Specify a continuous and differentiable function that satisfies the following characteristics:

f(x) is decreasing at 6 f(x)

has a local minimum at x=-3

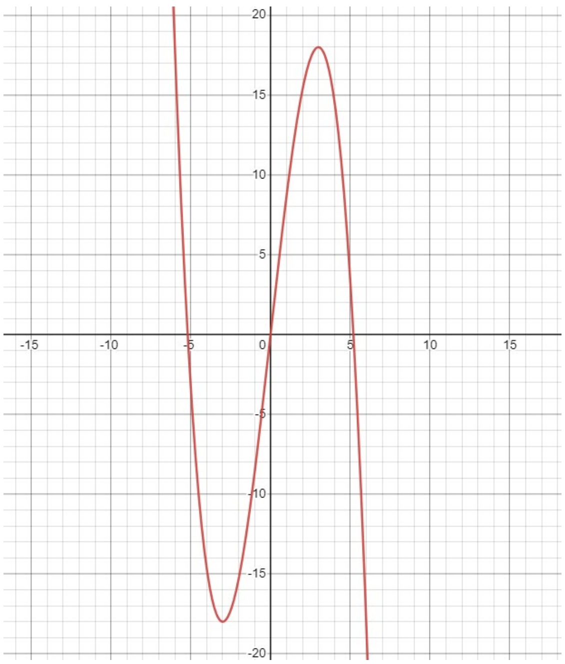
f(x) has a local maximum at x=3.

Step one: Finding the first Derivative

setting -3) = 0 we find (3+x)(3-x) leads to critical points at 3 and 3. So the derivative is 9-x^2.

Step Two: Integration to find the equation of the function

Since we know the antiderivative of a number is (x^(n+1))/(n+1) + C: we know that the antiderivative of 9-x^ 2 is 9x - (x^3/3) + C. Finding the antiderivative of 9-x^ 2 So, the antiderivative of 9-x^ 2 is 9x- (x^3)/3 and so f(x)= 9x - (x^3)/3 + C is our function's equation f(x).



Graph of f(x)= 9x - (x^3)/3

Intervals of Increase and Decrease

Plugging in values of -6, 0 and 4 into the first derivative, f'(x), we see that the function is decreasing at 6, increasing at 0, and decreasing again at 4.

9-x^ 2 9 - (-6)^2 = -27 < 0 therefore so decreasing

9 - (0)^2= 9 = 9 therefore + so increasing

9 - (4)^2= -7 < 0 therefore - so decreasing

The function decreases at intervals (infinity, -3) and (3, infinity). It increases at the interval (3,3).

Plugging in 3 and 3 into our function f(x), we find the critical points are at (3,18) (-3,-18). Since the function is decreasing at 6 and increasing at 0, we know that (3, 18) is a local maximum and (-3, -18) is a local minimum. We can confirm this by finding the concavity intervals from the second derivative.

Additional Characteristics of F(x) Find the Second Derivative:

From our derivative equation, we can find the second derivative.

f''(x)= -2x is our Second Derivative.

We know that the second derivative gives inflection points and concavity intervals. Since we know that x=-3 is a minimum, this means that f(x) should be concave upward at this point. Plugging in values smaller than 3 and larger than 3 to verify the signs of the intervals. Likewise, since we know that x=3 is a maximum, f(x) should be concave downward at this point, and we would need to verify this with the signs of the intervals plugged into the second derivative.

-2(-6) = +12 therefore + and concave up

-2(0)=0

-2(4) = -8 therefore - and concave down

Now we have verified that F(x) is concave up at (-infinity, 0) and concave down at (0, infinity) this makes sense with what we know about x=3 as a maximum and x=-3 as a minimum.

Is this function a unique solution to the question?

No, it is not unique. There are other functions that would fit the criteria given for this problem. Any equation that fits the following condition 9x - (x^3)/3 + C would work. This means that 9x-x^3/3 +1, or 9x-x^3/3 +2, or 9x-x^3/3 +3, or 9x-x^3/3 +4... etc, would work. Any positive constant multiplied by (f') also works. f ' divided by a positive constant works. Also, an f ' to an odd power will fit this function.