For my problem I was given the following properties to match to a continuous and differentiable function:

$f(x)$ is decreasing at –6

$f(x)$ has a local minimum at $x=-3$

$f(x)$ has a local maximum at $x=3$

**Step one: Finding the first Derivative**

From this we know that we have been given two critical points. This means that $f'(3)$ and $f'(-3)$ will equal 0. So we will start with the first derivative to come up with factors that would lead to 0.

$(3+x)(3-x)$ would lead to critical points at –3 and 3.

By expanding, we find the derivative for our chosen factors is 9-x^2.

**f'(x) = 9-x^2 will be our derivative equation.**

**Step Two: Integration to find the equation of the function**

The next step once we have a derivative is integration. Since we know the antiderivative of a number is (x^(n+1))/(n+1) + C: we know that the antiderivative of 9-x^ 2 is 9x - (x^3/3) + C.

Finding the antiderivative of 9-x^ 2

The antiderivative of 9 is 9x^(1)/1 or 9x

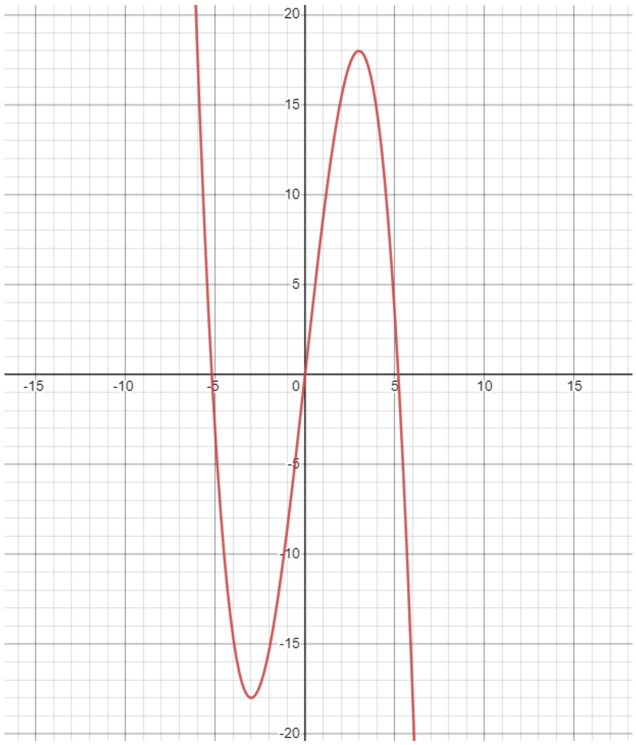
the antiderivative of x^2 is x^3/3

C is any constant, including zero so I am going with zero

So, the antiderivative of 9-x^ 2 is 9x- (x^3)/3 and so

**f(x)= 9x - (x^3)/3 + C is our function f(x)** but I will use f(x)= **9x - (x^3)/3**

The antiderivative gave us our equation for the function f(x) which we will plot in below




Graph of f(x)= 9x - (x^3)/3

(Desmos | Let's Learn Together., n.d.)

**Intervals of Increase and Decrease**

Plugging in values of -6, 0 and 4 into the first derivative, f'(x), we see that the function is decreasing at –6, increasing at 0, and decreasing again at 4. All this, we can verify on the graph.

9-x^ 2

9 - (-6)^2 = -27 < 0 therefore – so decreasing

9 - (0)^2= 9 = 9 therefore + so increasing

9 - (4)^2= -7 < 0 therefore - so decreasing

It decreases at intervals (–infinity, -3) and (3, infinity).

It increases at the interval (–3,3).

Plugging in –3 and 3 into our function f(x), we find the critical points are at (3,18) (-3,-18). Since the function is decreasing at –6 and increasing at 0, we know that (3,18) is a local maximum and (-3, -18) is a local minimum. We can confirm this by finding the concavity intervals from the second derivative.

**Additional Characteristics of F(x) Find the Second Derivative:**

From our derivative equation, we will calculate our second derivative knowing the derivative of $x^n = nx^{(n-1)}$. We find that:

**$f''(x)= -2x$ is our Second Derivative.**

We know that the second derivative gives inflection points and concavity intervals. Since we know that $x=-3$ is a minimum, this means that $f(x)$ should be concave upward at this point. We would need to plug in values smaller than –3 and larger than 3 to verify the signs of the intervals. Likewise, since we know that $x=3$ is a maximum, $f(x)$ should be concave downward at this point, and we would need to verify this with the signs of the intervals plugged into the second derivative.

$-2(-6) = +12$ therefore + and concave up

$-2(0)=0$

$-2(4) = -8$ therefore - and concave down

Now we have verified that $F(x)$ is concave up at (-infinity, 0) and concave down at (0, infinity) this makes sense with what we know about $x=3$ as a maximum and $x=-3$ as a minimum.

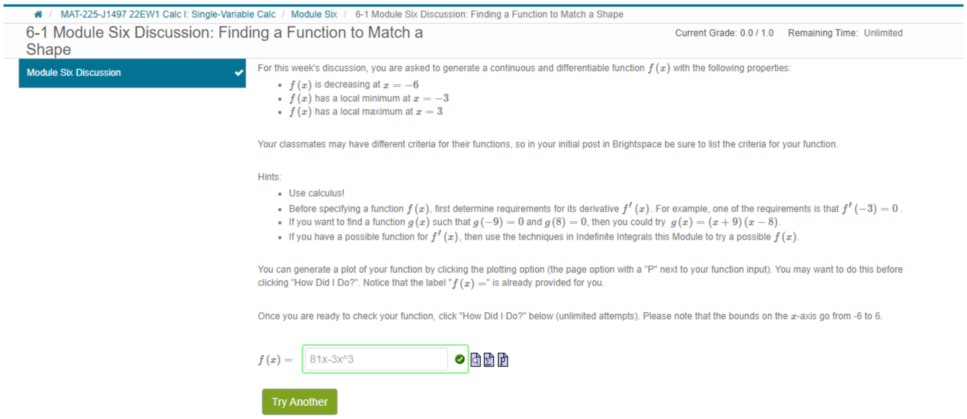
**Is this function a unique solution to the question?**

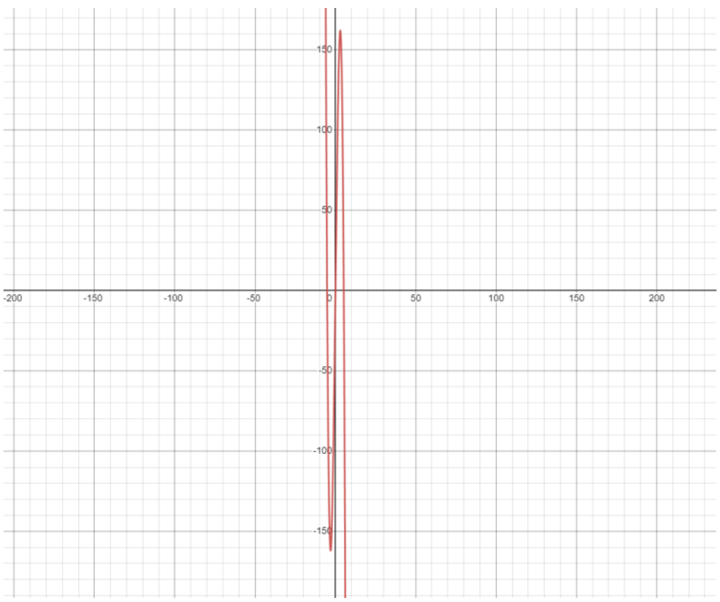
**What fits?**

No, the function that I found is not unique. There are other functions that would fit the criteria given for this problem. Specifically, any equation that fits the following condition **$9x - (x^3)/3 + C$** would work. This means that $9x-x^3/3 +1$, or $9x-x^3/3 +2$, or $9x-x^3/3 +3$, or $9x-x^3/3 +4$... etc, would work. I simply chose $9x-x^3/3$ (+0) to make the math and explanations easier.

I also checked function with derivative factors of: $9(3-x)(3+x)$ and this function *does* fill all of the criteria above. The derivative is $f'=81-9x^2$ and the function to match (found via antiderivative) is $f=81x-3x^3 + C$. This function has critical points at -3 (local minimum) and 3 (local maximum). It is also decreasing at -6. But, the local minimum here is at (-3,-162) and the maximum point is at (3,162). Visually this graph looks like the **$9x - (x^3)/3$** but much taller.

But not only this. Any C(**f'**) also works. For example any $5(3-x)(3+x)$, or $6(3-x)(3+x)$, or $7(3-x)(3+x)$, etc.





## What does not fit?

I checked the function with derivative factors of x(3-x)(3+x) and this function does not fit all of the criteria of the question. The derivative factors out to 9x-x^3 and the function is (9x^2)/2-((x^4)/4)+C. This function does have the same critical points but also at 0, and also with a *minimum* at 0 and a *maximum at 3 and* -3. Also, it is not decreasing at -6, it is increasing.

I checked (x+3)(x-3) and this does not fit the criteria above either. This derivative factors out to x^2-9. The function to match is (x^3/3)-9x+C. This function has a *minimum* at 3 and a *maximum at* -3. Also, it is not decreasing at -6, it is increasing.

All this I verified in Mobius as well as a part of my process, entering these factors individually to see what might fit and why or why not. Any value that fits the 9x-x^3/3 **+C** did work for mobius including 9x-x^3/3 +1, or 9x-x^3/3 +2, or 9x-x^3/3 +3, or 9x-x^3/3 +4... etc and also any **C**(f') worked including 108x-12x^3/3, 45x-5x^3/3, 99x-11x^3/3, and 63x-7x^3/3... etc.

*Desmos | Let's learn together.* (n.d.). Retrieved October 2, 2022, from