Lec 01:

Matrix:

A rectangular array of numbers into rows and coulmn is called matrix. It is usually denoted by a captial letter A, B, C

Order of Matrix:

No of rows × No of coulmn

Abbreviation.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

$$= \left[a_{ij} \right]_{m \times n}, \ i = 1, 2 \ldots m, \ j = 1, 2 \ldots n$$

Types of Matrix:

i) Rectangular Matrix

A matrix in which no of row is not equal to no of coulomns.

$$A = \begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 4 & 5 \end{bmatrix}$$

Square Matrix:

A matrix in whic no. of rows is equal to the number of coulumn $m = n$

Example: $A = \begin{bmatrix} 7 & 3 \\ 5 & 8 \end{bmatrix}$

Zero OR Null Matrix:

If all entries in a matrix are zero Denoted by $O_{m \times n}$

Example: $B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Diagonal Matrix:

A square matrix in whic all element of matrix except the main or principle diagonal are zero and atleast one element of diagonal is non-zero called diagonal matrix.

Example:

$D_1 = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $D_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

Scalar Matrix:

The diagonal matrix in which all elements of principle diagonal is equal to non-zero constant number is called scalar Matrix

Example: $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & & 2 \end{bmatrix}$

Identity Matrix:

The scalar matrix in which all element of diagonal equal to 1

Example: $I_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $I_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Addition of Matrices:

Two matrices can be added if they have same number of orders

Example: $A + B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

$= \begin{bmatrix} 1+5 & 2+6 \\ 3+7 & 4+8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$

Subtraction of Matrices:

Two matrices can be subtracted if they have same number of orders

Example: $A - B = \begin{bmatrix} 9 & 8 \\ 3 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 8 & 7 \\ 1 & 2 \end{bmatrix}$

3+4=5+6+7 /Shaked Azhas

Lec 02:

Upper Triangular Matrix:

A square matrix $A = [a_{ij}]$ of order n is said to be upper triangular matrix if all element below the main diagonal are zero if $\forall\ i > j$

Example:
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$$

Lower Triangular matrix:

A square matrix $A = [a_{ij}]$ of order n is said to be lower triangular if all element above the main diagonal are zero i.e $a_{ij} = 0\ \forall\ i < j$

Example:
$$A = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Transpose of Matrix:

Let $A = [a_{ij}]$ be $m \times n$ matrix over the field F. The transpose of A denoted by A^t is a $n \times m$ matrix by interchanging rows into coulomns.

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $A^t = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

Symmetric Matrix

A square matrix $A = [a_{ij}]$ is said to be symmetric if $A = A^t$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = A' = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Skew Symmetric Matrix:

A square matrix $A = [a_{ij}]$ is said to be skew symmetric if $A = -A^t$

Conjugate Matrix:

Let A be a matrix with complex entries, then the matrix $A = [a_{ij}]$ obtained by replacing each a_{ij} by its conjugate

$$A = \begin{bmatrix} 1+i & \sqrt{3}+i \\ i & 2 \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 1-i & \sqrt{3}-i \\ -i & 2 \end{bmatrix}$$

Hermitian Matrix:

A square matrix $A = [a_{ij}]$ over field of complex number is called Hermition matrix if $(\bar{A})^t = A$

Example: $A = \begin{bmatrix} 1 & 1-i \\ 1+i & 3 \end{bmatrix}$

Skew Hermitian:

A square matrix $A = [a_{ij}]$ over the field of complex number is called skew Hermitian if $(\bar{A})^t = -A$

Example:

Scalar multiplication:

Let $A = [a_{ij}]$ be a matrix of order $m \times n$ and $K \in F$ then scalar multiplication of K with A is denoted by KA and is obtained by multiplying each element by K

$$A = \begin{bmatrix} 3 & -4 \\ 2 & 9 \end{bmatrix}$$

$$-3A = -3 \begin{bmatrix} 3 & -4 \\ 2 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & 12 \\ -6 & -27 \end{bmatrix}$$

Multiplication of Two matrices

Two matrices A and B are said to be confirmable for product AB if no of column of A is equal to no of rows of B

$$A_{m \times n} \; B_{n \times p} = (AB)_{m \times p}$$

$$A = \begin{bmatrix} 2 & -1 & 5 \\ 3 & 4 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$$

AB is possible but BA is not possible

$$AB = \begin{bmatrix} (2)(3) + (-1)(1) + (5)(-2) \\ (3)(3) + (4)(1) + (0)(-2) \end{bmatrix}$$

$$AB = \begin{bmatrix} -5 \\ 13 \end{bmatrix}$$

Q. Find matrix A if

$$\begin{bmatrix} 2 & -3 \\ 0 & 0 \\ 1 & 4 \end{bmatrix} A = \begin{bmatrix} 4 & 1 \\ 0 & 0 \\ 3 & 2 \end{bmatrix}$$

Inverse method is used only for square matrix

$$A_{m \times n} \times B_{n \times p} = (AP)_{m \times p}$$

$m = 3, n = 2, p = 2$

$$\begin{bmatrix} 2 & -3 \\ 0 & 0 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 0 & 0 \\ 3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2a-3c & 2b-3d \\ 0+0 & 0+0 \\ a+4c & b+4d \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 0 & 0 \\ 3 & 2 \end{bmatrix}$$

$2a-3c = 4$ —— (1)

$a+4c = 3$ —— (2)

$2b-3d = 1$ —— (3)

$b+4d = 2$ —— (4)

Multiply eq (2) by -2 and add in eq (1)

$\cancel{-2a} - 8c = -6$

$\cancel{2a} - 3c = 4$

$\cancel{-}11c = \cancel{-}2 \Rightarrow \boxed{c = \frac{2}{11}}$

Put $c = \frac{2}{11}$ in eq (2)

$a + 4\left(\frac{2}{11}\right) = 3 \Rightarrow \boxed{a = \frac{25}{11}}$

Multiply eq (4) by -2 & add to eq (3)

$\cancel{-2b} - 8d = -4$

$\cancel{2b} - 3d = 1$

$\cancel{-}11d = \cancel{-}3 \Rightarrow \boxed{d = \frac{3}{11}}$

Put $d = \frac{3}{11}$ eq (4)

$b + 4\left(\frac{3}{11}\right) = 2 \Rightarrow \boxed{b = \frac{10}{11}}$

$$A = \begin{bmatrix} \frac{25}{11} & \frac{10}{11} \\ \frac{2}{11} & \frac{3}{11} \end{bmatrix}$$

Determinent of Square Matrix.

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Minor of an element of square matrix of order $n \geq 3$

Let $A = [a_{ij}]$ be a square matrix of order $n \geq 3$ Then minor of an element is denoted by M_{ij} and is given by the determinent of the $(n-1) \times (n-1)$ matrix obtained determinent th row

Example: $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 2 \\ 4 & 3 & -2 \end{bmatrix}$

Minor of $a_{22} = M_{22}$

$= \begin{vmatrix} 1 & 3 \\ 4 & -2 \end{vmatrix} = 1(-2) - 3(4) = -1$

Co-factor of an element of a square matrix of order $n \geq 3$

Let $A = [a_{ij}]$ be a square matrix of order $n \geq 3$, then co-factor of a_{ij} is denoted by A_{ij}

and is given by

$$A_{ij} = (-1)^{i+j} M_{ij}$$

°Determinant of Square matrix of order n

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots & a_{1j} \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} \cdots & a_{2j} \cdots & a_{2n} \\ a_{i1} & a_{i2} & a_{i3} \cdots & a_{ij} \cdots & a_{in} \\ a_{n1} & a_{n2} & a_{n3} \cdots & a_{nj} \cdots & a_{n4} \end{bmatrix}$

be a square matrix of order n.

Determinant of A is denoted by $|A|$ and is giving by adding the product of all element of a row (or coulmn with its cofactors

$$|A| = a_{i1}A_{i1} + a_{i2}A_{i2} + \cdots a_{ij}A_{ij} \cdots + a_{in}A_{in}$$

$a_{ij}\ A_{ij}$

Properties of Determinent:

1) $|A| = |A^t|$

2) If we interchange two rows or coulmn then for resulting matrix

$$|B| = -|A|$$

3) If any two rows or coulmn of a square matrix are identical then value of determinent is zero.

4) If we multiply each element of row or column by a non-zero scalar K. Then value of determinent ~~is K times the original~~ resulting matrix B is

$$|B| = K|A|$$

5) If a row (or column) consist of two terms, then its determinent is equal to the sum of two determinent having one term in its respective row or column

6) If all the element of a row or column is zero, then value of determinent is zero.

7) If we multiply any row or coulmn of a square matrix with some

scalar K and add resulting value to the corresponding element of any other row, then the value of determinant remain same

8) If a square matrix A is upper or lower triangular matrices, then value of determinant is equal to the product of all the element of main diagonal.

Q.
$$\begin{vmatrix} 1 & 3 & 2 \\ 2 & 1 & 0 \\ 1 & 0 & 3 \end{vmatrix} = ?$$

$$1\begin{vmatrix} 1 & 0 \\ 0 & 3 \end{vmatrix} - 2\begin{vmatrix} 3 & 2 \\ 0 & 3 \end{vmatrix} + 1\begin{vmatrix} 3 & 2 \\ 1 & 0 \end{vmatrix}$$

$1(3-0) - 2(9-0) + 1(0-2)$

$3 - 18 - 2$

$-15 - 2$

$= -17$

Q: Without expansion show that

$$\begin{vmatrix} a-2l & b-2m & c-2n \\ l & m & n \\ a & b & c \end{vmatrix} = 0$$

Sol
$$= \begin{vmatrix} a & b & c \\ l & m & n \\ a & b & c \end{vmatrix} + \begin{vmatrix} -2l & -2m & -2n \\ l & m & n \\ a & b & c \end{vmatrix}$$

R_1 & R_3 are identical

Taking (-2) common R_1

$$0 + (-2)\begin{vmatrix} l & m & n \\ l & m & n \\ a & b & c \end{vmatrix}$$

As R_1 & R_2 same

$0 + 0 = 0 = R.H.S$

New topic

$2x - 3y = -1$

$x + y = 1$

Elementry Row Operation

1) Interchange ith and jth row R_{ij}
2) Multiplying ith row with K. KR_i
3) Add Kth multiple of ith row with the corresponding element of jth row $R_i + KR_j$

Elementry coulmn operation
i) C_{ij}
ii) KC_i
iii) $C_i + KC_j$

Echelon Form of a Matrix

1) Make every element of Row 1
2) Make zero to every element under leading coefficient.

Reduced Echelon

1) Echelon form
2) Make every element zero under and upper side of leading coefficient.

Rank of Matrix

No. of non-zero rows of a matrix when it is reduced to echelon form
or
No. of independent rows of matrix is called rank of Matrix.

Q $\begin{bmatrix} 2 & 5 & 7 \\ 1 & 2 & -1 \\ -3 & -6 & 3 \end{bmatrix}$

$= R_{12} \begin{bmatrix} 1 & 2 & -1 \\ 2 & 5 & 7 \\ -3 & -6 & 3 \end{bmatrix}$

$\begin{array}{l} R_2 - 2R_1 \\ R_3 + 3R_1 \end{array} \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 9 \\ 0 & 0 & 0 \end{bmatrix}$

So its rank is 2

Question:

$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 4 & 5 & 6 & 7 \end{bmatrix}$

$\begin{array}{l} R_2 - 2R_1 \\ R_3 - 4R_1 \end{array} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -3 \\ 0 & -3 & -6 & -9 \end{bmatrix}$

$-R_2 \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & -3 & -6 & -9 \end{bmatrix}$

$R_3 + 3R_2 \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Rank = 2

Q. $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & 7 \\ 3 & 5 & 8 \\ -2 & -1 & -4 \end{bmatrix} \begin{matrix} R_2-2R_1 \\ = R_3-3R_1 \\ R_4+2R_1 \end{matrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 13 \\ 0 & -1 & -17 \\ 0 & 3 & -10 \end{bmatrix}$

$\begin{matrix} = \\ R_{23} \end{matrix} \begin{bmatrix} 1 & 2 & -3 \\ 0 & -1 & 17 \\ 0 & 0 & 13 \\ 0 & 3 & -10 \end{bmatrix} \Rightarrow -R_2 \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & -17 \\ 0 & 0 & 13 \\ 0 & 3 & -10 \end{bmatrix}$

$R_4-3R_2 \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & -17 \\ 0 & 0 & 13 \\ 0 & 0 & 41 \end{bmatrix} \Rightarrow \frac{1}{13}R_3 \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & -17 \\ 0 & 0 & 1 \\ 0 & 0 & 41 \end{bmatrix}$

$R_4-41R_3 \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -17 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ Rank = 3

New topic

Consistant:

If a system of linear eq^n. has a solution. Then it is called ---

in consistent:

If a system of linear eq^n. has no solution, it is called ..

Homogenous linear eq^n:

~~If~~ Consider for a linear equation $ax+by+cz=0$ if $d=0$, then it is called homogenous linear equation

Non-homogenous Linear equation:

if $d \neq 0$ then it is called ---

Consider m linear eq^n in n variable

$a_{11}x_1 + a_{12}x_2 + \dots\dots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \dots\dots + a_{2n}x_n = b_2$

$\vdots$

$a_{m1}x_1 + a_{m2}x_2 + \dots\dots + a_{mn}x_n = b_m$

In matrix form

$$\begin{bmatrix} a_{11} & a_{12} \dots & a_{1n} \\ a_{21} & a_{22} \dots & a_{2n} \\ \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{mn} \end{bmatrix}_{m\times n} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}_{m\times 1}$$

First Theorem:

Let $AX=B(\neq 0)$ be a non-homogenous system of m linear eq^n in n unknown, then system has a solution iff

Rank A = rank AB

where AB is augmented matrix

Second Theorem:

Let $AX=B$ be a non-homogenous system of n linear equation in n variable then

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System has a solution iff

Rank A = Rank AB = No. of variable = No. of equation

(i.e then $|A| \neq 0$, then unique solution

($X = A^{-1}B$) in unique solution

Third Theorem:

Let $AX = 0$ be homogenous system of Linear equation, then system has a non-trivial solution iff

rank A < No. of variable

In other words, $|A| = 0$ then system has non-trivial solution.

Question of Crammer Rule

$$x_1 + 2x_2 + 3x_3 = 6$$
$$2x_1 - x_2 + x_3 = 2$$
$$3x_1 + x_2 + x_3 = 5$$

In matrix form

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 5 \end{bmatrix}$$

$\cancel{AX = B}$

~~By inverse method.~~

By Crammer Rule:

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 1 & 1 \end{vmatrix} = 1(-1-1) - 2(2-3) + 3(\quad = 15$$

$$|A_1| = \begin{vmatrix} 6 & 2 & 3 \\ 2 & -1 & 1 \\ 5 & 1 & 1 \end{vmatrix} = 6(-1-1) - 2(2-5) + 3(2+5) = 15$$

$|A_2| = 15$ $\qquad$ $|A_3| = 15$

$$\begin{vmatrix} 1 & 6 & 3 \\ 2 & 2 & 1 \\ 3 & 5 & 1 \end{vmatrix} = 15 \qquad \begin{vmatrix} 1 & 2 & 6 \\ 2 & -1 & 2 \\ 3 & 1 & 5 \end{vmatrix} = 15$$

$$x_1 = \frac{|A_1|}{|A|} = \frac{15}{15} = 1 \qquad x_2 = \frac{|A_2|}{|A|} = \frac{15}{15} = 1$$

$$x_3 = \frac{|A_3|}{|A|} = \frac{15}{15} = 1$$

Matrix inversion method

b) ~~By Crammer Rule~~

$$\text{Adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \qquad AX = B, \quad X = A^{-1}B$$

where $A_{ij} = (-1)^{i+j} M_{ij}$

$$= \begin{bmatrix} M_{11} & -M_{12} & M_{13} \\ -M_{21} & M_{22} & -M_{23} \\ M_{31} & -M_{32} & M_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} \\ -\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} \end{bmatrix}$$

$$= A^t = \begin{bmatrix} -2 & +1 & 5 \\ +1 & -8 & 5 \\ 5 & 5 & -5 \end{bmatrix}^t$$

$$Adj\,A = \begin{bmatrix} -2 & 1 & 5 \\ 1 & -8 & 5 \\ 5 & 5 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} Adj\ A$$

$$= \frac{1}{15}\begin{bmatrix} -2 & 1 & 5 \\ 1 & -8 & 5 \\ 5 & 5 & -5 \end{bmatrix} = \begin{bmatrix} -2/15 & 1/15 & 5/15 \\ 1/15 & -8/15 & 5/15 \\ 5/15 & 5/15 & -5/15 \end{bmatrix}$$

$$X = A^{-1}B = \begin{bmatrix} -2/15 & 1/15 & 1/3 \\ 1/15 & -8/15 & 1/3 \\ 1/3 & 1/3 & -1/3 \end{bmatrix} \begin{bmatrix} 6 \\ 2 \\ 5 \end{bmatrix}$$

After multiplying

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

c) By using ~~Jordan~~ Gauss Method (Echlon form)

Take augmented matrix

$$A_B = \begin{bmatrix} 1 & 2 & 3 & 6 \\ 2 & -1 & 1 & 2 \\ 3 & 1 & 1 & 5 \end{bmatrix}$$

$$\begin{matrix} R_2 - 2R_1 \\ = R_3 - 3R_1 \end{matrix} \begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & -5 & -5 & -10 \\ 0 & -5 & -8 & -13 \end{bmatrix} = -\frac{1}{5}R_2 \begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & 1 & 1 & 2 \\ 0 & -5 & -8 & -13 \end{bmatrix}$$

$$R_3 + 5R_2 \begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & -3 & -3 \end{bmatrix} = -\frac{1}{3} \begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$x_1 + x_2 + 3x_3 = 6$ --- (4)

$x_2 + x_3 = 2$ --- (5)

$x_3 = 1$ --- (6)

Put eq '6' in (5)

$x_2 + 1 = 2 \Rightarrow \boxed{x_2 = 1}$

Put $x_3 = 1$ & $x_2 = 1$ in eq (4)

$x_1 + 2(1) + 3(1) = 6$

$x_1 = 1$

d) By using Guass Jorden Method (Reduced Echelon form)

CamScanner

$$= \begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \Rightarrow \cancel{R_1 - 2R_2}$$

$$R_1 - 2R_2 \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \Rightarrow \begin{array}{c} R_1 - R_3 \\ R_2 - R_3 \end{array} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{array}{l} x_1 = 1 \\ x_2 = 1 \\ x_3 = 1 \end{array} \quad \text{ANS}$$

CH 4

Question: Ex $\cancel{3.4}$ 4.1 (M.M)

Q3 $x_1 - x_2 + x_3 - x_4 + x_5 = 1$

$2x_1 + x_2 + 3x_3 + 4x_5 = 3$

$3x_1 - 2x_2 + 2x_3 + x_4 + x_5 = 1$

$x_2 + x_4 + x_5 = 0$

Write in matrix form

$$\begin{bmatrix} 1 & -1 & 1 & -1 & 1 \\ 2 & 1 & 3 & 0 & 4 \\ 3 & -2 & 2 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 0 \end{bmatrix}$$

Take augemented matrix

$$A_B = \begin{bmatrix} 1 & -1 & 1 & -1 & 1 \\ 2 & 1 & 3 & 0 & 3 \\ 3 & -2 & 2 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

We reduced it to reduced Echelon form By row operation

$$\begin{array}{c} R_2 - 2R_1 \\ R_3 - 3R_1 \end{array} \begin{bmatrix} 1 & -1 & 1 & -1 & 1 & 1 \\ 0 & 3 & 1 & 2 & 2 & 1 \\ 0 & 1 & -1 & 4 & -2 & -2 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$$R_{24} = \begin{bmatrix} 1 & -1 & 1 & -1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 4 & -2 & -2 \\ 0 & 3 & 1 & 2 & 2 & 1 \end{bmatrix}$$

$$\begin{array}{c} R_1 + R_2 \\ R_3 - R_2 \\ R_4 - 3R_2 \end{array} = \begin{bmatrix} 1 & 0 & 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 3 & -3 & -2 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix}$$

$$R_{34} = \begin{bmatrix} 1 & 0 & 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 3 & -3 & -2 \end{bmatrix}$$

$$\begin{array}{c} R_1 - R_3 \\ R_4 + R_3 \end{array} = \begin{bmatrix} 1 & 0 & 0 & 1 & 3 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 2 & -4 & -1 \end{bmatrix}$$

$$\frac{1}{2} R_4 = \begin{bmatrix} 1 & 0 & 0 & 1 & 3 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 1 & -2 & -\frac{1}{2} \end{bmatrix}$$

$$\text{Rank } A = \text{rank } A_B = 4$$

$$x_1 + x_4 + 3x_5 = 0$$
$$x_2 + x_4 + x_5 = 0$$
$$x_3 - x_4 - x_5 = 1$$
$$x_4 - 2x_5 = -\frac{1}{2}$$

$$\begin{array}{l} R_1 - R_4 \\ R_2 - R_4 = \\ R_3 + R_4 \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 & 5 & 1/2 \\ 0 & 1 & 0 & 0 & 3 & 1/2 \\ 0 & 0 & 1 & 0 & -3 & 1/2 \\ 0 & 0 & 0 & 1 & -2 & 1/2 \end{bmatrix}$$

$$\text{rank } A = \text{rank } A_B = 4$$

$$x_1 + 5x_5 = \frac{1}{2} \Rightarrow x_1 = 1/2 - 5x_5$$
$$x_2 + 3x_5 = 1/2 \Rightarrow x_2 = \frac{1}{2} - 3x_5$$
$$x_3 - 3x_5 = 1/2 \Rightarrow x_3 = \frac{1}{2} + 3x_5$$
$$x_4 - 2x_5 = -1/2 \Rightarrow x_4 = -\frac{1}{2} + 2x_5$$

For any arbitrary value of x_5 we can get corresponding value of x_1, x_2, x_3 & x_4

Let $x_5 = 0$	Let $x_5 = t$
$x_1 = \frac{1}{2}$	$x_1 = \frac{1}{2} - 5t$
$x_2 = \frac{1}{2}$	$x_2 = \frac{1}{2} - 3t$
$x_3 = \frac{1}{2}$	$x_3 = \frac{1}{2} + 3t$
$x_4 = -\frac{1}{2}$	$x_4 = -\frac{1}{2} + 2t$

Question 4

$$x_1 + x_2 - x_3 = 1$$
$$x_2 + x_3 - x_4 = 1$$
$$x_3 + x_4 - x_5 = 1$$
$$-x_3 + x_4 + x_5 = 1$$
$$-x_2 + x_3 + x_4 = 1$$

In matrix form

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & -1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Take augemeted Matrix, we get

$$A_B = \begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 & -1 & 1 \\ 0 & 0 & -1 & 1 & 1 & 1 \\ 0 & -1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$R_2 - R_1 = \begin{bmatrix} 2 & -1 & 3 & a \\ 1 & 2 & -8 & b-a \\ -5 & -5 & 21 & c \end{bmatrix} \Rightarrow R_{12} \begin{bmatrix} 1 & 2 & -8 & b-a \\ 2 & -1 & 3 & a \\ -5 & -5 & 21 & c \end{bmatrix}$$

$$\begin{array}{l} R_2 - 2R_1 \\ R_3 + 5R_1 \end{array} \begin{bmatrix} 1 & 2 & -8 & b-a \\ 0 & -5 & 19 & a-2b+2a \\ 0 & 5 & -19 & c+5b-5a \end{bmatrix}$$

$$-\frac{1}{5}R_2 = \begin{bmatrix} 1 & 2 & -8 & b-a \\ 0 & 1 & -\frac{19}{5} & \frac{3a-2b}{-5} \\ 0 & 5 & -19 & c+5b-5a \end{bmatrix}$$

$$\begin{array}{l} R_1 - 2R_2 = \\ R_3 - 5R_2 \end{array} \begin{bmatrix} 1 & 0 & -\frac{2}{5} & b-a+\frac{6a-4b}{5} \\ 0 & 1 & -\frac{19}{5} & \boxed{\frac{+2b-3a}{5}} \\ 0 & 0 & 0 & c+5b-5a+3a-2b \end{bmatrix}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -\frac{2}{5} & \frac{a+b}{5} \\ 0 & 1 & -\frac{19}{5} & \frac{2b-3a}{5} \\ 0 & 0 & 0 & c-2a+3b \end{array}\right]$$

Rank $A = 2$

The system is inconsistent iff

rank $A_B \neq 2$

$c - 2a + 3b \neq 0$

$c \neq 2a - 3b$

Question:

$2x_1 - x_2 - x_3 = 4$

$3x_1 + 4x_2 - 2x_3 = 11$

$3x_1 - 2x_2 + 4x_3 = 11$

In augemented matrix

$$A_b = \begin{bmatrix} 2 & -1 & -1 & 4 \\ 3 & 4 & -2 & 11 \\ 3 & -2 & 4 & 11 \end{bmatrix}$$

$$R_{12} = \begin{bmatrix} 3 & 4 & -2 & 11 \\ 2 & -1 & -1 & 4 \\ 3 & -2 & 4 & 11 \end{bmatrix}$$

$$R_1 - R_2 \begin{bmatrix} 1 & 5 & -1 & 7 \\ 2 & -1 & -1 & 4 \\ 3 & -2 & 4 & 11 \end{bmatrix}$$

$$\begin{array}{l} R_2 - 2R_1 \\ R_3 - 3R_1 \end{array} \begin{bmatrix} 1 & 5 & -1 & 7 \\ 0 & -11 & 1 & -10 \\ 0 & -17 & 7 & -10 \end{bmatrix}$$

$b - a + 2\left(\frac{3a-2b}{-5}\right)$

$b - a + \frac{6a-4b}{5}$

$\frac{5b-5a+6a-4b}{5}$

$\frac{b+a}{5}$

$c + 5b - 5a + 3a - 2b$

$= c - 2a + 3b$

$c + 5b - 5a$

}

return 0;

} Ruff

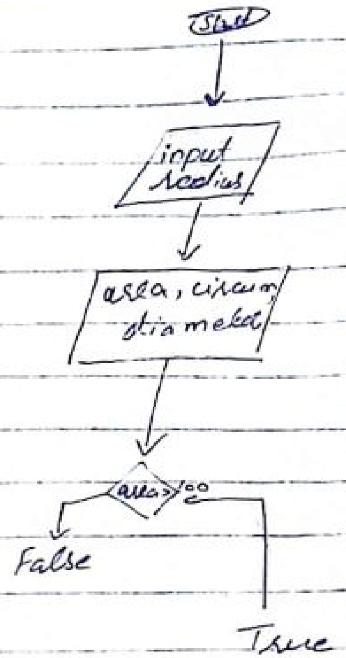
$-4+5\lambda-\lambda^2=0$

$\lambda^2-5\lambda+4=0$

$\lambda^2-4\lambda-\lambda+4=0$

$\lambda(\lambda-4)-1(\lambda-4)$

$(\lambda-1)(\lambda-4)=0$



Q.

$$\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{a^2+b^2}{a} & a \\ b & b & \frac{c^2+a^2}{b} \end{vmatrix} = 4abc$$

$$\frac{1}{abc}\begin{vmatrix} a^2+b^2 & c^2 & c^2 \\ a^2 & a^2+b^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$$ Multiply R_1 by c, R_2 by a, R_3 by b

$$\frac{1}{abc}\begin{vmatrix} a^2+b^2-c^2 & 0 & c^2 \\ 0 & b^2 & a^2 \\ b^2-c^2-a^2 & b^2-c^2-a^2 & c^2+a^2 \end{vmatrix}$$ C_1-C_3, C_2-C_3

$$=\frac{1}{abc}\begin{vmatrix} a^2+b^2-c^2 & 0 & c^2 \\ 0 & b^2+c^2-a^2 & a^2 \\ -2a^2 & -2c^2 & 0 \end{vmatrix}$$ $R_3-(R_1+R_2)$

Expand from R_1

$$=\frac{1}{abc}\left[a^2+b^2-c^2\begin{vmatrix} b^2+c^2-a^2 & a^2 \\ -2c^2 & 0 \end{vmatrix} - 0 + c^2\begin{vmatrix} 0 & b^2+c^2- \\ -2a^2 & -2 \end{vmatrix}\right.$$

$$=\frac{1}{abc}\left[a^2+b^2-c^2(2a^2c^2)+c^2(2a^2b^2-2a^2c^2+2a^4\right.$$

$$\frac{1}{abc}\left(2a^4c^2+2a^2b^2c^2-2a^2c^4+2a^2b^2c^2-2a^2c^4+2a^4c\right.$$

$$=\frac{1}{abc}\left[\cancel{4a^4c^2}+4a^2b^2c^2-\cancel{4a^2c^4}\right]$$

$$=\frac{1}{\cancel{abc}}\left(4a^2b^2c^2\right)$$

$$=4abc = R.H.S$$

↳ Periodic Matrix

for a square matrix if $A^{K+1}=A$ where K is +ve integer, then A is called periodic matrix.

Idempotent Matrix

For a square matrix A if $A^2 = A$ then it is called...

Nulpotent Matrix

For a matrix A if $A^P = 0$ then it is called...

Involuntry Matrix

$A^2 = I$

Q No. 1

35,282, 44,759, 58,916, 80,652, 92469 are all multiple of 13 show that determinent:

$$\begin{vmatrix} 3 & 5 & 2 & 8 & 2 \\ 4 & 4 & 7 & 5 & 9 \\ 5 & 8 & 9 & 1 & 6 \\ 8 & 0 & 6 & 5 & 2 \\ 9 & 2 & 4 & 6 & 9 \end{vmatrix}$$

$$\begin{vmatrix} 3 & 5 & 2 & 8 & 35282 \\ 4 & 4 & 7 & 5 & 44759 \\ 5 & 8 & 9 & 1 & 58916 \\ 8 & 0 & 6 & 5 & 80652 \\ 9 & 2 & 4 & 6 & 92469 \end{vmatrix} \quad C_5 + 10C_4 + 100C_3 + 1000C_2 + 10000C_1$$

$$= 13 \begin{vmatrix} 3 & 5 & 2 & 8 & 2714 \\ 4 & 4 & 7 & 5 & 3443 \\ 5 & 8 & 9 & 1 & 4532 \\ 8 & 0 & 6 & 5 & 6204 \\ 9 & 2 & 4 & 6 & 7113 \end{vmatrix}$$

$\therefore$ So this matrix is also multiple of 13

Ex 3.1

Q 1b imp Qution ★ imp

Show that any square matrix A can be written in a unique way as $A = B + C$

where $B = \frac{1}{2}(A + A^t)$ is hermitan

and $C = \frac{1}{2}(A - A^{-1})$ skew hermitian

Sol, Let $A = \frac{1}{2}(2A)$

$A = \frac{1}{2}(A + A) \Rightarrow A = \frac{1}{2}(A + \bar{A}^t + A - \bar{A}^t)$

$= \frac{1}{2}(A + \bar{A}^t) + i \cdot \frac{1}{2i}(A - \bar{A}^t)$

$A = B + iC$

where $B = \frac{1}{2}(A + \bar{A}^t)$ & $C = \frac{1}{2i}(A - \bar{A}^t)$

(10, 11, 12, 13, 14, 15, 16, ...

We are to prove that B is hermitian

$$\bar{B}^{t} = \left(\overline{\tfrac{1}{2}(A+\bar{A}^{t})}\right)^{t}$$

$$= \frac{1}{2}\left(\overline{A+\bar{A}^{t}}\right)^{t}$$

$$\frac{1}{2}\left(\bar{A} + (\bar{\bar{A}}^{t})\right)^{t} = \frac{1}{2}\left(\bar{A} + (\bar{\bar{A}})^{t}\right)^{t}$$

$$= \frac{1}{2}\left(\bar{A}^{t} + A^{t}\right)^{t} = \frac{1}{2}\left(\bar{A}^{t} + (A^{t})^{t}\right)$$

$$= \frac{1}{2}\left(\bar{A}^{t} + A\right) = \frac{1}{2}\left(A + \bar{A}^{t}\right)$$

$$= B$$

Consider:

$$\bar{C}^{t} = \left(\overline{\tfrac{1}{2i}(A-\bar{A}^{t})}\right)^{t}$$

$$= \frac{1}{-2i}\left(\overline{A-\bar{A}^{t}}\right)^{t} = -\frac{1}{2i}\left(\bar{A} - (\bar{\bar{A}}^{t})\right)^{t}$$

$$= -\frac{1}{2i}\left(\bar{A} - (\bar{\bar{A}})^{t}\right)^{t}$$

$$= -\frac{1}{2i}\left(\bar{A} - A^{t}\right)^{t} = -\frac{1}{2i}\left(\bar{A}^{t} - (A^{t})^{t}\right)$$

$$= -\frac{1}{2i}\left(\bar{A}^{t} - A\right) = \frac{1}{2i}\left(A - \bar{A}^{t}\right)$$

for uniqness, we suppose on contrary that there is another way, say

$$A = R + iS \quad (2)$$

where R & S are hermitian

$$\bar{A}^{t} = (\overline{R+iS})^{t}$$

$$= (\bar{R} + i\bar{S})^{t}$$

$$\bar{A}^{t} = \bar{R}^{t} - i(\bar{S})^{t}$$

$\bar{A}^{t} = R - iS$ — (3) since R and S are hermitian

Adding eq (2) & (3)

$$A + \bar{A}^{t} = 2R$$

$$R = \frac{1}{2}(A + \bar{A}^{t}) = B = P$$

Subtracting eq (3) from (2)

$$A - \bar{A}^{t} = 2iS$$

$$S = \frac{1}{2i}(A - \bar{A}^{t}) = C = Q$$

Questions

CH # 3 Q10 to Q17 + Examp 7, 8, 9
Ex 3.1

CH # 4 · Q4, 5, 8, 10, 11, 20 Examp 7, 8, 9
Ex 4.1

CH # 5 Example 8, 9
Ex 5.1 Q 6, 7, 9, 10, 11, 14, 15

A	B	C	Σ	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Σ :-

$= \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + A.BC$

$= \bar{A}(\bar{B}C + B\bar{C}) + A(\bar{B}\bar{C} + BC$

$\bar{A}\bar{B}C + \bar{A}B\bar{C} + AB\bar{C} + ABC.$

$= A(\bar{B}C + B\bar{C}) + AB(\bar{C} + C)$

$\Rightarrow A\ (B \oplus C) + AB.$

Ex 3.2

①. # 5 iv)

Find inverse of

i) $\begin{bmatrix} 1 & 0 & K \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ = sol: $\begin{bmatrix} 1 & 0 & K \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} R_1 - KR_3 \begin{bmatrix} 1 & 0 & -K \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

iv) $\begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & 3 \end{bmatrix}$

$\begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & 3 \end{bmatrix}; \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 1 & 0 & 5 \end{bmatrix} R_3 - 2R_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 2 & 1 \\ 2 & 1 & -1 \end{bmatrix} R_{13} \begin{bmatrix} -2 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 2 & 1 \\ 0 & 1 & -11 \end{bmatrix} R_3 - 2R_1 \begin{bmatrix} -2 & 0 & 1 \\ 0 & 1 & 0 \\ 5 & 0 & -2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -11 \\ 0 & 2 & 1 \end{bmatrix} \xrightarrow{R_{23}} \begin{bmatrix} -2 & 0 & 1 \\ 5 & 0 & -2 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -11 \\ 0 & 0 & 23 \end{bmatrix} R_3-2R_2 \begin{bmatrix} -2 & 0 & 1 \\ 5 & 0 & -2 \\ -10 & 1 & 4 \end{bmatrix}$$

~~ix)~~

Ex. 3.1

Q.11 Show that every square matrix A with entries from C can be written as a Hermitian matrix $B = \frac{1}{2}(A+\bar{A}^t)$ & as a skew Hermitian matrix $D = \frac{1}{2}(A-\bar{A}^t)$

Sol,

$$A = \frac{1}{2}(2A)$$
$$= \frac{1}{2}(A+A)$$
$$= \frac{1}{2}(A+\bar{A}^t+A-\bar{A}^t)$$
$$\frac{1}{2}(A+\bar{A}^t)+\frac{1}{2}(A-\bar{A}^t)$$
$$= B + D$$

where: $B = \frac{1}{2}(A+\bar{A}^t)$ &

$$D = \frac{1}{2}(A-\bar{A}^t)$$

Consider $\bar{B}^t = \frac{1}{2}\overline{(A+\bar{A}^t)}^t$

$$\bar{B}^t = \frac{1}{2}\overline{(A+\bar{A}^t)}^t = \frac{1}{2}((\bar{A}+(\bar{\bar{A}})^t)^t$$
$$\frac{1}{2}(\bar{A}^t+A^t)^t = \frac{1}{2}(\bar{A}^t+(A^t)^t$$
$$\frac{1}{2}(\bar{A}^t+A) = B \text{ is Hermitian}$$

$$\bar{D}^t = (\overline{\tfrac{1}{2}(A-\bar{A}^t)})^t$$
$$= \frac{1}{2}(\overline{A-\bar{A}^t})^t = \frac{1}{2}(\bar{A}-(\bar{\bar{A}})^t)^t$$
$$= \frac{1}{2}(\bar{A}-A^t)^t = \frac{1}{2}(\bar{A}^t-(A^t)^t)$$
$$= \frac{1}{2}(\bar{A}^t-A) = -\frac{1}{2}(A-\bar{A}^t)$$
$$= -D$$

D is skew ~~symmetric~~ Hermitian

Q12 If A & B are symmetric matrix then prove that AB is symmetric if & only if A and B are symetric.

Symmetric matrix $A^t = A$ commute $AB = BA$

Sol:

Proof: If AB is symmetric then

$(AB)^t = AB$

L.H.S $= (AB)^t$ using transpose property

$(AB)^t = B^t A^t$

Since A & B are symmetric $A^t = A$ $B^t = B$

$(AB)^t = BA \Rightarrow (AB)^t = AB$

Proof:

If A and B commute $(AB = BA)$ then:

$(AB)^t = B^t A^t$ using transpose pro

~~Since A & B are symmetric~~

$= BA$ (since A and B commute)

$= AB$ (since A and B commute)

so $(AB)^t = BA$

$(AB)^t = AB$

Q.10 Show that every square matrix A with entries from R, can be written as a symmetric matrix $B = \frac{1}{2}(A + A^t)$ and skew symmetric $D = \frac{1}{2}(A - A^t)$

Sol $A = \frac{1}{2}(2A) \Rightarrow \frac{1}{2}(A + A)$

$= \frac{1}{2}(A + A^t) + \frac{1}{2}(A - A^t)$

$= B + D$

where $B = \frac{1}{2}(A + A^t)$

$D = \frac{1}{2}(A - A^t)$

Consider $B^t = \left(\frac{1}{2}(A + A^t)\right)^t$

$B^t = \frac{1}{2}(A^t + (A^t)^t)$

$B^t = \frac{1}{2}(A^t + A)$

$B^t = B$ Symmetric matrix

Consider $D^t = \left(\frac{1}{2}(A - A^t)\right)^t$

$D^t = \frac{1}{2}(A^t - (A^t)^t)$

$= \frac{1}{2}(A^t - A)$

$= -\frac{1}{2}(A - A^t)$

$D^t = -D$ Skew symmetric

Q.13 Show that AA^t and A^tA are symmetric for any square matrix A.

AA^t is symmetric:

1) $(AA^t)^t = (A^t)^t A^t = AA^t$

$(AA^t)^t = AA^t$ so symmetric

2) $(A^tA)^t = A^t (A^t)^t = A^tA$

$(A^tA)^t = A^tA$ so symmetric.

$-1+2\lambda$ $\quad 1-\lambda-\lambda+\lambda \quad 1-2(1+\lambda)$

$-1-(-\lambda+\lambda^2) \quad -1+\lambda^2+\lambda^2 \quad 2-(1-\lambda) \quad 1-2-2\lambda$

Q.11 $(1-\lambda)x_1 + x_2 - x_3 = 0$

$x_1 - \lambda x_2 - 2x_3 = 0$

$x_1 + 2x_3 - \lambda x_3 = 0$

$$A = \begin{bmatrix} 1-\lambda & 1 & -1 \\ 1 & -\lambda & -2 \\ 1 & 2 & -\lambda \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & -\lambda \\ 1 & -\lambda & -2 \\ 1-\lambda & 1 & -1 \end{bmatrix} R13$$

$$A = \begin{bmatrix} 1 & 2 & -\lambda \\ 0 & -\lambda-2 & -2+\lambda \\ 0 & 1-2(1-\lambda) & -1+\lambda(1-\lambda) \end{bmatrix} \begin{matrix} R_2-R_1 \\ R_3-(1-\lambda)R_1 \end{matrix}$$

$$A = \begin{bmatrix} 1 & 2 & -\lambda \\ 0 & 1 & \frac{-2+\lambda}{-\lambda-2} \\ 0 & -1+2\lambda & -1+\lambda-\lambda^2 \end{bmatrix} \begin{matrix} \cancel{R_2-R_1} \\ \left(\frac{1}{-\lambda-2}\right)R_2 \end{matrix}$$

$$A = \begin{bmatrix} 1 & 2 & -\lambda \\ 0 & 1 & \frac{-2+\lambda}{-\lambda-2} \\ 0 & 0 & -1+\lambda-\lambda^2-(-1+2\lambda)\left(\frac{-2+\lambda}{-\lambda+2}\right) \end{bmatrix} \begin{matrix} \cancel{R_3-1} \\ R_3-(-1+2\lambda)R_2 \end{matrix}$$

$$-1+\lambda+\lambda^2-(-1+2\lambda)\frac{-2+\lambda}{-\lambda-2} = 0$$

$\cancel{-\lambda+2}$

$-\lambda-2$

$$-1+\lambda-\lambda^2+\left\{\frac{(-1+2\lambda)(-2+\lambda)}{+(\lambda+2)}\right\} = 0$$

$$\cancel{(-\lambda-2)(-1+\lambda-\lambda^2)-(-1+2\lambda)(-2+\lambda)=0(-\lambda-2)}$$

$$\cancel{-\{(\lambda+2)(1-\lambda+\lambda^2)+(1+2\lambda)(-2+\lambda)\}=0}$$

$$\cancel{(\lambda+2)(1-\lambda+\lambda^2)+(1+2\lambda)(-2+\lambda)=0}$$

$$\cancel{\lambda-\lambda^2+\lambda^3+2-2\lambda+2\lambda^2-2+\lambda-4\lambda+2\lambda^2=0}$$

$$\cancel{2-2-4\lambda+\lambda-2\lambda+\lambda-\lambda^2+2\lambda^2-4\lambda^2+\lambda^3=0}$$

$$\cancel{-4\lambda-\lambda^2+\lambda^3=0}$$

$$\cancel{\lambda(-3\lambda+\lambda^2)}$$

$$\cancel{\lambda^3-\lambda^2-4\lambda=0}$$

$$\frac{-1+\lambda-\lambda^2+(-1+2\lambda)(-2+\lambda)}{+(\lambda+2)} = 0$$

$$(\lambda+2)(-1+\lambda-\lambda^2)+(-1+2\lambda)(-2+\lambda) = 0$$

$$-\lambda+\lambda^2-\lambda^3-2+2\lambda-2\lambda^2+2-\lambda-4\lambda+2\lambda^2 = 0$$

$$-4\lambda+\lambda^2-\lambda^3 = 0$$

$$\lambda^3-\lambda^2+4\lambda = 0$$

$$\lambda(\lambda^2-\lambda+4) = 0$$

$\lambda = 0 \qquad \lambda^2-\lambda+4 = 0$

$$(-1+\lambda-\lambda^2)+\left\{\frac{-(-1+2\lambda)(-2+\lambda)}{+(\lambda+2)}\right\} = 0$$

CH#3 Canonical form

Q $\begin{bmatrix} 1 & 4 & 5 & 7 \\ 2 & 3 & 1 & 0 \\ 3 & -1 & 2 & 5 \end{bmatrix}$

Sol,

$$\overset{I_3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} 1 & 4 & 5 & 7 \\ 2 & 3 & 1 & 0 \\ 3 & -1 & 2 & 5 \end{bmatrix} \overset{I_4}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \begin{matrix} R_2-2R_1 \\ R_3-3R_1 \end{matrix} \begin{bmatrix} 1 & 4 & 5 & 7 \\ 0 & -5 & -9 & -14 \\ 0 & -13 & -13 & -16 \end{bmatrix} \begin{bmatrix} & & \\ & = & \\ & & \end{bmatrix}$$

$$\begin{bmatrix} & & \\ & " & \\ & & \end{bmatrix} \begin{matrix} C_2-4C_1 \\ C_3-5C_1 \\ C_4-7C_1 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -5 & -9 & -14 \\ 0 & -13 & -13 & -16 \end{bmatrix} \begin{bmatrix} 1 & -4 & -5 & -7 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

search from Youtube

CH#6 Vector Space

Field:

A set F is called field under two binary operations Addition "+" and multiplication "×" if F satisfies the following axioms.

i) Closure law of addition

$\forall\ a, b \in F,\ a+b \in F$

ii) Associative Law of addition:

$\forall\ a, b, c \in F$

$\Rightarrow (a+b)+c = a+(b+c)$

iii) Existent of identity element

$\forall\ a \in F,\ \exists\ 0 \in F$ such that

$a + 0 = 0 + a = a$

iv) Existence of inverse element

$\forall\ a \in F,\ \exists\ -a \in F$ such that

$a+(-a) = (-a)+a = 0$

$\therefore (F, +)$ is a group

v) Commutative Law

$\forall\ a, b \in F$

$a+b = b+a$

$\therefore (F, +)$ is an abelian group.

vi) Closure Law of multiplication

$\forall\ a,b \in F$

$\Rightarrow ab \in F$

vii) Associative Law of multiplication:

$\forall\ a,b,c \in F$

$\Rightarrow (ab)c = a(bc)$

viii) Existence of identity element:

$\forall$ ~~a,b,c~~ $a \in F, \exists\ 1 \in F$

$a.1 = 1.a = a$

ix) Existence of inverse element multiplicative

$\forall\ a \in F, \exists, \frac{1}{a} \in F$

$a \times \frac{1}{a} = \frac{1}{a} \cdot a = 1$

x) Commutative Law of multiplication.

$\forall\ a,b \in F$

~~$a \times b = b \times a$~~ $a.b = b.a$

xi) Distributive law of multiplication over addition

$\forall\ a,b,c \in F$

Left: $a.(b+c) = ab + a.c$

Right: $(a+b).c = a.c + b.c$

Then $(F,+,\cdot)$ is called Field

Example: Rational number 2) Real no.

3) Complex no.

Vector space:

Let F be a field and V a non-empty set whose elements an operation of addition is defined.

Suppose that for every $a \in F$ and every $v \in V$, av is an element of V. Then V is called vector space over "F" if the following axioms are satisfied.

i) V, is an abelian group under addition.

ii) $a(u+v) = au + av$ for $a \in F$ & $u,v \in$

iii) $(a+b)v = av + bv$ $b,a \in F$, $u,v \in V$

iv) $a(bv) = ab(v)$

v) $1.v = v$, 1 being multiplicative identity of F, where a,b are called scalless and u,v are called vectors.

Example ① set of vector form a vector space over set of real numbers i.e $V/\mathbb{R}$

② F(F) i.e is set of field F over F also form a V.S

3) Let $V = \{a + b\sqrt{2} \mid a, b \in Q\}$ is also a V.S over Q.

4) Let M_{mn} denote the set of all $m \times n$ matrices with entries from the field ~~and~~ of real number.

5) $F^n = \{(x_1, x_2, ---, x_n) \mid x_i \in F\}$

6) The set $P_n(x)$ of all polynomials of degree $\leq n$ together with the zero polynomial is also V.S over R.

7) Let X be a non empty subset of R. Let V be the set of all function $f: X \to R$

for $f, g \in V$, put

i) $(f+g)(x) = f(x) + g(x)$

ii) $(af)(x) = af(x)$

then V is also a V

Subspace:

A non-empty subset W of a vector space V over a field F is said to be subspace of V if W is itself a vector space over F under the same operation as defined in V.

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 8 & 9 \\ -1 & -5 & 4 \\ 4 & 7 & 23 \end{bmatrix}$$

$R_2 - 3R_1$, $R_3 + R_1$, $R_4 - 4R_1$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & 3 \\ 0 & -2 & 6 \\ 0 & -5 & 15 \end{bmatrix}$$

$-R_2$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & -3 \\ 0 & -2 & 6 \\ 0 & -5 & 15 \end{bmatrix}$$

$R_1 - 3R_2$, $R_3 + 2R_2$, $R_4 + 5R_2$

$$\begin{bmatrix} 1 & 0 & 11 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Rank $A = 2 < 3$

Non-trivial solution exist

$a + 11c = 0$

$b - 3c = 0$

$a = -11c$

$b = 3c$

put $c = 1$

$a = -11$

$b = 3$

$$\begin{pmatrix} 1 & 2 & 1 \\ -2 & 1 & -6 \\ 4 & 0 & 1 \\ 1 & -3 & 4 \end{pmatrix}$$

$R_2 + 2R_1$
$R_3 - 4R_1$
$R_4 - R_1$

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 5 & -4 \\ 0 & -8 & -3 \\ 0 & -5 & 3 \end{pmatrix}$$

$\frac{1}{5}R_2$

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & -4/5 \\ 0 & -8 & -3 \\ 0 & -5 & -3 \end{pmatrix}$$

$R_3 + 8R_2$
$R_4 + 5R_2$

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & -4/5 \\ 0 & 0 & -\frac{47}{5} \\ 0 & 0 & -7 \end{pmatrix}$$

Rank $A = 4 >$ No. of variables

$-3 - \frac{32}{5}$

$\frac{-15 - 32}{5}$

Non-trivial does not exist

$\therefore$ it is ~~B~~ L.I

Base of Vector:
A linear independent set which generates or spans a vectorspace "V" is called basis of vector for V.

Example:

$$R^2 = \{(x,y) / x,y \in R\}$$

$i = (1,0)$, $j = (0,1)$

$(x,y) = x(1,0) + y(0,1)$

$(3-7) = 3(1,0) + (-7)(0,1)$

$(0,0) = 0(1,0) + 0(0,1)$ Linear independent

Example 2:

$$R^3 = \{(x,y,z) / x,y,z \in R\}$$

$i = (1,0,0)$ $j = (0,1,0)$, $k = (0,0,1)$

$(0,0,0) = 0(1,0,0) + 0(0,1,0) + 0(0,0,1)$ L.I

$(x,y,z) = x(1,0,0) + y(0,1,0) + z(0,0,1)$ generate

Q.10 Find Determinant whether or not the given set of vectors is a basis for R^3

$$\{(1,2,-1),(0,3,1),(1,-5,3)\}$$

Let $(x,y,z) \in R^3$, for $a,b,c \in F = R$

$(x,y,z) = a(1,2,-1) + b(0,3,1) + c(1,-5,3)$

$= (a, 2a, -a) + (0, 3b, b) + (c, -5c, 3c)$

$(a+0+c, 2a+3b-5c, -a+b+3c)$

$a+0+c = x$ — ①

$2a+3b-5c = y$ — ②

$-a+b+3c = z$ — ③

Write in matrix form

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & x \\ 2 & 3 & -5 & y \\ -1 & 1 & 3 & z \end{array}\right]$$

Taking augmented matrix

$$\begin{array}{c} R_2 - 2R_1 \\ R_3 + R_1 \end{array} \left[\begin{array}{cccc} 1 & 0 & 1 & x \\ 0 & 3 & -7 & y-2x \\ 0 & 1 & 4 & z+x \end{array}\right]$$

$$R_{23} \left[\begin{array}{cccc} 1 & 0 & 1 & x \\ 0 & 1 & 4 & z+x \\ 0 & 3 & -7 & y-2x \end{array}\right] \xRightarrow{R_3 - 3R_2} \left[\begin{array}{cccc} 1 & 0 & 1 & x \\ 0 & 1 & 4 & z+x \\ 0 & 0 & -19 & y-5x-3z \end{array}\right]$$

$$-\frac{1}{19}R_3 \left[\begin{array}{cccc} 1 & 0 & 1 & x \\ 0 & 1 & 4 & z+x \\ 0 & 0 & 1 & \frac{-y+5x+3z}{19} \end{array}\right]$$

$$\begin{array}{c} R_1 - R_3 \\ R_2 - 4R_3 \end{array} \left[\begin{array}{cccc} 1 & 0 & 0 & \frac{14x+y-3z}{19} \\ 0 & 1 & 0 & \frac{-x+4y+7z}{19} \\ 0 & 0 & 1 & \frac{5x-y+3z}{19} \end{array}\right]$$

$\text{rank } A = \text{rank } A_B = 3$

unique solutions exist

$$a = \frac{14x - y - 38}{19}, \quad b = \frac{-x + 4y + 73}{19}$$

$$c = \frac{5x - y + 38}{19}$$

$$\therefore (x, y, z) = \frac{14x - y - 38}{19}(1, 2, -1) + \frac{-x + 4y + 73}{19}(0, 3, 1) + \frac{5x - y + 38}{19}(1, -5, 3)$$

$\therefore$ Given vector generates R^3

if $a(1, 2, -1), b(0, 3, 1) + c(1, -5, 3) = 0 = (0, 0, 0)$

$\Rightarrow a = 0, \ b = 0, \ c = 0$

given vector are linear independent.

Linear Transformation:

Let U and V are vector space or F and $T: U \to V$ be a function which satisfies, F..

$u_1, u_2 \in U, a \in F$ (conditions)

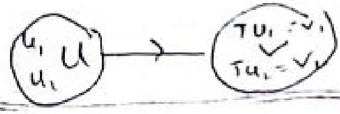
i) $T(u_1 + u_2) = T(u_1) + T(u_2)$

ii) $T(au_1) = aT(u_1)$

or combining above properties

for $u_1, u_2 \in U$, $a, b \in F$

$T(au_1 + bu_2) = aT(u_1) + bT(u_2)$



Ex 6.3 Q 1.)

check which of the following defined linear Transformation (L.T) from R^3 to R^2

i) $T(x_1, x_2, x_3) = (x_1 - x_2, x_1 - x_3)$

Let $u_1 = (x_1, x_2, x_3)$, $u_2 = (y_1, y_2, y_3) \in R^3$

$T(u_1 + u_2) =$

$= T(x_1, x_2, x_3 + y_1, y_2, y_3)$

$= T(x_1 + y_1, x_2 + y_2, x_3 + y_3)$

$= (x_1 + y_1 - x_2 - y_2, x_1 + y_1 - x_3 - y_3)$

$= (x_1 - x_2 + y_1 - y_2, x_1 - x_3 + y_1 - y_3)$

$= (x_1 - x_2, x_1 - x_3) + (y_1 - y_2, y_1 - y_3)$

$= T(x_1, x_2, x_3) + T(y_1, y_2, y_3)$

$= T(u_1) + T(u_2)$

$\forall a \in R, u_1 = (x_1, x_2, x_3) \in R^3$

$T(au_1)$

$= T(a(x_1, x_2, x_3)) = T(ax_1, ax_2, ax_3)$

$= (ax_1 - ax_2, ax_1 - ax_3)$

$= a(x_1 - x_2, x_1 - x_3)$

$= aT(x_1, x_2, x_3)$

$= aT(u_1)$

$\therefore T$ is Linear Transform

CamScanner

Past paper preparation

Q iv) Show that the vector $(6,11,6)$ can be written with the linear combination of the following vectors:

$v_1 = (2,1,4)$, $v_2 = (1,-1,3)$, $v_3 = (3,2,5)$

Sol, Let $a,b,c \in R$

$(6,11,6) = aV_1 + bV_2 + cV_3$

$= a(2,1,4) + b(1,-1,3) + c(3,2,5)$

$= (2a+b+3c, a-b+2c, 4a+3b+5c)$

$2a+b+3c = 6$ — (i)

$a-b+2c = 11$ — (ii)

$4a+3b+5c = 6$ — (iii)

Writing equation in matrix form

$$\begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 2 \\ 4 & 3 & 5 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 6 \end{bmatrix}$$

Take augmented matrix

$AX = B$

$$A_B = \begin{bmatrix} 2 & 1 & 3 & 6 \\ 1 & -1 & 2 & 11 \\ 4 & 3 & 5 & 6 \end{bmatrix}$$

$$\overset{R_{12}}{=} \begin{bmatrix} 1 & -1 & 2 & 11 \\ 2 & 1 & 3 & 6 \\ 4 & 3 & 5 & 6 \end{bmatrix} \underset{R_3-4R_1}{\overset{R_2-2R_1}{\Rightarrow}} \begin{bmatrix} 1 & -1 & 2 & 11 \\ 0 & 3 & -1 & -16 \\ 0 & 7 & -3 & -38 \end{bmatrix}$$

$$\overset{R_3-2R_2}{} \begin{bmatrix} 1 & -1 & 2 & 11 \\ 0 & 3 & -1 & -16 \\ 0 & 1 & -1 & -6 \end{bmatrix} \overset{R_{23}}{\Rightarrow} \begin{bmatrix} 1 & -1 & 2 & 11 \\ 0 & 1 & -1 & -6 \\ 0 & 3 & -1 & -16 \end{bmatrix}$$

$$\underset{R_3-3R_2}{\overset{R_1+R_2}{}} \begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & 1 & -1 & -6 \\ 0 & 0 & 2 & 2 \end{bmatrix} \overset{\frac{1}{2}R_3}{\Rightarrow} \begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & 1 & -1 & -6 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\underset{R_2+R_3}{\overset{R_1-R_3}{}} \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$a = 4$

$b = -5$

$c = 1$

2023 Q.5 Check whether the transformation $T: R^3 \to R^3$ defined by

$T(x_1, x_2, x_3) = (x_1 - x_2, x_2 - x_3, x_1)$

is linear or not

Solution: Let

$u_1 = (x_1, x_2, x_3), u_2 = (y_1, y_2, y_3) \in R^3$

$T(u_1 + u_2)$

$= T((x_1, x_2, x_3) + (y_1, y_2, y_3))$

$T(x_1+y_1, x_2+y_2, x_3+y_3)$

$(x_1+y_1-x_2-y_2, x_2+y_2-x_3-y_3, x_1+y_1)$

$(x_1 - x_2 + y_1 - y_2 , x_2 - x_3 + y_2 - y_3 , x_1 + y_1)$

$(x_1 - x_2 , x_2 - x_3 , x_1) + (y_1 - y_2 , y_2 - y_3 , y_1)$

$T(x_1, x_2, x_3) + T(y_1, y_2, y_3)$

$T(u_1) + T(u_2)$

Let $a \in F$, $u = (x_1, x_2, x_3) \in R^3$

$T(au)$

$= T(a(x_1, x_2, x_3))$

$T(ax_1, ax_2, ax_3)$

$(ax_1 - ax_2 , ax_2 - ax_3 , ax_1)$

$a(x_1 - x_2 , x_2 - x_3 , x_1)$

$a\ T(x_1, x_2, x_3)$

$aT(u)$ And

Linear Algebra

Inner Product

Let V be a vector space over field F.

$\langle , \rangle : V \times V \xrightarrow{\text{cartisian product}} F$ is called linear product if it is satisfy

i) $\langle v_1, v_2 \rangle = \langle v_2, v_1 \rangle$ for all $v_1, v_2 \in V$

ii) $\langle av_1 + bv_2, v_3 \rangle = a\langle v_1, v_2 \rangle + b\langle v_2, v_3 \rangle$

Example iii) $\langle v, v \rangle \geq 0$ & $\langle v_1, v \rangle = 0 \iff b = 0$

Let u, v belong $\in R^3$

$u = (u_1, u_2 \ldots u_n)$

$v = (v_1, v_2 \ldots v_n)$

$\langle u, v \rangle = u_1 v_1 + u_2 v_2 + \ldots + u_n v_n$ is called Euclinear inner product

Norm or Length

$u = (u_1, u_2 \ldots u_n) \in R^3$

$\|u\| = \sqrt{u_1^2 + u_2^2 + \ldots + u_n^2}$

unit vector

$u = \dfrac{u}{\|u\|}$

Q Ex 7.1 Past paper 2023

Q1 $u = (u_1, u_2)$, $v = (v_1, v_2)$

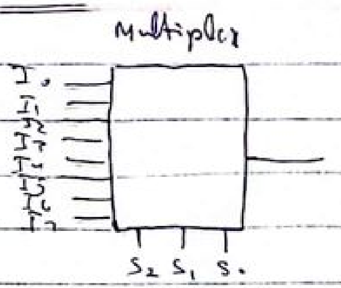
$\langle u, v \rangle = u_1 v_1 - 2u_1 v_2 - 2u_2 v_1 + 5u_2 v_2$

is an inner product on R

i) $\langle v, v \rangle = v_1^2 - 2v_1 v_2 - 2v_2 v_1 + 5v_2^2$

For 8 bit 8:1

Block diagram



2	4	= 8	
1	4	= 4	
3.	1	= 3	
3,	3.3	= 9.9	$= \frac{40.8}{15}$
3.	2	= 6	
3	3.3	= 9.9	
0	4	=	

$au_1w_1 - 2au_1w_2 - 2au_2w_1 + 5au_2w_2$
$+ bv_1w_1 - 2bv_1w_2 - 2bv_2w_1 + 5bv_2w_2$

$= a(u_1w_1 - 2u_1w_2 - 2u_2w_1 + 5u_2w_2)$
$+ b(v_1w_1 - 2v_1w_2 - 2v_2w_1 + 5v_2w_2)$
$= a(u,w) + b(v,w)$

So it is an inner product

Eigen values and Eigen vectors
If A is an $n \times n$ matrix over R, then a scalar $\lambda \in R$ is called an eigen value of A if there exists a non zero column vector $v \in R^3$ such that $Av = \lambda v$
In this case v is called a eigen ~~vector~~ value λ

$Av = \lambda v$
$= \lambda I v$

$Av - \lambda I v = 0$
$(A - \lambda I)v = 0$
Since $v \neq 0$, therefore $A - \lambda I$ is singular

$v_1^2 - 4v_1v_2 + 4v_2^2 + v_2^2$
$(v_1 - 2v_2)^2 + v_2^2 \geq 0$
$\langle v, v \rangle = 0$
$(v_1 - 2v_2)^2 + v_2^2 = 0$
$v_1 - 2v_2 = 0 \qquad v_2^2 = 0$
$v_2 = 0$

$v = (0, 0)$
Hence 1st property is satisfied.

ii) $\langle u, v \rangle = \langle v, u \rangle$
is $u, v \in R$
$\langle u, v \rangle = u_1v_1 - 2u_1v_2 - 2u_2v_1 + 5u_2v_2$
changing v and u
$= v_1u_1 - 2v_2u_1 - 2v_1u_2 + 5v_2u_2$
$\langle v, u \rangle$
So $\langle u, v \rangle = \langle v, u \rangle$ satisfied.

iii) $\langle au + bv, w \rangle = a\langle u, w \rangle + b\langle v, w \rangle$
$u = (u_1, u_2)$, $v = (v_1, v_2)$, $w = (w_1, w_2)$
$\langle a(u_1, u_2) + b(v_1, v_2), (w_1, w_2) \rangle$
$\langle (au_1 + bv_1, au_2 + bv_2), (w_1, w_2) \rangle$

$(au_1 + bv_1)w_1 - 2(au_1 + bv_1)w_2 - 2w_1(au_2 + bv_2)$
$+ 5(au_2 + bv_2)w_2$

$$\begin{bmatrix} 3-\lambda & 1-0 & 1-0 \\ 2-0 & 4-\lambda & 2-0 \\ 1-0 & 1-0 & 3-\lambda \end{bmatrix}$$

$\det(A - \lambda I) = 0$

$$\begin{bmatrix} 3-\lambda & 1 & 1 \\ 2 & 4-\lambda & 2 \\ 1 & 1 & 3-\lambda \end{bmatrix} = 0$$

$(3-\lambda)[(4-\lambda)(3-\lambda)-2] - 1[2(3-\lambda)-2] + 1[2-1(4-\lambda)] = 0$

$(3-\lambda)[12-7\lambda+\lambda^2-2] - 1[6-2\lambda-2] + 1[2-4+\lambda] = 0$

$(3-\lambda)(\lambda^2-7\lambda+10) - 1(4-2\lambda) + 1(-2+\lambda)$

$-\lambda^3 + 7\lambda^2 - 10\lambda + 3\lambda^2 - 21\lambda + 30 - 4$

$+2\lambda - 2 + \lambda = 0$

$-\lambda^3 + 10\lambda^2 - 28\lambda + 24 = 0$

By synthetic division

1	-10	28	-24
0	2	-16	24
1	-8	12	0

$\lambda^2 - 8\lambda + 12 = 0$

$(\lambda-6)(\lambda-2) = 0$

$\lambda = 2, 6$

For $\lambda = 2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} = 0$$

$\Rightarrow \det(A - \lambda I) = 0$

L.H.S of eq ① is known as characteristic polynomial of the matrix A. Every root of this equation is called an eigen value of A.

Past-paper 2023 Ex 7.3 Q.1

Q11 Find the eigen value and eigen vectors of the matrix

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix}$$

sol,

If λ is an eigen value of A then characteristic equation $\Rightarrow$

$\det(A - \lambda I) = 0$

$$\begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = 0$$

$$R_1 - 2R_2 \quad R_3 - R_1 \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = .$$

$$x_1 + x_2 + x_3 = 0$$

$$x_1 = -x_2 - x_3$$

Let $x_1 = -S - t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -S - t \\ S \\ t \end{bmatrix}$$

$$\begin{bmatrix} -S \\ S \\ 0 \end{bmatrix} \begin{bmatrix} -t \\ t \\ 0 \end{bmatrix} = S \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$