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Motivation Problem generation method Structure

## 1. Introduction

### 1.1 Motivation

My year 9 daughter kept asking me to write her new math problems, apart from those in the text book, so she can do more practice. I'd take a piece of paper and write problems, but it's a tedious process. So I thought: how about I **generate tons** of them?

### 1.2 Problem generation method

The problems and solutions have all been generated using the python programming language, and relied on a number of open source python packages, especially the SymPy package. Therefore, you can trust the accuracy of the solutions.

### 1.3 Structure

This document includes hundreds of mathematics problems that have been designed for year 11 students enrolled in VCE Mathematics Methods.

Most of the problems require the student to perform a single task, such as calculating the derivative of a function or finding the prime numbers within a range of integers. In addition, a number of questions have been designed based on the Mathematics Methods exam questions, which require the student to perform two or more tasks.

I hope you find them useful.

# 2. Functions

### 2.1 Problems

**Exercise 2.1** The line defined by the function

$$f(x) = ax + b$$

passes through the points (1,6) and (0,7). Find the values of a and b. **Exercise 2.2** The line defined by the function

$$f(x) = ax + b$$

passes through the points (-7, -5) and (4, 8). Find the values of a and b. **Exercise 2.3** The line defined by the function

$$f(x) = ax + b$$

passes through the points (4,9) and (5,7). Find the values of a and b. **Exercise 2.4** The line defined by the function

$$f(x) = ax + b$$

passes through the points (-3,10) and (7,-7). Find the values of a and b. **Exercise 2.5** The line defined by the function

$$f(x) = ax + b$$

passes through the points (3,-10) and (-7,0). Find the values of a and b. **Exercise 2.6** The line defined by the function

$$f(x) = ax + b$$

passes through the points (4,5) and (2,9). Find the values of a and b. **Exercise 2.7** The line defined by the function

$$f(x) = ax + b$$

passes through the points (5, -10) and (10, 0). Find the values of a and b.

Exercise 2.8 The line defined by the function

$$f(x) = ax + b$$

passes through the points (7,5) and (6,-4). Find the values of a and b. **Exercise 2.9** The line defined by the function

$$f(x) = ax + b$$

passes through the points (8,2) and (-6,6). Find the values of a and b. **Exercise 2.10** The line defined by the function

$$f(x) = ax + b$$

passes through the points (-2,-1) and (5,-9). Find the values of a and b. **Exercise 2.11** The graph defined by the function

$$f(x) = ax^2 + b$$

passes through the points (-4,25) and (-8,121). Find the values of a and b. **Exercise 2.12** The graph defined by the function

$$f(x) = ax^2 + b$$

passes through the points (-1,3) and (-6,-32). Find the values of a and b. **Exercise 2.13** The graph defined by the function

$$f(x) = ax^2 + b$$

passes through the points (5, -109) and (-6, -153). Find the values of a and b. **Exercise 2.14** The graph defined by the function

$$f(x) = ax^2 + b$$

passes through the points (-5, -22) and (-4, -13). Find the values of a and b. **Exercise 2.15** The graph defined by the function

$$f(x) = ax^2 + b$$

passes through the points (-3,36) and (9,396). Find the values of a and b. **Exercise 2.16** The graph defined by the function

$$f(x) = ax^2 + b$$

passes through the points (-5,106) and (-3,42). Find the values of a and b. **Exercise 2.17** The graph defined by the function

$$f(x) = ax^2 + b$$

passes through the points (5,60) and (-10,210). Find the values of a and b. **Exercise 2.18** The graph defined by the function

$$f(x) = ax^2 + b$$

passes through the points (0,6) and (5,-94). Find the values of a and b.

Exercise 2.19 The graph defined by the function

$$f(x) = ax^2 + b$$

passes through the points (-3,16) and (-10,198). Find the values of a and b. **Exercise 2.20** The graph defined by the function

$$f(x) = ax^2 + b$$

passes through the points (-5,110) and (0,10). Find the values of a and b. **Exercise 2.21** The graph defined by the function

$$f(x) = a^2x + b$$

passes through the points (2,24) and (-4,-30). Find the values of a and b. **Exercise 2.22** The graph defined by the function

$$f(x) = a^2x + b$$

passes through the points (-1, -17) and (-3, -35). Find the values of a and b. **Exercise 2.23** The graph defined by the function

$$f(x) = a^2x + b$$

passes through the points (1,31) and (-9,-219). Find the values of a and b. **Exercise 2.24** The graph defined by the function

$$f(x) = a^2x + b$$

passes through the points (2,56) and (-10,-244). Find the values of a and b. **Exercise 2.25** The graph defined by the function

$$f(x) = a^2x + b$$

passes through the points (2,56) and (-10,-244). Find the values of a and b. **Exercise 2.26** The graph defined by the function

$$f(x) = a^2x + b$$

passes through the points (-5, -40) and (0,5). Find the values of a and b. **Exercise 2.27** The graph defined by the function

$$f(x) = a^2x + b$$

passes through the points (4,101) and (3,76). Find the values of a and b. **Exercise 2.28** The graph defined by the function

$$f(x) = a^2x + b$$

passes through the points (-4, -14) and (-7, -17). Find the values of a and b. **Exercise 2.29** The graph defined by the function

$$f(x) = a^2x + b$$

passes through the points (2,21) and (-8,-69). Find the values of a and b.

Exercise 2.30 The graph defined by the function

$$f(x) = a^2x + b$$

passes through the points (-4, -27) and (-1, 0). Find the values of a and b. **Exercise 2.31** The graph defined by the function

$$f(x) = a^2x + ab$$

passes through the points (4,0) and (-8,-12). Find the values of a and b. **Exercise 2.32** The graph defined by the function

$$f(x) = a^2x + ab$$

passes through the points (4,90) and (5,115). Find the values of a and b. **Exercise 2.33** The graph defined by the function

$$f(x) = a^2x + ab$$

passes through the points (0,5) and (-2,-13). Find the values of a and b. **Exercise 2.34** The graph defined by the function

$$f(x) = a^2x + ab$$

passes through the points (-5, -42) and (-9, -78). Find the values of a and b. **Exercise 2.35** The graph defined by the function

$$f(x) = a^2x + ab$$

passes through the points (1,2) and (-1,-6). Find the values of a and b. **Exercise 2.36** The graph defined by the function

$$f(x) = a^2x + ab$$

passes through the points (1,17) and (-9,-73). Find the values of a and b. **Exercise 2.37** The graph defined by the function

$$f(x) = a^2x + ab$$

passes through the points (-4, -54) and (4, 74). Find the values of a and b. **Exercise 2.38** The graph defined by the function

$$f(x) = a^2x + ab$$

passes through the points (-5, -87) and (7, 105). Find the values of a and b. **Exercise 2.39** The graph defined by the function

$$f(x) = a^2x + ab$$

passes through the points (-3, -49) and (10, 159). Find the values of a and b. **Exercise 2.40** The graph defined by the function

$$f(x) = a^2x + ab$$

passes through the points (1, -8) and (0, -9). Find the values of a and b.

Exercise 2.41 The function

$$f(x) = m + 16x^2 - 16x + 5$$

is greater than -4 for all values of x for a certain value of k. Find the value of k. Exercise 2.42 The function

$$f(x) = m + x^2 - 14x + 37$$

is greater than -2 for all values of x for a certain value of k. Find the value of k. **Exercise 2.43** The function

$$f(x) = m + 64x^2 - 128x + 59$$

is greater than -4 for all values of x for a certain value of k. Find the value of k. **Exercise 2.44** The function

$$f(x) = m + 100x^2 - 100x + 28$$

is greater than 2 for all values of x for a certain value of k. Find the value of k. **Exercise 2.45** The function

$$f(x) = m + 9x^2 - 18x + 18$$

is greater than 3 for all values of x for a certain value of k. Find the value of k. Exercise 2.46 The function

$$f(x) = m + 81x^2 - 126x + 52$$

is greater than 9 for all values of x for a certain value of k. Find the value of k. **Exercise 2.47** The function

$$f(x) = m + 36x^2 - 84x + 50$$

is greater than 7 for all values of x for a certain value of k. Find the value of k. Exercise 2.48 The function

$$f(x) = m + 81x^2 + 90x + 15$$

is greater than -1 for all values of x for a certain value of k. Find the value of k. Exercise 2.49 The function

$$f(x) = m + 36x^2 + 24x + 14$$

is greater than 3 for all values of x for a certain value of k. Find the value of k. **Exercise 2.50** The function

$$f(x) = m + 16x^2 + 80x + 91$$

is greater than -2 for all values of x for a certain value of k. Find the value of k. **Exercise 2.51** A line is parallel to the graph

$$16x + 8y + 3 = 0$$

and passes through the point (-7,9). What is the equation of that line?

Exercise 2.52 A line is parallel to the graph

$$3x + y + 1 = 0$$

and passes through the point (18, -44). What is the equation of that line? **Exercise 2.53** A line is parallel to the graph

$$-50x - 5y + 7 = 0$$

and passes through the point (-8,76). What is the equation of that line? **Exercise 2.54** A line is parallel to the graph

$$-70x - 10y + 2 = 0$$

and passes through the point (11, -71). What is the equation of that line? **Exercise 2.55** A line is parallel to the graph

$$90x - 10y + 3 = 0$$

and passes through the point (2,23). What is the equation of that line? **Exercise 2.56** A line is parallel to the graph

$$-12x + 6y - 8 = 0$$

and passes through the point (-5, -12). What is the equation of that line? **Exercise 2.57** A line is parallel to the graph

$$56x + 7y - 7 = 0$$

and passes through the point (17, -144). What is the equation of that line? **Exercise 2.58** A line is parallel to the graph

$$-40x + 10y - 5 = 0$$

and passes through the point (8,41). What is the equation of that line? **Exercise 2.59** A line is parallel to the graph

$$-8x - 4y + 7 = 0$$

and passes through the point (9,-23). What is the equation of that line? **Exercise 2.60** A line is parallel to the graph

$$4x - 2y - 7 = 0$$

and passes through the point (6,22). What is the equation of that line? **Exercise 2.61** A line is perpendicular to the graph

$$-10x - 60y - 10 = 0$$

and passes through the point (19, -118). What is the equation of that line? **Exercise 2.62** A line is perpendicular to the graph

$$-10x - 70y + 8 = 0$$

and passes through the point (-14,97). What is the equation of that line?

Exercise 2.63 A line is perpendicular to the graph

$$2x + 2y + 10 = 0$$

and passes through the point (-6, 13). What is the equation of that line? **Exercise 2.64** A line is perpendicular to the graph

$$10x - 80y + 2 = 0$$

and passes through the point (12,92). What is the equation of that line? **Exercise 2.65** A line is perpendicular to the graph

$$-7x + 49y + 9 = 0$$

and passes through the point (18,127). What is the equation of that line? **Exercise 2.66** A line is perpendicular to the graph

$$4x + 28y + 4 = 0$$

and passes through the point (8, -64). What is the equation of that line? **Exercise 2.67** A line is perpendicular to the graph

$$5x - 10y + 1 = 0$$

and passes through the point (-16, -36). What is the equation of that line? **Exercise 2.68** A line is perpendicular to the graph

$$-x-y-8=0$$

and passes through the point (18, -9). What is the equation of that line? **Exercise 2.69** A line is perpendicular to the graph

$$8x + 80y - 8 = 0$$

and passes through the point (20, -203). What is the equation of that line? **Exercise 2.70** A line is perpendicular to the graph

$$6x + 42y - 2 = 0$$

and passes through the point (-12,83). What is the equation of that line? **Exercise 2.71** Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -5a - 4x$$

and

$$g(x) = -a - 5x$$

**Exercise 2.72** Find the value of x at which the graphs of the following functions intersect:

$$f(x) = 5a - 4x$$

and

$$g(x) = 2a - 2x$$

**Exercise 2.73** Find the value of x at which the graphs of the following functions intersect:

$$f(x) = 4a + 2x$$

and

$$g(x) = 2a - 3x$$

**Exercise 2.74** Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -a + 5x$$

and

$$g(x) = -2a - 3x$$

**Exercise 2.75** Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -a + x$$

and

$$g(x) = 2a + 4x$$

**Exercise 2.76** Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -2x - 3$$

and

$$g(x) = -a - 5x$$

**Exercise 2.77** Find the value of x at which the graphs of the following functions intersect:

$$f(x) = 2x + 1$$

and

$$g(x) = -2a + 5x$$

**Exercise 2.78** Find the value of x at which the graphs of the following functions intersect:

$$f(x) = 5a - 2x$$

and

$$g(x) = -a + x$$

**Exercise 2.79** Find the value of x at which the graphs of the following functions intersect:

$$f(x) = 4x - 2$$

and

$$g(x) = -4a + 5x$$

**Exercise 2.80** Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -4a + 5x$$

and

$$g(x) = -a + x$$

**Exercise 2.81** Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -ax^2 - 3a + 3x$$

and

$$g(x) = a(2x-5)$$

**Exercise 2.82** Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -4ax^2 + 3a + x$$

and

$$g(x) = a(2-x)$$

**Exercise 2.83** Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -2ax^2 - 2a - 3x$$

and

$$g(x) = 2a(1-2x)$$

**Exercise 2.84** Find the value of x at which the graphs of the following functions intersect:

$$f(x) = ax^2 - 3a + 3x$$

and

$$g(x) = a(x+3)$$

**Exercise 2.85** Find the value of x at which the graphs of the following functions intersect:

$$f(x) = 4ax^2 - 2a + 2x$$

and

$$g(x) = a(-5x - 4)$$

**Exercise 2.86** Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -4ax^2 + 2a + 3x$$

and

$$g(x) = a(5x - 2)$$

**Exercise 2.87** Find the value of x at which the graphs of the following functions intersect:

$$f(x) = ax^2 + 2a + x$$

and

$$g(x) = a(x-2)$$

**Exercise 2.88** Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -3ax^2 + 5a + 5x$$

and

$$g(x) = a(x+1)$$

**Exercise 2.89** Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -3ax^2 - 2a + 3x$$

and

$$g(x) = a(x+1)$$

**Exercise 2.90** Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -ax^2 + a - 5x$$

and

$$g(x) = a(-x-1)$$

**Exercise 2.91** If f(x) is defined as

$$f(x) = 4 - x$$

and g(x) is defined as

$$g(x) = 5x^2 - 5x - 5$$

simplify g(f(x)).

**Exercise 2.92** If f(x) is defined as

$$f(x) = -3x - 3$$

and g(x) is defined as

$$g(x) = -2x^2 - x - 3$$

simplify g(f(x)).

**Exercise 2.93** If f(x) is defined as

$$f(x) = x - 2$$

and g(x) is defined as

$$g(x) = -x^2 - 2x - 2$$

simplify g(f(x)).

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**Exercise 2.94** If f(x) is defined as

$$f(x) = 2x - 2$$

and g(x) is defined as

$$g(x) = 5x^2 - x - 3$$

simplify g(f(x)).

**Exercise 2.95** If f(x) is defined as

$$f(x) = 3x - 1$$

and g(x) is defined as

$$g(x) = -3x^2 - x + 2$$

simplify g(f(x)).

**Exercise 2.96** If f(x) is defined as

$$f(x) = x - 5$$

and g(x) is defined as

$$g(x) = -4x^2 + 3x - 5$$

simplify g(f(x)).

**Exercise 2.97** If f(x) is defined as

$$f(x) = x + 5$$

and g(x) is defined as

$$g(x) = 4x^2 + 2x + 3$$

simplify g(f(x)).

**Exercise 2.98** If f(x) is defined as

$$f(x) = -4x - 4$$

and g(x) is defined as

$$g(x) = 4x^2 - 2x + 3$$

simplify g(f(x)).

**Exercise 2.99** If f(x) is defined as

$$f(x) = 4 - 4x$$

and g(x) is defined as

$$g(x) = -4x^2 + 2x + 5$$

simplify g(f(x)).

**Exercise 2.100** If f(x) is defined as

$$f(x) = 4x + 2$$

and g(x) is defined as

$$g(x) = -5x^2 - 5x + 2$$

simplify g(f(x)).

**Exercise 2.101** If f(x) is defined as

$$f(x) = -4ax - 3x^2$$

and g(x) is defined as

$$g(x) = 2 - 4x$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.102** If f(x) is defined as

$$f(x) = -2ax + 2x^2$$

and g(x) is defined as

$$g(x) = -4x - 3$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.103** If f(x) is defined as

$$f(x) = -3ax - 2x^2$$

and g(x) is defined as

$$g(x) = 5 - 5x$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.104** If f(x) is defined as

$$f(x) = -3ax - 2x^2$$

and g(x) is defined as

$$g(x) = -3x - 2$$

solve the equation

$$g(f(x)) = 0$$

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**Exercise 2.105** If f(x) is defined as

$$f(x) = -2ax - 3x^2$$

and g(x) is defined as

$$g(x) = 2x + 1$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.106** If f(x) is defined as

$$f(x) = -2ax - 2x^2$$

and g(x) is defined as

$$g(x) = -4x - 4$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.107** If f(x) is defined as

$$f(x) = 4ax + 5x^2$$

and g(x) is defined as

$$g(x) = 2 - 4x$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.108** If f(x) is defined as

$$f(x) = 4ax + 2x^2$$

and g(x) is defined as

$$g(x) = 2x - 4$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.109** If f(x) is defined as

$$f(x) = ax - 5x^2$$

and g(x) is defined as

$$g(x) = 3x + 2$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.110** If f(x) is defined as

$$f(x) = -4ax - 3x^2$$

and g(x) is defined as

$$g(x) = 2x - 3$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.111** If f(x) is defined as

$$f(x) = \frac{3-x}{-5x-3}$$

find  $f^{-1}(x)$ .

**Exercise 2.112** If f(x) is defined as

$$f(x) = \frac{-x-4}{3x+2}$$

find  $f^{-1}(x)$ .

**Exercise 2.113** If f(x) is defined as

$$f(x) = \frac{1-x}{5x-3}$$

find  $f^{-1}(x)$ .

**Exercise 2.114** If f(x) is defined as

$$f(x) = \frac{-2x - 3}{-5x - 4}$$

find  $f^{-1}(x)$ .

**Exercise 2.115** If f(x) is defined as

$$f(x) = \frac{2x+4}{1-x}$$

find  $f^{-1}(x)$ .

**Exercise 2.116** If f(x) is defined as

$$f(x) = \frac{-5x - 3}{-5x - 2}$$

find  $f^{-1}(x)$ .

**Exercise 2.117** If f(x) is defined as

$$f(x) = \frac{4x - 5}{5x - 1}$$

find  $f^{-1}(x)$ .

**Exercise 2.118** If f(x) is defined as

$$f(x) = \frac{4 - 4x}{4x + 5}$$

find  $f^{-1}(x)$ .

**Exercise 2.119** If f(x) is defined as

$$f(x) = \frac{-5x-4}{4x-3}$$

find  $f^{-1}(x)$ .

**Exercise 2.120** If f(x) is defined as

$$f(x) = \frac{5 - 5x}{4x + 2}$$

find  $f^{-1}(x)$ .

**Exercise 2.121** If f(x) is defined as

$$f(x) = \frac{-4ax - 3a}{-2a - x}$$

find the value of a if  $f^{-1}(1) = 4$ .

**Exercise 2.122** If f(x) is defined as

$$f(x) = \frac{3ax - 2a}{3a + x}$$

find the value of a if  $f^{-1}(7) = 5$ .

**Exercise 2.123** If f(x) is defined as

$$f(x) = \frac{-4ax + 3a}{-3a - x}$$

find the value of a if  $f^{-1}(7) = 2$ .

**Exercise 2.124** If f(x) is defined as

$$f(x) = \frac{ax - a}{-5a - x}$$

find the value of a if  $f^{-1}(3) = 6$ .

**Exercise 2.125** If f(x) is defined as

$$f(x) = \frac{-ax + 4a}{-2a - 2x}$$

find the value of a if  $f^{-1}(9) = 10$ .

**Exercise 2.126** If f(x) is defined as

$$f(x) = \frac{-4ax + a}{-2a + 2x}$$

find the value of a if  $f^{-1}(7) = 3$ .

**Exercise 2.127** If f(x) is defined as

$$f(x) = \frac{5ax - 3a}{a + x}$$

find the value of a if  $f^{-1}(7) = 8$ . Exercise 2.128 If f(x) is defined as

$$f(x) = \frac{-2ax + a}{4a - x}$$

find the value of a if  $f^{-1}(3) = 9$ . Exercise 2.129 If f(x) is defined as

$$f(x) = \frac{-5ax - 2a}{a - 2x}$$

find the value of a if  $f^{-1}(4) = 4$ . Exercise 2.130 If f(x) is defined as

$$f(x) = \frac{2ax + a}{3a + 2x}$$

find the value of a if  $f^{-1}(6) = 2$ . Exercise 2.131 If f(x) is defined as

$$f(x) = \frac{3x - 5}{4x - 5}$$

what is the domain and range of f(x)? Exercise 2.132 If f(x) is defined as

$$f(x) = \frac{3-x}{4-x}$$

what is the domain and range of f(x)? Exercise 2.133 If f(x) is defined as

$$f(x) = \frac{-3x - 5}{x - 3}$$

what is the domain and range of f(x)? **Exercise 2.134** If f(x) is defined as

$$f(x) = \frac{1 - 5x}{4x - 4}$$

what is the domain and range of f(x)? **Exercise 2.135** If f(x) is defined as

$$f(x) = \frac{-5x - 3}{-2x - 4}$$

what is the domain and range of f(x)? **Exercise 2.136** If f(x) is defined as

$$f(x) = \frac{x-5}{4x-4}$$

what is the domain and range of f(x)?

**Exercise 2.137** If f(x) is defined as

$$f(x) = \frac{x-5}{x-3}$$

what is the domain and range of f(x)?

**Exercise 2.138** If f(x) is defined as

$$f(x) = \frac{3x+2}{-4x-2}$$

what is the domain and range of f(x)?

**Exercise 2.139** If f(x) is defined as

$$f(x) = \frac{2-x}{2x-4}$$

what is the domain and range of f(x)?

**Exercise 2.140** If f(x) is defined as

$$f(x) = \frac{-3x-2}{1-4x}$$

what is the domain and range of f(x)?

Exercise 2.141 A tranformation is defined by the matrix

$$\begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$$

Find the expression for the graph

$$y = -4x - 2$$

after applying the transformation.

Exercise 2.142 A tranformation is defined by the matrix

$$\begin{bmatrix} -3 & 0 \\ 0 & -5 \end{bmatrix}$$

Find the expression for the graph

$$y = 2 - 2x$$

after applying the transformation.

Exercise 2.143 A tranformation is defined by the matrix

$$\begin{bmatrix} 0 & -2 \\ -3 & 0 \end{bmatrix}$$

Find the expression for the graph

$$y = -5x - 4$$

Exercise 2.144 A tranformation is defined by the matrix

$$\begin{bmatrix} 0 & 2 \\ 5 & 0 \end{bmatrix}$$

Find the expression for the graph

$$y = 4 - 3x$$

after applying the transformation.

Exercise 2.145 A tranformation is defined by the matrix

$$\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$$

Find the expression for the graph

$$y = 4x + 2$$

after applying the transformation.

Exercise 2.146 A tranformation is defined by the matrix

$$\begin{bmatrix} 5 & 0 \\ 0 & -3 \end{bmatrix}$$

Find the expression for the graph

$$y = -3x - 5$$

after applying the transformation.

Exercise 2.147 A tranformation is defined by the matrix

$$\begin{bmatrix} 0 & -3 \\ 4 & 0 \end{bmatrix}$$

Find the expression for the graph

$$y = -x - 4$$

after applying the transformation.

**Exercise 2.148** A tranformation is defined by the matrix

$$\begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}$$

Find the expression for the graph

$$y = 5x - 3$$

after applying the transformation.

Exercise 2.149 A tranformation is defined by the matrix

$$\begin{bmatrix} -5 & 0 \\ 0 & 2 \end{bmatrix}$$

Find the expression for the graph

$$y = 3 - 5x$$

Exercise 2.150 A tranformation is defined by the matrix

$$\begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$$

Find the expression for the graph

$$y = 3 - x$$

after applying the transformation.

Exercise 2.151 A tranformation is defined by the matrix

$$\begin{bmatrix} 4 & 0 \\ 0 & -3 \end{bmatrix}$$

Find the expression for the graph

$$y = -2x^2 + 3x + 1$$

after applying the transformation.

**Exercise 2.152** A tranformation is defined by the matrix

$$\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$$

Find the expression for the graph

$$y = -3x^2 + 5x - 1$$

after applying the transformation.

Exercise 2.153 A tranformation is defined by the matrix

$$\begin{bmatrix} 5 & 0 \\ 0 & -5 \end{bmatrix}$$

Find the expression for the graph

$$y = 3x^2 - x - 2$$

after applying the transformation.

Exercise 2.154 A tranformation is defined by the matrix

$$\begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}$$

Find the expression for the graph

$$y = 5x^2 + x + 4$$

after applying the transformation.

**Exercise 2.155** A tranformation is defined by the matrix

$$\begin{bmatrix} -3 & 0 \\ 0 & -4 \end{bmatrix}$$

Find the expression for the graph

$$y = -x^2 - 5x + 5$$

Exercise 2.156 A tranformation is defined by the matrix

$$\begin{bmatrix} -5 & 0 \\ 0 & 5 \end{bmatrix}$$

Find the expression for the graph

$$y = x^2 - x - 1$$

after applying the transformation.

Exercise 2.157 A tranformation is defined by the matrix

$$\begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}$$

Find the expression for the graph

$$y = -4x^2 + 4x - 3$$

after applying the transformation.

Exercise 2.158 A tranformation is defined by the matrix

$$\begin{bmatrix} 0 & -4 \\ -3 & 0 \end{bmatrix}$$

Find the expression for the graph

$$y = -4x^2 + x + 3$$

after applying the transformation.

Exercise 2.159 A tranformation is defined by the matrix

$$\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$

Find the expression for the graph

$$y = -2x^2 + 3x + 3$$

after applying the transformation.

Exercise 2.160 A tranformation is defined by the matrix

$$\begin{bmatrix} -4 & 0 \\ 0 & -1 \end{bmatrix}$$

Find the expression for the graph

$$y = -3x^2 + 4x + 4$$

27 2.2 Solutions

### **Solutions** 2.2

1.

$${a:-1, b:7}$$

2.

$$\left\{a: \frac{13}{11}, b: \frac{36}{11}\right\}$$

3.

$${a:-2, b:17}$$

4.

$$\left\{a: -\frac{17}{10}, \ b: \frac{49}{10}\right\}$$

5.

$${a:-1, b:-7}$$

6.

$${a:-2, b:13}$$

7.

$${a:2, b:-20}$$

8.

$${a:9, b:-58}$$

9.

$$\left\{a:-\frac{2}{7},\ b:\frac{30}{7}\right\}$$

10.

$$\left\{a:-\frac{8}{7},\ b:-\frac{23}{7}\right\}$$

11.

$${a:-2, b:-7}$$

12.

$${a:1, b:4}$$

$$\left\{a: \frac{44}{61}, \ b: -\frac{7749}{61}\right\}$$

14.

$${a:1, b:3}$$

15.

$${a:4, b:72}$$

16.

$${a:-4, b:6}$$

17.

$$\left\{a: -\frac{6}{5}, b: 90\right\}$$

18.

$${a:-4, b:6}$$

19.

$${a:-2, b:-2}$$

20.

$${a:-4, b:10}$$

21.

$$[(-3, 6), (3, 6)]$$

22.

$$[(-3, -8), (3, -8)]$$

23.

$$[(-5, 6), (5, 6)]$$

24.

$$[(-5, 6), (5, 6)]$$

25.

$$[(-5, 6), (5, 6)]$$

26.

$$[(-3, 5), (3, 5)]$$

$$[(-5, 1), (5, 1)]$$

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28.

$$[(-1, -10), (1, -10)]$$

29.

$$[(-3, 3), (3, 3)]$$

30.

$$[(-3, 9), (3, 9)]$$

31.

$$[(-1, 4), (1, -4)]$$

32.

$$[(-5,\,2)\,,\,(5,\,-2)]$$

33.

$$\left[\left(-3, -\frac{5}{3}\right), \left(3, \frac{5}{3}\right)\right]$$

34.

$$[(-3, -1), (3, 1)]$$

35.

$$[(-2, 1), (2, -1)]$$

36.

$$\left[\left(-3, -\frac{8}{3}\right), \left(3, \frac{8}{3}\right)\right]$$

37.

$$\left[\left(-4,\,-\frac{5}{2}\right),\,\left(4,\,\frac{5}{2}\right)\right]$$

38.

$$\left[\left(-4,\,\frac{7}{4}\right),\,\left(4,\,-\frac{7}{4}\right)\right]$$

39.

$$\left[\left(-4,\,\frac{1}{4}\right),\,\left(4,\,-\frac{1}{4}\right)\right]$$

40.

$$[(-1, 9), (1, -9)]$$

41. -5

- 42. 10
- 43. 1
- 44. -1
- 45. -6
- 46. 6
- 47. 6
- 48. 9
- 49. -7
- 50. 7
- 51.

$$y = -2x - 5$$

52.

$$y = 10 - 3x$$

53.

$$y = -10x - 4$$

54.

$$y = 6 - 7x$$

55.

$$y = 9x + 5$$

56.

$$y = 2x - 2$$

57.

$$y = -8x - 8$$

58.

$$y = 4x + 9$$

59.

$$y = -2x - 5$$

60.

$$y = 2x + 10$$

61.

$$y = -6x - 4$$

$$y = -7x - 1$$

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63.

$$y = 7 - x$$

64.

$$y = 8x - 4$$

65.

$$y = 7x + 1$$

66.

$$y = -7x - 8$$

67.

$$y = 2x - 4$$

68.

$$y = 9 - x$$

69.

$$y = -10x - 3$$

70.

$$y = -7x - 1$$

71.

72.

$$\left\{\frac{3a}{2}\right\}$$

73.

$$\left\{-\frac{2a}{5}\right\}$$

74.

$$\left\{-\frac{a}{8}\right\}$$

75.

$$\{-a\}$$

$$\left\{1-\frac{a}{3}\right\}$$

77. 
$$\left\{ \frac{2a}{3} + \frac{1}{3} \right\}$$

79. 
$$\{4a-2\}$$

80. 
$$\left\{\frac{3a}{4}\right\}$$

81. 
$$\left\{ -\frac{2a-3}{2a} - \frac{\sqrt{3}\sqrt{4a^2 - 4a + 3}}{2a}, -\frac{2a-3}{2a} + \frac{\sqrt{3}\sqrt{4a^2 - 4a + 3}}{2a} \right\}$$

82. 
$$\left\{ \frac{a+1}{8a} - \frac{\sqrt{17a^2 + 2a + 1}}{8a}, \frac{a+1}{8a} + \frac{\sqrt{17a^2 + 2a + 1}}{8a} \right\}$$

83. 
$$\left\{ \frac{4a-3}{4a} - \frac{\sqrt{-16a^2 - 24a + 9}}{4a}, \frac{4a-3}{4a} + \frac{\sqrt{-16a^2 - 24a + 9}}{4a} \right\}$$

84. 
$$\left\{ \frac{a-3}{2a} - \frac{\sqrt{25a^2 - 6a + 9}}{2a}, \frac{a-3}{2a} + \frac{\sqrt{25a^2 - 6a + 9}}{2a} \right\}$$

85. 
$$\left\{ -\frac{5a+2}{8a} - \frac{\sqrt{-7a^2 + 20a + 4}}{8a}, -\frac{5a+2}{8a} + \frac{\sqrt{-7a^2 + 20a + 4}}{8a} \right\}$$

86. 
$$\left\{ -\frac{5a-3}{8a} - \frac{\sqrt{89a^2 - 30a + 9}}{8a}, -\frac{5a-3}{8a} + \frac{\sqrt{89a^2 - 30a + 9}}{8a} \right\}$$

87. 
$$\left\{ -\frac{\sqrt{-(3a+1)(5a-1)}}{2a} + \frac{a-1}{2a}, \frac{\sqrt{-(3a+1)(5a-1)}}{2a} + \frac{a-1}{2a} \right\}$$

88. 
$$\left\{ -\frac{a-5}{6a} - \frac{\sqrt{49a^2 - 10a + 25}}{6a}, -\frac{a-5}{6a} + \frac{\sqrt{49a^2 - 10a + 25}}{6a} \right\}$$

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89.

$$\left\{ -\frac{\sqrt{-(5a+3)(7a-3)}}{6a} - \frac{a-3}{6a}, \frac{\sqrt{-(5a+3)(7a-3)}}{6a} - \frac{a-3}{6a} \right\}$$

90.

$$\left\{ \frac{a-5}{2a} - \frac{\sqrt{9a^2 - 10a + 25}}{2a}, \frac{a-5}{2a} + \frac{\sqrt{9a^2 - 10a + 25}}{2a} \right\}$$

91.

$$5x^2 - 35x + 55$$

92.

$$-18x^2 - 33x - 18$$

93.

$$-x^2 + 2x - 2$$

94.

$$20x^2 - 42x + 19$$

95.

$$-27x^2 + 15x$$

96.

$$-4x^2 + 43x - 120$$

97.

$$4x^2 + 42x + 113$$

98.

$$64x^2 + 136x + 75$$

99.

$$-64x^2 + 120x - 51$$

100.

$$-80x^2 - 100x - 28$$

$$\left\{-\frac{2a}{3} - \frac{\sqrt{2}\sqrt{8a^2 - 3}}{6}, -\frac{2a}{3} + \frac{\sqrt{2}\sqrt{8a^2 - 3}}{6}\right\}$$

102.

$$\left\{ \frac{a}{2} - \frac{\sqrt{2}\sqrt{2a^2 - 3}}{4}, \frac{a}{2} + \frac{\sqrt{2}\sqrt{2a^2 - 3}}{4} \right\}$$

103.

$$\left\{ -\frac{3a}{4} - \frac{\sqrt{9a^2 - 8}}{4}, -\frac{3a}{4} + \frac{\sqrt{9a^2 - 8}}{4} \right\}$$

104.

$$\left\{ -\frac{3a}{4} - \frac{\sqrt{3}\sqrt{27a^2 + 16}}{12}, -\frac{3a}{4} + \frac{\sqrt{3}\sqrt{27a^2 + 16}}{12} \right\}$$

105.

$$\left\{ -\frac{a}{3} - \frac{\sqrt{2}\sqrt{2a^2 + 3}}{6}, -\frac{a}{3} + \frac{\sqrt{2}\sqrt{2a^2 + 3}}{6} \right\}$$

106.

$$\left\{ -\frac{a}{2} - \frac{\sqrt{a^2 + 2}}{2}, -\frac{a}{2} + \frac{\sqrt{a^2 + 2}}{2} \right\}$$

107.

$$\left\{-\frac{2a}{5} - \frac{\sqrt{2}\sqrt{8a^2 + 5}}{10}, -\frac{2a}{5} + \frac{\sqrt{2}\sqrt{8a^2 + 5}}{10}\right\}$$

108.

$$\left\{-a-\sqrt{a^2+1}, -a+\sqrt{a^2+1}\right\}$$

109.

$$\left\{ \frac{a}{10} - \frac{\sqrt{3}\sqrt{3a^2 + 40}}{30}, \frac{a}{10} + \frac{\sqrt{3}\sqrt{3a^2 + 40}}{30} \right\}$$

110.

$$\left\{-\frac{2a}{3} - \frac{\sqrt{2}\sqrt{8a^2 - 9}}{6}, -\frac{2a}{3} + \frac{\sqrt{2}\sqrt{8a^2 - 9}}{6}\right\}$$

111.

$$\left\{-\frac{3(y+1)}{5y-1}\right\}$$

$$\left\{-\frac{2(y+2)}{3y+1}\right\}$$

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113.

$$\left\{\frac{3y+1}{5y+1}\right\}$$

114.

$$\left\{-\frac{4y-3}{5y-2}\right\}$$

115.

$$\left\{\frac{y-4}{y+2}\right\}$$

116.

$$\left\{-\frac{2y-3}{5(y-1)}\right\}$$

117.

$$\left\{\frac{y-5}{5y-4}\right\}$$

118.

$$\left\{-\frac{5y-4}{4(y+1)}\right\}$$

119.

$$\left\{\frac{3y-4}{4y+5}\right\}$$

120.

$$\left\{-\frac{2y-5}{4y+5}\right\}$$

121.

122.

$$-4.375$$

123.

$$-0.875$$

124.

$$-0.9$$

$$-15.0$$

126.

14.0

127.

1.867

128.

0.931

129.

1.231

130.

-1.846

131. Domain:

$$\left(-\infty,\frac{5}{4}\right)\cup\left(\frac{5}{4},\infty\right)$$

Range:

$$\left(-\infty, \frac{3}{4}\right) \cup \left(\frac{3}{4}, \infty\right)$$

132. Domain:

$$(-\infty,4)\cup(4,\infty)$$

Range:

$$(-\infty,1)\cup(1,\infty)$$

133. Domain:

$$(-\infty,3)\cup(3,\infty)$$

Range:

$$(-\infty, -3) \cup (-3, \infty)$$

134. Domain:

$$(-\infty,1)\cup(1,\infty)$$

Range:

$$\left(-\infty,-\frac{5}{4}\right)\cup\left(-\frac{5}{4},\infty\right)$$

135. Domain:

$$(-\infty, -2) \cup (-2, \infty)$$

Range:

$$\left(-\infty,\frac{5}{2}\right)\cup\left(\frac{5}{2},\infty\right)$$

136. Domain:

$$(-\infty,1)\cup(1,\infty)$$

Range:

$$\left(-\infty,\frac{1}{4}\right)\cup\left(\frac{1}{4},\infty\right)$$

137. Domain:

$$(-\infty,3)\cup(3,\infty)$$

Range:

$$(-\infty,1)\cup(1,\infty)$$

138. Domain:

$$\left(-\infty,-\frac{1}{2}\right)\cup\left(-\frac{1}{2},\infty\right)$$

Range:

$$\left(-\infty, -\frac{3}{4}\right) \cup \left(-\frac{3}{4}, \infty\right)$$

139. Domain:

$$(-\infty,2)\cup(2,\infty)$$

Range:

$$\left\{-\frac{x}{2x-4} + \frac{2}{2x-4}\right\}$$

140. Domain:

$$\left(-\infty,\frac{1}{4}\right)\cup\left(\frac{1}{4},\infty\right)$$

Range:

$$\left(-\infty,\frac{3}{4}\right)\cup\left(\frac{3}{4},\infty\right)$$

$$y = -\frac{16x}{3} - \frac{2}{3}$$

142.

$$y = -\frac{6x}{5} - \frac{2}{5}$$

143.

$$y = \frac{2}{5} - \frac{3x}{10}$$

144.

$$y = \frac{2}{3} - \frac{5x}{6}$$

145.

$$y = -\frac{x}{4} - \frac{1}{6}$$

146.

$$y = 5x + \frac{5}{3}$$

147.

$$y = \frac{4x}{3} + \frac{4}{3}$$

148.

$$y = 25x - 3$$

149.

$$y = \frac{25x}{2} + \frac{3}{2}$$

150.

$$y = -x - 1$$

151.

$$y = \frac{32x^2}{3} - 4x - \frac{1}{3}$$

152.

$$y = \frac{5}{18} - \frac{\sqrt{36x + 13}}{18}$$

153.

$$y = -15x^2 + x + \frac{2}{5}$$

$$y = -\frac{\sqrt{60x - 79}}{30} - \frac{1}{30}$$

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155.

$$y = \frac{9x^2}{4} - \frac{15x}{4} - \frac{5}{4}$$

156.

$$y = 5x^2 + x - \frac{1}{5}$$

157.

$$y = \frac{16x^2}{3} - \frac{8x}{3} + 1$$

158.

$$y = -\frac{\sqrt{48x + 49}}{32} - \frac{1}{32}$$

159.

$$y = x^2 + \frac{3x}{2} - \frac{3}{2}$$

$$y = 48x^2 + 16x - 4$$

# 3. Algebra

## 3.1 Problems

**Exercise 3.1** Express the fraction

$$\frac{20mn+6}{5mn+1}$$

in the form

$$\frac{A}{5mn+1} + B$$

**Exercise 3.2** Express the fraction

$$\frac{16uv-4}{-2uv-5}$$

in the form

$$\frac{A}{-2uv-5} + B$$

**Exercise 3.3** Express the fraction

$$\frac{-4uw+6}{2uw-4}$$

in the form

$$\frac{A}{2uw-4} + B$$

**Exercise 3.4** Express the fraction

$$\frac{25z - 14}{-5z - 4}$$

in the form

$$\frac{A}{-5z-4} + B$$

#### **Exercise 3.5** Express the fraction

$$\frac{4-35p}{-5p-2}$$

in the form

$$\frac{A}{-5p-2} + B$$

#### **Exercise 3.6** Express the fraction

$$\frac{45ln+6}{-5ln-4}$$

in the form

$$\frac{A}{-5ln-4} + B$$

## **Exercise 3.7** Express the fraction

$$\frac{1-8q}{4q+3}$$

in the form

$$\frac{A}{4q+3} + B$$

## **Exercise 3.8** Express the fraction

$$\frac{24pqr+1}{3pqr-1}$$

in the form

$$\frac{A}{3pqr-1} + B$$

## **Exercise 3.9** Express the fraction

$$\frac{30pq+5}{-5pq-4}$$

in the form

$$\frac{A}{-5pq-4} + B$$

## **Exercise 3.10** Express the fraction

$$\frac{-40rst + 2}{-5rst - 2}$$

in the form

$$\frac{A}{-5rst-2} + B$$

**Exercise 3.11** Simplify

$$\frac{-2a^5b^2-3a^4}{a^3}$$

**Exercise 3.12** Simplify

$$\frac{-5r^3s^6 - 2r^3s^5}{(-1)r^3s^2}$$

**Exercise 3.13** Simplify

$$\frac{-16p^3q^7r^4 - 12q^6}{4q^3}$$

**Exercise 3.14** Simplify

$$\frac{6a^3b^6cd^3 + 15ab^5c}{(-1)\,3ab^3c}$$

**Exercise 3.15** Simplify

$$\frac{-6p^4q^4r^5 - 12q^2r^4}{3q^2r^4}$$

**Exercise 3.16** Simplify

$$\frac{2m^7n^5 + 5m^3n^5}{m^3n^3}$$

**Exercise 3.17** Simplify

$$\frac{4q^4r^5 - 16r^7}{(-1)4r^4}$$

**Exercise 3.18** Simplify

$$\frac{12a^4bc^2 - 9a^2}{3a}$$

**Exercise 3.19** Simplify

$$\frac{6r^4s^2 + 12r^4}{3r}$$

**Exercise 3.20** Simplify

$$\frac{-3u^7v^3w^6 + 2u^3v^3w^6}{(-1)u^3v^3w^4}$$

**Exercise 3.21** Simplify

$$\frac{3p^4q^4r}{9p^5q^8r^4-15p^4q^4r^3}$$

**Exercise 3.22** Simplify

$$\frac{(-1)\,s^2}{5r^4s^5 - rs^5}$$

**Exercise 3.23** Simplify

$$\frac{2r^2s^4}{-2r^4s^4+10r^2s^7}$$

**Exercise 3.24** Simplify

$$\frac{5b^2c^3}{25a^4b^3c^4 - 10b^2c^5}$$

**Exercise 3.25** Simplify

$$\frac{4a^2b^4c^4}{-16a^2b^4c^5 - 20a^2b^4c^4}$$

**Exercise 3.26** Simplify

$$\frac{(-1)\,2t}{6s^3t^2+2t^3}$$

**Exercise 3.27** Simplify

$$\frac{5vw^4}{-10v^3w^4 + 20v^2w^6}$$

**Exercise 3.28** Simplify

$$\frac{c^2}{-5b^4c^5 - c^2}$$

**Exercise 3.29** Simplify

$$\frac{4r^3}{16r^6 - 8r^3}$$

**Exercise 3.30** Simplify

$$\frac{4v^4}{-8u^4v^8 + 8u^4v^4}$$

**Exercise 3.31** Simplify

$$\frac{-12p^2q^4 - 6p^2q - 16q^4 - 8q}{-3p^2q - 4q}$$

**Exercise 3.32** Simplify

$$\frac{-9r^4s^6+9r^4s^5-9s^4+9s^3}{3s^3-3s^2}$$

**Exercise 3.33** Simplify

$$\frac{-12 l^4 m^6 + 6 l^4 m^5 + 20 l^2 m^2 - 10 l^2 m}{-4 m^2 + 2 m}$$

**Exercise 3.34** Simplify

$$\frac{4p^5q^2r^3 + 5p^3q^2r^7 - 16p^2q - 20qr^4}{p^3q^2r^3 - 4q}$$

**Exercise 3.35** Simplify

$$\frac{-r^8t^4 + 4r^7s^4t^7 - r^6s^3 + 4r^5s^7t^3}{-r^4t^4 - r^2s^3}$$

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**Exercise 3.36** Simplify

$$\frac{15rs^4 + 9rs - 10s^6 - 6s^3}{-3r + 2s^2}$$

**Exercise 3.37** Simplify

$$\frac{-3r^4s^7 - r^4s^4 + 12rs^6 + 4rs^3}{-3s^4 - s}$$

**Exercise 3.38** Simplify

$$\frac{-2r^6st^3 - r^4st^7 - 6r^2 - 3t^4}{2r^2 + t^4}$$

**Exercise 3.39** Simplify

$$\frac{20a^4d^4 - 10a^3c^3 + 8a^3d^4 - 4a^2c^3}{4ad^4 - 2c^3}$$

**Exercise 3.40** Simplify

$$\frac{2pq^8r^2 - 4pq^6r^2 + 4q^6r^3 - 8q^4r^3}{2pq^4r^2 + 4q^2r^3}$$

**Exercise 3.41** Simplify

$$\frac{m^4n - m^2n^4}{-m^8n^3 + m^6n^6 + m^5n - m^3n^4}$$

**Exercise 3.42** Simplify

$$\frac{5p^4q^4 - 4p^4q^3r}{-5p^6q^4r^4 + 4p^6q^3r^5 - 10p^5q^4r^4 + 8p^5q^3r^5}$$

**Exercise 3.43** Simplify

$$\frac{r^2s - 2s^4}{-2r^6s^3 + 4r^4s^6 - 2r^4s^2 + 4r^2s^5}$$

**Exercise 3.44** Simplify

$$\frac{3p^4q^4 - 3p}{-9p^8q^8r^2 + 3p^6q^4 + 9p^5q^4r^2 - 3p^3}$$

**Exercise 3.45** Simplify

$$\frac{-3p^3q^3 - 4p}{-12p^3q^4 - 16pq}$$

**Exercise 3.46** Simplify

$$\frac{-4p^4 + 3p^3r}{-12p^8q^3 + 9p^7q^3r + 8p^6q^3r^4 - 6p^5q^3r^5}$$

**Exercise 3.47** Simplify

$$\frac{5p^4r^3 - 2q^3}{-25p^6q^3r^6 + 20p^4r^3 + 10p^2q^6r^3 - 8q^3}$$

**Exercise 3.48** Simplify

$$\frac{-3pr^3-2q^3r^4}{12p^3r^7+8p^2q^3r^8+3pr^3+2q^3r^4}$$

**Exercise 3.49** Simplify

$$\frac{-r^4 + 5r^2}{-3r^8 + 5r^6s^3 + 15r^6 - 25r^4s^3}$$

**Exercise 3.50** Simplify

$$\frac{8r^3}{-24r^6s^4 - 24r^3s}$$

Exercise 3.51 If we divide the polynomial

$$-16l^3 + 40l^2 - 3lv - 17l + v - 10$$

by

$$4l - 5$$

the remainder is 5. What is the value of v? **Exercise 3.52** If we divide the polynomial

$$2vz - 3v + 4z^3 + 8z^2 - 28z + 16$$

by

$$4 - 4z$$

the remainder is 6. What is the value of v? **Exercise 3.53** If we divide the polynomial

$$-12n^3 + 32n^2 + nu - 8n + 3u - 20$$

by

$$5 - 3n$$

the remainder is 10. What is the value of u? **Exercise 3.54** If we divide the polynomial

$$-4sy - 2s - 12y^3 - 19y^2 + 8y + 16$$

by

$$3y + 4$$

the remainder is 7. What is the value of s?

**Exercise 3.55** If we divide the polynomial

$$20n^3 + 3n^2 - 2nx - 5n - x + 3$$

by

$$-4n - 3$$

the remainder is 7. What is the value of x? **Exercise 3.56** If we divide the polynomial

$$-3cu - 5c + 10u^3 - 24u^2 + 33u - 10$$

by

$$2-5u$$

the remainder is 3. What is the value of c? **Exercise 3.57** If we divide the polynomial

$$10p^3 - 12p^2 - 3pu - 18p + 4u + 4$$

by

$$2p - 4$$

the remainder is 6. What is the value of u? **Exercise 3.58** If we divide the polynomial

$$-5c^3 + 17c^2 - 5cl - c + 2l - 2$$

by

$$5c - 2$$

the remainder is 1. What is the value of 1? **Exercise 3.59** If we divide the polynomial

$$-8w^3 - 6w^2 + 5wz + 15w - 4z + 4$$

by

$$4w + 1$$

the remainder is 2. What is the value of z? **Exercise 3.60** If we divide the polynomial

$$-4sz + 4s + 10z^3 + 17z^2 - 30z + 8$$

by

$$4 - 5z$$

the remainder is 6. What is the value of s?

Exercise 3.61 Expand

$$(1-4t)(2r^4s-5r^2s^4t^3-2t^3)$$

Exercise 3.62 Expand

$$(3r^4s^2+4r^3s^3)(4rs^4+5r+5)$$

Exercise 3.63 Expand

$$(-a^3b^3c^4d^2-5a^2b^4c)(2a^4b^4d-3ac^2-2b^3d^4)$$

Exercise 3.64 Expand

$$(-u^3v^3+2)(2u^3w^4+5uv^4w^4+uw)$$

Exercise 3.65 Expand

$$(r^3s^4t^3 + 5rs^4t)(2r^3s^4t^2 - 3r^2s^4t^2 + 3r^2t^4)$$

Exercise 3.66 Expand

$$(5p^3-5q)(4p^2q^4+5q^3+5)$$

Exercise 3.67 Expand

$$(a^3b+ab^2c)(2a^3b+a^2c^3-3a^2)$$

Exercise 3.68 Expand

$$(-5u^2v^4+3u^2v^3w^3)(u^3vw^3+v^3-1)$$

Exercise 3.69 Expand

$$(3p^2-1)(3p^3q^4+5p^2q^4)$$

Exercise 3.70 Expand

$$(3mn^3 + 5n^4)(-4m^4n^2 + 2m^3n^4 + n)$$

Exercise 3.71 Expand

$$(-u^4 - 5v^2)^2 \cdot (4u^2w^3 + 5u)$$

Exercise 3.72 Expand

$$(5l^3n^4-3m^3)(-3l^4m^4n^2-5l^3)^2$$

Exercise 3.73 Expand

$$(s^4-2s^3)^2 \cdot (5r^3s^2-r^3)$$

Exercise 3.74 Expand

$$(5-5x)^2(xy^4z-2y^3z)$$

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Exercise 3.75 Expand

$$(2p+r^2)(2q^4-q^3)^2$$

Exercise 3.76 Expand

$$(4r^2s^3+2rs^3)(2r^3s+2)^2$$

Exercise 3.77 Expand

$$(5r+2)(-s^4-4s^3)^2$$

Exercise 3.78 Expand

$$(-2r^3+3s^3)^2(-4r^3s+2r^2)$$

Exercise 3.79 Expand

$$(-2mn^2+3n^3)(m^4n^2+5m^2)^2$$

Exercise 3.80 Expand

$$(2lm^3n-2n^2)(-4l^2m^3n^4-3l^2n^2)^2$$

Exercise 3.81 Expand

$$(p^4qr^3 - p + 5q^2r^2)^2$$

Exercise 3.82 Expand

$$(-2xy^3-2z^3+4)^2$$

Exercise 3.83 Expand

$$(2p^3qr^2+3p^3r+5q^4r^2)^2$$

Exercise 3.84 Expand

$$(-m^3n^3-m^3+1)^2$$

Exercise 3.85 Expand

$$(-5r^3s^4t^3 + 5r^3s^3t^4 + 2r^3st^4)^2$$

Exercise 3.86 Expand

$$(2l^2m^3n^2-3m^4+4)^2$$

Exercise 3.87 Expand

$$(5p^2q^3r^3 - 5p - 2q^3r)^2$$

Exercise 3.88 Expand

$$(3q+5)^2$$

Exercise 3.89 Expand

$$(-2c+3d+5)^2$$

Exercise 3.90 Expand

$$(4ab^3c^2d + 2ad + 2)^2$$

Exercise 3.91 Expand

$$(-5uw+1)(-2u^4v^2-4u^3w)(-2u^4w+3u^2)$$

Exercise 3.92 Expand

$$(2l^3 + 5lm^2n^4)(-mn^3 + 4)(5l^4m^4n - 5l^3)$$

Exercise 3.93 Expand

$$(2r^3-4s^3)(4r^4+s^2)(2s^2-3s)$$

Exercise 3.94 Expand

$$(rt^2-1)(3r^4t^4-rs^2t)(2s^2t^2+2)$$

Exercise 3.95 Expand

$$(-xz^4-5y^4)(4x^3y^4+4xz)(3x^3y^2z^2-2xz)$$

Exercise 3.96 Expand

$$\left(-2m^3+4n^3\right)\left(2m^4+2n^2\right)\left(-5mn^3-3n^4\right)$$

Exercise 3.97 Expand

$$(-4ln^2+m^2)(-3l^2n-ln)(-5m^2n^3+5n^3)$$

Exercise 3.98 Expand

$$\left(4l^4+2l^3m\right)\left(3l^4m^2-3m^2\right)\left(-5l^3m^4n^4+3m^3n\right)$$

Exercise 3.99 Expand

$$(5lm^4 - 4l) (ln^3 + 2) (3l^3mn^3 + 4n^3)$$

Exercise 3.100 Expand

$$(-2t-2)(3r^2t+4r^2)(-2s^3t^4+2)$$

**Exercise 3.101** Expand and simplify

$$(2q^4-2)(p^3q^4-5q^4)^2$$

Exercise 3.102 Expand and simplify

$$100b^6 \cdot \left(5a + 3b^2\right)$$

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Exercise 3.103 Expand and simplify

$$(3acd^4 - 3b^3c^3)^2 (-5ac^2d - 3c^4)$$

Exercise 3.104 Expand and simplify

$$(5p^2r^4+3)^2(p^3q^4r-p^2q^2r^2)$$

**Exercise 3.105** Expand and simplify

$$(5a^4 - a^2b^3c^2)(3a^2c + 4c^3)^2$$

**Exercise 3.106** Expand and simplify

$$(-x-3y^3)^2 \cdot (2x^4+4x^2y^2)$$

Exercise 3.107 Expand and simplify

$$(-2m^3n^3+4n^3)^2(m^4n^4-5n^3)$$

Exercise 3.108 Expand and simplify

$$(-5l^4n^4-1)^2(l^2mn+lm^3)$$

Exercise 3.109 Expand and simplify

$$(p^4+5q^2)(4p^2q+q^2)^2$$

**Exercise 3.110** Expand and simplify

$$(-v^2w^2-2)^2(-3u^4vw^4-5w^2)$$

Exercise 3.111 Expand

$$\left(-4p^4q^3-\frac{1}{3p^4}\right)^2$$

Exercise 3.112 Expand

$$\left(2b^3d^4 + \frac{1}{4a^2}\right)^2$$

Exercise 3.113 Expand

$$\left(4p^3q-\frac{1}{p^2}\right)^2$$

Exercise 3.114 Expand

$$\left(4x^3yz + \frac{1}{y^3z^4}\right)^2$$

Exercise 3.115 Expand

$$\left(u^2vw - \frac{1}{2u^4vw^4}\right)^2$$

Exercise 3.116 Expand

$$\left(-5a^4bc^2 + \frac{1}{a^4b^2c^3}\right)^2$$

Exercise 3.117 Expand

$$\left(-3+\frac{1}{mn^4}\right)^2$$

Exercise 3.118 Expand

$$\left(3r^4s^2 - \frac{1}{r^4s^4}\right)^2$$

Exercise 3.119 Expand

$$\left(-p^3 + \frac{1}{5p^2q^2}\right)^2$$

Exercise 3.120 Expand

$$\left(-3-\frac{1}{5u^4v^2}\right)^2$$

**Exercise 3.121** If  $a = 5b^4 + 2c^4$ , show that

$$(a-x)^2 + (a+x)^2 = 50b^8 + 40b^4c^4 + 8c^8 + 2x^2$$

**Exercise 3.122** If  $a = 5r^2t^2 - r^2$ , show that

$$(a-x)^2 + (a+x)^2 = 50r^4t^4 - 20r^4t^2 + 2r^4 + 2x^2$$

**Exercise 3.123** If  $a = 4d^2 + 2$ , show that

$$(a-x)^2 + (a+x)^2 = 32d^4 + 32d^2 + 2x^2 + 8$$

**Exercise 3.124** If  $a = -x^2y^2 - xyz^3$ , show that

$$(a-x)^{2} + (a+x)^{2} = x^{2} \left( (xy^{2} + yz^{3} - 1)^{2} + (xy^{2} + yz^{3} + 1)^{2} \right)$$

**Exercise 3.125** If  $a = -5p^4 + 2q^4$ , show that

$$(a-x)^2 + (a+x)^2 = 50p^8 - 40p^4q^4 + 8q^8 + 2x^2$$

**Exercise 3.126** If  $a = 4x^3y^3 - y^4$ , show that

$$(a-x)^2 + (a+x)^2 = 32x^6y^6 - 16x^3y^7 + 2x^2 + 2y^8$$

**Exercise 3.127** If  $a = 7r^3$ , show that

$$(a-x)^2 + (a+x)^2 = 98r^6 + 2x^2$$

**Exercise 3.128** If  $a = -5u^4w^4 + 4$ , show that

$$(a-x)^2 + (a+x)^2 = 50u^8w^8 - 80u^4w^4 + 2x^2 + 32$$

**Exercise 3.129** If  $a = -4b^4c + 2bc^2$ , show that

$$(a-x)^2 + (a+x)^2 = 32b^8c^2 - 32b^5c^3 + 8b^2c^4 + 2x^2$$

**Exercise 3.130** If  $a = x^{3}yz^{4} - 5z$ , show that

$$(a-x)^{2} + (a+x)^{2} = (-x^{3}yz^{4} + x + 5z)^{2} + (x^{3}yz^{4} + x - 5z)^{2}$$

**Exercise 3.131** If a = -4m - 4, show that

$$-(a-x)^3 + (a+x)^3 = -(4m-x+4)^3 - (4m+x+4)^3$$

**Exercise 3.132** If  $a = 3r^4s^3 - 3r^2$ , show that

$$-(a-x)^3 + (a+x)^3 = -(-3r^4s^3 + 3r^2 + x)^3 + (3r^4s^3 - 3r^2 + x)^3$$

**Exercise 3.133** If  $a = -2b^4c + b^4$ , show that

$$-(a-x)^{3} + (a+x)^{3} = (-2b^{4}c + b^{4} + x)^{3} - (2b^{4}c - b^{4} + x)^{3}$$

**Exercise 3.134** If  $a = -3b^3c^2 + 2b^2c^3$ , show that

$$-(a-x)^3 + (a+x)^3 = (-3b^3c^2 + 2b^2c^3 + x)^3 - (3b^3c^2 - 2b^2c^3 + x)^3$$

**Exercise 3.135** If  $a = -4p^4q - 3$ , show that

$$-(a-x)^3 + (a+x)^3 = -(4p^4q - x + 3)^3 - (4p^4q + x + 3)^3$$

**Exercise 3.136** If  $a = 2l^2n^4 - 5m^3$ , show that

$$-(a-x)^3 + (a+x)^3 = -(-2l^2n^4 + 5m^3 + x)^3 + (2l^2n^4 - 5m^3 + x)^3$$

**Exercise 3.137** If  $a = -2r^2s^4 - 2r^2s$ , show that

$$-(a-x)^3 + (a+x)^3 = -(2r^2s^4 + 2r^2s - x)^3 - (2r^2s^4 + 2r^2s + x)^3$$

**Exercise 3.138** If  $a = -3qr^3 - 2$ , show that

$$-(a-x)^{3} + (a+x)^{3} = -(3qr^{3} - x + 2)^{3} - (3qr^{3} + x + 2)^{3}$$

**Exercise 3.139** If  $a = -5m^2n - 1$ , show that

$$-(a-x)^{3} + (a+x)^{3} = -(5m^{2}n - x + 1)^{3} - (5m^{2}n + x + 1)^{3}$$

**Exercise 3.140** If  $a = -5rs^2 + 1$ , show that

$$-(a-x)^3 + (a+x)^3 = (-5rs^2 + x + 1)^3 - (5rs^2 + x - 1)^3$$

**Exercise 3.141** Solve the equation in x:

$$-4a^2x^2 - 5x^2 - 8x = 0$$

**Exercise 3.142** Solve the equation in x:

$$4a^2x - 5a + x = 0$$

**Exercise 3.143** Solve the equation in x:

$$-2ax + 5x^2 - 2x = 0$$

**Exercise 3.144** Solve the equation in x:

$$-4ax^2 - 5ax - 4x = 0$$

**Exercise 3.145** Solve the equation in x:

$$-7a - 1 = 0$$

**Exercise 3.146** Solve the equation in x:

$$-5ax^2 - a + 5x^2 + 2 = 0$$

**Exercise 3.147** Solve the equation in x:

$$5a^2 - 5ax + 2x = 0$$

**Exercise 3.148** Solve the equation in x:

$$-5a^2x + 5a^2 - 2a - 4x^2 = 0$$

**Exercise 3.149** Solve the equation in x:

$$-6ax - 3x^2 - 2 = 0$$

**Exercise 3.150** Solve the equation in x:

$$5x - 4 = 0$$

**Exercise 3.151** Solve the equation in x:

$$2a - 7x + 5 = 0$$

**Exercise 3.152** Solve the equation in x:

$$-ax - 6a - 3x = 0$$

**Exercise 3.153** Solve the equation in x:

$$6ax - 7a = 0$$

**Exercise 3.154** Solve the equation in x:

$$7ax + 3 = 0$$

**Exercise 3.155** Solve the equation in x:

$$2a + 4x + 1 = 0$$

**Exercise 3.156** Solve the equation in x:

$$-5ax - 2a + x + 8 = 0$$

**Exercise 3.157** Solve the equation in x:

$$6a = 0$$

**Exercise 3.158** Solve the equation in x:

$$-2ax + 1 = 0$$

**Exercise 3.159** Solve the equation in x:

$$-2ax + 3a - 8x = 0$$

**Exercise 3.160** Solve the equation in x:

$$5a + x - 1 = 0$$

**Exercise 3.161** Solve the following simultaneous equations for x and y:

$$3x + 4y + 4 = 0$$

$$-5x + 3y + 6 = 0$$

**Exercise 3.162** Solve the following simultaneous equations for x and y:

$$x - 3y + 3 = 0$$

$$-5x + 3y + 2 = 0$$

**Exercise 3.163** Solve the following simultaneous equations for x and y:

$$x + y = 0$$

$$-5x - 4y - 6 = 0$$

**Exercise 3.164** Solve the following simultaneous equations for x and y:

$$3x + 2y = 0$$

$$-4x - 3y - 3 = 0$$

**Exercise 3.165** Solve the following simultaneous equations for x and y:

$$-5x - 4y + 1 = 0$$

$$-3x - 4y + 8 = 0$$

**Exercise 3.166** Solve the following simultaneous equations for x and y:

$$4x + 3y + 8 = 0$$

$$x - 5y - 2 = 0$$

**Exercise 3.167** Solve the following simultaneous equations for x and y:

$$4x - 2y - 6 = 0$$

$$x - y - 6 = 0$$

**Exercise 3.168** Solve the following simultaneous equations for x and y:

$$-2x + 5y - 7 = 0$$

$$-4x + y + 9 = 0$$

**Exercise 3.169** Solve the following simultaneous equations for x and y:

$$2x + y - 3 = 0$$

$$-3x - 2y + 8 = 0$$

**Exercise 3.170** Solve the following simultaneous equations for x and y:

$$4x + 2y - 10 = 0$$

$$-2x - 2y = 0$$

**Exercise 3.171** Solve the following simultaneous equations for *x* and *y*:

$$4ax + 6y + 2 = 0$$

$$-12ay - 12x + 5 = 0$$

**Exercise 3.172** Solve the following simultaneous equations for x and y:

$$-5a - 3x + 12y = 0$$

$$4ax - 3a + 20y = 0$$

**Exercise 3.173** Solve the following simultaneous equations for x and y:

$$-2ax - 25ay - 3a = 0$$

$$8x - y - 4 = 0$$

**Exercise 3.174** Solve the following simultaneous equations for x and y:

$$3ay - 3a + 16x = 0$$

$$-3ax + 12y + 4 = 0$$

**Exercise 3.175** Solve the following simultaneous equations for x and y:

$$-6ay - 9x + 3 = 0$$

$$2ay + 2x + 2 = 0$$

**Exercise 3.176** Solve the following simultaneous equations for x and y:

$$-5ax + 20y - 4 = 0$$

$$8ax - 4a - 25y = 0$$

**Exercise 3.177** Solve the following simultaneous equations for x and y:

$$5a + 5x + 10y = 0$$

$$-4ax - 5a - 10y = 0$$

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**Exercise 3.178** Solve the following simultaneous equations for x and y:

$$-10ay + 3a + 2x = 0$$

$$2ay - 4a - 25x = 0$$

**Exercise 3.179** Solve the following simultaneous equations for x and y:

$$15ay - 5x - 1 = 0$$

$$10ax + 5y - 2 = 0$$

**Exercise 3.180** Solve the following simultaneous equations for x and y:

$$-15ax - 5a + 20y = 0$$

$$-5a - 20x - 20y = 0$$

**Exercise 3.181** Simplify

$$-\frac{6c}{4c+1} + \frac{4c-5}{3c}$$

**Exercise 3.182** Simplify

$$\frac{-5n-1}{2n+5} + \frac{6}{4n+1}$$

**Exercise 3.183** Simplify

$$\frac{2-4m}{-3m-5} - \frac{5}{2-2m}$$

**Exercise 3.184** Simplify

$$-\frac{5q}{7} + \frac{2-q}{3q+4}$$

**Exercise 3.185** Simplify

$$\frac{3u}{2} - \frac{1}{6}$$

**Exercise 3.186** Simplify

$$-\frac{b+5}{-4b-2} + \frac{5}{2-5b}$$

**Exercise 3.187** Simplify

$$\frac{6y}{5y-1} - \frac{6}{3y+4}$$

**Exercise 3.188** Simplify

$$-\frac{2u+5}{3u+4} - \frac{-5u-3}{2u}$$

**Exercise 3.189** Simplify

$$\frac{1-q}{2q+1} + \frac{5q-2}{-3q-4}$$

**Exercise 3.190** Simplify

$$\frac{a+5}{6a} - \frac{4a-5}{5a}$$

**Exercise 3.191** Simplify

$$\frac{2p-1}{5u-3} + \frac{-3u-5}{4p+5} - \frac{-p-5u}{4p}$$

**Exercise 3.192** Simplify

$$-\frac{qy}{-2q-y} + \frac{-3q-4}{3qy+2q} + 6$$

Exercise 3.193 Simplify

$$\frac{-5nx - 2n}{nx - 5n} - \frac{-4nx - 2n}{4n + 2x} + \frac{5nx - x}{2n}$$

**Exercise 3.194** Simplify

$$\frac{9}{7} + \frac{2ps - 5s}{-4ps - 5s} - \frac{-ps - 5p}{4p}$$

**Exercise 3.195** Simplify

$$-\frac{4dm}{5} + \frac{d}{2} + \frac{17m}{20} - 1$$

**Exercise 3.196** Simplify

$$-\frac{-3sx+2}{-3sx+x} + \frac{4x+3}{s+5} - \frac{-s-2x}{3-5x}$$

**Exercise 3.197** Simplify

$$-\frac{2x}{7} + \frac{8x}{p-x} - \frac{4x-4}{-3p-1}$$

**Exercise 3.198** Simplify

$$\frac{3n}{5} + \frac{-3m-2}{3mn-5n} + \frac{mn+n}{2m-2}$$

**Exercise 3.199** Simplify

$$\frac{5v-4}{5v-3} - \frac{6}{4-2p} + \frac{-5pv+5}{2-p}$$

**Exercise 3.200** Simplify

$$\frac{-5w-4}{4w+1} + \frac{5w+2}{-5w-3} - \frac{5}{2m}$$

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**Exercise 3.201** Simplify

$$\frac{\frac{4n}{5} + \frac{4}{3n+4}}{-\frac{n}{2n+3} + 2}$$

**Exercise 3.202** Simplify

$$\frac{\frac{v}{5} + \frac{9}{2}}{\frac{2v - 2}{3v + 5} - \frac{5 - 5v}{7v}}$$

**Exercise 3.203** Simplify

$$\frac{-\frac{p}{8} + \frac{-2p-3}{-2p-2} + \frac{1}{8}}{-\frac{p}{3-3p} - \frac{5p+1}{-4p-1}}$$

**Exercise 3.204** Simplify

$$\frac{-\frac{5c}{3} + \frac{3c-5}{4c-5} - \frac{1}{3}}{-\frac{4-5c}{5c} + \frac{5c-5}{5c}}$$

**Exercise 3.205** Simplify

$$\frac{-1 - \frac{-t-4}{7t}}{\frac{t}{-3t-5} - \frac{2-4t}{2t+1}}$$

**Exercise 3.206** Simplify

$$\frac{\frac{3m-5}{m} + \frac{3}{2m}}{-\frac{5}{3-m} - \frac{4}{m}}$$

**Exercise 3.207** Simplify

$$\frac{\frac{4b-4}{b+2} + \frac{4-5b}{3b}}{\frac{1-3b}{-2b-2} + \frac{b-1}{-3b-5}}$$

**Exercise 3.208** Simplify

$$\frac{\frac{8x}{5x-3} + \frac{-3x-4}{4-x}}{-\frac{4x}{4x-2} + \frac{-2x-4}{4x+1}}$$

**Exercise 3.209** Simplify

$$\frac{-a + \frac{8}{3a}}{\frac{2-2a}{-2a-1} + \frac{2}{4a+5}}$$

**Exercise 3.210** Simplify

$$\frac{-\frac{q}{5} + \frac{2q+3}{7q}}{\frac{6}{-3q-5} - \frac{-3q-2}{2-5q}}$$

**Exercise 3.211** Divide the polynomial

$$3x^2 - 3x - 3$$

by

$$4-x$$

Exercise 3.212 Divide the polynomial

$$2x^2 - 4x + 1$$

by

$$2x - 3$$

**Exercise 3.213** Divide the polynomial

$$4x^2 + 2x - 1$$

by

$$-x-2$$

**Exercise 3.214** Divide the polynomial

$$5x^2 - 3x + 5$$

by

$$-3x - 1$$

**Exercise 3.215** Divide the polynomial

$$x^2 + 2x + 4$$

by

$$3x + 1$$

**Exercise 3.216** Divide the polynomial

$$5x^2 + x - 4$$

by

$$-x-3$$

**Exercise 3.217** Divide the polynomial

$$-4x^2 - x - 2$$

$$5-x$$

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Exercise 3.218 Divide the polynomial

$$3x^2 + 4x + 5$$

by

$$-4x - 3$$

Exercise 3.219 Divide the polynomial

$$-4x^2 + x - 3$$

by

$$2-5x$$

**Exercise 3.220** Divide the polynomial

$$x^2 + 2x - 4$$

by

$$3x + 3$$

Exercise 3.221 Divide the polynomial

$$x^3 + 4x^2 - 2x + 2$$

by

$$3-x$$

Exercise 3.222 Divide the polynomial

$$-2x^3 + x^2 - x + 2$$

by

$$-4x - 3$$

**Exercise 3.223** Divide the polynomial

$$-x^3 - 2x^2 + 4x - 1$$

by

$$3x + 1$$

Exercise 3.224 Divide the polynomial

$$2x^3 + x^2 + 5x + 3$$

$$2-x$$

Exercise 3.225 Divide the polynomial

$$4x^3 - 2x^2 - 2x - 1$$

by

$$-2x - 3$$

Exercise 3.226 Divide the polynomial

$$-4x^3 - x^2 + 5x + 4$$

by

$$5x - 3$$

**Exercise 3.227** Divide the polynomial

$$-2x^3 + 2x^2 + 4x - 5$$

by

$$5-x$$

Exercise 3.228 Divide the polynomial

$$3x^3 - 2x^2 + 3x - 2$$

by

$$3x + 5$$

Exercise 3.229 Divide the polynomial

$$-5x^3 + x^2 + 2x + 5$$

by

$$2x - 3$$

**Exercise 3.230** Divide the polynomial

$$2x^3 + 5x^2 - 3x + 2$$

by

$$-4x - 4$$

Exercise 3.231 Divide the polynomial

$$-3x^3 - 5x^2 - 5x + 3$$

$$5x^2 - 2x - 3$$

Exercise 3.232 Divide the polynomial

$$2x^3 - 4x^2 + 2x - 3$$

by

$$-5x^2 - 4x + 1$$

Exercise 3.233 Divide the polynomial

$$-x^3 - x^2 + 4x - 1$$

by

$$-3x^2 - 5x - 2$$

Exercise 3.234 Divide the polynomial

$$-4x^3 + 5x^2 + 5x - 2$$

by

$$-2x^2 - x + 2$$

Exercise 3.235 Divide the polynomial

$$5x^3 - 3x^2 + x + 4$$

by

$$4x^2 - 3x + 2$$

Exercise 3.236 Divide the polynomial

$$-x^3 + 2x^2 - 4x - 1$$

by

$$-3x^2 + 4x + 1$$

Exercise 3.237 Divide the polynomial

$$2x^3 + x^2 + 3x + 2$$

by

$$-2x^2 + 4x - 5$$

Exercise 3.238 Divide the polynomial

$$-5x^3 - 2x^2 + 3x + 4$$

$$5x^2 - 3x - 5$$

Exercise 3.239 Divide the polynomial

$$-x^3 - x^2 - 4x - 3$$

by

$$-5x^2 + 4x + 5$$

Exercise 3.240 Divide the polynomial

$$3x^3 + x^2 + 3x - 3$$

by

$$5x^2 + 2x + 1$$

Exercise 3.241 Divide the polynomial

$$-x^2 + x + 2$$

by

$$2x^2 + 4x + 3$$

Exercise 3.242 Divide the polynomial

$$-5x^2 + x - 5$$

by

$$-3x^2 - 3x + 1$$

Exercise 3.243 Divide the polynomial

$$-4x^2 - x + 2$$

by

$$5x^2 + x - 5$$

**Exercise 3.244** Divide the polynomial

$$-4x^2 + x + 1$$

by

$$5x^2 + 3x + 3$$

**Exercise 3.245** Divide the polynomial

$$4x^2 - 4x + 1$$

$$-2x^2 + 2x + 5$$

65

Exercise 3.246 Divide the polynomial

$$3x^2 + 4x + 5$$

by

$$4x^2 - x - 1$$

Exercise 3.247 Divide the polynomial

$$-x^2 - x + 1$$

by

$$-5x^2 + 4x - 5$$

Exercise 3.248 Divide the polynomial

$$-4x^2 + 3x - 2$$

by

$$x^2 - x - 2$$

Exercise 3.249 Divide the polynomial

$$-2x^2 - 4x - 2$$

by

$$-2x^2 - 2x + 1$$

Exercise 3.250 Divide the polynomial

$$3x^2 - 3x - 2$$

by

$$-5x^2 + 5x + 4$$

Exercise 3.251 Factorize the polynomial

$$5x^2 + 22x - 15$$

**Exercise 3.252** Factorize the polynomial

$$4x^2 + 14x + 10$$

**Exercise 3.253** Factorize the polynomial

$$15x^2 - 14x - 8$$

Exercise 3.254 Factorize the polynomial

$$-5x^2 + 17x + 12$$

Exercise 3.255 Factorize the polynomial

$$-10x^2 - 6x + 4$$

Exercise 3.256 Factorize the polynomial

$$2x^2 + 10x + 12$$

Exercise 3.257 Factorize the polynomial

$$-20x^2 - 3x + 9$$

Exercise 3.258 Factorize the polynomial

$$10x^2 - 23x - 5$$

Exercise 3.259 Factorize the polynomial

$$15x^2 - 15$$

Exercise 3.260 Factorize the polynomial

$$x^2 + x - 2$$

Exercise 3.261 Factorize the polynomial

$$25x^3 + 15x^2 + 16x + 16$$

Exercise 3.262 Factorize the polynomial

$$20x^3 + 16x^2 - 24x + 4$$

Exercise 3.263 Factorize the polynomial

$$-3x^3 - 10x^2 + 9$$

Exercise 3.264 Factorize the polynomial

$$20x^3 - 21x^2 + 16x - 3$$

**Exercise 3.265** Factorize the polynomial

$$-10x^3 + 17x^2 + 35x + 12$$

**Exercise 3.266** Factorize the polynomial

$$9x^3 + 24x^2 + 30x + 15$$

**Exercise 3.267** Factorize the polynomial

$$-8x^3 + 14x^2 + 27x + 9$$

Exercise 3.268 Factorize the polynomial

$$2x^3 + 13x^2 + 14x - 5$$

Exercise 3.269 Factorize the polynomial

$$20x^3 - 19x^2 - 22x + 5$$

**Exercise 3.270** Factorize the polynomial

$$-2x^3 - 3x^2 + 7x + 5$$

Exercise 3.271 Factorize the polynomial

$$3x^4 - 15x^3 + 11x^2 + x + 12$$

Exercise 3.272 Factorize the polynomial

$$-16x^4 - 16x^3 + 17x^2 - 7x - 3$$

Exercise 3.273 Factorize the polynomial

$$-9x^4 + 3x^3 + 5x^2 - 5x - 2$$

Exercise 3.274 Factorize the polynomial

$$-4x^4 + 9x^3 - 21x^2 + 28x - 10$$

**Exercise 3.275** Factorize the polynomial

$$5x^4 + 23x^3 + 2x^2 - 16x - 6$$

Exercise 3.276 Factorize the polynomial

$$-9x^4 + 3x^3 + 3x^2 + 24x - 16$$

Exercise 3.277 Factorize the polynomial

$$8x^4 - 24x^3 + 30x^2 - 18x + 4$$

Exercise 3.278 Factorize the polynomial

$$-15x^4 + 13x^3 - 22x^2 + 24x - 4$$

Exercise 3.279 Factorize the polynomial

$$-4x^4 + 18x^3 - 18x^2 - x - 10$$

**Exercise 3.280** Factorize the polynomial

$$8x^4 + 6x^3 - 12x^2 - 6x + 4$$

Exercise 3.281 Factorize the polynomial

$$-45u^3 + 168u^2 - 185u + 50$$

Exercise 3.282 Factorize the polynomial

$$-18w^3 - 24w^2 + 70w + 100$$

Exercise 3.283 Factorize the polynomial

$$18q^3 - 30q^2 - 16q + 32$$

Exercise 3.284 Factorize the polynomial

$$64s^3 - 80s^2 + 28s - 3$$

Exercise 3.285 Factorize the polynomial

$$4l^3 - 19l^2 + 6l + 45$$

Exercise 3.286 Factorize the polynomial

$$-25q^3 - 5q^2 + 16q - 4$$

Exercise 3.287 Factorize the polynomial

$$-25v^3 + 115v^2 - 136v + 48$$

Exercise 3.288 Factorize the polynomial

$$2c^3 + 11c^2 + 12c - 9$$

Exercise 3.289 Factorize the polynomial

$$16l^3 - 56l^2 + 57l - 18$$

Exercise 3.290 Factorize the polynomial

$$80r^3 - 48r^2 - 12r + 8$$

Exercise 3.291 Solve the following equation

$$x^3 - 5x^2 + 3x + 9 = 0$$

Exercise 3.292 Solve the following equation

$$x^3 + 8x^2 + 5x - 50 = 0$$

Exercise 3.293 Solve the following equation

$$x^3 - 12x^2 + 45x - 50 = 0$$

**Exercise 3.294** Solve the following equation

$$x^3 + 10x^2 + 32x + 32 = 0$$

**Exercise 3.295** Solve the following equation

$$x^3 - 7x^2 + 15x - 9 = 0$$

Exercise 3.296 Solve the following equation

$$x^3 - 5x^2 + 3x + 9 = 0$$

Exercise 3.297 Solve the following equation

$$x^3 + 9x^2 + 15x - 25 = 0$$

**Exercise 3.298** Solve the following equation

$$x^3 - 3x^2 + 3x - 1 = 0$$

Exercise 3.299 Solve the following equation

$$x^3 + 5x^2 + 8x + 4 = 0$$

Exercise 3.300 Solve the following equation

$$x^3 + 9x^2 + 24x + 16 = 0$$

Exercise 3.301 Solve the following equation

$$x^3 - 7x^2 + 7x + 15 = 0$$

**Exercise 3.302** Solve the following equation

$$x^3 - 4x^2 - 11x + 30 = 0$$

**Exercise 3.303** Solve the following equation

$$x^3 - x^2 - 8x + 12 = 0$$

Exercise 3.304 Solve the following equation

$$x^3 + 3x^2 - 9x + 5 = 0$$

Exercise 3.305 Solve the following equation

$$x^3 - 3x + 2 = 0$$

Exercise 3.306 Solve the following equation

$$x^3 - 2x^2 - 4x + 8 = 0$$

**Exercise 3.307** Solve the following equation

$$x^3 + x^2 - x - 1 = 0$$

**Exercise 3.308** Solve the following equation

$$x^3 + x^2 - 17x + 15 = 0$$

Exercise 3.309 Solve the following equation

$$x^3 - 12x + 16 = 0$$

Exercise 3.310 Solve the following equation

$$x^3 - 9x^2 + 26x - 24 = 0$$

**Exercise 3.311** Complete the square for the following expression and then solve the equation:

$$-10m^2 - 7m$$

**Exercise 3.312** Complete the square for the following expression and then solve the equation:

$$-9x^2 - 15x$$

**Exercise 3.313** Complete the square for the following expression and then solve the equation:

$$16l^2 - 4l$$

**Exercise 3.314** Complete the square for the following expression and then solve the equation:

$$5n^2 - 21n$$

**Exercise 3.315** Complete the square for the following expression and then solve the equation:

$$-3q^2 - 7q$$

**Exercise 3.316** Complete the square for the following expression and then solve the equation:

$$4a^2 + 8a$$

**Exercise 3.317** Complete the square for the following expression and then solve the equation:

$$20q^2 + 13q$$

**Exercise 3.318** Complete the square for the following expression and then solve the equation:

$$-20x^2 + 21x$$

**Exercise 3.319** Complete the square for the following expression and then solve the equation:

$$20n^2 - 17n$$

**Exercise 3.320** Complete the square for the following expression and then solve the equation:

$$10v^2 + 8v$$

**Exercise 3.321** Solve the following inequality

$$3c + 2 \le 4$$

**Exercise 3.322** Solve the following inequality

$$2 - 4w > -1$$

**Exercise 3.323** Solve the following inequality

$$4y - 1 < -2$$

**Exercise 3.324** Solve the following inequality

$$n+5 \ge -1$$

Exercise 3.325 Solve the following inequality

$$3 - 3v > 3$$

**Exercise 3.326** Solve the following inequality

$$4 - 3l < 2$$

**Exercise 3.327** Solve the following inequality

$$-2l - 5 \le -1$$

Exercise 3.328 Solve the following inequality

$$5n - 4 \le -1$$

Exercise 3.329 Solve the following inequality

$$-4m - 1 > 4$$

**Exercise 3.330** Solve the following inequality

$$p + 3 < -2$$

**Exercise 3.331** Solve the following inequality for *u* 

$$u - 4y^2 > 3u + 5$$

**Exercise 3.332** Solve the following inequality for q

$$2q + 2y < -4q - 3y^2$$

**Exercise 3.333** Solve the following inequality for *a* 

$$4a + n < -2a - 3n^2$$

**Exercise 3.334** Solve the following inequality for n

$$-n + 2z < 5n - z$$

**Exercise 3.335** Solve the following inequality for q

$$-q + 2t^2 \le 3q - 2$$

**Exercise 3.336** Solve the following inequality for y

$$-2c + 2y \ge 3y - 1$$

**Exercise 3.337** Solve the following inequality for w

$$4t - 4w \ge -5t + w$$

**Exercise 3.338** Solve the following inequality for u

$$-4u + 3x \ge u - 1$$

**Exercise 3.339** Solve the following inequality for *s* 

$$3l^2 - 5s < -3l + 4s$$

**Exercise 3.340** Solve the following inequality for x

$$-5b^2 + 4x < -4b + 3x$$

# 3.2 Solutions

1.

$$4 + \frac{5}{5mn+1}$$

2.

$$-8 + \frac{1}{-2uv - 5}$$

3.

$$-2 + \frac{10}{2uw - 4}$$

4.

$$-5 - \frac{10}{-5z - 4}$$

5.

$$7 + \frac{6}{-5p-2}$$

6.

$$-9 + \frac{10}{-5ln - 4}$$

7.

$$-2 - \frac{2}{4q+3}$$

8.

$$8 + \frac{2}{3pqr - 1}$$

9.

$$-6 + \frac{9}{-5pq - 4}$$

10.

$$8 + \frac{4}{-5rst - 2}$$

11.

$$a\left(-2ab^2-3\right)$$

12.

$$s^3 \cdot (5s+2)$$

$$q^{3}\left(-4p^{3}qr^{4}-3\right)$$

14.

$$b^2\left(-2a^2bd^3-5\right)$$

15.

$$-2p^4q^2r-4$$

16.

$$n^2\cdot \left(2m^4+5\right)$$

17.

$$r\left(-q^4+4r^2\right)$$

18.

$$a\left(4a^2bc^2-3\right)$$

19.

$$2r^3\left(s^2+2\right)$$

20.

$$w^2 \cdot \left(3u^4 - 2\right)$$

21.

$$\frac{1}{r^2 \cdot (3pq^4r - 5)}$$

22.

$$-\frac{1}{rs^3 \cdot (5r^3 - 1)}$$

23.

$$-\frac{1}{r^2-5s^3}$$

24.

$$\frac{1}{c\left(5a^4b - 2c\right)}$$

25.

$$-\frac{1}{4c+5}$$

$$-\frac{1}{t(3s^3+t)}$$

$$-\frac{1}{2v(v-2w^2)}$$

$$-\frac{1}{5b^4c^3+1}$$

$$\frac{1}{2\cdot(2r^3-1)}$$

$$-\frac{1}{2u^4(v^4-1)}$$

$$4q^3 + 2$$

$$3s\left(-r^4s^2-1\right)$$

$$l^2 \cdot \left(3l^2m^4 - 5\right)$$

$$4p^2 + 5r^4$$

$$r^3\left(r-4s^4t^3\right)$$

$$s\left(-5s^3-3\right)$$

$$rs^2(r^3s-4)$$

$$-r^4st^3-3$$

$$a^2 \cdot (5a+2)$$

$$q^2\left(q^2-2\right)$$

75

41.

$$-\frac{1}{m^4n^2-m}$$

42.

$$-\frac{1}{pr^4(p+2)}$$

43.

$$-\frac{1}{2r^2s\left(r^2s+1\right)}$$

44.

$$-\frac{1}{3p^4q^4r^2 - p^2}$$

45.

$$\frac{1}{4q}$$

46.

$$\frac{1}{p^2q^3 \cdot (3p^2 - 2r^4)}$$

47.

$$-\frac{1}{5p^2q^3r^3-4}$$

48.

$$-\frac{1}{4p^2r^4+1}$$

49.

$$\frac{1}{r^2 \cdot (3r^2 - 5s^3)}$$

50.

$$-\frac{1}{3s\left(r^3s^3+1\right)}$$

51.

$$\left\{-\frac{5}{3l-1}\right\}$$

$$\left\{\frac{6}{2z-3}\right\}$$

$$\left\{\frac{10}{n+3}\right\}$$

$$\left\{-\frac{7}{2\cdot(2y+1)}\right\}$$

$$\left\{-\frac{7}{2n+1}\right\}$$

$$\left\{-\frac{3}{3u+5}\right\}$$

$$\left\{-\frac{6}{3p-4}\right\}$$

$$\left\{-\frac{1}{5c-2}\right\}$$

$$\left\{\frac{2}{5w-4}\right\}$$

$$\left\{-\frac{3}{2(z-1)}\right\}$$

61. 
$$-8r^4st + 2r^4s + 20r^2s^4t^4 - 5r^2s^4t^3 + 8t^4 - 2t^3$$

62. 
$$12r^5s^6 + 15r^5s^2 + 16r^4s^7 + 20r^4s^3 + 15r^4s^2 + 20r^3s^3$$

63. 
$$-2a^7b^7c^4d^3 - 10a^6b^8cd + 3a^4b^3c^6d^2 + 2a^3b^6c^4d^6 + 15a^3b^4c^3 + 10a^2b^7cd^4$$

64. 
$$-2u^6v^3w^4 - 5u^4v^7w^4 - u^4v^3w + 4u^3w^4 + 10uv^4w^4 + 2uw$$

65. 
$$2r^6s^8t^5 - 3r^5s^8t^5 + 3r^5s^4t^7 + 10r^4s^8t^3 - 15r^3s^8t^3 + 15r^3s^4t^5$$

66.

$$20p^5q^4 + 25p^3q^3 + 25p^3 - 20p^2q^5 - 25q^4 - 25q$$

67.

$$2a^6b^2 + a^5bc^3 - 3a^5b + 2a^4b^3c + a^3b^2c^4 - 3a^3b^2c$$

68.

$$-5u^5v^5w^3 + 3u^5v^4w^6 - 5u^2v^7 + 3u^2v^6w^3 + 5u^2v^4 - 3u^2v^3w^3$$

69.

$$9p^5q^4 + 15p^4q^4 - 3p^3q^4 - 5p^2q^4$$

70.

$$-12m^5n^5 + 6m^4n^7 - 20m^4n^6 + 10m^3n^8 + 3mn^4 + 5n^5$$

71.

$$4u^{10}w^3 + 5u^9 + 40u^6v^2w^3 + 50u^5v^2 + 100u^2v^4w^3 + 125uv^4$$

72.

$$45l^{11}m^8n^8 + 150l^{10}m^4n^6 + 125l^9n^4 - 27l^8m^{11}n^4 - 90l^7m^7n^2 - 75l^6m^3$$

73.

$$5r^3s^{10} - 20r^3s^9 + 19r^3s^8 + 4r^3s^7 - 4r^3s^6$$

74.

$$25x^3y^4z - 50x^2y^4z - 50x^2y^3z + 25xy^4z + 100xy^3z - 50y^3z$$

75.

$$8pq^8 - 8pq^7 + 2pq^6 + 4q^8r^2 - 4q^7r^2 + q^6r^2$$

76.

$$16r^8s^5 + 8r^7s^5 + 32r^5s^4 + 16r^4s^4 + 16r^2s^3 + 8rs^3$$

77.

$$5rs^8 + 40rs^7 + 80rs^6 + 2s^8 + 16s^7 + 32s^6$$

78.

$$-16r^9s + 8r^8 + 48r^6s^4 - 24r^5s^3 - 36r^3s^7 + 18r^2s^6$$

$$-2m^9n^6 + 3m^8n^7 - 20m^7n^4 + 30m^6n^5 - 50m^5n^2 + 75m^4n^3$$

$$32l^5m^9n^9 + 48l^5m^6n^7 + 18l^5m^3n^5 - 32l^4m^6n^{10} - 48l^4m^3n^8 - 18l^4n^6$$

$$p^8q^2r^6 - 2p^5qr^3 + 10p^4q^3r^5 + p^2 - 10pq^2r^2 + 25q^4r^4$$

$$4x^2y^6 + 8xy^3z^3 - 16xy^3 + 4z^6 - 16z^3 + 16$$

$$4p^6q^2r^4 + 12p^6qr^3 + 9p^6r^2 + 20p^3q^5r^4 + 30p^3q^4r^3 + 25q^8r^4$$

$$m^6n^6 + 2m^6n^3 + m^6 - 2m^3n^3 - 2m^3 + 1$$

$$25r^6s^8t^6 - 50r^6s^7t^7 + 25r^6s^6t^8 - 20r^6s^5t^7 + 20r^6s^4t^8 + 4r^6s^2t^8$$

$$4l^4m^6n^4 - 12l^2m^7n^2 + 16l^2m^3n^2 + 9m^8 - 24m^4 + 16$$

$$25p^4q^6r^6 - 50p^3q^3r^3 - 20p^2q^6r^4 + 25p^2 + 20pq^3r + 4q^6r^2$$

$$9q^2 + 30q + 25$$

$$4c^2 - 12cd - 20c + 9d^2 + 30d + 25$$

90.

$$16a^2b^6c^4d^2 + 16a^2b^3c^2d^2 + 4a^2d^2 + 16ab^3c^2d + 8ad + 4$$

91.

$$-20u^9v^2w^2 + 4u^8v^2w - 40u^8w^3 + 30u^7v^2w + 8u^7w^2 - 6u^6v^2 + 60u^6w^2 - 12u^5w$$

92.

$$-10l^7m^5n^4 + 40l^7m^4n + 10l^6mn^3 - 40l^6 - 25l^5m^7n^8 + 100l^5m^6n^5 + 25l^4m^3n^7 - 100l^4m^2n^4$$

$$16r^7s^2 - 24r^7s - 32r^4s^5 + 48r^4s^4 + 4r^3s^4 - 6r^3s^3 - 8s^7 + 12s^6$$

94.

$$6r^5s^2t^8 + 6r^5t^6 - 6r^4s^2t^6 - 6r^4t^4 - 2r^2s^4t^5 - 2r^2s^2t^3 + 2rs^4t^3 + 2rs^2t$$

95.

$$-12x^7y^6z^6 - 60x^6y^{10}z^2 + 8x^5y^4z^5 - 12x^5y^2z^7 + 40x^4y^8z - 60x^4y^6z^3 + 8x^3z^6 + 40x^2y^4z^2$$

96.

$$20m^8n^3 + 12m^7n^4 - 40m^5n^6 - 24m^4n^7 + 20m^4n^5 + 12m^3n^6 - 40mn^8 - 24n^9$$

97.

$$-60l^3m^2n^6 + 60l^3n^6 + 15l^2m^4n^4 - 20l^2m^2n^6 - 15l^2m^2n^4 + 20l^2n^6 + 5lm^4n^4 - 5lm^2n^4$$

98.

$$-60l^{11}m^6n^4 - 30l^{10}m^7n^4 + 36l^8m^5n + 60l^7m^6n^4 + 18l^7m^6n + 30l^6m^7n^4 - 36l^4m^5n - 18l^3m^6n$$

99.

$$15l^5m^5n^6 - 12l^5mn^6 + 30l^4m^5n^3 - 24l^4mn^3 + 20l^2m^4n^6 - 16l^2n^6 + 40lm^4n^3 - 32ln^3$$

100.

$$12r^2s^3t^6 + 28r^2s^3t^5 + 16r^2s^3t^4 - 12r^2t^2 - 28r^2t - 16r^2$$

101.

$$2q^{8}\left(p^{6}q^{4}-p^{6}-10p^{3}q^{4}+10p^{3}+25q^{4}-25\right)$$

102.

$$b^6 \cdot (500a + 300b^2)$$

103.

$$c^4 \left(-45 a^3 d^9+90 a^2 b^3 c^2 d^5-27 a^2 c^2 d^8-45 a b^6 c^4 d+54 a b^3 c^4 d^4-27 b^6 c^6\right)$$

104.

$$p^2q^2r\left(25p^5q^2r^8-25p^4r^9+30p^3q^2r^4-30p^2r^5+9pq^2-9r\right)$$

105.

$$a^{2}c^{2} \cdot (45a^{6} - 9a^{4}b^{3}c^{2} + 120a^{4}c^{2} - 24a^{2}b^{3}c^{4} + 80a^{2}c^{4} - 16b^{3}c^{6})$$

106.

$$2x^{2}(x^{4}+6x^{3}y^{3}+9x^{2}y^{6}+2x^{2}y^{2}+12xy^{5}+18y^{8})$$

$$4n^9 (m^{10}n - 4m^7n - 5m^6 + 4m^4n + 20m^3 - 20)$$

$$lm(25l^9n^9 + 25l^8m^2n^8 + 10l^5n^5 + 10l^4m^2n^4 + ln + m^2)$$

109.

$$q^2 \cdot (16p^8 + 8p^6q + 81p^4q^2 + 40p^2q^3 + 5q^4)$$

110.

$$w^{2} \left(-3 u^{4} v^{5} w^{6}-12 u^{4} v^{3} w^{4}-12 u^{4} v w^{2}-5 v^{4} w^{4}-20 v^{2} w^{2}-20\right)$$

111.

$$16p^8q^6 + \frac{8q^3}{3} + \frac{1}{9p^8}$$

112.

$$4b^6d^8 + \frac{b^3d^4}{a^2} + \frac{1}{16a^4}$$

113.

$$16p^6q^2 - 8pq + \frac{1}{p^4}$$

114.

$$16x^6y^2z^2 + \frac{8x^3}{y^2z^3} + \frac{1}{y^6z^8}$$

115.

$$u^4v^2w^2 - \frac{1}{u^2w^3} + \frac{1}{4u^8v^2w^8}$$

116.

$$25a^8b^2c^4 - \frac{10}{bc} + \frac{1}{a^8b^4c^6}$$

117.

$$9 - \frac{6}{mn^4} + \frac{1}{m^2n^8}$$

118.

$$9r^8s^4 - \frac{6}{s^2} + \frac{1}{r^8s^8}$$

119.

$$p^6 - \frac{2p}{5q^2} + \frac{1}{25p^4q^4}$$

$$9 + \frac{6}{5u^4v^2} + \frac{1}{25u^8v^4}$$

- 121. Provided in problem statement
- 122. Provided in problem statement
- 123. Provided in problem statement
- 124. Provided in problem statement
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- 137. Provided in problem statement
- 138. Provided in problem statement
- 139. Provided in problem statement
- 140. Provided in problem statement
- 141.

$$\left\{0, -\frac{8}{4a^2 + 5}\right\}$$

$$\left\{\frac{5a}{4a^2+1}\right\}$$

143.

$$\left\{0, \frac{2(a+1)}{5}\right\}$$

144.

$$\left\{0, -\frac{5a+4}{4a}\right\}$$

145.

Ø

146.

$$\left\{-\frac{\sqrt{5}\sqrt{-\frac{a-2}{a-1}}}{5}, \frac{\sqrt{5}\sqrt{-\frac{a-2}{a-1}}}{5}\right\}$$

$$\left\{\frac{5a^2}{5a-2}\right\}$$

$$\left\{-\frac{5a^{2}}{8}-\frac{\sqrt{a\left(25a^{3}+80a-32\right)}}{8},-\frac{5a^{2}}{8}+\frac{\sqrt{a\left(25a^{3}+80a-32\right)}}{8}\right\}$$

$$\left\{-a - \frac{\sqrt{3}\sqrt{3a^2 - 2}}{3}, -a + \frac{\sqrt{3}\sqrt{3a^2 - 2}}{3}\right\}$$

$$\left\{\frac{4}{5}\right\}$$

$$\left\{\frac{2a}{7} + \frac{5}{7}\right\}$$

$$\left\{-\frac{6a}{a+3}\right\}$$

$$\left\{\frac{7}{6}\right\}$$

$$\left\{-\frac{3}{7a}\right\}$$

$$\left\{-\frac{a}{2} - \frac{1}{4}\right\}$$

$$\left\{-\frac{2(a-4)}{5a-1}\right\}$$

$$\left\{\frac{1}{2a}\right\}$$

$$\left\{\frac{3a}{2(a+4)}\right\}$$

160.

$$\{1-5a\}$$

161.

$$\left\{ x : \frac{12}{29}, \ y : -\frac{38}{29} \right\}$$

162.

$$\left\{x:\frac{5}{4},\,y:\frac{17}{12}\right\}$$

163.

$${x:-6, y:6}$$

164.

$${x:6, y:-9}$$

165.

$$\left\{x: -\frac{7}{2}, y: \frac{37}{8}\right\}$$

166.

$$\left\{ x : -\frac{34}{23}, \ y : -\frac{16}{23} \right\}$$

167.

$${x:-3, y:-9}$$

168.

$$\left\{x:\frac{26}{9},\,y:\frac{23}{9}\right\}$$

169.

$${x:-2, y:7}$$

170.

$${x:5, y:-5}$$

171.

$$\left\{ x : \frac{-4a-5}{8a^2-12}, \ y : \frac{5a+6}{12a^2-18} \right\}$$

$$\left\{x: -\frac{16a}{12a+15}, y: \frac{20a^2+9a}{48a+60}\right\}$$

$$\left\{x: \frac{97}{202}, y: -\frac{16}{101}\right\}$$

$$\left\{ x : \frac{16a}{3a^2 + 64}, \ y : \frac{9a^2 - 64}{9a^2 + 192} \right\}$$

$$\left\{x:3,\,y:-\frac{4}{a}\right\}$$

$$\left\{ x: \frac{16a+20}{7a}, y: \frac{4a}{7} + \frac{32}{35} \right\}$$

$$\left\{x:0,\,y:-\frac{a}{2}\right\}$$

$$\left\{ x : -\frac{17a}{123}, \ y : \frac{67}{246} \right\}$$

$$\left\{ x : \frac{6a-1}{30a^2+5}, \ y : \frac{2a+2}{30a^2+5} \right\}$$

$$\left\{ x : -\frac{2a}{3a+4}, \ y : \frac{-3a^2+4a}{12a+16} \right\}$$

$$\frac{-2c^2 - 16c - 5}{12c^2 + 3c}$$

$$\frac{-20n^2 + 3n + 29}{8n^2 + 22n + 5}$$

$$\frac{8m^2 + 3m + 29}{6m^2 + 4m - 10}$$

$$\frac{-15q^2 - 27q + 14}{21q + 28}$$

185.

$$\frac{3u}{2} - \frac{1}{6}$$

186.

$$\frac{5b^2 + 3b - 20}{20b^2 + 2b - 4}$$

187.

$$\frac{18y^2 - 6y + 6}{15y^2 + 17y - 4}$$

188.

$$\frac{11u^2 + 19u + 12}{6u^2 + 8u}$$

189.

$$\frac{-13q^2 - 2q + 6}{6q^2 + 11q + 4}$$

190.

$$\frac{55 - 19a}{30a}$$

191.

$$\frac{32p^3 + 20p^2u + 12p^2 + 40pu^2 - 99pu + 25p + 125u^2 - 75u}{80p^2u - 48p^2 + 100pu - 60p}$$

192.

$$\frac{3q^2y^2 + 38q^2y + 18q^2 + 18qy^2 + 9qy - 8q - 4y}{6q^2y + 4q^2 + 3qy^2 + 2qy}$$

193.

$$\frac{14n^2x^2 - 88n^2x - 18n^2 + 5nx^3 - 37nx^2 + 6nx - x^3 + 5x^2}{4n^2x - 20n^2 + 2nx^2 - 10nx}$$

194.

$$\frac{28ps + 228p + 35s + 495}{112p + 140}$$

195.

$$-\frac{4dm}{5} + \frac{d}{2} + \frac{17m}{20} - 1$$

$$\frac{-3 s^3 x-21 s^2 x^2-5 s^2 x+60 s x^3-94 s x^2+33 s x-6 s-20 x^3+7 x^2+59 x-30}{15 s^2 x^2-9 s^2 x+70 s x^2-42 s x-25 x^2+15 x}$$

$$\frac{6p^2x - 6px^2 - 194px + 28p + 26x^2 - 84x}{-21p^2 + 21px - 7p + 7x}$$

198.

$$\frac{33m^2n^2 - 30m^2 - 58mn^2 + 10m + 5n^2 + 20}{30m^2n - 80mn + 50n}$$

199.

$$\frac{25pv^2 - 10pv - 4p - 20v + 14}{5pv - 3p - 10v + 6}$$

200.

$$\frac{-90mw^2 - 96mw - 28m - 100w^2 - 85w - 15}{40mw^2 + 34mw + 6m}$$

201.

$$\frac{24n^3 + 68n^2 + 88n + 60}{45n^2 + 150n + 120}$$

202.

$$\frac{42v^3 + 1015v^2 + 1575v}{290v^2 - 40v - 250}$$

203.

$$\frac{-12p^4+105p^3+87p^2-141p-39}{152p^3+64p^2-112p-24}$$

204.

$$\frac{-100c^3 + 150c^2 - 50c}{120c^2 - 258c + 135}$$

205.

$$\frac{-36t^3 - 54t^2 + 22t + 20}{70t^3 + 91t^2 - 70t}$$

206.

$$\frac{6m^2 - 25m + 21}{2m + 24}$$

207.

$$\frac{42b^4 + 4b^3 - 170b^2 - 52b + 80}{21b^4 + 78b^3 + 63b^2 - 18b}$$

$$\frac{-184x^4 + 214x^3 + 77x^2 - 45x - 12}{60x^4 - 236x^3 - 60x^2 + 188x - 48}$$

209.

$$\frac{-24a^4 - 42a^3 + 49a^2 + 112a + 40}{24a^3 + 18a^2 - 24a}$$

$$\frac{105q^4 - 17q^3 - 485q^2 - 185q + 150}{315q^3 + 1785q^2 - 70q}$$

213. 
$$\begin{array}{r}
-4x + 6 \\
-x-2) \overline{4x^2 + 2x - 1} \\
\underline{-4x^2 - 8x} \\
-6x - 1 \\
\underline{6x + 12}
\end{array}$$

215. 
$$\frac{\frac{1}{3}x + \frac{5}{9}}{x^2 + 2x + 4} \\
\underline{-x^2 - \frac{1}{3}x} \\
\underline{\frac{5}{3}x + 4} \\
\underline{-\frac{5}{3}x - \frac{5}{9}} \\
\underline{\frac{31}{3}}$$

216. 
$$\begin{array}{r}
 \frac{-\frac{3}{3}x - \frac{5}{9}}{\frac{31}{9}} \\
 -5x + 14 \\
 -x - 3) \overline{\smash{\big)}\ 5x^2 + x - 4} \\
 \underline{-5x^2 - 15x} \\
 -14x - 4 \\
 \underline{14x + 42} \\
 \hline
 38}$$

223. 
$$\frac{-\frac{1}{3}x^{2} - \frac{5}{9}x + \frac{41}{27}}{-x^{3} - 2x^{2} + 4x - 1}$$

$$\frac{x^{3} + \frac{1}{3}x^{2}}{-\frac{5}{3}x^{2} + 4x}$$

$$\frac{\frac{5}{3}x^{2} + \frac{5}{9}x}{-\frac{41}{9}x - 1}$$

$$-\frac{4^{1}}{9}x - \frac{41}{27}$$

$$-\frac{68}{27}$$
224. 
$$-2x^{2} - 5x - 15$$

$$-x + 2) \frac{2x^{3} + x^{2} + 5x + 3}{-2x^{3} + 4x^{2}}$$

$$5x^{2} + 5x$$

$$-5x^{2} + 10x$$

$$15x + 3$$

$$-15x + 30$$

$$33$$
225. 
$$-2x^{2} + 4x - 5$$

$$-2x^{3} - 6x^{2}$$

$$-8x^{2} - 2x$$

$$-8x^{2} - 2x$$

$$-8x^{2} + 12x$$

$$10x - 1$$

$$-10x - 15$$

$$-16$$
226. 
$$-\frac{4}{5}x^{2} - \frac{17}{25}x + \frac{74}{125}$$

$$5x - 3) - 4x^{3} - x^{2} + 5x + 4$$

$$-\frac{4x^{3} - \frac{12}{5}x^{2}}{-\frac{17}{5}x^{2} + 5x}$$

$$-\frac{17}{5}x^{2} - \frac{51}{25}x$$

$$-\frac{17}{5}x^{2} - \frac{51}{25}x$$

$$-\frac{17}{25}x - \frac{51}{25}x$$

$$-\frac{2x^{2}}{25}x + 4x$$

$$-\frac{74}{25}x + \frac{74}{125}$$

$$-\frac{17}{25}x - \frac{51}{25}x$$

$$-\frac{17}{25}x - \frac{17}{25}x - \frac{17}{25}x$$

$$-\frac{17}{25}x - \frac{17}{25}x -$$

228. 
$$\frac{x^{2} - \frac{7}{3}x + \frac{44}{9}}{3x^{3} - 2x^{2} + 3x - 2} - \frac{-3x^{3} - 5x^{2}}{-7x^{2} + 3x} - 2
\frac{-3x^{3} - 5x^{2}}{4\frac{4}{3}x - 2} - \frac{44}{3}x - 2
\frac{-\frac{44}{3}x - \frac{220}{9}}{-\frac{238}{9}} - \frac{238}{9}$$
229. 
$$\frac{-\frac{5}{2}x^{2} - \frac{13}{4}x - \frac{31}{8}}{2x^{2} - \frac{13}{2}x^{2} + 2x + 5}
\frac{\frac{13}{2}x^{2} - \frac{39}{4}x}{-\frac{13}{4}x - \frac{93}{8}} - \frac{53}{8}$$
230. 
$$\frac{-\frac{1}{2}x^{2} - \frac{3}{4}x + 5}{-\frac{31}{4}x - \frac{93}{8}} - \frac{53}{8}$$
231. 
$$\frac{-\frac{1}{2}x^{2} - \frac{3}{4}x + \frac{3}{2}}{-6x + 2}
\frac{-6x + 2}{-2x^{3} - 2x^{2}}
\frac{6x + 6}{8}$$
231. 
$$\frac{-\frac{3}{5}x - \frac{31}{25}x}{-\frac{31}{25}x^{2} - \frac{34}{5}x + 3}
\frac{\frac{31}{5}x^{2} - \frac{35}{25}x - \frac{93}{25}}{-\frac{235}{25}x - \frac{135}{25}}$$
232. 
$$-5x^{2} - 4x + 1) \frac{2x^{3} - 4x^{2} + 2x - 3}{-\frac{2x^{3}}{8}x^{2} + \frac{12}{5}x} - \frac{3}{25}
\frac{\frac{23}{5}x^{2} + \frac{12}{5}x - \frac{23}{25}}{-\frac{23}{25}x - \frac{103}{25}}$$
233. 
$$-3x^{2} - 5x - 2) \frac{-x^{3} - x^{2} + 4x - 1}{\frac{x^{3} + \frac{5}{3}x^{2} + \frac{13}{3}x - \frac{9}{9}}{\frac{32}{6}x - \frac{13}{9}}$$
234. 
$$-3x^{2} - 5x - 2) \frac{-x^{3} - x^{2} + 4x - 1}{\frac{x^{3} + \frac{5}{3}x^{2} + \frac{13}{3}x - \frac{9}{9}}{\frac{32}{6}x - \frac{13}{9}}}$$
235. 
$$-3x^{2} - 5x - 2) \frac{-x^{3} - x^{2} + 4x - 1}{\frac{x^{3} + \frac{5}{3}x^{2} + \frac{13}{3}x - \frac{9}{9}}{\frac{32}{6}x - \frac{13}{9}}}$$

242. 
$$\begin{array}{r}
 3x + \frac{7}{2} \\
 -3x^2 - 3x + 1) \overline{) -5x^2 + x - 5} \\
 \underline{5x^2 + 5x - \frac{5}{3}} \\
 \underline{6x - \frac{20}{3}} \\
 243. \\
 \underline{5x^2 + x - 5} - 4x^2 - x + 2 \\
 \underline{4x^2 + \frac{4}{5}x - 4} \\
 -\frac{1}{5}x - 2 \\
 244.
 \end{array}$$

245. 
$$-2x^{2} + 2x + 5) \frac{3}{4x^{2} - 4x + 1}$$
$$-4x^{2} + 4x + 10$$
$$0x + 11$$

247. 
$$\begin{array}{r}
\frac{1}{4}x + \frac{2}{4} \\
-5x^2 + 4x - 5) \overline{)-x^2 - x + 1} \\
\underline{x^2 - \frac{4}{5}x + 1} \\
-\frac{9}{5}x + 2
\end{array}$$
248. 
$$x^2 - x - 2) \overline{)-4x^2 + 3x - 2} \\
\underline{4x^2 - 4x - 8} \\
-x - 10$$

93

and

-x - 5

252.

-2x-5

and

-2x - 2

253.

5x + 2

and

3x - 4

254.

5x + 3

and

4-x

255.

-2x - 2

and

5x - 2

256.

x+3

and

2x + 4

257.

5x - 3

and

-4x - 3

258.

2x - 5

and

5x + 1

$$3x + 3$$

and

$$5x - 5$$

260.

$$x+2$$

and

$$x-1$$

261.

$$5x + 4$$

and

$$5x^2 - x + 4$$

262.

$$1 - 5x$$

and

$$-4x^2 - 4x + 4$$

263.

$$x+3$$

and

$$-3x^2 - x + 3$$

264.

$$1-4x$$

and

$$-5x^2 + 4x - 3$$

265.

$$5x + 4$$

and

$$-2x^2 + 5x + 3$$

95

266.

$$-3x - 3$$

and

$$-3x^2 - 5x - 5$$

267.

$$-4x - 3$$

and

$$2x^2 - 5x - 3$$

268.

$$-x - 5$$

and

$$-2x^2 - 3x + 1$$

269.

$$5x - 1$$

and

$$4x^2 - 3x - 5$$

270.

$$2x + 5$$

and

$$-x^2+x+1$$

271.

$$x-4$$

and

$$3x^3 - 3x^2 - x - 3$$

272.

$$-4x - 1$$

and

$$4x^3 + 3x^2 - 5x + 3$$

$$-3x - 1$$

and

$$3x^3 - 2x^2 - x + 2$$

274.

$$5 - 4x$$

and

$$x^3 - x^2 + 4x - 2$$

275.

$$-5x - 3$$

and

$$-x^3 - 4x^2 + 2x + 2$$

276.

$$3x - 4$$

and

$$-3x^3 - 3x^2 - 3x + 4$$

277.

$$4x - 2$$

and

$$2x^3 - 5x^2 + 5x - 2$$

278.

$$1 - 5x$$

and

$$3x^3 - 2x^2 + 4x - 4$$

279.

$$5 - 2x$$

and

$$2x^3 - 4x^2 - x - 2$$

97

280.

2x + 2

and

 $4x^3 - x^2 - 5x + 2$ 

281.

5 - 3u

and

5 - 3u

and

2-5u

282.

3w + 5

and

3w + 5

and

4 - 2w

283.

3q - 4

and

3q - 4

and

2q + 2

284.

1 - 4s

and

1 - 4s

and

4s - 3

3-l

and

3-l

and

4l + 5

286.

5q - 2

and

5q - 2

and

-q - 1

287.

4 - 5v

and

4 - 5v

and

3-v

288.

-c - 3

and

-c - 3

and

2c - 1

289.

3 - 4l

and

3 - 4l

and

l-2

99

290.

$$4r - 2$$

and

$$4r - 2$$

and

$$5r + 2$$

- 291. 3, 3 and -1
- 292. -5, -5 and 2
- 293. 5, 5 and 2
- 294. -4, -4 and -2
- 295. 3, 3 and 1
- 296. 3, 3 and -1
- 297. -5, -5 and 1
- 298. 1, 1 and 1
- 299. -2, -2 and -1
- 300. -4, -4 and -1
- 301. -1, 5 and 3
- 302. 2, 5 and -3
- 303. 2, -3 and 2
- 304. 1, 1 and -5
- 305. 1, -2 and 1
- 306. -2, 2 and 2
- 307. 1, -1 and -1
- 308. 1, -5 and 3
- 309. 2, -4 and 2
- 310. 2, 3 and 4
- 311. Add the following term to it:

The solution of the equation becomes:

$$\left[-\frac{1}{2},\,-\frac{1}{5}\right]$$

312. Add the following term to it:

-4

The solution of the equation becomes:

$$\left[-\frac{4}{3},\,-\frac{1}{3}\right]$$

313. Add the following term to it:

$$-12$$

The solution of the equation becomes:

$$\left[-\frac{3}{4}, 1\right]$$

314. Add the following term to it:

4

The solution of the equation becomes:

$$\left[\frac{1}{5}, 4\right]$$

315. Add the following term to it:

\_4

The solution of the equation becomes:

$$\left[-\frac{4}{3}, -1\right]$$

316. Add the following term to it:

3

The solution of the equation becomes:

$$\left[-\frac{3}{2}, -\frac{1}{2}\right]$$

317. Add the following term to it:

$$-15$$

The solution of the equation becomes:

$$\left[-\frac{5}{4},\,\frac{3}{5}\right]$$

318. Add the following term to it:

-4

The solution of the equation becomes:

$$\left[\frac{1}{4}, \frac{4}{5}\right]$$

319. Add the following term to it:

$$-10$$

The solution of the equation becomes:

$$\left[-\frac{2}{5},\,\frac{5}{4}\right]$$

320. Add the following term to it:

-2

The solution of the equation becomes:

$$\left[-1, \frac{1}{5}\right]$$

101

321.

$$c \leq \frac{2}{3}$$

322.

$$w < \frac{3}{4}$$

323.

$$y < -\frac{1}{4}$$

324.

$$-6 \le n$$

325.

$$v \le 0$$

326.

$$\frac{2}{3} < l$$

327.

$$-2 \le l$$

328.

$$n \le \frac{3}{5}$$

329.

$$m<-\frac{5}{4}$$

330.

$$p < -5$$

331.

$$u < \frac{1}{4} - \frac{5y}{4}$$

332.

$$q < \frac{y^2}{5}$$

$$a > \frac{4n}{9}$$

$$n < \frac{5z}{4} - \frac{5}{4}$$

335.

True

336.

$$y \ge -\frac{2c^2}{5} + c$$

337.

$$w \le \frac{t^2}{4} - \frac{3t}{4}$$

338.

$$u \ge \frac{4x}{3} - \frac{1}{3}$$

339.

$$s \ge -5l^2 - 5l$$

$$x \le \frac{3b^2}{2}$$

## 4. Differential Calculus

## 4.1 Problems

Exercise 4.1 Differentiate the polynomial with respect to w

$$-2w^2 + 5w - 3$$

**Exercise 4.2** Differentiate the polynomial with respect to m

$$-m^2 + 4m - 3$$

**Exercise 4.3** Differentiate the polynomial with respect to y

$$5y^2 - 3y + 4$$

**Exercise 4.4** Differentiate the polynomial with respect to x

$$2x^2 - 2x + 2$$

**Exercise 4.5** Differentiate the polynomial with respect to z

$$-5z^2 - z - 3$$

Exercise 4.6 Differentiate the polynomial with respect to n

$$5n^2 + 2n - 1$$

**Exercise 4.7** Differentiate the polynomial with respect to v

$$v^2 + 4v + 3$$

Exercise 4.8 Differentiate the polynomial with respect to d

$$d^2 - 2d + 3$$

Exercise 4.9 Differentiate the polynomial with respect to b

$$2b^2 + 2b - 3$$

**Exercise 4.10** Differentiate the polynomial with respect to c

$$-c^2 - 3c + 3$$

**Exercise 4.11** Differentiate the following expression with respect to p

$$3p^{\frac{2}{5}} + 5p^{\frac{2}{3}} - \sqrt[3]{p} - 2$$

Exercise 4.12 Differentiate the following expression with respect to d

$$-4d^{\frac{5}{4}} - 3d^5 - 1$$

**Exercise 4.13** Differentiate the following expression with respect to s

$$4s^{\frac{5}{3}} + s^{\frac{5}{2}} - s^3 - s^2$$

Exercise 4.14 Differentiate the following expression with respect to u

$$-2u^{\frac{5}{4}}-2u^{\frac{4}{3}}+4\sqrt[3]{u}-u$$

Exercise 4.15 Differentiate the following expression with respect to q

$$5q^{\frac{6}{5}} + 7q^4$$

**Exercise 4.16** Differentiate the following expression with respect to q

$$5q^{\frac{5}{3}} + 5q^4 + 10q$$

Exercise 4.17 Differentiate the following expression with respect to 1

$$-\sqrt[6]{l}-2l^4+2l^2+4$$

**Exercise 4.18** Differentiate the following expression with respect to y

$$5y^{\frac{5}{2}} + 2y + 2$$

**Exercise 4.19** Differentiate the following expression with respect to r

$$5r^{\frac{2}{3}} + 4\sqrt{r} - 10r$$

Exercise 4.20 Differentiate the following expression with respect to u

$$\sqrt[5]{u} + 4u^{\frac{4}{3}} + 3u^{\frac{2}{3}} - u^3$$

Exercise 4.21 Differentiate the following expression with respect to b

$$4 \tan (4v^2)$$

**Exercise 4.22** Differentiate the following expression with respect to n

$$3\csc(4nv^2)$$

Exercise 4.23 Differentiate the following expression with respect to 1

$$3\cot^{2}(3c^{2}l)$$

4.1 Problems 105

**Exercise 4.24** Differentiate the following expression with respect to t  $5 \sec (4t)$ 

**Exercise 4.25** Differentiate the following expression with respect to m  $-\cot(4c^2m)$ 

**Exercise 4.26** Differentiate the following expression with respect to c  $-4\tan(2x)$ 

**Exercise 4.27** Differentiate the following expression with respect to c  $5 \sec^2(s)$ 

**Exercise 4.28** Differentiate the following expression with respect to  $x - 2\sin^2(4v^2)$ 

**Exercise 4.29** Differentiate the following expression with respect to t  $\tan(4t^2)$ 

Exercise 4.30 Differentiate the following expression with respect to v  $4\tan(5bv)$ 

**Exercise 4.31** Differentiate the following expression with respect to a  $-\cos(5u^2) - \tan^2(3a^2u^2)$ 

**Exercise 4.32** Differentiate the following expression with respect to y  $2\cos^2(2y) - 3\csc^2(5v^2y)$ 

**Exercise 4.33** Differentiate the following expression with respect to q  $-4\sin^2(2aq^2) + 4\sec^2(2q)$ 

Exercise 4.34 Differentiate the following expression with respect to z  $4\cos^2(5uz) + 4\cot^2(4u^2)$ 

**Exercise 4.35** Differentiate the following expression with respect to a  $-2\sin^2(2a^2) - 4\tan(5b)$ 

**Exercise 4.36** Differentiate the following expression with respect to c  $\cot(2r^2) + 5\cot(5cr)$ 

**Exercise 4.37** Differentiate the following expression with respect to x

$$-4\sin\left(5m^2x\right) - \frac{1}{\tan^2\left(5m\right)}$$

Exercise 4.38 Differentiate the following expression with respect to r

$$-2\cos\left(2pr\right)-4\csc\left(4p^2r\right)$$

Exercise 4.39 Differentiate the following expression with respect to a

$$4\cot\left(a^2\right) - 3\cot^2\left(5a^2y^2\right)$$

**Exercise 4.40** Differentiate the following expression with respect to s

$$-3\cos^2(3s^2) - 3\cot(4a)$$

Exercise 4.41 Differentiate the following expression with respect to w

$$-12e^{6w^2}\tan(w^2)$$

Exercise 4.42 Differentiate the following expression with respect to q

$$12e^{q^2s^2}\cot\left(4q^2s\right)$$

**Exercise 4.43** Differentiate the following expression with respect to p

$$-2e^{np^2}\cos\left(p^2\right)$$

**Exercise 4.44** Differentiate the following expression with respect to t

$$15e^{3rt}\cos^2\left(3r^2\right)$$

**Exercise 4.45** Differentiate the following expression with respect to r

$$-16e^{10b^2}\sin^2(2br)$$

Exercise 4.46 Differentiate the following expression with respect to w

$$-4e^{5tw^2}\cot^2(3t)$$

**Exercise 4.47** Differentiate the following expression with respect to c

$$-10e^{7w^2}\cot\left(3w^2\right)$$

Exercise 4.48 Differentiate the following expression with respect to a

$$4e^x\cos^2(a^2)$$

Exercise 4.49 Differentiate the following expression with respect to d

$$12e^{9r}\csc^2(5d^2r)$$

**Exercise 4.50** Differentiate the following expression with respect to q

$$-10e^{6n^2}\csc(2n^2)$$

**Exercise 4.51** Differentiate the following expression with respect to x

$$12e^{3nx}\cos(2nx^2) + \cos^2(3nx^2)$$

4.1 Problems

Exercise 4.52 Differentiate the following expression with respect to a

$$15e^a\cos^2(a^2m) + 3\sec^2(4m^2)$$

**Exercise 4.53** Differentiate the following expression with respect to v

$$-4e^{10c^2}\sin^2(2v) + 3\csc^2(3c^2)$$

**Exercise 4.54** Differentiate the following expression with respect to p

$$-12e^{5p}\cot^2(4np^2) + 2\tan(4p)$$

**Exercise 4.55** Differentiate the following expression with respect to 1

$$8e^{4l}\csc(3l^2t^2) - 2\cot^2(4lt^2)$$

**Exercise 4.56** Differentiate the following expression with respect to n

$$6e^{10nt^2}\sec(5n^2) + 2\cos(5n^2t^2)$$

Exercise 4.57 Differentiate the following expression with respect to d

$$-16e^{6d^2z}\tan(2d) + 2\sec(3z)$$

**Exercise 4.58** Differentiate the following expression with respect to s

$$-2e^{3n^2}\tan^2(2n^2s) - 5\cos^2(3s^2)$$

Exercise 4.59 Differentiate the following expression with respect to w

$$-12e^{7w}\cos(3w^2) + 2\sec(4c^2w)$$

Exercise 4.60 Differentiate the following expression with respect to r

$$3e^{5r^2}\cos^2(2pr^2) + 3\sec(3pr^2)$$

Exercise 4.61 Let

$$f'(x) = 10x - 2$$

If f(1) = 5, find f(x).

Exercise 4.62 Let

$$f'(x) = 10x - 2$$

If f(10) = 482, find f(x).

Exercise 4.63 Let

$$f'(x) = 2x - 3$$

If f(1) = -1, find f(x).

Exercise 4.64 Let

$$f'(x) = 4 - 2x$$

If 
$$f(6) = -15$$
, find  $f(x)$ .

Exercise 4.65 Let

$$f'(x) = 4x + 5$$

If f(2) = 22, find f(x).

Exercise 4.66 Let

$$f'(x) = -10x - 1$$

If f(7) = -247, find f(x).

Exercise 4.67 Let

$$f'(x) = 5 - 6x$$

If f(9) = -201, find f(x).

Exercise 4.68 Let

$$f'(x) = 4x - 4$$

If f(0) = 5, find f(x).

Exercise 4.69 Let

$$f'(x) = 1 - 2x$$

If f(2) = -7, find f(x).

Exercise 4.70 Let

$$f'(x) = -6x - 5$$

If f(0) = 3, find f(x).

Exercise 4.71 Let

$$f'(x) = -9x^2 - 6x + 1$$

If f(9) = -2423, find f(x).

Exercise 4.72 Let

$$f'(x) = 6x^2 - 4x - 4$$

If f(4) = 76, find f(x).

Exercise 4.73 Let

$$f'(x) = 15x^2 + 10x - 1$$

If f(0) = -2, find f(x).

Exercise 4.74 Let

$$f'(x) = -9x^2 - 4x - 3$$

If f(3) = -112, find f(x).

Exercise 4.75 Let

$$f'(x) = -6x^2 + 6x - 1$$

If 
$$f(7) = -551$$
, find  $f(x)$ .

4.1 Problems

Exercise 4.76 Let

$$f'(x) = 6x^2 + 8x - 2$$

If f(5) = 342, find f(x).

Exercise 4.77 Let

$$f'(x) = 9x^2 + 10x + 1$$

If f(2) = 51, find f(x).

Exercise 4.78 Let

$$f'(x) = -9x^2 - 6x + 5$$

If f(2) = -24, find f(x).

Exercise 4.79 Let

$$f'(x) = -9x^2 + 10x + 3$$

If f(8) = -1196, find f(x).

Exercise 4.80 Let

$$f'(x) = 15x^2 - 4x - 3$$

If f(4) = 277, find f(x).

Exercise 4.81 Let

$$f'(x) = x - 5$$

If (5, -14.5) is a stationary point: find the expression of f(x), and determine the nature of the stationary point.

Exercise 4.82 Let

$$f'(x) = x - 7$$

If (7, -29.5) is a stationary point: find the expression of f(x), and determine the nature of the stationary point.

Exercise 4.83 Let

$$f'(x) = 20 - 5x$$

If (4,38.0) is a stationary point: find the expression of f(x), and determine the nature of the stationary point.

Exercise 4.84 Let

$$f'(x) = 4x - 36$$

If (9, -164) is a stationary point: find the expression of f(x), and determine the nature of the stationary point.

Exercise 4.85 Let

$$f'(x) = 15 - 5x$$

If (3,23.5) is a stationary point: find the expression of f(x), and determine the nature of the stationary point.

Exercise 4.86 Let

$$f'(x) = 4x - 4$$

If (1,-6) is a stationary point: find the expression of f(x), and determine the nature of the stationary point.

Exercise 4.87 Let

$$f'(x) = 2x - 12$$

If (6,-38) is a stationary point: find the expression of f(x), and determine the nature of the stationary point.

Exercise 4.88 Let

$$f'(x) = x - 4$$

If (4, -3.0) is a stationary point: find the expression of f(x), and determine the nature of the stationary point.

Exercise 4.89 Let

$$f'(x) = 2x - 2$$

If (1, -4) is a stationary point: find the expression of f(x), and determine the nature of the stationary point.

Exercise 4.90 Let

$$f'(x) = 18 - 2x$$

If (9,84) is a stationary point: find the expression of f(x), and determine the nature of the stationary point.

**Exercise 4.91** If f(x) is defined as

$$f(x) = k - x^3 - 9x^2 - 27x - 34$$

for what value of k will f(x) be invertible?

**Exercise 4.92** If f(x) is defined as

$$f(x) = k - 64x^3 - 96x^2 - 48x - 7$$

for what value of k will f(x) be invertible?

**Exercise 4.93** If f(x) is defined as

$$f(x) = k - 125x^3 - 75x^2 - 15x - 9$$

for what value of k will f(x) be invertible?

**Exercise 4.94** If f(x) is defined as

$$f(x) = k - 27x^3 + 108x^2 - 144x + 74$$

for what value of k will f(x) be invertible?

**Exercise 4.95** If f(x) is defined as

$$f(x) = k - 8x^3 + 24x^2 - 24x + 4$$

for what value of k will f(x) be invertible?

4.1 Problems

**Exercise 4.96** If f(x) is defined as

$$f(x) = k + 8x^3 - 48x^2 + 96x - 69$$

for what value of k will f(x) be invertible? **Exercise 4.97** If f(x) is defined as

$$f(x) = k - 8x^3 - 36x^2 - 54x - 20$$

for what value of k will f(x) be invertible? **Exercise 4.98** If f(x) is defined as

$$f(x) = k + x^3 + 3x^2 + 3x - 1$$

for what value of k will f(x) be invertible? **Exercise 4.99** If f(x) is defined as

$$f(x) = k - 125x^3 + 150x^2 - 60x + 16$$

for what value of k will f(x) be invertible? **Exercise 4.100** If f(x) is defined as

$$f(x) = k - x^3 - 9x^2 - 27x - 19$$

for what value of k will f(x) be invertible?

Exercise 4.101 For the function

$$f(x) = \frac{x^3}{3} + 2x^2 - 2$$

find the two points at which the tangents of the graph of f(x) are parallel to the line connecting the two points (5,5) and (9,25).

Exercise 4.102 For the function

$$f(x) = \frac{x^3}{3} + \frac{5x^2}{2} + 4x + 5$$

find the two points at which the tangents of the graph of f(x) are parallel to the line connecting the two points (-2,4) and (2,-4).

Exercise 4.103 For the function

$$f(x) = \frac{x^3}{3} - x^2 - 6x + 1$$

find the two points at which the tangents of the graph of f(x) are parallel to the line connecting the two points (-5, -4) and (-1, 4).

Exercise 4.104 For the function

$$f(x) = \frac{x^3}{3} - x^2 + 2x - 3$$

find the two points at which the tangents of the graph of f(x) are parallel to the line connecting the two points (-2,4) and (2,24).

Exercise 4.105 For the function

$$f(x) = \frac{x^3}{3} - x^2 - 4$$

find the two points at which the tangents of the graph of f(x) are parallel to the line connecting the two points (4,1) and (8,13).

Exercise 4.106 For the function

$$f(x) = \frac{x^3}{3} - \frac{x^2}{2} - 5x - 4$$

find the two points at which the tangents of the graph of f(x) are parallel to the line connecting the two points (-1,-1) and (3,-13).

Exercise 4.107 For the function

$$f(x) = \frac{x^3}{3} - 3x^2 + 14x + 3$$

find the two points at which the tangents of the graph of f(x) are parallel to the line connecting the two points (-1, -5) and (3, 15).

Exercise 4.108 For the function

$$f(x) = \frac{x^3}{3} - \frac{3x^2}{2} + 2$$

find the two points at which the tangents of the graph of f(x) are parallel to the line connecting the two points (3,2) and (7,-6).

Exercise 4.109 For the function

$$f(x) = \frac{x^3}{3} + \frac{3x^2}{2} - 7x - 3$$

find the two points at which the tangents of the graph of f(x) are parallel to the line connecting the two points (-3, -1) and (1, 11).

Exercise 4.110 For the function

$$f(x) = \frac{x^3}{3} + x^2 + x + 5$$

find the two points at which the tangents of the graph of f(x) are parallel to the line connecting the two points (5,2) and (9,18).

Exercise 4.111 For the function

$$f(x) = \frac{x^3}{3} - x^2 - 13x - 2$$

find the two points at which the tangents of the graph of f(x) are perpendicular to the line connecting the two points (5,-1) and (9,-1/5).

Exercise 4.112 For the function

$$f(x) = \frac{x^3}{3} - 2x^2 + 7x - 4$$

find the two points at which the tangents of the graph of f(x) are perpendicular to the line connecting the two points (-2,1) and (2,0).

4.1 Problems

Exercise 4.113 For the function

$$f(x) = \frac{x^3}{3} + \frac{7x^2}{2} + 9x + 3$$

find the two points at which the tangents of the graph of f(x) are perpendicular to the line connecting the two points (-1, -4) and (3, -8/3).

Exercise 4.114 For the function

$$f(x) = \frac{x^3}{3} - 2x + 2$$

find the two points at which the tangents of the graph of f(x) are perpendicular to the line connecting the two points (-2,1) and (2,5).

Exercise 4.115 For the function

$$f(x) = \frac{x^3}{3} - 9x + 3$$

find the two points at which the tangents of the graph of f(x) are perpendicular to the line connecting the two points (-2,2) and (2,14/5).

Exercise 4.116 For the function

$$f(x) = \frac{x^3}{3} - \frac{x^2}{2} - 7x - 2$$

find the two points at which the tangents of the graph of f(x) are perpendicular to the line connecting the two points (-1,2) and (3,6/5).

Exercise 4.117 For the function

$$f(x) = \frac{x^3}{3} + \frac{7x^2}{2} + 14x - 3$$

find the two points at which the tangents of the graph of f(x) are perpendicular to the line connecting the two points (1,1) and (5,-1).

Exercise 4.118 For the function

$$f(x) = \frac{x^3}{3} + \frac{x^2}{2} - x + 1$$

find the two points at which the tangents of the graph of f(x) are perpendicular to the line connecting the two points (3,-2) and (7,-14/5).

Exercise 4.119 For the function

$$f(x) = \frac{x^3}{3} - \frac{3x^2}{2} - 13x - 1$$

find the two points at which the tangents of the graph of f(x) are perpendicular to the line connecting the two points (-1,2) and (3,10/3).

Exercise 4.120 For the function

$$f(x) = \frac{x^3}{3} + \frac{3x^2}{2} - 9x + 5$$

find the two points at which the tangents of the graph of f(x) are perpendicular to the line connecting the two points (1,4) and (5,24/5).

1.

$$5 - 4w$$

2.

$$4 - 2m$$

3.

$$10y - 3$$

4.

$$4x - 2$$

5.

$$-10z - 1$$

6.

$$10n + 2$$

7.

$$2v + 4$$

8.

$$2d - 2$$

9.

$$4b + 2$$

10.

$$-2c - 3$$

11.

$$\frac{50p^{\frac{19}{15}} - 5p^{\frac{14}{15}} + 18p}{15p^{\frac{8}{5}}}$$

12.

$$-5\sqrt[4]{d}-15d^4$$

13.

$$\frac{20s^{\frac{2}{3}}}{3} + \frac{5s^{\frac{3}{2}}}{2} - 3s^2 - 2s$$

14.

$$\frac{-15u^{\frac{11}{12}} - 6u^{\frac{2}{3}} - 16u + 8}{6u^{\frac{2}{3}}}$$

15.

$$6\sqrt[5]{q} + 28q^3$$

16.

$$\frac{25q^{\frac{2}{3}}}{3} + 20q^3 + 10$$

17.

$$-8l^3 + 4l - \frac{1}{6l^{\frac{5}{6}}}$$

18.

$$\frac{25y^{\frac{3}{2}}}{2} + 2$$

19.

$$-10 + \frac{2}{\sqrt{r}} + \frac{10}{3\sqrt[3]{r}}$$

20.

$$\frac{16\sqrt[3]{u}}{3} - 3u^2 + \frac{2}{\sqrt[3]{u}} + \frac{1}{5u^{\frac{4}{5}}}$$

21.

0

22.

$$-12v^2\cot\left(4nv^2\right)\csc\left(4nv^2\right)$$

23.

$$-\frac{18c^2\cot\left(3c^2l\right)}{\sin^2\left(3c^2l\right)}$$

24.

$$20\tan(4t)\sec(4t)$$

25.

$$\frac{4c^2}{\sin^2\left(4c^2m\right)}$$

26.

0

27.

0

28.

0

29.

$$\frac{8t}{\cos^2\left(4t^2\right)}$$

30.

$$\frac{20b}{\cos^2{(5bv)}}$$

31.

$$-\frac{12au^2\tan{(3a^2u^2)}}{\cos^2{(3a^2u^2)}}$$

32.

$$30v^2\cot\left(5v^2y\right)\csc^2\left(5v^2y\right) - 4\sin\left(4y\right)$$

33.

$$-16aq\sin\left(4aq^2\right) + 16\tan\left(2q\right)\sec^2\left(2q\right)$$

34.

$$-20u\sin(10uz)$$

35.

$$-8a\sin\left(4a^2\right)$$

36.

$$-\frac{25r}{\sin^2{(5cr)}}$$

37.

$$-20m^2\cos\left(5m^2x\right)$$

38.

$$4p\left(4p\cot\left(4p^2r\right)\csc\left(4p^2r\right) + \sin\left(2pr\right)\right)$$

39.

$$4a\left(\frac{15y^2\cos(5a^2y^2)}{\sin^3(5a^2y^2)} - \frac{2}{\sin^2(a^2)}\right)$$

40.  $18s\sin\left(6s^2\right)$ 

$$-24e^{6w^2}w\left(6\log(e)\tan(w^2) + \frac{1}{\cos^2(w^2)}\right)$$

42. 
$$24e^{q^2s^2}qs\left(\frac{s\log{(e)}}{\tan{(4q^2s)}} - \frac{4}{\sin^2{(4q^2s)}}\right)$$

43. 
$$4e^{np^2}p\left(-n\log\left(e\right)\cos\left(p^2\right) + \sin\left(p^2\right)\right)$$

44. 
$$45e^{3rt}r\log\left(e\right)\cos^2\left(3r^2\right)$$

45. 
$$-32be^{10b^2}\sin{(4br)}$$

46. 
$$-40e^{5tw^2}tw\log(e)\cot^2(3t)$$

$$-8ae^x \sin(2a^2)$$

49. 
$$-240de^{9r}r\cot(5d^2r)\csc^2(5d^2r)$$

51. 
$$12n\left(-4e^{3nx}x\sin\left(2nx^2\right) + 3e^{3nx}\log\left(e\right)\cos\left(2nx^2\right) - \frac{x\sin\left(6nx^2\right)}{2}\right)$$

52. 
$$15e^{a}\left(-2am\sin\left(2a^{2}m\right)+\frac{\log\left(e\right)\cos\left(2a^{2}m\right)}{2}+\frac{\log\left(e\right)}{2}\right)$$

53. 
$$-8e^{10c^2}\sin(4v)$$

54. 
$$\frac{192e^{5p}np\cot\left(4np^2\right)}{\sin^2\left(4np^2\right)} - 60e^{5p}\log\left(e\right)\cot^2\left(4np^2\right) + 8\tan^2\left(4p\right) + 8$$

55. 
$$-48e^{4l}lt^2\cot\left(3l^2t^2\right)\csc\left(3l^2t^2\right) + 32e^{4l}\log\left(e\right)\csc\left(3l^2t^2\right) + \frac{16t^2\cot\left(4lt^2\right)}{\sin^2\left(4lt^2\right)}$$

56. 
$$60e^{10nt^2}n\tan(5n^2)\sec(5n^2) + 60e^{10nt^2}t^2\log(e)\sec(5n^2) - 20nt^2\sin(5n^2t^2)$$

57. 
$$e^{6d^2z} \left( -192dz \log(e) \tan(2d) - \frac{32}{\cos^2(2d)} \right)$$

58. 
$$-\frac{8e^{3n^2}n^2\tan(2n^2s)}{\cos^2(2n^2s)} + 30s\sin(6s^2)$$

59. 
$$8c^2 \tan (4c^2w) \sec (4c^2w) + 72e^{7w}w \sin (3w^2) - 84e^{7w} \log (e) \cos (3w^2)$$

60. 
$$6r\left(-2e^{5r^2}p\sin\left(4pr^2\right) + 5e^{5r^2}\log\left(e\right)\cos^2\left(2pr^2\right) + 3p\tan\left(3pr^2\right)\sec\left(3pr^2\right)\right)$$

61. 
$$f(x) = 5x^2 - 2x + 2$$

62. 
$$f(x) = 5x^2 - 2x + 2$$

63. 
$$f(x) = x^2 - 3x + 1$$

64. 
$$f(x) = -x^2 + 4x - 3$$

65. 
$$f(x) = 2x^2 + 5x + 4$$

66.

$$f(x) = -5x^2 - x + 5$$

67.

$$f(x) = -3x^2 + 5x - 3$$

68.

$$f(x) = 2x^2 - 4x + 5$$

69.

$$f(x) = -x^2 + x - 5$$

70.

$$f(x) = -3x^2 - 5x + 3$$

71.

$$f(x) = -3x^3 - 3x^2 + x - 2$$

72.

$$f(x) = 2x^3 - 2x^2 - 4x - 4$$

73.

$$f(x) = 5x^3 + 5x^2 - x - 2$$

74.

$$f(x) = -3x^3 - 2x^2 - 3x - 4$$

75.

$$f(x) = -2x^3 + 3x^2 - x - 5$$

76.

$$f(x) = 2x^3 + 4x^2 - 2x + 2$$

77.

$$f(x) = 3x^3 + 5x^2 + x + 5$$

78.

$$f(x) = -3x^3 - 3x^2 + 5x + 2$$

79.

$$f(x) = -3x^3 + 5x^2 + 3x - 4$$

80.

$$f(x) = 5x^3 - 2x^2 - 3x + 1$$

81.

$$f(x) = \frac{x^2}{2} - 5x - 2$$

and the stationary point is a minimum

82.

$$f(x) = \frac{x^2}{2} - 7x - 5$$

and the stationary point is a minimum

83.

$$f(x) = -\frac{5x^2}{2} + 20x - 2$$

and the stationary point is a maximum

84.

$$f(x) = 2x^2 - 36x - 2$$

and the stationary point is a minimum

85.

$$f(x) = -\frac{5x^2}{2} + 15x + 1$$

and the stationary point is a maximum

86.

$$f(x) = 2x^2 - 4x - 4$$

and the stationary point is a minimum

87.

$$f(x) = x^2 - 12x - 2$$

and the stationary point is a minimum

88.

$$f(x) = \frac{x^2}{2} - 4x + 5$$

and the stationary point is a minimum

89.

$$f(x) = x^2 - 2x - 3$$

and the stationary point is a minimum

90.

$$f(x) = -x^2 + 18x + 3$$

and the stationary point is a maximum

- 91. 7
- 92. -1
- 93. 8
- 94. -10
- 95. 4
- 96. 5
- 97. -7
- 98. 2
- 99. -8
- 100. -8
- 101. -5 and 1
- 102. -3 and -2
- 103. 4 and -2
- 104. 3 and -1
- 105. -1 and 3
- $106. \ \text{-}1 \ \text{and} \ 2$
- 107. 3 and 3
- 108. 1 and 2
- 109. 2 and -5
- 110. 1 and -3
- 111. 4 and -2
- 112. 3 and 1
- 113. -4 and -3
- 114. 1 and -1
- 115. 2 and -2
- 116. 4 and -3
- 117. -4 and -3
- 118. 2 and -3
- 119. 5 and -2
- 120. 1 and -4