تاریخ: دن:

Ex# 5.1

Q: 05

(i)

$$\underbrace{\begin{vmatrix} a & b & c \\ d & e & F \\ g & h & K \end{vmatrix}}_{A} = \underbrace{\begin{vmatrix} e & b & h \\ d & a & g \\ F & c & K \end{vmatrix}}_{B}$$

Show that determinate equal to "B".

L.H.S:-

$$\begin{vmatrix} a & b & c \\ d & e & F \\ g & h & K \end{vmatrix}$$

Following determinate properties:-

$$\begin{vmatrix} d & e & F \\ a & b & c \\ g & h & K \end{vmatrix} \quad R_{12}$$

$$\begin{vmatrix} e & d & F \\ b & a & c \\ c & g & K \end{vmatrix} \quad C_{12}$$

Taking Transpose:-

$$\begin{vmatrix} e & b & c \\ d & a & g \\ F & c & K \end{vmatrix} \rightarrow R.H.S$$

تاریخ:

(ii) $$\begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix} = 0$$

Taking transpose:-

$$\begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

$$(-1)\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

$$A = -A$$

$$A + A = 0$$

$$2A = 0$$

$$A = 0/2$$

$$\boxed{A = 0}$$

(iii) $$\begin{vmatrix} a+b & c & 1 \\ b+c & a & 1 \\ c+a & b & 1 \end{vmatrix}$$

تاریخ: ______ دن: ______

$$\begin{bmatrix} a+b+c & c & 1 \\ a+b+c & a & 1 \\ a+b+c & b & 1 \end{bmatrix} \quad C_1+C_2$$

Taking common from C_1

$$(a+b+c)\begin{bmatrix} 1 & c & 1 \\ 1 & a & 1 \\ 1 & b & 1 \end{bmatrix}$$

by property of determinant

$$C_1 \cong C_3$$

So, $(A+b+c)(0)$

$$\boxed{A=0}$$

Imp Note:

Determinant میں اگر کوئی بھی Row یا Coloumn
zero ہو جائے یا same آ جائے تو اس کا
Determinant جو ہے وہ "Zero" کا equal
ہو گا۔

Q:06

(i)
$$\begin{vmatrix} bc & ca & ab \\ 1/a & 1/b & 1/c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} bc & ca & ab \\ \frac{abc}{a} & \frac{abc}{b} & \frac{abc}{c} \\ a^2 & b^2 & c^2 \end{vmatrix} R_2(a$$

$$\begin{vmatrix} bc & ca & ab \\ bc & ca & ab \\ a^2 & b^2 & c^2 \end{vmatrix}$$

By determinant property:

$R_1 \cong R_2$

so,

$A = 0$

(iii) $\begin{vmatrix} a & a^2 & a/bc \\ b & b^2 & b/ca \\ c & c^2 & c/ab \end{vmatrix}$

$$= \frac{1}{abc}\begin{vmatrix} a & a^2 & \frac{a.abc}{bc} \\ b & b^2 & \frac{abc.b}{ca} \\ c & c^2 & \frac{abc.c}{ab} \end{vmatrix} (abc)C_3$$

$$\begin{vmatrix} a & a^2 & a^2 \\ b & b^2 & b^2 \\ c & c^2 & c^2 \end{vmatrix}$$

تاریخ: ______ دن: ______

$$\begin{vmatrix} a & a^2 & a^2 \\ b & b^2 & b^2 \\ c & c^2 & c^2 \end{vmatrix}$$

for following property

$$C_2 \cong C_3$$

$$A = \frac{1}{abc}(0) = 0$$

$$A = 0$$

(iv) $$\begin{vmatrix} \sin^2\theta & 1 & \cos^2\theta \\ \sin^2\phi & 1 & \cos^2\phi \\ \sin^2\phi & 1 & \cos^2\phi \end{vmatrix}$$

$$\begin{vmatrix} \sin^2\theta+\cos^2\theta & 1 & \cos^2\theta \\ \sin^2\phi+\cos^2\phi & 1 & \cos^2\phi \\ \sin^2\psi+\cos^2\psi & 1 & \cos^2\psi \end{vmatrix} \quad C_1 + C_3$$

$$\begin{vmatrix} 1 & 1 & \cos^2\theta \\ 1 & 1 & \cos^2\phi \\ 1 & 1 & \cos^2\psi \end{vmatrix}$$

تاریخ:

$C_1 \cong C_3$

$\Rightarrow |A| = 0$

(ii) $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$

$\begin{vmatrix} a-c & b-c & c-a \\ b-a & c-a & a-b \\ c-b & a-b & b-c \end{vmatrix}$ $C_1 + C_2$

Xing (-1) with C_1

$-1 \begin{vmatrix} c-a & b-c & c-a \\ a-b & c-a & a-b \\ b-c & a-b & b-c \end{vmatrix}$

$C_1 \cong C_3$

$A = 0$

(v) $\begin{vmatrix} \sin^2\alpha & \cos 2\alpha & \cos^2\alpha \\ \sin^2\beta & \cos 2\beta & \cos^2\beta \\ \sin^2\gamma & \cos 2\gamma & \cos^2\gamma \end{vmatrix}$

$$\boxed{\therefore \cos 2\theta = \cos^2\theta - \sin^2\theta}$$

$$\begin{vmatrix} \sin^2\alpha & \cos 2\alpha & \cos^2\alpha - \sin^2\alpha \\ \sin^2\beta & \cos 2\beta & \cos^2\beta - \sin^2\beta \\ \sin^2\gamma & \cos 2\gamma & \cos^2\gamma - \sin^2\gamma \end{vmatrix} \quad C_3 - C$$

$$\begin{vmatrix} \sin^2\alpha & \cos 2\alpha & \cos 2\alpha \\ \sin^2\beta & \cos 2\beta & \cos 2\beta \\ \sin^2\gamma & \cos 2\gamma & \cos 2\gamma \end{vmatrix}$$

$$C_2 \cong C_3$$

$$A = 0$$

Q:07

(i)

Multiply $R_1(a)$
$R_2(b)$
$R_3(c)$

Hint:- C_1 common taking abc common:-

$$\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a \\ 1 & b^2 & b \\ 1 & c^2 & c^3 \end{vmatrix}$$

L.H.S:-

$$\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix}$$

$$\begin{vmatrix} abc & a^2 & a^3 \\ abc & b^2 & b^3 \\ abc & c^2 & c^3 \end{vmatrix} \begin{matrix} R_1(a) \\ R_2(b) \\ R_3(c) \end{matrix}$$

Taking common (abc) from C_1

$$\frac{abc}{abc}\begin{vmatrix}1 & a^2 & a^3\\ 1 & b^2 & b^3\\ 1 & c^2 & c^3\end{vmatrix}$$

So, Prove that
L.H.S = R.H.S

Q: 03

(i)
$$\begin{vmatrix}1 & 0 & 2\\ 3 & 4 & 5\\ 5 & 6 & 7\end{vmatrix}$$

$$1\begin{vmatrix}4 & 5\\ 6 & 7\end{vmatrix} - (0)\begin{vmatrix}3 & 5\\ 5 & 7\end{vmatrix} + 2\begin{vmatrix}3 & 4\\ 5 & 6\end{vmatrix}$$

$1(28-30) - 0(21-25) + 2(18-20)$

$-2 - 4$

$|A| = -6$

(ii)
$$\begin{vmatrix}2 & -1 & 1\\ 3 & 2 & 4\\ -1 & 0 & 3\end{vmatrix}$$

$$2\begin{vmatrix}2 & 4\\ 0 & 3\end{vmatrix} - (-1)\begin{vmatrix}3 & 4\\ -1 & 3\end{vmatrix} + 1\begin{vmatrix}3 & 2\\ -1 & 0\end{vmatrix}$$

$2(6-0)+1(9+4)+1(0+2)$

$12+13+2$

$|A| = 27$

(iii)
$$\begin{vmatrix} 6 & -6 & 6 \\ 2 & 4 & -6 \\ 10 & -5 & 5 \end{vmatrix}$$

$$6\begin{vmatrix} 4 & -6 \\ -5 & 5 \end{vmatrix} - (-6)\begin{vmatrix} 2 & -6 \\ 10 & 5 \end{vmatrix} + 6\begin{vmatrix} 2 & 4 \\ 10 & -5 \end{vmatrix}$$

$6(20-30)+6(10+60)+6(-10-40)$

$6(-10)+6(70)+6(-50)$

$-60+420-300$

$|A| = 60$

Q:06

(vi)
$$A = \begin{vmatrix} \cos\alpha & \sin\alpha & \sin(\alpha+\beta) \\ \cos\beta & \sin\beta & \sin(\beta+\gamma) \\ \cos\gamma & \sin\gamma & \sin(\gamma+\delta \end{vmatrix} = 0$$

$$\begin{vmatrix} \cos\alpha & \sin\alpha & \sin\alpha\cos\beta+\cos\alpha\sin\beta \\ \cos\beta & \sin\beta & \sin\beta\cos\gamma+\cos\beta\sin\gamma \\ \cos\gamma & \sin\gamma & \sin\gamma\cos\delta+\cos\gamma\sin\gamma \end{vmatrix}$$

تاریخ: دن:

$$\begin{vmatrix} \cos\alpha & \sin\alpha & \sin\alpha\cos\delta \\ \cos\beta & \sin\beta & \sin\beta\cos\delta \\ \cos\gamma & \sin\gamma & \sin\gamma\cos\delta \end{vmatrix} + \begin{vmatrix} \cos\alpha & \sin\alpha & \cos\alpha\sin\delta \\ \cos\beta & \sin\beta & \cos\beta\sin\delta \\ \cos\gamma & \sin\gamma & \cos\gamma\sin\delta \end{vmatrix}$$

$$= \cos\delta \begin{vmatrix} \cos\alpha & \sin\alpha & \sin\alpha \\ \cos\beta & \sin\beta & \sin\beta \\ \cos\gamma & \sin\gamma & \sin\gamma \end{vmatrix} + \sin\delta \begin{vmatrix} \cos\alpha & \sin\alpha & \cos\alpha \\ \cos\beta & \sin\beta & \cos\beta \\ \cos\gamma & \sin\gamma & \cos\gamma \end{vmatrix}$$

$A = \cos(0) + \sin(0)$

$A = 0$

$$\boxed{A = 0}$$

(vii) $\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos(\alpha+\beta) \\ \cos\beta & \cos(\alpha+\beta) & 1 \end{vmatrix} = 0$

$C_2 - \cos\alpha C_1$

$C_3 - \cos\beta C_1$

$$\begin{vmatrix} 1 & 0 & 0 \\ \cos\alpha & 1-\cos^2\alpha & \cos(\alpha+\beta)-\cos\alpha\cos\beta \\ \cos\beta & \cos(\alpha+\beta)-\cos\alpha\cos\beta & 1-\cos^2\beta \end{vmatrix}$$

$\therefore \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\therefore 1-\cos^2\alpha = \sin^2\alpha$

تاریخ:

$$= \begin{vmatrix} 1 & 0 & 0 \\ \cos\alpha & \sin^2\alpha & \cancel{\cos\alpha}\cos\beta - \sin\alpha\sin\beta - \\ \cos\beta & \cancel{\cos\alpha}\cos\beta - \sin\alpha\sin\beta - \frac{\cancel{\cos\alpha}}{\cos\beta} & \sin^2\beta \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ \cos\alpha & \sin^2\alpha & -\sin\alpha\sin\beta \\ \cos\beta & -\sin\alpha\sin\beta & \sin^2\beta \end{vmatrix}$$

taking common "$\sin\alpha$" from C_2

Taking common "$\sin\beta$" from C

$$\begin{array}{c} \sin\alpha \\ -\sin\beta \end{array} \begin{vmatrix} 1 & 0 & 0 \\ \cos\alpha & \sin\alpha & \sin\alpha \\ \cos\beta & -\sin\beta & -\sin\beta \end{vmatrix}$$

$C_2 \cong C_3$

$-\sin\alpha \cdot \sin\beta \cdot (0)$

$$\boxed{\Delta = 0}$$

(Viii)

$$\begin{vmatrix} (a+b)^2 & a^2+b^2 & ab \\ (c+d)^2 & c^2+d^2 & cd \\ (g+h)^2 & g^2+h^2 & gh \end{vmatrix} = 0$$

تاریخ: دن:

$$\begin{vmatrix} a^2+b^2+2ab & a^2+b^2 & ab \\ c^2+d^2+2cd & c^2+d^2 & cd \\ g^2+h^2+2gh & g^2+h^2 & gh \end{vmatrix}$$

$$\begin{vmatrix} \cancel{a^2}+\cancel{b^2}+2ab-\cancel{a^2}-\cancel{b^2} & a^2+b^2 & ab \\ \cancel{c^2}+\cancel{d^2}+2cd-\cancel{c^2}-\cancel{d^2} & c^2+d^2 & cd \\ \cancel{g^2}+\cancel{h^2}+2gh-\cancel{g^2}-\cancel{h^2} & g^2+h^2 & gh \end{vmatrix} \quad C_1-C_2$$

$$\Delta = \begin{vmatrix} 2ab & a^2+b^2 & ab \\ 2cd & c^2+d^2 & cd \\ 2gh & g^2+h^2 & gh \end{vmatrix}$$

taking common from C_1

$$2\begin{vmatrix} ab & a^2+b^2 & ab \\ cd & c^2+d^2 & cd \\ gh & g^2+h^2 & gh \end{vmatrix}$$

$C_1 \cong C_3$

$2(0)$

$$\boxed{\Delta = 0}$$

(ix)
$$\begin{vmatrix} (a^m+a^{-m})^2 & (a^m-a^{-m})^2 & abc \\ (b^n+b^{-n})^2 & (b^n-b^{-n})^2 & abc \\ (c^p+c^{-p})^2 & (c^p-c^{-p})^2 & abc \end{vmatrix} = 0$$

$$\Delta = \begin{vmatrix} a^{2m}+a^{-2m}+2 & a^{2m}+a^{-2m}-2 & abc \\ b^{2n}+b^{-2n}+2 & b^{2n}+b^{-2n}-2 & abc \\ c^{2p}+c^{-2p}+2 & c^{2p}+c^{-2p}-2 & abc \end{vmatrix}$$

Talking common from C_3 "abc"

$$\begin{vmatrix} a^{2m}+a^{-2m}+2 & a^{2m}+a^{-2m}-2 & 1 \\ b^{2n}+b^{-2n}+2 & b^{2n}+b^{-2n}-2 & 1 \\ c^{2p}+c^{-2p}+2 & c^{2p}+c^{-2p}-2 & 1 \end{vmatrix}$$

$$\begin{vmatrix} a^{2m}+a^{-2m}+\cancel{2}-\cancel{2} & a^{2m}+a^{-2m}-\cancel{2}+\cancel{2} & 1 \\ b^{2m}+b^{-2m}+\cancel{2}-\cancel{2} & b^{2n}+b^{-2n}-\cancel{2}+\cancel{2} & 1 \\ c^{2p}+c^{-2p}+\cancel{2}-\cancel{2} & c^{2p}+c^{-2p}-\cancel{2}+\cancel{2} & 1 \end{vmatrix} \quad \begin{matrix} C_1-2 \\ C_2+2 \end{matrix}$$

$$abc \begin{vmatrix} a^{2m}+a^{-2m} & a^{2m}+a^{-2m} & 1 \\ b^{2n}+b^{-2n} & b^{2n}+b^{-2n} & 1 \\ c^{2p}+c^{-2p} & c^{2p}+c^{-2p} & 1 \end{vmatrix}$$

$C_1 \cong C_2$

$abc(0)$

$$\boxed{\Delta = 0}$$

15

$$\Delta = \begin{vmatrix} a^{2m}+a^{-2m}+2 & a^{2m}+a^{-2m}-2 & abc \\ b^{2n}+b^{-2n}+2 & b^{2n}+b^{-2n}-2 & abc \\ c^{2p}+c^{-2p}+2 & c^{2p}+c^{-2p}-2 & abc \end{vmatrix}$$

Talking common from C_3 "abc"

$$\begin{vmatrix} a^{2m}+a^{-2m}+2 & a^{2m}+a^{-2m}-2 & 1 \\ b^{2n}+b^{-2n}+2 & b^{2n}+b^{-2n}-2 & 1 \\ c^{2p}+c^{-2p}+2 & c^{2p}+c^{-2p}-2 & 1 \end{vmatrix}$$

$$\begin{vmatrix} a^{2m}+a^{-2m}+\cancel{2}-\cancel{2} & a^{2m}+a^{-2m}-\cancel{2}+\cancel{2} & 1 \\ b^{2n}+b^{-2n}+\cancel{2}-\cancel{2} & b^{2n}+b^{-2n}-\cancel{2}+\cancel{2} & 1 \\ c^{2p}+c^{-2p}+\cancel{2}-\cancel{2} & c^{2p}+c^{-2p}-\cancel{2}+\cancel{2} & 1 \end{vmatrix} \quad \begin{matrix} C_1-2C_3 \\ C_2+2C_3 \end{matrix}$$

$$abc\begin{vmatrix} a^{2m}+a^{-2m} & a^{2m}+a^{-2m} & 1 \\ b^{2n}+b^{-2n} & b^{2n}+b^{-2n} & 1 \\ c^{2p}+c^{-2p} & c^{2p}+c^{-2p} & 1 \end{vmatrix}$$

$C_1 \cong C_2$

$abc(0)$

$$\boxed{\Delta = 0}$$

تاریخ:　　　　　　　　　　دن:

(x) $\begin{vmatrix} \frac{1}{2!} & 1 & 0 \\ \frac{1}{3!} & \frac{1}{2!} & 1 \\ \frac{1}{4!} & \frac{1}{3!} & \frac{1}{2!} \end{vmatrix} = 0$

$$\begin{vmatrix} \frac{1}{2\cdot 1} & 1 & 0 \\ \frac{1}{2\cdot 3\cdot 1!} & \frac{1}{2\cdot 1!} & 1 \\ \frac{1}{4!3!2!1} & \frac{1}{3\cdot 2\cdot 1!} & \frac{1}{2\cdot 1!} \end{vmatrix}$$

$$\begin{vmatrix} \frac{1}{2!} & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & 1 \\ \frac{1}{24} & \frac{1}{6} & \frac{1}{2} \end{vmatrix}$$

$$\frac{1}{2\cdot 6\cdot 24} \begin{array}{c} \begin{vmatrix} 1 & 2 & 0 \\ 1 & 3 & 6 \\ 1 & 4 & 12 \end{vmatrix} \end{array} \begin{array}{l} 2R_1 \\ 6R_2 \\ 24R_3 \end{array}$$

$$\frac{\cancel{6}}{2 \cdot \cancel{6} \cdot 24}\begin{vmatrix} 1 & 2 & 0 \\ 1 & 3 & 1 \\ 1 & 4 & 2 \end{vmatrix}$$ Taking "6" common from C_3

$$\frac{1}{2 \cdot 24}\begin{vmatrix} 1 & 2 & 0 \\ 1 & 3 & 1 \\ 1 & 4 & 2 \end{vmatrix}$$

$$\frac{1}{48}\begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 2 \end{vmatrix}$$ $C_2 - 2C_1$

By Property
$C_2 \cong C_3$

So,

$$\frac{1}{48}(0)$$

$$\boxed{\Delta = 0}$$

(xii)

$$\begin{vmatrix} a & b & c & d & 1 \\ b & c & d & a & 1 \\ c & d & a & b & 1 \\ d & a & b & c & 1 \\ b & a & d & c & 1 \end{vmatrix}$$

$$\begin{vmatrix} a+b+c+d & b & c & d & 1 \\ a+b+c+d & c & d & a & 1 \\ a+b+c+d & d & a & b & 1 \\ a+b+c+d & a & b & c & 1 \\ a+b+c+d & a & d & c & 1 \end{vmatrix} \quad (C_1+C_2+C_3+C_4+C_5$$

Taking common from C_1

$$(a+b+c+d)\begin{vmatrix} 1 & b & c & d & 1 \\ 1 & c & d & a & 1 \\ 1 & d & a & b & 1 \\ 1 & a & b & c & 1 \\ 1 & a & d & c & 1 \end{vmatrix}$$

By Property

$C_1 \cong C_5$

So,

$(a+b+c+d)(0)$

$$\boxed{\Delta = 0}$$

14(1) same

(30) (ii)

$$\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{c^2+a^2}{b} \end{vmatrix}$$

$$\frac{1}{abc}\begin{vmatrix} a^2+b^2 & c^2 & c^2 \\ a^2 & b^2+c^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix} \begin{matrix} R_1(c) \\ R_2(a) \\ R_3(b) \end{matrix}$$

$$\frac{1}{abc}\begin{vmatrix} a^2+b^2-c^2 & 0 & c^2 \\ 0 & b^2+c^2-a^2 & a^2 \\ b^2-c^2-a^2 & b^2-c^2-a^2 & c^2+a^2 \end{vmatrix} \begin{matrix} C_2-C_3 \\ C_1-C_3 \end{matrix}$$

$$\frac{1}{abc}\begin{vmatrix} a^2+b^2-c^2 & 0 & c^2 \\ 0 & b^2+c^2-a^2 & a^2 \\ -2a^2 & -2c^2 & 0 \end{vmatrix} R_3-(R_1+R_2)$$

$$\frac{1}{abc}\left[a^2+b^2-c^2(0+2a^2c^2)-0+c^2(0+2c \atop b^2+c^2-a^2)\right.$$

$$\frac{1}{abc}\left\{(0+2a^4c^2+0+2a^2b^2c^2-0-2a^2c^4)+c^2(2a^2b^2 \atop -2a^4)\right.$$

12, 13, 14(a), 24

$$\frac{1}{abc}\left[\cancel{2a^4c^2}+2a^2b^2c^2-\cancel{2a^2c^4}+2a^2b^2c^2+\cancel{2a^2c^4}-\cancel{2c^2a^4}\right]$$

$$\frac{1}{abc}\left[4a^2b^2c^2\right]$$

$$\boxed{\Delta = 4abc}$$

Q: 11 v.Imp (6-Times repeated)

$$\begin{vmatrix} x & a & a & a \\ a & x & a & a \\ a & a & x & a \\ a & a & a & x \end{vmatrix} = (x-a)^3(x+3a)$$

$$\begin{vmatrix} x+3a & x+3a & x+3a & x+3a \\ a & x & a & a \\ a & a & x & a \\ a & a & a & x \end{vmatrix} \quad R_1+(R_2+R_3+R_4)$$

$$(x+3a)\begin{vmatrix} 1 & 1 & 1 & 1 \\ a & x & a & a \\ a & a & x & a \\ a & a & a & x \end{vmatrix}$$

$$(x+3a)\begin{vmatrix} 1 & 0 & 0 & 0 \\ a & x-a & 0 & 0 \\ a & 0 & x-a & 0 \\ a & 0 & 0 & x-a \end{vmatrix}$$

$\because$ determinent of upper / lower matrix is product of diagonal of this Matrix.

$$(x+3a)(1)(x-a)(x-a)(x-a)$$

$$= (x+3a)(x-a)^3$$

Hence Proved;

L.H.S = R.H.S