VCE Mathematics Methods: Practice problems and solutions (Book 1)

Contents

1. Introduction

1.1 Motivation

My year 9 daughter kept asking me to write her new math problems, apart from those in the text book, so she can do more practice. I'd take a piece of paper and write problems, but it's a tedious process. So I thought: how about I **generate tons** of them?

1.2 Problem generation method

The problems and solutions have all been generated using the python programming language, and relied on a number of open source python packages, especially the SymPy package. Therefore, you can trust the accuracy of the solutions.

1.3 Structure

This document includes hundreds of mathematics problems that have been designed for year 11 students enrolled in VCE Mathematics Methods.

Most of the problems require the student to perform a single task, such as calculating the derivative of a function or finding the prime numbers within a range of integers. In addition, a number of questions have been designed based on the Mathematics Methods exam questions, which require the student to perform two or more tasks.

I hope you find them useful.

2. Numbers

2.1 Problems

Exercise 2.1 Simplify the expression

$$\sqrt[3]{152}$$

.

Exercise 2.2 What are the prime numbers in the range 123 and 135?
Exercise 2.3 What are the prime numbers in the range 625 and 645?
Exercise 2.4 What are the lowest common multiple (LCM) of 733 and 518?
Exercise 2.5 What are the highest common factor (HCF) of 865 and 750?
Exercise 2.6 What are the highest common factor (HCF) of 243 and 859?
Exercise 2.7 What are the prime numbers in the range 75 and 86?
Exercise 2.8 What are the prime numbers in the range 119 and 130?
Exercise 2.9 What are the prime numbers in the range 78 and 93?
Exercise 2.10 What are the prime numbers in the range 65 and 76?
Exercise 2.11 Simplify the expression

$$\sqrt{160}$$

.

Exercise 2.12 What are the lowest common multiple (LCM) of 167 and 761?
Exercise 2.13 What are the prime numbers in the range 357 and 374?
Exercise 2.14 Find the prime factor decomposition of 165.
Exercise 2.15 Find the prime factor decomposition of 131.
Exercise 2.16 What are the lowest common multiple (LCM) of 829 and 593?
Exercise 2.17 What are the prime numbers in the range 729 and 748?
Exercise 2.18 What are the prime numbers in the range 141 and 161?
Exercise 2.19 Simplify the expression

$$\sqrt{180}$$

.

Exercise 2.20 Find the prime factor decomposition of 148.

Exercise 2.21 Find the prime factor decomposition of 191.
Exercise 2.22 Simplify the expression

$$\sqrt[3]{136}$$

.
Exercise 2.23 Find the prime factor decomposition of 152.
Exercise 2.24 Simplify the expression

$$\sqrt{150}$$

.
Exercise 2.25 What are the prime numbers in the range 74 and 92?
Exercise 2.26 Find the prime factor decomposition of 32.
Exercise 2.27 Find the prime factor decomposition of 161.
Exercise 2.28 What are the lowest common multiple (LCM) of 52 and 707?
Exercise 2.29 Find the prime factor decomposition of 88.
Exercise 2.30 What are the highest common factor (HCF) of 658 and 344?
Exercise 2.31 Simplify the expression

$$\sqrt[3]{1296}$$

.
Exercise 2.32 Find the prime factor decomposition of 174.
Exercise 2.33 Simplify the expression

$$\sqrt[3]{2376}$$

.
Exercise 2.34 What are the highest common factor (HCF) of 403 and 56?
Exercise 2.35 What are the prime numbers in the range 416 and 429?
Exercise 2.36 What are the lowest common multiple (LCM) of 646 and 36?
Exercise 2.37 What are the highest common factor (HCF) of 956 and 740?
Exercise 2.38 Simplify the expression

$$\sqrt{1600}$$

.
Exercise 2.39 Find the prime factor decomposition of 83.
Exercise 2.40 What are the prime numbers in the range 97 and 112?
Exercise 2.41 Simplify the expression

$$\sqrt{576}$$

.
Exercise 2.42 Simplify the expression

$$\sqrt{931}$$

.
Exercise 2.43 Find the prime factor decomposition of 19.
Exercise 2.44 What are the prime numbers in the range 639 and 649?
Exercise 2.45 What are the prime numbers in the range 32 and 47?

Exercise 2.46 What are the prime numbers in the range 125 and 138?
Exercise 2.47 Simplify the expression

$$\sqrt[3]{243}$$

.
Exercise 2.48 What are the prime numbers in the range 196 and 215?
Exercise 2.49 What are the lowest common multiple (LCM) of 803 and 812?
Exercise 2.50 Simplify the expression

$$\sqrt{490}$$

.
Exercise 2.51 Find the prime factor decomposition of 190.
Exercise 2.52 What are the prime numbers in the range 603 and 617?
Exercise 2.53 What are the lowest common multiple (LCM) of 524 and 599?
Exercise 2.54 What are the prime numbers in the range 136 and 146?
Exercise 2.55 What are the prime numbers in the range 84 and 95?
Exercise 2.56 What are the highest common factor (HCF) of 757 and 361?
Exercise 2.57 Find the prime factor decomposition of 149.
Exercise 2.58 What are the prime numbers in the range 288 and 301?
Exercise 2.59 What are the lowest common multiple (LCM) of 210 and 875?
Exercise 2.60 What are the highest common factor (HCF) of 331 and 424?
Exercise 2.61 What are the highest common factor (HCF) of 580 and 44?
Exercise 2.62 What are the highest common factor (HCF) of 748 and 126?
Exercise 2.63 Find the prime factor decomposition of 124.
Exercise 2.64 What are the lowest common multiple (LCM) of 416 and 650?
Exercise 2.65 Find the prime factor decomposition of 191.
Exercise 2.66 What are the prime numbers in the range 153 and 170?
Exercise 2.67 What are the prime numbers in the range 90 and 101?
Exercise 2.68 What are the lowest common multiple (LCM) of 778 and 804?
Exercise 2.69 What are the highest common factor (HCF) of 466 and 984?
Exercise 2.70 What are the prime numbers in the range 417 and 427?
Exercise 2.71 What are the highest common factor (HCF) of 960 and 226?
Exercise 2.72 Simplify the expression

$$\sqrt[3]{1512}$$

.
Exercise 2.73 Find the prime factor decomposition of 30.
Exercise 2.74 Simplify the expression

$$\sqrt[3]{13122}$$

.
Exercise 2.75 What are the highest common factor (HCF) of 486 and 794?
Exercise 2.76 What are the highest common factor (HCF) of 881 and 294?
Exercise 2.77 What are the prime numbers in the range 443 and 453?
Exercise 2.78 What are the prime numbers in the range 23 and 33?
Exercise 2.79 What are the highest common factor (HCF) of 220 and 756?

Exercise 2.80 Simplify the expression

$$\sqrt{240}$$

.

Exercise 2.81 Find the prime factor decomposition of 61.
Exercise 2.82 What are the prime numbers in the range 68 and 81?
Exercise 2.83 What are the highest common factor (HCF) of 626 and 9?
Exercise 2.84 What are the highest common factor (HCF) of 734 and 63?
Exercise 2.85 Find the prime factor decomposition of 200.
Exercise 2.86 What are the highest common factor (HCF) of 107 and 491?
Exercise 2.87 What are the lowest common multiple (LCM) of 415 and 788?
Exercise 2.88 Simplify the expression

$$\sqrt[3]{2560}$$

.

Exercise 2.89 What are the prime numbers in the range 30 and 50?
Exercise 2.90 What are the prime numbers in the range 164 and 180?
Exercise 2.91 What are the highest common factor (HCF) of 957 and 721?
Exercise 2.92 Find the prime factor decomposition of 67.
Exercise 2.93 What are the prime numbers in the range 180 and 190?
Exercise 2.94 What are the prime numbers in the range 14 and 34?
Exercise 2.95 Find the prime factor decomposition of 34.
Exercise 2.96 Find the prime factor decomposition of 69.
Exercise 2.97 What are the highest common factor (HCF) of 329 and 594?
Exercise 2.98 What are the lowest common multiple (LCM) of 118 and 825?
Exercise 2.99 What are the prime numbers in the range 46 and 63?
Exercise 2.100 Find the prime factor decomposition of 155.

2.2 Solutions

1. $$2 \cdot \sqrt[3]{19}$$
2. 127, 131
3. 631, 641, 643
4. 379694
5. 5
6. 1
7. 79, 83
8. 127
9. 79, 83, 89
10. 67, 71, 73
11. $$4\sqrt{10}$$
12. 127087
13. 359, 367, 373

14.

$$11^1 \cdot 3^1 \cdot 5^1$$

15.

$$131^1$$

16. 491597
17. 733, 739, 743
18. 149, 151, 157
19.

$$6\sqrt{5}$$

20.

$$2^2 \cdot 37^1$$

21.

$$191^1$$

22.

$$2 \cdot \sqrt[3]{17}$$

23.

$$19^1 \cdot 2^3$$

24.

$$5\sqrt{6}$$

25. 79, 83, 89
26.

$$2^5$$

27.

$$23^1 \cdot 7^1$$

28. 36764
29.

$$11^1 \cdot 2^3$$

30. 2
31.

$$6 \cdot \sqrt[3]{6}$$

32.

$$2^1 \cdot 29^1 \cdot 3^1$$

33.

$$6 \cdot \sqrt[3]{11}$$

34. 1
35. 419, 421
36. 11628
37. 4
38.

$$40$$

39.

$$83^{1}$$

40. 97, 101, 103, 107, 109
41.

$$24$$

42.

$$7\sqrt{19}$$

43.

$$19^{1}$$

44. 641, 643, 647
45. 37, 41, 43
46. 127, 131, 137
47.

$$3 \cdot 3^{\frac{2}{3}}$$

48. 197, 199, 211
49. 652036
50.

$$7\sqrt{10}$$

51.

$$19^{1} \cdot 2^{1} \cdot 5^{1}$$

52. 607, 613
53. 313876
54. 137, 139
55. 89
56. 1
57.

$$149^{1}$$

58. 293
59. 5250
60. 1
61. 4
62. 2
63.

$$2^2 \cdot 31^1$$

64. 10400
65.

$$191^1$$

66. 157, 163, 167
67. 97
68. 312756
69. 2
70. 419, 421
71. 2
72.

$$6 \cdot \sqrt[3]{7}$$

73.

$$2^1 \cdot 3^1 \cdot 5^1$$

74.

$$9 \cdot \sqrt[3]{18}$$

75. 2
76. 1
77. 443, 449
78. 23, 29, 31
79. 4
80.

$$4\sqrt{15}$$

81.

$$61^1$$

82. 71, 73, 79
83. 1
84. 1
85.

$$2^3 \cdot 5^2$$

86. 1
87. 327020

88.

$$8 \cdot \sqrt[3]{5}$$

89. 31, 37, 41, 43, 47
90. 167, 173, 179
91. 1
92.

$$67^1$$

93. 181
94. 17, 19, 23, 29, 31
95.

$$17^1 \cdot 2^1$$

96.

$$23^1 \cdot 3^1$$

97. 1
98. 97350
99. 47, 53, 59, 61
100.

$$31^1 \cdot 5^1$$

3. Algebra

3.1 Problems

Exercise 3.1 Simplify

$$\frac{1}{12} - \frac{4}{5-10p}$$

Exercise 3.2 Simplify

$$\frac{9r^3s^3 + 8r^3s^2 - 27s^4 - 24s^3}{-9s^3 - 8s^2}$$

Exercise 3.3 Evaluate the first 4 terms of the following recurrance sequence:

$$M(u) = u\left(6u - M(u-1)\right)$$

,

where $M(0) = 3$.

Exercise 3.4 Simplify

$$\frac{\frac{9a+8}{9a-9} + \frac{10}{5-4a}}{-4a-5}$$

Exercise 3.5 Expand

$$\left(6c^3 + 6d\right)\left(-5a^4b^4d^3 + 2ac^3d^4\right)^2$$

Exercise 3.6 Expand and simplify

$$\left(-9m^4 + 6n^3\right)^2\left(-4m^2n^4 + 9n^4\right)$$

Exercise 3.7 Complete the square for the following expression and then solve the equation:

$$-15x^2 - 51x$$

Exercise 3.8 Evaluate the first 4 terms of the following recurrance sequence:

$$M(t) = -6t^2 - \frac{M(t-1)}{9}$$

,

where $M(0) = 6$.

Exercise 3.9 Expand

$$\left(-10n^2 - \frac{1}{7m^2}\right)^2$$

Exercise 3.10 Solve the following simultaneous equations for x and y:

$$-9a + 4x + 4y = 0$$

$$12ax - 12ay + 8a = 0$$

Exercise 3.11 Expand and simplify

$$\left(7p^2r - 4r^3\right)\left(p^3q^3r^2 + 7pr^2\right)^2$$

Exercise 3.12 If $a = -9m^2 - 7n^2$, show that

$$(a-x)^2 + (a+x)^2 = 162m^4 + 252m^2n^2 + 98n^4 + 2x^2$$

Exercise 3.13 Solve the equation in x:

$$4ax + 6a - 2x = 0$$

Exercise 3.14 Simplify

$$\frac{\frac{-8b-8}{2-7b} + \frac{10b+6}{7b}}{-\frac{3}{-10b-9} - \frac{8b+4}{5b}}$$

Exercise 3.15 Simplify

$$\frac{10-5s}{-4s-4} + \frac{6}{2-5s}$$

Exercise 3.16 Solve the following simultaneous equations for x and y:

$$6x - 5y + 1 = 0$$

$$-8x + 7y + 4 = 0$$

Exercise 3.17 Expand

$$\left(10u^2w - 8\right)\left(-8u^4v^3 + uv^4\right)\left(2v^3w^3 - 2v^3w^2\right)$$

Exercise 3.18 Simplify

$$\frac{7s^4}{-42r^4s^6 - 49r^2s^8 + 42r^2s^6 - 28r^2s^4}$$

Exercise 3.19 Evaluate the first 4 terms of the following recurrance sequence:

$$G(s) = -8s - 3G(s-1)$$

,

where $G(0) = 8$.

Exercise 3.20 Expand

$$\left(3r + \frac{1}{2rs^2}\right)^2$$

Exercise 3.21 Simplify

$$\frac{-s^3 - 5s^2}{7s^2 - 9s} + \frac{-6s^3 + s}{-2s^2 + 4s}$$

Exercise 3.22 Expand

$$\left(2p^4 + 7q^4\right)\left(4p^4q^2 + 3p^4 + 4\right)$$

Exercise 3.23 Solve the following inequality

$$4c + 5 > 10$$

Exercise 3.24 Factorize the polynomial

$$30v^2 + 64v - 14$$

Exercise 3.25 Simplify

$$\frac{81x^2y^4z^5 + 54xy^4z^3}{9y^4z}$$

Exercise 3.26 Simplify

$$\frac{-\frac{-3s-8}{18s} - \frac{1}{11s}}{-\frac{12s}{11} + \frac{-7s-9}{4s-3}}$$

Exercise 3.27 Solve the following inequality

$$3t + 8 \geq -9$$

Exercise 3.28 Solve the equation in x:

$$-ax - 16 = 0$$

Exercise 3.29 If $a = -10b^4c^4 + 3$, show that

$$-(a-x)^3 + (a+x)^3 = 200b^8c^8 - 120b^4c^4 + 2x^2 + 18$$

Exercise 3.30 Evaluate the first 4 terms of the following recurrance sequence:

$$q(n) = -n\left(q(n-1) + 9\right)$$

,

where $q(0) = 3$.

Exercise 3.31 Simplify

$$-\frac{7-8b^3}{b^3-2}+\frac{-8b^3-6b}{6b^2+1}$$

Exercise 3.32 Solve the following simultaneous equations for x and y:

$$7x-3y+7=0$$

$$5x+9y-4=0$$

Exercise 3.33 Evaluate the first 4 terms of the following recurrance sequence:

$$M(r)=2r^2+2M(r-1)$$

,
where $M(0)=0$.

Exercise 3.34 Solve the equation in x:

$$-13ax+7x=0$$

Exercise 3.35 Evaluate the first 4 terms of the following recurrance sequence:

$$F(l)=8l-\frac{2}{F(l-1)}$$

, where $F(0)=10$.

Exercise 3.36 Evaluate the first 4 terms of the following recurrance sequence:

$$t(m)=-m+2t(m-1)$$

,
where $t(0)=5$.

Exercise 3.37 Expand

$$\left(-3x^2z^3-y\right)\left(-8xy^2z^3+3\right)^2$$

Exercise 3.38 Evaluate the first 4 terms of the following recurrance sequence:

$$Y(s)=-7s-Y(s-1)$$

,
where $Y(0)=5$.

Exercise 3.39 Expand and simplify

$$\left(-7p^4r+7r^4\right)\left(5pqr^4-9r^2\right)^2$$

Exercise 3.40

Exercise 3.41 Expand

$$\left(10m+\frac{1}{7}\right)^2$$

Exercise 3.42 Complete the square for the following expression and then solve the equation:

$$12v^2 - 26v$$

Exercise 3.43 Solve the equation in x:

$$7ax + 7a + 8 = 0$$

Exercise 3.44 Expand

$$\left(4s^4 - 9s^3\right)\left(3t^3 - t^2\right)^2$$

Exercise 3.45 Evaluate the first 4 terms of the following recurrance sequence:

$$T(v) = 2v - \frac{8v}{T(v-1)}$$

, where $T(0) = 6$.

Exercise 3.46 Evaluate the first 4 terms of the following recurrance sequence:

$$Y(v) = -2v + 6Y(v-1)$$

,
where $Y(0) = 0$.

Exercise 3.47 Evaluate the first 4 terms of the following recurrance sequence:

$$P(m) = 9m - \frac{m}{P(m-1)}$$

, where $P(0) = 5$.

Exercise 3.48 Complete the square for the following expression and then solve the equation:

$$-15c^2 + 25c$$

Exercise 3.49 Simplify

$$\frac{\frac{8-2w}{6w+5} - \frac{12}{7}}{\frac{8w}{7w-3} - \frac{17}{5w+3}}$$

Exercise 3.50 Solve the following inequality for u

$$10u + 3v \leq 4 - 8u$$

Exercise 3.51 Factorize the polynomial

$$3y^2 - 8y - 16$$

Exercise 3.52 If $a = 9b - 8c^4$, show that

$$(a-x)^2 + (a+x)^2 = 162b^2 - 288bc^4 + 128c^8 + 2x^2$$

Exercise 3.53 Solve the following inequality

$$2w - 8 < 5$$

Exercise 3.54

Exercise 3.55 Simplify

$$\frac{(-1)\,12bc^2}{6c^2}$$

Exercise 3.56 If $a = 5b^2d^3 - 8d^4$, show that

$$(a-x)^2 + (a+x)^2 = 50b^4d^6 - 160b^2d^7 + 128d^8 + 2x^2$$

Exercise 3.57 Expand

$$\left(8r^4s^4t - 8s\right)\left(8r^4t^3 + 10s^4 + 6\right)$$

Exercise 3.58 Factorize the polynomial

$$-28p^2 - 8p + 20$$

Exercise 3.59 Simplify

$$\frac{\frac{7c}{13} + \frac{5}{13} + \frac{-c-6}{2-c}}{-\frac{c+3}{4c} + \frac{6c-10}{16c}}$$

Exercise 3.60 Simplify

$$\frac{(-1)\,5r^3}{-5r^5s - 15r^3s^2 - 35r^3s}$$

Exercise 3.61 Expand

$$(a-10)\left(7b^3 - 2b\right)\left(3b^2d^4 + 10\right)$$

Exercise 3.62 Solve the following inequality

$$8b - 8 \leq -1$$

Exercise 3.63 Expand

$$\left(-3r^4t - 3rs^2\right)\left(3r^4t + 2st^2\right)^2$$

Exercise 3.64 Expand

$$\left(-5a^3b^2c^3 - \frac{1}{6a^4}\right)^2$$

Exercise 3.65 If $a = -2m^2n^2 + 9m^2$, show that

$$(a-x)^2 + (a+x)^2 = 8m^4n^4 - 72m^4n^2 + 162m^4 + 2x^2$$

Exercise 3.66 Solve the following inequality for q

$$3q + 10r > 9q + 5r^2$$

Exercise 3.67 Solve the following simultaneous equations for x and y:

$$7ax + 10a + 50y = 0$$

$$-3ax + 4a + 45y = 0$$

Exercise 3.68 Expand

$$\left(-6m-\frac{1}{2}\right)^2$$

Exercise 3.69 Simplify

$$v-\frac{-5v-4}{6v+9}-\frac{5}{9}$$

Exercise 3.70 Evaluate the first 6 terms of the following recurrance sequence:

$$R(a)=6a-6R(a-2)+2R(a-1)$$

,

where $R(0)=8$, $R(1)=2$.

Exercise 3.71 Factorize the polynomial

$$-16a^2-52a-40$$

Exercise 3.72 Complete the square for the following expression and then solve the equation:

$$49w^2+35w$$

Exercise 3.73 Expand

$$\left(-3y^3z+\frac{1}{9z^2}\right)^2$$

Exercise 3.74 If $a=4q^4-3q^3$, show that

$$(a-x)^2+(a+x)^2=32q^8-48q^7+18q^6+2x^2$$

Exercise 3.75 Factorize the polynomial

$$2z^2-2z-40$$

Exercise 3.76

Exercise 3.77 Simplify

$$\frac{-15u^6v^4w^2+25u^6v^2w^2-15u^2vw^5+10u^2w^5}{5u^2w^2}$$

Exercise 3.78 Expand

$$\left(2a^2b^2c^3d+5b^2d\right)\left(-2a^2d^2+5d^2+4\right)$$

Exercise 3.79 Simplify

$$\frac{(-1)\,8a^3}{-64a^6c^3+48a^6c+64a^5c^3-32a^4}$$

Exercise 3.80 Simplify

$$-\frac{10-6n}{-2n-4}-\frac{6n+10}{6n}$$

Exercise 3.81

Exercise 3.82 If $a = 2m^3 + 3n^4$, show that

$$(a-x)^2 + (a+x)^2 = 8m^6 + 24m^3n^4 + 18n^8 + 2x^2$$

Exercise 3.83 Evaluate the first 4 terms of the following recurrance sequence:

$$F(z) = -z - \frac{6z}{F(z-1)}$$

, where $F(0) = 4$.

Exercise 3.84 Simplify

$$\frac{-54a^4c^2 - 6ac^2}{-9a^3 - 1}$$

Exercise 3.85 Simplify

$$-\frac{3u+6}{-3u-2} + \frac{6u-4}{5u}$$

Exercise 3.86 Expand

$$\left(5p^3 - q^3\right)\left(7p^4q^2r^4 - 10r^2\right)^2$$

Exercise 3.87 Simplify

$$\frac{-35c^5d^4 + 40c^3d^3}{7c^3d^4 - 8cd^3}$$

Exercise 3.88 Solve the following simultaneous equations for x and y:

$$-10x + 8y - 4 = 0$$

$$8x + 2y + 4 = 0$$

Exercise 3.89 Simplify

$$\frac{\frac{6l+2}{-5l-10} - \frac{-7l-3}{6l}}{7 + \frac{3l+3}{8l}}$$

Exercise 3.90 Simplify

$$\frac{12p^4q^6 + 15p^3q^8 - 40p^2q^6 - 24p^2q^4 - 35pq^8 + 28q^6 + 56q^4}{-3p^2q^4 + 7q^4}$$

Exercise 3.91 Evaluate the first 4 terms of the following recurrance sequence:

$$X(q) = q\left(6q - X(q-1)\right)$$

,

where $X(0) = 6$.

Exercise 3.92 Evaluate the first 4 terms of the following recurrance sequence:

$$G(a) = \frac{a\left(-20a + G(a-1)\right)}{10}$$

,

where $G(0) = 9$.

Exercise 3.93 Simplify

$$\frac{(-1)\,8m^3n^4}{-16m^5n^4 - 16m^3n^8}$$

Exercise 3.94 Expand

$$\left(-10p + 6q^4\right)\left(5p^4 - pq^2\right)^2$$

Exercise 3.95 If $a = 1 - 7r^3$, show that

$$(a-x)^2 + (a+x)^2 = 98r^6 - 28r^3 + 2x^2 + 2$$

Exercise 3.96 Evaluate the first 4 terms of the following recurrance sequence:

$$T(d) = 8d - 4T(d-1)$$

,

where $T(0) = 2$.

Exercise 3.97 Solve the following simultaneous equations for x and y:

$$-49ax + 7y + 9 = 0$$

$$a - 12x + 4y = 0$$

Exercise 3.98 Evaluate the first 4 terms of the following recurrance sequence:

$$T(n) = -9n + T(n-1)$$

,

where $T(0) = 0$.

Exercise 3.99 Expand

$$\left(4p^2 + \frac{1}{8q^4}\right)^2$$

Exercise 3.100 Solve the equation in x:

$$-a + x + 14 = 0$$

3.2 Solutions

1. $$\frac{10p+43}{120p-60}$$

2. $$-r^3+3s$$

3. 3, 3, 18, 0, 96

4. $$\frac{-36a^2+103a-50}{144a^3-144a^2-225a+225}$$

5. $$150a^8b^8c^3d^6+150a^8b^8d^7-120a^5b^4c^6d^7-120a^5b^4c^3d^8+24a^2c^9d^8+24a^2c^6d^9$$

6. $$n^4\left(-324m^{10}+729m^8+432m^6n^3-972m^4n^3-144m^2n^6+324n^6\right)$$

7. $$\left[-3,\ -\frac{2}{5}\right]$$

8. 6, -6.6667, -23.2593, -51.4156, -90.2872

9. $$100n^4+\frac{20n^2}{7m^2}+\frac{1}{49m^4}$$

10. $$\left\{x:\frac{9a}{8}-\frac{1}{3},\ y:\frac{9a}{8}+\frac{1}{3}\right\}$$

11. $$p^2r^5\cdot\left(7p^6q^6-4p^4q^6r^2+98p^4q^3-56p^2q^3r^2+343p^2-196r^2\right)$$

12. Provided in problem statement

13. $$\left\{-\frac{3a}{2a-1}\right\}$$

14. $$\frac{-6300b^3-9570b^2-2910b+540}{3920b^3+3633b^2+406b-504}$$

15. $$\frac{25s^2-84s-4}{20s^2+12s-8}$$

16.

$$\left\{x : -\frac{27}{2},\ y : -16\right\}$$

17.

$$-160u^6v^6w^4+160u^6v^6w^3+128u^4v^6w^3-128u^4v^6w^2+20u^3v^7w^4-20u^3v^7w^3-16uv^7w^3+16uv^7w^2$$

18.

$$-\frac{1}{r^2\cdot(6r^2s^2+7s^4-6s^2+4)}$$

19. 8, -32, 80, -264, 760

20.

$$9r^2+\frac{3}{s^2}+\frac{1}{4r^2s^4}$$

21.

$$\frac{40s^3-60s^2+13s+9}{14s^2-46s+36}$$

22.

$$8p^8q^2+6p^8+28p^4q^6+21p^4q^4+8p^4+28q^4$$

23.

$$\frac{5}{4}<c$$

24.

$$-3v-7$$

and

$$2-10v$$

25.

$$3xz^2\cdot\left(3xz^2+2\right)$$

26.

$$\frac{-132s^2-181s+210}{864s^3+738s^2+1782s}$$

27.

$$-\frac{17}{3}\leq t$$

28.

$$\left\{-\frac{16}{a}\right\}$$

29. Provided in problem statement
30. 3, -12, 6, -45, 144
31. $$\frac{-8b^6+48b^5-6b^4+24b^3-42b^2+12b-7}{6b^5+b^3-12b^2-2}$$
32. $$\left\{x:-\frac{17}{26},\ y:\frac{21}{26}\right\}$$
33. 0, 2, 12, 42, 116
34. $$\{0\}$$
35. 10, 7.8000, 15.7436, 23.8730, 31.9162
36. 5, 9, 16, 29, 54
37. $$-192x^4y^4z^9+144x^3y^2z^6-64x^2y^5z^6-27x^2z^3+48xy^3z^3-9y$$
38. 5, -12, -2, -19, -9
39. $$r^5\left(-175p^6q^2r^4+630p^5qr^2-567p^4+175p^2q^2r^7-630pqr^5+567r^3\right)$$
40. $$\frac{180bz^2-570bz+6b-210z^2+590z+53}{90bz-105z}$$
41. $$100m^2+\frac{20m}{7}+\frac{1}{49}$$
42. $$\left[\frac{2}{3},\ \frac{3}{2}\right]$$
43. $$\left\{-\frac{7a+8}{7a}\right\}$$
44. $$36s^4t^6-24s^4t^5+4s^4t^4-81s^3t^6+54s^3t^5-9s^3t^4$$
45. 6, 0.6667, -20, 7.2000, 3.5556
46. 0, -2, -16, -102, -620
47. 5, 8.8000, 17.7727, 26.8312, 35.8509
48. $$\left[-1,\ \frac{8}{3}\right]$$

49.

$$\frac{-3010w^3 - 656w^2 + 750w + 36}{1680w^3 - 2590w^2 - 1183w + 1785}$$

50.

$$u \leq \frac{2}{9} - \frac{v}{6}$$

51.

$$-3y - 4$$

and

$$4 - y$$

52. Provided in problem statement

53.

$$w < \frac{13}{2}$$

54.

$$\frac{-450n^2 + 10np - 105n + 45p^2 - 453p + 477}{50np - 450n - 15p + 135}$$

55.

$$-2b$$

56. Provided in problem statement

57.

$$64r^8s^4t^4 + 80r^4s^8t + 48r^4s^4t - 64r^4st^3 - 80s^5 - 48s$$

58.

$$4p + 4$$

and

$$5 - 7p$$

59.

$$\frac{56c^3 + 32c^2 + 544c}{13c^2 - 169c + 286}$$

60.

$$\frac{1}{s(r^2 + 3s + 7)}$$

61.

$$21ab^5d^4 - 6ab^3d^4 + 70ab^3 - 20ab - 210b^5d^4 + 60b^3d^4 - 700b^3 + 200b$$

62.

$$b \leq \frac{7}{8}$$

63.

$$-27r^{12}t^3 - 27r^9s^2t^2 - 36r^8st^4 - 36r^5s^3t^3 - 12r^4s^2t^5 - 12rs^4t^4$$

64.

$$25a^6b^4c^6 + \frac{5b^2c^3}{3a} + \frac{1}{36a^8}$$

65. Provided in problem statement
66.

$$q < -\frac{5r^2}{6} + \frac{5r}{3}$$

67.

$$\left\{x : -\frac{50}{93},\ y : -\frac{58a}{465}\right\}$$

68.

$$36m^2 + 6m + \frac{1}{4}$$

69.

$$\frac{18v^2 + 32v - 3}{18v + 27}$$

70. 8, 2, -32, -58, 100, 578
71.

$$2a + 4$$

and

$$-8a - 10$$

72.

$$\left[-\frac{4}{7},\ -\frac{1}{7}\right]$$

73.

$$9y^6z^2 - \frac{2y^3}{3z} + \frac{1}{81z^4}$$

74. Provided in problem statement
75.

$$z + 4$$

and

$$2z - 10$$

76.

$$\frac{200n^2y+276n^2-175ny^2+121ny-210y^2}{-80n^2+70ny}$$

77.

$$-3u^4v^4+5u^4v^2-3vw^3+2w^3$$

78.

$$-4a^4b^2c^3d^3+10a^2b^2c^3d^3+8a^2b^2c^3d-10a^2b^2d^3+25b^2d^3+20b^2d$$

79.

$$\frac{1}{2a\left(4a^2c^3-3a^2c-4ac^3+2\right)}$$

80.

$$\frac{-12n^2+4n-10}{3n^2+6n}$$

81.

$$\frac{-200r^3u-380r^2u^2+155r^2u+755r^2-68ru^2+263ru+285r+90u}{400r^3u+350r^3+200r^2u^2+415r^2u+210r^2+120ru^2+105ru}$$

82. Provided in problem statement
83. 4, -2.5000, 2.8000, -9.4286, -1.4545
84.

$$6ac^2$$

85.

$$\frac{33u^2+30u-8}{15u^2+10u}$$

86.

$$245p^{11}q^4r^8-49p^8q^7r^8-700p^7q^2r^6+140p^4q^5r^6+500p^3r^4-100q^3r^4$$

87.

$$-5c^2$$

88.

$$\left\{x:-\frac{10}{21},\ y:-\frac{2}{21}\right\}$$

89.

$$\frac{-4l^2+292l+120}{885l^2+1815l+90}$$

90.

$$-4p^2q^2-5pq^4+4q^2+8$$

91. 6, 0, 24, -18, 168
92. 9, -1.1000, -8.2200, -20.4660, -40.1864
93. $$\frac{1}{2\left(m^2+n^4\right)}$$
94. $$-250p^9+150p^8q^4+100p^6q^2-60p^5q^6-10p^3q^4+6p^2q^8$$
95. Provided in problem statement
96. 2, 0, 16, -40, 192
97. $$\left\{x:\frac{36-7a}{196a-84},\ y:\frac{108-49a^2}{196a-84}\right\}$$
98. 0, -9, -27, -54, -90
99. $$16p^4+\frac{p^2}{q^4}+\frac{1}{64q^8}$$
100. $$\{a-14\}$$

4. Functions

4.1 Problems

Exercise 4.1 If $f(x)$ is defined as

$$f(x) = \frac{7x+10}{-x-3}$$

find $f^{-1}(x)$.

Exercise 4.2 If $f(x)$ is defined as

$$f(x) = 3x^2 - 8x + 2$$

and $g(x)$ is defined as

$$g(x) = -6f(10x^2 + 7x + 7)x^2 + 8x$$

simplify $g(x)$ in terms of x.

Exercise 4.3 If $f(x)$ is defined as

$$f(x) = -5x^2 + 6x + 4$$

and $g(x)$ is defined as

$$g(x) = f(6x^2 + 9x - 10)x^2 + 5x$$

simplify $g(x)$ in terms of x.

Exercise 4.4 If $f(x)$ is defined as

$$f(x) = -4x - 9$$

and $g(x)$ is defined as

$$g(x) = 3x^2 + 2x - 3$$

simplify $g(f(x))$.

Exercise 4.5 If $f(x)$ is defined as

$$f(x) = \frac{7x-4}{8x-9}$$

find $f^{-1}(x)$.

Exercise 4.6 For the following two functions,

$$f(x) = -4ax^2 - 10ax + 8$$

and

$$g(x) = a\left(-x^2 + 5x - 1\right)$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.7 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -10ax^2 + 7a - 7x$$

and

$$g(x) = -3ax + 9$$

Exercise 4.8 If $f(x)$ is defined as

$$f(x) = 9ax + 8x^2$$

and $g(x)$ is defined as

$$g(x) = 9x - 2$$

solve the equation

$$g(f(x)) = 0$$

Exercise 4.9 If $f(x)$ is defined as

$$f(x) = \frac{2-3x}{3x+8}$$

what is the domain and range of $f(x)$?

Exercise 4.10 If $f(x)$ is defined as

$$f(x) = -9x^2 - 6x + 8$$

and $g(x)$ is defined as

$$g(x) = -3f(-7x^2 + x + 5)x^2 - 9x$$

simplify $g(x)$ in terms of x.

Exercise 4.11 For the following two functions,

$$f(x) = -4ax^2 - 7ax - 7$$

and

$$g(x) = 6ax^2 + 6ax - 9$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.12 If $f(x)$ is defined as

$$f(x) = 3ax - 9x^2$$

and $g(x)$ is defined as

$$g(x) = 7x + 6$$

solve the equation

$$g(f(x)) = 0$$

Exercise 4.13 If $f(x)$ is defined as

$$f(x) = \frac{-2x - 9}{4x + 6}$$

what is the domain and range of $f(x)$?

Exercise 4.14 If $f(x)$ is defined as

$$f(x) = 7x^2 + 2x + 5$$

and $g(x)$ is defined as

$$g(x) = -9f(-x^2 - x + 6)x^2 - 3x$$

simplify $g(x)$ in terms of x.

Exercise 4.15 If $f(x)$ is defined as

$$f(x) = \frac{2x + 5}{7x + 10}$$

find $f^{-1}(x)$.

Exercise 4.16 If $f(x)$ is defined as

$$f(x) = -7ax - 9x^2$$

and $g(x)$ is defined as

$$g(x) = 8 - 7x$$

solve the equation

$$g(f(x)) = 0$$

Exercise 4.17 If $f(x)$ is defined as

$$f(x) = \frac{3 - 2x}{6 - 6x}$$

what is the domain and range of $f(x)$?

Exercise 4.18 For the following two functions,

$$f(x) = ax^2 - 5ax - 7$$

and

$$g(x) = 10ax^2 + 3ax + 10$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.19 If $f(x)$ is defined as

$$f(x) = \frac{-10ax - 4a}{8a + 5x}$$

find the value of a if $f^{-1}(4) = 9$.

Exercise 4.20 If $f(x)$ is defined as

$$f(x) = -6x^2 - 3x - 5$$

and $g(x)$ is defined as

$$g(x) = -8f(7x^2 + 2x - 3)x^2 - 8x$$

simplify $g(x)$ in terms of x.

Exercise 4.21 If $f(x)$ is defined as

$$f(x) = \frac{6 - 3x}{10x - 7}$$

what is the domain and range of $f(x)$?

Exercise 4.22 If $f(x)$ is defined as

$$f(x) = -x - 9$$

and $g(x)$ is defined as

$$g(x) = 8f(2 - 2x)x^2 - 3x$$

simplify $g(x)$ in terms of x.

Exercise 4.23 For the following two functions,

$$f(x) = 4a\left(x^2 - x - 1\right)$$

and

$$g(x) = -9ax^2 - 2ax + 8$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.24 If $f(x)$ is defined as

$$f(x) = \frac{2x - 1}{8x + 3}$$

what is the domain and range of $f(x)$?

Exercise 4.25 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -10a - 4x$$

and

$$g(x) = -a - 9x$$

Exercise 4.26 If $f(x)$ is defined as

$$f(x) = 8x + 6$$

and $g(x)$ is defined as

$$g(x) = 4x^2 + 5x - 1$$

simplify $g(f(x))$.

Exercise 4.27 If $f(x)$ is defined as

$$f(x) = \frac{4-9x}{9x-10}$$

what is the domain and range of $f(x)$?

Exercise 4.28 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -5a - 8x$$

and

$$g(x) = 3a + 6x$$

Exercise 4.29 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -5ax^2 + 7a + x$$

and

$$g(x) = 3a\,(x-3)$$

Exercise 4.30 If $f(x)$ is defined as

$$f(x) = -4x^2 + x + 2$$

and $g(x)$ is defined as

$$g(x) = 4f(-3x^2 - 3x - 3)x^2 - 8x$$

simplify $g(x)$ in terms of x.

Exercise 4.31 If $f(x)$ is defined as

$$f(x) = \frac{2-x}{6x+4}$$

what is the domain and range of $f(x)$?

Exercise 4.32 If $f(x)$ is defined as

$$f(x) = \frac{7-10x}{10-10x}$$

find $f^{-1}(x)$.

Exercise 4.33 For the following two functions,

$$f(x) = 2a\left(-5x^2 + 3x + 5\right)$$

and

$$g(x) = 2ax^2 + ax - 5$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.34 If $f(x)$ is defined as

$$f(x) = \frac{-7x-4}{-7x-3}$$

what is the domain and range of $f(x)$?

Exercise 4.35 If $f(x)$ is defined as

$$f(x) = \frac{3-7x}{10x+7}$$

find $f^{-1}(x)$.

Exercise 4.36 For the following two functions,

$$f(x) = a\left(-4x^2+8x+3\right)$$

and

$$g(x) = 3ax-4$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.37 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -3ax^2-5a-5x$$

and

$$g(x) = -9ax+3$$

Exercise 4.38 If $f(x)$ is defined as

$$f(x) = \frac{6x+1}{5x-2}$$

what is the domain and range of $f(x)$?

Exercise 4.39 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = 8-8x$$

and

$$g(x) = 6a+8x$$

Exercise 4.40 If $f(x)$ is defined as

$$f(x) = 5x^2+4x-7$$

and $g(x)$ is defined as

$$g(x) = -9f(8x^2+8x+8)x^2-9x$$

simplify $g(x)$ in terms of x.

Exercise 4.41 If $f(x)$ is defined as

$$f(x) = -10x^2-6x+10$$

and $g(x)$ is defined as

$$g(x) = 6f(4x^2-6x-7)x^2-10x$$

simplify $g(x)$ in terms of x.

Exercise 4.42 If $f(x)$ is defined as

$$f(x) = 5x - 10$$

and $g(x)$ is defined as

$$g(x) = -5x^2 - 10x - 10$$

simplify $g(f(x))$.

Exercise 4.43 For the following two functions,

$$f(x) = 7ax^2 + 10ax - 8$$

and

$$g(x) = -4ax - 4$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.44 If $f(x)$ is defined as

$$f(x) = 6x - 10$$

and $g(x)$ is defined as

$$g(x) = -2f(5 - 2x)x^2 - 10x$$

simplify $g(x)$ in terms of x.

Exercise 4.45 For the following two functions,

$$f(x) = a\left(6x^2 - 9x - 5\right)$$

and

$$g(x) = 7ax^2 + 8ax + 10$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.46 If $f(x)$ is defined as

$$f(x) = \frac{3 - x}{4x + 7}$$

what is the domain and range of $f(x)$?

Exercise 4.47 If $f(x)$ is defined as

$$f(x) = -9x - 7$$

and $g(x)$ is defined as

$$g(x) = -10f(10x - 6)x^2 + 3x$$

simplify $g(x)$ in terms of x.

Exercise 4.48 If $f(x)$ is defined as

$$f(x) = 3ax - 6x^2$$

and $g(x)$ is defined as

$$g(x) = 3x + 6$$

solve the equation

$$g(f(x)) = 0$$

Exercise 4.49 If $f(x)$ is defined as

$$f(x) = 2 - 9x$$

and $g(x)$ is defined as

$$g(x) = -3x^2 + 6x - 7$$

simplify $g(f(x))$.

Exercise 4.50 For the following two functions,

$$f(x) = 5ax^2 - 3ax + 8$$

and

$$g(x) = -7ax^2 + 10ax + 10$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.51 For the following two functions,

$$f(x) = a\left(x^2 + 4x - 5\right)$$

and

$$g(x) = -6ax^2 - 6ax - 2$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.52 If $f(x)$ is defined as

$$f(x) = 2ax - 3x^2$$

and $g(x)$ is defined as

$$g(x) = -9x - 7$$

solve the equation

$$g(f(x)) = 0$$

Exercise 4.53 If $f(x)$ is defined as

$$f(x) = -10x - 1$$

and $g(x)$ is defined as

$$g(x) = -3f(4 - 7x)x^2 - x$$

simplify $g(x)$ in terms of x.

Exercise 4.54 If $f(x)$ is defined as

$$f(x) = 7 - x$$

and $g(x)$ is defined as

$$g(x) = -3f(6x + 8)x^2 + 9x$$

simplify $g(x)$ in terms of x.

Exercise 4.55 If $f(x)$ is defined as

$$f(x) = \frac{9-10x}{5x+6}$$

find $f^{-1}(x)$.

Exercise 4.56 If $f(x)$ is defined as

$$f(x) = 2x^2 + 8x - 4$$

and $g(x)$ is defined as

$$g(x) = 3f(9x^2 + 8x - 5)x^2 + 6x$$

simplify $g(x)$ in terms of x.

Exercise 4.57 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = 4ax^2 + 5a - 2x$$

and

$$g(x) = -9ax + 8$$

Exercise 4.58 If $f(x)$ is defined as

$$f(x) = -10x - 1$$

and $g(x)$ is defined as

$$g(x) = -8f(6 - 10x)x^2 - 6x$$

simplify $g(x)$ in terms of x.

Exercise 4.59 If $f(x)$ is defined as

$$f(x) = -3ax - 5x^2$$

and $g(x)$ is defined as

$$g(x) = -2x - 7$$

solve the equation

$$g(f(x)) = 0$$

Exercise 4.60 If $f(x)$ is defined as

$$f(x) = 9x^2 + 5x - 6$$

and $g(x)$ is defined as

$$g(x) = 10f(-4x^2 + 5x - 7)x^2 + 9x$$

simplify $g(x)$ in terms of x.

Exercise 4.61 For the following two functions,

$$f(x) = ax^2 - 4ax + 5$$

and

$$g(x) = a\left(-10x^2 - 9x + 3\right)$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.62 For the following two functions,

$$f(x) = -7ax^2 + 2ax - 7$$

and

$$g(x) = a\left(-x - 8\right)$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.63 If $f(x)$ is defined as

$$f(x) = -8ax + x^2$$

and $g(x)$ is defined as

$$g(x) = -9x - 5$$

solve the equation

$$g(f(x)) = 0$$

Exercise 4.64 For the following two functions,

$$f(x) = a\left(-9x^2 - 7x + 9\right)$$

and

$$g(x) = -4ax^2 + 4ax + 8$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.65 If $f(x)$ is defined as

$$f(x) = 9ax + 5x^2$$

and $g(x)$ is defined as

$$g(x) = 4x + 2$$

solve the equation

$$g(f(x)) = 0$$

Exercise 4.66 For the following two functions,

$$f(x) = a\left(x^2 - 5x + 1\right)$$

and

$$g(x) = -7ax - 8$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.67 If $f(x)$ is defined as

$$f(x) = \frac{9ax - 6a}{9a + 8x}$$

find the value of a if $f^{-1}(1) = 2$.

Exercise 4.68 If $f(x)$ is defined as

$$f(x) = \frac{-6ax + 3a}{-7a - x}$$

find the value of a if $f^{-1}(4) = 3$.

Exercise 4.69 If $f(x)$ is defined as

$$f(x) = -2x - 9$$

and $g(x)$ is defined as

$$g(x) = x^2 + 10x - 6$$

simplify $g(f(x))$.

Exercise 4.70 If $f(x)$ is defined as

$$f(x) = -10x^2 - 4x - 8$$

and $g(x)$ is defined as

$$g(x) = -6f(5x^2 - 5x + 8)x^2 + 4x$$

simplify $g(x)$ in terms of x.

Exercise 4.71 If $f(x)$ is defined as

$$f(x) = 7ax - x^2$$

and $g(x)$ is defined as

$$g(x) = 10x + 6$$

solve the equation

$$g(f(x)) = 0$$

Exercise 4.72 If $f(x)$ is defined as

$$f(x) = -10x - 3$$

and $g(x)$ is defined as

$$g(x) = -5x^2 + 10x + 4$$

simplify $g(f(x))$.

Exercise 4.73 For the following two functions,

$$f(x) = 4ax^2 + 7ax - 6$$

and

$$g(x) = 2a(-5x - 3)$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.74 If $f(x)$ is defined as

$$f(x) = -6x - 9$$

and $g(x)$ is defined as

$$g(x) = -3x^2 + x + 4$$

simplify $g(f(x))$.

Exercise 4.75 If $f(x)$ is defined as

$$f(x) = 9x + 6$$

and $g(x)$ is defined as

$$g(x) = 9f(3x - 1)x^2 + 2x$$

simplify $g(x)$ in terms of x.

Exercise 4.76 If $f(x)$ is defined as

$$f(x) = 5 - 2x$$

and $g(x)$ is defined as

$$g(x) = -4x^2 - 2x - 1$$

simplify $g(f(x))$.

Exercise 4.77 If $f(x)$ is defined as

$$f(x) = 3ax + 3x^2$$

and $g(x)$ is defined as

$$g(x) = -6x - 4$$

solve the equation

$$g(f(x)) = 0$$

Exercise 4.78 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -3x - 8$$

and

$$g(x) = -a + 5x$$

Exercise 4.79 For the following two functions,

$$f(x) = 10ax^2 + 7ax - 2$$

and

$$g(x) = a\,(8 - 5x)$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.80 For the following two functions,

$$f(x) = 6ax^2 - 5ax - 7$$

and

$$g(x) = a(6x + 5)$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.81 For the following two functions,

$$f(x) = a\left(6x^2 + 7x - 9\right)$$

and

$$g(x) = 5ax^2 + 3ax - 3$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.82 If $f(x)$ is defined as

$$f(x) = \frac{-7ax + 8a}{6a - 3x}$$

find the value of a if $f^{-1}(6) = 6$.

Exercise 4.83 For the following two functions,

$$f(x) = -4ax^2 + ax + 9$$

and

$$g(x) = a\left(-8x^2 - 9x - 5\right)$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.84 If $f(x)$ is defined as

$$f(x) = 10x + 9$$

and $g(x)$ is defined as

$$g(x) = 6x^2 - 6x - 5$$

simplify $g(f(x))$.

Exercise 4.85 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = 3x - 9$$

and

$$g(x) = -10a - 10x$$

Exercise 4.86 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -2ax^2 + 9a - 3x$$

and

$$g(x) = -4ax + 2$$

Exercise 4.87 If $f(x)$ is defined as

$$f(x) = 8x - 5$$

and $g(x)$ is defined as

$$g(x) = -8x^2 + 5x - 3$$

simplify $g(f(x))$.

Exercise 4.88 If $f(x)$ is defined as

$$f(x) = 8x + 1$$

and $g(x)$ is defined as

$$g(x) = -2x^2 + 7x + 8$$

simplify $g(f(x))$.

Exercise 4.89 For the following two functions,

$$f(x) = a\left(-5x^2 - 7x + 2\right)$$

and

$$g(x) = 5ax^2 + 9ax + 10$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.90 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = 3x + 10$$

and

$$g(x) = -5a + 7x$$

Exercise 4.91 If $f(x)$ is defined as

$$f(x) = \frac{-3ax + 10a}{-2a - 2x}$$

find the value of a if $f^{-1}(9) = 3$.

Exercise 4.92 If $f(x)$ is defined as

$$f(x) = x - 10$$

and $g(x)$ is defined as

$$g(x) = x^2 + 5x - 9$$

simplify $g(f(x))$.

Exercise 4.93 For the following two functions,

$$f(x) = -8ax^2 - 5ax + 6$$

and

$$g(x) = -8ax + 10$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.94 If $f(x)$ is defined as

$$f(x) = \frac{-9ax+5a}{7a+x}$$

find the value of a if $f^{-1}(6) = 5$.

Exercise 4.95 For the following two functions,

$$f(x) = 10ax^2 - 4ax + 1$$

and

$$g(x) = -6ax^2 - 2ax + 8$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.96 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = 7ax^2 + a + 3x$$

and

$$g(x) = 5ax + 3$$

Exercise 4.97 If $f(x)$ is defined as

$$f(x) = -4x^2 - 4x + 6$$

and $g(x)$ is defined as

$$g(x) = 2f(-9x^2 + 3x - 4)x^2 - 10x$$

simplify $g(x)$ in terms of x.

Exercise 4.98 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = 10ax^2 - a - 3x$$

and

$$g(x) = -3ax - 9$$

Exercise 4.99 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -ax^2 + 7a - 8x$$

and

$$g(x) = 4ax + 8$$

Exercise 4.100 If $f(x)$ is defined as

$$f(x) = \frac{9x - 10}{-3x - 8}$$

what is the domain and range of $f(x)$?

4.2 Solutions

1. $$\left\{-\frac{3y+10}{y+7}\right\}$$

2. $$-1800x^6-2520x^5-2922x^4-1428x^3-558x^2+8x$$

3. $$-180x^6-540x^5+231x^4+954x^3-556x^2+5x$$

4. $$48x^2+208x+222$$

5. $$\left\{\frac{9y-4}{8y-7}\right\}$$

6. $$-\frac{32}{79}<a, a<0$$

7. $$\left\{\frac{3a-7}{20a}-\frac{\sqrt{289a^2-402a+49}}{20a}, \frac{3a-7}{20a}+\frac{\sqrt{289a^2-402a+49}}{20a}\right\}$$

8. $$\left\{-\frac{9a}{16}-\frac{\sqrt{729a^2+64}}{48}, -\frac{9a}{16}+\frac{\sqrt{729a^2+64}}{48}\right\}$$

9. Domain:
$$\left(-\infty,-\frac{8}{3}\right)\cup\left(-\frac{8}{3},\infty\right)$$
Range:
$$(-\infty,-1)\cup(-1,\infty)$$

10. $$1323x^6-378x^5-1989x^4+288x^3+741x^2-9x$$

11. $$-\frac{80}{169}<a, a<0$$

12.

$$\left\{\frac{a}{6}-\frac{\sqrt{7}\sqrt{7a^2+24}}{42},\frac{a}{6}+\frac{\sqrt{7}\sqrt{7a^2+24}}{42}\right\}$$

13. Domain:

$$\left(-\infty,-\frac{3}{2}\right)\cup\left(-\frac{3}{2},\infty\right)$$

Range:

$$\left(-\infty,-\frac{1}{2}\right)\cup\left(-\frac{1}{2},\infty\right)$$

14.

$$-63x^6-126x^5+711x^4+774x^3-2421x^2-3x$$

15.

$$\left\{-\frac{5\cdot(2y-1)}{7y-2}\right\}$$

16.

$$\left\{-\frac{7a}{18}-\frac{\sqrt{7}\sqrt{343a^2-288}}{126},-\frac{7a}{18}+\frac{\sqrt{7}\sqrt{343a^2-288}}{126}\right\}$$

17. Domain:

$$(-\infty,1)\cup(1,\infty)$$

Range:

$$\left(-\infty,\frac{1}{3}\right)\cup\left(\frac{1}{3},\infty\right)$$

18.

$$0<a,a<\frac{153}{16}$$

19.

$$-1.429$$

20.

$$2352x^6+1344x^5-1656x^4-528x^3+400x^2-8x$$

21. Domain:

$$\left(-\infty,\frac{7}{10}\right)\cup\left(\frac{7}{10},\infty\right)$$

Range:

$$\left(-\infty,-\frac{3}{10}\right)\cup\left(-\frac{3}{10},\infty\right)$$

22.

$$16x^3 - 88x^2 - 3x$$

23.

$$-\frac{104}{53} < a, a < 0$$

24. Domain:

$$\left(-\infty, -\frac{3}{8}\right) \cup \left(-\frac{3}{8}, \infty\right)$$

Range:

$$\left(-\infty, \frac{1}{4}\right) \cup \left(\frac{1}{4}, \infty\right)$$

25.

$$\left\{\frac{9a}{5}\right\}$$

26.

$$256x^2 + 424x + 173$$

27. Domain:

$$\left(-\infty, \frac{10}{9}\right) \cup \left(\frac{10}{9}, \infty\right)$$

Range:

$$(-\infty, -1) \cup (-1, \infty)$$

28.

$$\left\{-\frac{4a}{7}\right\}$$

29.

$$\left\{-\frac{3a-1}{10a} - \frac{\sqrt{329a^2 - 6a + 1}}{10a}, -\frac{3a-1}{10a} + \frac{\sqrt{329a^2 - 6a + 1}}{10a}\right\}$$

30.

$$-144x^6 - 288x^5 - 444x^4 - 300x^3 - 148x^2 - 8x$$

31. Domain:

$$\left(-\infty, -\frac{2}{3}\right) \cup \left(-\frac{2}{3}, \infty\right)$$

Range:

$$\left(-\infty, -\frac{1}{6}\right) \cup \left(-\frac{1}{6}, \infty\right)$$

32.

$$\left\{\frac{10y-7}{10(y-1)}\right\}$$

33.

$$-\frac{48}{101} < a, a < 0$$

34. Domain:

$$\left(-\infty, -\frac{3}{7}\right) \cup \left(-\frac{3}{7}, \infty\right)$$

Range:

$$(-\infty, 1) \cup (1, \infty)$$

35.

$$\left\{-\frac{7y-3}{10y+7}\right\}$$

36.

$$-\frac{64}{73} < a, a < 0$$

37.

$$\left\{\frac{9a-5}{6a} - \frac{\sqrt{21a^2-126a+25}}{6a}, \frac{9a-5}{6a} + \frac{\sqrt{21a^2-126a+25}}{6a}\right\}$$

38. Domain:

$$\left(-\infty, \frac{2}{5}\right) \cup \left(\frac{2}{5}, \infty\right)$$

Range:

$$\left(-\infty, \frac{6}{5}\right) \cup \left(\frac{6}{5}, \infty\right)$$

39.

$$\left\{\frac{1}{2} - \frac{3a}{8}\right\}$$

40.

$$-2880x^6 - 5760x^5 - 8928x^4 - 6048x^3 - 3105x^2 - 9x$$

41.

$$-960x^6 + 2880x^5 + 1056x^4 - 4824x^3 - 2628x^2 - 10x$$

42.

$$-125x^2 + 450x - 410$$

43.

$$-\frac{4}{7} < a, a < 0$$

44.

$$24x^3 - 40x^2 - 10x$$

45.

$$0 < a, a < \frac{40}{269}$$

46. Domain:

$$\left(-\infty, -\frac{7}{4}\right) \cup \left(-\frac{7}{4}, \infty\right)$$

Range:

$$\left(-\infty, -\frac{1}{4}\right) \cup \left(-\frac{1}{4}, \infty\right)$$

47.

$$900x^3 - 470x^2 + 3x$$

48.

$$\left\{\frac{a}{4} - \frac{\sqrt{3}\sqrt{3a^2+16}}{12}, \frac{a}{4} + \frac{\sqrt{3}\sqrt{3a^2+16}}{12}\right\}$$

49.

$$-243x^2 + 54x - 7$$

50.

$$-\frac{96}{169} < a, a < 0$$

51.

$$0 < a, a < \frac{7}{30}$$

52.

$$\left\{\frac{a}{3} - \frac{\sqrt{3}\sqrt{3a^2+7}}{9}, \frac{a}{3} + \frac{\sqrt{3}\sqrt{3a^2+7}}{9}\right\}$$

53.

$$-210x^3 + 123x^2 - x$$

54.

$$18x^3 + 3x^2 + 9x$$

55.

$$\left\{-\frac{3\cdot(2y-3)}{5\,(y+2)}\right\}$$

56.

$$486x^6+864x^5+60x^4-288x^3+18x^2+6x$$

57.

$$\left\{-\frac{9a-2}{8a}-\frac{\sqrt{a^2+92a+4}}{8a},-\frac{9a-2}{8a}+\frac{\sqrt{a^2+92a+4}}{8a}\right\}$$

58.

$$-800x^3+488x^2-6x$$

59.

$$\left\{-\frac{3a}{10}-\frac{\sqrt{9a^2+70}}{10},-\frac{3a}{10}+\frac{\sqrt{9a^2+70}}{10}\right\}$$

60.

$$1440x^6-3600x^5+7090x^4-6050x^3+4000x^2+9x$$

61.

$$0<a,a<\frac{220}{157}$$

62.

$$0<a,a<\frac{196}{233}$$

63.

$$\left\{4a-\frac{\sqrt{144a^2-5}}{3},4a+\frac{\sqrt{144a^2-5}}{3}\right\}$$

64.

$$0<a,a<\frac{160}{301}$$

65.

$$\left\{-\frac{9a}{10}-\frac{\sqrt{81a^2-10}}{10},-\frac{9a}{10}+\frac{\sqrt{81a^2-10}}{10}\right\}$$

66.

$$0<a$$

67.

$$5.333$$

68.

$$-0.923$$

69.

$$4x^2+16x-15$$

70.

$$1500x^6-3000x^5+6420x^4-4920x^3+4080x^2+4x$$

71.

$$\left\{\frac{7a}{2}-\frac{\sqrt{5}\sqrt{245a^2+12}}{10},\frac{7a}{2}+\frac{\sqrt{5}\sqrt{245a^2+12}}{10}\right\}$$

72.

$$-500x^2-400x-71$$

73.

$$-\frac{96}{193}<a,a<0$$

74.

$$-108x^2-330x-248$$

75.

$$243x^3-27x^2+2x$$

76.

$$-16x^2+84x-111$$

77.

$$\left\{-\frac{a}{2}-\frac{\sqrt{9a^2-8}}{6},-\frac{a}{2}+\frac{\sqrt{9a^2-8}}{6}\right\}$$

78.

$$\left\{\frac{a}{8}-1\right\}$$

79.

$$-\frac{5}{29}<a,a<0$$

80.

$$-\frac{168}{241}<a,a<0$$

81.

$$0 < a, a < \frac{3}{13}$$

82.

$$1.543$$

83.

$$0 < a, a < \frac{36}{5}$$

84.

$$600x^2 + 1020x + 427$$

85.

$$\left\{ \frac{9}{13} - \frac{10a}{13} \right\}$$

86.

$$\left\{ \frac{4a-3}{4a} - \frac{\sqrt{88a^2 - 40a + 9}}{4a}, \frac{4a-3}{4a} + \frac{\sqrt{88a^2 - 40a + 9}}{4a} \right\}$$

87.

$$-512x^2 + 680x - 228$$

88.

$$-128x^2 + 24x + 13$$

89.

$$0 < a, a < \frac{25}{21}$$

90.

$$\left\{ \frac{5a}{4} + \frac{5}{2} \right\}$$

91.

$$-2.842$$

92.

$$x^2 - 15x + 41$$

93.

$$0 < a, a < \frac{128}{9}$$

94.

$$-0.366$$

95.

$$-112 < a, a < 0$$

96.

$$\left\{\frac{5a-3}{14a} - \frac{\sqrt{3}\sqrt{-a^2+18a+3}}{14a}, \frac{5a-3}{14a} + \frac{\sqrt{3}\sqrt{-a^2+18a+3}}{14a}\right\}$$

97.

$$-648x^6 + 432x^5 - 576x^4 + 168x^3 - 84x^2 - 10x$$

98.

$$\left\{-\frac{3(a-1)}{20a} - \frac{\sqrt{49a^2-378a+9}}{20a}, -\frac{3(a-1)}{20a} + \frac{\sqrt{49a^2-378a+9}}{20a}\right\}$$

99.

$$\left\{-\frac{2(a+2)}{a} - \frac{\sqrt{11a^2+8a+16}}{a}, -\frac{2(a+2)}{a} + \frac{\sqrt{11a^2+8a+16}}{a}\right\}$$

100. Domain:

$$\left(-\infty, -\frac{8}{3}\right) \cup \left(-\frac{8}{3}, \infty\right)$$

Range:

$$(-\infty, -3) \cup (-3, \infty)$$

5. Differential calculus

5.1 Problems

Exercise 5.1 Differentiate the polynomial with respect to d

$$-3d^3 - d^2 - 4d - 9$$

Exercise 5.2 Differentiate the following expression with respect to y

$$9w^5 + y^5 - y^2$$

Exercise 5.3 Differentiate the following expression with respect to c

$$-7c^6 - 3cx^5 + 3$$

Exercise 5.4 Differentiate the following expression with respect to u

$$-u^{\frac{2}{5}} + 4uy^3 + 3\sqrt[3]{y}$$

Exercise 5.5 Differentiate the following expression with respect to p

$$3\sqrt[5]{p}w^{\frac{4}{5}} - 10\sqrt[4]{p} + 10p^{\frac{3}{2}} + w + 2$$

Exercise 5.6 Differentiate the following expression with respect to b

$$-7\sqrt[5]{b} + 10p^{\frac{2}{3}} - p$$

Exercise 5.7 Differentiate the polynomial with respect to m

$$-9m^2 + 3m + 5$$

Exercise 5.8 Double derivative: Differentiate the following expression with respect to x twice

$$b^4x^4 - 10b^2x^4 - 9bx^2 + 9b - x^3$$

Exercise 5.9 Differentiate the polynomial with respect to a

$$8a^2 - 3a - 1$$

Exercise 5.10 Differentiate the polynomial with respect to y

$$-3y^4 + y^3 + 9y^2 + 4y + 7$$

Exercise 5.11 Differentiate the following expression with respect to c

$$3c^6q^4 - 8c^5q - 3c^4q^3 + 8c^4 - 9q^4$$

Exercise 5.12 Differentiate the polynomial with respect to t

$$-4t^2 - 10t + 1$$

Exercise 5.13 Differentiate the following expression with respect to p

$$p^3\left(-8p^2s^2 + 9ps^2 - 2s - 3\right)$$

Exercise 5.14 Differentiate the following expression with respect to b

$$-8b^4t^6 + 2b^3t^2 - 8b^3 + 5b^2 + 3t^6$$

Exercise 5.15 Differentiate the following expression with respect to p

$$-5d^5 + 6d^2 - p^5$$

Exercise 5.16 Differentiate the following expression with respect to a

$$-5a^{\frac{3}{4}}\sqrt[3]{u} + 8\sqrt[4]{a} + 8a^5\sqrt[3]{u} + 6au^{\frac{2}{3}} + 7$$

Exercise 5.17 Differentiate the polynomial with respect to r

$$-2r^2 + r + 5$$

Exercise 5.18 Differentiate the following expression with respect to w

$$-3q^5w^6 + 4q^5w^3 - 3q^3w^4 - 2q^3 - 7w$$

Exercise 5.19 Differentiate the polynomial with respect to b

$$2b^4 + 4b^3 - 7b^2 - 6b - 1$$

Exercise 5.20 Differentiate the following expression with respect to x

$$2d^5 + 3dx^6 - d + 10$$

Exercise 5.21 Differentiate the following expression with respect to t

$$2s^4t^2 - 8s^4 + 5t^5 - 8t^3$$

Exercise 5.22 Differentiate the following expression with respect to q

$$5c^5 + 9cq^2 + 10q^6$$

Exercise 5.23 Differentiate the polynomial with respect to d

$$-7d^2 + d + 9$$

Exercise 5.24 Differentiate the polynomial with respect to n

$$n^4 + 10n^3 - 9n^2 + 9n - 10$$

Exercise 5.25 Differentiate the following expression with respect to a

$$10a^4 + 4a^3 + 5a + 8c^3$$

Exercise 5.26 Differentiate the following expression with respect to s

$$-7b^{\frac{2}{5}}s^{\frac{3}{5}} - 2\sqrt[5]{b} - 10s^{\frac{2}{3}} - 5s^6 - 3$$

Exercise 5.27 Differentiate the polynomial with respect to r

$$2r^4 + 2r^3 - r^2 + r - 8$$

Exercise 5.28 Differentiate the following expression with respect to u

$$6\sqrt[3]{b} + b^{\frac{3}{2}}u^2 + 8u^{\frac{4}{5}}$$

Exercise 5.29 Differentiate the following expression with respect to l

$$l^4 \cdot \left(3l^2m^3 + 9l^2 - 5m\right)$$

Exercise 5.30 Differentiate the following expression with respect to d

$$-9b^6 - b^4 + 2bd^3 + 6$$

Exercise 5.31 Differentiate the polynomial with respect to m

$$m^3 - 3m^2 - 10m - 3$$

Exercise 5.32 Double derivative: Differentiate the following expression with respect to x twice

$$10w^3x + 2w^2 + 10x^4 + 2x$$

Exercise 5.33 Differentiate the polynomial with respect to l

$$-l^4 - 5l^3 - 7l^2 + 10l + 7$$

Exercise 5.34 Differentiate the following expression with respect to y

$$-5m^{\frac{5}{2}} - 6\sqrt{m} + 9$$

Exercise 5.35 Differentiate the following expression with respect to p

$$2p\left(5p^5 + p^4 - 5z^5\right)$$

Exercise 5.36 Differentiate the following expression with respect to t

$$-3t^{\frac{6}{5}} + 8t^{\frac{4}{5}} - 3\sqrt[3]{t} - 8t^5z^{\frac{2}{5}} + 8z^{\frac{5}{6}}$$

Exercise 5.37 Double derivative: Differentiate the following expression with respect to m twice

$$s^4\left(-5s^2 - 6s + 8\right)$$

Exercise 5.38 Differentiate the following expression with respect to m

$$-5m^{\frac{3}{5}}\sqrt[3]{v}+8m^{\frac{2}{3}}-9m^{\frac{5}{2}}v^{\frac{3}{2}}-6m^5$$

Exercise 5.39 Differentiate the polynomial with respect to l

$$10l^3-8l^2-l+2$$

Exercise 5.40 Differentiate the following expression with respect to y

$$-3\sqrt[6]{p}+\sqrt{p}\sqrt[5]{y}-8p^3y^3$$

Exercise 5.41 Differentiate the following expression with respect to l

$$-3l^{\frac{5}{6}}+6l^3p-5l\sqrt[5]{p}-lp^{\frac{2}{3}}-\sqrt[3]{p}$$

Exercise 5.42 Differentiate the polynomial with respect to m

$$6m^4-4m^3+m^2+m-5$$

Exercise 5.43 Differentiate the following expression with respect to l

$$l^{\frac{5}{4}}+6w^{\frac{2}{5}}-10w^{\frac{5}{4}}$$

Exercise 5.44 Differentiate the following expression with respect to d

$$-9a-d^{\frac{2}{3}}-9$$

Exercise 5.45 Differentiate the polynomial with respect to x

$$-5x^4-10x^3+5x^2+5x+1$$

Exercise 5.46 Differentiate the following expression with respect to m

$$2cm^5-10c+3m^5+4m^4$$

Exercise 5.47 Differentiate the following expression with respect to b

$$2b^{\frac{3}{2}}q^{\frac{5}{3}}+9bq+b$$

Exercise 5.48 Differentiate the following expression with respect to x

$$-2q^{\frac{2}{5}}x^{\frac{5}{4}}-9q^2-4x^{\frac{5}{2}}$$

Exercise 5.49 Differentiate the polynomial with respect to u

$$-4u^3-5u^2+10u+7$$

Exercise 5.50 Double derivative: Differentiate the following expression with respect to q twice

$$p^4\left(-10p-q^3+1\right)$$

Exercise 5.51 Differentiate the following expression with respect to m

$$4\sqrt[5]{mq}-3m^{\frac{4}{3}}q^6-5\sqrt[3]{mq}^{\frac{2}{3}}+5q^{\frac{3}{4}}+6\sqrt{q}$$

Exercise 5.52 Differentiate the following expression with respect to c

$$9\sqrt[4]{cz} - 7c + 6z^{\frac{5}{4}} - 2z^2 - 3z$$

Exercise 5.53 Differentiate the following expression with respect to w

$$\sqrt[4]{x} - 6x^3 - 2$$

Exercise 5.54 Double derivative: Differentiate the following expression with respect to y twice

$$-6u^2y^6 + 7y^4 + 7$$

Exercise 5.55 Differentiate the following expression with respect to a

$$a\left(-4a^4 + 9a^2q + 2\right)$$

Exercise 5.56 Differentiate the polynomial with respect to n

$$-9n^3 - 6n^2 + 3n + 6$$

Exercise 5.57 Double derivative: Differentiate the following expression with respect to u twice

$$-4m^6u^6 + 9m^6u^3 + 4mu - 3u - 8$$

Exercise 5.58 Double derivative: Differentiate the following expression with respect to d twice

$$-13c^5 - 9d^3 + 7d$$

Exercise 5.59 Differentiate the following expression with respect to z

$$-6\sqrt{pz^{\frac{3}{2}}} + 3p^3 + 9z^2 + 2$$

Exercise 5.60 Double derivative: Differentiate the following expression with respect to x twice

$$x^2 \cdot \left(7s^5x^4 + 2s^3 - 6s^2x^3 + 7s^2x - 9x^4\right)$$

Exercise 5.61 Differentiate the polynomial with respect to u

$$-6u^2 - 10u + 4$$

Exercise 5.62 Differentiate the following expression with respect to u

$$s^5u^6 - 3s^4 - 8u^6 - 8u$$

Exercise 5.63 Differentiate the following expression with respect to c

$$-10a^6c^4 + 9a^5 - 5$$

Exercise 5.64 Double derivative: Differentiate the following expression with respect to z twice

$$8d^4 - 9d^2 - 8$$

Exercise 5.65 Differentiate the following expression with respect to p

$$x\left(-7p^3 - 4x^4 + 7x^3\right)$$

Exercise 5.66 Differentiate the following expression with respect to l

$$7l^4 + 2lq + 7q^3$$

Exercise 5.67 Differentiate the following expression with respect to a

$$3a^6 - 2a^4 - 8s^6 - 7s^4 - s^3$$

Exercise 5.68 Differentiate the polynomial with respect to p

$$2p^3 + 3p^2 - 9p - 5$$

Exercise 5.69 Double derivative: Differentiate the following expression with respect to d twice

$$-10d^6u + 6d^5 + 9d^4u^3 + 8u^2 + 10$$

Exercise 5.70 Double derivative: Differentiate the following expression with respect to l twice

$$-4l^3 - 2w$$

Exercise 5.71 Differentiate the polynomial with respect to a

$$3a^4 + 5a^3 - 8a^2 + 9a - 7$$

Exercise 5.72 Differentiate the polynomial with respect to a

$$-6a^3 + a^2 + 3a + 1$$

Exercise 5.73 Differentiate the polynomial with respect to m

$$-7m^3 - 8m^2 + 10m + 4$$

Exercise 5.74 Differentiate the following expression with respect to w

$$5t^{\frac{6}{5}} + 3t^{\frac{3}{2}} + 10t + 5w^{\frac{5}{2}} - 3$$

Exercise 5.75 Double derivative: Differentiate the following expression with respect to w twice

$$7u^6w^6 - 4u^3 + 10w^5$$

Exercise 5.76 Double derivative: Differentiate the following expression with respect to r twice

$$-9s^4 + 7s^2 - 4$$

Exercise 5.77 Differentiate the following expression with respect to p

$$-4d^{\frac{5}{4}}\sqrt[3]{p} + 10d^{\frac{5}{4}}p - 7d\sqrt[4]{p} + 6d$$

Exercise 5.78 Double derivative: Differentiate the following expression with respect to b twice

$$-5b^5 + 9br^2 - 15r^5 - 5$$

Exercise 5.79 Differentiate the following expression with respect to l

$$4c^{\frac{5}{4}} + 6c^{\frac{2}{3}} + 6l^3 + 9l^2 + 8l$$

Exercise 5.80 Double derivative: Differentiate the following expression with respect to t twice

$$2b^6t^5 - 5b^4 - b^2 + 9t^5 - 2$$

Exercise 5.81 Differentiate the following expression with respect to u

$$10c^6u^3 + 7c^3u^3 - 5cu^2 + 8c - 8$$

Exercise 5.82 Differentiate the polynomial with respect to y

$$-8y^2 - y + 3$$

Exercise 5.83 Double derivative: Differentiate the following expression with respect to x twice

$$3x^2 \cdot \left(2w^3 - 3w^2 - x^4\right)$$

Exercise 5.84 Double derivative: Differentiate the following expression with respect to r twice

$$-6n^2 + 5n + 10r$$

Exercise 5.85 Differentiate the polynomial with respect to z

$$8z^4 - 7z^3 + 10z^2 + 7z - 3$$

Exercise 5.86 Differentiate the following expression with respect to z

$$5dz^6 + 3d - 10$$

Exercise 5.87 Differentiate the following expression with respect to c

$$-2b^6 + 7b^5 + 3bc + 5b - 7c^2$$

Exercise 5.88 Double derivative: Differentiate the following expression with respect to r twice

$$d\left(-3d^5 + 6d^4r^3 - 10r^5\right)$$

Exercise 5.89 Double derivative: Differentiate the following expression with respect to q twice

$$-3c^6q^5 - 8c^3q^3 - 7q^6 + 7$$

Exercise 5.90 Differentiate the following expression with respect to p

$$5p^6\sqrt{u} + 4p^4u^{\frac{5}{4}} + 7u^{\frac{3}{2}}$$

Exercise 5.91 Differentiate the following expression with respect to z

$$-6s^{\frac{5}{3}}z^{\frac{2}{3}} + 10s^2z^{\frac{4}{5}} - 2z^3$$

Exercise 5.92 Differentiate the polynomial with respect to d

$$-d^4 + 9d^3 - 3d^2 - 4d + 9$$

Exercise 5.93 Differentiate the following expression with respect to v

$$10\sqrt[6]{v}w^2 - 3vw^{\frac{5}{4}} - 7vw^5$$

Exercise 5.94 Double derivative: Differentiate the following expression with respect to d twice

$$-10d^2n^3+2d^2-n^6-9n^3$$

Exercise 5.95 Differentiate the following expression with respect to x

$$t^6x^3-8t^5x^6+4t^2x-17$$

Exercise 5.96 Differentiate the polynomial with respect to y

$$y^3-4y^2+2y+1$$

Exercise 5.97 Differentiate the polynomial with respect to z

$$-4z^2-10z+10$$

Exercise 5.98 Differentiate the following expression with respect to q

$$10\sqrt[5]{q}y-10\sqrt{q}+8y^{\frac{4}{5}}-5y^{\frac{3}{2}}+9$$

Exercise 5.99 Differentiate the polynomial with respect to d

$$2d^2-6d-3$$

Exercise 5.100 Differentiate the following expression with respect to d

$$7\sqrt[4]{d}z+5d^{\frac{5}{2}}z^{\frac{2}{3}}+4d^{\frac{3}{2}}+10d^2z^3$$

5.2 Solutions

1.

$$-9d^2-2d-4$$

2.

$$y\left(5y^3-2\right)$$

3.

$$-42c^5-3x^5$$

4.

$$4y^3-\frac{2}{5u^{\frac{3}{5}}}$$

5.

$$15\sqrt{p}-\frac{5}{2p^{\frac{3}{4}}}+\frac{3w^{\frac{4}{5}}}{5p^{\frac{4}{5}}}$$

6.

$$-\frac{7}{5b^{\frac{4}{5}}}$$

7.

$$3-18m$$

8.

$$12b^4x^2-120b^2x^2-18b-6x$$

9.

$$16a-3$$

10.

$$-12y^3+3y^2+18y+4$$

11.

$$c^3\cdot\left(18c^2q^4-40cq-12q^3+32\right)$$

12.

$$-8t-10$$

13.

$$p^2\left(-40p^2s^2+36ps^2-6s-9\right)$$

14.

$$2b\left(-16b^2t^6+3bt^2-12b+5\right)$$

15.

$$-5p^4$$

16.

$$40a^4\sqrt[3]{u}+6u^{\frac{2}{3}}-\frac{15\sqrt[3]{u}}{4\sqrt[4]{a}}+\frac{2}{a^{\frac{3}{4}}}$$

17.

$$1-4r$$

18.

$$-18q^5w^5+12q^5w^2-12q^3w^3-7$$

19.

$$8b^3+12b^2-14b-6$$

20.

$$18dx^5$$

21.

$$t\left(4s^4+25t^3-24t\right)$$

22.

$$18cq+60q^5$$

23.

$$1-14d$$

24.

$$4n^3+30n^2-18n+9$$

25.

$$40a^3+12a^2+5$$

26.

$$-\frac{21b^{\frac{2}{5}}}{5s^{\frac{2}{5}}}-30s^5-\frac{20}{3\sqrt[3]{s}}$$

27.

$$8r^3+6r^2-2r+1$$

28.

$$2b^{\frac{3}{2}}u+\frac{32}{5\sqrt[5]{u}}$$

29.

$$l^3\cdot\left(18l^2m^3+54l^2-20m\right)$$

30.

$$6bd^2$$

31.

$$3m^2-6m-10$$

32.

$$120x^2$$

33.

$$-4l^3-15l^2-14l+10$$

34.

$$0$$

35.

$$60p^5 + 10p^4 - 10z^5$$

36.

$$-\frac{18\sqrt[5]{t}}{5} - 40t^4 z^{\frac{2}{5}} - \frac{1}{t^{\frac{2}{3}}} + \frac{32}{5\sqrt[5]{t}}$$

37.

$$0$$

38.

$$-\frac{45m^{\frac{3}{2}}v^{\frac{3}{2}}}{2} - 30m^4 + \frac{16}{3\sqrt[3]{m}} - \frac{3\sqrt[3]{v}}{m^{\frac{2}{5}}}$$

39.

$$30l^2 - 16l - 1$$

40.

$$\frac{\sqrt{p}}{5y^{\frac{4}{5}}} - 24p^3y^2$$

41.

$$18l^2p - 5\sqrt[5]{p} - p^{\frac{2}{3}} - \frac{5}{2\sqrt[6]{l}}$$

42.

$$24m^3 - 12m^2 + 2m + 1$$

43.

$$\frac{5\sqrt[4]{l}}{4}$$

44.

$$-\frac{2}{3\sqrt[3]{d}}$$

45.

$$-20x^3 - 30x^2 + 10x + 5$$

46.

$$m^3 \cdot (10cm + 15m + 16)$$

47.

$$3\sqrt{b}q^{\frac{5}{3}} + 9q + 1$$

48.

$$-\frac{5q^{\frac{2}{5}}\sqrt[4]{x}}{2}-10x^{\frac{3}{2}}$$

49.

$$-12u^2-10u+10$$

50.

$$-6p^4q$$

51.

$$-4\sqrt[3]{m}q^6-\frac{5q^{\frac{2}{3}}}{3m^{\frac{2}{3}}}+\frac{4q}{5m^{\frac{4}{5}}}$$

52.

$$-7+\frac{9z}{4c^{\frac{3}{4}}}$$

53.

$$0$$

54.

$$y^2\left(-180u^2y^2+84\right)$$

55.

$$-20a^4+27a^2q+2$$

56.

$$-27n^2-12n+3$$

57.

$$6m^6u\left(9-20u^3\right)$$

58.

$$-54d$$

59.

$$-9\sqrt{p}\sqrt{z}+18z$$

60.

$$210s^5x^4+4s^3-120s^2x^3+42s^2x-270x^4$$

61.

$$-12u-10$$

62.

$$6s^5u^5 - 48u^5 - 8$$

63.

$$-40a^6c^3$$

64.

$$0$$

65.

$$-21p^2x$$

66.

$$28l^3 + 2q$$

67.

$$a^3 \cdot \left(18a^2 - 8\right)$$

68.

$$6p^2 + 6p - 9$$

69.

$$d^2\left(-300d^2u + 120d + 108u^3\right)$$

70.

$$-24l$$

71.

$$12a^3 + 15a^2 - 16a + 9$$

72.

$$-18a^2 + 2a + 3$$

73.

$$-21m^2 - 16m + 10$$

74.

$$\frac{25w^{\frac{3}{2}}}{2}$$

75.

$$w^3 \cdot \left(210u^6w + 200\right)$$

76.

$$0$$

77.

$$10d^{\frac{5}{4}} - \frac{4d^{\frac{5}{4}}}{3p^{\frac{2}{3}}} - \frac{7d}{4p^{\frac{3}{4}}}$$

78.

$$-100b^3$$

79.

$$18l^2 + 18l + 8$$

80.

$$t^3 \cdot \left(40b^6 + 180\right)$$

81.

$$cu\left(30c^5u + 21c^2u - 10\right)$$

82.

$$-16y - 1$$

83.

$$12w^3 - 18w^2 - 90x^4$$

84.

$$0$$

85.

$$32z^3 - 21z^2 + 20z + 7$$

86.

$$30dz^5$$

87.

$$3b - 14c$$

88.

$$36d^5r - 200dr^3$$

89.

$$-60c^6q^3 - 48c^3q - 210q^4$$

90.

$$p^3 \cdot \left(30p^2\sqrt{u} + 16u^{\frac{5}{4}}\right)$$

91.

$$-\frac{4s^{\frac{5}{3}}}{\sqrt[3]{z}} + \frac{8s^2}{\sqrt[5]{z}} - 6z^2$$

92.

$$-4d^3 + 27d^2 - 6d - 4$$

93.

$$-3w^{\frac{5}{4}} - 7w^5 + \frac{5w^2}{3v^{\frac{5}{6}}}$$

94.

$$4 - 20n^3$$

95.

$$t^2 \cdot \left(3t^4x^2 - 48t^3x^5 + 4\right)$$

96.

$$3y^2 - 8y + 2$$

97.

$$-8z - 10$$

98.

$$-\frac{5}{\sqrt{q}} + \frac{2y}{q^{\frac{4}{5}}}$$

99.

$$4d - 6$$

100.

$$\frac{25d^{\frac{3}{2}}z^{\frac{2}{3}}}{2} + 6\sqrt{d} + 20dz^3 + \frac{7z}{4d^{\frac{3}{4}}}$$

6. Integral calculus

6.1 Problems

Exercise 6.1 Evaluate the following indefinite integral

$$\int 10w^{\frac{6}{5}}y^{\frac{3}{5}} - w^{\frac{3}{2}} - 8w^2y^2 + 5y^{\frac{3}{2}} + 10dy$$

.

Exercise 6.2 Evaluate the following indefinite integral

$$\int -8m^5w - 9m^3 + 4m^2w^4 + 15w^4dw$$

.

Exercise 6.3 Evaluate the following indefinite integral

$$\int 2c^{\frac{4}{5}}s^6 - 10c^{\frac{4}{5}}s + 8\sqrt{c} - 6\sqrt[4]{s}dc$$

.

Exercise 6.4 Integrate the polynomial with respect to q

$$7q^3 - 2q^2 + 6q - 4$$

Exercise 6.5 Integrate the polynomial with respect to d

$$4d^4 + 4d^3 - 6d^2 + 10d + 2$$

Exercise 6.6 Evaluate the definite integral for the following function, where the limits are 2 and 4.

$$f(r) = 4r^{\frac{5}{4}} + 4r^2 - 7r + 7$$

Exercise 6.7 Evaluate the following indefinite integral

$$\int -6a^6 - 5a^4z - a^2z - 10az^2 - 2da$$

.

Exercise 6.8 Integrate the polynomial with respect to w

$$6w^4 - 2w^3 - w^2 - 6w + 5$$

Exercise 6.9 Evaluate the definite integral for the following function, where the limits are 1 and 2.

$$f(d) = 6d^{\frac{5}{6}} + 10\sqrt[4]{d} + 6d^4 + 8d + 9$$

Exercise 6.10 Integrate the polynomial with respect to v

$$4v^2 + 9v + 3$$

Exercise 6.11 Integrate the polynomial with respect to c

$$8c^2 - 2c + 8$$

Exercise 6.12 Evaluate the following indefinite integral

$$\int -10b^{\frac{4}{5}} + 4y^{\frac{5}{3}} + 10y^6 + 4db$$

.

Exercise 6.13 Integrate the polynomial with respect to z

$$6z^4 - 9z^3 + 3z^2 - 3z + 8$$

Exercise 6.14 Evaluate the following indefinite integral

$$\int -\sqrt[6]{a} - 9\sqrt[5]{a} + 8\sqrt[4]{a}\sqrt[5]{u} + 2a^5 + 6\sqrt[3]{u}da$$

.

Exercise 6.15 Evaluate the following indefinite integral

$$\int 5q^5 + q^4t + 4q^4 + 9t^6 dt$$

.

Exercise 6.16 Evaluate the definite integral for the following function, where the limits are 3 and 5.

$$f(w) = -4\sqrt[5]{w} - 9$$

Exercise 6.17 Evaluate the following indefinite integral

$$\int y\left(2c^6y^4 - 4c^3y + 9y^5 + 5y^2 - 6\right) dc$$

.

Exercise 6.18 Integrate the polynomial with respect to m

$$-4m^3 - 7m^2 + 9m + 1$$

Exercise 6.19 Evaluate the following indefinite integral

$$\int -3\sqrt{n} - 4n^6 + 7da$$

.

Exercise 6.20 Evaluate the following indefinite integral

$$\int 3w^5 - 5w^4 - 5w^2z^5 + 10z^3 dz$$

.

Exercise 6.21 Evaluate the following indefinite integral

$$\int -3r^{\frac{5}{4}}y - 2\sqrt{r}\sqrt{y} + 7r dr$$

.

Exercise 6.22 Evaluate the following indefinite integral

$$\int r^2\left(r^4t^6 - 10r - 9t^2\right) dt$$

.

Exercise 6.23 Evaluate the following indefinite integral

$$\int -6a^6q^6 + 9a^6 - a^3q + 3q^6 - 3q^2 da$$

.

Exercise 6.24 Evaluate the following indefinite integral

$$\int 5u^{\frac{6}{5}}x^6 + 4\sqrt[4]{u}x + 3u dx$$

.

Exercise 6.25 Evaluate the definite integral for the following function, where the limits are 1 and 5.

$$f(m) = 3\sqrt[6]{m} - m^{\frac{2}{5}} + 2\sqrt[5]{m} + 8m^{\frac{4}{3}} + 9m$$

Exercise 6.26 Integrate the polynomial with respect to m

$$m^4 + 9m^3 - 7m^2 + 7m + 6$$

Exercise 6.27 Evaluate the following indefinite integral

$$\int y^3z - 9y^2 + 10yz^5 + 2z^3 - 2z dy$$

.

Exercise 6.28 Evaluate the definite integral for the following function, where the limits are 2 and 4.

$$f(s) = 10\sqrt[3]{s} - 2s^{\frac{3}{2}} - 5\sqrt{s} - s^6 + 7s$$

Exercise 6.29 Integrate the polynomial with respect to y

$$-10y^3 + 3y^2 + y + 2$$

Exercise 6.30 Evaluate the following indefinite integral

$$\int 4\sqrt[5]{q}\sqrt[3]{u} - 7q^{\frac{4}{3}} + 4u^2 - 7dq$$

.

Exercise 6.31 Integrate the polynomial with respect to t

$$6t^4 - 2t^3 + 5t^2 - 3t + 4$$

Exercise 6.32 Integrate the polynomial with respect to q

$$7q^4 - 4q^3 + 5q^2 + 2q + 8$$

Exercise 6.33 Evaluate the definite integral for the following function, where the limits are 3 and 4.

$$f(r) = -4r^{\frac{6}{5}} - 8r^{\frac{2}{3}} + 9$$

Exercise 6.34 Integrate the polynomial with respect to x

$$2x^3 - 6x^2 + 4x - 10$$

Exercise 6.35 Integrate the polynomial with respect to q

$$9q^4 - 2q^3 - 10q^2 + 10q - 1$$

Exercise 6.36 Evaluate the definite integral for the following function, where the limits are 1 and 2.

$$f(c) = 5c^{\frac{5}{2}} - 3c^{\frac{3}{2}} - 8c$$

Exercise 6.37 Evaluate the following indefinite integral

$$\int 5a^5 + 6w da$$

.

Exercise 6.38 Evaluate the definite integral for the following function, where the limits are 1 and 5.

$$f(n) = 13$$

Exercise 6.39 Evaluate the following indefinite integral

$$\int yz^{\frac{3}{2}} - 3y - 4z^{\frac{4}{5}} - 6z^2 dz$$

.

Exercise 6.40 Evaluate the definite integral for the following function, where the limits are 2 and 5.

$$f(p) = 5p^{\frac{4}{5}} - p^{\frac{5}{3}} - p^{\frac{5}{2}} - p$$

Exercise 6.41 Evaluate the following indefinite integral

$$\int -7r^6 - 8t^6 - 8dr$$

.

Exercise 6.42 Evaluate the following indefinite integral

$$\int -7a^5 - 10a^3n^4 - 8a^3 - 8n^6 - 6n^4 da$$

.

Exercise 6.43 Evaluate the following indefinite integral

$$\int 8c^{\frac{2}{3}}\sqrt[3]{x}+c\sqrt{x}+3c-7\sqrt[6]{x}dc$$

.

Exercise 6.44 Evaluate the definite integral for the following function, where the limits are 0 and 1.

$$f(w)=-\sqrt[6]{w}-\sqrt[5]{w}+w^{\frac{3}{4}}-4\sqrt[4]{w}-2w$$

Exercise 6.45 Evaluate the following indefinite integral

$$\int 6c^{\frac{6}{5}}\sqrt[6]{x}+\sqrt[5]{c}\sqrt{x}-5c^{\frac{2}{3}}x^{\frac{5}{2}}+2\sqrt{c}x^{\frac{4}{3}}-5\sqrt[6]{x}dx$$

.

Exercise 6.46 Evaluate the following indefinite integral

$$\int -6q^6z^{\frac{6}{5}}+3q^4+z^{\frac{5}{4}}-8\sqrt{z}-4dz$$

.

Exercise 6.47 Evaluate the following indefinite integral

$$\int s^4-2v^6ds$$

.

Exercise 6.48 Integrate the polynomial with respect to z

$$10z^2-7z+7$$

Exercise 6.49 Evaluate the following indefinite integral

$$\int z^2\cdot\left(8c^4z^2+3c^3+8c^2+2z^2\right)dz$$

.

Exercise 6.50 Integrate the polynomial with respect to n

$$2n^2+2n+8$$

Exercise 6.51 Evaluate the following indefinite integral

$$\int 8n^{\frac{2}{5}}x^{\frac{4}{5}}-8nx^{\frac{2}{3}}+\sqrt[3]{x}+7x-8dn$$

.

Exercise 6.52 Evaluate the following indefinite integral

$$\int 17p^4-8v^3dp$$

.

Exercise 6.53 Evaluate the following indefinite integral

$$\int s^{\frac{3}{2}}x^5+5\sqrt{s}+5sds$$

.

Exercise 6.54 Evaluate the following indefinite integral

$$\int 7w^5 + w^4 + 5wx^5 - 5dw$$

.

Exercise 6.55 Evaluate the following indefinite integral

$$\int 4n^{\frac{6}{5}} + 6\sqrt[3]{n}\sqrt[5]{u} + 5\sqrt{n} + 2n^2u^2 - 5\sqrt[3]{u}dn$$

.

Exercise 6.56 Evaluate the following indefinite integral

$$\int q\left(-4c^2q^3 + 6q^3 - 3\right)dq$$

.

Exercise 6.57 Evaluate the definite integral for the following function, where the limits are 2 and 4.

$$f(p) = -4p^{\frac{3}{4}} + 10\sqrt{p} + 16p - 3$$

Exercise 6.58 Integrate the polynomial with respect to d

$$6d^4 - 10d^3 + 8d^2 - 7d - 10$$

Exercise 6.59 Evaluate the following indefinite integral

$$\int 8r^{\frac{5}{4}}z^5 + 3\sqrt{r}\sqrt{z} + 5z^{\frac{2}{3}} + 3zdz$$

.

Exercise 6.60 Integrate the polynomial with respect to n

$$-n^2 + 6n + 5$$

Exercise 6.61 Evaluate the definite integral for the following function, where the limits are 2 and 5.

$$f(d) = 6\sqrt[5]{d} + d^{\frac{5}{4}} - 9\sqrt[4]{d} + 10d^{\frac{4}{3}} + 2\sqrt[3]{d}$$

Exercise 6.62 Evaluate the following indefinite integral

$$\int -8m^{\frac{6}{5}}s^{\frac{5}{6}} - 9m^{\frac{2}{3}}\sqrt{s} + 9s^{\frac{4}{5}} + 5dm$$

.

Exercise 6.63 Evaluate the definite integral for the following function, where the limits are 2 and 3.

$$f(t) = 10t^{\frac{3}{4}} + 9t^{\frac{2}{3}} - t^2 + 12$$

Exercise 6.64 Evaluate the following indefinite integral

$$\int -u^6w^4 + 7u^4 - 10u^3w^3 + 10uw^3 + 4w^2dw$$

.

Exercise 6.65 Integrate the polynomial with respect to d

$$-7d^{2} - 8d + 1$$

Exercise 6.66 Evaluate the definite integral for the following function, where the limits are 1 and 2.

$$f(r) = 7r^{\frac{4}{5}} - 6r^{\frac{3}{2}} + 9$$

Exercise 6.67 Evaluate the following indefinite integral

$$\int -7b^{\frac{3}{2}}c^{6} + 9c - 9dc$$

.

Exercise 6.68 Evaluate the following indefinite integral

$$\int 2w^{6}z^{6} - 9w^{5}z - 5z^{6} + 4z^{4} + 9dw$$

.

Exercise 6.69 Evaluate the following indefinite integral

$$\int -6\sqrt[5]{s}u^{6} + 8s^{\frac{5}{4}} + 8su^{\frac{2}{3}}ds$$

.

Exercise 6.70 Evaluate the following indefinite integral

$$\int -8c^{\frac{3}{4}} - 3\sqrt{c} - 3l^{\frac{3}{5}}dc$$

.

Exercise 6.71 Evaluate the following indefinite integral

$$\int 4m^{4}q^{2} + 7q^{6} + 7dq$$

.

Exercise 6.72 Evaluate the following indefinite integral

$$\int 8\sqrt{z} + 4z^{2} - 9dm$$

.

Exercise 6.73 Integrate the polynomial with respect to r

$$-2r^{4} - 10r^{3} + 7r^{2} + 4r + 10$$

Exercise 6.74 Evaluate the definite integral for the following function, where the limits are 2 and 6.

$$f(t) = 10t^{\frac{5}{2}} - 6\sqrt{t} + 5t^{5} + 10t^{4} - 10$$

Exercise 6.75 Evaluate the following indefinite integral

$$\int -8l^{4}z^{4} + 4l^{3}z^{2} - 8l^{3} - 6l^{2} - 6dz$$

.

Exercise 6.76 Evaluate the following indefinite integral

$$\int -8q^{\frac{6}{5}}\sqrt[3]{w} - 10q^{\frac{4}{5}} - 7q^{\frac{5}{3}} - 3q + 9\sqrt{w}dq$$

.

Exercise 6.77 Integrate the polynomial with respect to w

$$-9w^4 + 10w^3 - 9w^2 - 9w - 4$$

Exercise 6.78 Evaluate the following indefinite integral

$$\int -m^6 - m^5x^5 - 8m + 10x^3 + 2dx$$

.

Exercise 6.79 Evaluate the following indefinite integral

$$\int -4m^5 - 3m^3x^6 + m^2x^2 - 7m + 6xdx$$

.

Exercise 6.80 Integrate the polynomial with respect to p

$$7p^3 + 2p^2 - p + 2$$

Exercise 6.81 Evaluate the following indefinite integral

$$\int a^2 \cdot \left(9a^3 + 10as^6 - 3as^3 + 10s^6\right) ds$$

.

Exercise 6.82 Evaluate the following indefinite integral

$$\int m\left(4m^5 - 2m^4 + 9mw + 10w^4\right) dw$$

.

Exercise 6.83 Evaluate the definite integral for the following function, where the limits are 0 and 3.

$$f(d) = -d^{\frac{5}{3}} - 4\sqrt{d} + 4d$$

Exercise 6.84 Evaluate the definite integral for the following function, where the limits are 0 and 2.

$$f(n) = -7n^{\frac{5}{4}} - 6n^{\frac{5}{3}} - 10n^2 - 2n$$

Exercise 6.85 Evaluate the following indefinite integral

$$\int 9a - x^6 + 8x^4 + x^2dx$$

.

Exercise 6.86 Evaluate the following indefinite integral

$$\int 5\sqrt[3]{c}\sqrt[3]{q} + 2\sqrt{c} + 9c^2q - 6cdc$$

.

Exercise 6.87 Evaluate the following indefinite integral

$$\int rw\left(-10r^4+10r^2w-9w^3\right)dw$$

.

Exercise 6.88 Evaluate the following indefinite integral

$$\int w^4\left(m^6+1\right)dw$$

.

Exercise 6.89 Evaluate the definite integral for the following function, where the limits are 1 and 4.

$$f(d)=-2d^{\frac{2}{3}}-d^{\frac{5}{2}}-8d^{\frac{3}{2}}-11d^4$$

Exercise 6.90 Integrate the polynomial with respect to b

$$-9b^4-9b^3-b^2-9b+4$$

Exercise 6.91 Evaluate the following indefinite integral

$$\int 10p^4w+5p^3w^{\frac{2}{3}}-5wdp$$

.

Exercise 6.92 Integrate the polynomial with respect to z

$$4z^3-4z^2+10z-1$$

Exercise 6.93 Evaluate the definite integral for the following function, where the limits are 2 and 4.

$$f(l)=7l^{\frac{5}{3}}-8\sqrt[3]{l}-2l^6$$

Exercise 6.94 Evaluate the following indefinite integral

$$\int 10t^{\frac{3}{5}}\sqrt[6]{y}-9t^{\frac{2}{3}}y^{\frac{5}{4}}+3t^{\frac{3}{2}}y^5-y^{\frac{3}{2}}dt$$

.

Exercise 6.95 Evaluate the following indefinite integral

$$\int -3s^{\frac{2}{5}}-2s-8z^{\frac{3}{2}}dz$$

.

Exercise 6.96 Evaluate the following indefinite integral

$$\int -2\sqrt[6]{v}y^5-3v^{\frac{5}{2}}+3\sqrt{v}-5ydv$$

.

Exercise 6.97 Evaluate the following indefinite integral

$$\int 9s^6-9s^2v^3-4s+5v^3dv$$

.

Exercise 6.98 Evaluate the definite integral for the following function, where the limits are 0 and 3.

$$f(t) = 8t^{\frac{4}{3}} + 9\sqrt{t} - t$$

Exercise 6.99 Integrate the polynomial with respect to t

$$-5t^2 + 6t + 9$$

Exercise 6.100 Evaluate the definite integral for the following function, where the limits are 1 and 4.

$$f(z) = 9z^{\frac{3}{5}} + 10z^{\frac{5}{3}} + 8z$$

6.2 Solutions

1. $$\frac{25w^{\frac{6}{5}}y^{\frac{8}{5}}}{4} - w^{\frac{3}{2}}y - \frac{8w^2y^3}{3} + 2y^{\frac{5}{2}} + 10y$$

2. $$\frac{w\left(-20m^5w - 45m^3 + w^4 \cdot \left(4m^2 + 15\right)\right)}{5}$$

3. $$\frac{10c^{\frac{9}{5}}s^6}{9} - \frac{50c^{\frac{9}{5}}s}{9} + \frac{16c^{\frac{3}{2}}}{3} - 6c\sqrt[4]{s}$$

4. $$\frac{q\left(21q^3 - 8q^2 + 36q - 48\right)}{12}$$

5. $$\frac{d\left(4d^4 + 5d^3 - 10d^2 + 25d + 10\right)}{5}$$

6. The indefinite integral is

   $$\frac{16r^{\frac{9}{4}}}{9} + \frac{4r^3}{3} - \frac{7r^2}{2} + 7r$$

   The value of the definite integral is 78.4366018452598.

7. $$\frac{a\left(-18a^6 - 21a^4z - 7a^2z - 105az^2 - 42\right)}{21}$$

8. $$\frac{w\left(36w^4 - 15w^3 - 10w^2 - 90w + 150\right)}{30}$$

9. The indefinite integral is

$$\frac{36d^{\frac{11}{6}}}{11}+8d^{\frac{5}{4}}+\frac{6d^5}{5}+4d^2+9d$$

The value of the definite integral is 77.6172606956989.

10.

$$\frac{v\left(8v^2+27v+18\right)}{6}$$

11.

$$\frac{c\left(8c^2-3c+24\right)}{3}$$

12.

$$-\frac{50b^{\frac{9}{5}}}{9}+4by^{\frac{5}{3}}+10by^6+4b$$

13.

$$\frac{z\left(24z^4-45z^3+20z^2-30z+160\right)}{20}$$

14.

$$-\frac{6a^{\frac{7}{6}}}{7}-\frac{15a^{\frac{6}{5}}}{2}+\frac{32a^{\frac{5}{4}}\sqrt[5]{u}}{5}+\frac{a^6}{3}+6a\sqrt[3]{u}$$

15.

$$\frac{t\left(7q^4t+q^4\cdot(70q+56)+18t^6\right)}{14}$$

16. The indefinite integral is

$$-\frac{10w^{\frac{6}{5}}}{3}-9w$$

The value of the definite integral is -28.5381849615317.

17.

$$\frac{cy\left(2c^6y^4-7c^3y+63y^5+35y^2-42\right)}{7}$$

18.

$$\frac{m\left(-6m^3-14m^2+27m+6\right)}{6}$$

19.

$$a\left(-3\sqrt{n}-4n^6+7\right)$$

20.

$$\frac{z\left(w^4\cdot(18w-30)-5w^2z^5+15z^3\right)}{6}$$

21.

$$-\frac{4r^{\frac{9}{4}}y}{3}-\frac{4r^{\frac{3}{2}}\sqrt{y}}{3}+\frac{7r^2}{2}$$

22.

$$\frac{r^2t\left(r^4t^6-70r-21t^2\right)}{7}$$

23.

$$a\left(\frac{3a^6\cdot\left(3-2q^6\right)}{7}-\frac{a^3q}{4}+3q^2\left(q^4-1\right)\right)$$

24.

$$\frac{x\left(5u^{\frac{6}{5}}x^6+14\sqrt[4]{u}x+21u\right)}{7}$$

25. The indefinite integral is

$$\frac{18m^{\frac{7}{6}}}{7}-\frac{5m^{\frac{7}{5}}}{7}+\frac{5m^{\frac{6}{5}}}{3}+\frac{24m^{\frac{7}{3}}}{7}+\frac{9m^2}{2}$$

The value of the definite integral is 269.128746694918.

26.

$$\frac{m\left(12m^4+135m^3-140m^2+210m+360\right)}{60}$$

27.

$$\frac{y\left(y^3z-12y^2+20yz^5+8z\left(z^2-1\right)\right)}{4}$$

28. The indefinite integral is

$$\frac{15s^{\frac{4}{3}}}{2}-\frac{4s^{\frac{5}{2}}}{5}-\frac{10s^{\frac{3}{2}}}{3}-\frac{s^7}{7}+\frac{7s^2}{2}$$

The value of the definite integral is -2289.87559132634.

29.

$$\frac{y\left(-5y^3+2y^2+y+4\right)}{2}$$

30.

$$\frac{10q^{\frac{6}{5}}\sqrt[3]{u}}{3}-3q^{\frac{7}{3}}+4qu^2-7q$$

31.

$$\frac{t\left(36t^4-15t^3+50t^2-45t+120\right)}{30}$$

32.

$$\frac{q\left(21q^4-15q^3+25q^2+15q+120\right)}{15}$$

33. The indefinite integral is

$$-\frac{20r^{\frac{11}{5}}}{11}-\frac{24r^{\frac{5}{3}}}{5}+9r$$

The value of the definite integral is -27.4287578382460.

34.

$$\frac{x\left(x^3-4x^2+4x-20\right)}{2}$$

35.

$$\frac{q\left(54q^4-15q^3-100q^2+150q-30\right)}{30}$$

36. The indefinite integral is

$$\frac{10c^{\frac{7}{2}}}{7}-\frac{6c^{\frac{5}{2}}}{5}-4c^2$$

The value of the definite integral is -2.85435581512691.

37.

$$\frac{a\left(5a^5+36w\right)}{6}$$

38. The indefinite integral is

$$13n$$

The value of the definite integral is 52.0000000000000.

39.

$$\frac{2yz^{\frac{5}{2}}}{5}-3yz-\frac{20z^{\frac{9}{5}}}{9}-2z^3$$

40. The indefinite integral is

$$\frac{25p^{\frac{9}{5}}}{9}-\frac{3p^{\frac{8}{3}}}{8}-\frac{2p^{\frac{7}{2}}}{7}-\frac{p^2}{2}$$

The value of the definite integral is -71.4995101481135.

41.

$$r\left(-r^6-8t^6-8\right)$$

42.

$$\frac{a\left(-7a^5+a^3\left(-15n^4-12\right)+n^4\left(-48n^2-36\right)\right)}{6}$$

43.

$$\frac{24c^{\frac{5}{3}}\sqrt[3]{x}}{5}+\frac{c^2\sqrt{x}}{2}+\frac{3c^2}{2}-7c\sqrt[6]{x}$$

44. The indefinite integral is

$$-\frac{6w^{\frac{7}{6}}}{7}-\frac{5w^{\frac{6}{5}}}{6}+\frac{4w^{\frac{7}{4}}}{7}-\frac{16w^{\frac{5}{4}}}{5}-w^2$$

The value of the definite integral is -5.31904761904762.

45.

$$\frac{36c^{\frac{6}{5}}x^{\frac{7}{6}}}{7}+\frac{2\sqrt[5]{c}x^{\frac{3}{2}}}{3}-\frac{10c^{\frac{2}{3}}x^{\frac{7}{2}}}{7}+\frac{6\sqrt{c}x^{\frac{7}{3}}}{7}-\frac{30x^{\frac{7}{6}}}{7}$$

46.

$$-\frac{30q^6z^{\frac{11}{5}}}{11}+3q^4z+\frac{4z^{\frac{9}{4}}}{9}-\frac{16z^{\frac{3}{2}}}{3}-4z$$

47.

$$\frac{s\left(s^4-10v^6\right)}{5}$$

48.

$$\frac{z\left(20z^2-21z+42\right)}{6}$$

49.

$$z^3\left(\frac{c^2\cdot(15c+40)}{15}+\frac{z^2\cdot\left(24c^4+6\right)}{15}\right)$$

50.

$$\frac{n\left(2n^2+3n+24\right)}{3}$$

51.

$$\frac{40n^{\frac{7}{5}}x^{\frac{4}{5}}}{7}-4n^2x^{\frac{2}{3}}+n\sqrt[3]{x}+7nx-8n$$

52.

$$\frac{p\left(17p^4-40v^3\right)}{5}$$

53.

$$\frac{2s^{\frac{5}{2}}x^5}{5}+\frac{10s^{\frac{3}{2}}}{3}+\frac{5s^2}{2}$$

54.

$$\frac{w\left(35w^5+6w^4+75wx^5-150\right)}{30}$$

55.

$$\frac{20n^{\frac{11}{5}}}{11}+\frac{9n^{\frac{4}{3}}\sqrt[5]{u}}{2}+\frac{10n^{\frac{3}{2}}}{3}+\frac{2n^3u^2}{3}-5n\sqrt[3]{u}$$

56.

$$\frac{q^2\cdot\left(4q^3\cdot\left(3-2c^2\right)-15\right)}{10}$$

57. The indefinite integral is

$$-\frac{16p^{\frac{7}{4}}}{7}+\frac{20p^{\frac{3}{2}}}{3}+8p^2-3p$$

The value of the definite integral is 106.305443157761.

58.

$$\frac{d\left(36d^4-75d^3+80d^2-105d-300\right)}{30}$$

59.

$$\frac{4r^{\frac{5}{4}}z^6}{3}+2\sqrt{r}z^{\frac{3}{2}}+3z^{\frac{5}{3}}+\frac{3z^2}{2}$$

60.

$$\frac{n\left(-n^2+9n+15\right)}{3}$$

61. The indefinite integral is

$$5d^{\frac{6}{5}}+\frac{4d^{\frac{9}{4}}}{9}-\frac{36d^{\frac{5}{4}}}{5}+\frac{30d^{\frac{7}{3}}}{7}+\frac{3d^{\frac{4}{3}}}{2}$$

The value of the definite integral is 171.457243247689.

62.

$$-\frac{40m^{\frac{11}{5}}s^{\frac{5}{6}}}{11}-\frac{27m^{\frac{5}{3}}\sqrt{s}}{5}+9ms^{\frac{4}{5}}+5m$$

63. The indefinite integral is

$$\frac{40t^{\frac{7}{4}}}{7}+\frac{27t^{\frac{5}{3}}}{5}-\frac{t^3}{3}+12t$$

The value of the definite integral is 42.0768675808480.

64.

$$\frac{w\left(-6u^6w^4+210u^4+75uw^3\cdot\left(1-u^2\right)+40w^2\right)}{30}$$

65.

$$\frac{d\left(-7d^2-12d+3\right)}{3}$$

66. The indefinite integral is

$$\frac{35r^{\frac{9}{5}}}{9} - \frac{12r^{\frac{5}{2}}}{5} + 9r$$

The value of the definite integral is 7.47655856360244.

67.

$$\frac{c\left(-2b^{\frac{3}{2}}c^6 + 9c - 18\right)}{2}$$

68.

$$\frac{w\left(4w^6z^6 - 21w^5z - 70z^6 + 56z^4 + 126\right)}{14}$$

69.

$$-5s^{\frac{6}{5}}u^6 + \frac{32s^{\frac{9}{4}}}{9} + 4s^2u^{\frac{2}{3}}$$

70.

$$-\frac{32c^{\frac{7}{4}}}{7} - 2c^{\frac{3}{2}} - 3cl^{\frac{3}{5}}$$

71.

$$q\left(\frac{4m^4q^2}{3} + q^6 + 7\right)$$

72.

$$m\left(8\sqrt{z} + 4z^2 - 9\right)$$

73.

$$\frac{r\left(-12r^4 - 75r^3 + 70r^2 + 60r + 300\right)}{30}$$

74. The indefinite integral is

$$\frac{20t^{\frac{7}{2}}}{7} - 4t^{\frac{3}{2}} + \frac{5t^6}{6} + 2t^5 - 10t$$

The value of the definite integral is 55706.5528383165.

75.

$$\frac{2z\left(-12l^4z^4 + 10l^3z^2 - 60l^3 - 45l^2 - 45\right)}{15}$$

76.

$$-\frac{40q^{\frac{11}{5}}\sqrt[3]{w}}{11} - \frac{50q^{\frac{9}{5}}}{9} - \frac{21q^{\frac{8}{3}}}{8} - \frac{3q^2}{2} + 9q\sqrt{w}$$

77.

$$\frac{w\left(-18w^4 + 25w^3 - 30w^2 - 45w - 40\right)}{10}$$

78.

$$\frac{x\left(-6m^6-m^5x^5-48m+15x^3+12\right)}{6}$$

79.

$$\frac{x\left(-84m^5-9m^3x^6+7m^2x^2-147m+63x\right)}{21}$$

80.

$$\frac{p\left(21p^3+8p^2-6p+24\right)}{12}$$

81.

$$\frac{a^2s\left(252a^3-21as^3+40s^6\left(a+1\right)\right)}{28}$$

82.

$$\frac{mw\left(m^4\cdot\left(8m-4\right)+9mw+4w^4\right)}{2}$$

83. The indefinite integral is

$$-\frac{3d^{\frac{8}{3}}}{8}-\frac{8d^{\frac{3}{2}}}{3}+2d^2$$

The value of the definite integral is -2.87668936335119.

84. The indefinite integral is

$$-\frac{28n^{\frac{9}{4}}}{9}-\frac{9n^{\frac{8}{3}}}{4}-\frac{10n^3}{3}-n^2$$

The value of the definite integral is -59.7522980099699.

85.

$$\frac{x\left(945a-15x^6+168x^4+35x^2\right)}{105}$$

86.

$$\frac{15c^{\frac{4}{3}}\sqrt[3]{q}}{4}+\frac{4c^{\frac{3}{2}}}{3}+3c^3q-3c^2$$

87.

$$\frac{rw^2\left(-75r^4+50r^2w-27w^3\right)}{15}$$

88.

$$\frac{w^5\left(m^6+1\right)}{5}$$

89. The indefinite integral is

$$-\frac{6d^{\frac{5}{3}}}{5}-\frac{2d^{\frac{7}{2}}}{7}-\frac{16d^{\frac{5}{2}}}{5}-\frac{11d^5}{5}$$

The value of the definite integral is -2396.98095636470.

90.

$$\frac{b\left(-108b^4-135b^3-20b^2-270b+240\right)}{60}$$

91.

$$\frac{p\left(8p^4w+5p^3w^{\frac{2}{3}}-20w\right)}{4}$$

92.

$$z\left(z^3-\frac{4z^2}{3}+5z-1\right)$$

93. The indefinite integral is

$$\frac{21l^{\frac{8}{3}}}{8}-6l^{\frac{4}{3}}-\frac{2l^7}{7}$$

The value of the definite integral is -4578.38434407442.

94.

$$\frac{25t^{\frac{8}{5}}\sqrt[6]{y}}{4}-\frac{27t^{\frac{5}{3}}y^{\frac{5}{4}}}{5}+\frac{6t^{\frac{5}{2}}y^5}{5}-ty^{\frac{3}{2}}$$

95.

$$-3s^{\frac{2}{5}}z-2sz-\frac{16z^{\frac{5}{2}}}{5}$$

96.

$$-\frac{12v^{\frac{7}{6}}y^5}{7}-\frac{6v^{\frac{7}{2}}}{7}+2v^{\frac{3}{2}}-5vy$$

97.

$$v\left(s\left(9s^5-4\right)+\frac{v^3\cdot\left(5-9s^2\right)}{4}\right)$$

98. The indefinite integral is

$$\frac{24t^{\frac{7}{3}}}{7}+6t^{\frac{3}{2}}-\frac{t^2}{2}$$

The value of the definite integral is 71.1806155628684.

99.

$$\frac{t\left(-5t^2+9t+27\right)}{3}$$

100. The indefinite integral is

$$\frac{45z^{\frac{8}{5}}}{8}+\frac{15z^{\frac{8}{3}}}{4}+4z^2$$

The value of the definite integral is 253.506951962251.