Memristive Pupillography: A Memristive Circuit Model of the Eye's Response to Changes in Illumination Intensity

David Sheppard
Advanced Memristors and Nanoelectronic VLSI
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***Abstract*—Since its introduction, the memristor has shown great promise as a method by which nonlinear biological systems may be modeled using nothing more than a circuit configuration. While many have undertaken the challenge of producing memristive models for biological elements like neurons, there exists a lack of established memristive circuits for pupillary modeling. This paper outlines a basic circuit that makes use of the memristor's nonlinearities to produce a system that can respond to changes in a source voltage in the same way that a pupil responds to changes in light. The results of the new model are analyzed and compared with the properties previously established in biological pupillograms. The strengths of the model are highlighted and possible future improvements are suggested.**

I. Introduction and Objectives

A. Background

Since the first description of the conceptual memristor, researchers have attempted to take advantage of the nonlinear properties produced by this unique device. Indeed, not only does this nonvolatile circuit element hold promise for use in memory systems, but the memristor is also being used to model biological systems in ways not previously imagined [1]. One of the most prominent sources of attention amongst those studying memristors is the neuron [1], [2]. Many models have been proposed that model the neuron's electrical behavior as it generates action potentials. Nonetheless, much less effort has been placed on the possibility of modeling many other biological systems using memristors.

When modeling a neuron, many factors are taken into account in order to achieve desired results. Proposed in 1952, the Hodgkin-Huxley neuron model has shown renewed interest as a basis for a memristive model of the neuron [3], [4]. The nonlinearity and plasticity of the memristor lends the device well for such a model. Neuromorphic researchers have used methods such as leaky integrate-and-fire (LIF) systems to achieve thresholding for neuronal firing [2], [5]. Additionally, memristive neuronal models that allow for "fully asynchronous architectures with neurons sending their action potentials not only forward but also backward" have been developed [2]. While the memristor certainly lends itself well to neuron modeling, its dynamics provide the potential for use in many other nonlinear biological models. While biological systems like the human pupil have been modeled mathematically [6], no significant work has been published regarding a memristive model of the pupil's reactions to variations in illumination.

B. Goals

This paper proposes a memristor-based pupil model that mimics the eye's response to changes in illumination. The illumination stimulus is given as a voltage value between 0 V and 1 V while the output is given as a voltage value between approximately 0.5 V and 1 V. The output voltage value corresponds specifically to the diameter of the pupil. This allows the model to produce results that can easily be compared to biological pupillograms. The proposed model makes use of a memristor with an asymmetric rate function to optimally model pupillary responses. The pupil model is tested and analyzed with regard to biological results.

II. A Brief Overview of Pupillography

Pupillography involves the study of the pupil's reaction to changes illumination. Since the invention of infrared video pupillographs, studying the pupil's characteristics has become increasingly easier. Not only does pupillography serve as a diagnostic method in opthalmology, it can also serve to aid in psychological studies [7].

When performing a pupillographic test, the subject is acclimated to a dark environment and then exposed to a brief burst of light. The diameter of one of their pupils is observed before, during, and after this stimulus. During the test, an infrared video camera is positioned near their eye in order to record the pupil's response to the changing illumination without creating additional visible illumination [7], [8]. Using this data, the subject's pupil diameter can then be plotted with respect to time. This data can be used to determine the responsiveness of the subject's pupil(s) and can be used to diagnose conditions like Horner syndrome [7].

Many factors can affect the pupil's response to stimuli. These factors can include stimulus intensity, stimulus color, psychological properties, and motor activity [7], [6], [9]. Among these factors is wavelength of the stimulus. It has been found that blue-green stimuli ($<$490 nm) primarily stimulates the eye's rods while red lighting ($>$610 nm) primarily stimulates the cones [7]. An example plot of pupil responses to red and blue light stimuli is shown in Fig. 1.

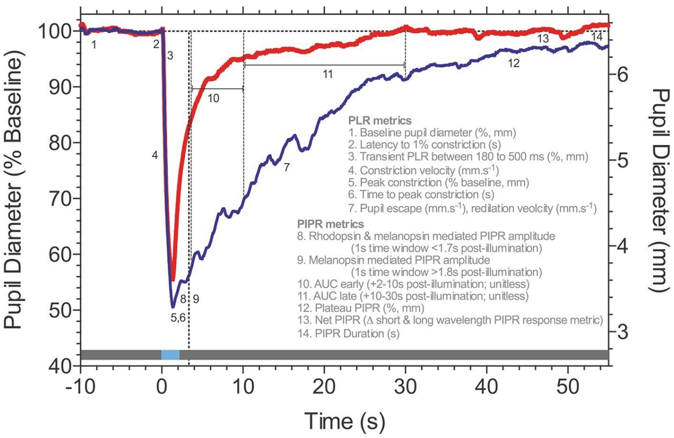

Fig. 1. Plot of pupil diameter over time. Both traces are the average of three trials on a single healthy subject where a 1 second pulse of light was applied at time=0. The red trace corresponds to a stimulus of 637 nm red light and the blue trace corresponds to a stimulus of 465 nm blue light. Image obtained from [7].

III. Design and Methodology

The design process was completed in the Cadence Virtuoso design environment. The memristor model was written first in Verilog-A and tested with a sinusoidal voltage source to confirm its functionality. This model was then incorporated into the overall pupil model which was constructed as a schematic. The simulation results were exported as CSV files and analyzed in MATLAB for ease of data handling.

A. The Memristor Model

This proposed model makes use of a simple memristor model created in Verilog-A. This voltage-controlled memristor conforms to an imbalanced linear ion drift model with the threshold effect. As explained in [1], the linear ion drift model is one of the simplest memristor models. It assumes a rate of memristance change (r_M) that is linearly related to the voltage across the memristor (V_m).

The memristor model is defined such that it retains its memristance value when the voltage across the memristor is within specified threshold values, i.e. $r_M(V_m) = 0$ when $V_{reset} \leq V_m \leq V_{set}$. The threshold values are defined as $V_{set} = 0.3$ V and $V_{reset} = -0.3$ V. Additionally, the memristor's memristance value was assigned boundaries. The maximum possible resistance achievable by the memristor model is 500 kΩ while the minimum memristance is 10 kΩ. The model's memristance value is initialized to 500 kΩ.

In order to achieve the desired results, the rate function r_M is defined such that the memristor model experiences a faster change in memristance when a positive voltage is applied than it does when a negative voltage with the same magnitude is applied. The constant memristance change rate (γ) is defined as $\gamma_1 = -2.5\frac{\text{M}\Omega}{\text{V}\cdot\text{s}}$ for positive voltage values and $\gamma_1 = -20\frac{\text{k}\Omega}{\text{V}\cdot\text{s}}$ for negative voltage values. Taking into account the threshold voltages, the rate of change of memristance with respect to the voltage across the memristor can be given as:

$$r_M(V_m) = \begin{cases} (-2.00\,V_m - 0.60) \times 10^4\ \Omega s^{-1} & \text{for } V_m < -0.5 \text{ and } M < 500\text{ k}\Omega \\ (-2.50\,V_m + 0.75) \times 10^6\ \Omega s^{-1} & \text{for } V_m > 0.5 \text{ and } M > 10\text{ k}\Omega \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where V_m is the voltage across the memristor and M is the present memristance value.

This particular model produces a memristor that is more sensitive to positive voltages than negative voltages, i.e. it has a high γ_1/γ_2 ratio. When connecting the memristor model to a sinusoidal voltage source, the pinched hysteresis behavior is apparent, but varies with successive periods due to the imbalance in the model's sensitivity. Fig. 2 shows the behavior of this model when connected to a 1 Hz sinusoidal voltage source with an amplitude of 1 V.

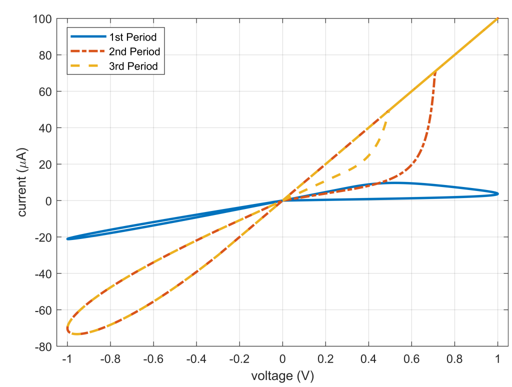

Fig. 2. Hysteresis behavior of the memristor model when connected to a sinusoidal voltage source with an amplitude of 1 V and a frequency of 1 Hz. The data was plotted over three periods.

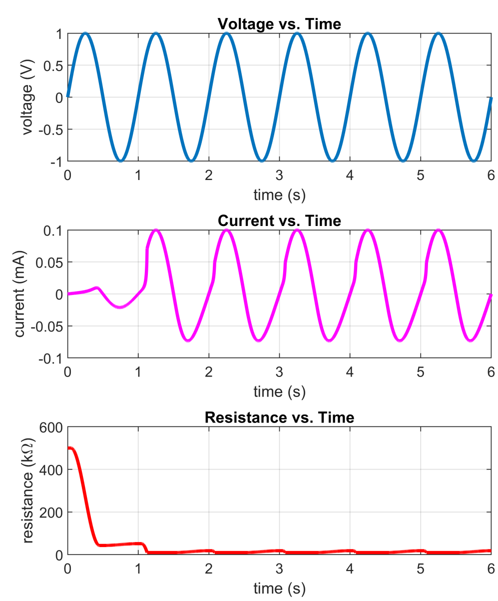

Fig. 3. Time domain behavior of the given memristor model when connected to a sinusoidal voltage source with an amplitude of 1 V and a frequency of 1 Hz. The data was plotted over six periods.

The behavior of this model is further outlined in Fig. 3. This plot demonstrates that the model's imbalance causes the memristance to decrease faster than it increases when connected to a sinusoidal voltage source. In this case, the memristance value exists in a narrow range for a sinusoidal input once the minimum memristance value has been reached.

B. The Pupil System Model

The model of the pupil system is designed to be straightforward in terms of usage and modifiability. The input voltage signal ranges from 0 V to 1 V which corresponds to minimum and maximum illumination. The output of the circuit is taken as a voltage value ranging from 0.5 V to 0.98 V where 0.5 V corresponds to minimum pupil diameter and 0.98 V corresponds to maximum pupil diameter (the reasons behind these parameters are discussed later). The output voltage is taken as the quotient of two voltages along a simple voltage divider circuit consisting of a fixed resistor and a memristor.

In order for the circuit to operate properly, a relationship between input voltage and steady state output voltage was devised. For simplicity, the steady state output voltage is modeled as a linear function of input voltage. In other words, it is assumed that if the steady state pupil diameter is d_1 at illumination i_1 and the steady state pupil diameter is d_2 at illumination i_2, then the pupil diameter at $\frac{i_1+i_2}{2}$ can be given as the midpoint between d_1 and d_2 (equivalent to $\frac{d_1+d_2}{2}$). Using the previously defined input and output voltage ranges, the steady state output voltage (V_{ss}) as a function of input voltage is approximated as:

$$V_{ss} = -0.48v_{in} + 0.98 \tag{2}$$

While a linear relationship may be a liberal assumption, it serves as a good basis for a simple and consistent model. (If a more advanced representation of steady state pupil diameter as a function of illumination is desired, it could be implemented by adding a single "preprocessing" stage immediately after the stimulus voltage source.)

The overall process of the pupil model is a straight-forward closed-loop design. The input voltage is read and the steady state output voltage is calculated. The steady state output voltage is then subtracted from the present output voltage to produce an error value (V_{ri}). This error value is amplified and sent through a low-pass filter to smooth the affects of voltage spikes. This filtered error value (V_r) is then sent to the voltage divider circuit. If this voltage value is high enough, it will create a change in memristance within the memristor, otherwise the memristance will remain constant. This process is summarized in Fig. 4.

As previously discussed, the output of the circuit is taken as a ratio of two voltages: $V_{out} = \frac{V_m}{V_r}$. The two voltages are taken from a simple voltage divider consisting of a resistor in series with a memristor. The resistor R was set to 10 kΩ which is equal to the minimum memristance value of the memristor. As a result, the voltage across the memristor lies in the range $0.5V_r \leq V_m \leq 0.98V_r$. Therefore, the value of the model's output (V_m/V_r) lies in the range $0.5 \leq V_{out} \leq 0.98$. It is for

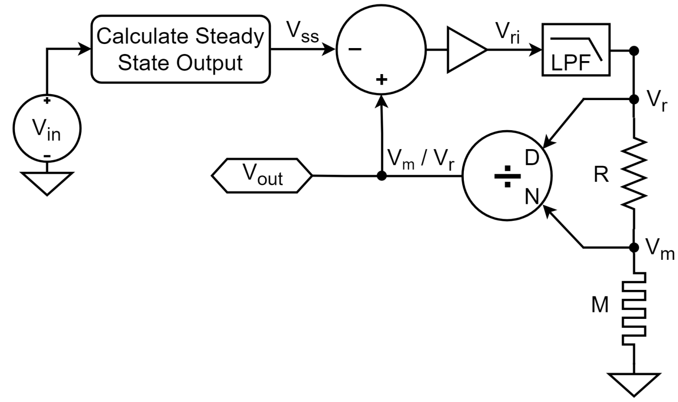

Fig. 4. Block diagram of the memristive pupil model

this reason that (2) was defined for an output in the range of 0.5 V to 0.98 V. To sum up these relations, the output voltage as a function of memristance can be given as

$$V_{out}(M) = 1 - \frac{R}{R+M} = 1 - \frac{10000}{10000+M} \tag{3}$$

To achieve these results, various subcircuits had to be defined. A custom difference amplifier was created to calculate V_{ss}, a difference amplifier with a non-unity gain was created to determine the error value V_{ri}, a basic RC lowpass filter was constructed to diminish the effects of voltage spikes, and a division block was defined in Verilog-A. Each amplifier is powered with a 2 V and -2 V DC power supply.

1) Steady State Output Voltage Calculator: The steady state voltage calculator is the first stage of the pupil model. This portion of the circuit is an analog implementation of the calculation given in (2). This was designed using a difference amplifier like the one seen in Fig. 5. The output of the difference amplifier can be given as $V_{out} = \left[\frac{R_4(R_1+R_3)}{R_1(R_2+R_4)}V_A - \frac{R_3}{R_1}V_B\right]$ as explained in [10]. To produce an output corresponding to (2), the resistance values of $R_1, R_2, R_3,$ and R_4 are set to 680 kΩ, 200 kΩ, 330 kΩ, and 390 kΩ, respectively. All of these values are standard resistor values. The model's input voltage V_{in} is connected to V_B and a 1 V DC voltage source is connected to V_A. This produces the relationship $V_{ss} \approx -0.485v_{in} + 0.982$ which very closely approximates the relationship given in (2).

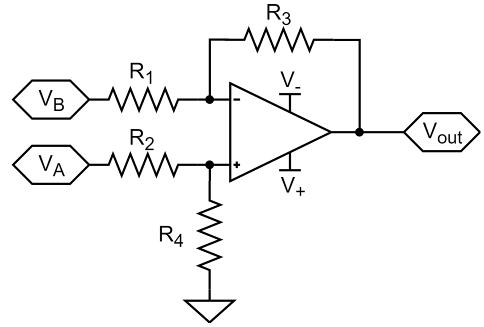

Fig. 5. Difference amplifier design

2) Difference Amplifier for Calculating Error: The difference block that calculates and amplifies the difference between V_{out} and V_{ss} was constructed in a similar fashion. To achieve an equal gain for both inputs, the resistance values of $R_1, R_2, R_3,$ and R_4 are set to 10 MΩ, 10 MΩ, 100 kΩ, and 100 kΩ, respectively. This gives the difference amplifier a total gain of 100. Therefore, the equation corresponding to this amplifier is given as $V_{out} = 100(V_A - V_B)$. Without this high gain, the magnitude of V_r and V_m would only rise above the threshold voltage of the memristor when the error is extremely high. Having a gain of 100 allows V_r and V_m to be high enough to tweak the memristance value as long as the error value is $\approx \pm 3$ mV which is about 6% of the range of possible values of V_{out}.

3) Lowpass Filter: The lowpass filter is used to smooth sudden voltage spikes and is constructed as a simple RC lowpass filter [10]. The resistor has a value of 100 Ω and the capacitor is 4.7 μF. This gives the filter a cutoff frequency of about 340 Hz. Without this lowpass filter, the simulation process would often suffer from a lack of convergence due to sudden voltage spikes. While this error could often be remedied by reducing the simulator's time step size, the lowpass filter greatly improved the circuit's resilience to simulation convergence errors.

4) Division Block: The final stage of the model determines the output voltage. This is accomplished by dividing V_m by V_r. This configuration was decided upon because the result is not affected by the polarity of the values and because its resulting equation given in (3) works well at simulating a natural pupil response. For simplicity, this division process was defined in Verilog-A code. If this entire pupil model were to be physically constructed, the division block could be implemented either in the analog domain with a sophisticated division circuit or digitally with a basic microprocessor.

5) Overall System: Once each of these subcircuits were constructed, they were combined into the top-level schematic. The schematic as created in Cadence is shown in Fig. 6. This schematic is laid out in a similar fashion as the block diagram in Fig. 4 with the signals primarily progressing from left to right.

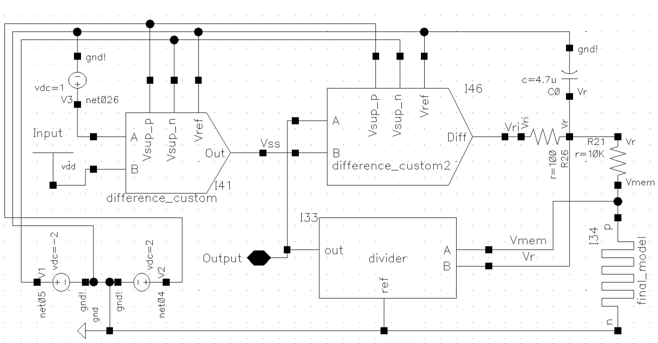

Fig. 6. Top-level schematic as seen in Cadence

IV. Results and Evaluation

To understand the successes and limitations of the model, six tests were performed on the model, each with a different input signal. For explanation purposes, a nonzero input voltage value will be referred to as "light" and an input value of 0 V will be referred to as "darkness." The tests were as follows: a sudden light pulse after acclimation to darkness, a light pulse train after acclimation to darkness, a mild light pulse (0.7 V) after acclimation to dim light (0.3 V), a sudden light outage after acclimation to the light, a 1 Hz sinusoidal light intensity after acclimation to darkness, and a 10 Hz sinusoidal light intensity after acclimation to darkness.

A. Response to Light Pulses

The first test was conducted in a manner similar to that outlined in [7] for the biological test that produced the data seen in Fig. 1. The test performed on the proposed memristive pupil model began with two seconds of darkness (0 V) followed by one second of maximum light intensity (1 V) followed by 54 seconds of darkness. The light pulse was given a rise and fall time of 1 μs (virtually instantaneous). Note that a long acclimation period is not needed for the simulation because the memristive model is initialized to steady state darkness conditions. Additionally, the test does not assume any specific wavelength of light stimuli as [7] did. (If one desired to simulate a response to various light wavelengths, a variety of memristor models with varying γ values could be used to each mimic responses to a different light wavelength.)

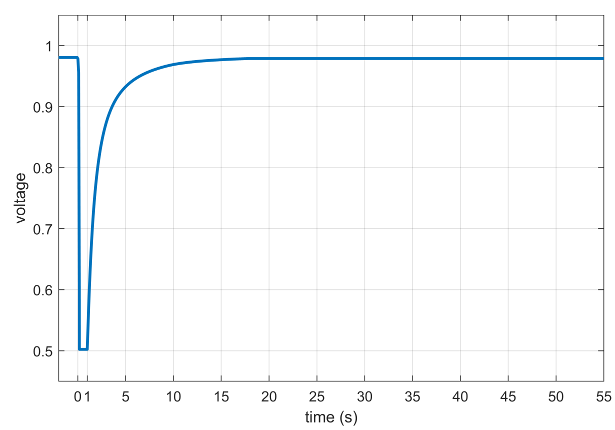

Fig. 7. Pupil model's response to a 1 second maximum light pulse occurring at t=0

The results for this light pulse test are shown in Fig. 7. As this figure shows, the model's response very closely resembles the results of the biological tests shown on the red (upper) trace in Fig. 1 as well as results obtained by [8], [11], and [12]. The output voltage very quickly drops in the presence of light and takes much longer to return to the steady-state darkness level. This result is primarily due to the imbalanced nature of the memristor memristor model, i.e. the high γ_1/γ_2 ratio. The nature of the voltage divider circuit also allows for the very gradual return to maximum pupil diameter (output voltage) because the rate of change of V_{out} with respect to M

declines as M increases. The derivative of (3) with respect to M can be given as $\frac{d}{dM}V_{out}(M) = \frac{10000}{(10000+M)^2}$.

To further demonstrate the repetitive nature of the model, a one second light pulse was applied to the model periodically. The model was acclimated to the dark (0 V) and then exposed to a one second maximum intensity light pulse followed by nine seconds of darkness before repeating. The results given in Fig. 8 show that the model does behave periodically when provided with a periodic stimulus. Each time, the acclimation to the light is quick while the acclimation to the dark is much slower. In each period, the output is closely approaching steady state as the light pulse turns back on. This is consistent with Ellis' biological results in [8].

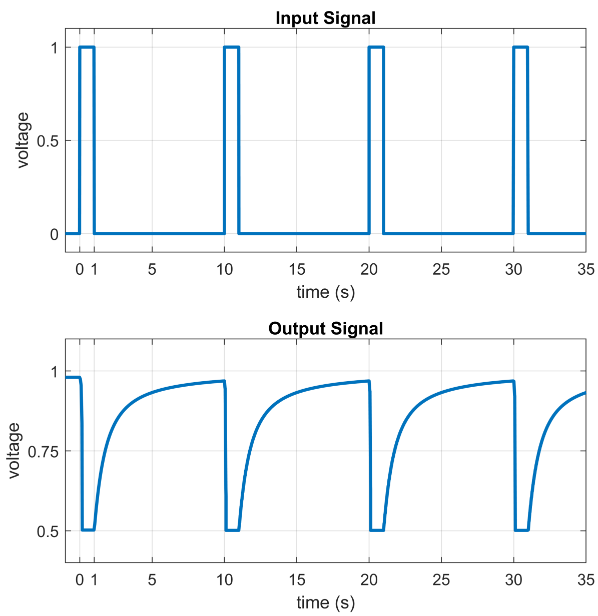

Fig. 8. Pupil model's response to a periodic 1 second maximum light pulse (10 second period with 10% duty cycle)

To ensure that the model responds properly to intermediate V_{in} values, the model was tested with a mild one-second light pulse in the midst of dim light. The model was first acclimated to dim light (0.3 V) for two seconds and then exposed to a one second pulse of brighter light (0.7 V). The model behaved as expected. The output adjusted to the 0.3 V source by quickly adjusting to the steady state value of about 0.84 V in accordance with (2). Once the stimulus increased to 0.7 V at time=0, the model quickly reached the steady state value of about 0.65 V. Once the source returned to the dim light (0.3 V), the output then gradually increased to its previous steady state value of about 0.84 V. Since the range of the stimuli values was smaller in this test than in the first light pulse test, the model took less time to reach steady state behavior. While the model took about 15 seconds to return to steady state after the pulse in the first test (0 V - 1 V pulse), the model needed just over 1 second to return to steady state after the pulse in this test (0.3 V - 0.7 V).

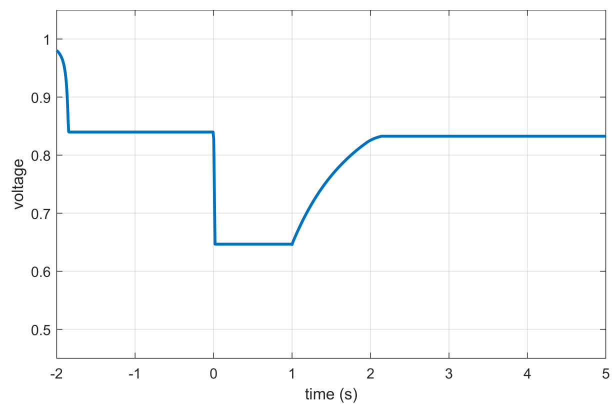

Fig. 9. Pupil model's response to a 1 second mild light pulse (0.7 V) at t=0 after acclimation to dim light (0.3 V)

B. Response to a Light Outage

The inverse of the singular light pulse test was also modeled. In this test, the model was acclimated to maximum light intensity (1 V) before experiencing one second of darkness (0 V) at time=0. The stimulus then returned to maximum light intensity at time=1. The results are shown in Fig. 10. In this test, the model further demonstrated its quick response to increasing illumination and slower response to decreasing illumination. In other words, the model mimics the idea that the pupil takes more time to reach steady state when entering the dark than it does when being exposed to light.

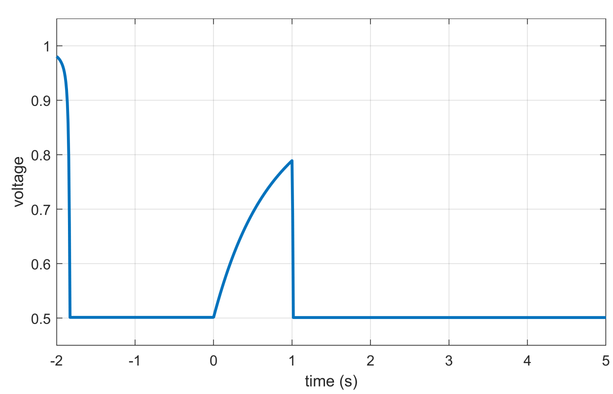

Fig. 10. Pupil model's response to a one second light outage occurring at t=0 after acclimation to maximum light intensity

C. Repose to Sinusoidal Sources

The final two tests to be performed each involved a sinusoidal light stimulus. The model was tested with both a 1 Hz and a 10 Hz sinusoidal light source. The input waveform was given an amplitude of 0.5 V, an offset of 0.5 V, and a phase delay that allowed the signal to begin at 0 V. The waveforms at both frequencies are shown in Fig. 11. According to [7], the human pupil response to a sinusoidal stimulus "has a low pass characteristic" with a cutoff frequency of about 9 Hz. In other words, the pupil diameter does not noticeably produce periodic variations when exposed to a sinusoidal stimulus with a frequency of about 9 Hz or greater. Once this cutoff is reached, the pupil treats the light source more like a constant value.

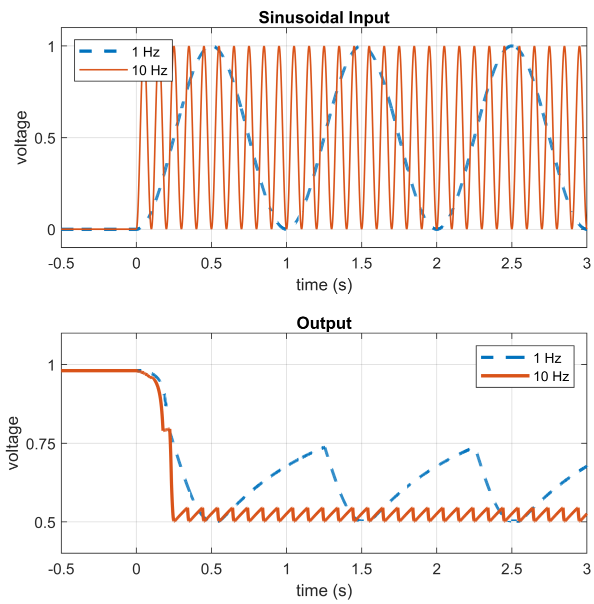

Fig. 11. Pupil model's response to sinusoidal waveforms with frequencies of 1 Hz and 10 Hz

The results shown in Fig. 11 demonstrate that the model does indeed produce a sinusoidal response to the 1 Hz input and produces a small periodic variation when exposed to the 10 Hz stimulus. After further testing, it was found that the model does not produce steady output in the presence of a sinusoidal input until the input signal frequency begins to approach approximately 100 Hz. This stands out as a notable contrast from a biological pupil which experiences a cutoff at about 9 Hz [7].

V. Discussion of Limitations and Possible Future Work

Overall, the model appears to perform reasonably well. The asymmetry of the pupil's response correlates well with the biological results given in [7], [8], and [11]. Additionally, the response to the light pulse fits very well with the plot from [7] shown in Fig. 1. The model has been shown to adapt to illumination fluctuations well, regardless of the intensity of the light. Additionally, the model shows a lowpass response in the presence of sinusoidal stimuli, albeit with a higher cutoff than biological descriptions [7]. Furthermore, the model can be easily adapted to accommodate faster or slower pupillary responses by modifying parameters like γ and R. This allows the model to be adapted to simulate responses corresponding to different wavelengths of light (see Fig. 1) as well as various diseases that affect the pupil's response time and response magnitude [7].

Nonetheless, the model is not perfect. The first point of discrepancy between the memristive pupil model and biological results deals with latency. As [7] and [8] point out, a biological pupil experiences a measurable latency between a change in stimulus and its reaction to that change. In [8], Ellis shows that this latency is within the order of magnitude of hundreds of milliseconds and that latency decreases as stimulus intensity increases. Conversely, this memristive pupil model begins its response without delay. While the simulated pupil diameter takes a measurable amount of time to reach steady state (as do biological pupils), the response begins immediately. With further development, the model could be modified and expanded to include a delay that is inversely related to stimulus intensity.

Another additional discrepancy between the memristive pupil model and biological results is the presence of abrupt plateauing in the memristive model's output during the presence of a sudden light pulse. This can be noticed in Fig. 7, 8, and 9. Biological results like those shown in 1 seem to reach a singular minimum pupil diameter value following the light pulse before returning to a greater value. This minor difference could perhaps be corrected with further development of the model. It is hypothesized that adding a latency effect to the model and altering the memristance range could perhaps help to remedy this problem.

Finally, a number of assumptions made during the design process could be further improved upon. As previously stated, a linear approximation was used to represent the relationship between steady state pupil diameter and stimulus intensity. This representation given in (2) serves as a very general approximation and could likely be improved upon with the use of more extensive biological data. Likewise, the simulation does not take into account imperfections of components and noisy signals. If the complete model were to be realized as a physical circuit, care would need to be taken to minimize noise and ensure proper tolerance ranges of components. Since the second difference amplifier has a gain of 100, a small amount of noise in the V_{out} or V_{ss} signals would be increased 100-fold and alter the behavior of the circuit. Since the gain value of the difference amplifier impacts how close the output gets to steady state before dropping the magnitude of V_m below the magnitude of the threshold voltages, then reducing the gain of the difference amplifier (all else equal) would reduce the steady state accuracy of the model. This could be compensated by altering the value of R or by changing the range of memristance values. Additionally, it is worth mentioning that this model only accepts a single input value that corresponds to illumination intensity. Further development could lead to a model that accounts for factors such as emotions and muscle activity, both of which have been shown to affect pupil responses [6], [7], [9].

VI. Conclusions

In light of the results, the model appears to perform reasonably well. The model manifests the asymmetric relations present in biological results and responds to illumination changes properly. Light intensity was shown to influence the speed at which the model reached steady state. The lowpass behavior exhibited in biological pupils was also present in the memristive model, albeit with a notably higher cutoff frequency. Furthermore, the model can be easily adapted to accommodate faster or slower responses by modifying parameters like γ and R. This opens up the possibility of simulating responses that correspond to different wavelengths of light (see Fig. 1) as well as various diseases that affect the pupil's response time and magnitude [7].

Ultimately, the proposed memristive pupil model has shown to provide a response that is consistent with biological results. The model was tested with various types of light pulses as well as sinusoidal stimuli. In each case, the model provided output values that matched expectations well. While the model has been shown to have imperfections, the simple structure of the model provides room for improvement and modifications to suit various simulation applications. With further development, the proposed memristive pupil model can be used to create an accurate and straightforward system that mimics the human eye like no circuit has before.

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