and 7 ones. Each digit is connected by a line to its corresponding place value label. Below the labels, the mathematical expression for the number is shown as a sum of products:  $5 \times 10^3 + 4 \times 10^2 + 3 \times 10^1 + 7 \times 10^0$ .

thousands  
5  
hundreds  
4  
tens  
3  
ones  
7

$5 \times 10^3 + 4 \times 10^2 + 3 \times 10^1 + 7 \times 10^0$

Decimal

A diagram illustrating the binary positional numbering system for the number 1010. The number is broken down into its constituent parts: 1 eights, 0 fours, 1 twos, and 0 ones. Each digit is connected by a line to its corresponding place value label. Below the labels, the mathematical expression for the number is shown as a sum of products:  $1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$ .

eights  
1  
fours  
0  
twos  
1  
ones  
0

$1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

Binary

# Unsigned Binaries (example)

Largest unsigned number in four bits?

Binary Representation

1      1      1      1

Decimal Value

$$8 + 4 + 2 + 1 = 15$$

$$2^4 - 1 = 15$$

In Decimal notation we know that:

$$9999 = 10000 - 1 = 10^4 - 1$$

In binary we have:

$$1111_2 = 10000_2 - 1 = 2^4 - 1 = 16 - 1$$

# Unsigned Binaries (example)

Largest unsigned number in four bits?

Binary Representation

1      1      1      1

Decimal Value

$$8 + 4 + 2 + 1 = 15$$

$$2^4 - 1 = 15$$

Largest unsigned number in  $k$  bits?

Binary Representation

1    1    ...    1

Decimal Value

$$2^k - 1$$

# Some Powers of Two to Memorize

**Name**

**Exponent**

**Value**

# Decimal to Binary Conversion

Method of Repeated Division:

581		1
290		0
145		1
72		0
36		0
18		0
9		1
4		0
2		0
1		1
0		0



$$581 = 01001000101$$

# Decimal to Binary Conversion

Method of Repeated Subtraction:

k	$2^k$
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

10	9	8	7	6	5	4	3	2	1	0
0	1	0	0	1	0	0	0	1	0	1

$$\begin{array}{r} 581 \\ -512 \\ \hline 69 \\ -64 \\ \hline 5 \\ -4 \\ \hline 1 \\ -1 \\ \hline 0 \end{array}$$

$$581 = 01001000101$$





# Review of logic Operators

The complement of a binary number:

$$\begin{array}{rcccc} & x & = & 1 & 0 & 0 & 1 \\ + & \bar{x} & = & 0 & 1 & 1 & 0 \\ \hline x + \bar{x} & = & 1 & 1 & 1 & 1 \end{array}$$

By convention:

$$x + \bar{x} = -1$$

$$1 \ 1 \ 1 \ 1 = -1$$

# 2-Complement Notation

## By convention:

$$-1 = 1 \quad 1 \quad 1 \quad 1$$

## Some algebra:

$$X + \overline{X} = -1$$

$$x + \overline{x} + 1 = 0$$

$$\overline{x} + 1 = -x$$

$$-X = \overline{X} + 1$$

## Example:

$$6 = 0 \ 1 \ 1 \ 0_2$$

$-6 = ?$

$$\begin{array}{r} \phantom{0000}1 \\ \overline{6} = 1 \phantom{0} 0 \phantom{0} 1_2 \\ + \phantom{0000}1_2 \\ \hline -6 = \textcircled{1} \phantom{0} 0 \phantom{0} 0_2 \end{array}$$

A leading 1 indicates a negative number.

Given the binary number:

1010

How do you know if it is  $10_{10}$  or  $-6_{10}$ ?

You don't!

You need to be told if this is:

An unsigned representation

Or a 2-complement representation

# Meaning Depends on Context

April, May and June



Theater play: *April, May and June* are three sisters confronting their late mother's affair.

# Meaning Depends on Context

Given the binary representation:

01000001

What does it represent?

You don't know!

It could be:

+65<sub>10</sub>


The ASCII character 'A'

Something else

# Range of 2-Complement Notation

Largest positive number in 2-complement in four bits?

Binary Representation

0 1 1 1  
  
 $2^3$

Decimal Value

$$4 + 2 + 1 = 7$$

$$2^3 - 1 = 7$$

$$2^{4-1} - 1 = 7$$

Largest positive number in 2-complement in  $k$  bits?

Binary Representation

0 1 ... 1

Decimal Value

$$2^{k-1} - 1$$

# Range of 2-Complement Notation

Most negative number in 2-complement in four bits?

Binary Representation

1 0 0 0

Decimal Value

$$-2^3 = -8$$

$$-2^{4-1} = -8$$

$$\begin{array}{rcccccl} & & & & & 1 & 1 & 1 & & \\ \hline 1 & 0 & 0 & 0 & = & 0 & 1 & 1 & 1 & \\ & & & & + & & & & 1 & \\ & & & & & \hline & & & & & 1 & 0 & 0 & 0 & = 8_{10} \end{array}$$

# Range of 2-Complement Notation

Most negative number in 2-complement in four bits?

Binary Representation

1   0   0   0

Decimal Value

$$-2^3 = -8$$

$$-2^{4-1} = -8$$

Most negative number in 2-complement in  $k$  bits?

Binary Representation

1   0   ...   0

Decimal Value

$$-2^{k-1}$$



# Range of 2-Complement Notation

2-complement in  $k$  bits:

Largest positive number:

Binary Representation

0    1    ...    1

Decimal Value

$2^{k-1} - 1$

Most negative number:

Binary Representation

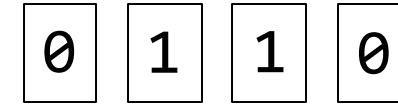
1    0    ...    0

Decimal Value

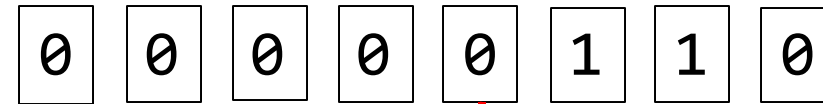
$-2^{k-1}$

# Sign Extension

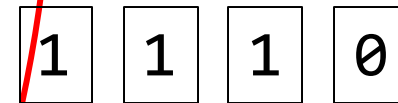
Number +6 represented in four digits:



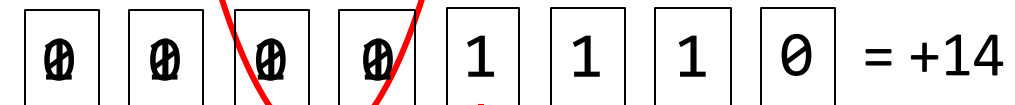
How to represent +6  
in eight digits?



Number -2 represented in four digits:



How to represent -2  
in eight digits?



# Sign Extension in RISC-V

In RISC-V a word has 32 bits

Sometimes a 12-bit (or a 20-bit)  
value has to be stored into a word

# Sign Extension in RISC-V

Instructions with Immediate value

Add Immediate

`addi`: extend immediate value

example: `t0 ← t0 + 15`



Registers are 32 bits

The diagram shows three red arrows originating from the text 'Registers are 32 bits'. One arrow points to the first 't0' in the example instruction. A second arrow points to the second 't0'. A third arrow points to the immediate value '15'.

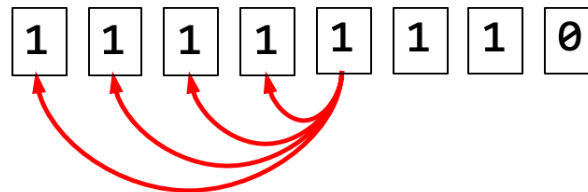
Immediate constants are 12 bits

Constant is sign-extended to 32 bits

What have we learned?

$$\begin{array}{cccc}
 \text{eights} & & \text{fours} & & \text{twos} & & \text{ones} \\
 1 & & 0 & & 1 & & 0 \\
 \swarrow & & \swarrow & & \swarrow & & \swarrow \\
 1 \times 2^3 + & 0 \times 2^2 + & 1 \times 2^1 + & 0 \times 2^0
 \end{array}$$

$$\begin{array}{r}
 \phantom{1} \phantom{0} \phantom{0} \phantom{1} \phantom{2} \\
 \phantom{1} \phantom{0} \phantom{0} \phantom{1} \phantom{2} \\
 \phantom{1} \phantom{0} \phantom{0} \phantom{1} \phantom{2} \\
 \phantom{1} \phantom{0} \phantom{0} \phantom{1} \phantom{2} \\
 \phantom{1} \phantom{0} \phantom{0} \phantom{1} \phantom{2} \\
 \hline
 -6 = 1 \ 0 \ 1 \ 0_2
 \end{array}$$



$$2^k - 1$$

581	1
290	0
145	1
72	0
36	0
18	0
9	1
4	0
2	0
1	1
0	0

$$-2^{k-1}$$

$$2^{k-1} - 1$$

April, May and June



0	1	
1	2	
2	4	
3	8	
4	16	
5	32	
6	64	
7	128	
8	256	
9	512	
10	1024	

581	
-512	
69	
-64	
5	
-4	
1	
-1	
0	