



# Product Importance Sampling for Light Transport Path Guiding

Sebastian Herholz<sup>1</sup>    Oskar Elek<sup>2</sup>    Jiří Vorba<sup>2,3</sup>

Hendrik Lensch<sup>1</sup>    Jaroslav Křivánek<sup>2</sup>

<sup>1</sup>University Tübingen

<sup>2</sup>Charles University Prague

<sup>3</sup>Weta Digital

# Motivation

Reference  
(4 weeks)



# BDPT

(1hr)



**BDPT**  
(1hr)



**Vorba2014**  
(1hr)

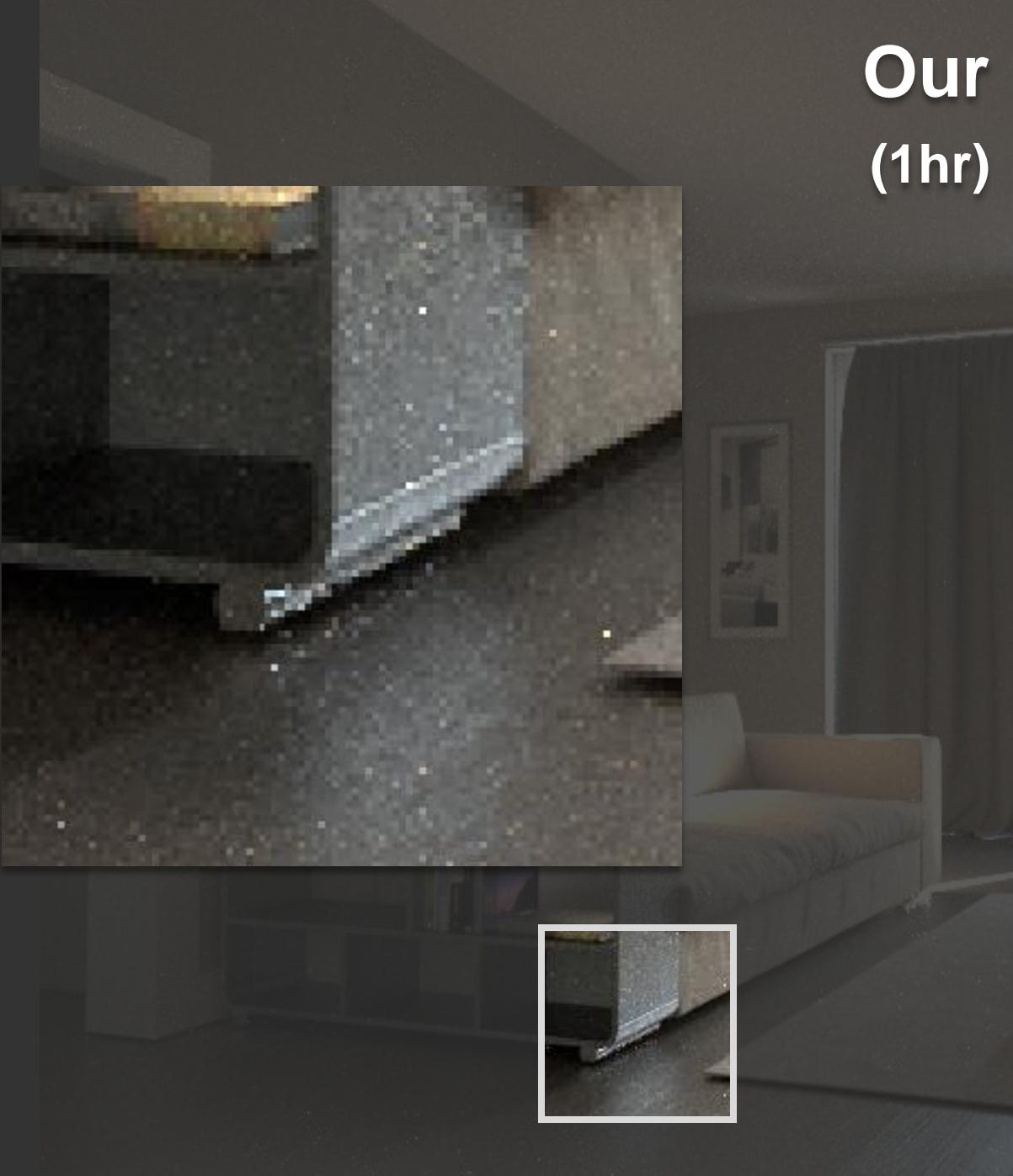


Our  
(1hr)



Vorba2014  
(1hr)





Our  
(1hr)

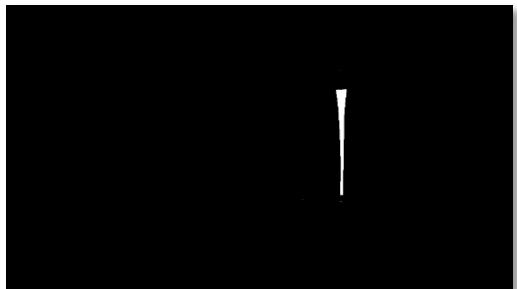


Vorba2014  
(1hr)

# Light Transport: Rendering Equation

$$L_o = L_e + \underbrace{\int_{\Omega} f_r \cdot L_i \cdot \cos\theta \cdot d\vec{\omega}_i}_{L_R}$$

emission



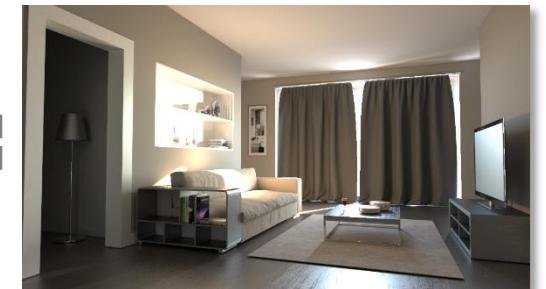
direct



indirect



combined

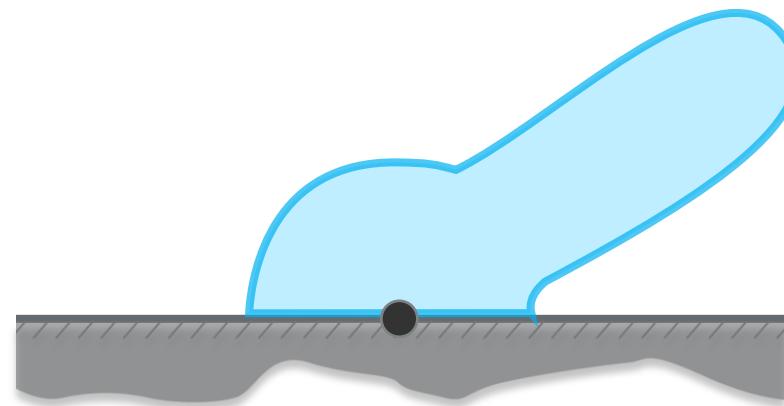


# Light Transport: Rendering Equation

$$L_o = L_e + \underbrace{\int_{\Omega} f_r \cdot L_i \cdot \cos\theta \cdot d\vec{\omega}_i}_{L_R}$$

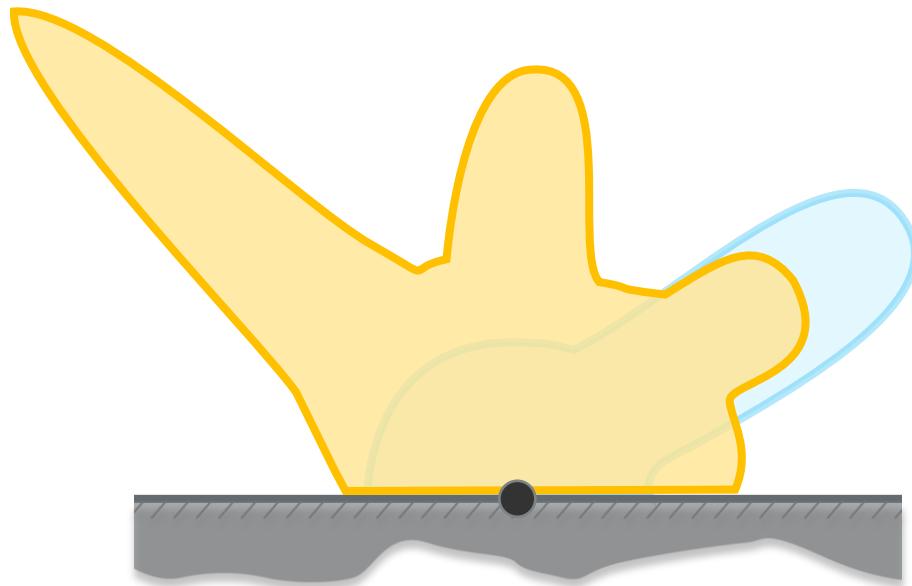


# Bidirectional Reflectance Distribution Function (BRDF)



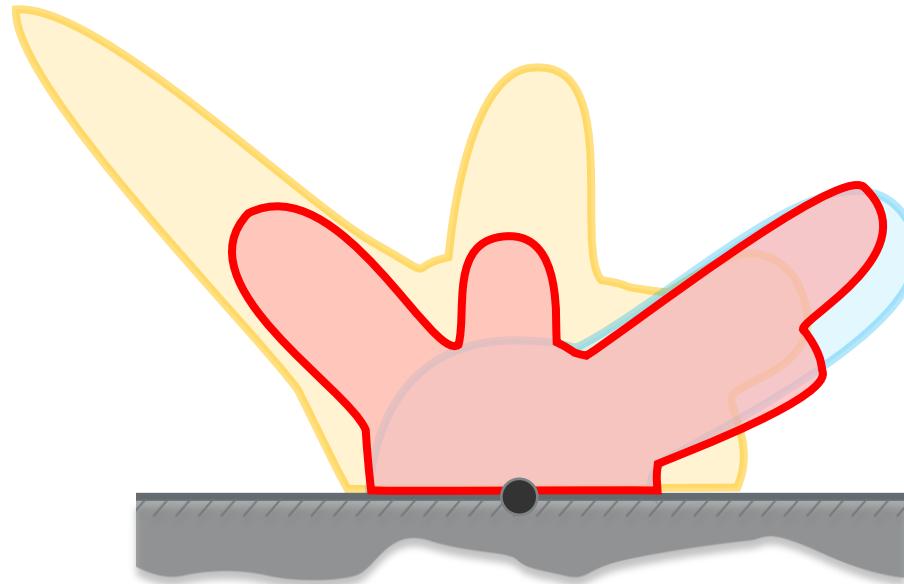
$$L_R = \int_{\Omega} f_r \cdot L_i \cdot \cos\theta \cdot d\vec{\omega}_i$$

# Incomming Illumination



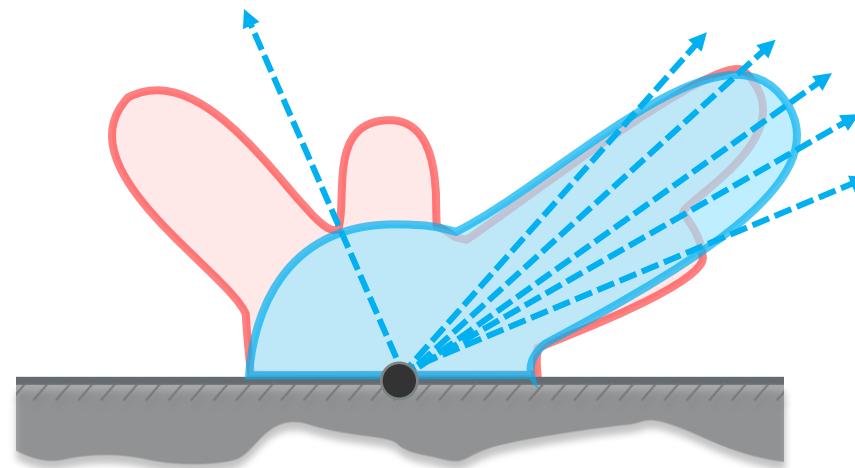
$$L_R = \int_{\Omega} f_r \cdot [L_i \cdot \cos\theta] \cdot d\vec{\omega}_i$$

# Reflectance Integral



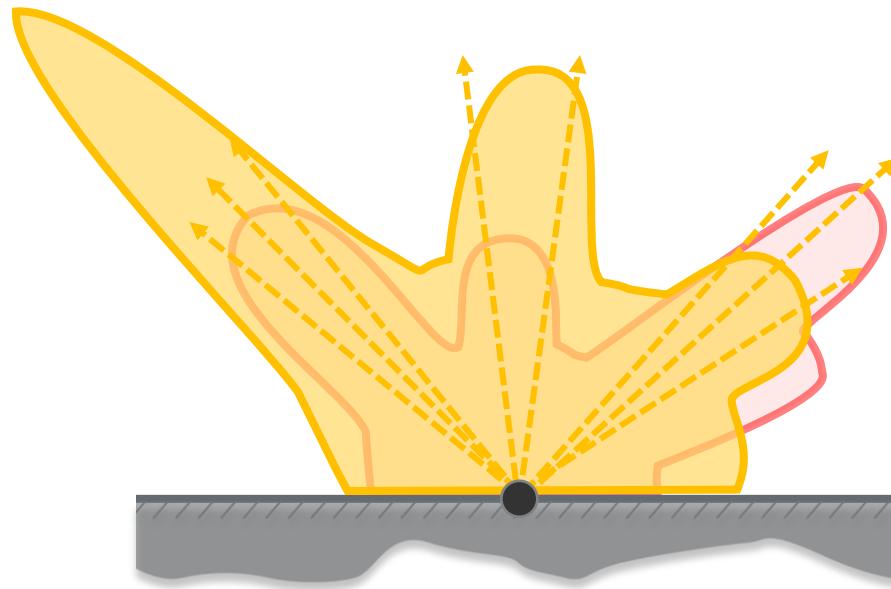
$$L_R = \int_{\Omega} [f_r \cdot L_i \cdot \cos\theta] \cdot d\vec{\omega}_i$$

# BRDF-based Sampling



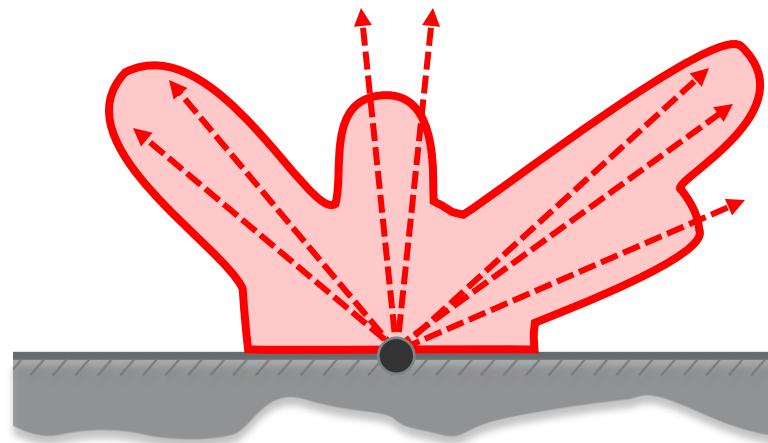
$$p_{f_r}(\omega_i | \omega_o, x) \propto f_r(x, \vec{\omega}_o, \vec{\omega}_i)$$

# Guided Illumination Sampling



$$p_L(\omega_i | \omega_o, x) \propto L_i(x, \vec{\omega}_i) \cdot \cos\theta$$

# Optimal (Product) Sampling

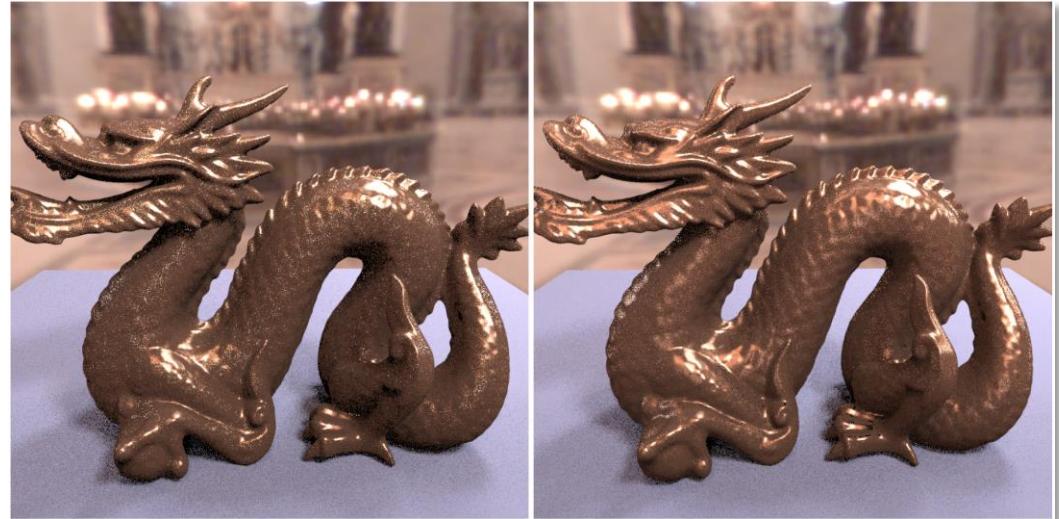


$$p_{opt}(\omega_i | \omega_o, x) \propto f_r(x, \vec{\omega}_o, \vec{\omega}_i) L_i(x, \vec{\omega}_i) \cdot \cos\theta$$

## Related Work



[CAM08]: Practical product importance sampling for direct illumination



[TCE05]: Importance resampling for global illumination



[CJAMJ05]: Wavelet importance sampling: efficiently evaluating products of complex functions  
[JCJ09]: Importance sampling spherical harmonics

# Product Importance Sampling

Motivation

Product Importance Sampling

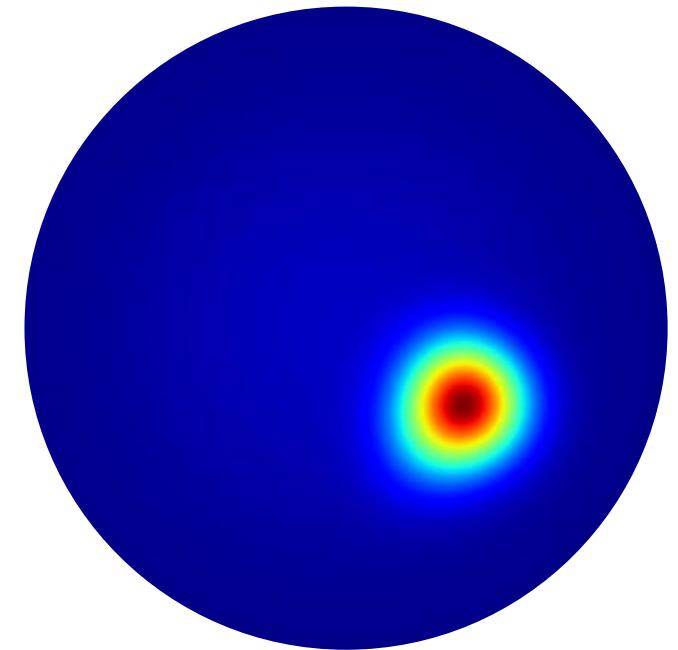
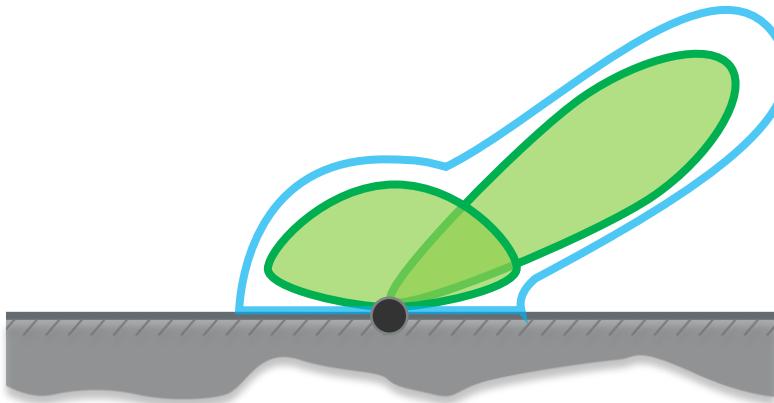
BRDF Fitting

Component Reduction

Results

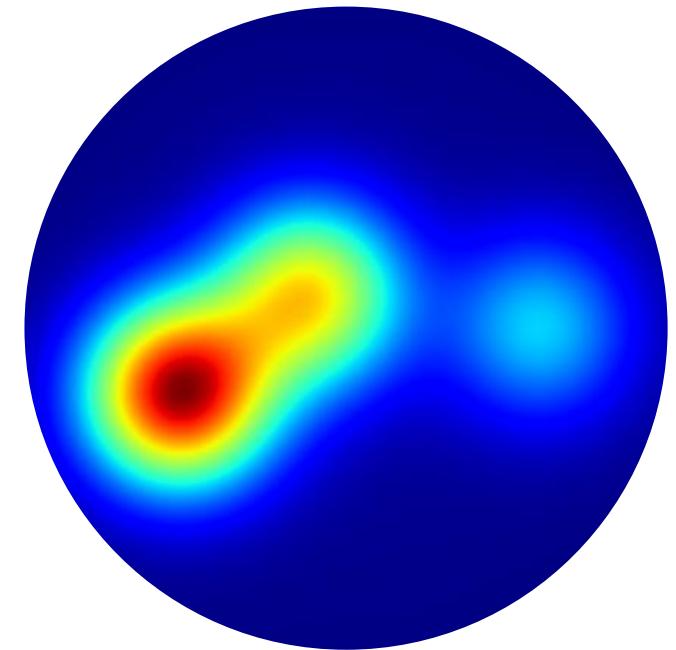
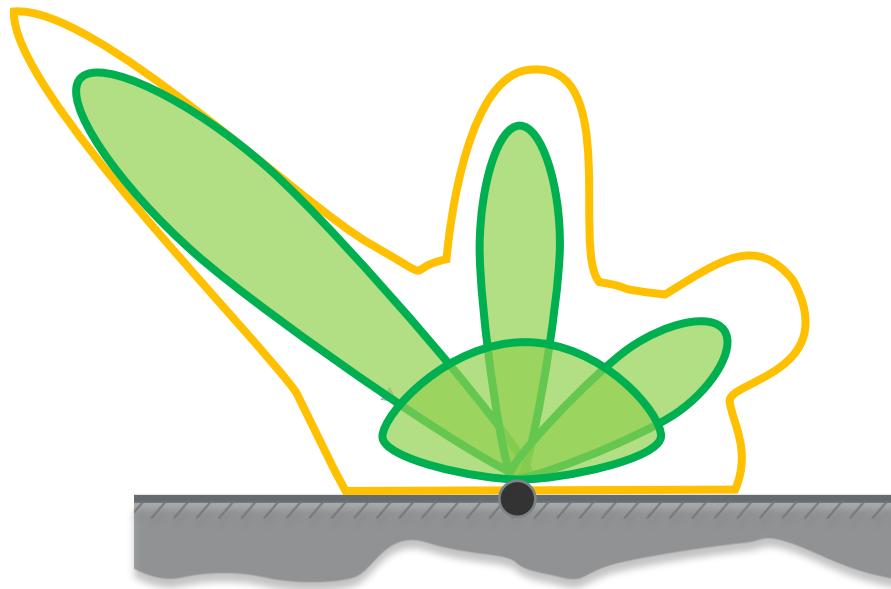
Future Work

# BRDF GMM Representation



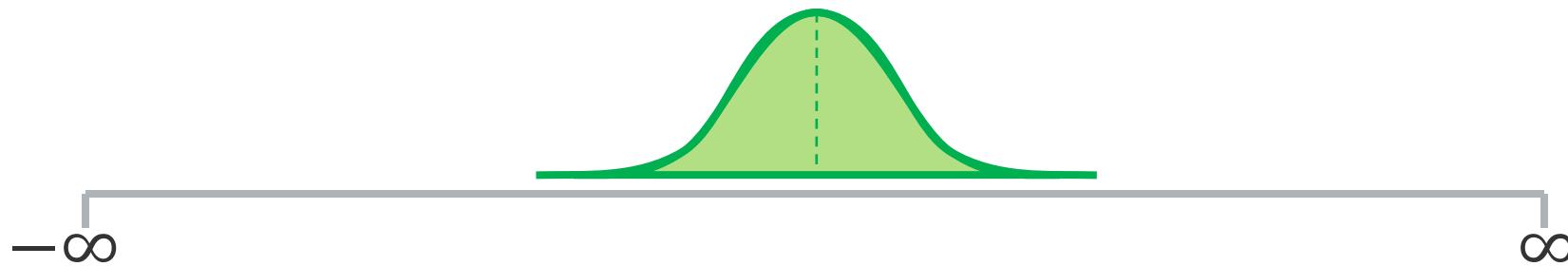
$$p_{f_r}(\omega_i | \omega_o, x) \approx G_{f_r}(y, \Theta)$$

# Illumination GMM Representation



$$p_L(\omega_i | \omega_o, x) \approx G_L(y, \Theta)$$

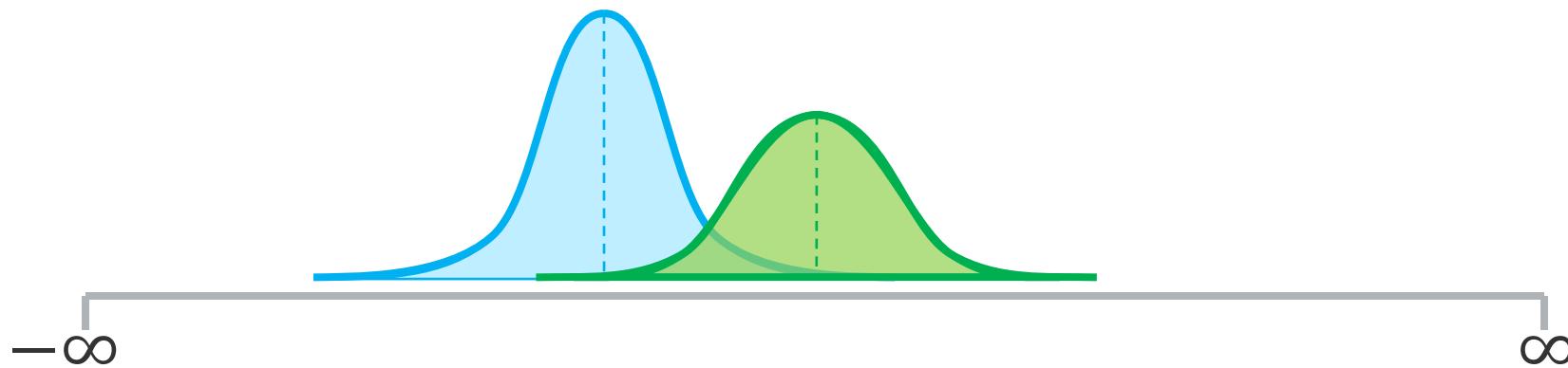
## Gaussian Mixture Model (GMM)



$$G(y, \Theta) = \sum^K \pi_i N(y, \mu_i, \Sigma_i)$$

$$\Theta = \{\pi_0 \dots, \mu_0 \dots, \Sigma_0 \dots\}$$

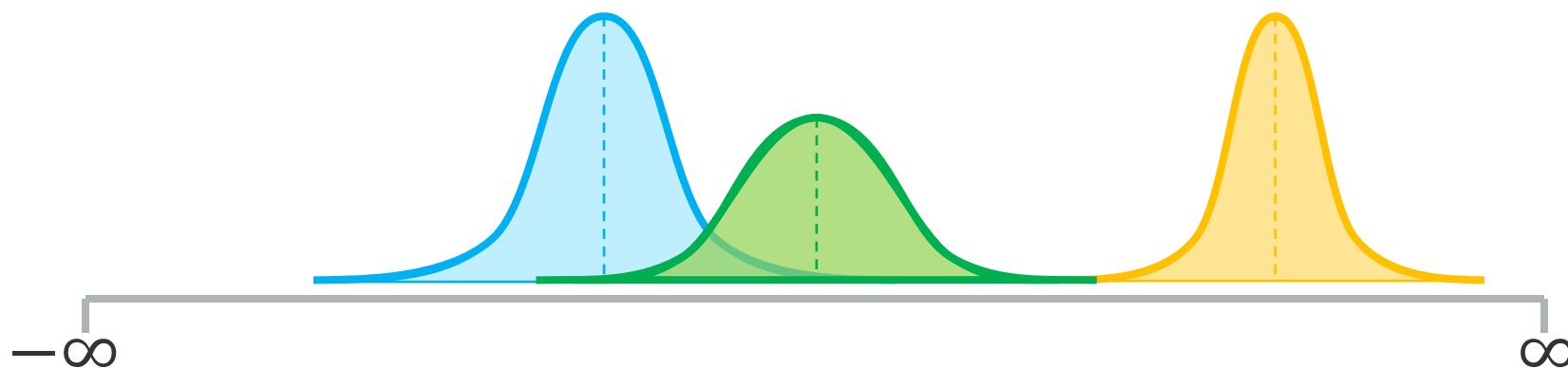
## Gaussian Mixture Model (GMM)



$$G(y, \Theta) = \sum_{i=1}^K \pi_i N(y, \mu_i, \Sigma_i)$$

$$\Theta = \{\pi_0, \dots, \mu_0, \dots, \Sigma_0, \dots\}$$

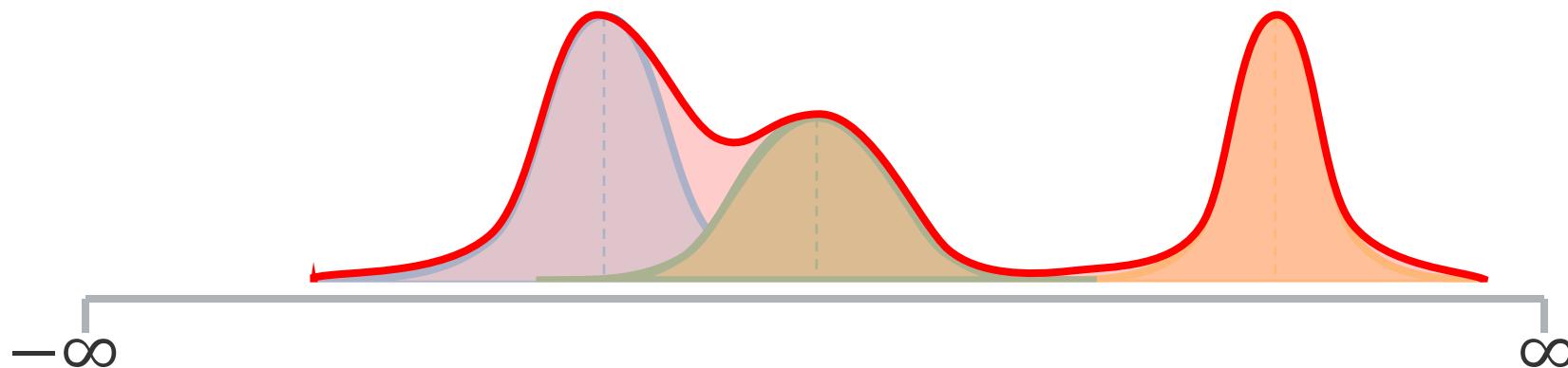
## Gaussian Mixture Model (GMM)



$$G(y, \Theta) = \sum_{i=1}^K \pi_i N(y, \mu_i, \Sigma_i)$$

$$\Theta = \{\pi_0 \dots, \mu_0 \dots, \Sigma_0 \dots\}$$

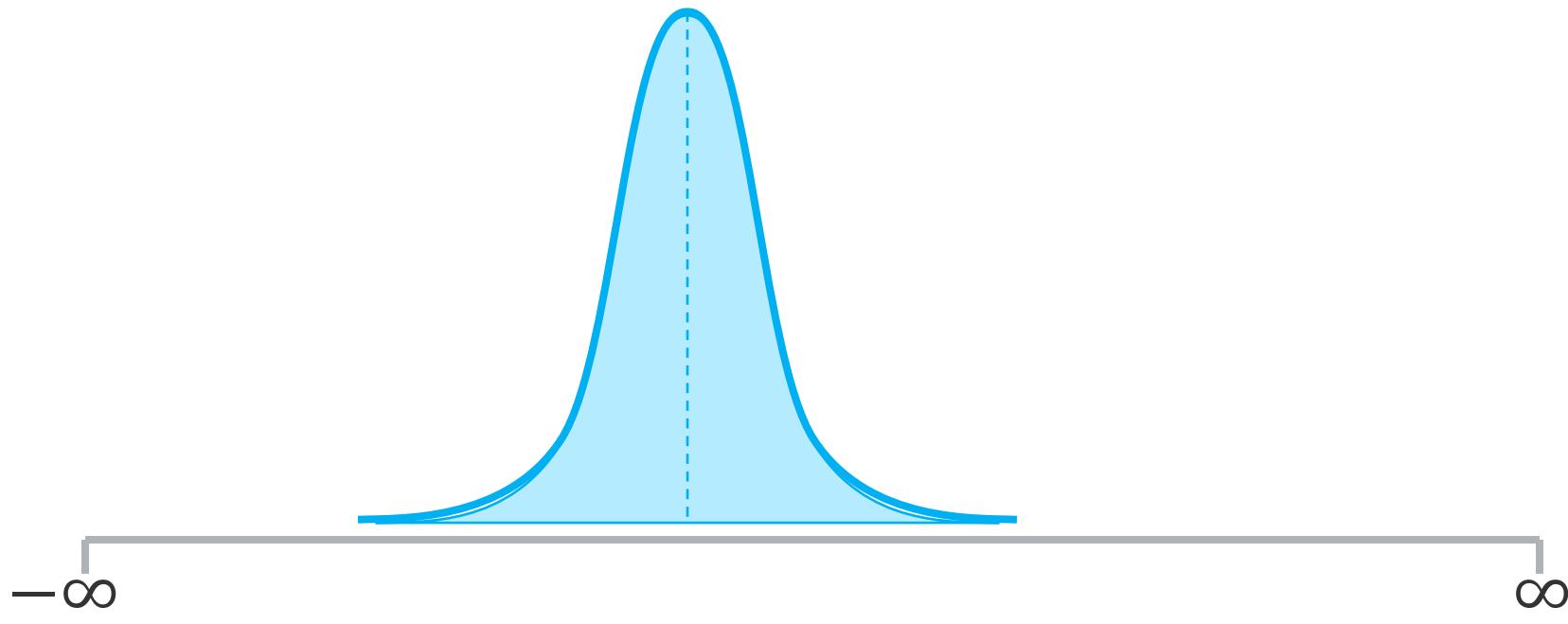
## Gaussian Mixture Model (GMM)



$$G(y, \Theta) = \sum_{i=1}^K \pi_i N(y, \mu_i, \Sigma_i)$$

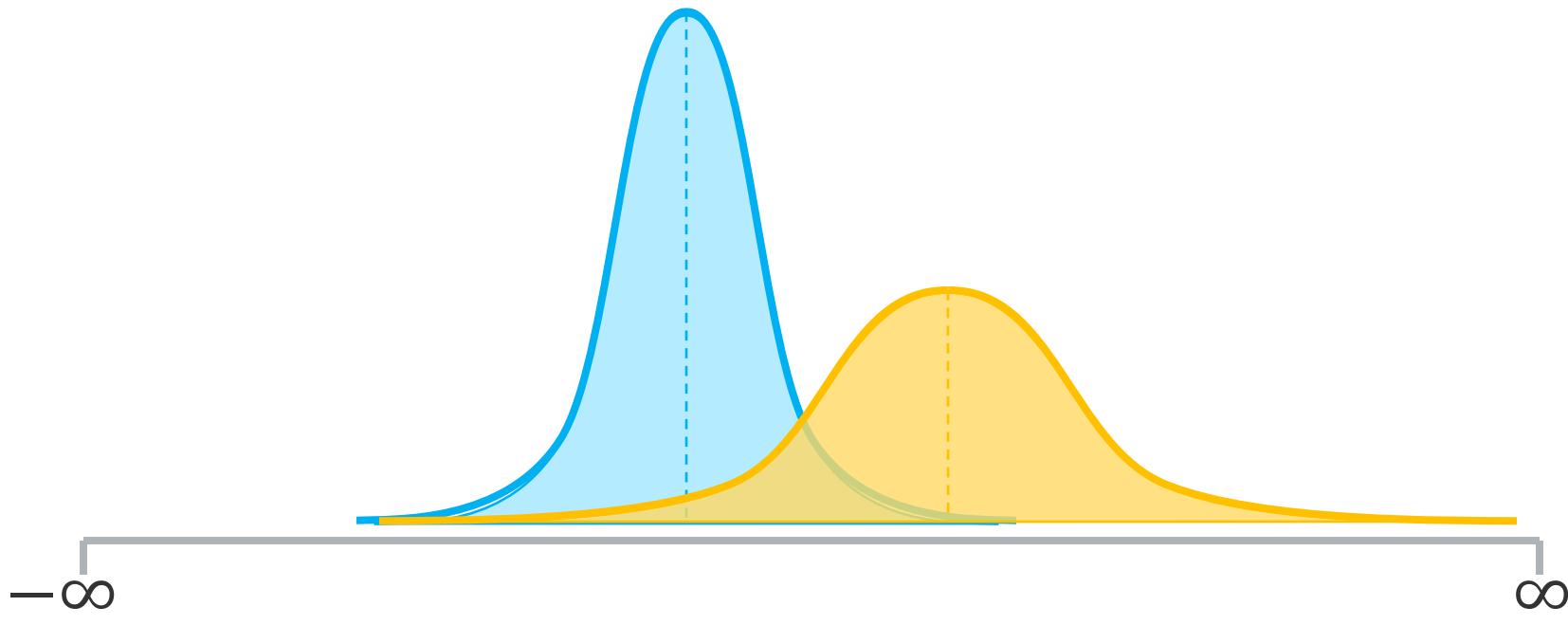
$$\Theta = \{\pi_0 \dots, \mu_0 \dots, \Sigma_0 \dots\}$$

## Product of two Gaussians is a Gaussian



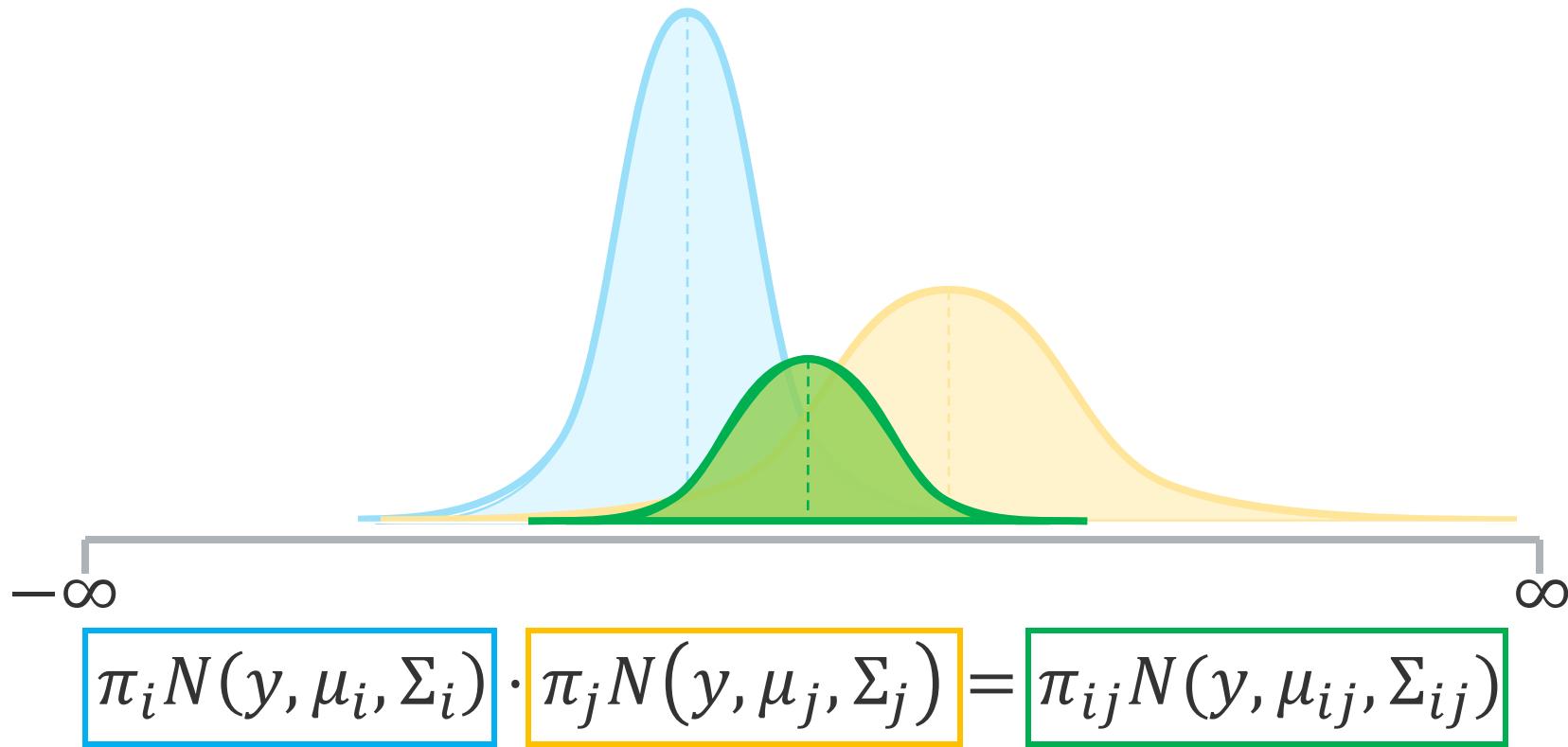
$$\pi_i N(y, \mu_i, \Sigma_i) \cdot \pi_j N(y, \mu_j, \Sigma_j) = \pi_{ij} N(y, \mu_{ij}, \Sigma_{ij})$$

## Product of two Gaussians is a Gaussian

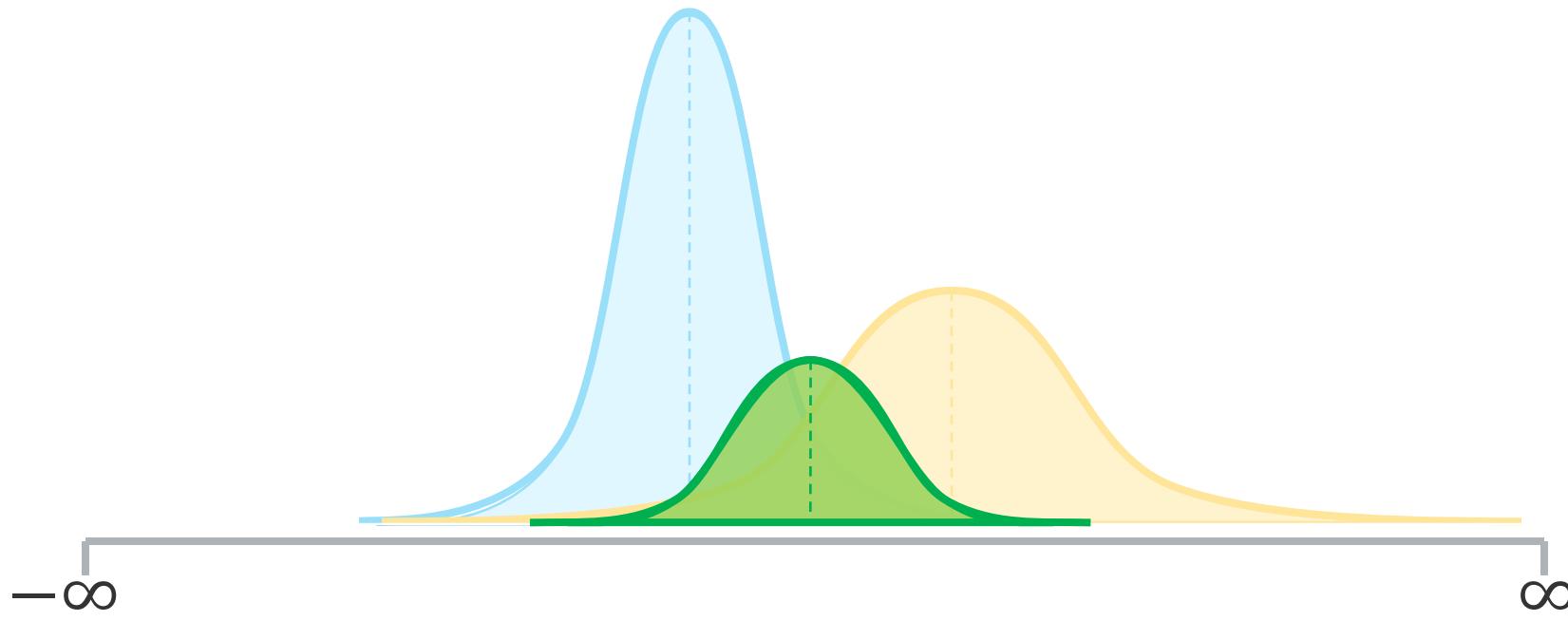


$$\boxed{\pi_i N(y, \mu_i, \Sigma_i)} \cdot \boxed{\pi_j N(y, \mu_j, \Sigma_j)} = \pi_{ij} N(y, \mu_{ij}, \Sigma_{ij})$$

# Product of two Gaussians is a Gaussian

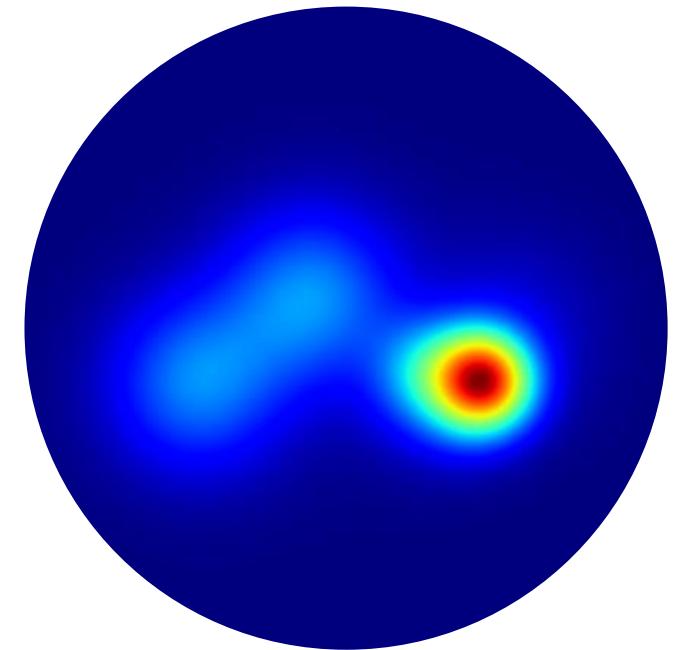
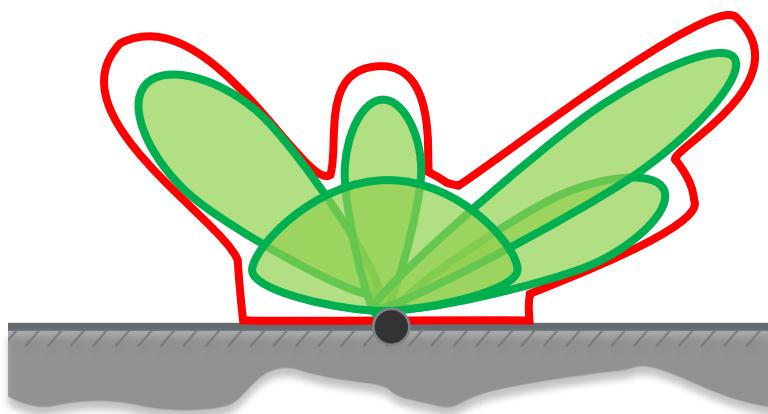


## Product of two Gaussians is a Gaussian



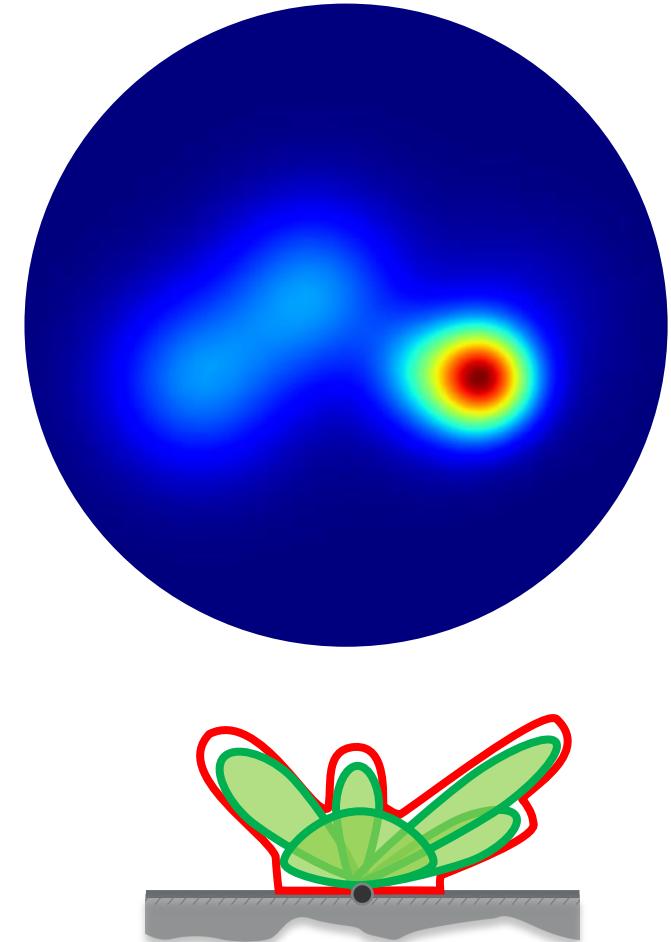
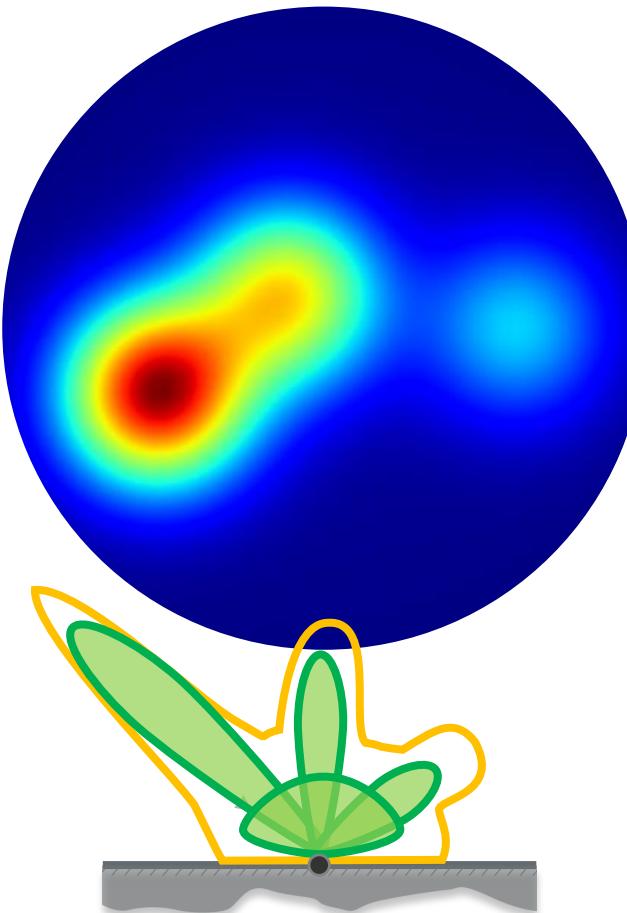
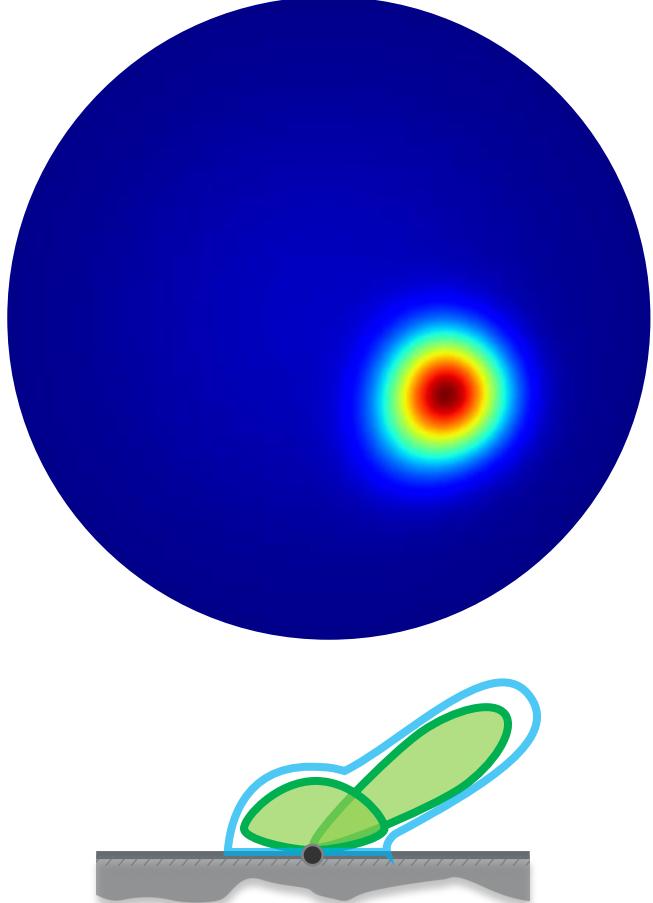
- Full GMM product contains  $K^2$  components

# Product GMM Representation



$$p_{fr} \otimes p_L \approx G_{fr} \otimes G_L = G_{\otimes}(y, \Theta)$$

# Product GMM Representation



# Pipeline

## Pre-Processing



## Rendering



# Pipeline

## Pre-Processing

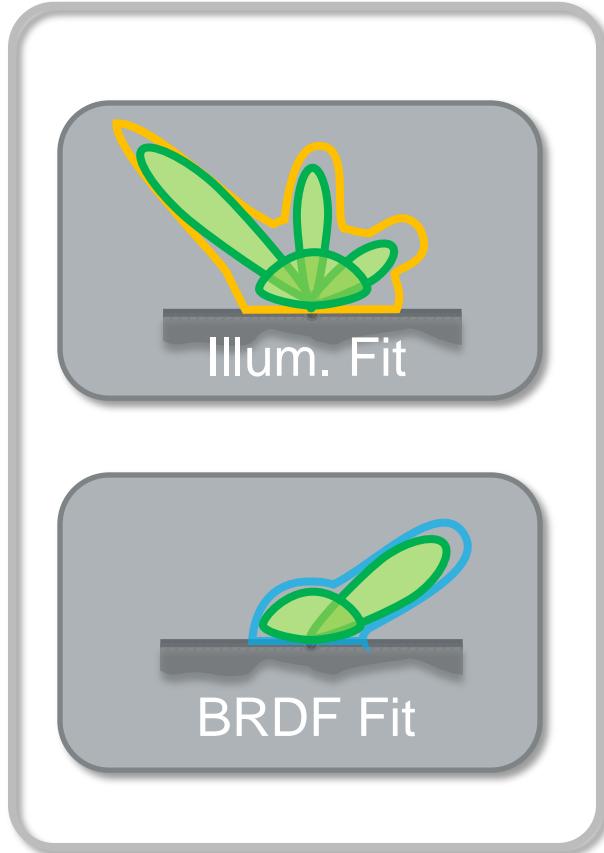


## Rendering



# Pipeline

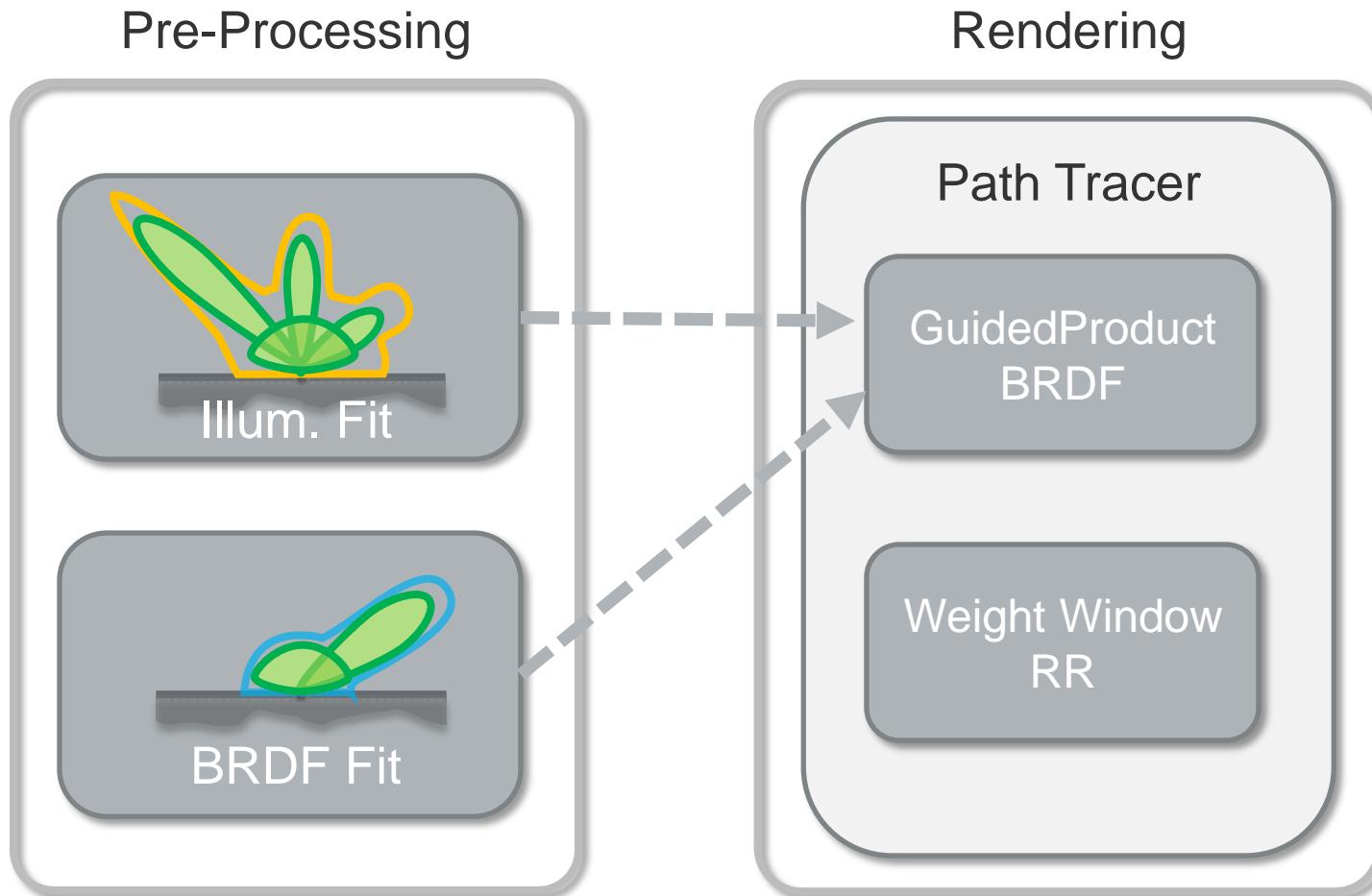
Pre-Processing



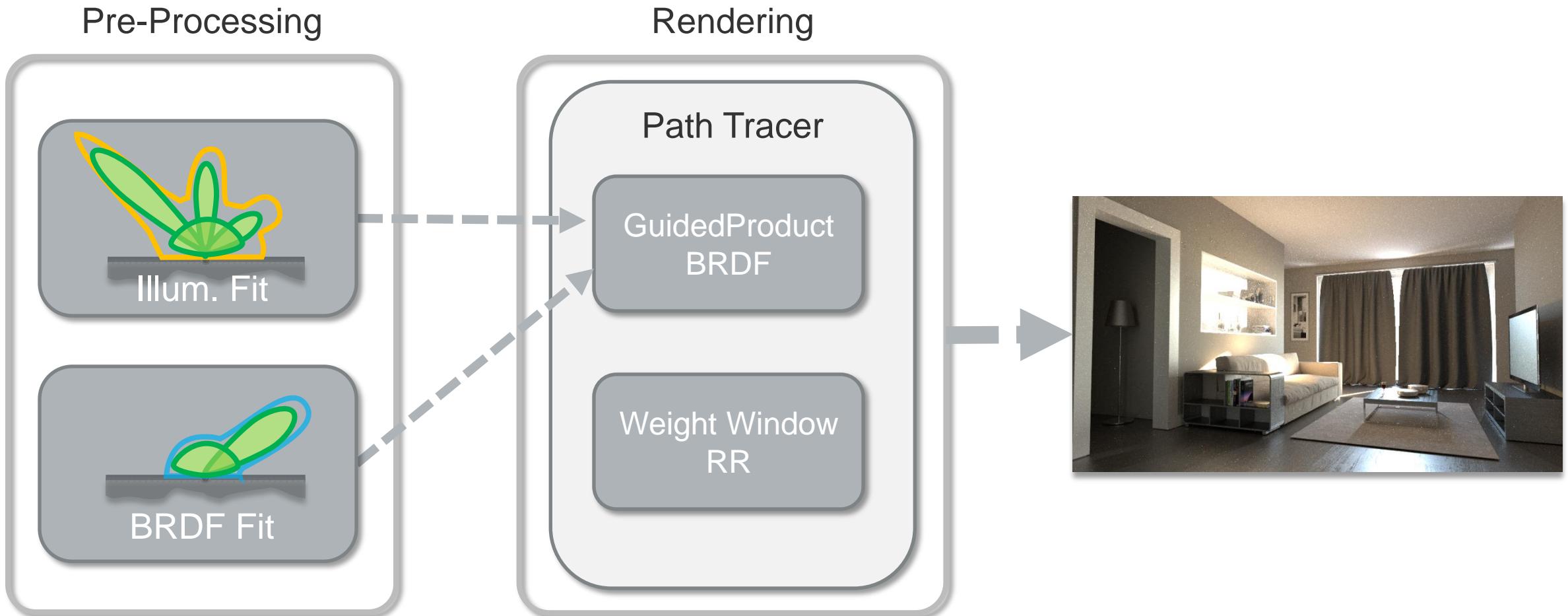
Rendering



# Pipeline

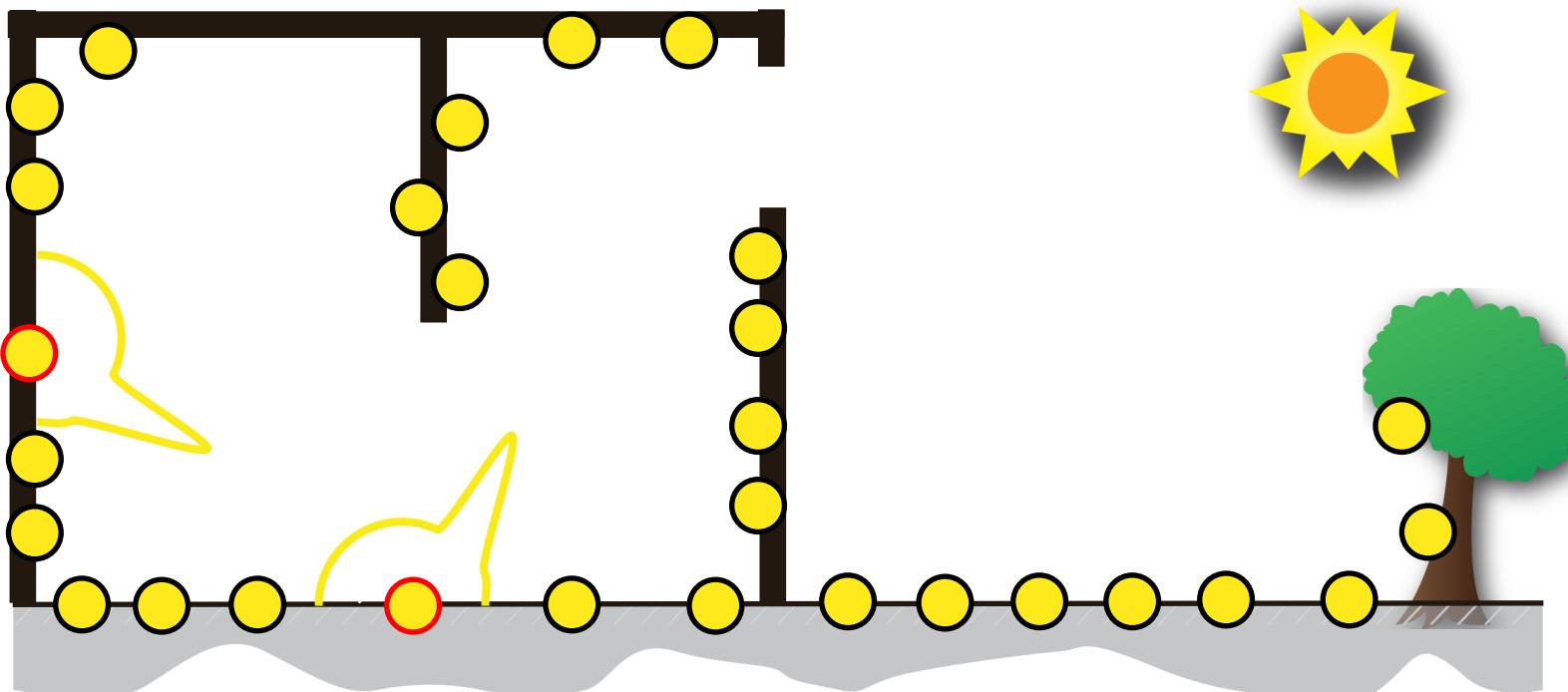


# Pipeline



# Illumination Fit: [Vorba2014]

## GMM Illumination caches



[Vorba2014]

### On-line Learning of Parametric Mixture Models for Light Transport Simulation

Jiří Vorba<sup>1\*</sup> Ondřej Karlík<sup>1\*</sup> Martin Šík<sup>1\*</sup> Tobias Ritschel<sup>2†</sup> Jaroslav Krivánek<sup>1‡</sup>  
<sup>1</sup>Charles University in Prague    <sup>2</sup>MPI Informatik, Saarbrücken

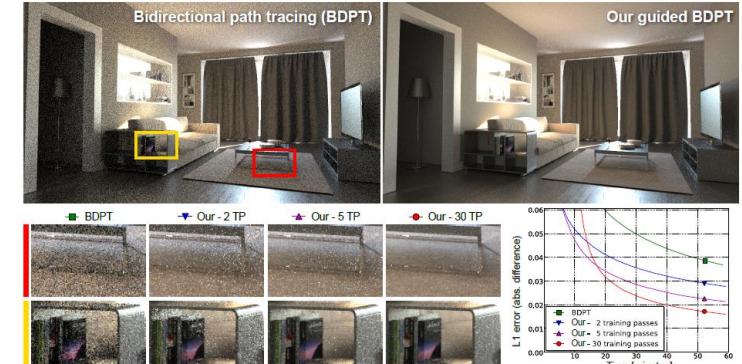


Figure 1: We render a scene featuring difficult visibility with bidirectional path tracing (BDPT) guided by our parametric distributions learned on-line in a number of training passes (TP). The insets show equal-time (1h) comparisons of images obtained with different numbers of training passes. The results reveal that the time spent on additional training passes is quickly amortized by the superior performance of the subsequent guided rendering.

### Abstract

Monte Carlo techniques for light transport simulation rely on importance sampling when constructing light transport paths. Previous work has shown that suitable sampling distributions can be recovered from particles distributed in the scene prior to rendering. We propose to represent the distributions by a parametric mixture model trained in an on-line (i.e., progressive) manner from a potentially infinite stream of particles. This enables recovering good sampling distributions in scenes with complex lighting, where the necessary number of particles may exceed available memory. Using these distributions for sampling scattering directions and light emission significantly improves the performance of state-of-the-art light transport simulation algorithms when dealing with complex lighting.

CR Categories: I.3.3 [Computer Graphics]: Three-Dimensional Graphics and Realism—Display Algorithms

Keywords: light transport simulation, importance sampling, parametric density estimation, on-line expectation maximization

Links: [DL](#) [PDF](#) [WEB](#) [CODE](#)

### 1 Introduction

Despite recent advances, robust and efficient light transport simulation is still a challenging open issue. Numerous algorithms have been proposed to solve the problem, but certain common lighting conditions, such as highly occluded scenes, remain difficult. Most existing unidirectional and bidirectional methods rely on incremental, local construction of transport sub-paths, which is oblivious to the global distribution of radiance or importance. As a result, the probability of obtaining a non-zero contribution upon sub-path connection in highly occluded scenes is low. This is the main reason why such scenes remain difficult to render. While Metropolis light transport and related methods [Veach and Guibas 1997; Kelemen et al. 2002; Cline et al. 2005] strive for importance sampling on the entire path space, they suffer from sample correlation and are often outperformed by the classic Monte Carlo approaches.

\*{jirka.karlik,martin.sik}@cg.mff.cuni.cz

†ritschel@mpi-inf.mpg.de

‡jaroslav.krivanek@mff.cuni.cz

# BRDF Fitting and Caching

# Fitting BRDF GMM

## Weighted EM

- Weighted MAP EM [Vorba2014]
- Sample BRDF (N=512)
- $w_i = \frac{f_r(x, \omega_i, \omega_o)}{p(\omega_o)}$
- Init components using K BRDF samples (QMC sampler)

## CERES

- Non-linear optimization
- Init with weighted EM
- Objective function:

$$\sum_i^N \left[ 1 - \frac{\hat{f}_r(\omega_i)}{G(y|\Theta)} \right]^2$$

# Fitting BRDF GMM

## Weighted EM

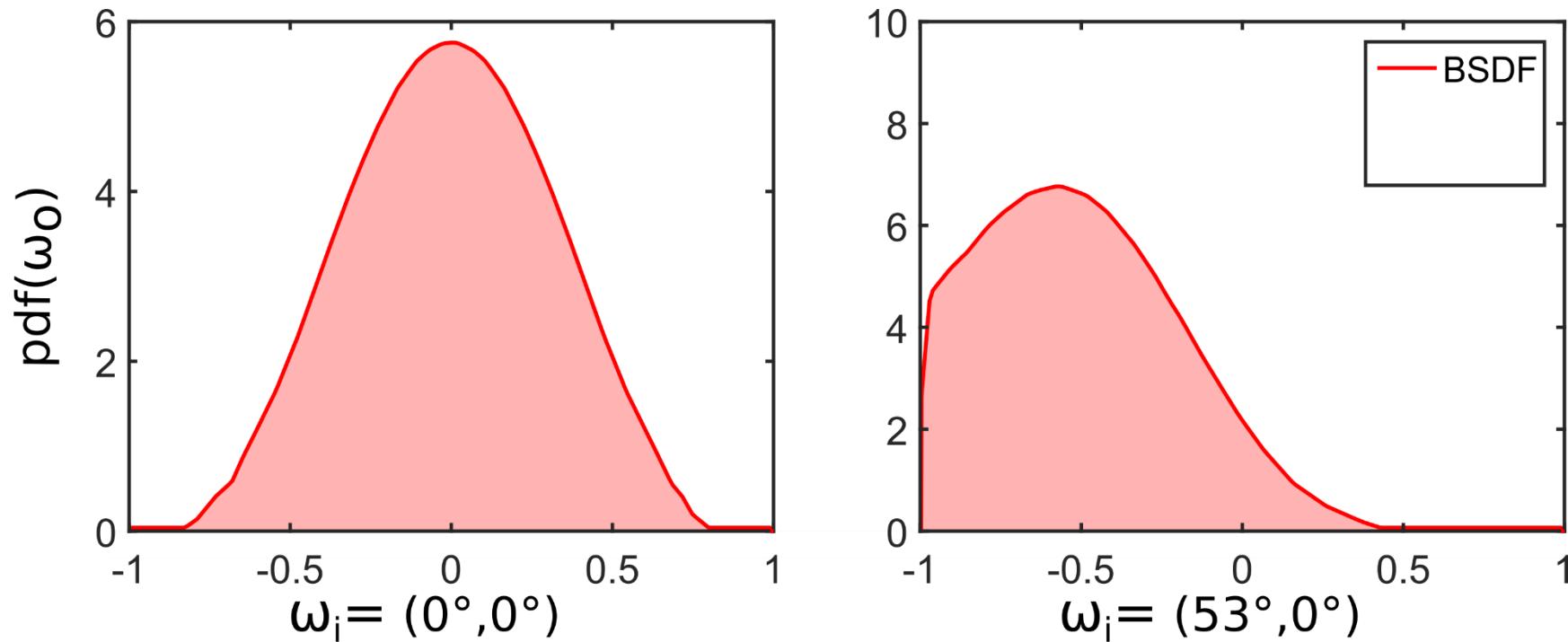
- Weighted MAP EM [Vorba2014]
- Sample BRDF (N=512)
- $w_i = \frac{f_r(x, \omega_i, \omega_o)}{p(\omega_o)}$
- Init components using K BRDF samples (QMC sampler)

## CERES

- Non-linear optimization
- Init with weighted EM
- Objective function:

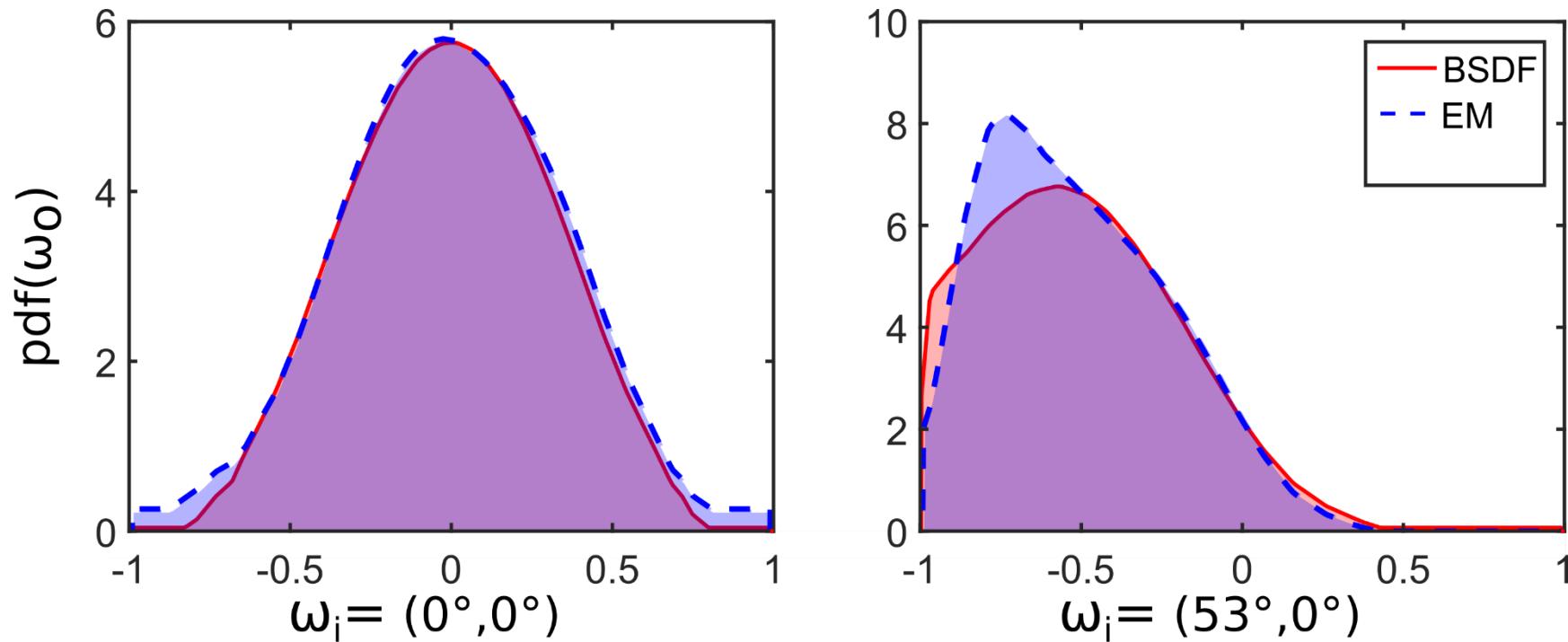
$$\sum_i^N \left[ 1 - \frac{\hat{f}_r(\omega_i)}{G(y|\Theta)} \right]^2$$

## wEM vs CERES



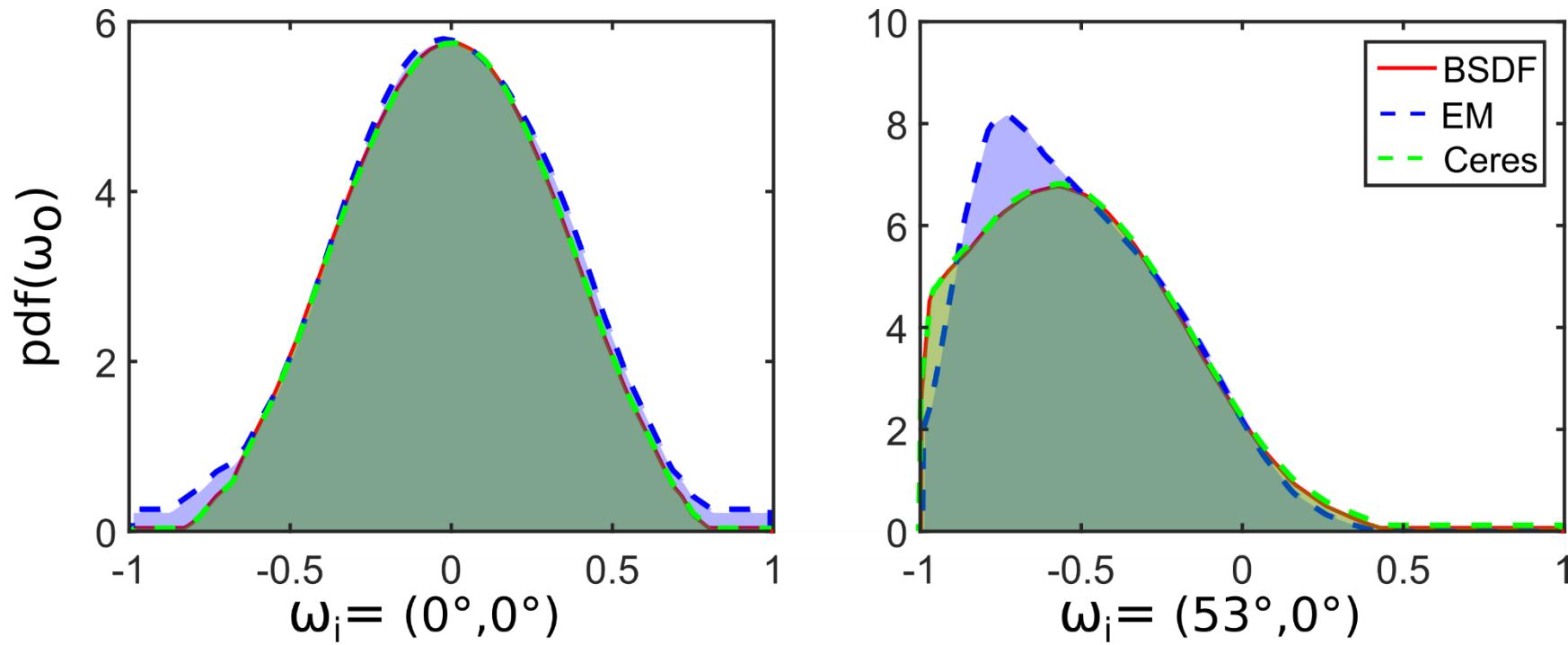
rough conductor  $\alpha = 0.3$   
 $K = 8$

## wEM vs CERES



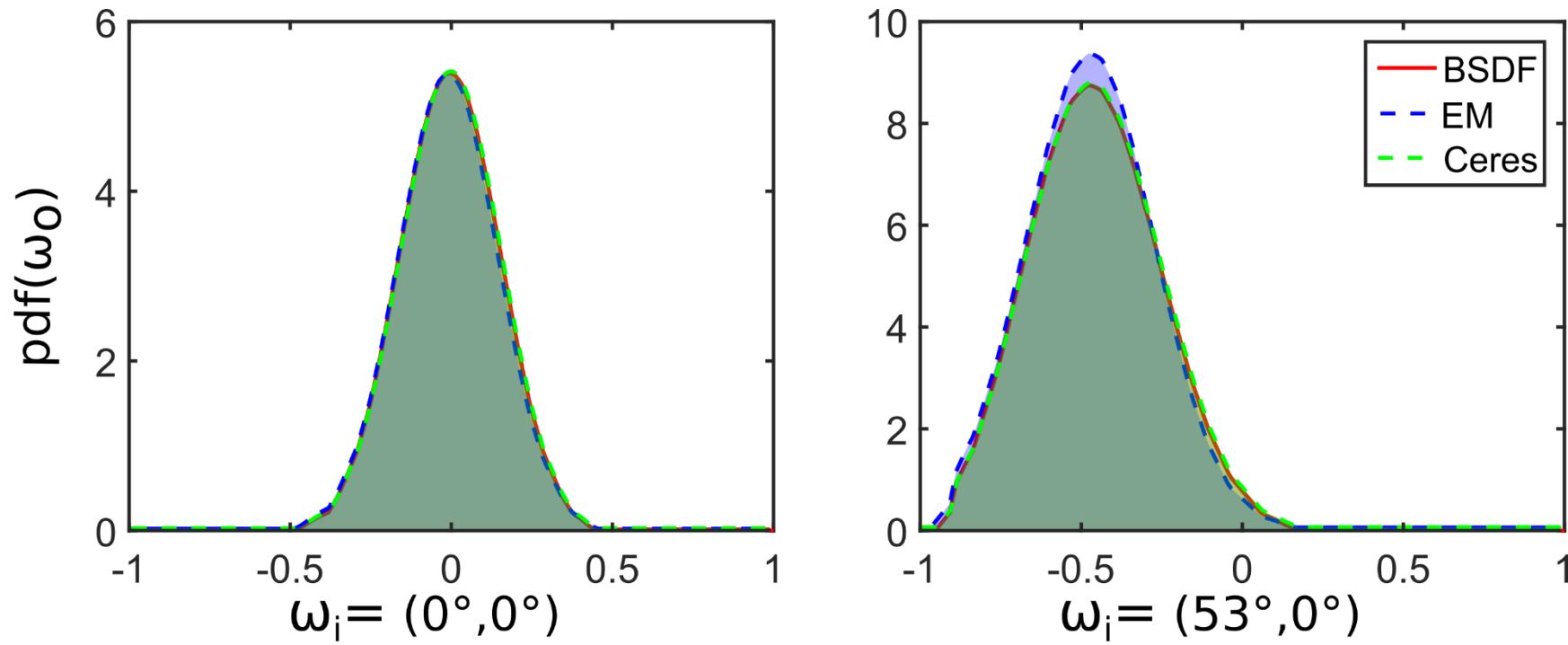
rough conductor  $\alpha = 0.3$   
 $K = 8$

## wEM vs CERES



rough conductor  $\alpha = 0.3$   
 $K = 8$

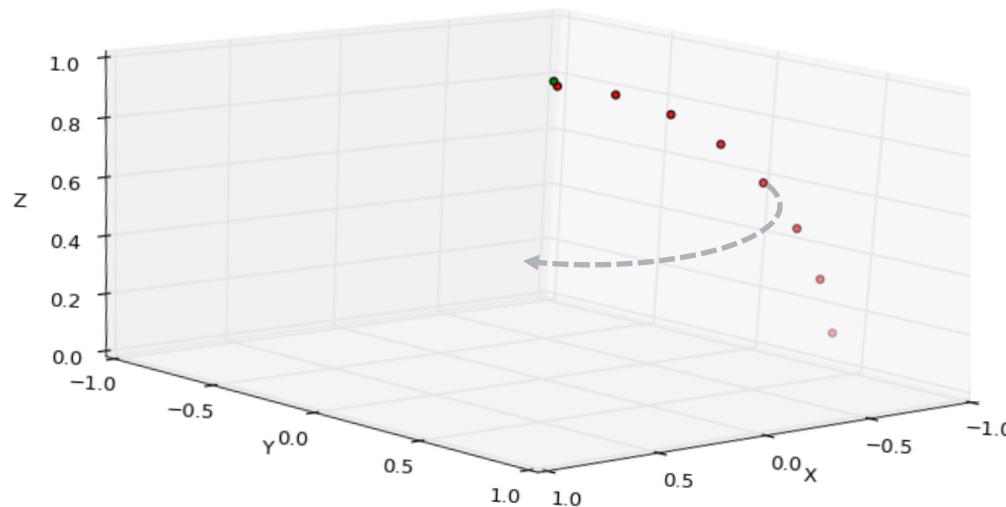
## wEM vs CERES



rough conductor  $\alpha = 0.15$   
 $K = 8$

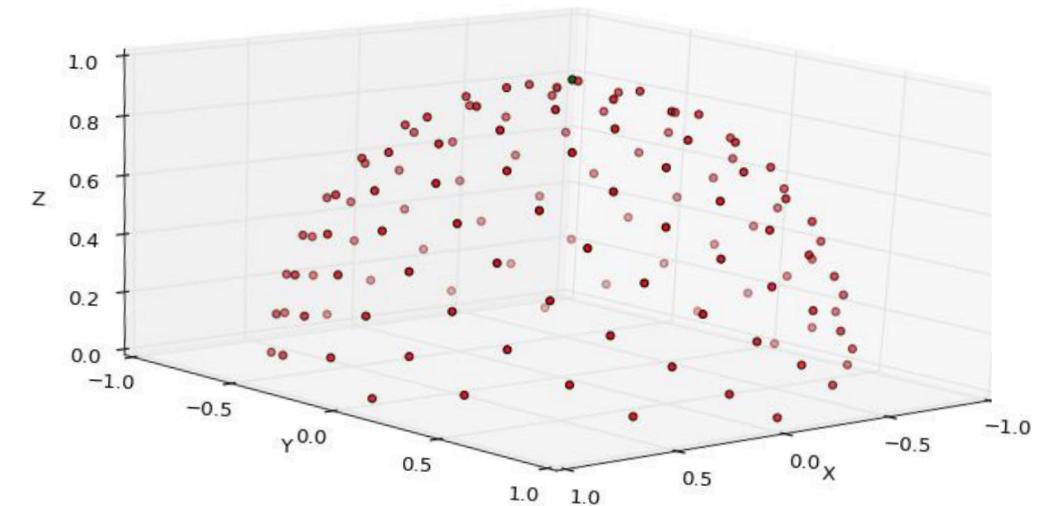
# Caching

- Isotropic



- 512 different elevation angles

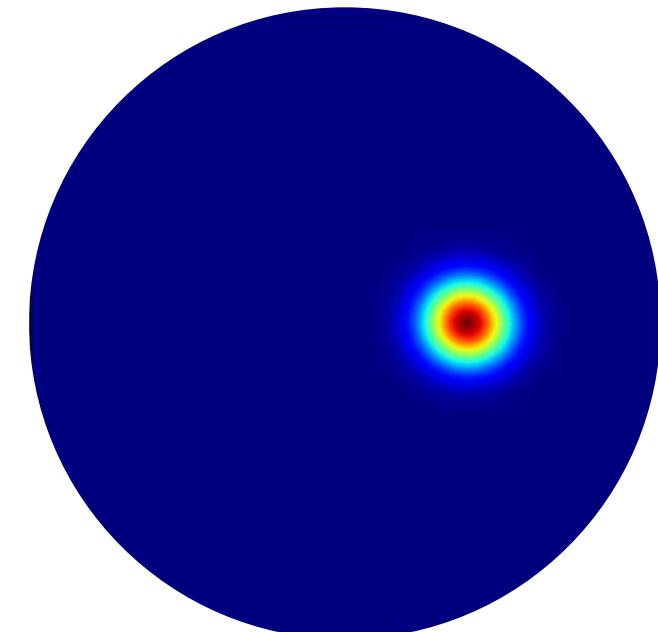
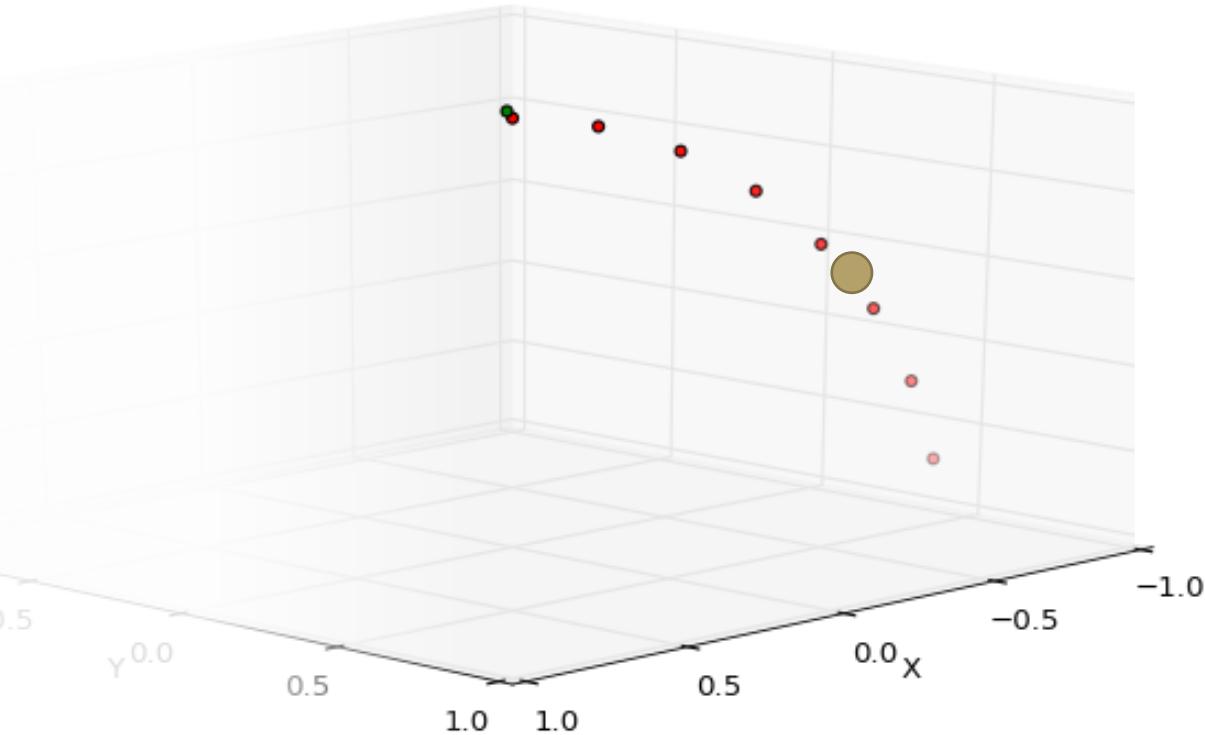
- Anisotropic



- 4096 spherical Fibonacci points [KISS15]

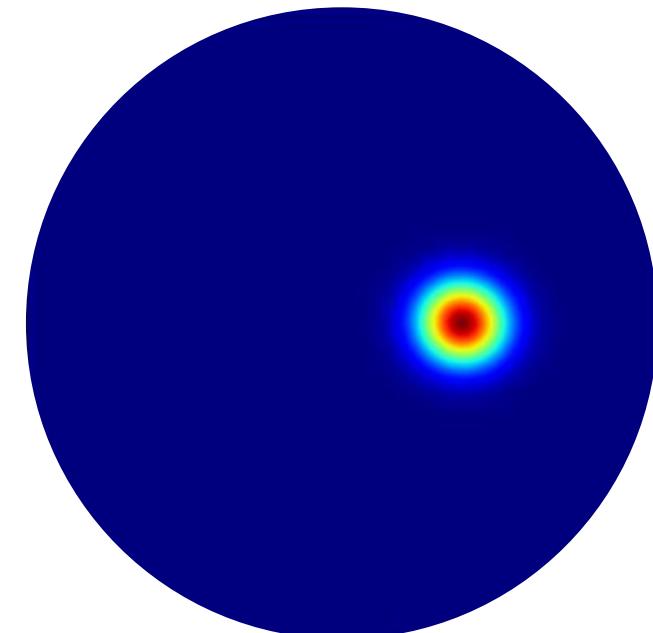
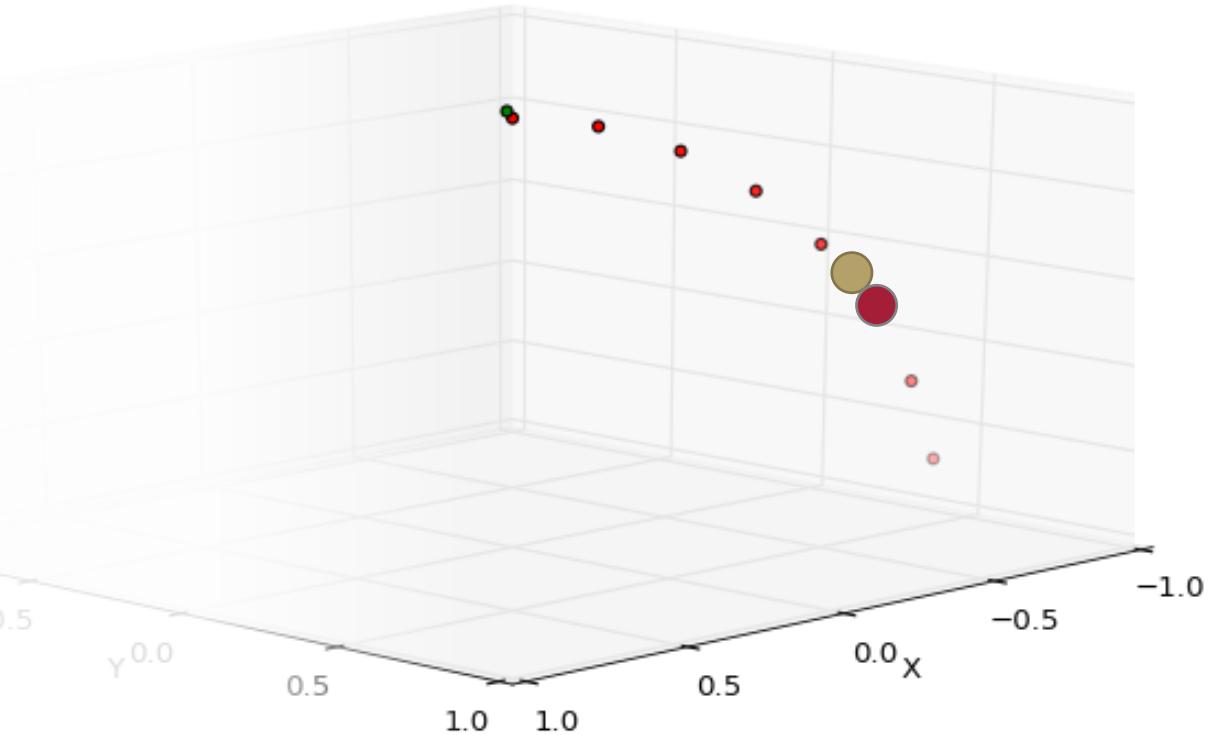
# Caching

- Isotropic



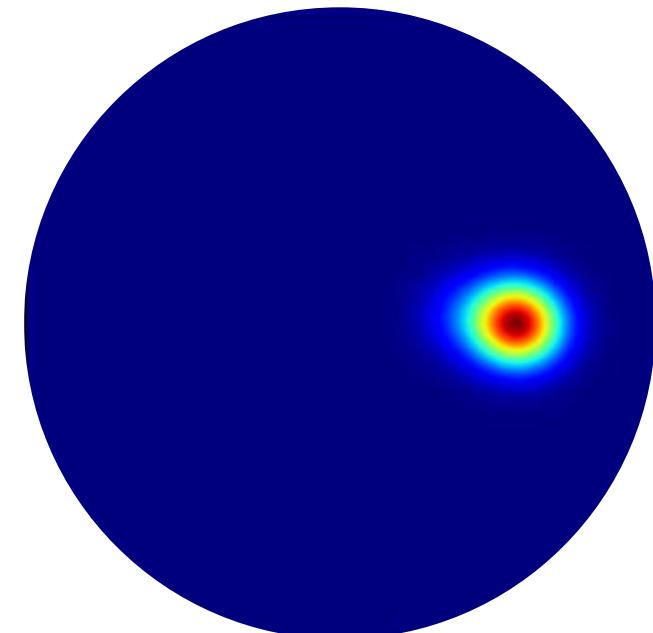
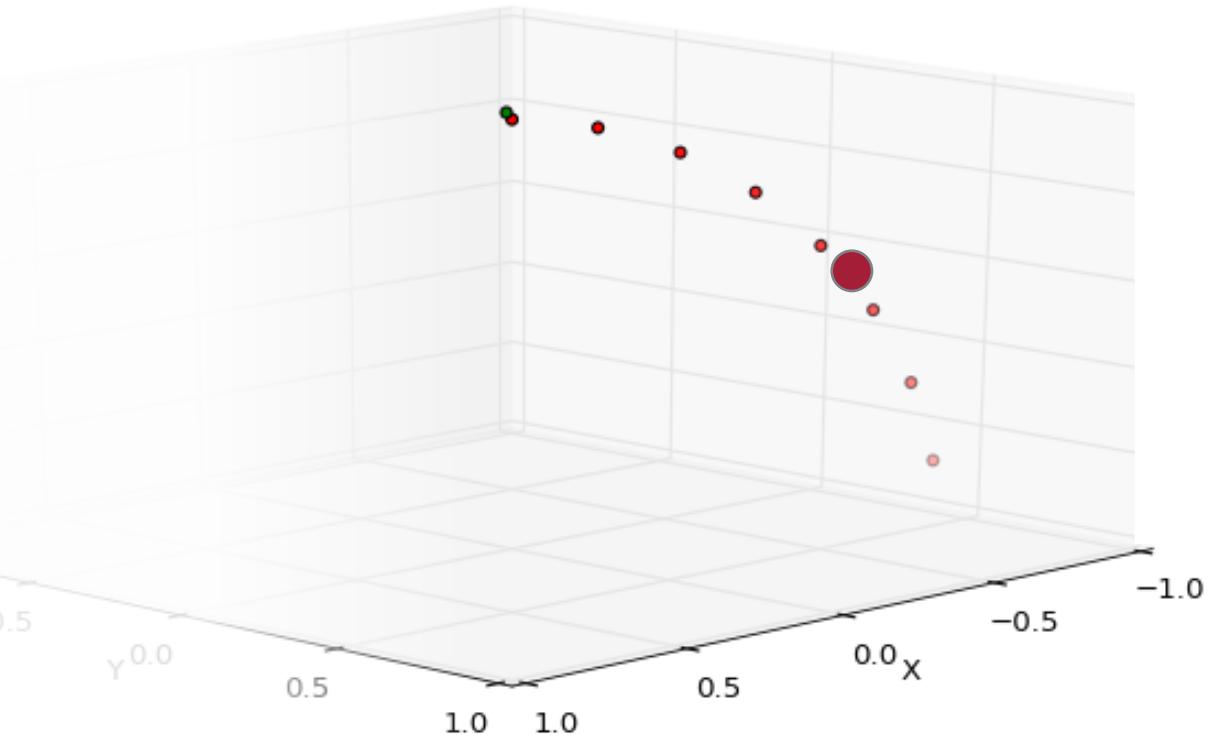
# Caching

- Isotropic



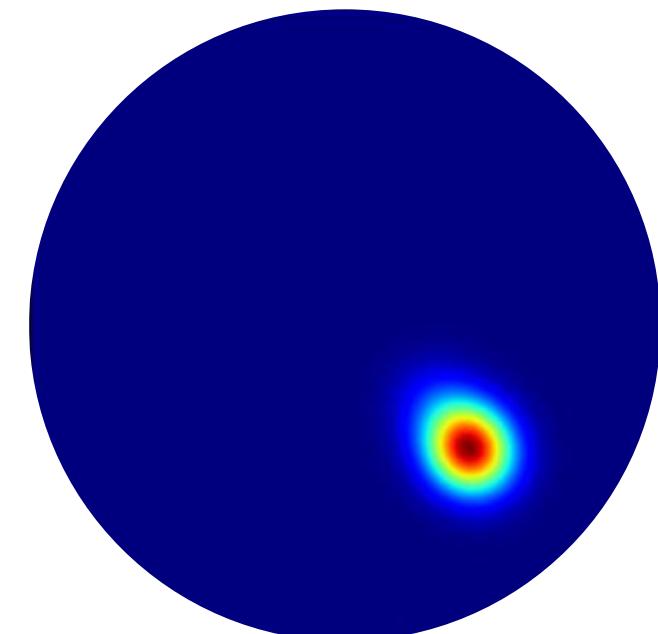
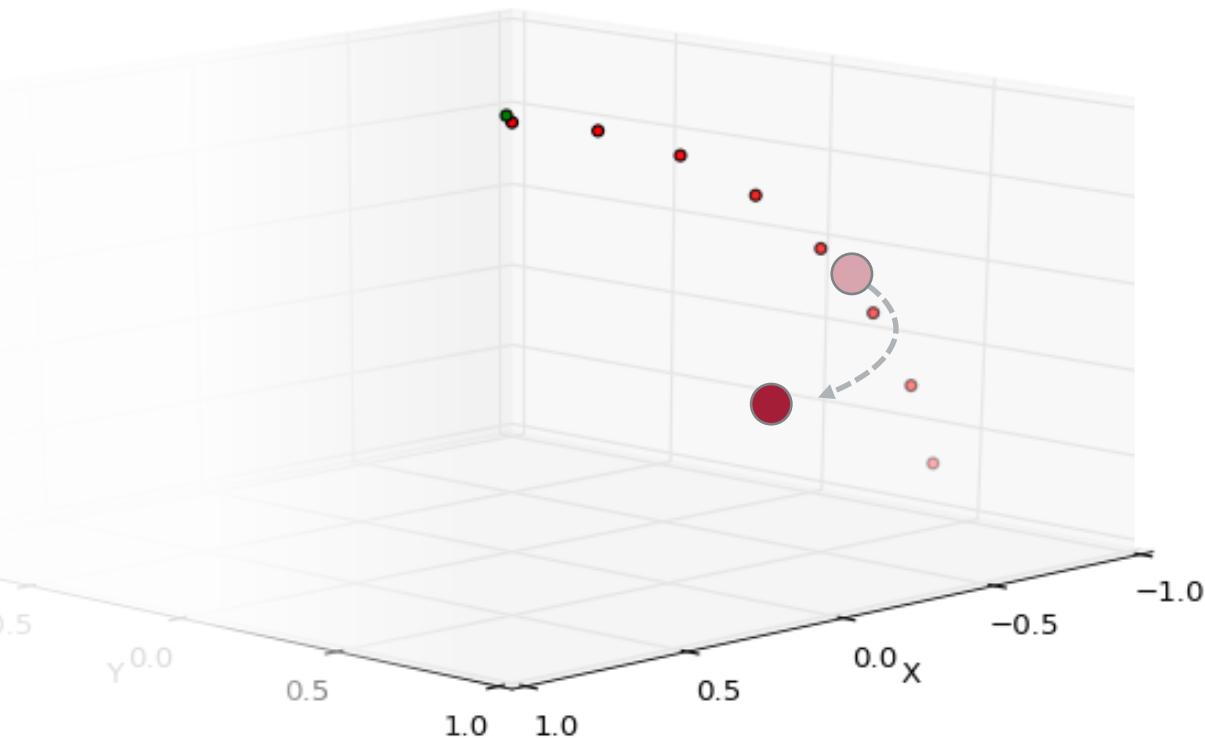
# Caching

- Isotropic



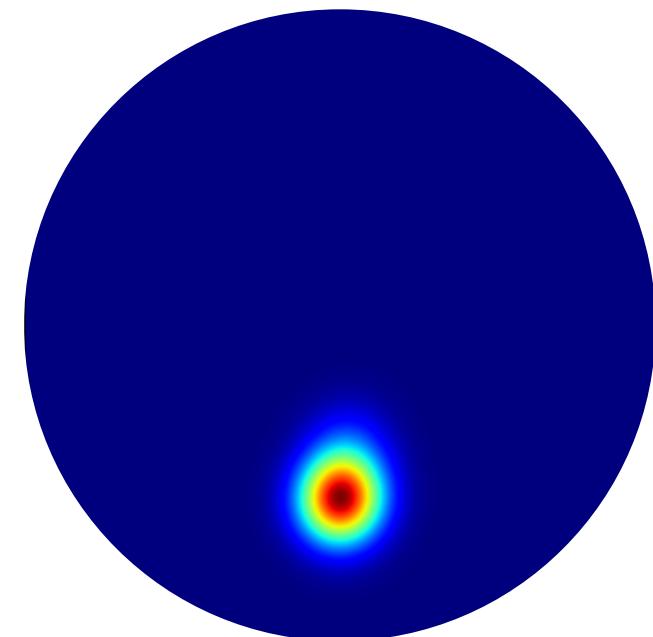
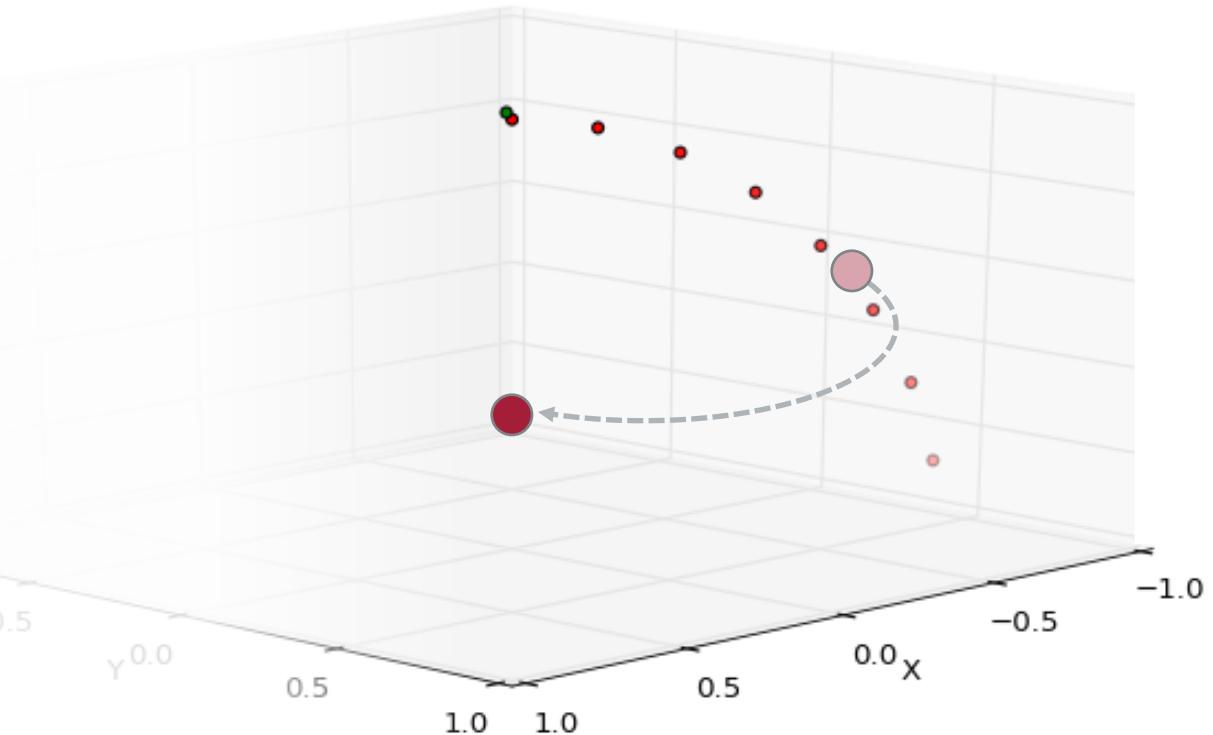
# Caching

- Isotropic



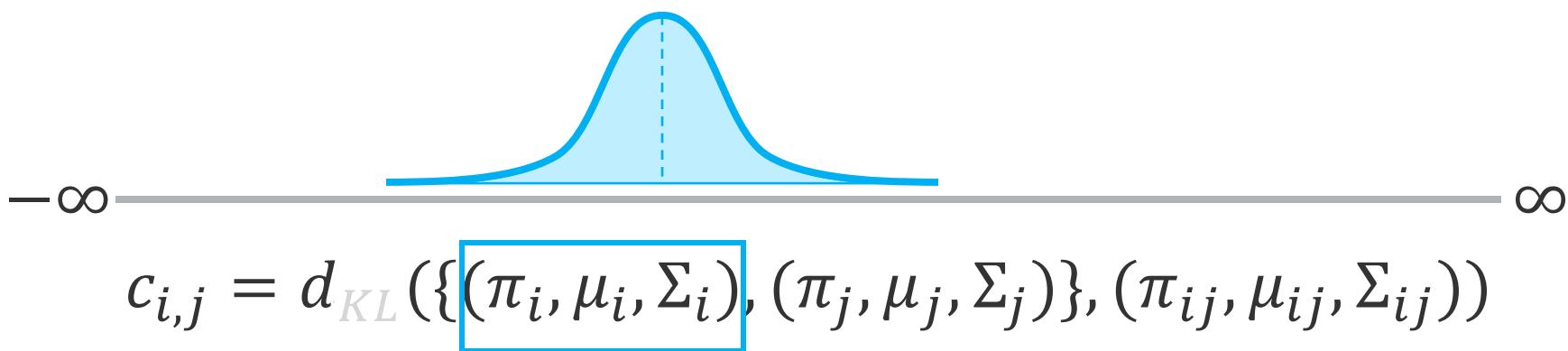
# Caching

- Isotropic

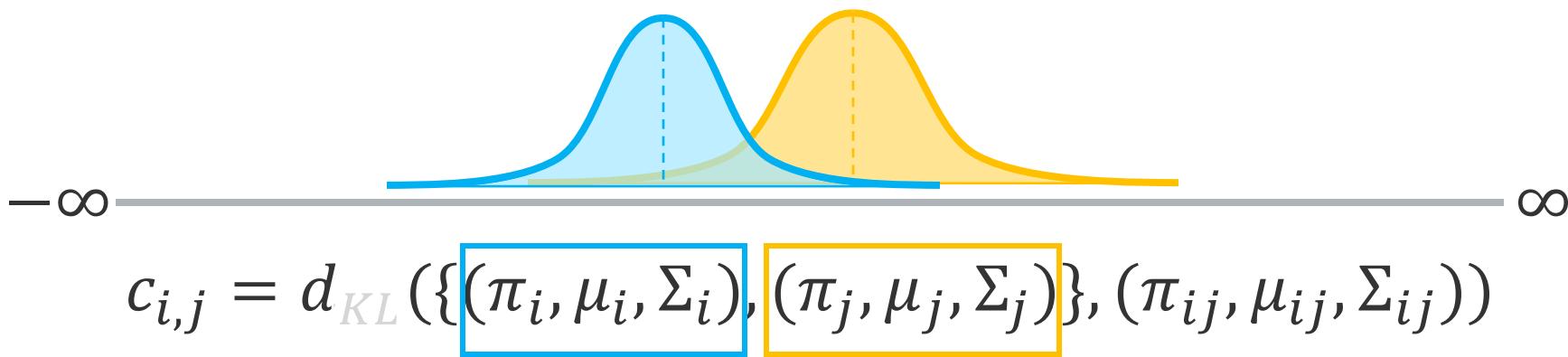


# GMM Component Reduction

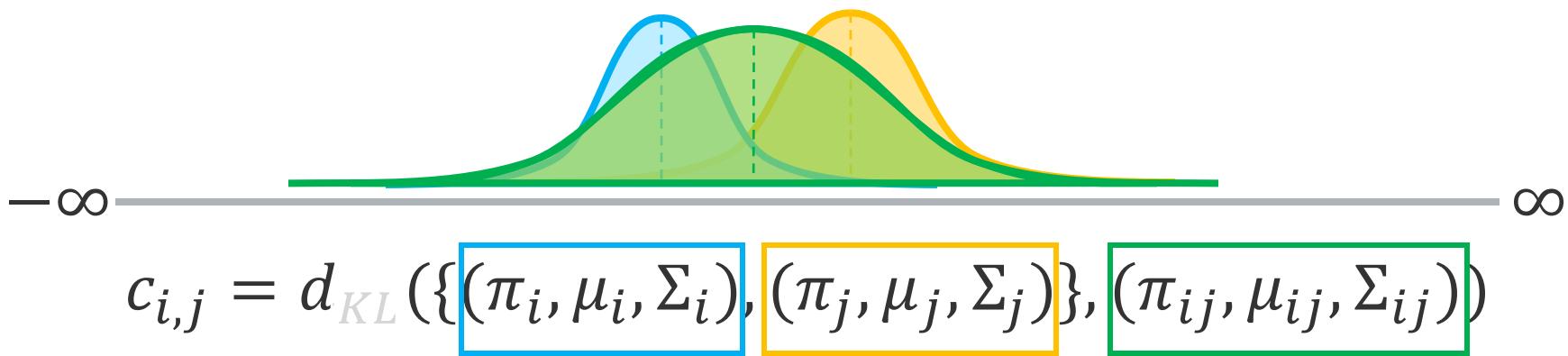
# Component Reduction



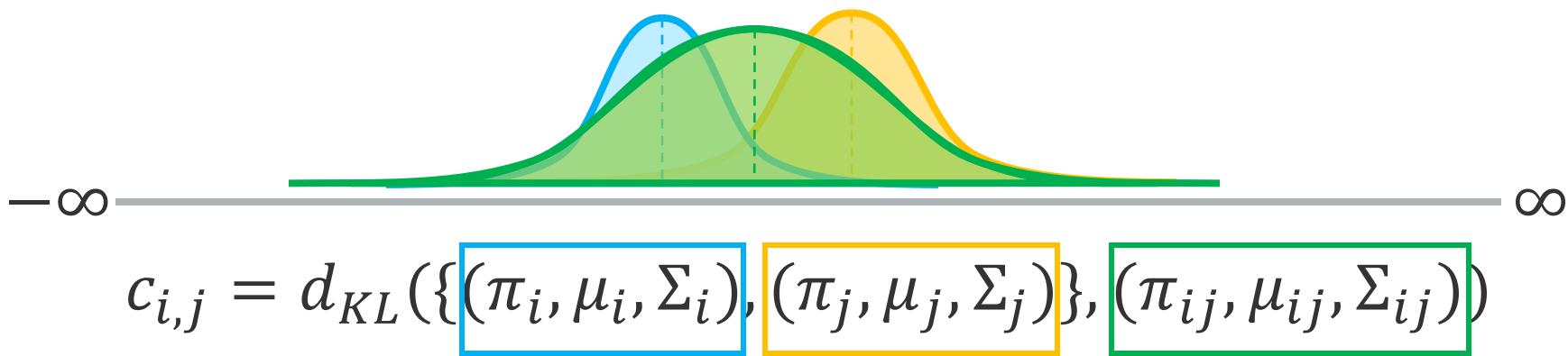
# Component Reduction



# Component Reduction

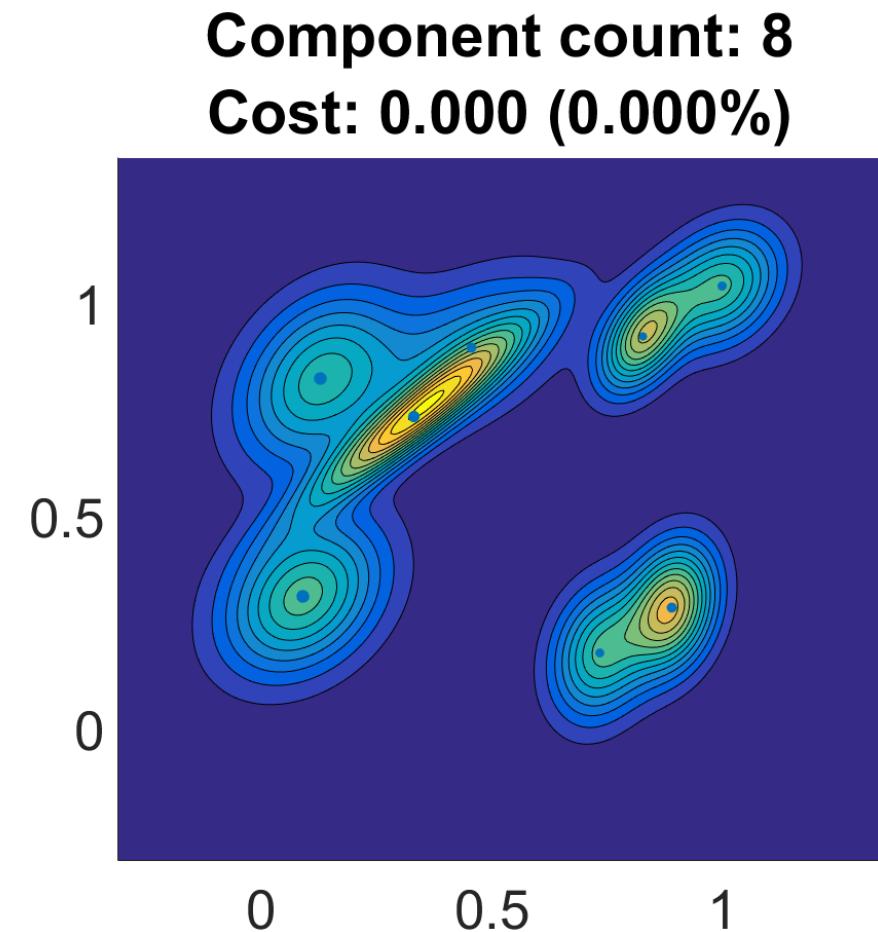
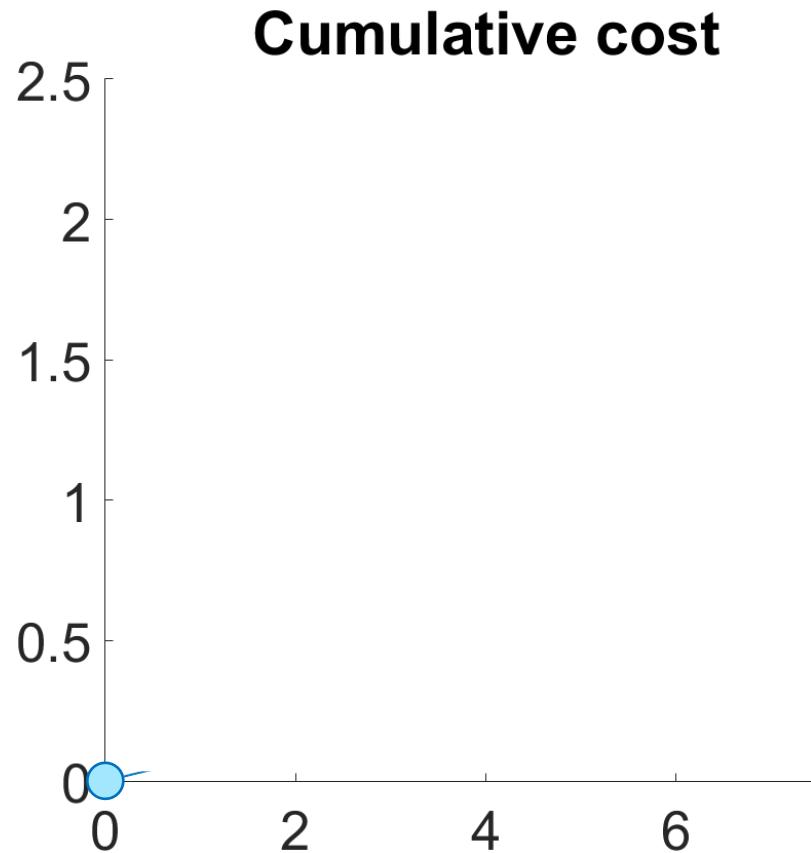


# Component Reduction

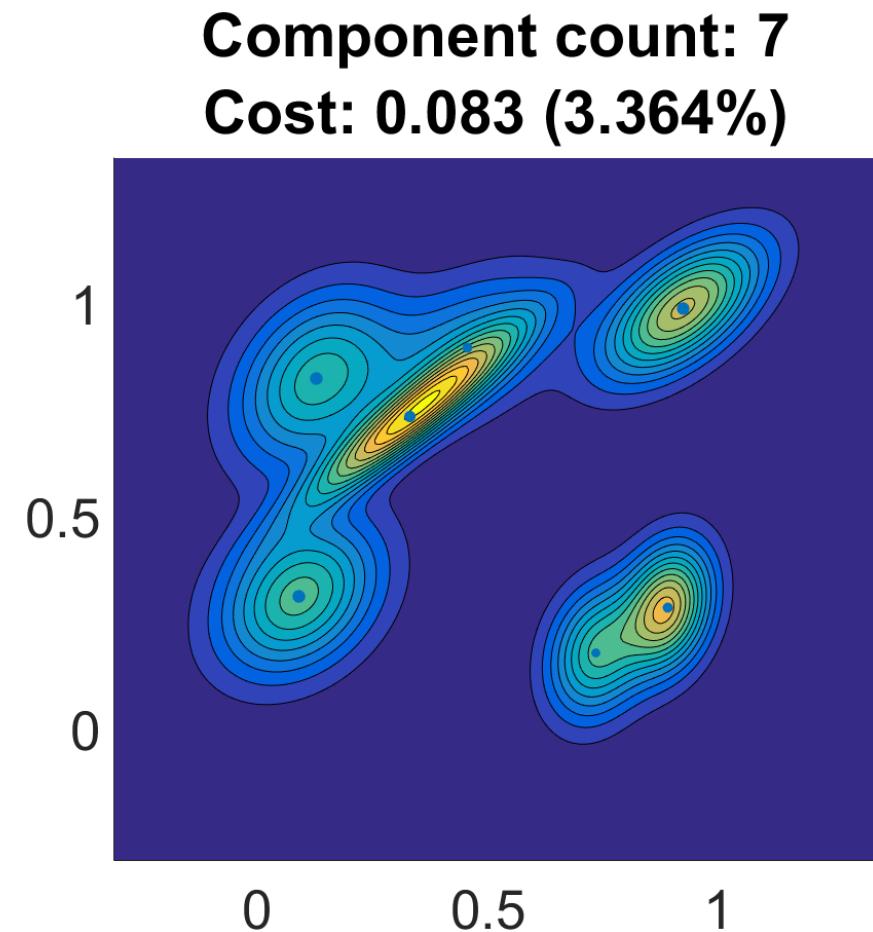
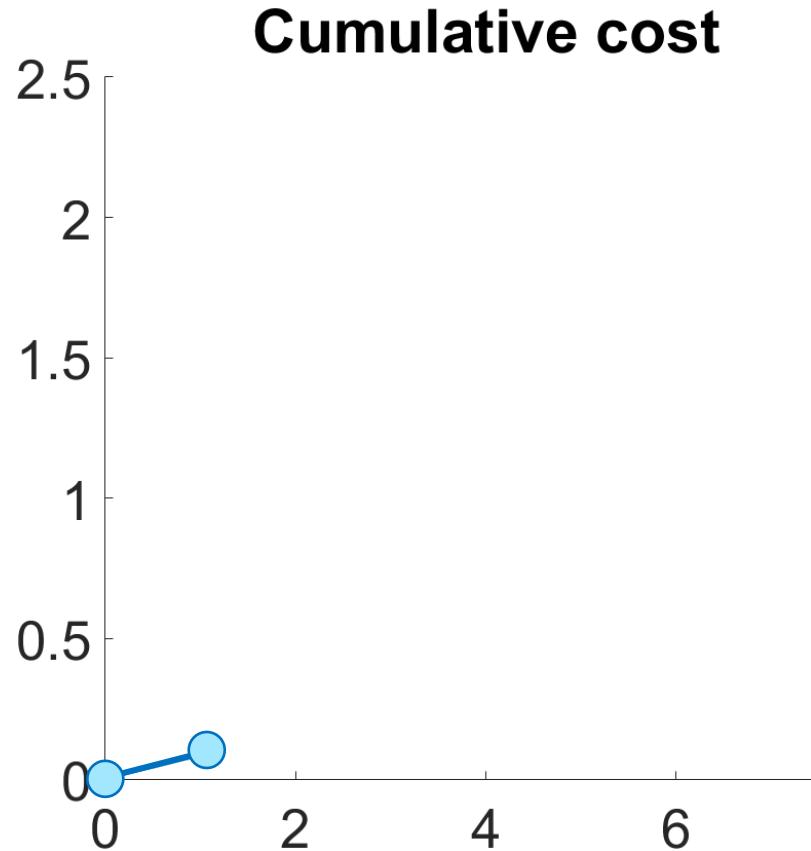


- Adapted from [Runnals2007], used also in [Jacob2011]
- Kullback-Leibler discrimination:  $d_{KL}$

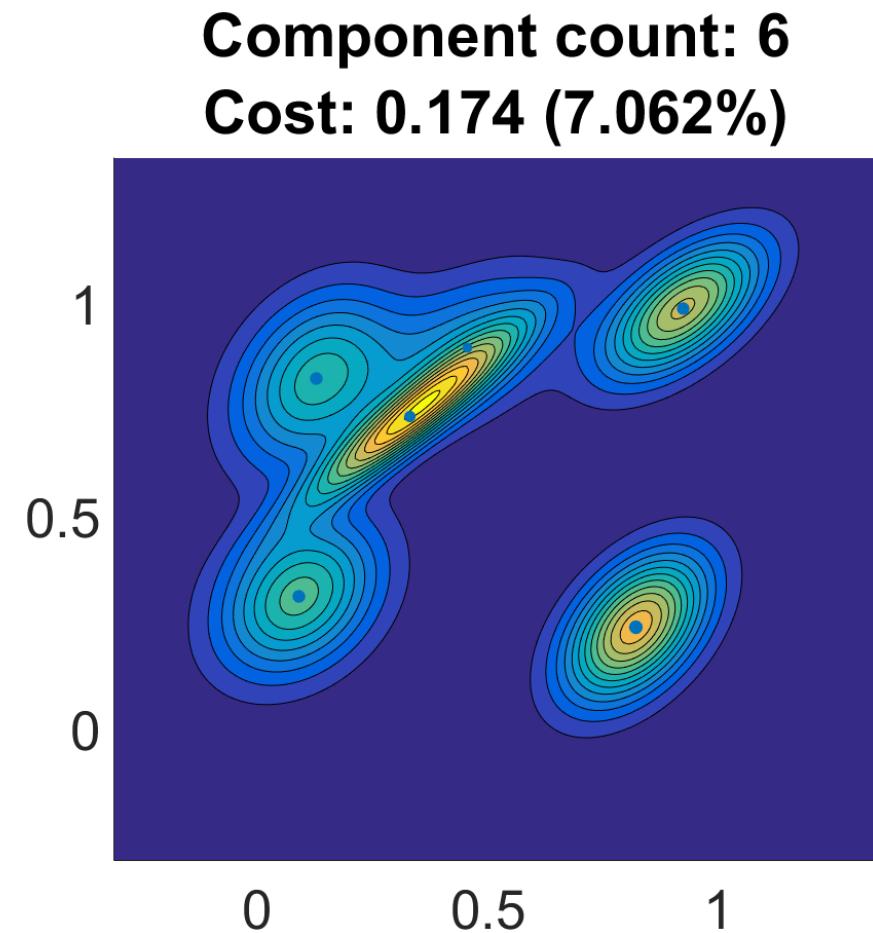
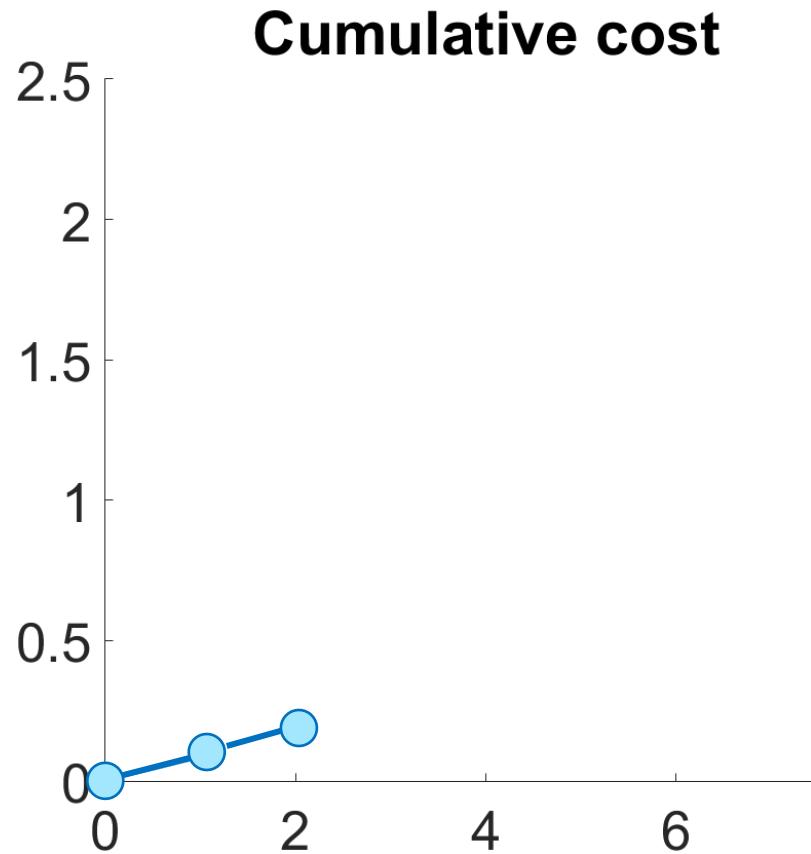
# Component Reduction



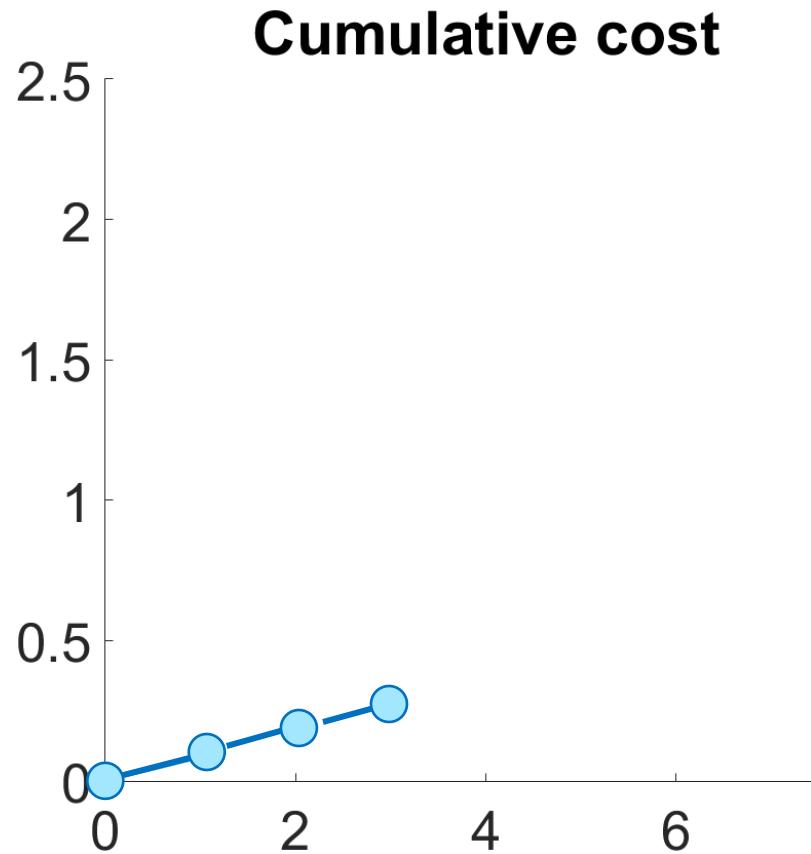
# Component Reduction



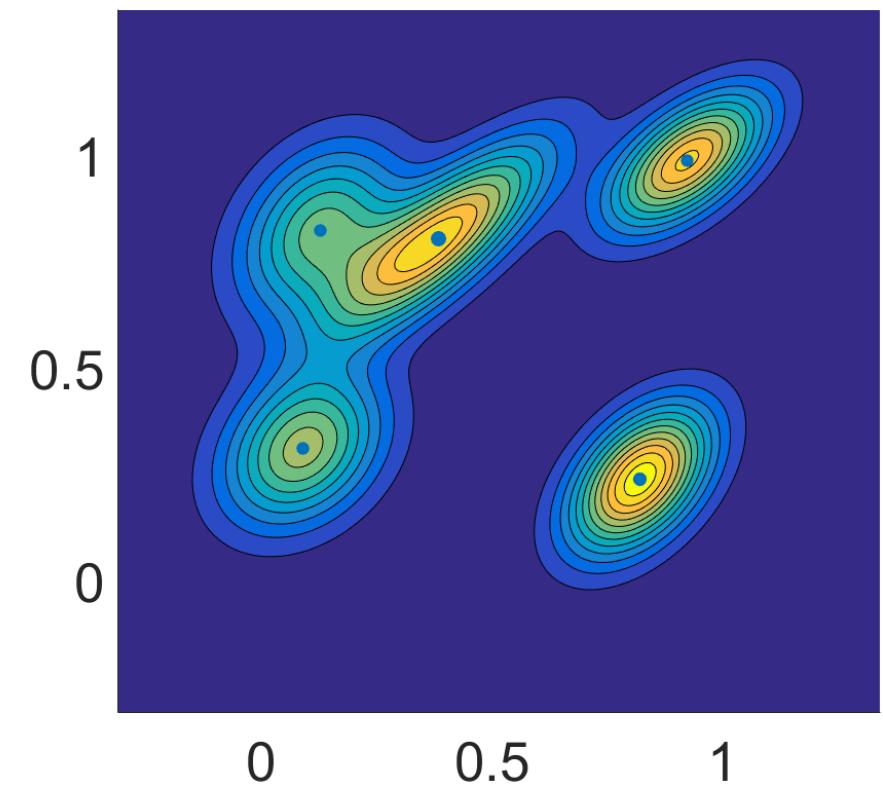
# Component Reduction



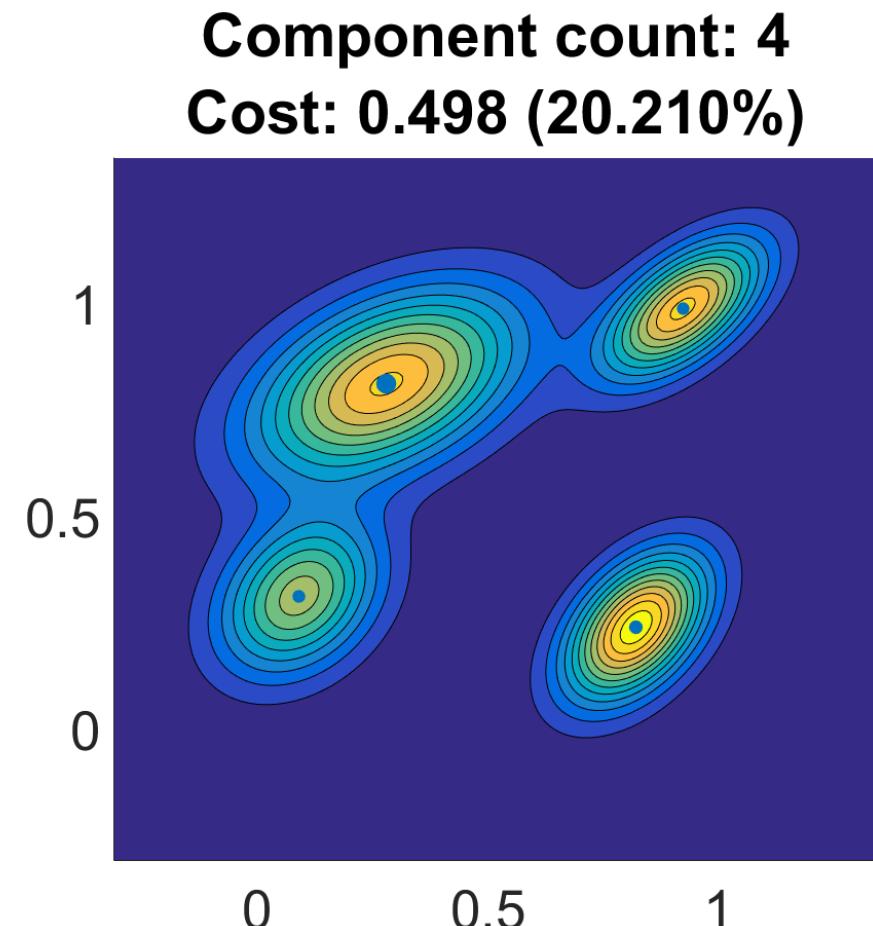
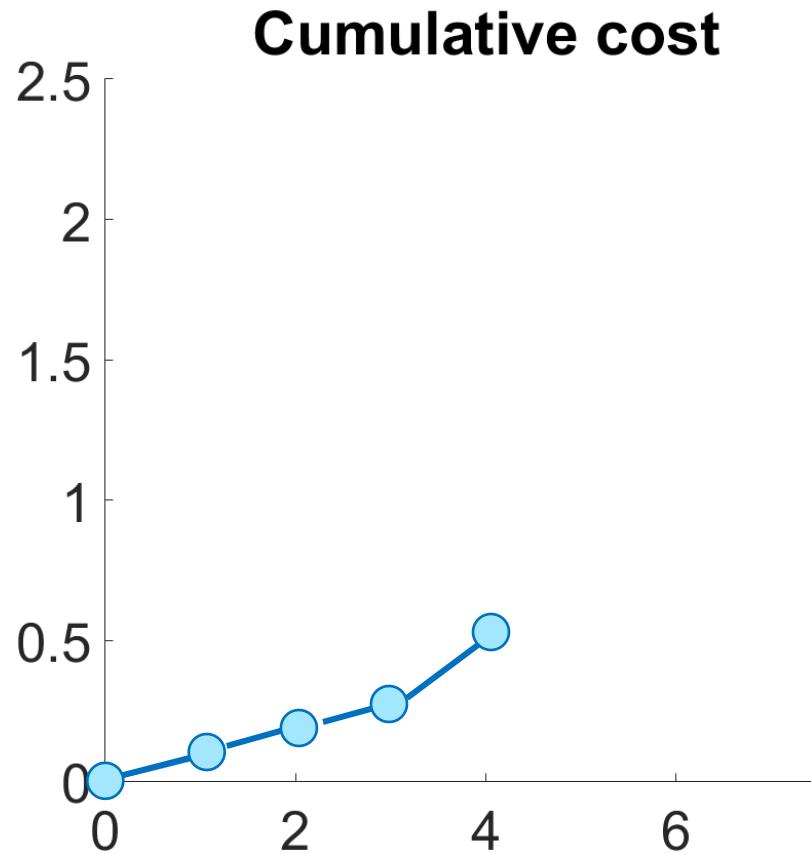
# Component Reduction



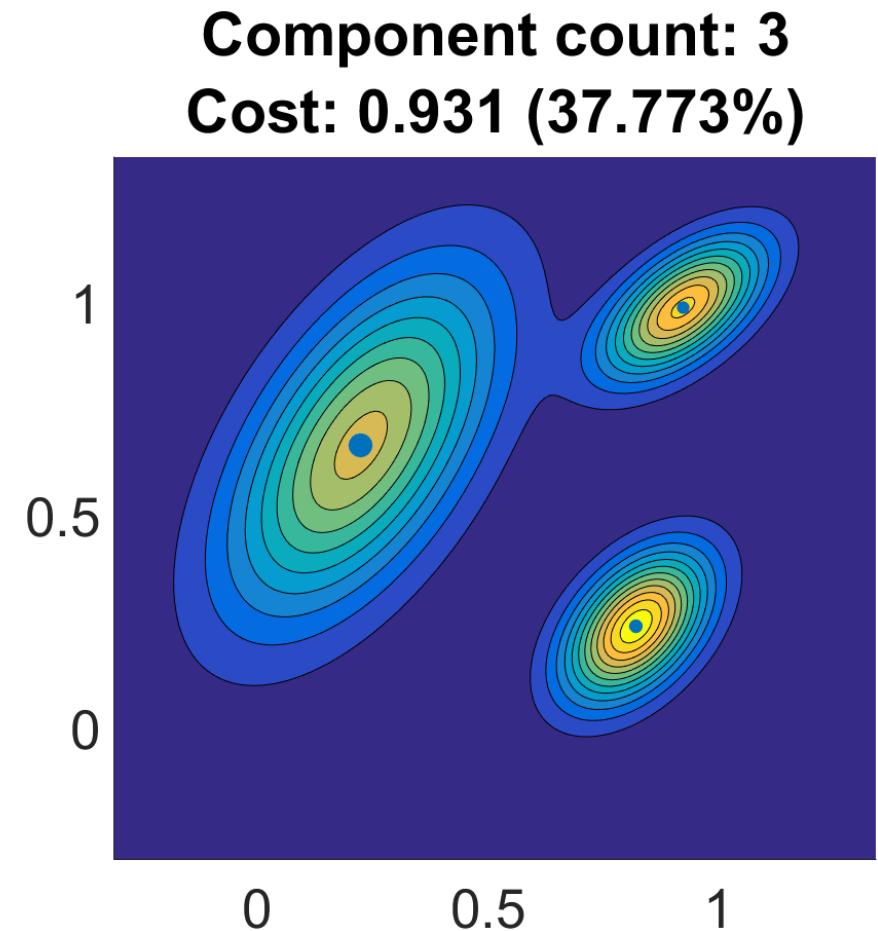
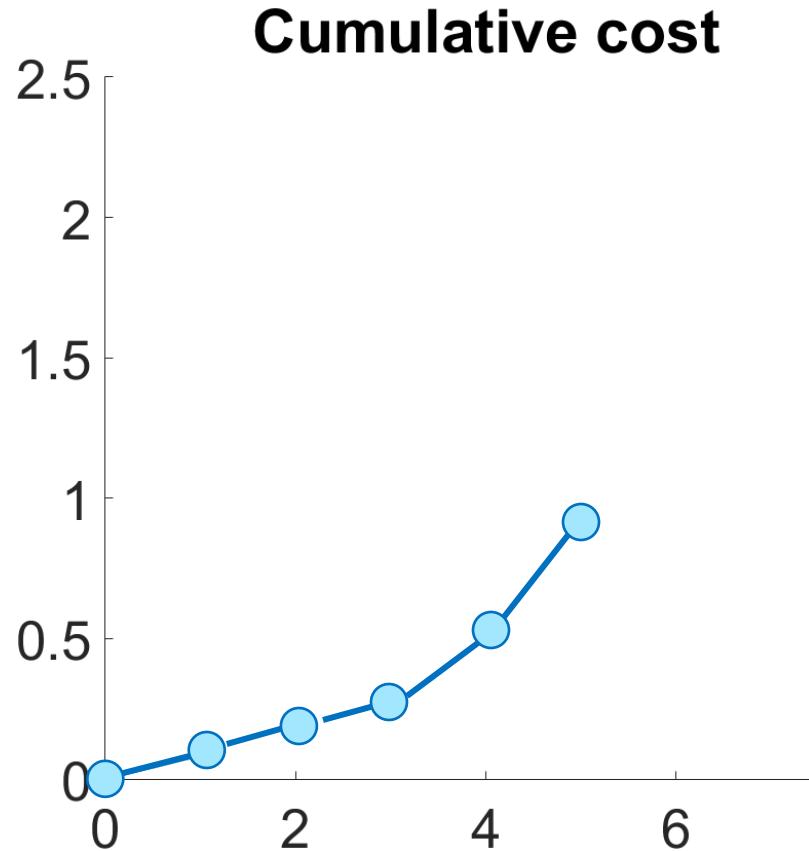
**Component count: 5**  
**Cost: 0.269 (10.917%)**



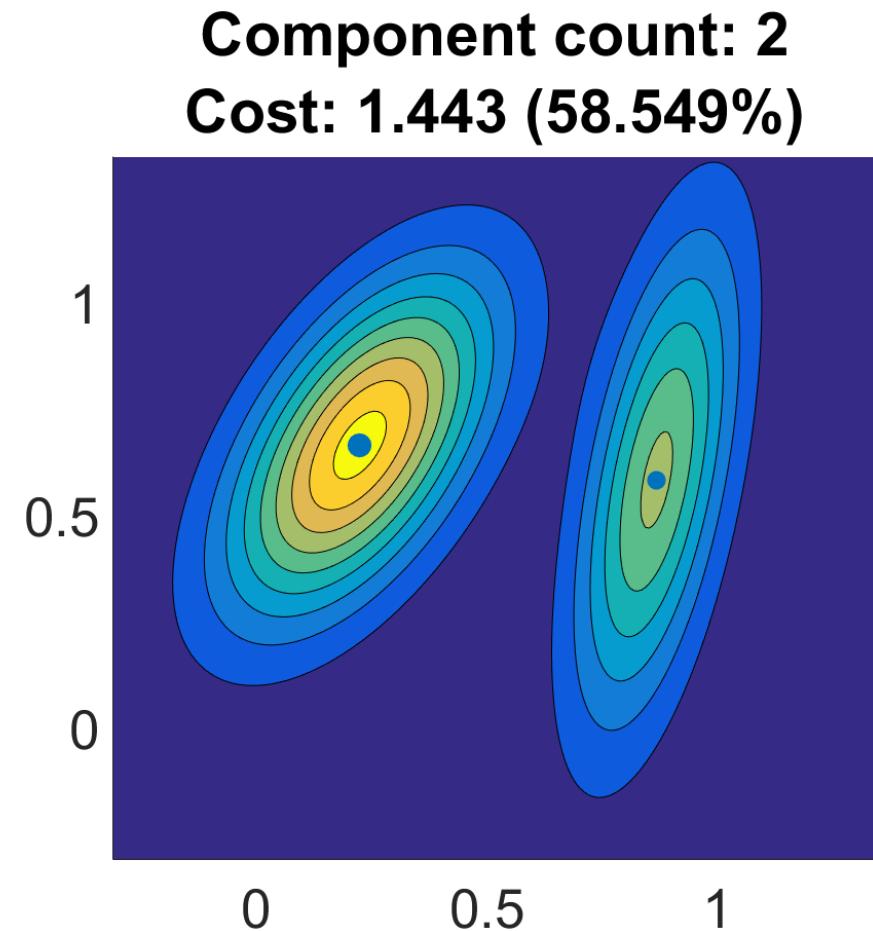
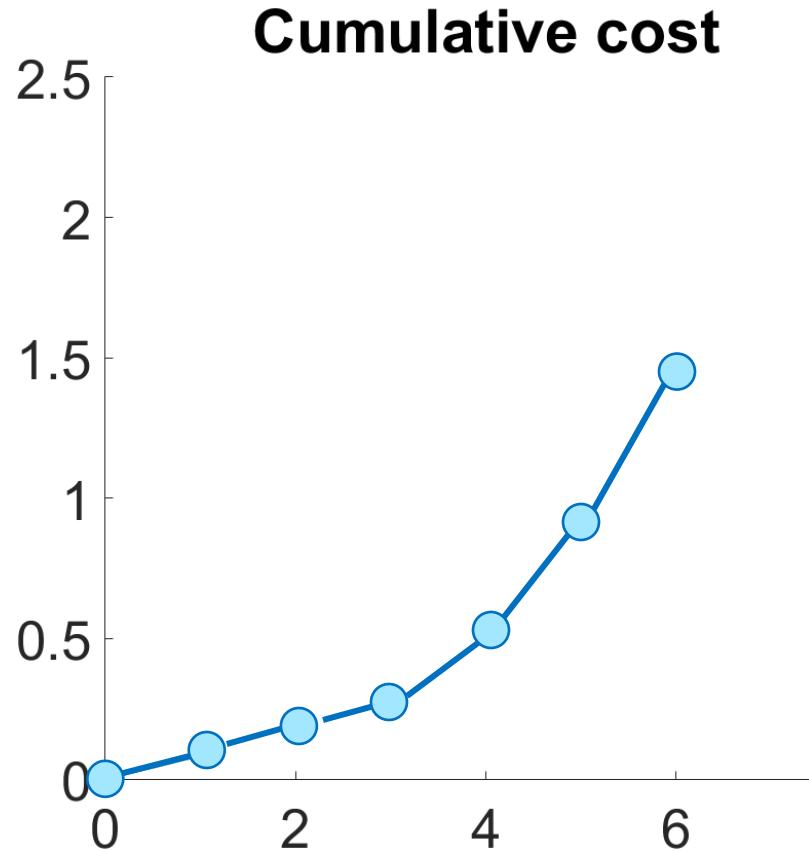
# Component Reduction



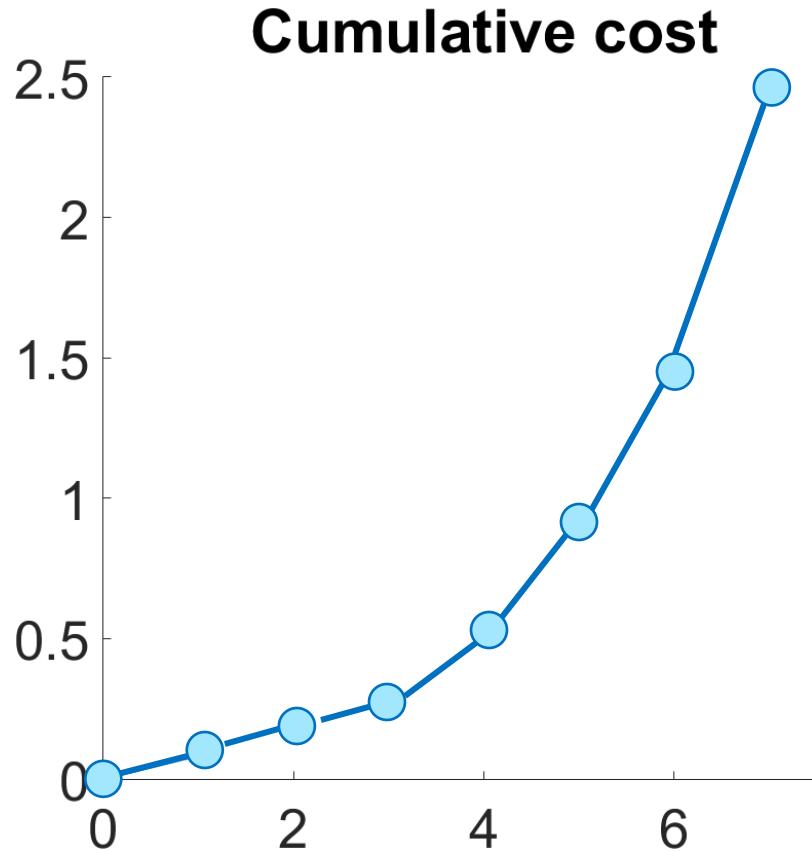
# Component Reduction



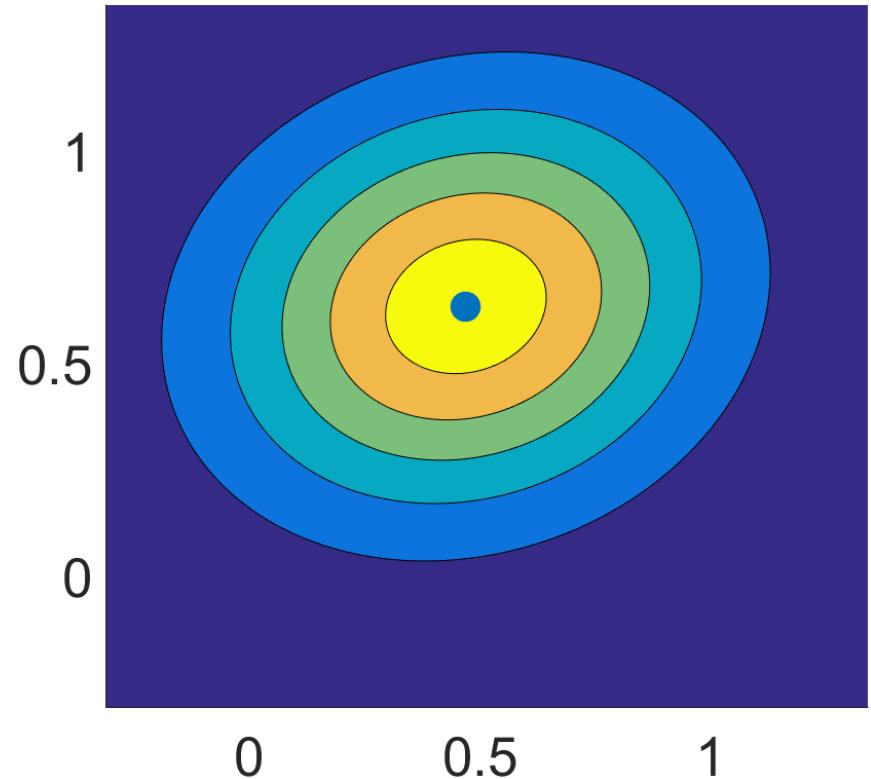
# Component Reduction



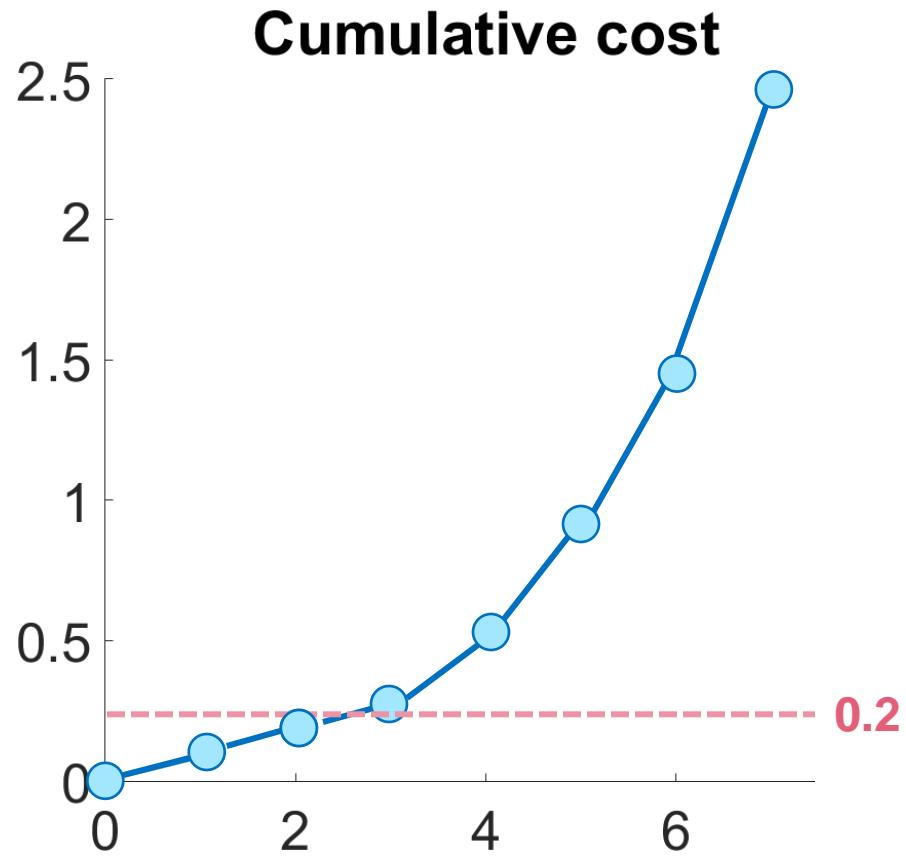
# Component Reduction



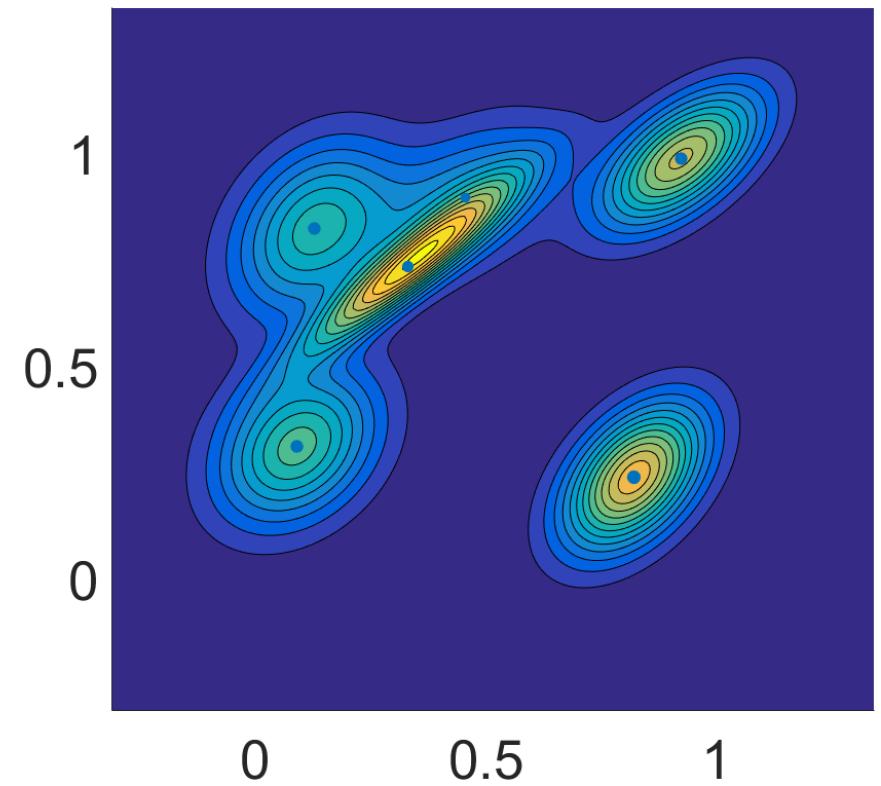
**Component count: 1**  
**Cost: 2.464 (100.000%)**



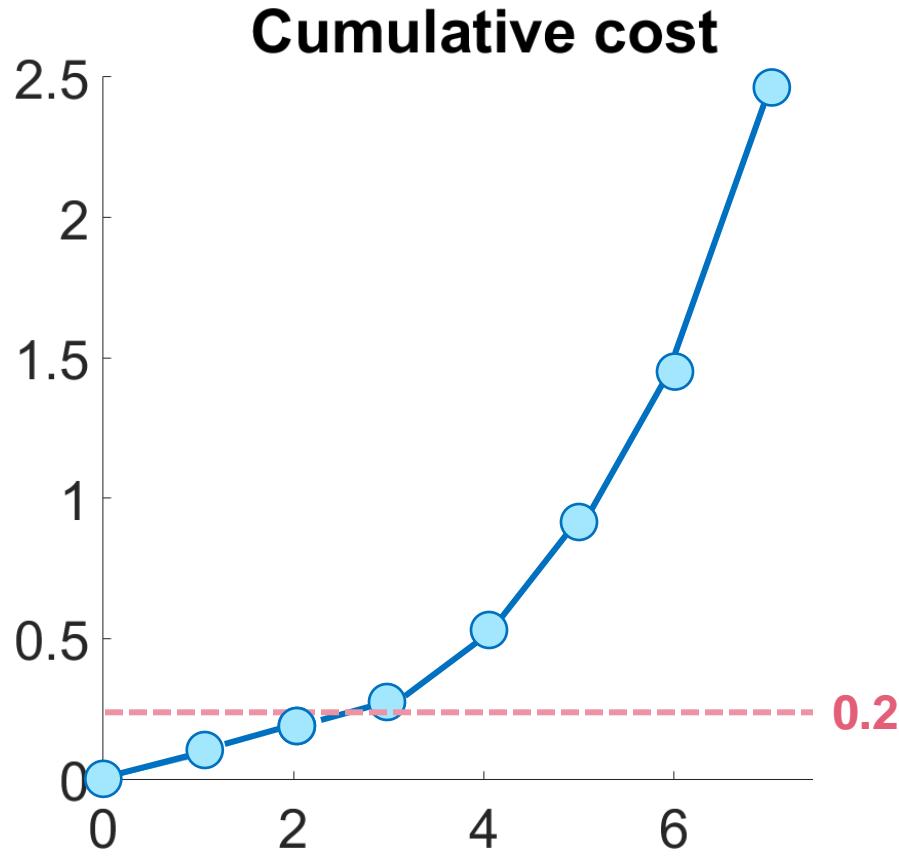
# Component Reduction



**Component count: 6**  
**Cost: 0.174 (7.062%)**



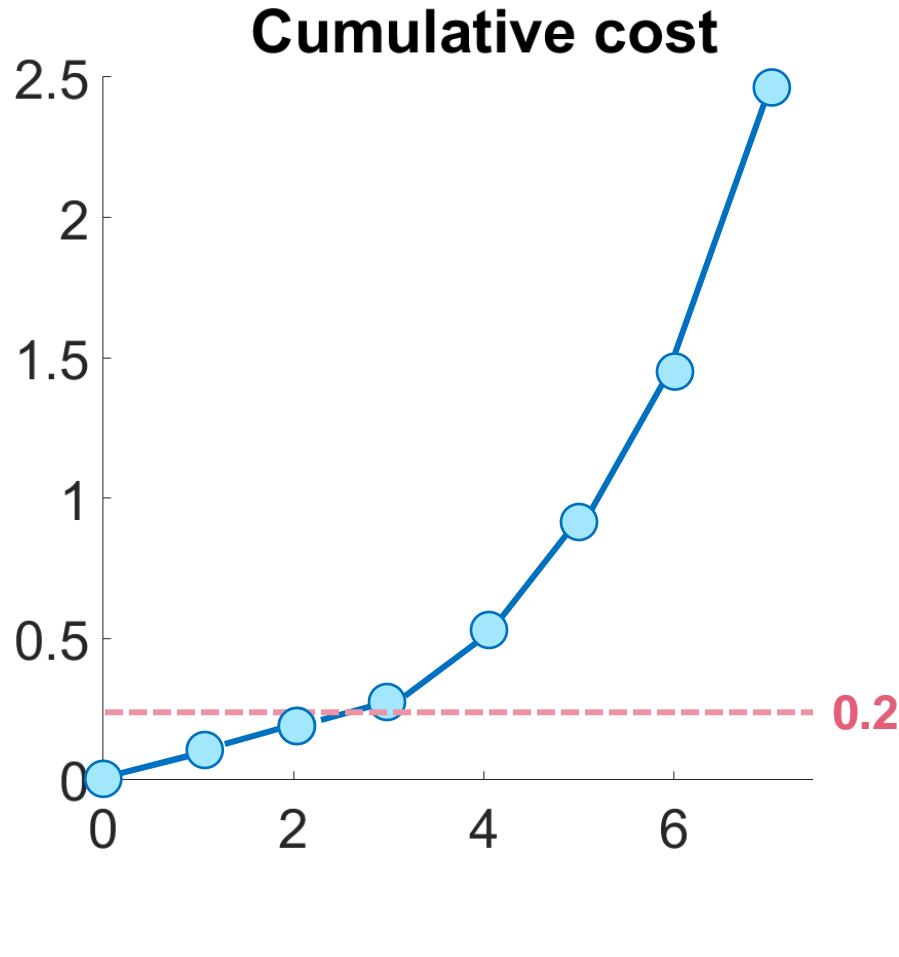
# Component Reduction



## Reduction

- BRDF:
  - Full  $K = 8$
- Illumination
  - Full  $K = 8$
- Product GMM:
  - Avg.  $K_{ij} = 64$

# Component Reduction



## Reduction

- BRDF:
  - Full  $K = 8$
  - **Red. avg.  $K = 2$**
- Illumination
  - Full  $K = 8$
  - **Red. 50% to 4 comp.**
- Product GMM:
  - Avg.  $K_{ij} = 64$
  - **Red. avg.  $K_{ij} = 12$**

# Results

# Reference



Reference



equal time 1hr

# Path Tracer

# Reference



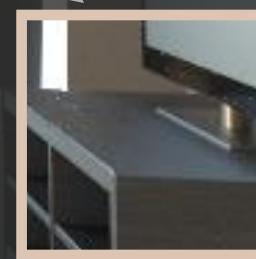
4736 / 2.1830



equal time 1hr



SPP / MSE



Path Tracer



4736 / 2.1830

882 / 0.0331

Vorba2014

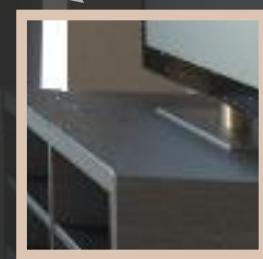


equal time 1hr

Reference



SPP / MSE



Path Tracer



4736 / 2.1830

882 / 0.0331

Our

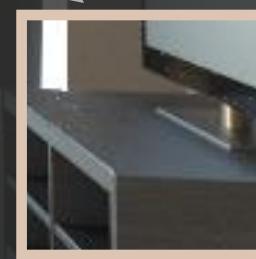


1128 / 0.0211

Reference



SPP / MSE



equal time 1hr

Path Tracer



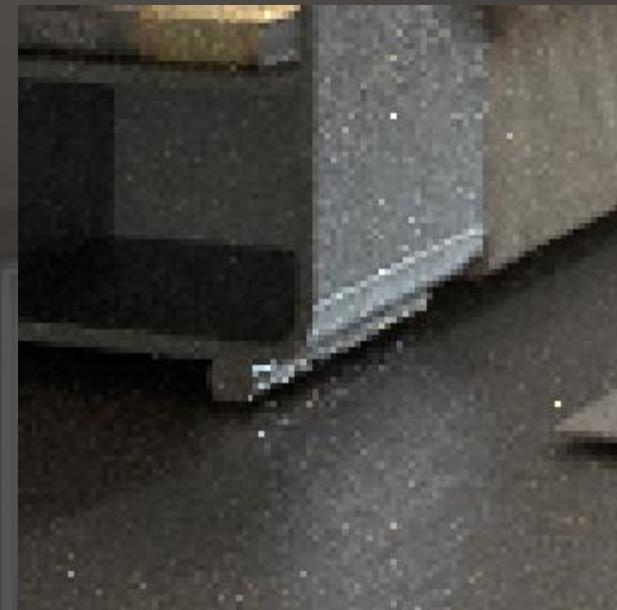
4736 / 2.1830

882 / 0.0331

Vorba2014



Our



Reference



SPP / MSE

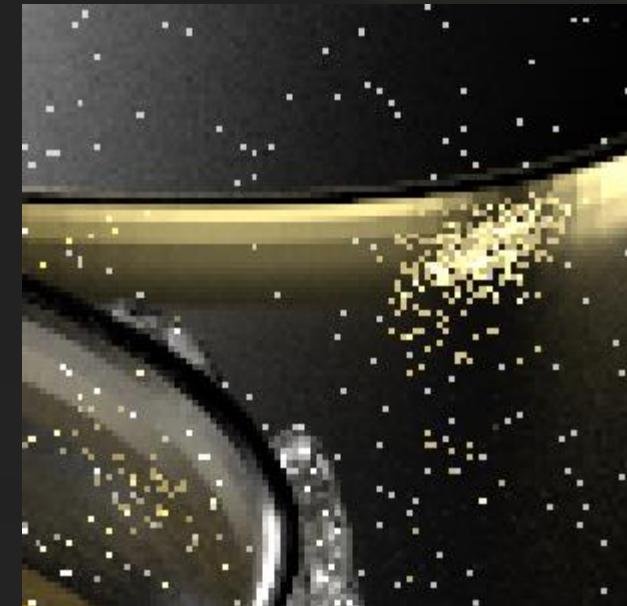
equal time 1hr



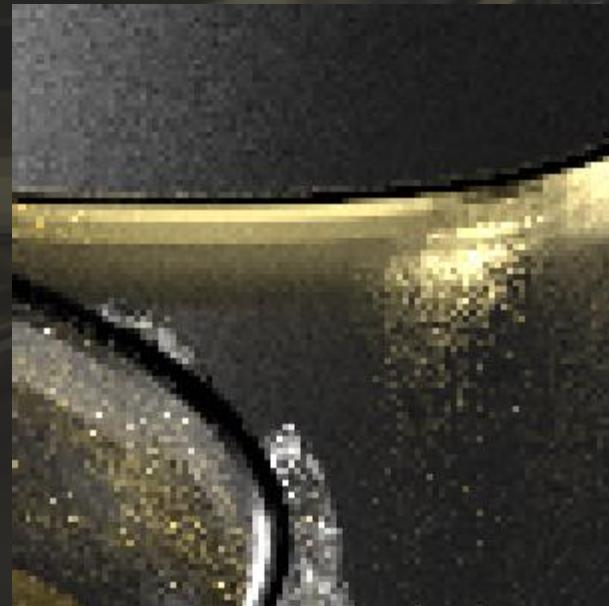
# Reference

equal time 1hr

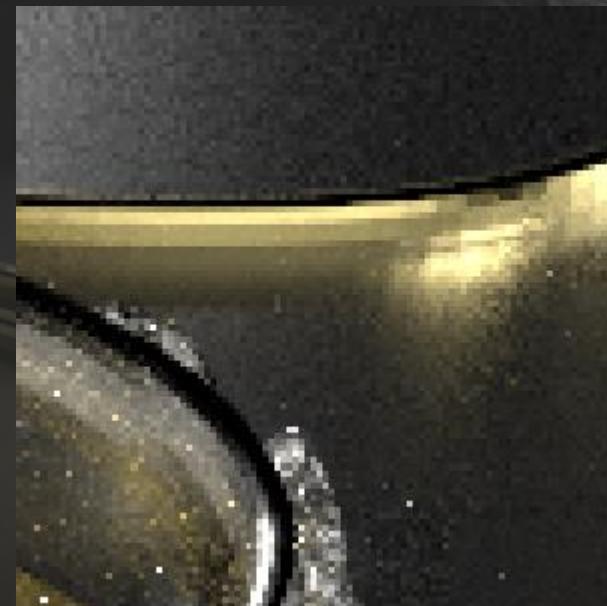
Path Tracer



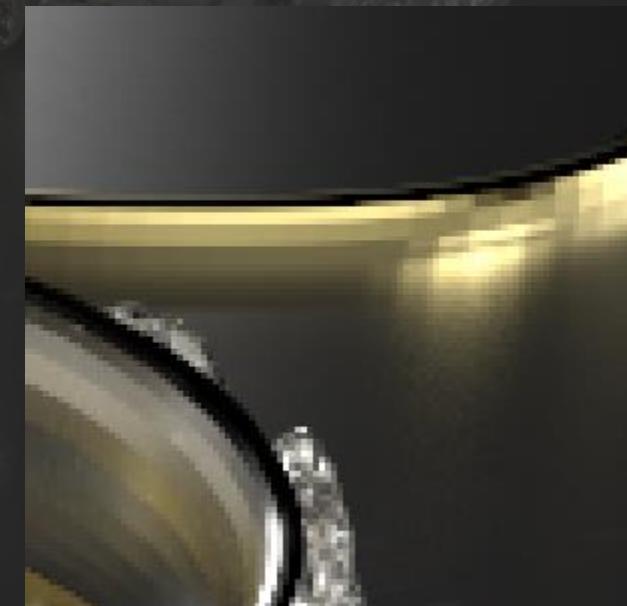
Vorba2014



Our



Reference



4335 / 0.0081

1528 / 0.0025

1322 / 0.0007

SPP / MSE

**Path Tracer**



4335 / 0.0081

1528 / 0.0025

**Our**

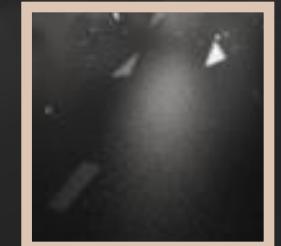


1322 / 0.0007

**Reference**



SPP / MSE

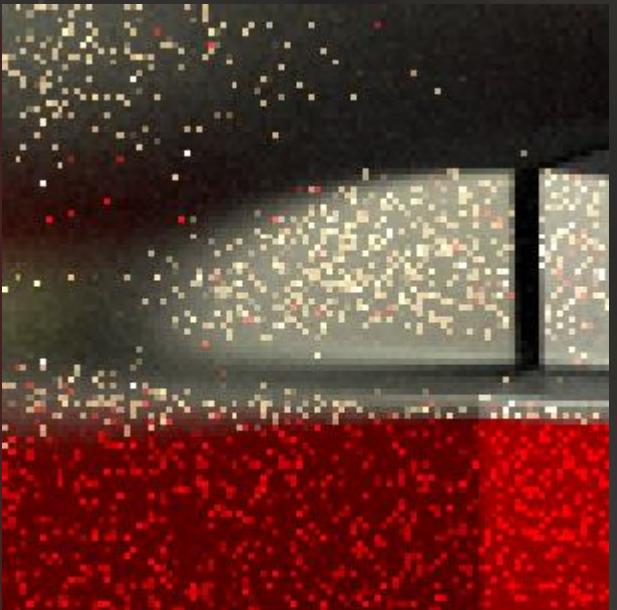


equal time 1hr



Reference

Path Tracer

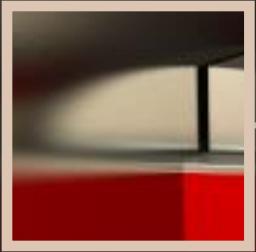


3120 / 0.9715

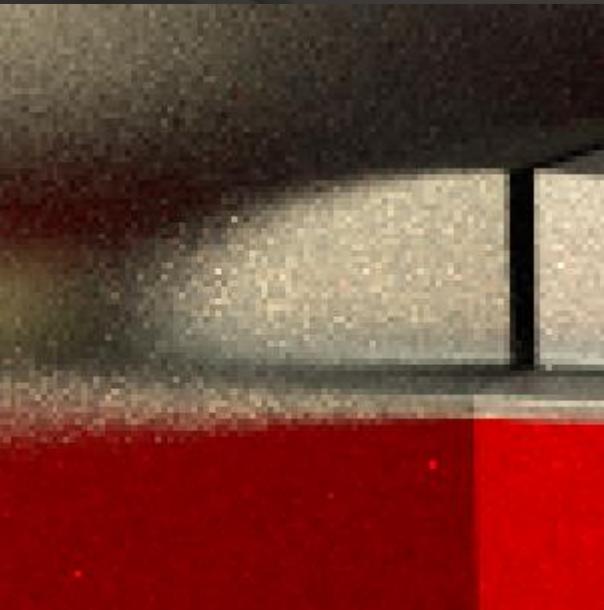
1616 / 0.006

712 / 0.007

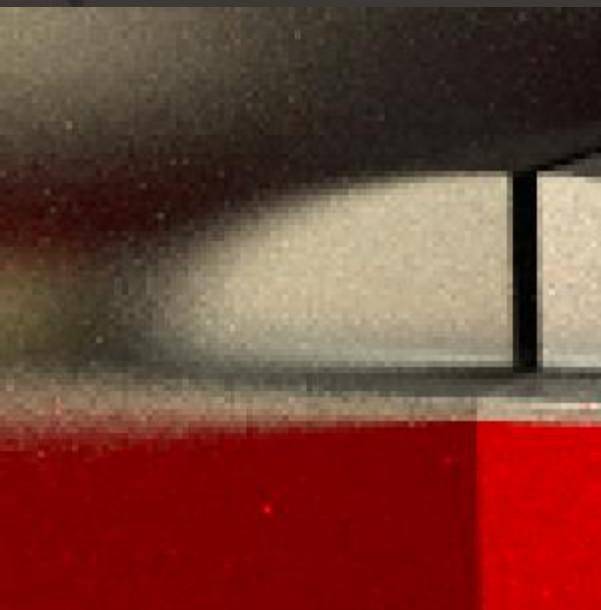
SPP / MSE



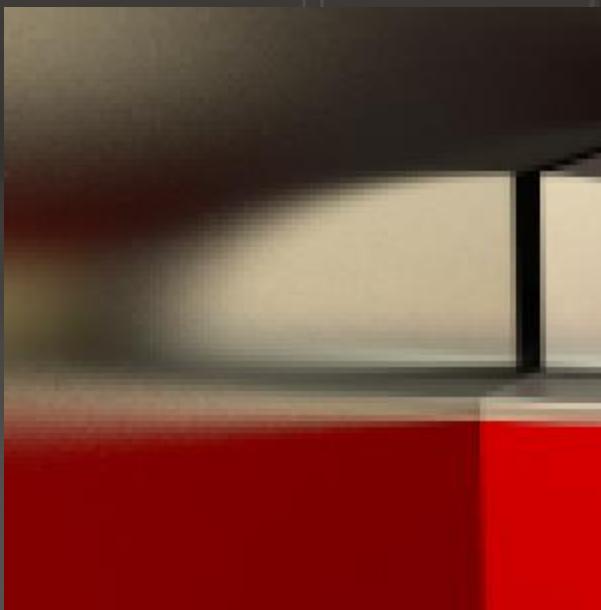
Vorba2014



Our



Reference



equal time 1hr

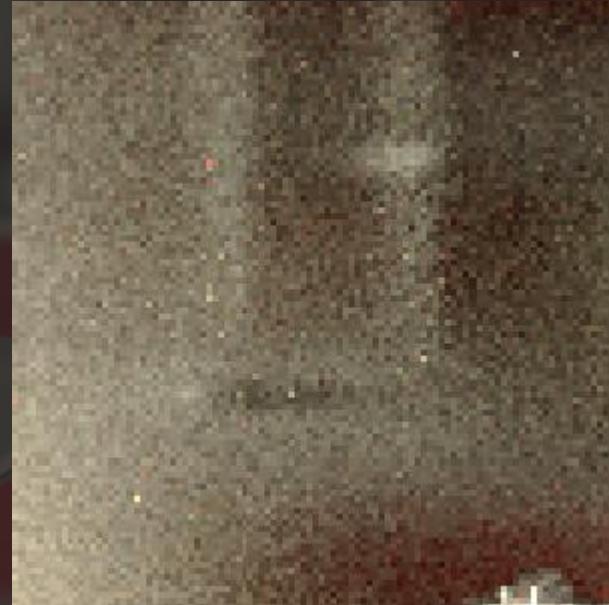
equal time 1hr

Path Tracer



3120 / 0.9715

Vorba2014



1616 / 0.006

Our



712 / 0.007

Reference



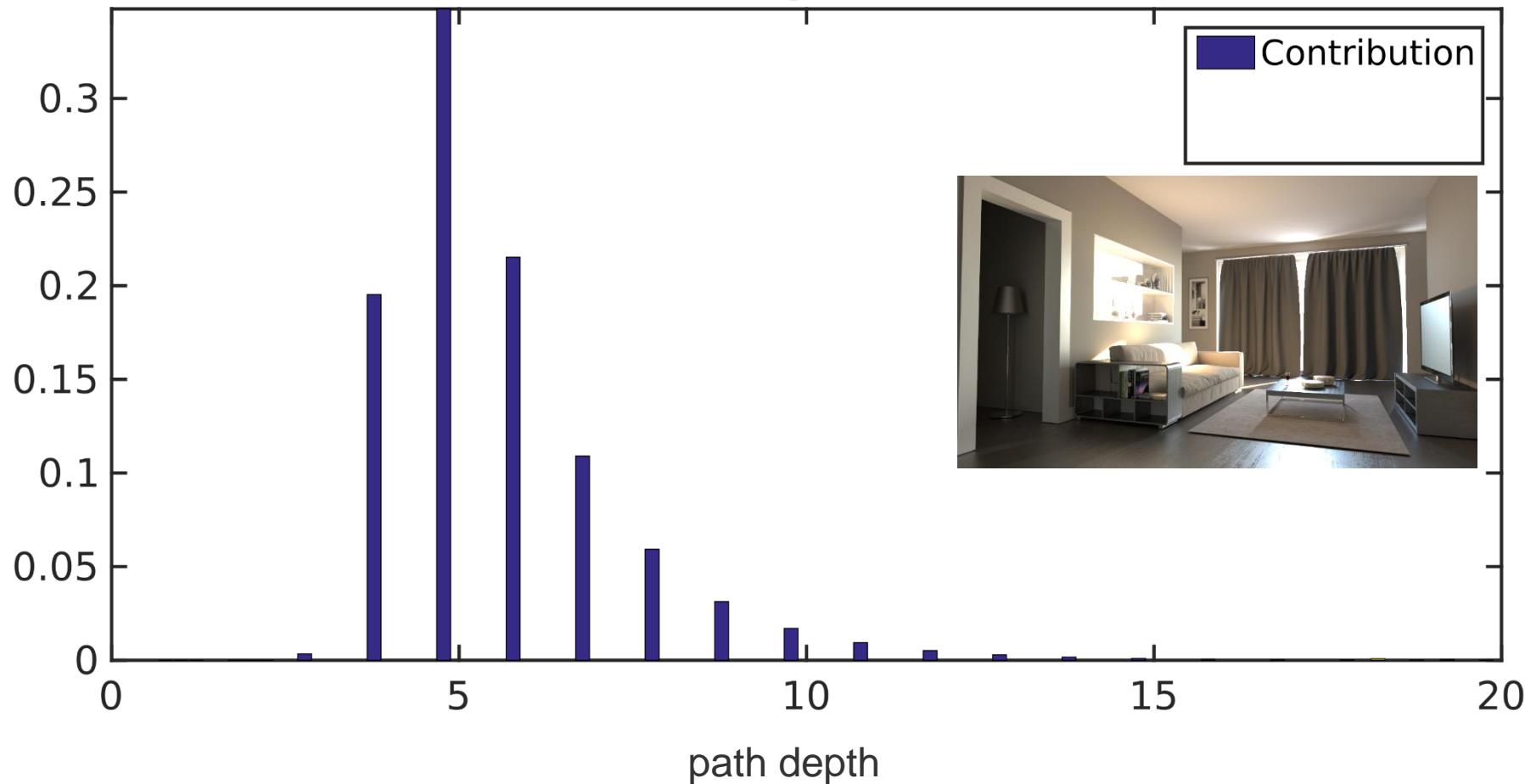
SPP / MSE

# Average Path Lengths

Vorba: 9.8      Ours: 4.9

normalized per-path segment contribution

## LivingRoom

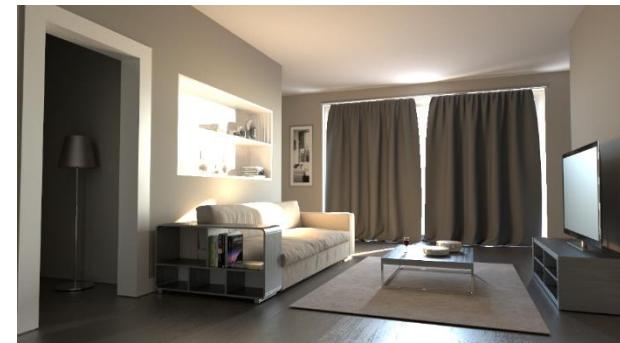
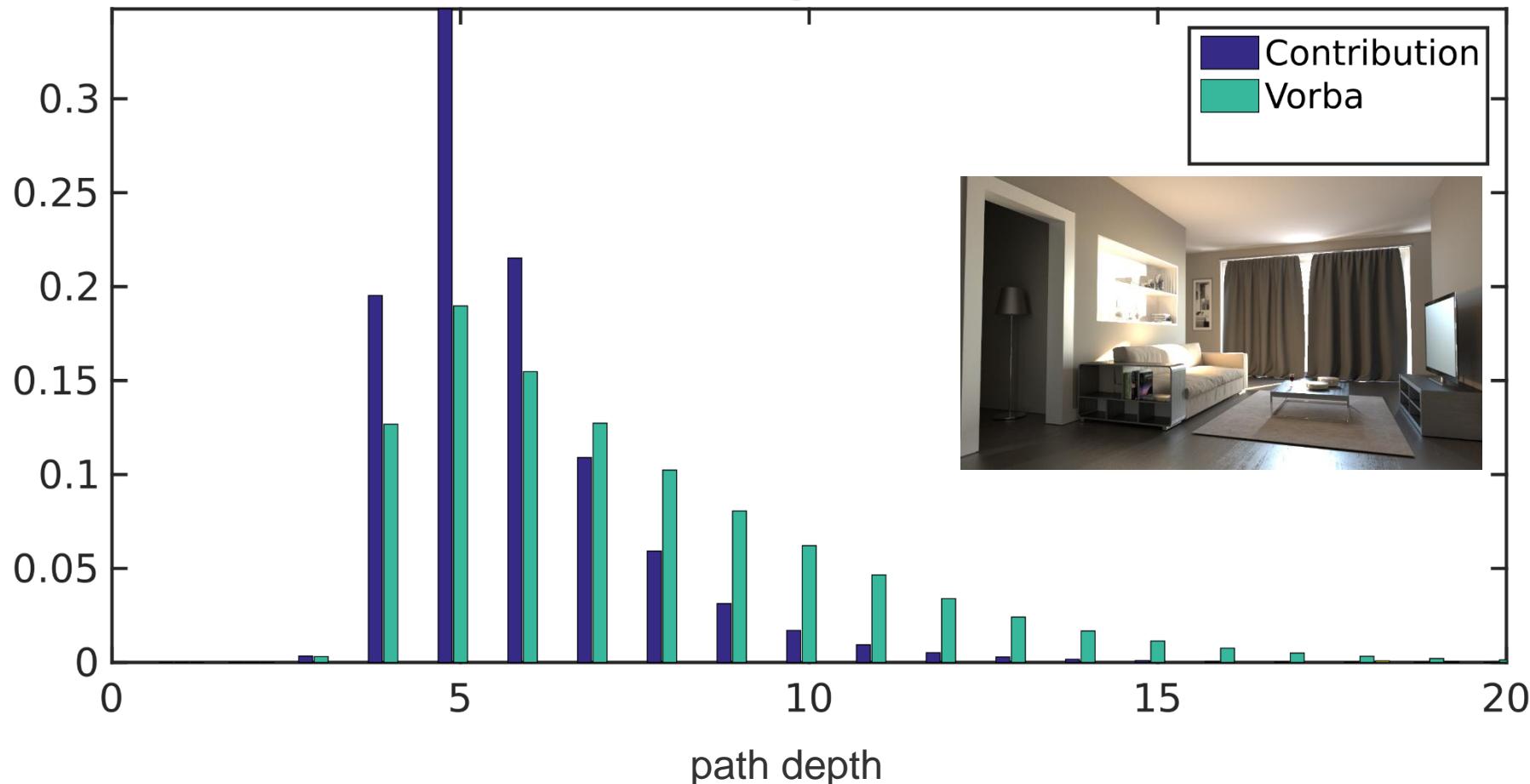


# Average Path Lengths

Vorba: 9.8      Ours: 4.9

normalized per-path segment contribution

## LivingRoom

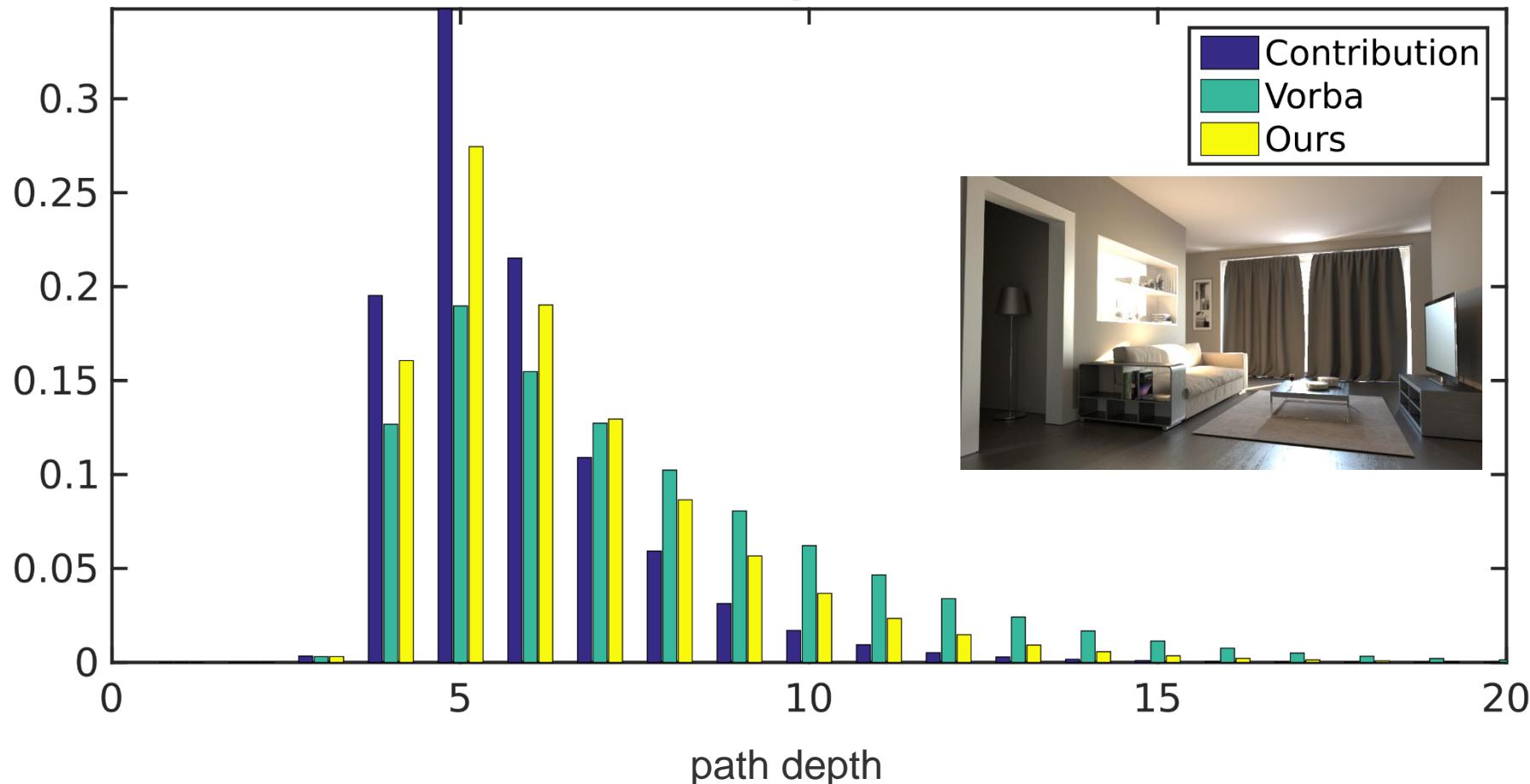


# Average Path Lengths

Vorba: 9.8      Ours: 4.9

normalized per-path segment contribution

## LivingRoom

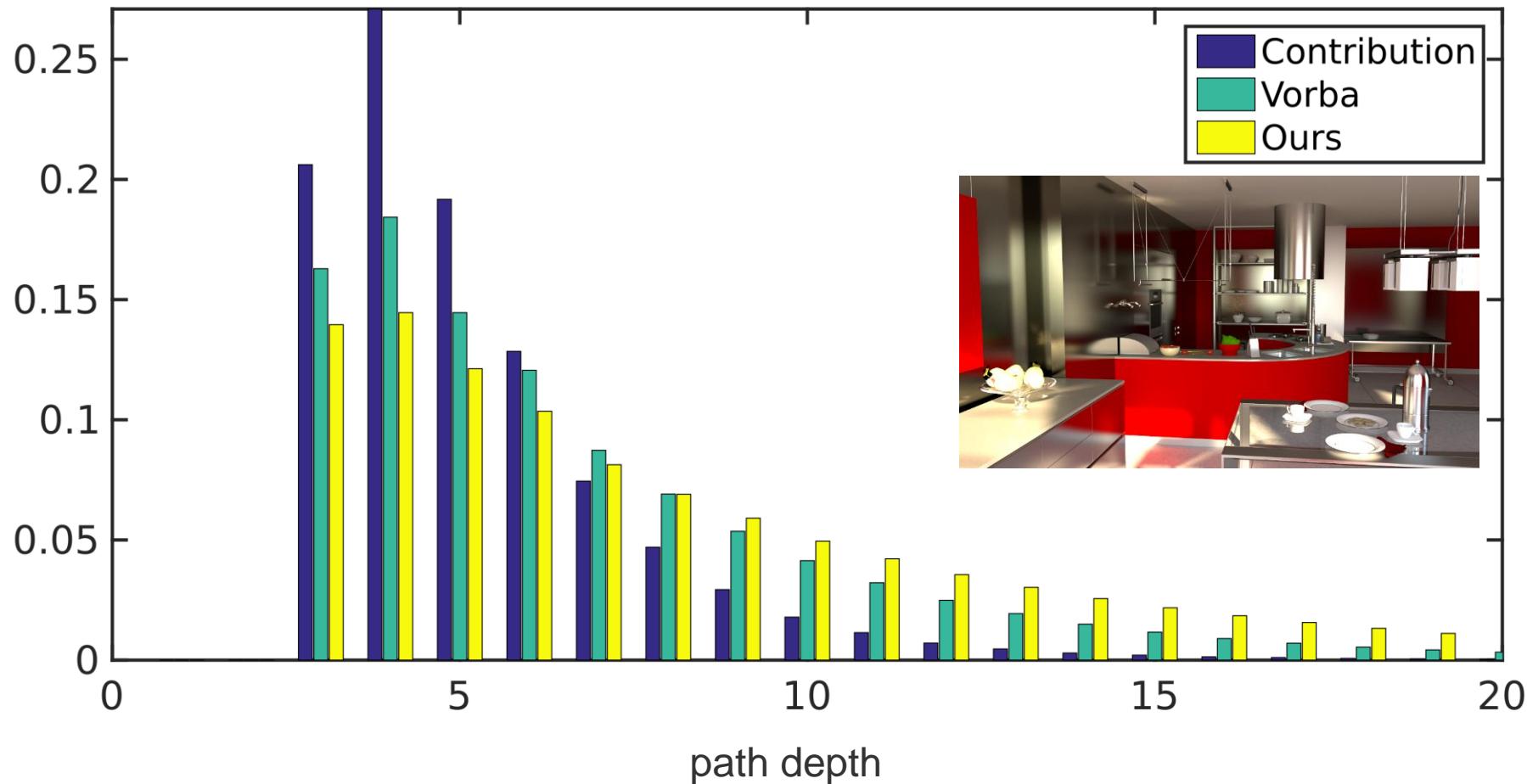


# Average Path Lengths

Vorba: 6.5      Ours: 9.0

normalized per-path segment contribution

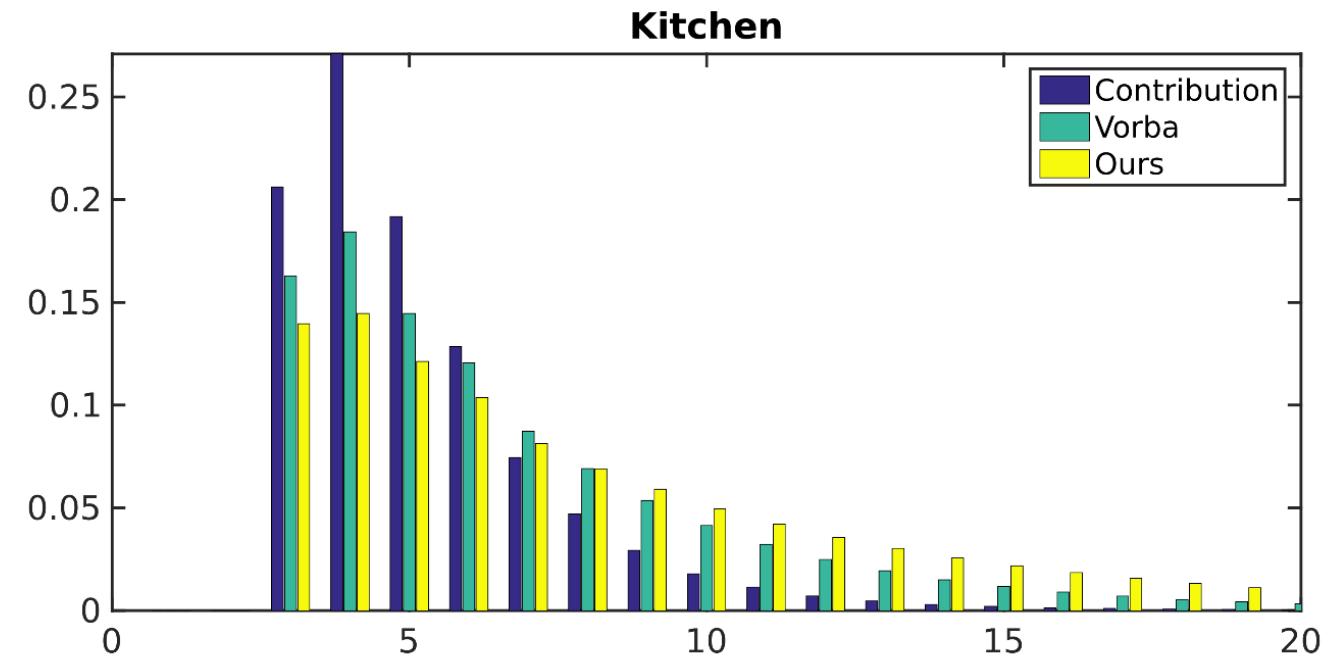
## Kitchen



# Discussion / Future work

# Discussion / Future Work

- Path Length and Russian Roulette
  - Adjoint-driven RR and Splitting  
[Vorba2016]



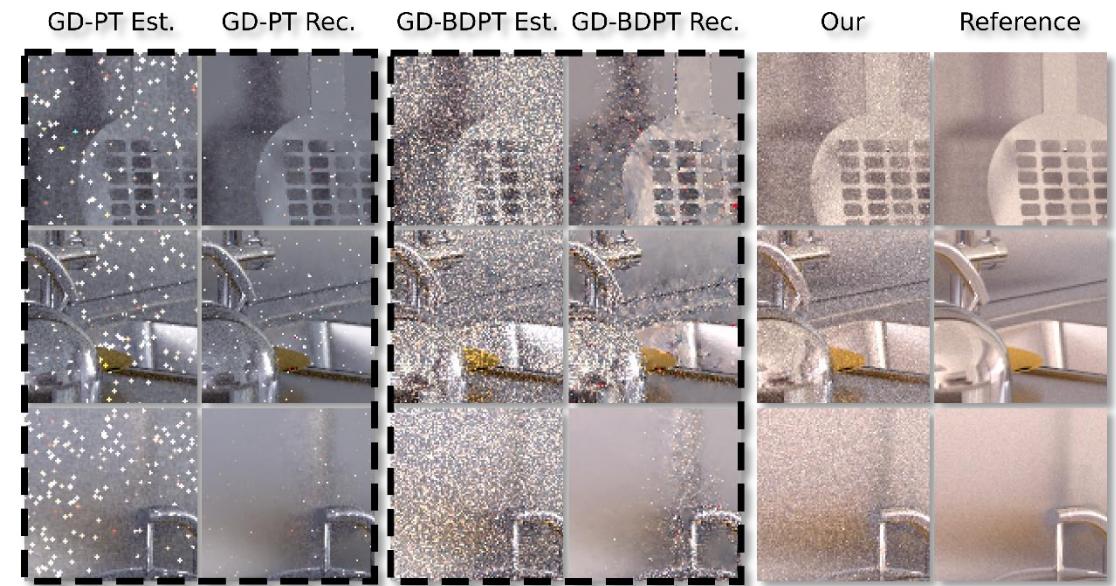
## Discussion / Future Work

- SVBRDFs
  - Enlarge BRDF caches
  - Direct function transform
- BRDF->GMM

Scene	BRDF caching			
	# BRDFs	# Caches	Avg. # comp.	Mem.
LIVINGROOM	41	15k	2.5	7.7 MB
KITCHEN	72	2.5k	1.8	10 MB
JEWELRY	6	1.5k	1.44	0.7 MB

# Discussion / Future Work

- Extension to other MC-algorithms
  - BDPT
  - MCMC
  - Gradient domain



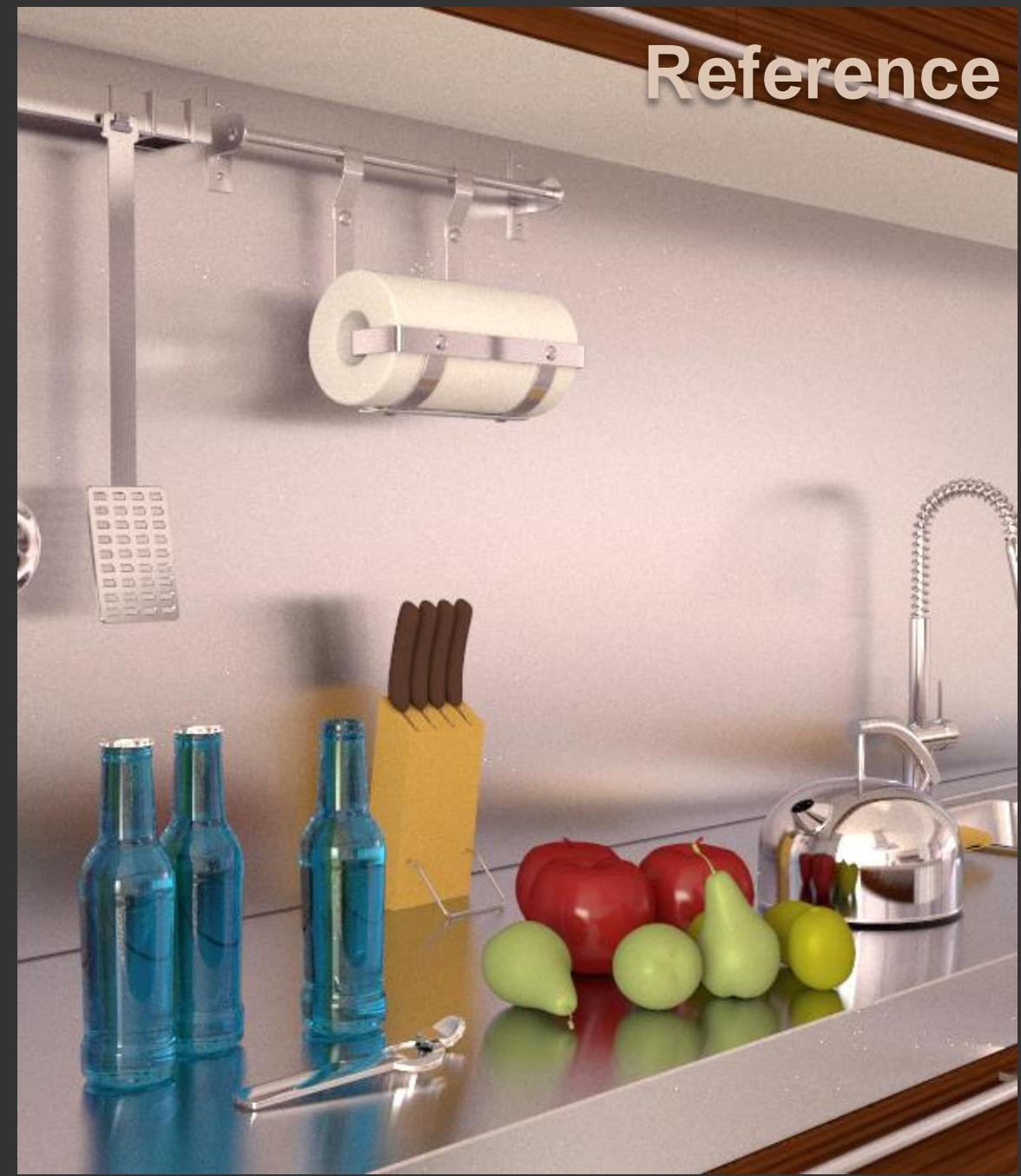
**GD-PT  
(full)**



**GD-BDPT  
(full)**



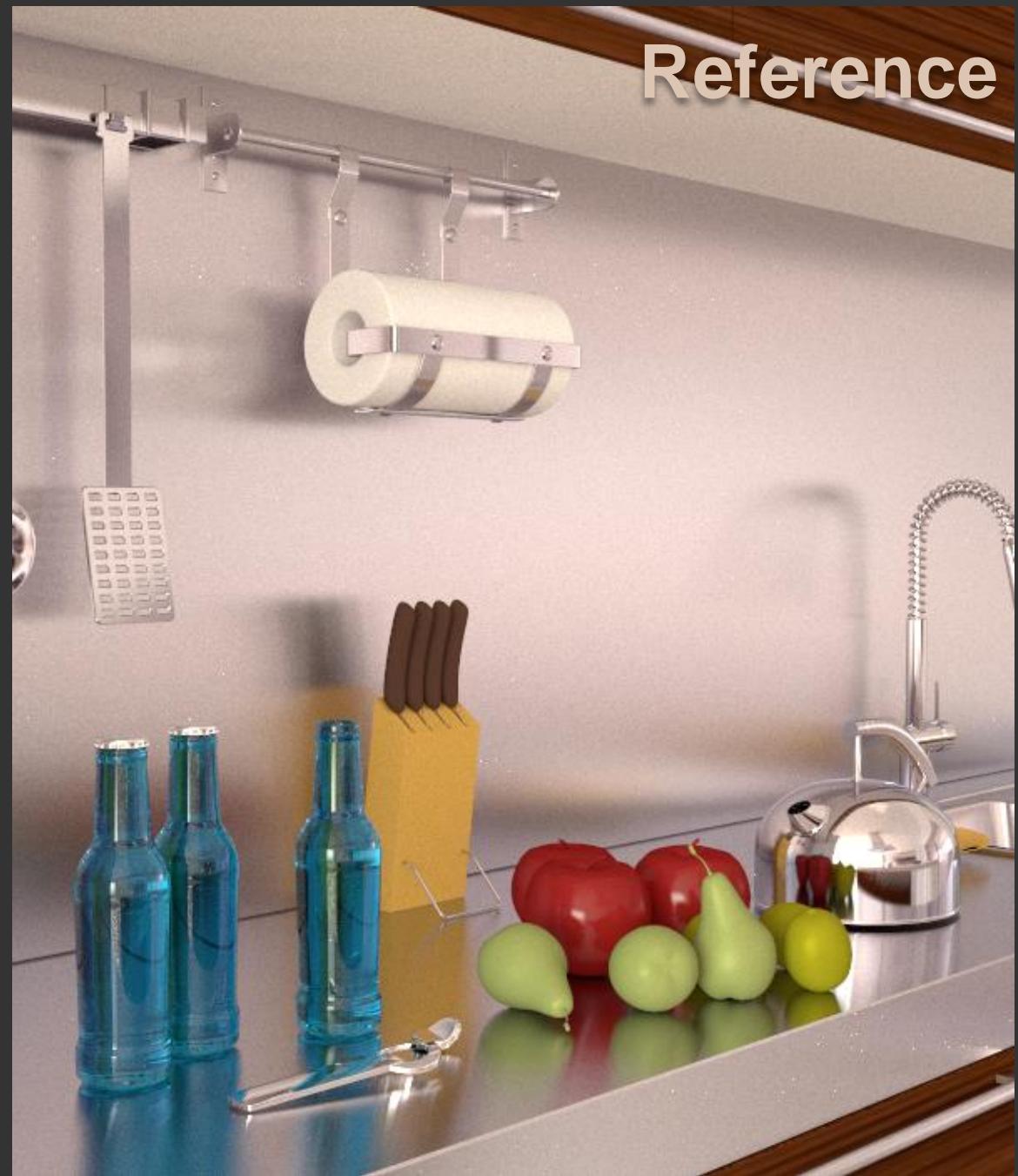
**Reference**



GD-PT  
(estimate)



GD-BDPT  
(estimate)



Reference

Our



Reference



## Discussion/Future Work

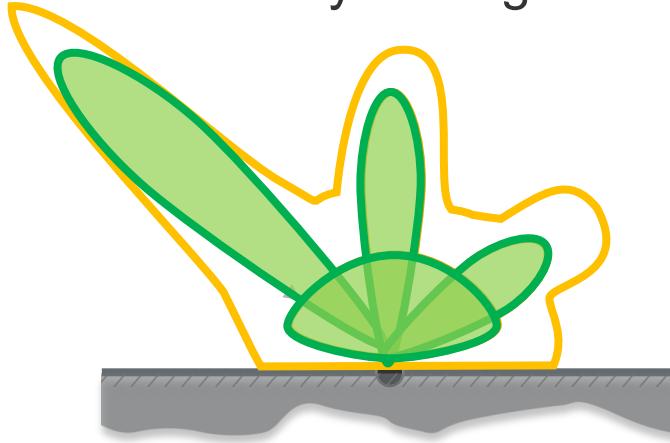
- Optimizing Illumination caches
  - Poorly fitted illumination caches cause inconsistent convergence rates



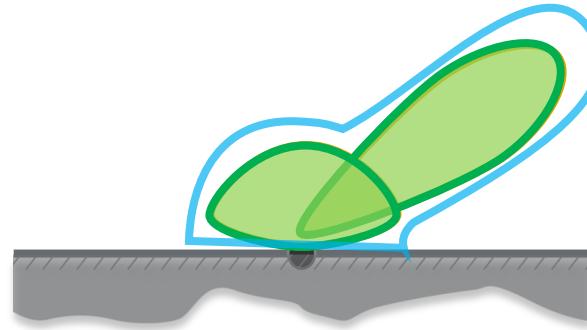
# Product Importance Sampling for Light Transport Path Guiding

Sebastian Herholz<sup>1</sup> Oskar Elek<sup>2</sup> Jiří Vorba<sup>2,3</sup>  
Hendrik Lensch<sup>1</sup> Jaroslav Křivánek<sup>2</sup>

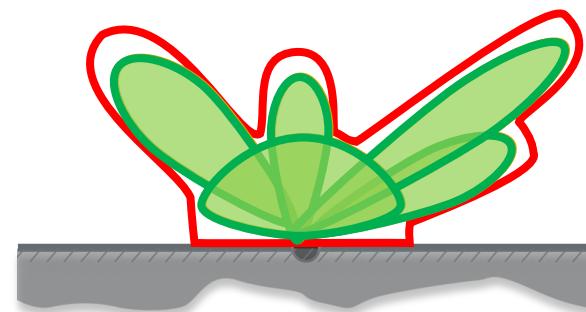
<sup>1</sup>University Tübingen



<sup>2</sup>Charles University Prague



<sup>3</sup>Weta Digital



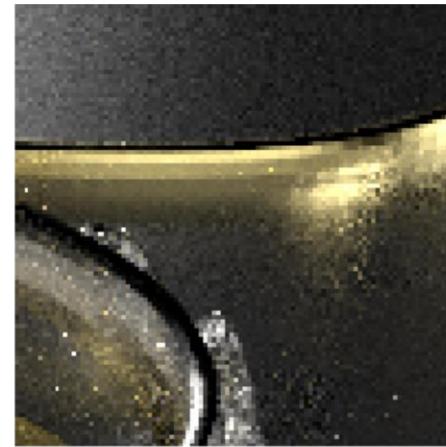
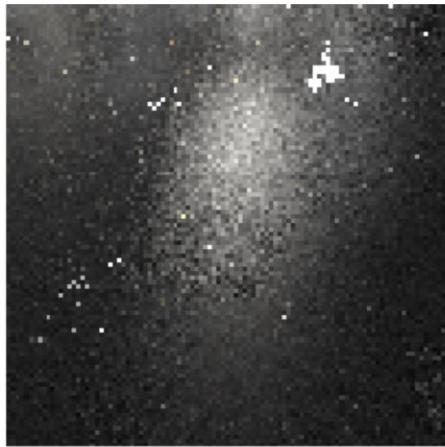
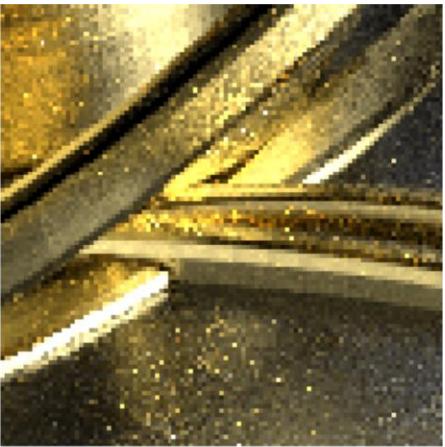
Thanks to:

- Martin Šik, Ivo Kondapaneni, Ludvík Koutný,  
Anton Kaplanyan, Johannes Hanika
- Anonymous reviewers
- You!

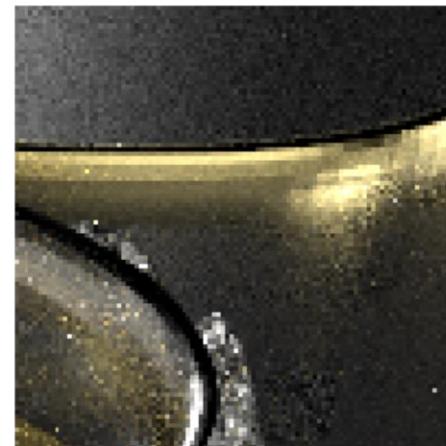
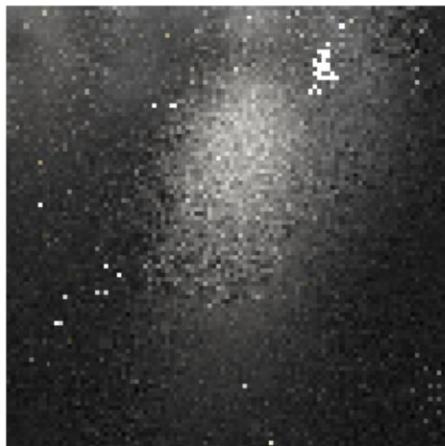
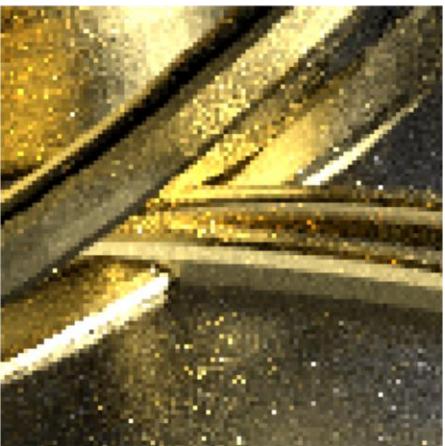
# **Backup**

## wEM vs CERES

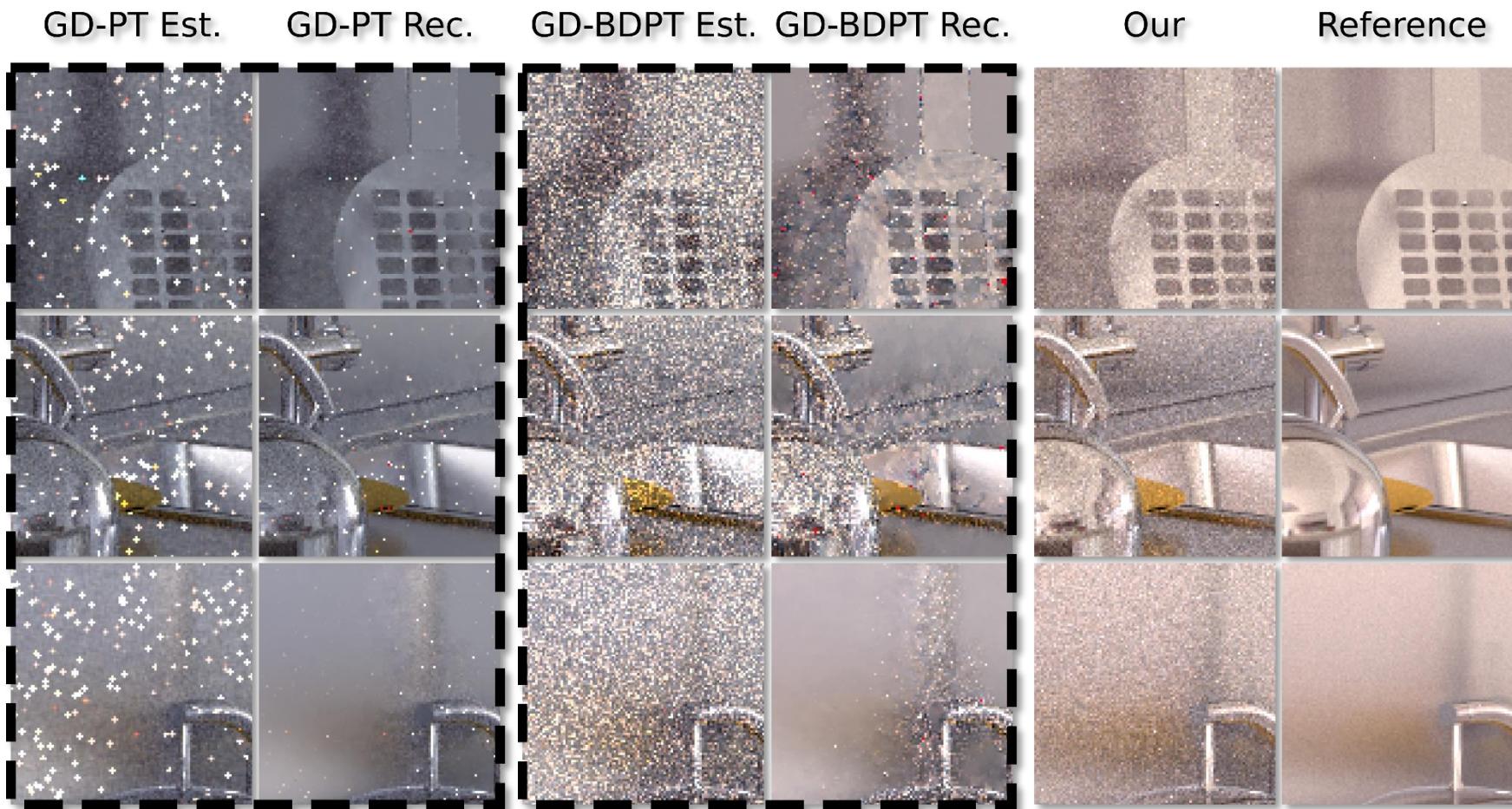
wEM

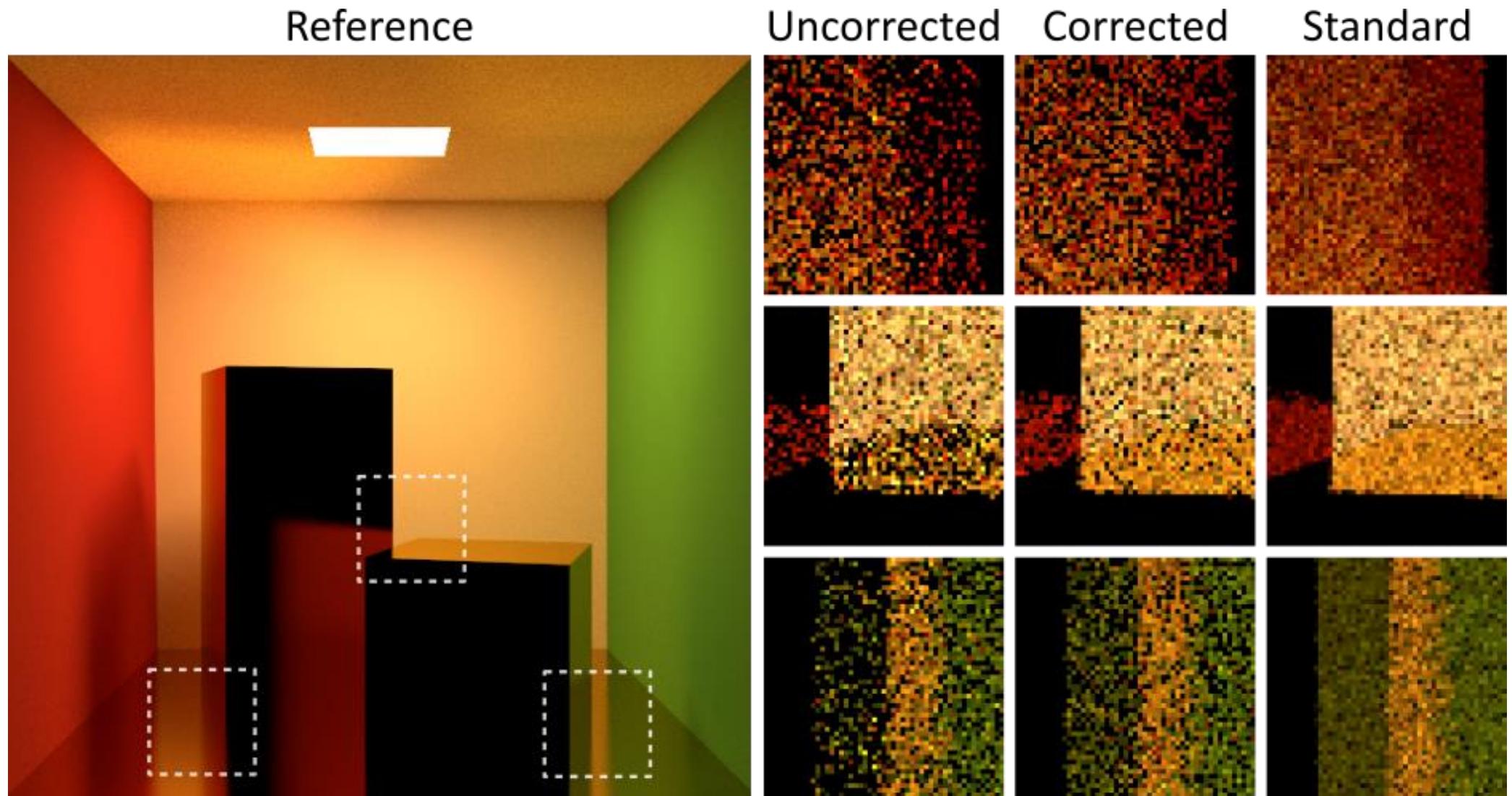


Ceres



# Gradient Domain





## Preporcessing times:

**Table 1:** *Timings for the cache fitting stages (in minutes).*

Scene	Illumination	BRDF	
		wEM	Ceres
LIVINGROOM	14.0	0.31	25.5
KITCHEN	20.1	0.44	42.3
JEWELRY	6.1	0.04	6.3

## Statistics:

**Table 2:** Left and middle: BRDF and illumination caching statistics for the scenes in Fig. 8. Right: Overhead of the product sampling relative to illumination-only sampling, without ('naïve') and with the reduction of the BRDF and illumination mixtures.

Scene	BRDF caching				Illumination caching			Sampling overhead [%]	
	# BRDFs	# Caches	Avg. # comp.	Mem.	# Caches	# Reduced	Mem.	Naïve	Reduced
LIVINGROOM	41	15k	2.5	7.7 MB	82k	57 %	192.9 MB	10.8	7.1
KITCHEN	72	2.5k	1.8	10 MB	107k	62 %	236.9 MB	26.7	9.9
JEWELRY	6	1.5k	1.44	0.7 MB	16k	33 %	19.5 MB	16.5	-1.1

**END**