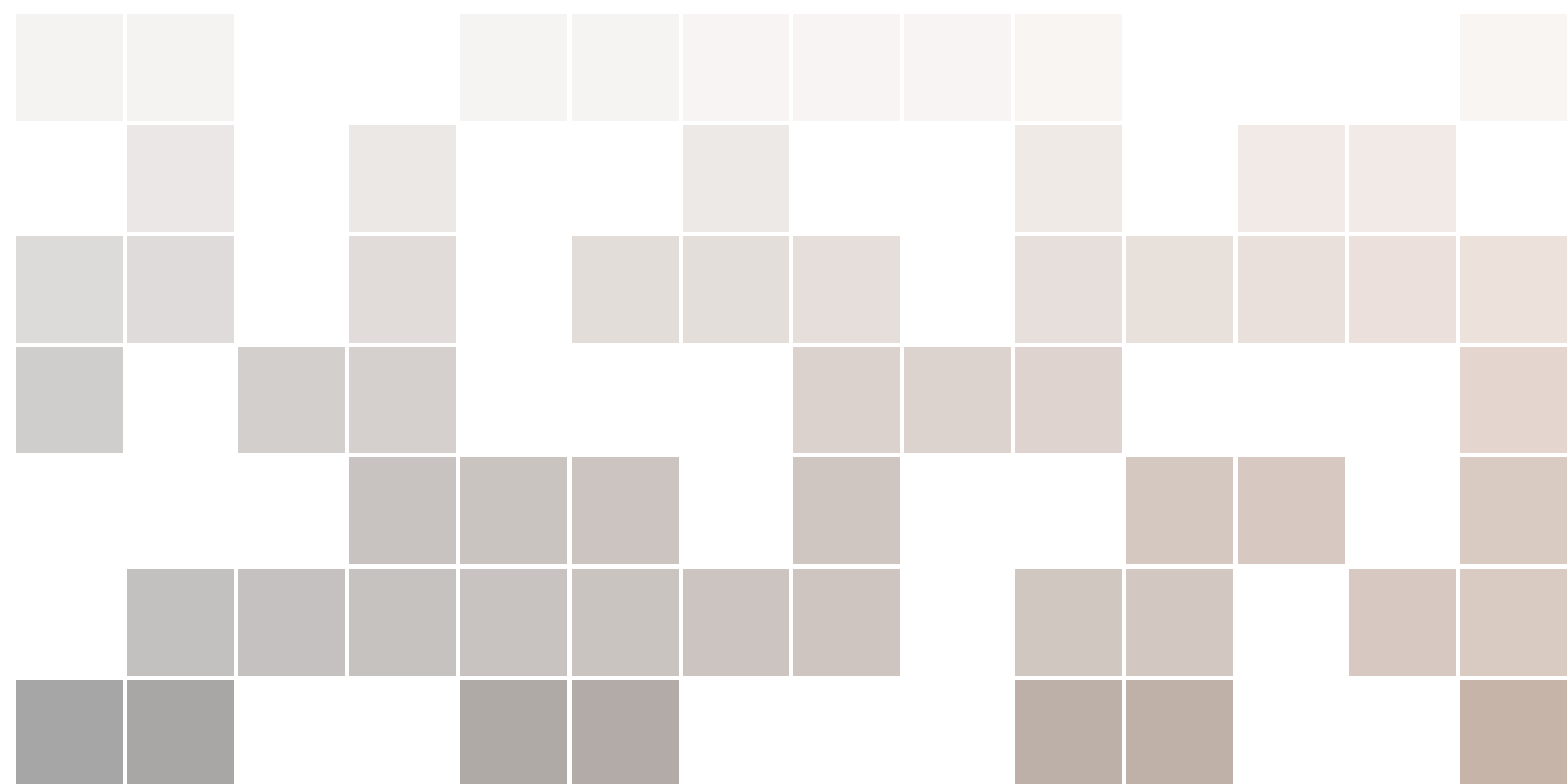


# VCE Mathematics Methods: Practice problems and solutions (Book 2)



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# 1. Introduction

## 1.1 Motivation

My year 9 daughter kept asking me to write her new math problems, apart from those in the text book, so she can do more practice. I'd take a piece of paper and write problems, but it's a tedious process. So I thought: how about I **generate tons** of them?

## 1.2 Problem generation method

The problems and solutions have all been generated using the python programming language, and relied on a number of open source python packages, especially the SymPy package. Therefore, you can trust the accuracy of the solutions.

## 1.3 Structure

This document includes hundreds of mathematics problems that have been designed for year 11 students enrolled in VCE Mathematics Methods.

Most of the problems require the student to perform a single task, such as calculating the derivative of a function or finding the prime numbers within a range of integers. In addition, a number of questions have been designed based on the Mathematics Methods exam questions, which require the student to perform two or more tasks.

I hope you find them useful.



## 2. Numbers

### 2.1 Problems

**Exercise 2.1** Find the prime factor decomposition of 109.

**Exercise 2.2** What are the lowest common multiple (LCM) of 346 and 770?

**Exercise 2.3** What are the prime numbers in the range 79 and 97?

**Exercise 2.4** Simplify the expression

$$\sqrt[3]{3024}$$

**Exercise 2.5** Find the prime factor decomposition of 130.

**Exercise 2.6** What are the lowest common multiple (LCM) of 728 and 401?

**Exercise 2.7** What are the prime numbers in the range 506 and 520?

**Exercise 2.8** What are the highest common factor (HCF) of 967 and 374?

**Exercise 2.9** Simplify the expression

$$\sqrt[3]{1250}$$

**Exercise 2.10** Simplify the expression

$$\sqrt[3]{20000}$$

**Exercise 2.11** Find the prime factor decomposition of 67.

**Exercise 2.12** Simplify the expression

$$\sqrt{225}$$

**Exercise 2.13** What are the prime numbers in the range 156 and 173?

**Exercise 2.14** What are the lowest common multiple (LCM) of 371 and 209?

**Exercise 2.15** What are the lowest common multiple (LCM) of 226 and 649?

**Exercise 2.16** What are the prime numbers in the range 165 and 184?

**Exercise 2.17** Simplify the expression

$$\sqrt[3]{1715}$$

**Exercise 2.18** What are the highest common factor (HCF) of 602 and 126?

**Exercise 2.19** What are the prime numbers in the range 350 and 366?

**Exercise 2.20** What are the prime numbers in the range 58 and 76?

**Exercise 2.21** What are the highest common factor (HCF) of 99 and 958?

**Exercise 2.22** What are the prime numbers in the range 161 and 172?

**Exercise 2.23** Simplify the expression

$$\sqrt[3]{13122}$$

**Exercise 2.24** What are the prime numbers in the range 139 and 159?

**Exercise 2.25** Find the prime factor decomposition of 130.

**Exercise 2.26** Simplify the expression

$$\sqrt{24}$$

**Exercise 2.27** Simplify the expression

$$\sqrt{1088}$$

**Exercise 2.28** What are the lowest common multiple (LCM) of 829 and 147?

**Exercise 2.29** Find the prime factor decomposition of 103.

**Exercise 2.30** What are the prime numbers in the range 622 and 642?

**Exercise 2.31** What are the highest common factor (HCF) of 473 and 816?

**Exercise 2.32** Simplify the expression

$$\sqrt{153}$$

**Exercise 2.33** What are the prime numbers in the range 520 and 539?

**Exercise 2.34** What are the lowest common multiple (LCM) of 75 and 87?

**Exercise 2.35** What are the lowest common multiple (LCM) of 378 and 218?

**Exercise 2.36** What are the prime numbers in the range 11 and 27?

**Exercise 2.37** Simplify the expression

$$\sqrt{225}$$

**Exercise 2.38** What are the prime numbers in the range 749 and 759?

**Exercise 2.39** Find the prime factor decomposition of 117.

**Exercise 2.40** Simplify the expression

$$\sqrt[3]{2125}$$



**Exercise 2.41** Simplify the expression

$$\sqrt[3]{3000}$$

**Exercise 2.42** Find the prime factor decomposition of 183.

**Exercise 2.43** Simplify the expression

$$\sqrt{1600}$$

**Exercise 2.44** What are the highest common factor (HCF) of 117 and 522?

**Exercise 2.45** What are the prime numbers in the range 137 and 149?

**Exercise 2.46** What are the prime numbers in the range 63 and 76?

**Exercise 2.47** What are the prime numbers in the range 48 and 66?

**Exercise 2.48** Simplify the expression

$$\sqrt[3]{48}$$

**Exercise 2.49** Simplify the expression

$$\sqrt[3]{19000}$$

**Exercise 2.50** Find the prime factor decomposition of 43.

**Exercise 2.51** What are the prime numbers in the range 156 and 175?

**Exercise 2.52** What are the prime numbers in the range 164 and 177?

**Exercise 2.53** What are the highest common factor (HCF) of 616 and 91?

**Exercise 2.54** Simplify the expression

$$\sqrt[3]{112}$$

**Exercise 2.55** Find the prime factor decomposition of 150.

**Exercise 2.56** What are the highest common factor (HCF) of 790 and 844?

**Exercise 2.57** Find the prime factor decomposition of 18.

**Exercise 2.58** Find the prime factor decomposition of 134.

**Exercise 2.59** What are the prime numbers in the range 171 and 187?

**Exercise 2.60** What are the lowest common multiple (LCM) of 711 and 59?

**Exercise 2.61** Simplify the expression

$$\sqrt[3]{56}$$

**Exercise 2.62** What are the prime numbers in the range 235 and 255?

**Exercise 2.63** What are the prime numbers in the range 470 and 487?

**Exercise 2.64** What are the prime numbers in the range 32 and 48?

**Exercise 2.65** Simplify the expression

$$\sqrt{56}$$

- Exercise 2.66** Find the prime factor decomposition of 65.  
**Exercise 2.67** What are the highest common factor (HCF) of 237 and 880?  
**Exercise 2.68** Find the prime factor decomposition of 98.  
**Exercise 2.69** What are the lowest common multiple (LCM) of 797 and 481?  
**Exercise 2.70** What are the highest common factor (HCF) of 8 and 958?  
**Exercise 2.71** What are the lowest common multiple (LCM) of 654 and 797?  
**Exercise 2.72** Simplify the expression

$$\sqrt{833}$$

- Exercise 2.73** What are the lowest common multiple (LCM) of 516 and 866?  
**Exercise 2.74** Find the prime factor decomposition of 106.  
**Exercise 2.75** Simplify the expression

$$\sqrt{208}$$

- Exercise 2.76** What are the highest common factor (HCF) of 392 and 39?  
**Exercise 2.77** Simplify the expression

$$\sqrt[3]{625}$$

- Exercise 2.78** What are the prime numbers in the range 848 and 858?  
**Exercise 2.79** What are the prime numbers in the range 200 and 215?  
**Exercise 2.80** Find the prime factor decomposition of 64.  
**Exercise 2.81** Simplify the expression

$$\sqrt[3]{832}$$

- Exercise 2.82** What are the prime numbers in the range 19 and 36?  
**Exercise 2.83** What are the prime numbers in the range 100 and 120?  
**Exercise 2.84** What are the lowest common multiple (LCM) of 115 and 234?  
**Exercise 2.85** Find the prime factor decomposition of 40.  
**Exercise 2.86** What are the prime numbers in the range 992 and 1007?  
**Exercise 2.87** What are the lowest common multiple (LCM) of 473 and 449?  
**Exercise 2.88** Simplify the expression

$$\sqrt{243}$$

- Exercise 2.89** Find the prime factor decomposition of 120.  
**Exercise 2.90** What are the highest common factor (HCF) of 121 and 680?  
**Exercise 2.91** What are the highest common factor (HCF) of 700 and 297?  
**Exercise 2.92** Simplify the expression

$$\sqrt{384}$$

**Exercise 2.93** Simplify the expression

$$\sqrt{216}$$

**Exercise 2.94** Simplify the expression

$$\sqrt{686}$$

**Exercise 2.95** Simplify the expression

$$\sqrt[3]{4608}$$

**Exercise 2.96** Simplify the expression

$$\sqrt{288}$$

**Exercise 2.97** What are the lowest common multiple (LCM) of 10 and 833?

**Exercise 2.98** What are the lowest common multiple (LCM) of 374 and 476?

**Exercise 2.99** What are the highest common factor (HCF) of 5 and 248?

**Exercise 2.100** What are the prime numbers in the range 171 and 183?

## 2.2 Solutions

1.

$$109^1$$

2. 133210

3. 79, 83, 89

4.

$$6 \cdot \sqrt[3]{14}$$

5.

$$13^1 \cdot 2^1 \cdot 5^1$$

6. 291928

7. 509

8. 1

9.

$$5 \cdot \sqrt[3]{10}$$

10.

$$10 \cdot \sqrt[3]{20}$$

11.

$$67^1$$

12.

15

13. 157, 163, 167

14. 77539

15. 146674

16. 167, 173, 179, 181

17.

$$7 \cdot \sqrt[3]{5}$$

18. 14

19. 353, 359

20. 59, 61, 67, 71, 73

21. 1

22. 163, 167

23.

$$9 \cdot \sqrt[3]{18}$$

24. 139, 149, 151, 157

25.

$$13^1 \cdot 2^1 \cdot 5^1$$

26.

$$2\sqrt{6}$$

27.

$$8\sqrt{17}$$

28. 121863

29.

$$103^1$$

30. 631, 641

31. 1

32.

$$3\sqrt{17}$$

33. 521, 523

34. 2175

35. 41202

36. 11, 13, 17, 19, 23

37.

15

38. 751, 757

39.

$$13^1 \cdot 3^2$$

40.

$$5 \cdot \sqrt[3]{17}$$

41.

$$10 \cdot \sqrt[3]{3}$$

42.

$$3^1 \cdot 61^1$$

43.

$$40$$

44. 9

45. 137, 139

46. 67, 71, 73

47. 53, 59, 61

48.

$$2 \cdot \sqrt[3]{6}$$

49.

$$10 \cdot \sqrt[3]{19}$$

50.

$$43^1$$

51. 157, 163, 167, 173

52. 167, 173

53. 7

54.

$$2 \cdot \sqrt[3]{14}$$

55.

$$2^1 \cdot 3^1 \cdot 5^2$$

56. 2

57.

$$2^1 \cdot 3^2$$

58.

$$2^1 \cdot 67^1$$

59. 173, 179, 181

60. 41949

61.

$$2 \cdot \sqrt[3]{7}$$

62. 239, 241, 251

63. 479

64. 37, 41, 43, 47

65.

$$2\sqrt{14}$$

66.

$$13^1 \cdot 5^1$$

67. 1

68.

$$2^1 \cdot 7^2$$

69. 383357

70. 2

71. 521238

72.

$$7\sqrt{17}$$

73. 223428

74.

$$2^1 \cdot 53^1$$

75.

$$4\sqrt{13}$$

76. 1

77.

$$5 \cdot \sqrt[3]{5}$$

78. 853, 857

79. 211

80.

$$2^6$$

81.

$$4 \cdot \sqrt[3]{13}$$

82. 19, 23, 29, 31

83. 101, 103, 107, 109, 113

84. 26910

85.

$$2^3 \cdot 5^1$$

86. 997

87. 212377

88.

$$9\sqrt{3}$$

89.

$$2^3 \cdot 3^1 \cdot 5^1$$

90. 1

91. 1

92.

$$8\sqrt{6}$$

93.

$$6\sqrt{6}$$

94.

$$7\sqrt{14}$$

95.

$$8 \cdot 3^{\frac{2}{3}}$$

96.

$$12\sqrt{2}$$

97. 8330

98. 5236

99. 1

100. 173, 179, 181





## 3. Algebra

### 3.1 Problems

**Exercise 3.1** Expand

$$(2a^2c^3 + 3ab^4c^4 - 2b^4c^2)^2$$

**Exercise 3.2** Evaluate the first 4 terms of the following recurrence sequence:

$$G(p) = -2p + 2G(p-1)$$

,  
where  $G(0) = 3$ .

**Exercise 3.3** Evaluate the first 6 terms of the following recurrence sequence:

$$M(r) = -4r^3 + 2M(r-2) - 4M(r-1)$$

,  
where  $M(0) = 2$ ,  $M(1) = 0$ .

**Exercise 3.4** Simplify

$$\frac{(-1)p^2q^3r^4}{(-1)7p^2q^4r^8}$$

**Exercise 3.5** Solve the following simultaneous equations for  $x$  and  $y$ :

$$16ax + 12y + 5 = 0$$

$$3a - 45x - 24y = 0$$

**Exercise 3.6** Simplify

$$\frac{36n^5}{6n^4}$$

**Exercise 3.7** Simplify

$$\frac{8m^3}{6m^3 - 2m^2} - \frac{2m}{-4m - 9}$$

**Exercise 3.8** Simplify

$$\frac{4 - 5c^3}{c^2 + 6} + \frac{2c^3 - c^2}{-3c - 10}$$

**Exercise 3.9** Expand

$$\left(-6p^2r^3 - \frac{1}{2r}\right)^2$$

**Exercise 3.10** Factorize the polynomial

$$-80v^2 + 66v - 7$$

**Exercise 3.11** Solve the equation in  $x$ :

$$-2a^2x + 6a - 2x^2 - 3 = 0$$

**Exercise 3.12** Solve the following simultaneous equations for  $x$  and  $y$ :

$$-8ay - 6a + 18x = 0$$

$$-81x - 27y - 10 = 0$$

**Exercise 3.13** Evaluate the first 4 terms of the following recurrence sequence:

$$q(v) = -9v - \frac{9}{q(v-1)}$$

, where  $q(0) = 2$ .

**Exercise 3.14**

**Exercise 3.15** Simplify

$$\frac{-12r^7s^5t^8 - 28r^7s^3t^5 - 21r^6s^4t^5 - 49r^6s^2t^2 - 12r^4st^8 - 21r^3t^5}{4r^4st^4 + 7r^3t}$$

**Exercise 3.16** Expand and simplify

$$(3m^2 - 8n^4)^2 \cdot (4m^2n^4 + 3mn)$$

**Exercise 3.17** Evaluate the first 4 terms of the following recurrence sequence:

$$M(l) = 3l(3l - M(l-1))$$

,

where  $M(0) = 1$ .

**Exercise 3.18** Simplify

$$\frac{-6x^2 - 3}{-6x^3 - 7x^2} - \frac{8x^2 - 4x}{-7x^2 - 9x}$$

**Exercise 3.19** Expand

$$\left(8 + \frac{1}{4q^2r^2}\right)^2$$

**Exercise 3.20** Evaluate the first 4 terms of the following recurrence sequence:

$$R(s) = 3s^2 - \frac{R(s-1)}{3}$$

,

where  $R(0) = 7$ .

**Exercise 3.21** Simplify

$$-\frac{-2l-10}{3l} + \frac{10l-5}{10l}$$

**Exercise 3.22** Expand

$$(-7p^2 + 6q^4)(10q^3 - 3q^2)^2$$

**Exercise 3.23** Evaluate the first 6 terms of the following recurrence sequence:

$$x(u) = -u^2 - ux(u-1) - 6x(u-2)$$

,

where  $x(0) = 8, x(1) = 10$ .

**Exercise 3.24** Factorize the polynomial

$$-30v^2 - 18v + 12$$

**Exercise 3.25** Evaluate the first 4 terms of the following recurrence sequence:

$$M(w) = -4w + M(w-1)$$

,

where  $M(0) = 3$ .

**Exercise 3.26** Simplify

$$\frac{-20r^2s^4 + 15r^2s^3 - 5r^2}{(-1)5r^2}$$

**Exercise 3.27** Evaluate the first 6 terms of the following recurrence sequence:

$$F(p) = 7p^3 - 8F(p-2) - 3F(p-1)$$

,

where  $F(0) = 6, F(1) = 0$ .

**Exercise 3.28** Solve the following simultaneous equations for  $x$  and  $y$ :

$$10ax - 6a + 48y = 0$$

$$16ay + 5a + 20x = 0$$

**Exercise 3.29** Solve the following inequality for  $l$

$$-10l + t \leq 9l - 6$$

**Exercise 3.30** Simplify

$$\frac{3l^3mn^4}{12l^4m^4n^4 + 24l^3m^4n^6 - 30l^3mn^6}$$

**Exercise 3.31** Expand and simplify

$$(6r^2 + 9rs)^2 (-7r^4s^2 - 2r^3s^4t)$$

**Exercise 3.32** Solve the following inequality

$$8u - 7 < 6$$

**Exercise 3.33** Evaluate the first 6 terms of the following recurrence sequence:

$$M(l) = -9l^2 - lM(l-1) - 6M(l-2)$$

,

where  $M(0) = 8$ ,  $M(1) = 7$ .

**Exercise 3.34** Solve the following simultaneous equations for  $x$  and  $y$ :

$$49ax + 35ay + 3 = 0$$

$$-6ax + 24y + 4 = 0$$

**Exercise 3.35** Evaluate the first 6 terms of the following recurrence sequence:

$$T(a) = 8a^2 + aT(a-2) - 4T(a-1)$$

,

where  $T(0) = 5$ ,  $T(1) = 8$ .

**Exercise 3.36** Simplify

$$\frac{3l^4mn^4}{30l^8m^5n^5 + 15l^8m^5n^4 + 30l^4m^5n^5 - 18l^4mn^6}$$

**Exercise 3.37** Expand

$$(-7p^4q^2 - 10q^3r^4)^2 \cdot (4q^4r^3 + 2)$$

**Exercise 3.38** Solve the following inequality for  $r$

$$9p^2 - r \geq -7p^2 - 5r$$

**Exercise 3.39** Solve the equation in  $x$ :

$$7ax - 24a = 0$$

**Exercise 3.40** Simplify

$$\frac{z}{3z+6} - \frac{13z}{-z-8}$$

**Exercise 3.41** Simplify

$$-\frac{13r}{4} - \frac{3}{4-4r}$$

**Exercise 3.42** Evaluate the first 4 terms of the following recurrence sequence:

$$M(t) = t^2 - 3M(t-1)$$

,

where  $M(0) = 2$ .

**Exercise 3.43** Solve the following simultaneous equations for  $x$  and  $y$ :

$$-60ax + 6ay + 2a = 0$$

$$12ay - 9a - 8x = 0$$

**Exercise 3.44** Solve the equation in  $x$ :

$$7ax - 4a + 9 = 0$$

**Exercise 3.45** Solve the equation in  $x$ :

$$-10ax^2 - 3a + 7x = 0$$

**Exercise 3.46** Factorize the polynomial

$$-16a^2 - 46a - 28$$

**Exercise 3.47** Expand

$$\left(8 - \frac{1}{5q^2}\right)^2$$

**Exercise 3.48** Evaluate the first 4 terms of the following recurrence sequence:

$$y(d) = d(-6d + y(d-1))$$

,

where  $y(0) = 0$ .

**Exercise 3.49** Simplify

$$\frac{-30l^5m^4 - 30l^4m^5n^3}{-5l^4 - 5l^3mn^3}$$

**Exercise 3.50** Simplify

$$-\frac{6q+1}{16q} - \frac{1}{2q}$$

**Exercise 3.51** Simplify

$$\frac{n^4}{(-1)2n^4}$$

**Exercise 3.52** Simplify

$$\frac{2mn^3 + 18mn^2 - 2n^5 - 18n^4}{-2mn^2 + 2n^4}$$

**Exercise 3.53** Solve the equation in  $x$ :

$$-4ax + 5a + 15x + 8 = 0$$

**Exercise 3.54** Solve the following inequality

$$6 - 5y \leq 2$$

**Exercise 3.55** Solve the following simultaneous equations for  $x$  and  $y$ :

$$7x + 8y = 0$$

$$6x + y - 8 = 0$$

**Exercise 3.56** Simplify

$$\frac{(-1)7n}{-42m^4n^2 - 49m^4n + 7m^2n^2 + 49mn^5}$$

**Exercise 3.57** Solve the equation in  $x$ :

$$6a^2x^2 - 3a^2 - 4x^2 - 6x = 0$$

**Exercise 3.58** Expand and simplify

$$(7x^2 + 10x)(6x^4 - 9z^2)^2$$

**Exercise 3.59** Simplify

$$\frac{4q}{6 - 6q} - \frac{-6q^3 - 7}{-q^2 + 4q}$$

**Exercise 3.60** Simplify

$$\frac{9c^3d^4}{90a^3bc^6d^5 - 54a^2c^7d^4 + 54a^2c^3d^4 + 36c^7d^8}$$

**Exercise 3.61** Simplify

$$\frac{27r^4s - 6r^3st^4 + 36r^3st - 8r^2st^5}{-3r^3s - 4r^2st}$$

**Exercise 3.62** Evaluate the first 4 terms of the following recurrence sequence:

$$G(q) = -5q^2 + 2G(q - 1)$$

,

where  $G(0) = 0$ .

**Exercise 3.63** Evaluate the first 4 terms of the following recurrence sequence:

$$M(x) = x(9x - M(x-1))$$

,

where  $M(0) = 0$ .

**Exercise 3.64** Simplify

$$\frac{8q+4}{-9q^3-2} + \frac{6q^2+6q}{5q}$$

**Exercise 3.65** Evaluate the first 4 terms of the following recurrence sequence:

$$T(m) = -2m^2 - T(m-1)$$

,

where  $T(0) = 2$ .

**Exercise 3.66** Simplify

$$\frac{3-9z}{10z-5} + \frac{7}{13z}$$

**Exercise 3.67** Simplify

$$\frac{8a^5b^4 + 3a^4b^4}{-8a-3}$$

**Exercise 3.68** Simplify

$$-2b + \frac{-4b-1}{8b-1} - \frac{3}{5}$$

**Exercise 3.69** Solve the following simultaneous equations for  $x$  and  $y$ :

$$-9x - 5y + 1 = 0$$

$$-3x - 7y + 1 = 0$$

**Exercise 3.70** Solve the following inequality

$$7w + 1 < 5$$

**Exercise 3.71** Expand

$$(5p^4q^3 + 5p^4 - 8)^2$$

**Exercise 3.72** Evaluate the first 6 terms of the following recurrence sequence:

$$R(w) = 3wR(w-2) - 3w - R(w-1)$$

,

where  $R(0) = 10$ ,  $R(1) = 8$ .

**Exercise 3.73** Expand

$$(7 - 10r^2)(6r^3 + 2s^3)(8r^3s^3 + s)$$

**Exercise 3.74** Solve the equation in  $x$ :

$$-2a^2 + 4ax^2 - 10ax = 0$$

**Exercise 3.75** Complete the square for the following expression and then solve the equation:

$$-7v^2 - 60v$$

**Exercise 3.76** Simplify

$$\frac{21l^5n^3 + 21l^3m^3 - 9l^2m^2n^3 - 9l^2n^6 - 15l^2n^3 - 9m^5 - 9m^3n^3 - 15m^3}{3l^2n^3 + 3m^3}$$

**Exercise 3.77** Evaluate the first 4 terms of the following recurrence sequence:

$$y(u) = -8u + y(u - 1)$$

,

where  $y(0) = 6$ .

**Exercise 3.78** Factorize the polynomial

$$-70s^2 - 60s + 10$$

**Exercise 3.79** Solve the following inequality

$$9z - 10 < 9$$

**Exercise 3.80** If  $a = 9b^4c^3d^2 + 8b^3c^3$ , show that

$$(a - x)^2 + (a + x)^2 = 162b^8c^6d^4 + 288b^7c^6d^2 + 128b^6c^6 + 2x^2$$

**Exercise 3.81** Solve the following inequality

$$3c + 5 > -4$$

**Exercise 3.82** Factorize the polynomial

$$54 - 54m^2$$

**Exercise 3.83** Expand

$$\left(b^4c^2 - \frac{1}{ab^4c^3}\right)^2$$

**Exercise 3.84** Expand

$$\left(2m^3n - \frac{1}{7m^2n^3}\right)^2$$

**Exercise 3.85** Expand

$$(-6l^2 - 9lm^4n^4 - 3m)^2$$



**Exercise 3.86** Evaluate the first 4 terms of the following recurrence sequence:

$$q(a) = -7a + q(a-1)$$

,

where  $q(0) = 6$ .

**Exercise 3.87** Solve the following simultaneous equations for  $x$  and  $y$ :

$$3x + 10y + 10 = 0$$

$$-9x + 7y + 9 = 0$$

**Exercise 3.88** Expand

$$\left(6q^2 - \frac{1}{2p^3q^3}\right)^2$$

**Exercise 3.89** Expand

$$(-6r^3 + 3s^3)(5r^4 - 2r^3 + 4s^4)$$

**Exercise 3.90** Solve the equation in  $x$ :

$$ax - 4x + 3 = 0$$

**Exercise 3.91** If  $a = -4bc^3d^3 - 10c$ , show that

$$-(a-x)^3 + (a+x)^3 = 32b^2c^6d^6 + 160bc^4d^3 + 200c^2 + 2x^2$$

**Exercise 3.92** Expand

$$(3ln - 8)(2l^3m^3 + n)(-2lm^2n^4 + 4m^4)$$

**Exercise 3.93** Expand

$$\left(-2w^3 - \frac{1}{v}\right)^2$$

**Exercise 3.94** Evaluate the first 6 terms of the following recurrence sequence:

$$x(b) = -5b - 6x(b-2) - 4x(b-1)$$

,

where  $x(0) = 2, x(1) = 6$ .

**Exercise 3.95** If  $a = 7b^2c^3 + 8c^4$ , show that

$$(a-x)^2 + (a+x)^2 = 98b^4c^6 + 224b^2c^7 + 128c^8 + 2x^2$$

**Exercise 3.96** Simplify

$$\frac{10m^3 - n^3}{-20m^3n^4 - 50m^3n + 2n^7 + 5n^4}$$

**Exercise 3.97** Evaluate the first 6 terms of the following recurrence sequence:

$$y(b) = -7b^2 - 2by(b-2) - 2y(b-1)$$

,

where  $y(0) = 1$ ,  $y(1) = 9$ .

**Exercise 3.98** Expand

$$(-2a^4 - 7a^2c^4d + 8d^3)^2$$

**Exercise 3.99** Solve the equation in  $x$ :

$$4ax - 4a - 10x = 0$$

**Exercise 3.100** Solve the following inequality

$$7 - 10y \leq 8$$

### 3.2 Solutions

1.

$$4a^4c^6 + 12a^3b^4c^7 + 9a^2b^8c^8 - 8a^2b^4c^5 - 12ab^8c^6 + 4b^8c^4$$

2. 3, 4, 4, 2, -4

3. 2, 0, -28, 4, -328, 820

4.

$$\frac{1}{7qr^4}$$

5.

$$\left\{ x : \frac{-3a-10}{32a-45}, y : \frac{16a^2+75}{128a-180} \right\}$$

6.

$$6n$$

7.

$$\frac{22m^2 + 34m}{12m^2 + 23m - 9}$$

8.

$$\frac{-2c^5 - 14c^4 - 62c^3 + 6c^2 + 12c + 40}{3c^3 + 10c^2 + 18c + 60}$$

9.

$$36p^4r^6 + 6p^2r^2 + \frac{1}{4r^2}$$

10.

$$7 - 10v$$

and

$$8v - 1$$

11.

$$\left\{ -\frac{a^2}{2} - \frac{\sqrt{a^4 + 12a - 6}}{2}, -\frac{a^2}{2} + \frac{\sqrt{a^4 + 12a - 6}}{2} \right\}$$

12.

$$\left\{ x: \frac{41a}{324a + 243}, y: \frac{-27a - 10}{36a + 27} \right\}$$

13. 2, -13.5000, -17.3333, -26.4808, -35.6601

14.

$$\frac{80n^2w^2 - 360n^2w - 200n^2 - 90nw^3 + 405nw^2 + 225nw - 126w - 140}{70nw^2 - 315nw - 175n}$$

15.

$$t(-3r^3s^4t^3 - 7r^3s^2 - 3t^3)$$

16.

$$mn(36m^5n^3 + 27m^4 - 192m^3n^7 - 144m^2n^4 + 256mn^{11} + 192n^8)$$

17. 1, 6, 0, 81, -828

18.

$$\frac{48x^4 + 74x^3 + 26x^2 + 21x + 27}{42x^4 + 103x^3 + 63x^2}$$

19.

$$64 + \frac{4}{q^2r^2} + \frac{1}{16q^4r^4}$$

20. 7, 0.6667, 11.7778, 23.0741, 40.3086

21.

$$\frac{10l + 17}{6l}$$

22.

$$-700p^2q^6 + 420p^2q^5 - 63p^2q^4 + 600q^{10} - 360q^9 + 54q^8$$

23. 8, 10, -72, 147, -172, -47

24.

$$6v + 6$$

and

$$2 - 5v$$

25. 3, -1, -9, -21, -37

26.

$$4s^4 - 3s^3 + 1$$

27. 6, 0, 8, 165, -111, -112

28.

$$\left\{ x: \frac{6a^2 + 15a}{10a^2 - 60}, y: \frac{-5a^2 - 12a}{16a^2 - 96} \right\}$$

29.

$$l \geq \frac{t}{19} + \frac{6}{19}$$

30.

$$\frac{1}{2 \cdot (2lm^3 + 4m^3n^2 - 5n^2)}$$

31.

$$r^5s^2(-252r^3 - 72r^2s^2t - 756r^2s - 216rs^3t - 567rs^2 - 162s^4t)$$

32.

$$u < \frac{13}{8}$$

33. 8, 7, -98, 171, -240, -51

34.

$$\left\{ x: \frac{70a - 36}{105a^2 + 588a}, y: -\frac{107}{105a + 588} \right\}$$

35. 5, 8, 10, 56, -56, 704

36.

$$\frac{1}{10l^4m^4n + 5l^4m^4 + 10m^4n - 6n^2}$$

37.

$$196p^8q^8r^3 + 98p^8q^4 + 560p^4q^9r^7 + 280p^4q^5r^4 + 400q^{10}r^{11} + 200q^6r^8$$

38.

$$r \geq -4p^2$$

39.

$$\left\{ \frac{24}{7} \right\}$$

40.

$$\frac{40z^2 + 86z}{3z^2 + 30z + 48}$$

41.

$$\frac{-13r^2 + 13r + 3}{4r - 4}$$

42. 2, -5, 19, -48, 160

43.

$$\left\{ x : \frac{13a}{120a - 8}, y : \frac{135a + 4}{180a - 12} \right\}$$

44.

$$\left\{ \frac{4a - 9}{7a} \right\}$$

45.

$$\left\{ -\frac{\sqrt{49 - 120a^2}}{20a} + \frac{7}{20a}, \frac{\sqrt{49 - 120a^2}}{20a} + \frac{7}{20a} \right\}$$

46.

$$-8a - 7$$

and

$$2a + 4$$

47.

$$64 - \frac{16}{5q^2} + \frac{1}{25q^4}$$

48. 0, -6, -36, -162, -744

49.

$$6lm^4$$

50.

$$\frac{-6q - 9}{16q}$$

51.

$$-\frac{1}{2}$$

52.

$$-n - 9$$

53.

$$\left\{ \frac{5a + 8}{4a - 15} \right\}$$

54.

$$\frac{4}{5} \leq y$$

55.

$$\left\{ x : \frac{64}{41}, y : -\frac{56}{41} \right\}$$

56.

$$\frac{1}{m(6m^3n + 7m^3 - mn - 7n^4)}$$

57.

$$\left\{ -\frac{\sqrt{3}\sqrt{6a^4 - 4a^2 + 3}}{2 \cdot (3a^2 - 2)} + \frac{3}{2 \cdot (3a^2 - 2)}, \frac{\sqrt{3}\sqrt{6a^4 - 4a^2 + 3}}{2 \cdot (3a^2 - 2)} + \frac{3}{2 \cdot (3a^2 - 2)} \right\}$$

58.

$$9x(28x^9 + 40x^8 - 84x^5z^2 - 120x^4z^2 + 63xz^4 + 90z^4)$$

59.

$$\frac{-18q^4 + 16q^3 + 8q^2 - 21q + 21}{3q^3 - 15q^2 + 12q}$$

60.

$$\frac{1}{2 \cdot (5a^3bc^3d - 3a^2c^4 + 3a^2 + 2c^4d^4)}$$

61.

$$-9r + 2t^4$$

62. 0, -5, -30, -105, -290

63. 0, 9, 18, 27, 36

64.

$$\frac{54q^4 + 54q^3 - 28q - 8}{45q^3 + 10}$$

65. 2, -4, -4, -14, -18

66.

$$\frac{-117z^2 + 109z - 35}{130z^2 - 65z}$$

67.

$$-a^4b^4$$

68.

$$\frac{-80b^2 - 34b - 2}{40b - 5}$$

69.

$$\left\{x : \frac{1}{24}, y : \frac{1}{8}\right\}$$

70.

$$w < \frac{4}{7}$$

71.

$$25p^8q^6 + 50p^8q^3 + 25p^8 - 80p^4q^3 - 80p^4 + 64$$

72. 10, 8, 46, 17, 523, -283

73.

$$-480r^8s^3 + 336r^6s^3 - 160r^5s^6 - 60r^5s + 112r^3s^6 + 42r^3s - 20r^2s^4 + 14s^4$$

74.

$$\left\{\frac{5}{4} - \frac{\sqrt{8a+25}}{4}, \frac{\sqrt{8a+25}}{4} + \frac{5}{4}\right\}$$

75.

$$\left[-8, -\frac{4}{7}\right]$$

76.

$$7l^3 - 3m^2 - 3n^3 - 5$$

77. 6, -2, -18, -42, -74

78.

$$-10s - 10$$

and

$$7s - 1$$

79.

$$z < \frac{19}{9}$$

80. Provided in problem statement

81.

$$-3 < c$$

82.

$$-9m - 9$$

and

$$6m - 6$$

83.

$$b^8c^4 - \frac{2}{ac} + \frac{1}{a^2b^8c^6}$$

84.

$$4m^6n^2 - \frac{4m}{7n^2} + \frac{1}{49m^4n^6}$$

85.

$$36l^4 + 108l^3m^4n^4 + 81l^2m^8n^8 + 36l^2m + 54lm^5n^4 + 9m^2$$

86. 6, -1, -15, -36, -64

87.

$$\left\{ x : \frac{20}{111}, y : -\frac{39}{37} \right\}$$

88.

$$36q^4 - \frac{6}{p^3q} + \frac{1}{4p^6q^6}$$

89.

$$-30r^7 + 12r^6 + 15r^4s^3 - 24r^3s^4 - 6r^3s^3 + 12s^7$$

90.

$$\left\{ -\frac{3}{a-4} \right\}$$

91. Provided in problem statement

92.

$$-12l^5m^5n^5 + 24l^4m^7n + 32l^4m^5n^4 - 64l^3m^7 - 6l^2m^2n^6 + 12lm^4n^2 + 16lm^2n^5 - 32m^4n$$

93.

$$4w^6 + \frac{4w^3}{v} + \frac{1}{v^2}$$

94. 2, 6, -46, 133, -276, 281

95. Provided in problem statement

96.

$$-\frac{1}{n(2n^3+5)}$$

97. 1, 9, -50, -17, 322, -649

98.

$$4a^8 + 28a^6c^4d + 49a^4c^8d^2 - 32a^4d^3 - 112a^2c^4d^4 + 64d^6$$

99.

$$\left\{ \frac{2a}{2a-5} \right\}$$

100.

$$-\frac{1}{10} \leq y$$



## 4. Functions

### 4.1 Problems

**Exercise 4.1** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 7 - 6x$$

and

$$g(x) = 7a - 6x$$

**Exercise 4.2** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -8ax^2 + 10a - 6x$$

and

$$g(x) = -3ax + 4$$

**Exercise 4.3** If  $f(x)$  is defined as

$$f(x) = 3ax - 10x^2$$

and  $g(x)$  is defined as

$$g(x) = 3x + 6$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 4.4** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -10a + 2x$$

and

$$g(x) = 2a - x$$

**Exercise 4.5** If  $f(x)$  is defined as

$$f(x) = \frac{x+1}{5-8x}$$

find  $f^{-1}(x)$ .

**Exercise 4.6** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 3ax^2 - 10a + 6x$$

and

$$g(x) = a(-5x - 4)$$

**Exercise 4.7** If  $f(x)$  is defined as

$$f(x) = \frac{9x-2}{-3x-1}$$

find  $f^{-1}(x)$ .

**Exercise 4.8** If  $f(x)$  is defined as

$$f(x) = 7x + 8$$

and  $g(x)$  is defined as

$$g(x) = -10f(-8x - 10)x^2 - 5x$$

simplify  $g(x)$  in terms of  $x$ .

**Exercise 4.9** If  $f(x)$  is defined as

$$f(x) = \frac{ax-a}{7a+5x}$$

find the value of  $a$  if  $f^{-1}(3) = 5$ .

**Exercise 4.10** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 6ax^2 - 8a - 7x$$

and

$$g(x) = a(5 - 6x)$$

**Exercise 4.11** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -4a + 10x$$

and

$$g(x) = 4a - 10x$$

**Exercise 4.12** If  $f(x)$  is defined as

$$f(x) = -4x^2 - x - 5$$

and  $g(x)$  is defined as

$$g(x) = -8f(-2x^2 - x + 1)x^2 - 8x$$

simplify  $g(x)$  in terms of  $x$ .

**Exercise 4.13** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 4ax^2 + 5a - 9x$$

and

$$g(x) = 6ax - 7$$

**Exercise 4.14** If  $f(x)$  is defined as

$$f(x) = \frac{x-6}{6-2x}$$

what is the domain and range of  $f(x)$ ?

**Exercise 4.15** For the following two functions,

$$f(x) = 3ax^2 + ax - 7$$

and

$$g(x) = a(x^2 - 10x - 2)$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 4.16** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -3a - x$$

and

$$g(x) = -7a - 4x$$

**Exercise 4.17** For the following two functions,

$$f(x) = a(-6x^2 + 9x - 8)$$

and

$$g(x) = -10ax - 5$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 4.18** If  $f(x)$  is defined as

$$f(x) = x - 1$$

and  $g(x)$  is defined as

$$g(x) = 7x^2 + 2x + 4$$

simplify  $g(f(x))$ .

**Exercise 4.19** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -5a + 4x$$

and

$$g(x) = -6a + 10x$$

**Exercise 4.20** For the following two functions,

$$f(x) = 7ax^2 + 4ax - 10$$

and

$$g(x) = 8a(1 - x)$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 4.21** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -6ax^2 + 4a + x$$

and

$$g(x) = 2a(-2x - 5)$$

**Exercise 4.22** If  $f(x)$  is defined as

$$f(x) = -10x^2 - x + 7$$

and  $g(x)$  is defined as

$$g(x) = 5f(-7x^2 + 9x - 4)x^2 - 6x$$

simplify  $g(x)$  in terms of  $x$ .

**Exercise 4.23** If  $f(x)$  is defined as

$$f(x) = 8x + 5$$

and  $g(x)$  is defined as

$$g(x) = -f(9x - 8)x^2 - 9x$$

simplify  $g(x)$  in terms of  $x$ .

**Exercise 4.24** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 9 - 7x$$

and

$$g(x) = 9a + 7x$$

**Exercise 4.25** If  $f(x)$  is defined as

$$f(x) = 7x + 4$$

and  $g(x)$  is defined as

$$g(x) = 10x^2 + x + 8$$

simplify  $g(f(x))$ .

**Exercise 4.26** If  $f(x)$  is defined as

$$f(x) = 6x + 4$$

and  $g(x)$  is defined as

$$g(x) = -7x^2 - 9x + 8$$

simplify  $g(f(x))$ .

**Exercise 4.27** For the following two functions,

$$f(x) = 7ax^2 - 8ax + 7$$

and

$$g(x) = 4a(2 - x)$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 4.28** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 5a + x$$

and

$$g(x) = -7a + 10x$$

**Exercise 4.29** If  $f(x)$  is defined as

$$f(x) = 9x - 3$$

and  $g(x)$  is defined as

$$g(x) = -8f(-9x - 7)x^2 + 2x$$

simplify  $g(x)$  in terms of  $x$ .

**Exercise 4.30** If  $f(x)$  is defined as

$$f(x) = \frac{6x + 7}{-8x - 7}$$

find  $f^{-1}(x)$ .

**Exercise 4.31** If  $f(x)$  is defined as

$$f(x) = \frac{1 - 4x}{2x + 1}$$

find  $f^{-1}(x)$ .

**Exercise 4.32** For the following two functions,

$$f(x) = a(-6x^2 + 5x + 2)$$

and

$$g(x) = 2ax + 1$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 4.33** If  $f(x)$  is defined as

$$f(x) = \frac{9 - 6x}{-3x - 3}$$

find  $f^{-1}(x)$ .

**Exercise 4.34** For the following two functions,

$$f(x) = 6ax^2 + 8ax + 4$$

and

$$g(x) = -9ax - 8$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 4.35** If  $f(x)$  is defined as

$$f(x) = \frac{4x - 5}{-9x - 2}$$

what is the domain and range of  $f(x)$ ?

**Exercise 4.36** If  $f(x)$  is defined as

$$f(x) = -2x^2 + 2x + 5$$

and  $g(x)$  is defined as

$$g(x) = -4f(3x^2 + 2x + 8)x^2 - 5x$$

simplify  $g(x)$  in terms of  $x$ .

**Exercise 4.37** If  $f(x)$  is defined as

$$f(x) = -x^2 - 4x - 4$$

and  $g(x)$  is defined as

$$g(x) = 3f(10x^2 - 10x - 10)x^2 + x$$

simplify  $g(x)$  in terms of  $x$ .

**Exercise 4.38** If  $f(x)$  is defined as

$$f(x) = 10x - 4$$

and  $g(x)$  is defined as

$$g(x) = -8x^2 + 10x - 1$$

simplify  $g(f(x))$ .

**Exercise 4.39** For the following two functions,

$$f(x) = -6ax^2 + ax - 5$$

and

$$g(x) = 8ax^2 - 9ax + 1$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 4.40** If  $f(x)$  is defined as

$$f(x) = \frac{7 - 6x}{2x + 8}$$

what is the domain and range of  $f(x)$ ?

**Exercise 4.41** If  $f(x)$  is defined as

$$f(x) = \frac{-9x - 6}{5x - 7}$$

what is the domain and range of  $f(x)$ ?

**Exercise 4.42** If  $f(x)$  is defined as

$$f(x) = 3x^2 - x + 10$$

and  $g(x)$  is defined as

$$g(x) = 10f(-3x^2 - 3x + 2)x^2 - 7x$$

simplify  $g(x)$  in terms of  $x$ .

**Exercise 4.43** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = ax^2 - 4a - 7x$$

and

$$g(x) = a(9x + 5)$$

**Exercise 4.44** For the following two functions,

$$f(x) = ax^2 + ax - 3$$

and

$$g(x) = ax - 10$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 4.45** For the following two functions,

$$f(x) = 10ax^2 - 6ax + 7$$

and

$$g(x) = -10ax^2 - 5ax - 4$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 4.46** If  $f(x)$  is defined as

$$f(x) = \frac{8 - 5x}{9x + 8}$$

find  $f^{-1}(x)$ .

**Exercise 4.47** If  $f(x)$  is defined as

$$f(x) = 4ax - 10x^2$$

and  $g(x)$  is defined as

$$g(x) = 7x - 2$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 4.48** If  $f(x)$  is defined as

$$f(x) = 9x^2 - 8x + 10$$

and  $g(x)$  is defined as

$$g(x) = -5f(x^2 + 9x - 8)x^2 - 2x$$

simplify  $g(x)$  in terms of  $x$ .

**Exercise 4.49** If  $f(x)$  is defined as

$$f(x) = 2x + 10$$

and  $g(x)$  is defined as

$$g(x) = 4x^2 - x - 3$$

simplify  $g(f(x))$ .

**Exercise 4.50** If  $f(x)$  is defined as

$$f(x) = 3ax - 6x^2$$

and  $g(x)$  is defined as

$$g(x) = -7x - 5$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 4.51** If  $f(x)$  is defined as

$$f(x) = 5x + 7$$

and  $g(x)$  is defined as

$$g(x) = -f(9x + 7)x^2 + 10x$$

simplify  $g(x)$  in terms of  $x$ .

**Exercise 4.52** If  $f(x)$  is defined as

$$f(x) = \frac{6x - 8}{5x - 10}$$

find  $f^{-1}(x)$ .



**Exercise 4.53** If  $f(x)$  is defined as

$$f(x) = 2 - 6x$$

and  $g(x)$  is defined as

$$g(x) = -x^2 + 2x - 5$$

simplify  $g(f(x))$ .

**Exercise 4.54** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -x - 6$$

and

$$g(x) = 3a + x$$

**Exercise 4.55** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 7ax^2 + 3a - 10x$$

and

$$g(x) = a(10x - 9)$$

**Exercise 4.56** If  $f(x)$  is defined as

$$f(x) = 4x^2 - 3x + 4$$

and  $g(x)$  is defined as

$$g(x) = -5f(9x^2 + 9x + 10)x^2 - x$$

simplify  $g(x)$  in terms of  $x$ .

**Exercise 4.57** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 8x - 9$$

and

$$g(x) = -2a - 8x$$

**Exercise 4.58** If  $f(x)$  is defined as

$$f(x) = \frac{-9x - 7}{10x + 6}$$

find  $f^{-1}(x)$ .

**Exercise 4.59** For the following two functions,

$$f(x) = 2ax^2 + 10ax + 3$$

and

$$g(x) = 6ax^2 + 4ax - 6$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 4.60** If  $f(x)$  is defined as

$$f(x) = 7x - 4$$

and  $g(x)$  is defined as

$$g(x) = -2f(6x + 10)x^2 + 7x$$

simplify  $g(x)$  in terms of  $x$ .

**Exercise 4.61** If  $f(x)$  is defined as

$$f(x) = \frac{8ax - 7a}{-7a + 2x}$$

find the value of  $a$  if  $f^{-1}(6) = 10$ .

**Exercise 4.62** If  $f(x)$  is defined as

$$f(x) = 6 - 3x$$

and  $g(x)$  is defined as

$$g(x) = 4x^2 + 3x + 8$$

simplify  $g(f(x))$ .

**Exercise 4.63** If  $f(x)$  is defined as

$$f(x) = 10 - 4x$$

and  $g(x)$  is defined as

$$g(x) = 6f(-7x - 4)x^2 + 9x$$

simplify  $g(x)$  in terms of  $x$ .

**Exercise 4.64** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -8ax^2 + 9a - 8x$$

and

$$g(x) = -7ax - 2$$

**Exercise 4.65** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 9ax^2 - 8a - 10x$$

and

$$g(x) = -ax + 8$$

**Exercise 4.66** If  $f(x)$  is defined as

$$f(x) = \frac{8x - 2}{5 - 6x}$$

find  $f^{-1}(x)$ .

**Exercise 4.67** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -2ax^2 - 5a - 3x$$

and

$$g(x) = 3ax + 5$$

**Exercise 4.68** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 5ax^2 - 7a - 4x$$

and

$$g(x) = 2a(3x + 5)$$

**Exercise 4.69** If  $f(x)$  is defined as

$$f(x) = 7x - 9$$

and  $g(x)$  is defined as

$$g(x) = -10f(6x - 7)x^2 - 4x$$

simplify  $g(x)$  in terms of  $x$ .

**Exercise 4.70** If  $f(x)$  is defined as

$$f(x) = 9ax + 9x^2$$

and  $g(x)$  is defined as

$$g(x) = -7x - 10$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 4.71** If  $f(x)$  is defined as

$$f(x) = \frac{-x - 3}{10x - 5}$$

find  $f^{-1}(x)$ .

**Exercise 4.72** If  $f(x)$  is defined as

$$f(x) = \frac{x - 4}{10 - 8x}$$

find  $f^{-1}(x)$ .

**Exercise 4.73** If  $f(x)$  is defined as

$$f(x) = 8x^2 - 3x - 3$$

and  $g(x)$  is defined as

$$g(x) = -7f(3x^2 - x - 7)x^2 + 4x$$

simplify  $g(x)$  in terms of  $x$ .

**Exercise 4.74** If  $f(x)$  is defined as

$$f(x) = 10x - 2$$

and  $g(x)$  is defined as

$$g(x) = 9f(x - 3)x^2 - 5x$$

simplify  $g(x)$  in terms of  $x$ .

**Exercise 4.75** If  $f(x)$  is defined as

$$f(x) = \frac{-9ax - 3a}{-6a + 10x}$$

find the value of  $a$  if  $f^{-1}(1) = 6$ .

**Exercise 4.76** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -10a - 5x$$

and

$$g(x) = a - 2x$$

**Exercise 4.77** For the following two functions,

$$f(x) = 6ax^2 - 3ax + 2$$

and

$$g(x) = 2a(4x^2 + 3x + 3)$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 4.78** For the following two functions,

$$f(x) = 8ax^2 - 5ax - 7$$

and

$$g(x) = ax^2 + 10ax + 5$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 4.79** For the following two functions,

$$f(x) = -4ax^2 - ax + 9$$

and

$$g(x) = -10ax^2 - 2ax - 2$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 4.80** If  $f(x)$  is defined as

$$f(x) = \frac{-6ax + 9a}{-7a - 10x}$$

find the value of  $a$  if  $f^{-1}(3) = 10$ .

**Exercise 4.81** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = ax^2 + 4a + 5x$$

and

$$g(x) = a(-2x - 3)$$

**Exercise 4.82** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 10ax^2 + 4a - 3x$$

and

$$g(x) = a(3x + 1)$$

**Exercise 4.83** If  $f(x)$  is defined as

$$f(x) = \frac{-7ax + 5a}{-3a - 6x}$$

find the value of  $a$  if  $f^{-1}(5) = 3$ .

**Exercise 4.84** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -6a + 9x$$

and

$$g(x) = -a - 7x$$

**Exercise 4.85** If  $f(x)$  is defined as

$$f(x) = -6x - 3$$

and  $g(x)$  is defined as

$$g(x) = 10f(4 - 2x)x^2 - x$$

simplify  $g(x)$  in terms of  $x$ .

**Exercise 4.86** If  $f(x)$  is defined as

$$f(x) = 10 - 4x$$

and  $g(x)$  is defined as

$$g(x) = -7f(2x + 3)x^2 - 6x$$

simplify  $g(x)$  in terms of  $x$ .

**Exercise 4.87** For the following two functions,

$$f(x) = 10ax^2 - 8ax + 5$$

and

$$g(x) = a(7 - 3x)$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 4.88** If  $f(x)$  is defined as

$$f(x) = -4x^2 - 9x - 4$$

and  $g(x)$  is defined as

$$g(x) = -6f(6x^2 - 10x - 2)x^2 + 7x$$

simplify  $g(x)$  in terms of  $x$ .

**Exercise 4.89** If  $f(x)$  is defined as

$$f(x) = -4x - 6$$

and  $g(x)$  is defined as

$$g(x) = 2x^2 - x + 7$$

simplify  $g(f(x))$ .

**Exercise 4.90** For the following two functions,

$$f(x) = 4ax^2 - 4ax + 1$$

and

$$g(x) = -5ax^2 - 4ax + 7$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 4.91** If  $f(x)$  is defined as

$$f(x) = -8x^2 - 5x + 7$$

and  $g(x)$  is defined as

$$g(x) = f(-7x^2 + 9x + 3)x^2 + 8x$$

simplify  $g(x)$  in terms of  $x$ .

**Exercise 4.92** If  $f(x)$  is defined as

$$f(x) = \frac{2x - 3}{6 - 2x}$$

find  $f^{-1}(x)$ .

**Exercise 4.93** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -3ax^2 + 5a + 9x$$

and

$$g(x) = a(3x + 5)$$

**Exercise 4.94** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 7ax^2 - 9a + 4x$$

and

$$g(x) = 2a(1 - x)$$

**Exercise 4.95** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -10a - 10x$$

and

$$g(x) = -a + 3x$$

**Exercise 4.96** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 5ax^2 - 3a - 3x$$

and

$$g(x) = 2a(4x - 3)$$

**Exercise 4.97** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -9a + 2x$$

and

$$g(x) = 4a - 8x$$

**Exercise 4.98** If  $f(x)$  is defined as

$$f(x) = -3ax + 2x^2$$

and  $g(x)$  is defined as

$$g(x) = 3 - 8x$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 4.99** If  $f(x)$  is defined as

$$f(x) = -6ax + 7x^2$$

and  $g(x)$  is defined as

$$g(x) = 5x + 5$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 4.100** If  $f(x)$  is defined as

$$f(x) = -10x^2 + x + 9$$

and  $g(x)$  is defined as

$$g(x) = 8f(-9x^2 - 8x + 3)x^2 - 5x$$

simplify  $g(x)$  in terms of  $x$ .

## 4.2 Solutions

1.

$$\emptyset$$

2.

$$\left\{ \frac{3(a-2)}{16a} - \frac{\sqrt{329a^2 - 164a + 36}}{16a}, \frac{3(a-2)}{16a} + \frac{\sqrt{329a^2 - 164a + 36}}{16a} \right\}$$

3.

$$\left\{ \frac{3a}{20} - \frac{\sqrt{9a^2 + 80}}{20}, \frac{3a}{20} + \frac{\sqrt{9a^2 + 80}}{20} \right\}$$

4.

$$\{4a\}$$

5.

$$\left\{ \frac{5y-1}{8y+1} \right\}$$

6.

$$\left\{ -\frac{5a+6}{6a} - \frac{\sqrt{97a^2 + 60a + 36}}{6a}, -\frac{5a+6}{6a} + \frac{\sqrt{97a^2 + 60a + 36}}{6a} \right\}$$

7.

$$\left\{ -\frac{y-2}{3(y+3)} \right\}$$

8.

$$560x^3 + 620x^2 - 5x$$

9.

$$-4.412$$

10.

$$\left\{ -\frac{6a-7}{12a} - \frac{\sqrt{348a^2 - 84a + 49}}{12a}, -\frac{6a-7}{12a} + \frac{\sqrt{348a^2 - 84a + 49}}{12a} \right\}$$

11.

$$\left\{ \frac{2a}{5} \right\}$$

12.

$$128x^6 + 128x^5 - 112x^4 - 72x^3 + 80x^2 - 8x$$



13.

$$\left\{ \frac{3 \cdot (2a+3)}{8a} - \frac{\sqrt{-44a^2 - 4a + 81}}{8a}, \frac{3 \cdot (2a+3)}{8a} + \frac{\sqrt{-44a^2 - 4a + 81}}{8a} \right\}$$

14. Domain:

$$(-\infty, 3) \cup (3, \infty)$$

Range:

$$\left( -\infty, -\frac{1}{2} \right) \cup \left( -\frac{1}{2}, \infty \right)$$

15.

$$-\frac{8}{15} < a, a < 0$$

16.

$$\left\{ -\frac{4a}{3} \right\}$$

17.

$$-\frac{120}{169} < a, a < 0$$

18.

$$7x^2 - 12x + 9$$

19.

$$\left\{ \frac{a}{6} \right\}$$

20.

$$-\frac{35}{46} < a, a < 0$$

21.

$$\left\{ \frac{4a+1}{12a} - \frac{\sqrt{352a^2 + 8a + 1}}{12a}, \frac{4a+1}{12a} + \frac{\sqrt{352a^2 + 8a + 1}}{12a} \right\}$$

22.

$$-2450x^6 + 6300x^5 - 6815x^4 + 3555x^3 - 745x^2 - 6x$$

23.

$$-72x^3 + 59x^2 - 9x$$

24.

$$\left\{ \frac{9}{14} - \frac{9a}{14} \right\}$$

25.

$$490x^2 + 567x + 172$$

26.

$$-252x^2 - 390x - 140$$

27.

$$0 < a, a < \frac{49}{60}$$

28.

$$\left\{ \frac{4a}{3} \right\}$$

29.

$$648x^3 + 528x^2 + 2x$$

30.

$$\left\{ -\frac{7(y+1)}{2 \cdot (4y+3)} \right\}$$

31.

$$\left\{ -\frac{y-1}{2(y+2)} \right\}$$

32.

$$0 < a, a < \frac{8}{19}$$

33.

$$\left\{ -\frac{y+3}{y-2} \right\}$$

34.

$$0 < a, a < \frac{288}{289}$$

35. Domain:

$$\left( -\infty, -\frac{2}{9} \right) \cup \left( -\frac{2}{9}, \infty \right)$$

Range:

$$\left( -\infty, -\frac{4}{9} \right) \cup \left( -\frac{4}{9}, \infty \right)$$

36.

$$72x^6 + 96x^5 + 392x^4 + 240x^3 + 428x^2 - 5x$$

37.

$$-300x^6 + 600x^5 + 180x^4 - 480x^3 - 192x^2 + x$$

38.

$$-800x^2 + 740x - 169$$

39.

$$0 < a, a < \frac{84}{25}$$

40. Domain:

$$(-\infty, -4) \cup (-4, \infty)$$

Range:

$$(-\infty, -3) \cup (-3, \infty)$$

41. Domain:

$$\left(-\infty, \frac{7}{5}\right) \cup \left(\frac{7}{5}, \infty\right)$$

Range:

$$\left(-\infty, -\frac{9}{5}\right) \cup \left(-\frac{9}{5}, \infty\right)$$

42.

$$270x^6 + 540x^5 - 60x^4 - 330x^3 + 200x^2 - 7x$$

43.

$$\left\{ \frac{9a+7}{2a} - \frac{\sqrt{117a^2+126a+49}}{2a}, \frac{9a+7}{2a} + \frac{\sqrt{117a^2+126a+49}}{2a} \right\}$$

44.

$$0 < a$$

45.

$$0 < a, a < 880$$

46.

$$\left\{ -\frac{8(y-1)}{9y+5} \right\}$$

47.

$$\left\{ \frac{a}{5} - \frac{\sqrt{7}\sqrt{7a^2-5}}{35}, \frac{a}{5} + \frac{\sqrt{7}\sqrt{7a^2-5}}{35} \right\}$$

48.

$$-45x^6 - 810x^5 - 2885x^4 + 6840x^3 - 3250x^2 - 2x$$

49.

$$16x^2 + 158x + 387$$

50.

$$\left\{ \frac{a}{4} - \frac{\sqrt{21}\sqrt{21a^2+40}}{84}, \frac{a}{4} + \frac{\sqrt{21}\sqrt{21a^2+40}}{84} \right\}$$

51.

$$-45x^3 - 42x^2 + 10x$$

52.

$$\left\{ \frac{2 \cdot (5y - 4)}{5y - 6} \right\}$$

53.

$$-36x^2 + 12x - 5$$

54.

$$\left\{ -\frac{3a}{2} - 3 \right\}$$

55.

$$\left\{ \frac{5(a+1)}{7a} - \frac{\sqrt{-59a^2+50a+25}}{7a}, \frac{5(a+1)}{7a} + \frac{\sqrt{-59a^2+50a+25}}{7a} \right\}$$

56.

$$-1620x^6 - 3240x^5 - 5085x^4 - 3465x^3 - 1870x^2 - x$$

57.

$$\left\{ \frac{9}{16} - \frac{a}{8} \right\}$$

58.

$$\left\{ -\frac{6y+7}{10y+9} \right\}$$

59.

$$-4 < a, a < 0$$

60.

$$-84x^3 - 132x^2 + 7x$$

61.

$$1.043$$

62.

$$36x^2 - 153x + 170$$

63.

$$168x^3 + 156x^2 + 9x$$

64.

$$\left\{ \frac{7a-8}{16a} - \frac{\sqrt{337a^2-48a+64}}{16a}, \frac{7a-8}{16a} + \frac{\sqrt{337a^2-48a+64}}{16a} \right\}$$

65.

$$\left\{ -\frac{a-10}{18a} - \frac{\sqrt{289a^2+268a+100}}{18a}, -\frac{a-10}{18a} + \frac{\sqrt{289a^2+268a+100}}{18a} \right\}$$

66.

$$\left\{ \frac{5y+2}{2 \cdot (3y+4)} \right\}$$

67.

$$\left\{ -\frac{\sqrt{-(a+1)(31a-9)}}{4a} - \frac{3(a+1)}{4a}, \frac{\sqrt{-(a+1)(31a-9)}}{4a} - \frac{3(a+1)}{4a} \right\}$$

68.

$$\left\{ \frac{3a+2}{5a} - \frac{\sqrt{2}\sqrt{47a^2+6a+2}}{5a}, \frac{3a+2}{5a} + \frac{\sqrt{2}\sqrt{47a^2+6a+2}}{5a} \right\}$$

69.

$$-420x^3 + 580x^2 - 4x$$

70.

$$\left\{ -\frac{a}{2} - \frac{\sqrt{7}\sqrt{63a^2-40}}{42}, -\frac{a}{2} + \frac{\sqrt{7}\sqrt{63a^2-40}}{42} \right\}$$

71.

$$\left\{ \frac{5y-3}{10y+1} \right\}$$

72.

$$\left\{ \frac{2 \cdot (5y+2)}{8y+1} \right\}$$

73.

$$-504x^6 + 336x^5 + 2359x^4 - 805x^3 - 2870x^2 + 4x$$

74.

$$90x^3 - 288x^2 - 5x$$

75.

$$-1.176$$

76.

$$\left\{ -\frac{11a}{3} \right\}$$

77.

$$-\frac{16}{33} < a, a < 0$$

78.

$$-\frac{112}{75} < a, a < 0$$

79.

$$0 < a, a < 264$$

80.

$$10.0$$

81.

$$\left\{ -\frac{2a+5}{2a} - \frac{\sqrt{-24a^2+20a+25}}{2a}, -\frac{2a+5}{2a} + \frac{\sqrt{-24a^2+20a+25}}{2a} \right\}$$

82.

$$\left\{ \frac{3(a+1)}{20a} - \frac{\sqrt{3}\sqrt{-37a^2+6a+3}}{20a}, \frac{3(a+1)}{20a} + \frac{\sqrt{3}\sqrt{-37a^2+6a+3}}{20a} \right\}$$

83.

$$90.0$$

84.

$$\left\{ \frac{5a}{16} \right\}$$

85.

$$120x^3 - 270x^2 - x$$

86.

$$56x^3 + 14x^2 - 6x$$

87.

$$0 < a, a < \frac{40}{61}$$

88.

$$864x^6 - 2880x^5 + 2148x^4 + 420x^3 + 12x^2 + 7x$$

89.

$$32x^2 + 100x + 85$$

90.

$$a < 0$$

91.

$$-392x^6 + 1008x^5 - 277x^4 - 477x^3 - 80x^2 + 8x$$

92.

$$\left\{ \frac{3 \cdot (2y+1)}{2(y+1)} \right\}$$

93.

$$\left\{ 0, -\frac{a-3}{a} \right\}$$

94.

$$\left\{ -\frac{a+2}{7a} - \frac{\sqrt{2}\sqrt{39a^2+2a+2}}{7a}, -\frac{a+2}{7a} + \frac{\sqrt{2}\sqrt{39a^2+2a+2}}{7a} \right\}$$

95.

$$\left\{ -\frac{9a}{13} \right\}$$

96.

$$\left\{ \frac{8a+3}{10a} - \frac{\sqrt{4a^2+48a+9}}{10a}, \frac{8a+3}{10a} + \frac{\sqrt{4a^2+48a+9}}{10a} \right\}$$

97.

$$\left\{ \frac{13a}{10} \right\}$$

98.

$$\left\{ \frac{3a}{4} - \frac{\sqrt{3}\sqrt{3a^2+1}}{4}, \frac{3a}{4} + \frac{\sqrt{3}\sqrt{3a^2+1}}{4} \right\}$$

99.

$$\left\{ \frac{3a}{7} - \frac{\sqrt{9a^2-7}}{7}, \frac{3a}{7} + \frac{\sqrt{9a^2-7}}{7} \right\}$$

100.

$$-6480x^6 - 11520x^5 - 872x^4 + 3776x^3 - 624x^2 - 5x$$





## 5. Differential calculus

### 5.1 Problems

**Exercise 5.1** Double derivative: Differentiate the following expression with respect to  $n$  twice

$$8n^5w + 8n^4 + 10w^4 + 8w - 7$$

**Exercise 5.2** Differentiate the following expression with respect to  $d$

$$2d^6 - 7dn^6 + n^6 - 4$$

**Exercise 5.3** Differentiate the following expression with respect to  $d$

$$9d^{\frac{2}{5}}n^{\frac{3}{5}} + 3d^{\frac{2}{3}}\sqrt{n} - 4n^{\frac{6}{5}} - 2n^2 + 5$$

**Exercise 5.4** Differentiate the polynomial with respect to  $c$

$$5c^3 + 7c^2 + 2c - 10$$

**Exercise 5.5** Double derivative: Differentiate the following expression with respect to  $n$  twice

$$-d^3 + 10d + 2n^6 - 8n^5 + 3$$

**Exercise 5.6** Differentiate the following expression with respect to  $n$

$$7n^{\frac{5}{6}} + \sqrt[6]{n} - n^{\frac{5}{4}} - 6$$

**Exercise 5.7** Double derivative: Differentiate the following expression with respect to  $t$  twice

$$n^3 \cdot (6n^3t^6 - 9n^2 - 5)$$

**Exercise 5.8** Differentiate the following expression with respect to  $y$

$$-9v^{\frac{2}{5}} + 7v^2y^3 - 9vy^6 + 9\sqrt[5]{y}$$

**Exercise 5.9** Differentiate the following expression with respect to  $a$

$$-9a^5 - 2an^4 - 12n$$

**Exercise 5.10** Differentiate the following expression with respect to  $r$

$$4\sqrt[5]{l} - 5r^{\frac{3}{5}} - 2r$$

**Exercise 5.11** Differentiate the following expression with respect to  $s$

$$9a^4s - 6a^4 + s^2 - 9$$

**Exercise 5.12** Differentiate the following expression with respect to  $t$

$$6c^{\frac{5}{6}}t^{\frac{2}{5}} + c^{\frac{3}{4}}t^{\frac{6}{5}} - 10\sqrt[3]{ct^{\frac{3}{5}}} - 5t^6 - 4$$

**Exercise 5.13** Differentiate the following expression with respect to  $z$

$$-8s^{\frac{6}{5}}z^2 - 8s^{\frac{3}{5}} - 9s^{\frac{2}{3}}z + 10\sqrt[3]{sz^{\frac{3}{2}}} + 6\sqrt[3]{sz^3}$$

**Exercise 5.14** Differentiate the following expression with respect to  $d$

$$d^{\frac{4}{3}} + 5d^{\frac{3}{2}} - 2\sqrt{s}$$

**Exercise 5.15** Differentiate the following expression with respect to  $w$

$$-6w$$

**Exercise 5.16** Differentiate the polynomial with respect to  $z$

$$-2z^4 + 6z^3 + 10z^2 - 6z - 9$$

**Exercise 5.17** Differentiate the following expression with respect to  $q$

$$-4l^{\frac{5}{6}} + 4l^{\frac{3}{2}} + 7\sqrt[3]{q}$$

**Exercise 5.18** Differentiate the polynomial with respect to  $r$

$$4r^4 + 5r^3 + 10r^2 + 2r - 4$$

**Exercise 5.19** Differentiate the following expression with respect to  $s$

$$3s^5 + s^3x^2 - s^2 - 1$$

**Exercise 5.20** Double derivative: Differentiate the following expression with respect to  $z$  twice

$$-y^3 - 7z^3 + 9z^2 + 2z$$

**Exercise 5.21** Double derivative: Differentiate the following expression with respect to  $c$  twice

$$c(8c^5 + 3c^3l^4 + 2c^3l + 4c - l^2)$$

**Exercise 5.22** Differentiate the following expression with respect to  $s$

$$4\sqrt[6]{ms^{\frac{4}{3}}} - 6m^{\frac{3}{4}}\sqrt[6]{s} + 5m^2\sqrt[5]{s}$$

**Exercise 5.23** Double derivative: Differentiate the following expression with respect to  $x$  twice

$$-6t^6 - 6t^4 + t^3 + 6x^2 - 8x$$

**Exercise 5.24** Differentiate the following expression with respect to  $l$

$$9l^{\frac{5}{6}}m - 2l^2\sqrt[4]{m} - 7lm^{\frac{6}{5}}$$

**Exercise 5.25** Double derivative: Differentiate the following expression with respect to  $s$  twice

$$l^3(-4ls^3 + 9ls^2 - 10)$$

**Exercise 5.26** Differentiate the following expression with respect to  $l$

$$3\sqrt[4]{l} - 9l^6n^5 + 8l^3n^3 + 10ln^4 + 5n^{\frac{4}{5}}$$

**Exercise 5.27** Differentiate the following expression with respect to  $s$

$$2\sqrt[4]{d}s + 2s^{\frac{2}{5}} + 9$$

**Exercise 5.28** Differentiate the following expression with respect to  $b$

$$-3b^4n^4 + 8b^3 - 9b^2n - n^2 - 3$$

**Exercise 5.29** Differentiate the following expression with respect to  $n$

$$b^2(-4b^3 - 7b^2n^6 + 8bn^4 + 8bn + 10n^5)$$

**Exercise 5.30** Differentiate the following expression with respect to  $d$

$$-2a^{\frac{5}{3}} - 4a^{\frac{2}{3}}\sqrt[3]{d} - 8\sqrt{a}\sqrt[3]{d} - 7\sqrt[6]{d}$$

**Exercise 5.31** Differentiate the following expression with respect to  $z$

$$z(8u^5z^4 + 9u^3z^4 - 8)$$

**Exercise 5.32** Differentiate the polynomial with respect to  $z$

$$2z^4 - 7z^3 - 10z^2 - 9z + 5$$

**Exercise 5.33** Double derivative: Differentiate the following expression with respect to  $r$  twice

$$r(-9r^4 - 4r^2 - 4)$$

**Exercise 5.34** Differentiate the polynomial with respect to  $y$

$$-8y^3 - 7y^2 - 3y + 1$$

**Exercise 5.35** Differentiate the following expression with respect to  $p$

$$p^2v^2 \cdot (4p^2v^3 + 8p - v^4)$$

**Exercise 5.36** Differentiate the polynomial with respect to  $s$

$$-2s^3 - 5s^2 + 8s - 6$$

**Exercise 5.37** Double derivative: Differentiate the following expression with respect to  $m$  twice

$$-2l - m^5 + 5m^4$$

**Exercise 5.38** Differentiate the following expression with respect to  $b$

$$-5b^6n^4 - 4b^5 - 8b^3 + 4n + 8$$

**Exercise 5.39** Differentiate the following expression with respect to  $n$

$$8n^2 \cdot (2 - n^4)$$

**Exercise 5.40** Differentiate the polynomial with respect to  $r$

$$8r^4 - 10r^3 - r^2 - 6r - 7$$

**Exercise 5.41** Differentiate the polynomial with respect to  $p$

$$6p^3 + 9p^2 - 5p - 2$$

**Exercise 5.42** Differentiate the following expression with respect to  $x$

$$9l^3x^3 - 8l^2 - 2x^2$$

**Exercise 5.43** Double derivative: Differentiate the following expression with respect to  $v$  twice

$$-3r^3v^4 - 5r^3 - 7r^2v^2 - 2r + 6v^5$$

**Exercise 5.44** Differentiate the following expression with respect to  $y$

$$-3u^{\frac{5}{4}} + 3u^{\frac{2}{3}} + \sqrt{u}y^3 + 5u$$

**Exercise 5.45** Differentiate the following expression with respect to  $d$

$$3d^6v^3 + 5d^2v^6 - 3v^2 - 1$$

**Exercise 5.46** Differentiate the following expression with respect to  $x$

$$\sqrt[3]{l}x^3 - 10x^{\frac{6}{5}} - 9\sqrt{x}$$

**Exercise 5.47** Differentiate the following expression with respect to  $m$

$$-4l^5m^6 + 3m^5 + 3m^3 + 10m - 3$$

**Exercise 5.48** Differentiate the following expression with respect to  $m$

$$-10b^5m^3 - 5b^5 - 2bm^6 + 3m^2$$

**Exercise 5.49** Differentiate the following expression with respect to  $n$

$$4d^{\frac{4}{3}}n^4 - 9dn^{\frac{3}{2}} + 9n$$

**Exercise 5.50** Differentiate the following expression with respect to  $p$

$$-6b^{\frac{2}{5}} - b^{\frac{5}{3}} + 9bp - 6\sqrt[3]{p}$$

**Exercise 5.51** Double derivative: Differentiate the following expression with respect to  $z$  twice

$$-6r - 9z^5 + 8z^3 + 10z^2 + 2$$

**Exercise 5.52** Differentiate the polynomial with respect to  $b$

$$5b^3 + 5b^2 + 3b + 6$$

**Exercise 5.53** Differentiate the polynomial with respect to  $s$

$$10s^4 + s^3 + 8s^2 - 9s + 5$$

**Exercise 5.54** Differentiate the following expression with respect to  $r$

$$a^3 \cdot (4a^3r + 3a - 9r^3 - 10r)$$

**Exercise 5.55** Differentiate the following expression with respect to  $n$

$$u^2 \cdot (10n^6u^2 + 4n^5u^2 + 10)$$

**Exercise 5.56** Differentiate the following expression with respect to  $m$

$$m(-10d^3m - 3d^2 - 6m)$$

**Exercise 5.57** Differentiate the polynomial with respect to  $a$

$$2a^3 - 8a^2 + 2a - 10$$

**Exercise 5.58** Differentiate the following expression with respect to  $w$

$$-5u^{\frac{3}{5}} + 9\sqrt[4]{u} - u^{\frac{5}{3}}w + 10u^{\frac{2}{3}}$$

**Exercise 5.59** Differentiate the following expression with respect to  $m$

$$-10\sqrt{m}\sqrt[6]{w} + 3w^{\frac{5}{3}} - 10w^{\frac{5}{2}} - 9w$$

**Exercise 5.60** Double derivative: Differentiate the following expression with respect to  $d$  twice

$$s(-3d^6 - 4d^5s + 5d^4s^5 - 9s)$$

**Exercise 5.61** Differentiate the following expression with respect to  $x$

$$6p^{\frac{5}{2}}x^3 + p^{\frac{3}{2}}\sqrt[6]{x} - 3p^2 + 5px^4 + 10x^{\frac{5}{2}}$$

**Exercise 5.62** Differentiate the following expression with respect to  $y$

$$3a^{\frac{5}{6}}\sqrt[5]{y} + 2\sqrt[3]{ay^{\frac{3}{5}}} - 6y$$

**Exercise 5.63** Double derivative: Differentiate the following expression with respect to  $v$  twice

$$v(7v^5z^2 - 4vz^6 - 9vz - 3)$$

**Exercise 5.64** Double derivative: Differentiate the following expression with respect to  $w$  twice

$$-5v^6 + 10w^2 - 6w$$

**Exercise 5.65** Double derivative: Differentiate the following expression with respect to  $z$  twice

$$-2n^4z^3 + 10n^4 + 10n^2 - 5z^5 + 5$$

**Exercise 5.66** Differentiate the following expression with respect to  $n$

$$-8\sqrt[4]{n}z^3 + 2n^{\frac{5}{3}}z - 8n^{\frac{3}{2}} - 4\sqrt{z} + 2$$

**Exercise 5.67** Double derivative: Differentiate the following expression with respect to  $p$  twice

$$-2p^5v^4 + 4p^5 - 6v^6 - 5$$

**Exercise 5.68** Double derivative: Differentiate the following expression with respect to  $r$  twice

$$b(-7b^5r + 4b^4 - 4r^4)$$

**Exercise 5.69** Differentiate the polynomial with respect to  $s$

$$-2s^3 - 4s^2 + 8s + 3$$

**Exercise 5.70** Double derivative: Differentiate the following expression with respect to  $z$  twice

$$2r^5 - 10r^2 - 7r - 8z^6 + 3z^3$$

**Exercise 5.71** Double derivative: Differentiate the following expression with respect to  $p$  twice

$$x(3p^5x^2 + 3p - 8x^2)$$

**Exercise 5.72** Differentiate the following expression with respect to  $c$

$$2c^5l^5 - 9c^5 + 9c^2 - 9l^2$$

**Exercise 5.73** Differentiate the following expression with respect to  $s$

$$10p^6 - 3p^5 - 7p - 3s^3$$

**Exercise 5.74** Differentiate the polynomial with respect to  $a$

$$-2a^4 + 3a^3 + 9a^2 - 7a - 8$$

**Exercise 5.75** Differentiate the polynomial with respect to  $b$

$$10b^4 + b^3 + 6b^2 + 5b - 4$$

**Exercise 5.76** Double derivative: Differentiate the following expression with respect to  $x$  twice

$$-9x^5z^6 + 4x^2 - 8x - 10z^4 + 5z^2$$

**Exercise 5.77** Differentiate the following expression with respect to  $b$

$$4b^6 + 3b^2x^2 - 2b + 9x^3 + 1$$

**Exercise 5.78** Differentiate the polynomial with respect to  $x$

$$2x^2 - 5x + 5$$

**Exercise 5.79** Differentiate the following expression with respect to  $s$

$$2s + 13$$

**Exercise 5.80** Double derivative: Differentiate the following expression with respect to  $d$  twice

$$d^4 - 10dv^3 + 3v^6 - 5v^3$$

**Exercise 5.81** Differentiate the following expression with respect to  $s$

$$-3s^{\frac{6}{5}} - 6s + 8w^5$$

**Exercise 5.82** Double derivative: Differentiate the following expression with respect to  $m$  twice

$$9d^5m^6 + 4d^4 + 6m^4$$

**Exercise 5.83** Differentiate the polynomial with respect to  $t$

$$-7t^3 + 9t^2 + 7t + 4$$

**Exercise 5.84** Differentiate the following expression with respect to  $q$

$$3q^{\frac{3}{4}}r^{\frac{5}{3}} + 7q^{\frac{5}{3}}r^{\frac{2}{5}} + q^{\frac{5}{3}} + 4\sqrt{q} + r^{\frac{3}{4}}$$

**Exercise 5.85** Differentiate the following expression with respect to  $z$

$$w^2 \cdot (6w^4 + 9z^6 - 2z^3)$$

**Exercise 5.86** Double derivative: Differentiate the following expression with respect to  $d$  twice

$$8d^6z^2 + 10d^6 + 4d^5 + 5$$

**Exercise 5.87** Differentiate the following expression with respect to  $r$

$$\sqrt{q}(10q^2 - 4q^4\sqrt{r} + 9)$$

**Exercise 5.88** Differentiate the following expression with respect to  $c$

$$-13c^5 - 10c^3 + 10n^3 + 4$$

**Exercise 5.89** Double derivative: Differentiate the following expression with respect to  $x$  twice

$$-5p^5x^6 + 10x^6 + 6$$

**Exercise 5.90** Differentiate the polynomial with respect to  $b$

$$10b^3 + 9b^2 - 7b + 2$$

**Exercise 5.91** Differentiate the polynomial with respect to  $a$

$$9a^4 - 4a^3 - 9a^2 - 6a - 3$$

**Exercise 5.92** Double derivative: Differentiate the following expression with respect to  $y$  twice

$$5y^3z^6 - 4y^2 - 7z^2 - 8$$

**Exercise 5.93** Differentiate the following expression with respect to  $c$

$$-9c^6w^6 - 10c^2w^3 + 9c^2 + 3$$

**Exercise 5.94** Differentiate the following expression with respect to  $z$

$$-4\sqrt[4]{s}z^{\frac{2}{5}} + 5s^{\frac{5}{2}} - 6s^{\frac{3}{2}}\sqrt[6]{z} - 9\sqrt{z}$$

**Exercise 5.95** Double derivative: Differentiate the following expression with respect to  $y$  twice

$$-9y^3z^4 - 5y^3z^3 - 5y^2 + 10z^4 + z^3$$

**Exercise 5.96** Double derivative: Differentiate the following expression with respect to  $n$  twice

$$-n^4 + 3n^3 + 6w^6$$

**Exercise 5.97** Differentiate the polynomial with respect to  $s$

$$-6s^4 - 6s^3 + 10s^2 - 3s - 6$$

**Exercise 5.98** Differentiate the polynomial with respect to  $l$

$$3l^3 - 2l^2 - 2l + 9$$

**Exercise 5.99** Double derivative: Differentiate the following expression with respect to  $x$  twice

$$-3r^4x^4 - 3r^3x - 10r^2x + 9r^2 + 2$$

**Exercise 5.100** Double derivative: Differentiate the following expression with respect to  $z$  twice

$$z^2 \cdot (5v^5z + 2z - 10)$$

## 5.2 Solutions

1.

$$n^2 \cdot (160nw + 96)$$

2.

$$12d^5 - 7n^6$$

3.

$$\frac{2\sqrt{n}}{\sqrt[3]{d}} + \frac{18n^{\frac{3}{5}}}{5d^{\frac{3}{5}}}$$

4.

$$15c^2 + 14c + 2$$

5.

$$n^3 \cdot (60n - 160)$$

6.

$$\frac{70n^{\frac{5}{6}} + 2\sqrt[6]{n} - 15n^{\frac{5}{4}}}{12n}$$



7.

$$180n^6t^4$$

8.

$$21v^2y^2 - 54vy^5 + \frac{9}{5y^{\frac{4}{5}}}$$

9.

$$-45a^4 - 2n^4$$

10.

$$-2 - \frac{3}{r^{\frac{2}{5}}}$$

11.

$$9a^4 + 2s$$

12.

$$\frac{6 \cdot (2c^{\frac{5}{6}}t^{\frac{2}{5}} + c^{\frac{3}{4}}t^{\frac{6}{5}} - 5\sqrt[3]{ct^{\frac{3}{5}}} - 25t^6)}{5t}$$

13.

$$-16s^{\frac{6}{5}}z - 9s^{\frac{2}{3}} + 15\sqrt[3]{s}\sqrt{z} + 18\sqrt[3]{s}z^2$$

14.

$$\frac{4\sqrt[3]{d}}{3} + \frac{15\sqrt{d}}{2}$$

15.

$$-6$$

16.

$$-8z^3 + 18z^2 + 20z - 6$$

17.

$$\frac{7}{3q^{\frac{2}{3}}}$$

18.

$$16r^3 + 15r^2 + 20r + 2$$

19.

$$s(15s^3 + 3sx^2 - 2)$$

20.

$$18 - 42z$$

21.

$$240c^4 + 36c^2l^4 + 24c^2l + 8$$

22.

$$\frac{16\sqrt[6]{m}\sqrt[3]{s}}{3} - \frac{m^{\frac{3}{4}}}{s^{\frac{5}{6}}} + \frac{m^2}{s^{\frac{4}{5}}}$$

23.

$$12$$

24.

$$-4l\sqrt[4]{m} - 7m^{\frac{6}{5}} + \frac{15m}{2\sqrt[6]{l}}$$

25.

$$l^4 \cdot (18 - 24s)$$

26.

$$-54l^5n^5 + 24l^2n^3 + 10n^4 + \frac{3}{4l^{\frac{3}{4}}}$$

27.

$$2\sqrt[4]{d} + \frac{4}{5s^{\frac{3}{5}}}$$

28.

$$6b(-2b^2n^4 + 4b - 3n)$$

29.

$$b^2(-42b^2n^5 + 32bn^3 + 8b + 50n^4)$$

30.

$$\frac{8d^{\frac{5}{6}}\left(-a^{\frac{2}{3}} - 2\sqrt{a}\right) - 7d^{\frac{2}{3}}}{6d^{\frac{3}{2}}}$$

31.

$$40u^5z^4 + 45u^3z^4 - 8$$

32.

$$8z^3 - 21z^2 - 20z - 9$$

33.

$$-180r^3 - 24r$$

34.

$$-24y^2 - 14y - 3$$

35.

$$2pv^2 \cdot (8p^2v^3 + 12p - v^4)$$

36.

$$-6s^2 - 10s + 8$$

37.

$$20m^2 \cdot (3 - m)$$

38.

$$b^2(-30b^3n^4 - 20b^2 - 24)$$

39.

$$-48n^5 + 32n$$

40.

$$32r^3 - 30r^2 - 2r - 6$$

41.

$$18p^2 + 18p - 5$$

42.

$$x(27l^3x - 4)$$

43.

$$-36r^3v^2 - 14r^2 + 120v^3$$

44.

$$3\sqrt{u}y^2$$

45.

$$18d^5v^3 + 10dv^6$$

46.

$$3\sqrt[3]{l}x^2 - 12\sqrt[5]{x} - \frac{9}{2\sqrt{x}}$$

47.

$$-24l^5m^5 + 15m^4 + 9m^2 + 10$$

48.

$$6m(-5b^5m - 2bm^4 + 1)$$

49.

$$16d^{\frac{4}{3}}n^3 - \frac{27d\sqrt{n}}{2} + 9$$

50.

$$9b - \frac{2}{p^{\frac{2}{3}}}$$

51.

$$-180z^3 + 48z + 20$$

52.

$$15b^2 + 10b + 3$$

53.

$$40s^3 + 3s^2 + 16s - 9$$

54.

$$a^3 \cdot (4a^3 - 27r^2 - 10)$$

55.

$$n^4u^4 \cdot (60n + 20)$$

56.

$$-20d^3m - 3d^2 - 12m$$

57.

$$6a^2 - 16a + 2$$

58.

$$-u^{\frac{5}{3}}$$

59.

$$-\frac{5\sqrt[6]{w}}{\sqrt{m}}$$

60.

$$10d^2s(-9d^2 - 8ds + 6s^5)$$

61.

$$18p^{\frac{5}{2}}x^2 + \frac{p^{\frac{3}{2}}}{6x^{\frac{5}{6}}} + 20px^3 + 25x^{\frac{3}{2}}$$

62.

$$\frac{3a^{\frac{5}{6}}}{5y^{\frac{4}{5}}} + \frac{6\sqrt[3]{a}}{5y^{\frac{2}{5}}} - 6$$

63.

$$2z(105v^4z - 4z^5 - 9)$$

64.

$$20$$

65.

$$-12n^4z - 100z^3$$

66.

$$\frac{2 \cdot \left(5n^{\frac{17}{12}}z - 18n^{\frac{5}{4}} - 3z^3\right)}{3n^{\frac{3}{4}}}$$

67.

$$40p^3 \cdot (2 - v^4)$$

68.

$$-48br^2$$

69.

$$-6s^2 - 8s + 8$$

70.

$$-240z^4 + 18z$$

71.

$$60p^3x^3$$

72.

$$c(10c^3l^5 - 45c^3 + 18)$$

73.

$$-9s^2$$

74.

$$-8a^3 + 9a^2 + 18a - 7$$

75.

$$40b^3 + 3b^2 + 12b + 5$$

76.

$$-180x^3z^6 + 8$$

77.

$$24b^5 + 6bx^2 - 2$$

78.

$$4x - 5$$

79.

$$2$$

80.

$$12d^2$$

81.

$$-\frac{18\sqrt[5]{s}}{5} - 6$$

82.

$$m^2 \cdot (270d^5m^2 + 72)$$

83.

$$-21t^2 + 18t + 7$$

84.

$$\frac{20q^{\frac{17}{12}} \cdot \left(7r^{\frac{2}{3}} + 1\right) + 24\sqrt[4]{q} + 27\sqrt{q}r^{\frac{5}{3}}}{12q^{\frac{3}{4}}}$$

85.

$$w^2z^2 \cdot (54z^3 - 6)$$

86.

$$d^3 \cdot (240dz^2 + 300d + 80)$$

87.

$$-\frac{q^{\frac{3}{2}}}{r^{\frac{3}{4}}}$$

88.

$$c^2(-65c^2 - 30)$$

89.

$$150x^4 \cdot (2 - p^5)$$

90.

$$30b^2 + 18b - 7$$

91.

$$36a^3 - 12a^2 - 18a - 6$$

92.

$$30yz^6 - 8$$

93.

$$2c(-27c^4w^6 - 10w^3 + 9)$$

94.

$$-\frac{8\sqrt[4]{s}}{5z^{\frac{3}{5}}} - \frac{s^{\frac{3}{2}}}{z^{\frac{5}{6}}} - \frac{9}{2\sqrt{z}}$$

95.

$$-54yz^4 - 30yz^3 - 10$$

96.

$$6n(3 - 2n)$$

97.

$$-24s^3 - 18s^2 + 20s - 3$$

98.

$$9l^2 - 4l - 2$$

99.

$$-36r^4x^2$$

100.

$$30v^5z + 12z - 20$$





## 6. Integral calculus

### 6.1 Problems

**Exercise 6.1** Evaluate the definite integral for the following function, where the limits are 0 and 3.

$$f(d) = d(d^3 + 12)$$

**Exercise 6.2** Integrate the polynomial with respect to  $a$

$$-a^4 + 9a^3 + 10a^2 - 7a + 3$$

**Exercise 6.3** Evaluate the following indefinite integral

$$\int -b^4 - 9b^2 + 2bv^6 - 1dv$$

**Exercise 6.4** Evaluate the definite integral for the following function, where the limits are 1 and 5.

$$f(y) = -2y^{\frac{5}{4}} - 7y^3 + 9y$$

**Exercise 6.5** Evaluate the following indefinite integral

$$\int q^3 \cdot (4q^2w^2 + 3q^2 - 2w^5) dw$$

**Exercise 6.6** Evaluate the following indefinite integral

$$\int 4\sqrt[5]{wz^{\frac{3}{5}}} - 9w^{\frac{3}{2}} - 9w^6\sqrt[4]{z} + 2z^6 + 4dz$$

**Exercise 6.7** Evaluate the following indefinite integral

$$\int -5b^{\frac{2}{3}}\sqrt{t} + b^3t + 6\sqrt{t} - 9tdb$$

**Exercise 6.8** Evaluate the following indefinite integral

$$\int -9n^{\frac{3}{5}}y^{\frac{6}{5}} + 8n^{\frac{5}{4}}y - 9n^{\frac{3}{2}}y^5 + 5y^{\frac{4}{3}}dy$$

**Exercise 6.9** Evaluate the definite integral for the following function, where the limits are 2 and 4.

$$f(v) = 8v^{\frac{4}{5}} + 6\sqrt[4]{v} - 7v^{\frac{4}{3}} - 8v^2 - 8$$

**Exercise 6.10** Evaluate the definite integral for the following function, where the limits are 1 and 5.

$$f(a) = -6\sqrt[4]{a} + 2a^2 - 4a$$

**Exercise 6.11** Evaluate the following indefinite integral

$$\int -10m - p^{\frac{5}{2}} + p^4 + 10dp$$

**Exercise 6.12** Integrate the polynomial with respect to  $z$

$$7z^3 + z^2 + 3z - 7$$

**Exercise 6.13** Evaluate the following indefinite integral

$$\int -4q^5u - q^5 + 6q^2u^2 + 8u^6dq$$

**Exercise 6.14** Integrate the polynomial with respect to  $r$

$$6r^2 + 10r - 9$$

**Exercise 6.15** Evaluate the following indefinite integral

$$\int -8r^6 - 5r^3y + 2ry^5 + 9r + 3dr$$

**Exercise 6.16** Evaluate the definite integral for the following function, where the limits are 3 and 7.

$$f(a) = -7a^{\frac{4}{3}} + 7a^{\frac{2}{3}} - 5\sqrt{a} + 8a^5 - 5a$$

**Exercise 6.17** Integrate the polynomial with respect to  $w$

$$-4w^4 + 2w^3 - 3w^2 + 9w + 4$$

**Exercise 6.18** Integrate the polynomial with respect to  $u$

$$-3u^2 + 3u + 8$$

**Exercise 6.19** Evaluate the following indefinite integral

$$\int 7t^{\frac{3}{4}} + 4t^3u - 8u^{\frac{6}{5}} - u^{\frac{4}{5}} + 8u^3dt$$

**Exercise 6.20** Evaluate the definite integral for the following function, where the limits are 3 and 4.

$$f(z) = 8z^4 - 2z - 2$$

**Exercise 6.21** Evaluate the following indefinite integral

$$\int x^3 (-10b^2x^3 - 10x + 1) db$$

**Exercise 6.22** Evaluate the following indefinite integral

$$\int 5a^5 - 4a^4 + 10z^5 - 1da$$

**Exercise 6.23** Evaluate the definite integral for the following function, where the limits are 2 and 3.

$$f(p) = p^{\frac{5}{6}} + 8p^{\frac{2}{3}} + 10p^{\frac{3}{2}} - 8p^4 + 3p^3$$

**Exercise 6.24** Evaluate the following indefinite integral

$$\int -4b^{\frac{3}{2}}q^{\frac{3}{2}} - 8b^2q^{\frac{4}{5}} + bq^{\frac{4}{3}} + 6q^{\frac{3}{5}} - 6db$$

**Exercise 6.25** Evaluate the following indefinite integral

$$\int -7a^2p^3 - 3a + 3p^{\frac{3}{4}} - 7p^3 da$$

**Exercise 6.26** Evaluate the following indefinite integral

$$\int -6c^{\frac{5}{3}} - 10c^{\frac{3}{2}} + 6\sqrt{c}s^2 - 9c^2 - 10s^{\frac{3}{4}}dc$$

**Exercise 6.27** Evaluate the following indefinite integral

$$\int 6q^{\frac{3}{2}}x + q^2 + 9q + 4dx$$

**Exercise 6.28** Evaluate the following indefinite integral

$$\int -2\sqrt{m} - 9m^2p + 6m\sqrt[5]{p} - 3dm$$

**Exercise 6.29** Evaluate the definite integral for the following function, where the limits are 3 and 7.

$$f(c) = 6c^3 + 9c$$

**Exercise 6.30** Evaluate the definite integral for the following function, where the limits are 2 and 3.

$$f(s) = -4s^{\frac{4}{5}} + 4s^{\frac{3}{5}} + 7s^{\frac{2}{5}} - 3s^4 - 5s$$

**Exercise 6.31** Evaluate the following indefinite integral

$$\int 6lr^2 - 7l + 2r^6 dr$$

**Exercise 6.32** Evaluate the definite integral for the following function, where the limits are 2 and 5.

$$f(t) = -10\sqrt[3]{t} + 5t^5 + t^2$$

**Exercise 6.33** Evaluate the following indefinite integral

$$\int -5\sqrt[9]{q}x^6 + 8q - 6dq$$

**Exercise 6.34** Integrate the polynomial with respect to  $v$

$$-7v^2 + 6v - 1$$

**Exercise 6.35** Evaluate the following indefinite integral

$$\int 4l^5 + 5l^4 + 8n^6 dn$$

**Exercise 6.36** Evaluate the following indefinite integral

$$\int r^{\frac{2}{5}} + r^{\frac{3}{2}}x^{\frac{5}{2}} + 7r^5x^{\frac{3}{2}} + 10rx^5 + 8xdx$$

**Exercise 6.37** Evaluate the definite integral for the following function, where the limits are 0 and 4.

$$f(w) = -w^{\frac{4}{5}} - 9\sqrt[3]{w} + 3\sqrt{w}$$

**Exercise 6.38** Evaluate the definite integral for the following function, where the limits are 3 and 6.

$$f(v) = 7v^{\frac{6}{5}} + 8v^{\frac{5}{4}} - 1$$

**Exercise 6.39** Integrate the polynomial with respect to  $u$

$$10u^2 - 6u - 5$$

**Exercise 6.40** Integrate the polynomial with respect to  $x$

$$-9x^4 + 5x^3 + 2x^2 + 2x + 5$$

**Exercise 6.41** Evaluate the definite integral for the following function, where the limits are 3 and 6.

$$f(n) = -8n^{\frac{5}{6}} - 10n^{\frac{2}{3}} - n^6 + 6$$

**Exercise 6.42** Evaluate the following indefinite integral

$$\int 2c^6 - 4c^3 + 7p^2 dp$$

**Exercise 6.43** Evaluate the definite integral for the following function, where the limits are 3 and 4.

$$f(l) = 10l^{\frac{4}{3}} + 4\sqrt[5]{l} + 8l^2 + 8$$

**Exercise 6.44** Evaluate the following indefinite integral

$$\int -4n^3 z^{\frac{2}{3}} - 4n^2 z + 9nz^{\frac{5}{2}} + 2dz$$

**Exercise 6.45** Integrate the polynomial with respect to  $r$

$$2r^4 - 3r^3 - 2r^2 + 5r - 5$$

**Exercise 6.46** Evaluate the definite integral for the following function, where the limits are 1 and 4.

$$f(u) = u^{\frac{3}{5}} - 3\sqrt[5]{u} + 8$$

**Exercise 6.47** Evaluate the following indefinite integral

$$\int -5\sqrt[6]{l}\sqrt{z} - 7\sqrt[3]{l}z^{\frac{2}{3}} + 6z^{\frac{2}{3}} dl$$

**Exercise 6.48** Evaluate the following indefinite integral

$$\int 3n^4 + 6n^3 r^6 - 7n^3 - 6r^3 dn$$

**Exercise 6.49** Evaluate the following indefinite integral

$$\int 4c^2 z^2 - 6c - 10dc$$

**Exercise 6.50** Integrate the polynomial with respect to  $s$

$$10s^2 - 5s - 9$$

**Exercise 6.51** Evaluate the following indefinite integral

$$\int -2\sqrt{rs}^{\frac{2}{3}} + 2r^2 - 4r + s^{\frac{3}{2}} + 4sdr$$

**Exercise 6.52** Evaluate the following indefinite integral

$$\int 8m^3 - 3m^2\sqrt{v} + 2v^{\frac{2}{5}} + 2vdm$$

**Exercise 6.53** Evaluate the following indefinite integral

$$\int 7b^6 - 5b^5w + 8b^2w^3 - 2b - 6w^5 dw$$

**Exercise 6.54** Evaluate the following indefinite integral

$$\int -6b^3 + 9r^2 + 9rdb$$

**Exercise 6.55** Evaluate the definite integral for the following function, where the limits are 0 and 4.

$$f(x) = 10x^{\frac{4}{5}} + 4x^{\frac{2}{5}} + x^6 + 3x$$

**Exercise 6.56** Evaluate the following indefinite integral

$$\int -8b^{\frac{5}{4}} - \sqrt[3]{b} + 8b^4 db$$

**Exercise 6.57** Evaluate the following indefinite integral

$$\int -s^5w^3 + 4s^5w^2 - 5s^5w + 3ds$$

**Exercise 6.58** Evaluate the following indefinite integral

$$\int 9c^{\frac{6}{5}} - 4\sqrt[5]{c} - q^{\frac{6}{5}} - 10q dq$$

**Exercise 6.59** Integrate the polynomial with respect to y

$$8y^3 + 9y^2 - 4y + 1$$

**Exercise 6.60** Evaluate the following indefinite integral

$$\int b^3 \cdot (4bz^2 - 7b - z^2) db$$

**Exercise 6.61** Integrate the polynomial with respect to y

$$-10y^4 - 10y^3 + 10y^2 - 2y + 1$$

**Exercise 6.62** Evaluate the following indefinite integral

$$\int 13l + 8udl$$

**Exercise 6.63** Evaluate the following indefinite integral

$$\int 2w^{\frac{5}{6}}y^{\frac{3}{2}} + w^{\frac{4}{5}} + 9w^{\frac{3}{4}}\sqrt[3]{y} - 10w^{\frac{5}{3}}\sqrt[3]{y} - 5\sqrt{y} dy$$

**Exercise 6.64** Evaluate the following indefinite integral

$$\int s(10s^5 - 2s^2 + 9t^5) dt$$

**Exercise 6.65** Integrate the polynomial with respect to  $c$

$$4c^4 + 9c^3 - 2c^2 - 2c + 2$$

**Exercise 6.66** Integrate the polynomial with respect to  $c$

$$9c^3 + c^2 + 5c + 2$$

**Exercise 6.67** Integrate the polynomial with respect to  $u$

$$-2u^4 + 7u^3 - 4u^2 + 2u - 10$$

**Exercise 6.68** Evaluate the following indefinite integral

$$\int -2c^4 l^3 - 4c^3 l^4 + 8l^6 - 5dl$$

**Exercise 6.69** Evaluate the following indefinite integral

$$\int -v^2 + 4\sqrt[3]{w} + 2\sqrt{w}dw$$

**Exercise 6.70** Integrate the polynomial with respect to  $v$

$$-7v^2 + 4v + 1$$

**Exercise 6.71** Evaluate the following indefinite integral

$$\int 6w^{\frac{3}{4}} + 10w^3 - 2w^2 + 9z^{\frac{3}{5}} + z^{\frac{2}{5}}dw$$

**Exercise 6.72** Evaluate the following indefinite integral

$$\int r(-3a^5 r^5 - 3ar^3 + 4r^4 - 8) dr$$

**Exercise 6.73** Evaluate the following indefinite integral

$$\int 7a^5 u^4 + 10a^5 u^3 - a^2 - 8au - 10udu$$

**Exercise 6.74** Evaluate the following indefinite integral

$$\int m^2(-2m^3 t - 3m - 8t^5) dm$$

**Exercise 6.75** Evaluate the following indefinite integral

$$\int 10b^{\frac{6}{5}}y^{\frac{3}{2}} - 3y^{\frac{5}{4}} - 3y^3 + 3y db$$

**Exercise 6.76** Integrate the polynomial with respect to  $w$

$$-7w^2 + w + 6$$

**Exercise 6.77** Evaluate the definite integral for the following function, where the limits are 1 and 3.

$$f(p) = 8p^{\frac{6}{5}} - 8\sqrt[5]{p} - 7p^4 + 3p - 1$$

**Exercise 6.78** Evaluate the following indefinite integral

$$\int 2\sqrt[4]{c} - 7m^{\frac{3}{5}} - 9\sqrt{m} dc$$

**Exercise 6.79** Evaluate the definite integral for the following function, where the limits are 0 and 2.

$$f(d) = -8d^{\frac{5}{6}} + 3d^{\frac{4}{3}}$$

**Exercise 6.80** Evaluate the definite integral for the following function, where the limits are 0 and 1.

$$f(q) = 6\sqrt[6]{q} + 10q^{\frac{5}{4}} - 5q^{\frac{3}{2}} + 9\sqrt{q} - 5q$$

**Exercise 6.81** Evaluate the definite integral for the following function, where the limits are 0 and 3.

$$f(r) = 7r^{\frac{5}{6}} + 2r^{\frac{3}{5}} - 7r^{\frac{2}{3}} - 3r^2 + 2$$

**Exercise 6.82** Evaluate the following indefinite integral

$$\int b^2 (-9b^4x^2 + 7b^4 + 6b^3x - 9b^2 - 4x^6) db$$

**Exercise 6.83** Evaluate the definite integral for the following function, where the limits are 0 and 4.

$$f(v) = v^{\frac{2}{5}} - 4\sqrt[4]{v} + 10v^{\frac{3}{2}} - 9v$$

**Exercise 6.84** Evaluate the following indefinite integral

$$\int 2\sqrt[6]{q} - q^5\sqrt{y} + 4q^3y^2 + 3y^{\frac{5}{4}} dq$$

**Exercise 6.85** Evaluate the definite integral for the following function, where the limits are 2 and 3.

$$f(c) = -10c^{\frac{6}{5}} - 4c^2 + 5c$$



**Exercise 6.86** Evaluate the definite integral for the following function, where the limits are 1 and 3.

$$f(l) = l^{\frac{5}{4}} + \sqrt[3]{l} - 5l^6 + 3$$

**Exercise 6.87** Evaluate the following indefinite integral

$$\int -6b^{\frac{4}{3}} - 3b^3 - 2w^{\frac{2}{3}} + 3w^3 dw$$

**Exercise 6.88** Evaluate the following indefinite integral

$$\int -4q^6 t^3 - 2q^4 + 2qt^3 + 4qt^2 - 10dq$$

**Exercise 6.89** Integrate the polynomial with respect to  $p$

$$-6p^3 + p^2 - 8p - 1$$

**Exercise 6.90** Evaluate the following indefinite integral

$$\int a^3 (-9a^3 - 2y^5 - 9y) da$$

**Exercise 6.91** Integrate the polynomial with respect to  $b$

$$-7b^4 - 8b^3 - b^2 - 4b + 4$$

**Exercise 6.92** Evaluate the following indefinite integral

$$\int 2n^6 p^4 + 7n^5 p^4 - 7n^5 - 7n^3 - 2dp$$

**Exercise 6.93** Evaluate the following indefinite integral

$$\int -10pq - 4q^{\frac{5}{6}} - 8q^{\frac{5}{3}} + 3dp$$

**Exercise 6.94** Evaluate the following indefinite integral

$$\int z (-7xz^5 + 4xz^3 + 7z - 5) dz$$

**Exercise 6.95** Evaluate the following indefinite integral

$$\int 8c^4 r^6 - 10c^4 + 4cr^2 + 5r^2 dc$$

**Exercise 6.96** Evaluate the definite integral for the following function, where the limits are 1 and 4.

$$f(a) = 4a^{\frac{4}{5}} + 4a^{\frac{5}{2}} - 6a^4 + 4a$$

**Exercise 6.97** Evaluate the following indefinite integral

$$\int a(5a^4u^3 + 2a^2 - 10u) du$$

**Exercise 6.98** Evaluate the following indefinite integral

$$\int -b^6s^3 - 9b^3 - 10db$$

**Exercise 6.99** Integrate the polynomial with respect to  $z$

$$z^2 - 4z - 9$$

**Exercise 6.100** Integrate the polynomial with respect to  $r$

$$2r^3 - 8r^2 - 4r + 2$$

## 6.2 Solutions

1. The indefinite integral is

$$\frac{d^2(d^3 + 30)}{5}$$

The value of the definite integral is 102.600000000000.

- 2.

$$\frac{a(-12a^4 + 135a^3 + 200a^2 - 210a + 180)}{60}$$

- 3.

$$v\left(-b^4 - 9b^2 + \frac{2bv^6}{7} - 1\right)$$

4. The indefinite integral is

$$-\frac{8y^{\frac{9}{4}}}{9} - \frac{7y^4}{4} + \frac{9y^2}{2}$$

The value of the definite integral is -1016.34108402714.

- 5.

$$\frac{q^3w(4q^2w^2 + 9q^2 - w^5)}{3}$$

- 6.

$$\frac{5\sqrt[5]{wz^{\frac{8}{5}}}}{2} - 9w^{\frac{3}{2}}z - \frac{36w^6z^{\frac{5}{4}}}{5} + \frac{2z^7}{7} + 4z$$

- 7.

$$-3b^{\frac{5}{3}}\sqrt{t} + \frac{b^4t}{4} + 6b\sqrt{t} - 9bt$$

8.

$$-\frac{45n^{\frac{3}{5}}y^{\frac{11}{5}}}{11} + 4n^{\frac{5}{4}}y^2 - \frac{3n^{\frac{3}{2}}y^6}{2} + \frac{15y^{\frac{7}{3}}}{7}$$

9. The indefinite integral is

$$\frac{40v^{\frac{9}{5}}}{9} + \frac{24v^{\frac{5}{4}}}{5} - 3v^{\frac{7}{3}} - \frac{8v^3}{3} - 8v$$

The value of the definite integral is -172.257329007093.

10. The indefinite integral is

$$-\frac{24a^{\frac{5}{4}}}{5} + \frac{2a^3}{3} - 2a^2$$

The value of the definite integral is 3.57829591735737.

11.

$$-10mp - \frac{2p^{\frac{7}{2}}}{7} + \frac{p^5}{5} + 10p$$

12.

$$\frac{z(21z^3 + 4z^2 + 18z - 84)}{12}$$

13.

$$\frac{q(-q^5 \cdot (4u + 1) + 12q^2u^2 + 48u^6)}{6}$$

14.

$$r(2r^2 + 5r - 9)$$

15.

$$\frac{r(-32r^6 - 35r^3y + 14r(2y^5 + 9) + 84)}{28}$$

16. The indefinite integral is

$$-3a^{\frac{7}{3}} + \frac{21a^{\frac{5}{3}}}{5} - \frac{10a^{\frac{3}{2}}}{3} + \frac{4a^6}{3} - \frac{5a^2}{2}$$

The value of the definite integral is 155588.034030386.

17.

$$\frac{w(-8w^4 + 5w^3 - 10w^2 + 45w + 40)}{10}$$

18.

$$\frac{u(-2u^2 + 3u + 16)}{2}$$

19.

$$4t^{\frac{7}{4}} + t^4u - 8tu^{\frac{6}{5}} - tu^{\frac{4}{5}} + 8tu^3$$

20. The indefinite integral is

$$\frac{z(8z^4 - 5z - 10)}{5}$$

The value of the definite integral is 1240.600000000000.

21.

$$\frac{bx^3(-10b^2x^3 - 30x + 3)}{3}$$

22.

$$\frac{a(25a^5 - 24a^4 + 300z^5 - 30)}{30}$$

23. The indefinite integral is

$$\frac{6p^{\frac{11}{6}}}{11} + \frac{40p^{\frac{7}{5}}}{7} + 4p^{\frac{5}{2}} - \frac{8p^5}{5} + \frac{3p^4}{4}$$

The value of the definite integral is -235.456672956732.

24.

$$-\frac{8b^{\frac{5}{2}}q^{\frac{3}{2}}}{5} - \frac{8b^3q^{\frac{4}{5}}}{3} + \frac{b^2q^{\frac{4}{3}}}{2} + 6bq^{\frac{3}{5}} - 6b$$

25.

$$\frac{a(-14a^2p^3 - 9a + 18p^{\frac{3}{4}} - 42p^3)}{6}$$

26.

$$-\frac{9c^{\frac{8}{3}}}{4} - 4c^{\frac{5}{2}} + 4c^{\frac{3}{2}}s^2 - 3c^3 - 10cs^{\frac{3}{4}}$$

27.

$$x(3q^{\frac{3}{2}}x + q^2 + 9q + 4)$$

28.

$$-\frac{4m^{\frac{3}{2}}}{3} - 3m^3p + 3m^2\sqrt[6]{p} - 3m$$

29. The indefinite integral is

$$\frac{3c^2(c^2 + 3)}{2}$$

The value of the definite integral is 3660.000000000000.

30. The indefinite integral is

$$-\frac{20s^{\frac{9}{5}}}{9} + \frac{5s^{\frac{8}{5}}}{2} + 5s^{\frac{7}{5}} - \frac{3s^5}{5} - \frac{5s^2}{2}$$

The value of the definite integral is -130.413717000468.

31.

$$\frac{r(14lr^2 - 49l + 2r^6)}{7}$$

32. The indefinite integral is

$$-\frac{15t^{\frac{4}{3}}}{2} + \frac{5t^6}{6} + \frac{t^3}{3}$$

The value of the definite integral is 12961.2747177480.

33.

$$-\frac{30q^{\frac{7}{6}}x^6}{7} + 4q^2 - 6q$$

34.

$$\frac{v(-7v^2 + 9v - 3)}{3}$$

35.

$$\frac{n(l^4 \cdot (28l + 35) + 8n^6)}{7}$$

36.

$$r^{\frac{2}{5}}x + \frac{2r^{\frac{3}{2}}x^{\frac{7}{2}}}{7} + \frac{14r^5x^{\frac{5}{2}}}{5} + \frac{5rx^6}{3} + 4x^2$$

37. The indefinite integral is

$$-\frac{5w^{\frac{9}{5}}}{9} - \frac{27w^{\frac{4}{3}}}{4} + 2w^{\frac{3}{2}}$$

The value of the definite integral is -33.5963464765209.

38. The indefinite integral is

$$\frac{35v^{\frac{11}{5}}}{11} + \frac{32v^{\frac{9}{4}}}{9} - v$$

The value of the definite integral is 283.454257490695.

39.

$$\frac{u(10u^2 - 9u - 15)}{3}$$

40.

$$\frac{x(-108x^4 + 75x^3 + 40x^2 + 60x + 300)}{60}$$

41. The indefinite integral is

$$-\frac{48n^{\frac{11}{6}}}{11} - \frac{50n^{\frac{7}{5}}}{7} - \frac{n^7}{7} + 6n$$

The value of the definite integral is -39798.7661423313.

42.

$$\frac{p(6c^3(c^3 - 2) + 7p^2)}{3}$$

43. The indefinite integral is

$$\frac{50l^{\frac{9}{5}}}{9} + \frac{10l^{\frac{6}{5}}}{3} + \frac{8l^3}{3} + 8l$$

The value of the definite integral is 139.030898726601.

44.

$$-\frac{12n^3z^{\frac{5}{3}}}{5} - 2n^2z^2 + \frac{18nz^{\frac{7}{2}}}{7} + 2z$$

45.

$$\frac{r(24r^4 - 45r^3 - 40r^2 + 150r - 300)}{60}$$

46. The indefinite integral is

$$\frac{5u^{\frac{8}{5}}}{8} - \frac{5u^{\frac{6}{5}}}{2} + 8u$$

The value of the definite integral is 18.4234126672562.

47.

$$-\frac{30l^{\frac{7}{6}}\sqrt{z}}{7} - \frac{21l^{\frac{4}{3}}z^{\frac{2}{3}}}{4} + 6lz^{\frac{2}{3}}$$

48.

$$\frac{n(12n^4 + n^3 \cdot (30r^6 - 35) - 120r^3)}{20}$$

49.

$$\frac{c(4c^2z^2 - 9c - 30)}{3}$$

50.

$$\frac{s(20s^2 - 15s - 54)}{6}$$

51.

$$-\frac{4r^{\frac{3}{2}}s^{\frac{2}{3}}}{3} + \frac{2r^3}{3} - 2r^2 + rs^{\frac{3}{2}} + 4rs$$

52.

$$m(2m^3 - m^2\sqrt{v} + 2v^{\frac{2}{5}} + 2v)$$

53.

$$\frac{w(14b^6 - 5b^5w + 4b^2w^3 - 4b - 2w^5)}{2}$$

54.

$$\frac{3b(-b^3 + 6r(r+1))}{2}$$

55. The indefinite integral is

$$\frac{50x^{\frac{9}{5}}}{9} + \frac{20x^{\frac{7}{5}}}{7} + \frac{x^7}{7} + \frac{3x^2}{2}$$

The value of the definite integral is 2451.83490789485.

56.

$$-\frac{32b^{\frac{9}{4}}}{9} - \frac{3b^{\frac{4}{3}}}{4} + \frac{8b^5}{5}$$

57.

$$\frac{s(s^5w(-w^2 + 4w - 5) + 18)}{6}$$

58.

$$9c^{\frac{6}{5}}q - 4\sqrt[5]{c}q - \frac{5q^{\frac{11}{5}}}{11} - 5q^2$$

59.

$$y(2y^3 + 3y^2 - 2y + 1)$$

60.

$$b^4 \left( \frac{b(4z^2 - 7)}{5} - \frac{z^2}{4} \right)$$

61.

$$\frac{y(-12y^4 - 15y^3 + 20y^2 - 6y + 6)}{6}$$

62.

$$\frac{l(13l + 16u)}{2}$$

63.

$$\frac{4w^{\frac{5}{6}}y^{\frac{5}{2}}}{5} + w^{\frac{4}{5}}y + \frac{27w^{\frac{3}{4}}y^{\frac{4}{3}}}{4} - \frac{15w^{\frac{5}{3}}y^{\frac{4}{3}}}{2} - \frac{10y^{\frac{3}{2}}}{3}$$

64.

$$\frac{st(20s^5 - 4s^2 + 3t^5)}{2}$$

65.

$$\frac{c(48c^4 + 135c^3 - 40c^2 - 60c + 120)}{60}$$

66.

$$\frac{c(27c^3 + 4c^2 + 30c + 24)}{12}$$

67.

$$\frac{u(-24u^4 + 105u^3 - 80u^2 + 60u - 600)}{60}$$

68.

$$\frac{l(-35c^4l^3 - 56c^3l^4 + 80l^6 - 350)}{70}$$

69.

$$-v^2w + 3w^{\frac{4}{3}} + \frac{4w^{\frac{3}{2}}}{3}$$

70.

$$\frac{v(-7v^2 + 6v + 3)}{3}$$

71.

$$\frac{24w^{\frac{7}{4}}}{7} + \frac{5w^4}{2} - \frac{2w^3}{3} + 9wz^{\frac{3}{5}} + wz^{\frac{2}{5}}$$

72.

$$\frac{r^2(-45a^5r^5 - 63ar^3 + 70r^4 - 420)}{105}$$

73.

$$\frac{u(14a^5u^4 + 25a^5u^3 - 10a^2 - 40au - 50u)}{10}$$

74.

$$\frac{m^3(-4m^3t - 9m - 32t^5)}{12}$$

75.

$$\frac{50b^{\frac{11}{5}}y^{\frac{3}{2}}}{11} - 3by^{\frac{5}{4}} - 3by^3 + 3by$$

76.

$$\frac{w(-14w^2 + 3w + 36)}{6}$$

77. The indefinite integral is

$$\frac{40p^{\frac{11}{5}}}{11} - \frac{20p^{\frac{6}{5}}}{3} - \frac{7p^5}{5} + \frac{3p^2}{2} - p$$

The value of the definite integral is -309.914939556409.



78.

$$\frac{8c^{\frac{5}{4}}}{5} - 7cm^{\frac{3}{5}} - 9c\sqrt{m}$$

79. The indefinite integral is

$$-\frac{48d^{\frac{11}{6}}}{11} + \frac{9d^{\frac{7}{3}}}{7}$$

The value of the definite integral is -9.07063820028891.

80. The indefinite integral is

$$\frac{36q^{\frac{7}{6}}}{7} + \frac{40q^{\frac{9}{4}}}{9} - 2q^{\frac{5}{2}} + 6q^{\frac{3}{2}} - \frac{5q^2}{2}$$

The value of the definite integral is 11.0873015873016.

81. The indefinite integral is

$$\frac{42r^{\frac{11}{6}}}{11} + \frac{5r^{\frac{8}{3}}}{4} - \frac{21r^{\frac{5}{3}}}{5} - r^3 + 2r$$

The value of the definite integral is -11.3456015788855.

82.

$$\frac{b^3 \cdot (15b^4 \cdot (7 - 9x^2) + 105b^3x - 189b^2 - 140x^6)}{105}$$

83. The indefinite integral is

$$\frac{5v^{\frac{7}{5}}}{7} - \frac{16v^{\frac{5}{4}}}{5} + 4v^{\frac{5}{2}} - \frac{9v^2}{2}$$

The value of the definite integral is 42.8726410490308.

84.

$$\frac{12q^{\frac{7}{6}}}{7} - \frac{q^6\sqrt{y}}{6} + q^4y^2 + 3qy^{\frac{5}{4}}$$

85. The indefinite integral is

$$-\frac{50c^{\frac{11}{5}}}{11} - \frac{4c^3}{3} + \frac{5c^2}{2}$$

The value of the definite integral is -42.9096289540220.

86. The indefinite integral is

$$\frac{4l^{\frac{9}{4}}}{9} + \frac{3l^{\frac{4}{3}}}{4} - \frac{5l^7}{7} + 3l$$

The value of the definite integral is -1548.11365828801.

87.

$$-6b^{\frac{4}{3}}w - 3b^3w - \frac{6w^{\frac{5}{3}}}{5} + \frac{3w^4}{4}$$

88.

$$\frac{q(-20q^6t^3 - 14q^4 + 35qt^2(t+2) - 350)}{35}$$

89.

$$\frac{p(-9p^3 + 2p^2 - 24p - 6)}{6}$$

90.

$$\frac{a^4(-36a^3 - 14y^5 - 63y)}{28}$$

91.

$$\frac{b(-21b^4 - 30b^3 - 5b^2 - 30b + 60)}{15}$$

92.

$$\frac{p(n^5p^4 \cdot (2n+7) - 35n^5 - 35n^3 - 10)}{5}$$

93.

$$p\left(-5pq - 4q^{\frac{5}{6}} - 8q^{\frac{5}{3}} + 3\right)$$

94.

$$\frac{z^2(-30xz^5 + 24xz^3 + 70z - 75)}{30}$$

95.

$$\frac{c(c^4 \cdot (8r^6 - 10) + 10cr^2 + 25r^2)}{5}$$

96. The indefinite integral is

$$\frac{20a^{\frac{9}{5}}}{9} + \frac{8a^{\frac{7}{2}}}{7} - \frac{6a^5}{5} + 2a^2$$

The value of the definite integral is -1027.73329278585.

97.

$$\frac{au(5a^4u^3 + 8a^2 - 20u)}{4}$$

98.

$$\frac{b(-4b^6s^3 - 63b^3 - 280)}{28}$$

99.

$$\frac{z(z^2 - 6z - 27)}{3}$$

100.

$$\frac{r(3r^3 - 16r^2 - 12r + 12)}{6}$$