

For my problem I was given the following properties to match to a continuous and differentiable function:

$f(x)$ is decreasing at -6

$f(x)$ has a local minimum at $x=-3$

$f(x)$ has a local maximum at $x=3$

Step one: Finding the first Derivative

From this we know that we have been given two critical points. This means that $f'(3)$ and $f'(-3)$ will equal 0. So we will start with the first derivative to come up with factors that would lead to 0.

$(3+x)(3-x)$ would lead to critical points at -3 and 3 .

By expanding, we find the derivative for our chosen factors is $9-x^2$.

$f'(x) = 9-x^2$ will be our derivative equation.

Step Two: Integration to find the equation of the function

The next step once we have a derivative is integration. Since we know the antiderivative of a number is $(x^{(n+1)})/(n+1) + C$: we know that the antiderivative of $9-x^2$ is $9x - (x^3/3) + C$.

Finding the antiderivative of $9-x^2$

The antiderivative of 9 is $9x^{(1)}/1$ or $9x$

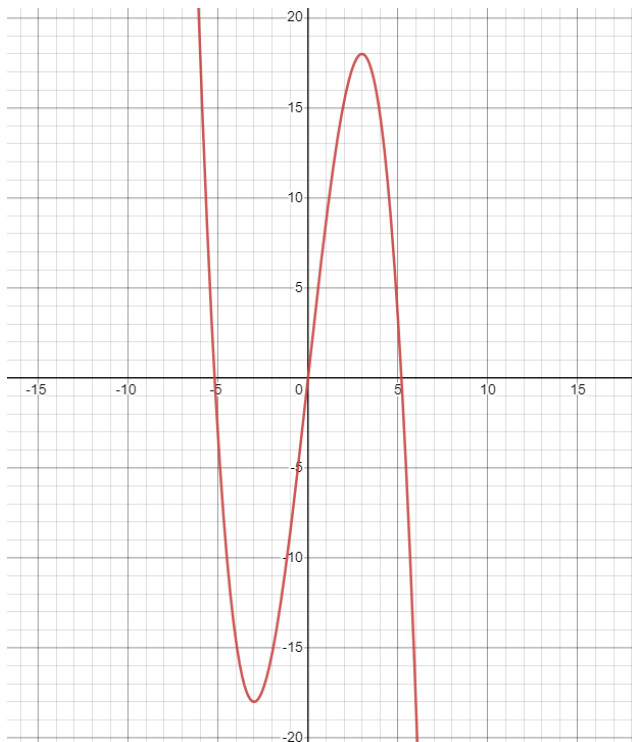
the antiderivative of x^2 is $x^3/3$

C is any constant, including zero so I am going with zero

So, the antiderivative of $9-x^2$ is $9x - (x^3)/3$ and so

$f(x) = 9x - (x^3)/3 + C$ is our function $f(x)$ but I will use $f(x) = 9x - (x^3)/3$

The antiderivative gave us our equation for the function $f(x)$ which we will plot in below



Graph of $f(x) = 9x - \frac{x^3}{3}$

(Desmos | Let's Learn Together., n.d.)

Intervals of Increase and Decrease

Plugging in values of -6, 0 and 4 into the first derivative, $f'(x)$, we see that the function is decreasing at -6, increasing at 0, and decreasing again at 4. All this, we can verify on the graph.

$$9 - x^2$$

$$9 - (-6)^2 = -27 < 0 \text{ therefore } - \text{ so decreasing}$$

$$9 - (0)^2 = 9 = 9 \text{ therefore } + \text{ so increasing}$$

$$9 - (4)^2 = -7 < 0 \text{ therefore } - \text{ so decreasing}$$

It decreases at intervals $(-\infty, -3)$ and $(3, \infty)$.

It increases at the interval $(-3, 3)$.

Plugging in -3 and 3 into our function $f(x)$, we find the critical points are at $(3, 18)$ $(-3, -18)$. Since the function is decreasing at -6 and increasing at 0, we know that $(3, 18)$ is a local maximum and $(-3, -18)$ is a local minimum. We can confirm this by finding the concavity intervals from the second derivative.

Additional Characteristics of $F(x)$ Find the Second Derivative:

From our derivative equation, we will calculate our second derivative knowing the derivative of $x^n = nx^{(n-1)}$. We find that:

$f''(x) = -2x$ is our Second Derivative.

We know that the second derivative gives inflection points and concavity intervals. Since we know that $x = -3$ is a minimum, this means that $f(x)$ should be concave upward at this point. We would need to plug in values smaller than -3 and larger than 3 to verify the signs of the intervals. Likewise, since we know that $x = 3$ is a maximum, $f(x)$ should be concave downward at this point, and we would need to verify this with the signs of the intervals plugged into the second derivative.

$-2(-6) = +12$ therefore $+$ and concave up

$-2(0) = 0$

$-2(4) = -8$ therefore $-$ and concave down

Now we have verified that $F(x)$ is concave up at $(-\infty, 0)$ and concave down at $(0, \infty)$ this makes sense with what we know about $x = 3$ as a maximum and $x = -3$ as a minimum.

Is this function a unique solution to the question?

What fits?

No, the function that I found is not unique. There are other functions that would fit the criteria given for this problem. Specifically, any equation that fits the following condition $9x - \frac{(x^3)}{3} + C$ would work. This means that $9x - \frac{x^3}{3} + 1$, or $9x - \frac{x^3}{3} + 2$, or $9x - \frac{x^3}{3} + 3$, or $9x - \frac{x^3}{3} + 4...$ etc, would work. I simply chose $9x - \frac{x^3}{3} (+0)$ to make the math and explanations easier.

I also checked function with derivative factors of: $9(3-x)(3+x)$ and this function *does* fill all of the criteria above. The derivative is $f' = 81 - 9x^2$ and the function to match (found via antiderivative) is $f = 81x - 3x^3 + C$. This function has critical points at -3 (local minimum) and 3 (local maximum). It is also decreasing at -6 . But, the local minimum here is at $(-3, -162)$ and the maximum point is at $(3, 162)$. Visually this graph looks like the $9x - \frac{(x^3)}{3}$ but much taller.

But not only this. Any $C(f')$ also works. For example any $5(3-x)(3+x)$, or $6(3-x)(3+x)$, or $7(3-x)(3+x)$, etc.

MAT-225-J1497 22EW1 Calc I: Single-Variable Calc / Module Six / 6-1 Module Six Discussion: Finding a Function to Match a Shape

6-1 Module Six Discussion: Finding a Function to Match a Shape

Current Grade: 0.0 / 1.0 Remaining Time: Unlimited

Module Six Discussion

For this week's discussion, you are asked to generate a continuous and differentiable function $f(x)$ with the following properties:

- $f(x)$ is decreasing at $x = -6$
- $f(x)$ has a local minimum at $x = -3$
- $f(x)$ has a local maximum at $x = 3$

Your classmates may have different criteria for their functions, so in your initial post in Brightspace be sure to list the criteria for your function.

Hints:

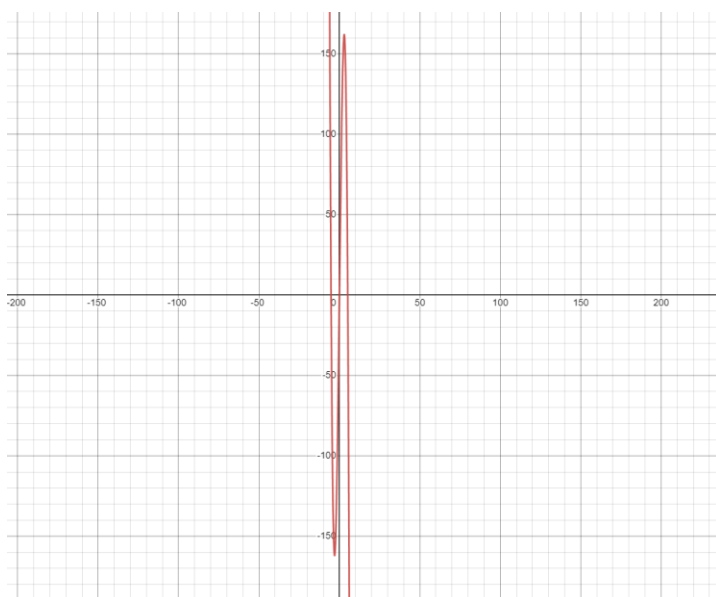
- Use calculus!
- Before specifying a function $f(x)$, first determine requirements for its derivative $f'(x)$. For example, one of the requirements is that $f'(-3) = 0$.
- If you want to find a function $g(x)$ such that $g(-9) = 0$ and $g(8) = 0$, then you could try $g(x) = (x+9)(x-8)$.
- If you have a possible function for $f'(x)$, then use the techniques in Indefinite Integrals this Module to try a possible $f(x)$.

You can generate a plot of your function by clicking the plotting option (the page option with a "P" next to your function input). You may want to do this before clicking "How Did I Do?". Notice that the label " $f(x) =$ " is already provided for you.

Once you are ready to check your function, click "How Did I Do?" below (unlimited attempts). Please note that the bounds on the x -axis go from -6 to 6.

$f(x) =$ ☒ ☐ ☐ ☐

Try Another



$f(x) =$ ☒ ☐ ☐ ☐

$f(x) =$ ☒ ☐ ☐ ☐

$f(x) =$ ☒ ☐ ☐ ☐

$f(x) =$ ☒ ☐ ☐ ☐

What does not fit?

I checked the function with derivative factors of $x(3-x)(3+x)$ and this function does not fit all of the criteria of the question. The derivative factors out to $9x-x^3$ and the function is $(9x^2)/2 - ((x^4)/4) + C$. This function does have the same critical points but also at 0, and also with a *minimum* at 0 and a *maximum* at 3 and -3. Also, it is not decreasing at -6, it is increasing.

I checked $(x+3)(x-3)$ and this does not fit the criteria above either. This derivative factors out to x^2-9 . The function to match is $(x^3/3)-9x+C$. This function has a *minimum* at 3 and a *maximum* at -3. Also, it is not decreasing at -6, it is increasing.

All this I verified in Mobius as well as a part of my process, entering these factors individually to see what might fit and why or why not. Any value that fits the $9x-x^3/3 + C$ did work for mobius including $9x-x^3/3 + 1$, or $9x-x^3/3 + 2$, or $9x-x^3/3 + 3$, or $9x-x^3/3 + 4$... etc and also any $C(f')$ worked including $108x-12x^3/3$, $45x-5x^3/3$, $99x-11x^3/3$, and $63x-7x^3/3$... etc.

Desmos | Let's learn together. (n.d.). Retrieved October 2, 2022, from

