EXERCISE 4

Solve the following systems of linear equations, the field of scalars being R:

1.
$$\begin{aligned} 2x_1 + x_3 &= 1 \\ 2x_1 + 4x_2 - x_3 &= -2 \\ x_1 - 8x_2 - 3x_3 &= 2 \end{aligned}$$

2.
$$\begin{aligned} x_1 + x_2 + x_3 &= a \\ x_1 + (1+a)x_2 + x_3 &= 2a \\ x_1 + x_2 + (1+a)x_3 &= 3a \end{aligned}$$

3.
$$\begin{aligned} x_1 - x_2 + x_3 - x_4 + x_5 &= 1 \\ 2x_1 + x_2 + 3x_3 + 4x_5 &= 3 \\ 3x_1 - 2x_2 + 2x_3 + x_4 + x_5 &= 1 \\ x_2 + x_4 + x_5 &= 0 \end{aligned}$$

4.
$$\begin{aligned} x_1 + x_2 - x_3 &= 1 \\ x_2 + x_3 - x_4 &= 1 \\ x_3 + x_4 - x_5 &= 1 \\ -x_1 + x_4 + x_5 &= 1 \\ -x_2 + x_3 + x_4 &= 1 \end{aligned}$$

5.
$$\begin{aligned} x_1 - 2x_2 - 7x_3 + 7x_4 &= 5 \\ -x_1 + 2x_2 + 8x_3 - 5x_4 &= -7 \\ 3x_1 - 4x_2 - 17x_3 + 13x_4 &= 14 \\ 2x_1 - 2x_2 - 11x_3 + 8x_4 &= 7 \end{aligned}$$

6.
$$\begin{aligned} x_1 + 2x_2 + x_3 &= -1 \\ 6x_1 + x_2 + x_3 &= -4 \\ 2x_1 - 3x_2 - x_3 &= 0 \\ x_1 - x_2 &= 1 \end{aligned}$$

7.
$$\begin{aligned} 2x_1 + x_2 + 5x_3 &= 4 \\ 3x_1 - 2x_2 + 2x_3 &= 2 \\ 5x_1 - 8x_2 - 4x_3 &= 1 \end{aligned}$$

8. **Solve the system of equations having the given matrices as their augmented matrices:**

(i) $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 \end{array}\right]$ (ii) $\left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 0 \end{array}\right]$

(iii) $\left[\begin{array}{ccc|c} 4 & 2 & -1 & 0 \\ 3 & 3 & 6 & 3 \\ 5 & 1 & -8 & -1 \end{array}\right]$ (iv) $\left[\begin{array}{ccc|c} 2 & -1 & 3 & 4 \\ 0 & 4 & 1 & 8 \\ 0 & 0 & 2 & 0 \end{array}\right]$

For what values of λ do the following homogeneous equations have nontrivial solutions? Find these solutions: (Problems 9 – 11):

9.
$$\begin{aligned} (1-\lambda)x_1 + x_2 &= 0 \\ x_1 + (1-\lambda)x_2 &= 0 \end{aligned}$$

10.
$$\begin{aligned} (3-\lambda)x_1 - x_2 + x_3 &= 0 \\ x_1 - (1-\lambda)x_2 + x_3 &= 0 \\ x_1 - x_2 + (1-\lambda)x_3 &= 0 \end{aligned}$$

11.
$$\begin{aligned} (1-\lambda)x_1 + x_2 - x_3 &= 0 \\ x_1 - \lambda x_2 - 2x_3 &= 0 \\ x_1 + 2x_2 - \lambda x_3 &= 0 \end{aligned}$$

In each of the following use Gauss-Jordan method to reduce the given system to reduced echelon form, indicating the operations performed and determine the solution if any: (Problems 12 – 19):

12.
$$\begin{aligned} 6x_1 - 6x_2 + 6x_3 &= 6 \\ 2x_1 - 4x_2 - 6x_3 &= 12 \\ 10x_1 - 5x_2 + 5x_3 &= 30 \end{aligned}$$

13.
$$\begin{aligned} 5x_1 + 5x_2 - x_3 &= 0 \\ 10x_1 + 5x_2 + 2x_3 &= 0 \\ 5x_1 + 15x_2 - 9x_3 &= 0 \end{aligned}$$

14.
$$\begin{aligned} 5x_1 - 2x_2 + x_3 &= 3 \\ 3x_1 + 2x_2 + 7x_3 &= 5 \\ x_1 + x_2 + 3x_3 &= 2 \end{aligned}$$

15.
$$\begin{aligned} 5x_1 - 2x_2 + x_3 &= 2 \\ 3x_1 + 2x_2 + 7x_3 &= 3 \\ x_1 + x_2 + 3x_3 &= 2 \end{aligned}$$

16.
$$\begin{aligned} 2x_1 - x_2 + 3x_3 &= 3 \\ 3x_1 + x_2 - 5x_3 &= 0 \\ 4x_1 - x_2 + x_3 &= 3 \end{aligned}$$

17.
$$\begin{aligned} x_1 + 3x_2 + 5x_3 - 4x_4 &= 1 \\ x_1 + 2x_2 + x_3 - x_4 + x_5 &= -1 \\ x_1 - 2x_2 + 3x_3 + 2x_4 - x_5 &= 3 \\ x_1 + 5x_2 + 3x_3 + x_4 + x_5 &= -11 \\ x_1 + 3x_2 - x_3 + x_4 + 2x_5 &= -3 \end{aligned}$$

18.
$$\begin{aligned} 3x_1 + 2x_2 + 4x_3 &= 7 \\ 2x_1 + x_2 + x_3 &= 4 \\ x_1 + 3x_2 + 5x_3 &= 3 \end{aligned}$$

19.
$$\begin{aligned} 5x_1 + 4x_3 + 2x_4 &= 3 \\ x_1 - x_2 + 2x_3 + x_4 &= 1 \\ 4x_1 + x_2 + 2x_3 &= 1 \\ x_1 + x_2 + x_3 + x_4 &= 0 \end{aligned}$$

20. Show that the system
$$\begin{aligned} 2x_1 - x_2 + 3x_3 &= a \\ 3x_1 + x_2 - 5x_3 &= b \\ -5x_1 - 5x_2 + 21x_3 &= c \end{aligned}$$
is inconsistent if $c \neq 2a - 3b$.

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1. $2x_1 + x_3 = 1$

$2x_1 + 4x_2 - x_3 = -2$

$x_1 - 8x_2 - 3x_3 = 2$

The augumented matrix is

$$A = \left[\begin{array}{ccc|c} 2 & 0 & 1 & 1 \\ 2 & 4 & -1 & -2 \\ 1 & -8 & -3 & 2 \end{array}\right]$$

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$$\underset{\sim}{R}\left[\begin{array}{ccc|c} 1 & -8 & -3 & 2 \\ 2 & 4 & -1 & -2 \\ 2 & 0 & 1 & 1 \end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c} 1 & -8 & -3 & 2 \\ 0 & 20 & 5 & -6 \\ 0 & 16 & 7 & -3 \end{array}\right] \begin{array}{l} R_2-2R_1 \\ R_3-2R_1 \end{array}$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c} 1 & -8 & -3 & 2 \\ 0 & 20 & 5 & -6 \\ 0 & 0 & 3 & 9/5 \end{array}\right] \quad R_3-\frac{16}{20}R_2$$

اس کو 1 بنانے کے بعد solve کریں گے تب بھی جواب یہی آئے گا۔

Rank = 3

By back substitution

$$x_1-8x_2-3x_3=2 \quad —①$$

$$20x_2+5x_3=-6 \quad —②$$

$$3x_3=\frac{9}{5} \quad —③$$

From equ ③

$$3x_3 = \frac{9}{5}$$

$$x_3 = \frac{\cancel{9}^{3}}{5} \times \frac{1}{\cancel{3}}$$

$$\boxed{x_3 = \frac{3}{5}}$$

put value of x_3 in equ ②

$$20x_2 + \cancel{5}\left(\frac{3}{\cancel{5}}\right) = -6$$

$$20x_2 + 3 = -6$$

$$20x_2 = -6 - 3$$

$$20x_2 = -9$$

$$\boxed{x_2 = \frac{-9}{20}}$$

put value of x_2, x_3 in equ ①

$$x_1 - \cancel{8}^{4\,2}\left(\frac{-9}{\cancel{20}_{\cancel{10}_{5}}}\right) - 3\left(\frac{3}{5}\right) = 2$$

$$x_1 + \frac{18}{5} - \frac{9}{5} = 2$$

$$x_1 + \frac{18-9}{5} = 2$$

$$x_1 + \frac{9}{5} = 2$$

$$x_1 = 2 - \frac{9}{5}$$

$$x_1 = \frac{10-9}{5}$$

$$\boxed{x_1 = \frac{1}{5}}$$

3. $x_1 - x_2 + x_3 - x_4 + x_5 = 1$

$2x_1 + x_2 + 3x_3 + 4x_5 = 3$

$3x_1 - 2x_2 + 2x_3 + x_4 + x_5 = 1$

$x_2 + x_4 + x_5 = 0$

$$\left[\begin{array}{ccccc|c} 1 & -1 & 1 & -1 & 1 & 1 \\ 2 & 1 & 3 & 0 & 4 & 3 \\ 3 & -2 & 2 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccccc|c} 1 & -1 & 1 & -1 & 1 & 1 \\ 0 & 3 & 1 & 2 & 2 & 1 \\ 0 & 1 & -1 & 4 & -2 & -2 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} \\ R_2-2R_1 \\ R_3-3R_1 \\ \\ \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & -1 & 1 & -1 & 1 & 1 \\ 0 & 3 & 1 & 2 & 2 & 1 \\ 0 & 0 & -4/3 & 10/3 & -8/3 & -7/3 \\ 0 & 0 & -1/3 & 1/3 & 1/3 & -1/3 \end{array}\right] \begin{array}{l} \\ R_3-\frac{1}{3}R_2 \\ \\ R_4-\frac{1}{3}R_2 \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & -1 & 1 & -1 & 1 & 1 \\ 0 & 3 & 1 & 2 & 2 & 1 \\ 0 & 0 & 4/3 & 10/3 & -8/3 & -7/3 \\ 0 & 0 & 0 & -1/2 & 1 & 1/4 \end{array}\right] \quad R_4 - \frac{1}{4}$$

Rank = 4

By back substitution

$x_1 - x_2 + x_3 - x_4 + x_5 = 1$ —— ①

$3x_2 + x_3 + 2x_4 + 2x_5 = 1$ —— ②

$-\frac{4}{3}x_3 + \frac{10}{3}x_4 - \frac{8}{3}x_5 = -\frac{7}{3}$ —— ③

$-\frac{1}{2}x_4 + x_5 = \frac{1}{4}$ —— ④

From equ ④

let $x_5 = t$

$-\frac{1}{2}x_4 + t = \frac{1}{4}$

$-\frac{1}{2}x_4 = \frac{1}{4} - t$

$$-x_4 = \left[\frac{1}{4} - t\right]2$$

$$-x_4 = \frac{\cancel{2}}{\cancel{4}_2} - 2t$$

$$-x_4 = \frac{1}{2} - 2t$$

$$\boxed{x_4 = -\frac{1}{2} + 2t}$$

From equ (3)

$$-\frac{4}{3}x_3 + \frac{10}{3}\left(\frac{-1}{2} + 2t\right) - \frac{8}{3}(t) = -\frac{7}{3}$$

$$-\frac{4}{3}x_3 - \frac{10}{6} + \frac{20t}{3} - \frac{8t}{3} = -\frac{7}{3}$$

$$-\frac{4}{3}x_3 - \frac{5}{3} + \frac{20t}{3} - \frac{8t}{3} = -\frac{7}{3}$$

$$\frac{-4x_3 - 5 + 20t - 8t}{3} = -\frac{7}{3}$$

$$\frac{-4x_3 - 5 + 12t}{3} = \frac{-7}{3}$$

$$-4x_3 - 5 + 12t = \frac{-7}{\cancel{3}} \times \cancel{3}$$

$$-4x_3 + 12t = -7 + 5$$

$$-4x_3 + 12t = -2$$

$$-4x_3 = -2 - 12t$$

$$-x_3 = \frac{-2 - 12t}{4}$$

$$-x_3 = \frac{-2}{4} - \frac{12t}{4}$$

$$-x_3 = \frac{-1}{2} - 3t$$

$$\boxed{x_3 = \frac{1}{2} + 3t}$$

From equ (2)

$$3x_2 + \frac{1}{2} + 3t + 2\left[-\frac{1}{2} + 2t\right] + 2t = 1$$

$3x_2 + \frac{1}{2} + 3t - 1 + 4t + 2t = 1$

$3x_2 + \frac{1}{2} - 1 + 9t = 1$

$\frac{6x_2 + 1 - 2 + 18t}{2} = 1$

$\frac{6x_2 - 1 + 18t}{2} = 1$

$6x_2 - 1 + 18t = 2$

$6x_2 + 18t = 2 + 1$

$6x_2 + 18t = 3$

$6(x_2 + 3t) = 3$

$x_2 + 3t = 3/\cancel{6}\,2$

$$\boxed{x_2 = \frac{1}{2} - 3t}$$

From equ ①

$$x_1 - \left[\frac{1}{2} - 3t\right] + \frac{1}{2} + 3t - \left[\frac{-1}{2} + 2t\right] + t = 1$$

$$x_1 - \frac{1}{2} + 3t + \frac{1}{2} + 3t + \frac{1}{2} - 2t + t = 1$$

$$\frac{2x_1 - \cancel{1} + 6t + \cancel{1} + 6t + 1 - 4t + 2t}{2} = 1$$

$$2x_1 + 10t + 1 = 2$$

$$2x_1 + 10t = 2 - 1$$

$$2x_1 + 10t = 1$$

$$2x_1 = 1 - 10t$$

$$x_1 = \frac{1}{2} - \frac{10}{2}t$$

$$\boxed{x_1 = \frac{1}{2} - 5t}$$

Replace t by x_5

7. $2x_1 + x_2 + 5x_3 = 4$

$3x_1 - 2x_2 + 2x_3 = 2$

$5x_1 - 8x_2 - 4x_3 = 1$

$$\left[\begin{array}{ccc|c} 2 & 1 & 5 & 4 \\ 3 & -2 & 2 & 2 \\ 5 & -8 & -4 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1/2 & 5/2 & 2 \\ 3 & -2 & 2 & 2 \\ 5 & -8 & -4 & 1 \end{array}\right] \quad \frac{R_1}{2}$$

$$\left[\begin{array}{ccc|c} 1 & 1/2 & 5/2 & 2 \\ 0 & -7/2 & -11/2 & -4 \\ 0 & -21/2 & -33/2 & -9 \end{array}\right] \quad \begin{array}{l} R_2 - 3R_1 \\ R_3 - 5R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1/2 & 5/2 & 2 \\ 0 & 1 & 11/7 & 8/7 \\ 0 & 1 & +33/21 & 18/21 \end{array}\right] \quad \begin{array}{l} R_2 \times -\frac{2}{7} \\ R_3 \times -\frac{2}{21} \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1/2 & 5/2 & 2 \\ 0 & 1 & 11/7 & 8/7 \\ 0 & 1 & +11/7 & 6/7 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1/2 & 5/2 & 2 \\ 0 & 1 & 11/7 & 8/7 \\ 0 & 0 & 0 & -2/7 \end{array}\right] \quad R_3 - R_2$$

This system has no solution.

Q. V.Imp $\lambda = ?$

$(1-\lambda)x_1 + x_2 = 0$
$x_1 + (1-\lambda)x_2 = 0$
System is homogeneous because these are 0

$$\left[\begin{array}{cc|c} 1 & 1-\lambda & 0 \\ 1-\lambda & 1 & 0 \end{array}\right]$$

trivial solution if $|A|$ is $\neq 0$

non-trivial solution if $|A| = 0$

$$A = \begin{bmatrix} 1 & 1-\lambda \\ 1-\lambda & 1 \end{bmatrix} \quad \text{— (1)}$$

$$|A| = \begin{vmatrix} 1 & 1-\lambda \\ 1-\lambda & 1 \end{vmatrix}$$

$= 1 - (1-\lambda)^2$

$= 1 - (1+\lambda^2 - 2\lambda)$

$= 1 - 1 - \lambda^2 + 2\lambda$

$= 2\lambda - \lambda^2$

System has non trivial solution if $|A| = 0$

$2\lambda - \lambda^2 = 0$

$\lambda(2-\lambda)=0$

$\boxed{\lambda=0}$ $\quad 2-\lambda=0$

$\boxed{\lambda=2}$

If $\lambda\neq 0$ and $\lambda\neq 2$ then system has trivial solution.

put $\lambda=0$ in equ ①

$\begin{bmatrix} 1 & 1-0 \\ 1-0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} R_2-R_1$

By back substitution

$x_1+x_2=0$

$x_2=-x_1$

Now put $\lambda=2$ in ①

$\begin{bmatrix} 1 & 1-2 \\ 1-2 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} R_2-R_1$

By back substitution

$x_1-x_2=0$

$x_1=x_2$

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$-\lambda = 0 \quad x_2 = x_1 \quad , \lambda = 2 \quad x_1 = x_2$

10. $(3-\lambda)x_1 - x_2 + x_3 = 0$

$x_1 - (1-\lambda)x_2 + x_3 = 0$

$x_1 - x_2 + (1-\lambda)x_3 = 0$

$$\begin{bmatrix} 3-\lambda & -1 & 1 \\ 1 & -(1-\lambda) & 1 \\ 1 & -1 & (1-\lambda) \end{bmatrix} \rightarrow ①$$

$$|A| = \begin{bmatrix} 1 & -1 & (1-\lambda) \\ (3-\lambda) & -1 & 1 \\ 1 & -(1-\lambda) & 1 \end{bmatrix}$$

$$|A| = 1\begin{vmatrix} -1 & 1 \\ -(1-\lambda) & 1 \end{vmatrix} - (-1)\begin{vmatrix} 3-\lambda & 1 \\ 1 & 1 \end{vmatrix} + 1-\lambda \begin{vmatrix} 3-\lambda & -1 \\ 1 & -(1-\lambda) \end{vmatrix}$$

$$= 1(-1+1-\lambda) + 1(3-\lambda-1) + (1-\lambda)[(3-\lambda)(-1+\lambda) + 1]$$

$= 1(-\lambda) + 1(2-\lambda) + (1-\lambda)[-3+3\lambda+\lambda-\lambda^2+1]$

$= -\lambda + 2 - \lambda + (1-\lambda)(-3+4\lambda-\lambda^2+1)$

$= -\lambda + 2 - \lambda + (-3) + 4\lambda - \lambda^2 + 1 + 3\lambda - 4\lambda^2 + \lambda^3 - \lambda$

$= \cancel{2} - 2\lambda - \cancel{3} + 7\lambda - 5\lambda^2 + \lambda^3 + 1 - \lambda$

$= 4\lambda - 5\lambda^2 + \lambda^3$

$|A| = 0$

$4\lambda - 5\lambda^2 + \lambda^3 = 0$

$\lambda(4 - 5\lambda + \lambda^2) = 0$

$\boxed{\lambda = 0}$, $\quad -5\lambda + 4 + \lambda^2 = 0$

$\lambda^2 - 5\lambda + 4 = 0$

$\lambda^2 - 4\lambda - \lambda + 4 = 0$

$\lambda(\lambda - 4) - 1(\lambda - 4) = 0$

$(\lambda - 4)(\lambda - 1) = 0$

$\lambda - 4 = 0 \qquad \lambda - 1 = 0$

$\boxed{\lambda = 4} \qquad \boxed{\lambda = 1}$

put $\lambda=0$ in equ ①

$$= \begin{bmatrix} 3-0 & -1 & 1 \\ 1 & -(1-0) & 1 \\ 1 & -1 & 1-0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 1 \\ 3 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix} R_1 \Longleftrightarrow R_2$$

$$= \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} R_2-3R_1 \\ R_3-R_1 \\ \end{matrix}$$

By back substitution

$x_1 - x_2 + x_3 = 0$ —②

$2x_2 - 2x_3 = 0$ —③

From equ ③

$2x_2 = 2x_3$

$x_2 = \frac{\cancel{2}}{\cancel{2}} x_3$

$x_2 = x_3$

put $x_2 = x_3$ in equ ②

$x_1 - \cancel{x_3} + \cancel{x_3} = 0$

$x_1 = 0$

Now put $\lambda = 1$

$$\begin{bmatrix} 3-1 & -1 & 1 \\ 1 & -(1-1) & 1 \\ 1 & -1 & 1-1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 1 \\ 1 & -1 & 0 \end{bmatrix} \quad R_1 \Longleftrightarrow R_2$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix} \quad \begin{matrix} \\ R_2 - 2R_1 \\ R_3 - R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix} \quad R_2 \times (-1)$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 + R_2$$

By back substitution

$x_1 + x_3 = 0$ — ②

$x_2 + x_3 = 0$ — ③

From equ ③

$x_2 = -x_3$

$-x_2 = x_3$

put value of x_3 in ②

$x_1 - x_2 = 0$

$x_1 = x_2$

So,

$x_1 = x_2 = -x_3$

Now put $\lambda = 4$ in ①

$$= \begin{bmatrix} 3-4 & -1 & 1 \\ 1 & -(1-4) & 1 \\ 1 & -1 & 1-4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & 1 \\ 1 & 3 & 1 \\ 1 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -1 \\ 1 & 3 & 1 \\ 1 & -1 & -3 \end{bmatrix} \quad R_1 \times (-1)$$

$$= \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 2 \\ 0 & -2 & -2 \end{bmatrix} \quad \begin{matrix} R_2 - R_1 \\ R_3 - R_1 \end{matrix}$$

$$= \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 + R_2$$

$$= \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \frac{R_2}{2}$$

By back substitution

$$x_1 + x_2 - x_3 = 0 \quad \text{--- } 2$$

$$x_2 + x_3 = 0 \quad \text{--- } 3$$

From equ 3

$$x_2 = -x_3 \quad \text{OR} \quad x_3 = -x_2$$

put this in equ 2

$$x_1 - x_3 - x_3 = 0$$

$$x_1 - 2x_3 = 0$$

$$x_1 = 2x_3$$

$$\boxed{\begin{array}{l} \lambda = 0, \; x_1 = 0, \; x_2 = x_3 \\ \lambda = 1, \; x_1 = x_2 = -x_3 \\ \lambda = 4, \; x_1 = 2x_3, \; x_3 = -x_2 \end{array}}$$

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11. $(1-\lambda)x_1 + x_2 - x_3 = 0$

$$x_1 - \lambda x_2 - 2x_3 = 0$$

$$x_1 + 2x_2 - \lambda x_3 = 0$$

$$A = \begin{bmatrix} 1-\lambda & 1 & -1 \\ 1 & -\lambda & -2 \\ 1 & 2 & -\lambda \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -\lambda & -2 \\ 1-\lambda & 1 & -1 \\ 1 & 2 & -\lambda \end{bmatrix} \text{ — } ①$$

$$|A| = 1\begin{vmatrix} 1 & -1 \\ 2 & -\lambda \end{vmatrix} + \lambda \begin{vmatrix} 1-\lambda & -1 \\ 1 & -\lambda \end{vmatrix} - 2\begin{vmatrix} 1-\lambda & 1 \\ 1 & 2 \end{vmatrix}$$

$$= 1(-\lambda+2) + \lambda[(1-\lambda)(-\lambda)+1] - 2(2-2\lambda-1)$$

$$= -\lambda + 2 + \lambda(-\lambda + \cancel{\lambda^2} + 1) - 2(1-2\lambda)$$

$$= \cancel{-\lambda} + \cancel{2} - \lambda^2 + \lambda^3 + \cancel{\lambda} - \cancel{2} + 4\lambda$$

$$= \lambda^3 - \lambda^2 + 4\lambda$$

$$|A| = 0$$

$$\lambda^3 - \lambda^2 + 4\lambda = 0$$

$$\lambda(\lambda^2 - \lambda + 4) = 0$$

$$\boxed{\lambda = 0} \qquad \lambda^2 - \lambda + 4 = 0$$

$$\lambda^2 - \lambda = -4$$

$$\lambda(\lambda - 1) = -4$$

$$\boxed{\lambda = -4} \qquad \lambda - 1 = -4$$

$$\lambda = -4 + 1$$

$$\boxed{\lambda = -3}$$

put $\lambda=0$ in equ ①

$$= \begin{bmatrix} 1 & 0 & -2 \\ 1-0 & 1 & -1 \\ 1 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -2 \\ 1 & 1 & -1 \\ 1 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix} \begin{matrix} R_2 - R_1 \\ R_3 - R_1 \\ \\ \end{matrix}$$

$$= \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} R_3 - 2R_2 \\ \\ \\ \end{matrix}$$

By back substitution

$x_1 - 2x_3 = 0$ —— 2

$x_2 + x_3 = 0$ —— 3

From 3

$x_2 = -x_3$

From equ 2

$x_1 + 2x_2 = 0$

$x_1 = -2x_2$

$\lambda = 0$, $x_1 = -2x_2$, $x_2 = -x_3$

Reduced echelon

12. $6x_1 - 6x_2 + 6x_3 = 6$

$2x_1 - 4x_2 - 6x_3 = 12$

$10x_1 - 5x_2 + 5x_3 = 30$

$$A = \left[\begin{array}{ccc|c} 6 & -6 & 6 & 6 \\ 2 & -4 & -6 & 12 \\ 10 & -5 & 5 & 30 \end{array}\right]$$

$$A = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 2 & -4 & -6 & 12 \\ 10 & -5 & 5 & 30 \end{array}\right] \quad \frac{R_1}{6}$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & -2 & -8 & 10 \\ 0 & 5 & -5 & 20 \end{array}\right] \quad \begin{array}{l} R_2 - 2R_1 \\ R_3 - 10R_1 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 1 & 4 & -5 \\ 0 & 5 & -5 & 20 \end{array}\right] \quad \frac{R_2}{-2}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 5 & -4 \\ 0 & 1 & 4 & -5 \\ 0 & 0 & -25 & 45 \end{array}\right] \quad \begin{array}{l} R_3 - 5R_2 \\ R_1 + R_2 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 5 & -4 \\ 0 & 1 & 4 & -5 \\ 0 & 0 & 1 & -\frac{9}{5} \end{array}\right] \quad \frac{R_3}{-25}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 11/5 \\ 0 & 0 & 1 & \frac{-9}{5} \end{array}\right] \begin{array}{l} R_1 - 5R_3 \\ R_2 - 4R_3 \\ \end{array}$$

$\boxed{x_1 = 5}$

$\boxed{x_2 = 11/5}$

$\boxed{x_3 = -9/5}$

Reduced echelon

13. $5x_1 + 5x_2 - x_3 = 0$

$10x_1 + 5x_2 + 2x_3 = 0$

$5x_1 + 15x_2 - 9x_3 = 0$

$$A = \begin{bmatrix} 5 & 5 & -1 \\ 10 & 5 & 2 \\ 5 & 15 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -1/5 \\ 10 & 5 & 2 \\ 5 & 15 & -9 \end{bmatrix} \frac{R_1}{5}$$

$$= \begin{bmatrix} 1 & 1 & -1/5 \\ 0 & -5 & 4 \\ 0 & 10 & -8 \end{bmatrix} \begin{matrix} R_2 - 10R_1 \\ R_3 - 5R_1 \\ \end{matrix}$$

$$= \begin{bmatrix} 1 & 1 & -1/5 \\ 0 & 1 & -4/5 \\ 0 & 10 & -8 \end{bmatrix} \frac{R_2}{-5}$$

$$= \begin{bmatrix} 1 & 1 & -1/5 \\ 0 & 1 & -4/5 \\ 0 & 0 & 0 \end{bmatrix} R_3 - 10R_2$$

By back substitution

$$x_1 + x_2 - \frac{1}{5}x_3 = 0 \quad \text{—} \ (1)$$

$$x_2 - \frac{4}{5}x_3 = 0 \quad \text{—} \ (2)$$

From (2)

$$\boxed{x_2 = 4/5\, x_3}$$

put value of x_2 in (1)

$$x_1 + \frac{4}{5}x_3 - \frac{1}{5}x_3 = 0$$

$$x_1 = \frac{3}{5}x_3 = 0$$

$$\boxed{x_1 = -\frac{3}{5}x_3}$$

x_3 arbitrary

CamScanner

18. $3x_1+2x_2+4x_3=7$

$2x_1+x_2+x_3=4$

$x_1+3x_2+5x_3=3$

$$A=\left[\begin{array}{ccc|c} 3 & 2 & 4 & 7 \\ 2 & 1 & 1 & 4 \\ 1 & 3 & 5 & 3 \end{array}\right]$$

$$=\left[\begin{array}{ccc|c} 1 & 1 & 3 & 3 \\ 2 & 1 & 1 & 4 \\ 1 & 3 & 5 & 3 \end{array}\right] \quad R_1-R_2$$

$$=\left[\begin{array}{ccc|c} 1 & 1 & 3 & 3 \\ 0 & -1 & -5 & -2 \\ 0 & 2 & 2 & 0 \end{array}\right] \quad \begin{array}{l} R_2-2R_1 \\ R_3-R_1 \end{array}$$

$$=\left[\begin{array}{ccc|c} 1 & 1 & 3 & 3 \\ 0 & 1 & 5 & 2 \\ 0 & 2 & 2 & 0 \end{array}\right] \quad R_2\times(-1)$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 3 & 3 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & -8 & -4 \end{array}\right] \quad R_3 - 2R_2$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & -8 & -4 \end{array}\right] \quad R_1 - R_2$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 1 & 1/2 \end{array}\right] \quad \frac{R_3}{-8}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & 1/2 \end{array}\right] \quad \begin{array}{l} R_1 + 2R_3 \\ R_2 - 5R_3 \end{array}$$

$$\boxed{x_1 = 2}$$

$$\boxed{x_2 = -1/2}$$

$$\boxed{x_3 = 1/2}$$

20

$2x_1 - x_2 + 3x_3 = a$

$3x_1 + x_2 - 5x_3 = b$ v.imp

$-5x_1 - 5x_2 + 21x_3 = c$

Show that system is inconsistent if $c \neq 2a - 3b$

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & a \\ 3 & 1 & -5 & b \\ -5 & -5 & 21 & c \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1/2 & 3/2 & a/2 \\ 3 & 1 & -5 & b \\ -5 & -5 & 21 & c \end{array}\right] \quad \frac{R_1}{2}$$

$$\left[\begin{array}{ccc|c} 1 & -1/2 & 3/2 & a/2 \\ 0 & 5/2 & -19/2 & \frac{2b-3a}{2} \\ 0 & -\frac{15}{2} & \frac{57}{2} & \frac{5a+2c}{2} \end{array}\right] \quad R_2 - 3R_1$$

$$\left[\begin{array}{ccc|c} 1 & -1/2 & 3/2 & a/0 \\ 0 & 5/2 & -19/2 & 2b-3a/2 \\ 0 & 0 & 0 & 3b-2a+c \end{array}\right] \quad R_3+3R_2$$

→ System is inconsistent if

$3b-2a+c \neq 0$

$c \neq 2a-3b$

→ system is consistent if

$3b-2a+c=0$

$c=2a-3b$