

HOMWORK 1

SETS AND FUNCTIONS *

10-606 MATHEMATICAL FOUNDATIONS FOR MACHINE LEARNING

START HERE: Instructions

- **Collaboration Policy:** Please read the collaboration policy in the syllabus.
- **Late Submission Policy:** See the late submission policy in the syllabus.
- **Submitting your work:** You will use Gradescope to submit answers to all questions.
 - **Written:** For written problems such as short answer, multiple choice, derivations, proofs, or plots, please use the provided template. Submissions can be handwritten onto the template, but should be labeled and clearly legible. If your writing is not legible, you will not be awarded marks. Alternatively, submissions can be written in \LaTeX . Each derivation/proof should be completed in the boxes provided. To receive full credit, you are responsible for ensuring that your submission contains exactly the same number of pages and the same alignment as our PDF template.
 - **Latex Template:** <https://www.overleaf.com/read/qvrjqvfxgtxh#f7f5>

Question	Points
Sets and Functions	15
Programming Sets and Functions	11
Total:	26

Instructions for Specific Problem Types

For "Select One" questions, please fill in the appropriate bubble completely:

Select One: Who taught this course?

- ☒ Matt Gormley
- ☐ Marie Curie
- ☐ Noam Chomsky

If you need to change your answer, you may cross out the previous answer and bubble in the new answer:

Select One: Who taught this course?

- ☒ Henry Chai
- ☐ Marie Curie
- ☒ Noam Chomsky

For "Select all that apply" questions, please fill in all appropriate squares completely:

Select all that apply: Which are scientists?

- ☒ Stephen Hawking
- ☒ Albert Einstein
- ☒ Isaac Newton
- ☐ I don't know

Again, if you need to change your answer, you may cross out the previous answer(s) and bubble in the new answer(s):

Select all that apply: Which are scientists?

- ☒ Stephen Hawking
- ☒ Albert Einstein
- ☒ Isaac Newton
- ☐ I don't know

For questions where you must fill in a blank, please make sure your final answer is fully included in the given space. You may cross out answers or parts of answers, but the final answer must still be within the given space.

Fill in the blank: What is the course number?

10-606

10-606~~7~~

1 Sets and Functions (15 points)

1. (1 point) **Select one:** $\{\}$ and $\{\{\}\}$ are the same set.

☐ True

☒ False

2. (1 point) **Select one:** $\{\}$ is a subset of $\{\{\}\}$.

☒ True

☐ False

3. (1 point) **Select one:** $\{\}$ is an element of $\{\{\}\}$.

☒ True

☐ False

4. (2 points) Using set builder notation, write the set of positive odd integers that are divisible by 5 or by 7.

$$\{ 2x+1 \mid x \in \mathbb{Z}, ((x \cdot 5) == 0) \vee ((x \cdot 7) == 0) \}$$

5. (2 points) **Select all that apply:** Which of the following are set operations are commutative?

☒ union

☒ intersection

☐ set difference

6. (2 points) **Select one:** Suppose we have two sets $\mathcal{A} = \{1, 2, 3\}$ and $\mathcal{B} = \{-1, -2, -3\}$. Which of the following is the set product $\mathcal{A} \times \mathcal{B}$?

- ☐ -14
☐ $\{1, 4, 9\}$
☐ $\{-1, -4, -9\}$
☐ $\{(1, -1), (2, -2), (3, -3)\}$
☐ $\{\{1, -1\}, \{2, -2\}, \{3, -3\}\}$
☐ $\{(1, -1), (1, -2), (1, -3), (2, -1), (2, -2), (2, -3), (3, -1), (3, -2), (3, -3)\}$
☒ $\{\{1, -1\}, \{1, -2\}, \{1, -3\}, \{2, -1\}, \{2, -2\}, \{2, -3\}, \{3, -1\}, \{3, -2\}, \{3, -3\}\}$

7. (2 points) Given two sets $\mathcal{X} = \{x \mid x \in \mathbb{R}, x > 7\}$ and $\mathcal{Y} = \{x \mid x \in \mathbb{Z}, x < 1000\}$, what is the set intersection $\mathcal{X} \cap \mathcal{Y}$ in set builder notation?

$$\mathcal{X} \cap \mathcal{Y} = \{n \mid n \in \mathbb{Z}, 7 < n < 1000\}$$

8. (2 points) Given two sets $\mathcal{S} = \{x \mid x \in \mathbb{Z}, x > 7\}$ and $\mathcal{T} = \{x \mid x \in \mathbb{R}, x < 10\}$, what is the set difference $\mathcal{S} \setminus \mathcal{T}$?

$$\mathcal{S} \setminus \mathcal{T} = \{n \mid n \in \mathbb{Z}, 7 < n < 10\}$$

9. (2 points) Let f be a function which accepts two arguments. The first argument is a primary color $\{\text{red, green, blue}\}$ and the second argument is a year in the 20th century. It returns a real number. Write a mathematical expression describing the type of this function.

$$f: \{\text{red, green, blue}\} \times \{n \in \mathbb{Z} \mid 1901 \leq n < 2000\} \rightarrow \mathbb{R}$$

2 Programming Sets and Functions (11 points)

1. (2 points) Write a Python set comprehension that yields the set of positive integers that are divisible by 5 or by 7, and which are less than 1000.

```
def divisible_by_5_or_7(number):
    return {n for n in range(number) if (n % 5 == 0)
        or (n % 7 == 0)}
```

2. (5 points) A database join collects records from two different tables, matches them on a given field, and outputs a new element for each possible match. Write a Python function called `join` that performs a join of two tables (`table1` and `table2`) by matching a given field (`field_to_match`). For full credit, you must use set comprehensions. For this problem you can assume that both tables have the field `field_to_match`, and this is the only field that they have in common. We implement tables as lists of dictionaries, where the keys in each dictionary are fields. Below is an example table with two entries with "course" and "final score" as the fields.

```
[{"course": "10606", "final score": "92"},
 {"course": "15213", "final score": "83"}]
```

Here is an example of join:

```
table1 = [{"name": "John", "University": "CMU"},
          {"name": "Mary", "University": "MIT"}]
table2 = [{"name": "John", "Residence": "Pennsylvania"},
          {"name": "Mary", "Residence": "Boston"}]
field_to_match = "name"
table3 = join(table1, table2, field_to_match)

correct_answer = [{"name": "John", "University": "CMU",
                    "Residence": "Pennsylvania"},
                  {"name": "Mary", "University": "MIT",
                    "Residence": "Boston"}]
assert table3 == correct_answer
```

```
def join(table1, table2, field_to_match):
    joined = []
    for v1, v2 in zip(table1, table2):
        if v1[field_to_match] == v2[field_to_match]:
            joined.append({**v1, **v2})
    return joined
```

3. (4 points) Write a function `is_multiple` that takes 3 integer variables, m , n , and x as input and returns an indicator of whether the predicate x is a multiple of both m and n .

```
def is_multiple(m, n, x):  
    if ((x % m) == 0 and (x % n) == 0):  
        return True  
    else:  
        return False
```

3 Collaboration Questions

After you have completed all other components of this assignment, report your answers to these questions regarding the collaboration policy. Details of the policy can be found in the syllabus.

1. Did you receive any help whatsoever from anyone in solving this assignment? If so, include full details.
2. Did you give any help whatsoever to anyone in solving this assignment? If so, include full details.
3. Did you find or come across code that implements any part of this assignment? If so, include full details.

Your Answer

1. for joining two dictionaries, I had to go online on seeks for - seeks to see different methods for merging.
2. No, I did the assignment by myself.
3. Yes for merging dictionaries using keywords **, forgot about how to do that, needed a refresher!