THE ANSWERS MUST BE ATTEMPTED ON THE ANSWER SHEET PROVIDED

Q.1. Write short note on the following:

(6x5=30)

(i) Find all values of k for which the given augmented matrices corresponds to a consistent linear system.

a) [1 k -1]

b) $\begin{bmatrix} k & 1 & -2 \\ 4 & -1 & 2 \end{bmatrix}$

(ii) Sketch the unit circle in R2 using the given inner product

 $\langle u, v \rangle = \frac{1}{4}u_1v_1 + \frac{1}{16}u_2v_2$

(iii) Prove that

 $\begin{vmatrix} a^2 + b^2 & c & c \\ a & \frac{b^2 + c^3}{c} & a \\ b & b & \frac{c^2 + a^2}{c} \end{vmatrix} = 4abc$

(iv) Show that the vector v = (6, 11, 6) can be written with the linear combination of the following vectors

 $v_1 = (2, 1, 4), \quad v_2 = (1, -1, 3), \quad v_3 = (3, 2, 5)$

- (v) Define $T: \mathbb{R}^3 \to \mathbb{R}^3$ by $T(x_1, x_2, x_3) = (-x_3, x_1, x_2 + x_3)$. Find N(T). Is T one-to-one?
- (vi) Show that the following matrix is involutery

 $\begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$

Answer the following questions.

(4x7.5=30)

Q.2 Solve the system of linear equations by Gauss Jorden method

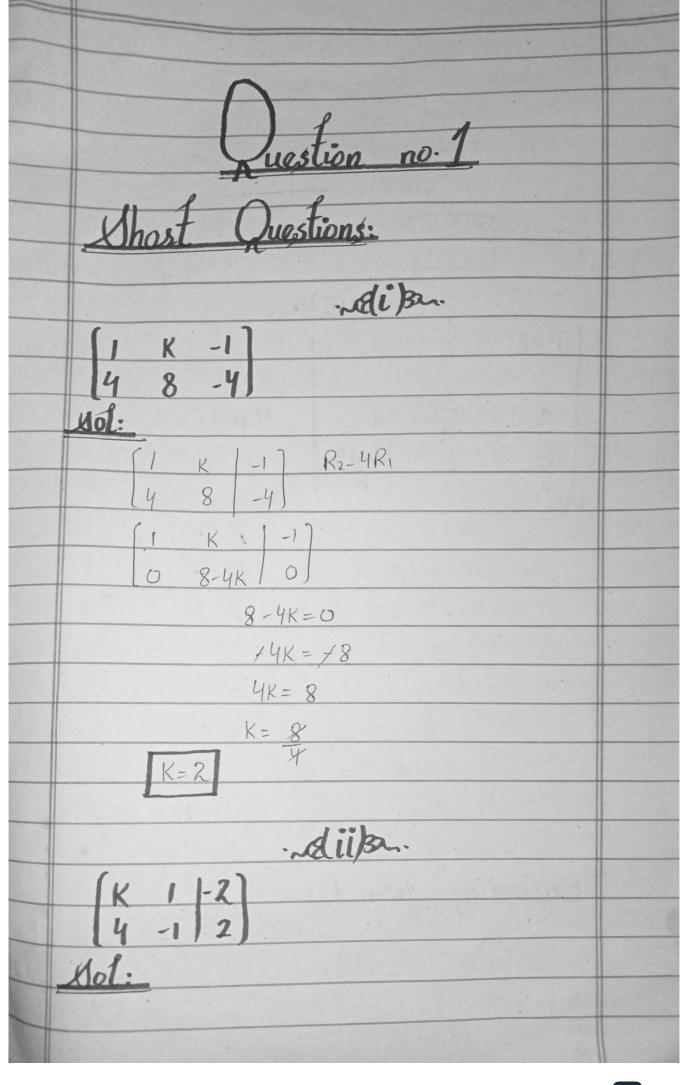
x-y+2z-w=-1 2x+y-2z-2w=-2 -x+2y-4z+w=13x-3w=-3

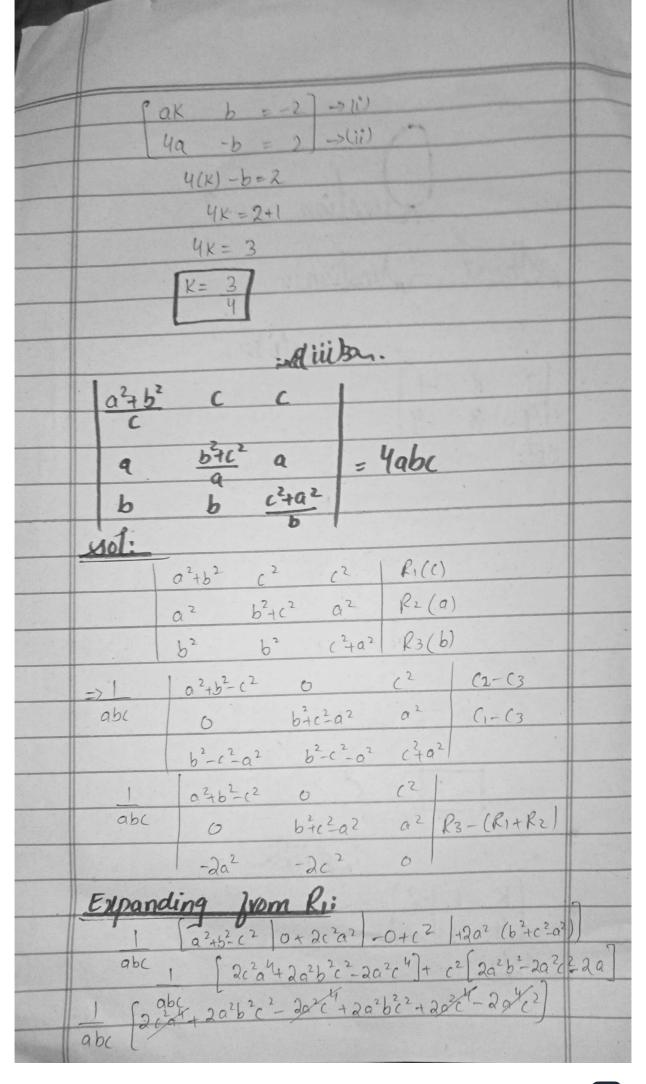
Q.3 Find the characteristic equation, the eigenvalues, and bases for the eigenspaces of the following matrix.

 $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$

- Q.4 Determine whether the vectors are linearly independent or not? $v_1 = (1, -2, 3), v_2 = (5, 6, -1), v_3 = (3, 2, 1)$.
- Q.5 Show that the following are sets are the subspaces of their corresponding vector spaces
 a) The set of lines passing through origin in R² Le.
 - $\{(x,y)\in R^2\mid y=ax, a \text{ is any real scalar}\}$ b) The set of places passing through origin in R^3 i.e.

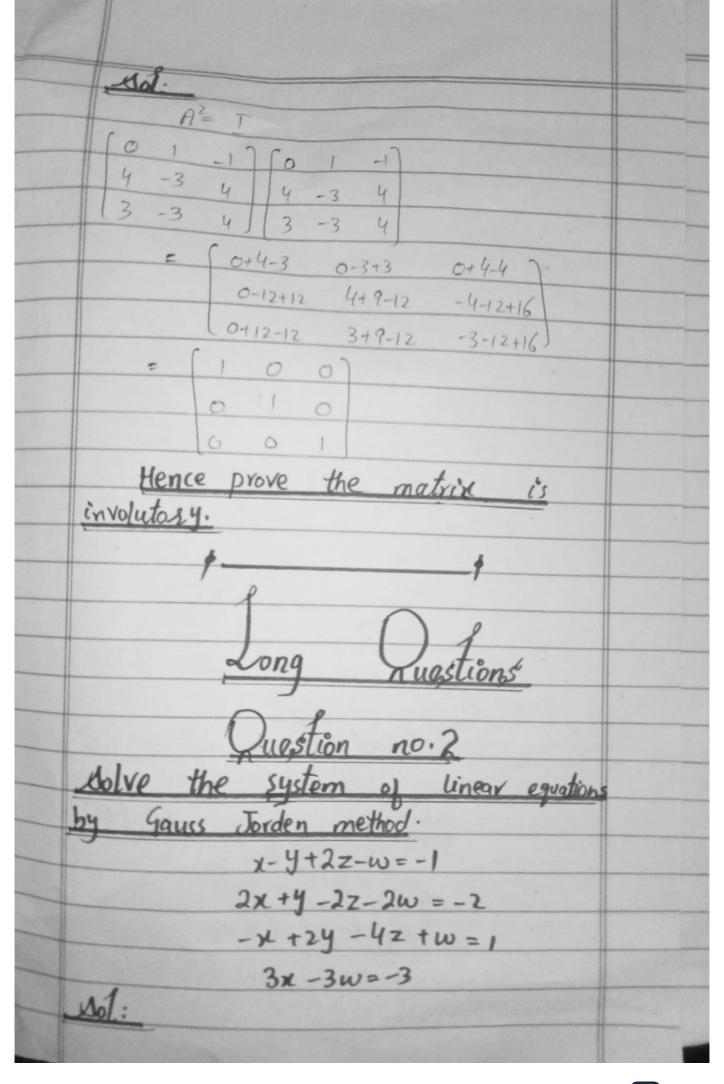
 $\{(x,y,z) \in \mathbb{R}^3 \mid ax+by+cz=0, a,b,c \text{ are any real scalars}\}$

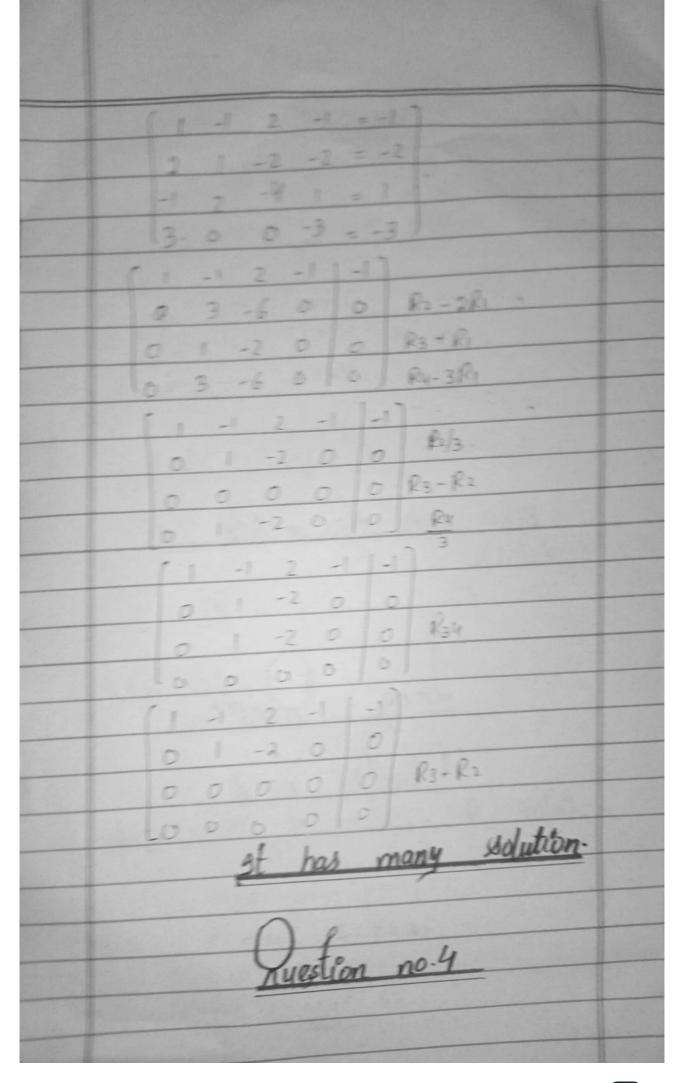




	1 [4a2b2(2)
	D= 4abc
	Aiv)3n.
	V= (6,11,6)
	V, = (2,1,4)
	$V_2 = (1,-1,3)$
	V3 = (3,2,5)
	Not:
	2 1 3 6
	4 3 5 6
-	
	2 / 812
-	2 1 3 5 6 TIZ
	(1 -1 2 117
	0 3 -1 -16 R2-2R1
1	6 7-3 -38 R3-4Ri
	(1 -1 > 111)
	0 1 -1/3 -16/3 R2
	[0 7 -3 -38]
1	(1-12 11)
	0 1 -1/3 -16/3 R3-7R2
	0 0 -2/3 -2/3

7-4-27 =11-4 (1) y- 1 2 = -16 -18) From eq. (1ii) 122=12 Z= +1 x3 Z=1 Put in eq. (ii) $9 - \frac{1}{3}(1) = -16$ $9 - \frac{1}{3} = -\frac{16}{3}$ y= -18 => Jy=-5 Put in eq. (i) 7-(-5)+2(1)=11 y+'5+2 = 11 X = -7+11 V= 4V1-5V2+V3 .dviba.





V1= (1,-2,3)	
V2= (5,6,-1)	
$V_3 = (3,2,1)$	
401:	
[1 5 3]	
-2 6 2	
3 -1 1)	
[1 5 3]	
-1 3 1 R2	
3 -1 1 2	
[153]	
0 8 4 R2+R1	
0 -16 -8 R3-3R1	
[153]	
0 8 4 6	
0 -16 -8	
(153)	
0 8 4 R3+2R2	
0 0 -4	
so, prove that vectors are	
so, prove that vectors are linearly independent.	
08	
w= $\frac{1}{2}(x,y) \in \mathbb{R}^2 \mid y = ax$ a is any seal scalas.	
$w = \left\{ (x, y) \in \mathbb{R}^2 \middle y = ax \right\}$	
a is any seal scalas.	

sol: U= (131) &w V= (2,2) EW U= (1,1) = a1 x,14 V = (2,2) = a2x,2yThen: U+V= (ax, y) + (a2x, 2y) = (1,1)+(2,2) = (3,3) & v Now: KV= K(2,2) KV= 2K, 2K This is the subspace of vector set w. A Part bBa. w= { (x,y,z) ER2 | ax+by+ cz=6 (i) Let xoy, Z are zero. a(0) +b(0)+((0) =0 0=0 EW NOW: 1) = (3,3,3) = a3, b3, (3 Ew V= (4, 4, 4) = ay sby s (4 & w U+V= (3,3,3) +(4,4,4) = (7,7,7) EW = 70,75,70 Then:

KV= K(4,4,4) = 4K, 4K, 4K & W so this is the subspace set vector W: Question no.3 Find eigenvalua: A= 1 -3 3 3 -5 3 Not: -3 3 3 -5 3 s(t) = det (tIn-A) *t(100 010 00 (1-33 o to. 3 -5 3 t-1 3 -3 -3 t+5 -3 6 t-4 Take determinant:

= (1-t)(-5-t)(4-t)-(-6)(3)-(-3)(3(4-t)-3(-5))+3	
(3(-6)-(-5-t)6)	
- 1-t (+3t-2)-(-3)(3t-6)+3(6t+30)	
= -t3-4+4t => Fox eigenvalues:	
det (+In-A)=0 => -t3-4t2+4t=0	
Take common (-):	
-(+3+4+2-11+)=0 => +3+4+2-4+=0	
t/12+4t-4)=0 => t=0	
12+44-11=0	
using quadratic jormula:	
3 3 3 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5	
n=-b+. 5b2-4ac	
10 - 10 - 200 / 10	
= -(1 ±)(4) 2-4(1) (-4)	
- 4+ 11/+1/ - 4+ 122	
= -4±16+16 -> -4±132	
= -4 ± 452	
= -4 (-1+52)	
X	
x=-2+52 ; x=-2-52	
so, the eigenvalues are:	1
(0,-2+529-2-52)	
1	