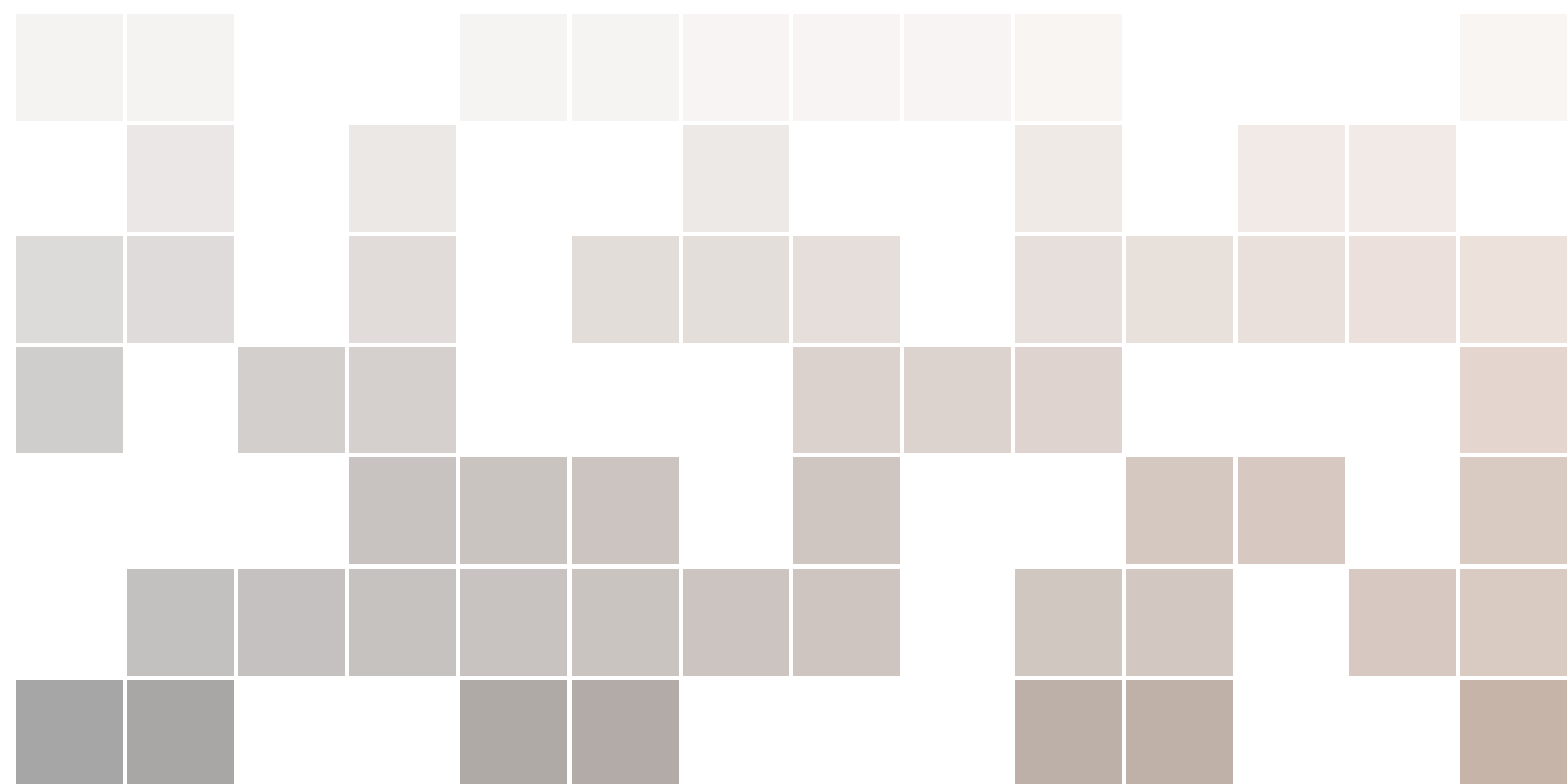


VCE Mathematics Methods: Practice problems and solutions (Book 2)



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1. Introduction

1.1 Motivation

My year 9 daughter kept asking me to write her new math problems, apart from those in the text book, so she can do more practice. I'd take a piece of paper and write problems, but it's a tedious process. So I thought: how about I **generate tons** of them?

1.2 Problem generation method

The problems and solutions have all been generated using the python programming language, and relied on a number of open source python packages, especially the SymPy package. Therefore, you can trust the accuracy of the solutions.

1.3 Structure

This document includes hundreds of mathematics problems that have been designed for year 11 students enrolled in VCE Mathematics Methods.

Most of the problems require the student to perform a single task, such as calculating the derivative of a function or finding the prime numbers within a range of integers. In addition, a number of questions have been designed based on the Mathematics Methods exam questions, which require the student to perform two or more tasks.

I hope you find them useful.

2. Numbers

2.1 Problems

Exercise 2.1 Find the prime factor decomposition of 109.

Exercise 2.2 What are the lowest common multiple (LCM) of 346 and 770?

Exercise 2.3 What are the prime numbers in the range 79 and 97?

Exercise 2.4 Simplify the expression

$$\sqrt[3]{3024}$$

Exercise 2.5 Find the prime factor decomposition of 130.

Exercise 2.6 What are the lowest common multiple (LCM) of 728 and 401?

Exercise 2.7 What are the prime numbers in the range 506 and 520?

Exercise 2.8 What are the highest common factor (HCF) of 967 and 374?

Exercise 2.9 Simplify the expression

$$\sqrt[3]{1250}$$

Exercise 2.10 Simplify the expression

$$\sqrt[3]{20000}$$

Exercise 2.11 Find the prime factor decomposition of 67.

Exercise 2.12 Simplify the expression

$$\sqrt{225}$$

Exercise 2.13 What are the prime numbers in the range 156 and 173?

Exercise 2.14 What are the lowest common multiple (LCM) of 371 and 209?

Exercise 2.15 What are the lowest common multiple (LCM) of 226 and 649?

Exercise 2.16 What are the prime numbers in the range 165 and 184?

Exercise 2.17 Simplify the expression

$$\sqrt[3]{1715}$$

Exercise 2.18 What are the highest common factor (HCF) of 602 and 126?

Exercise 2.19 What are the prime numbers in the range 350 and 366?

Exercise 2.20 What are the prime numbers in the range 58 and 76?

Exercise 2.21 What are the highest common factor (HCF) of 99 and 958?

Exercise 2.22 What are the prime numbers in the range 161 and 172?

Exercise 2.23 Simplify the expression

$$\sqrt[3]{13122}$$

Exercise 2.24 What are the prime numbers in the range 139 and 159?

Exercise 2.25 Find the prime factor decomposition of 130.

Exercise 2.26 Simplify the expression

$$\sqrt{24}$$

Exercise 2.27 Simplify the expression

$$\sqrt{1088}$$

Exercise 2.28 What are the lowest common multiple (LCM) of 829 and 147?

Exercise 2.29 Find the prime factor decomposition of 103.

Exercise 2.30 What are the prime numbers in the range 622 and 642?

Exercise 2.31 What are the highest common factor (HCF) of 473 and 816?

Exercise 2.32 Simplify the expression

$$\sqrt{153}$$

Exercise 2.33 What are the prime numbers in the range 520 and 539?

Exercise 2.34 What are the lowest common multiple (LCM) of 75 and 87?

Exercise 2.35 What are the lowest common multiple (LCM) of 378 and 218?

Exercise 2.36 What are the prime numbers in the range 11 and 27?

Exercise 2.37 Simplify the expression

$$\sqrt{225}$$

Exercise 2.38 What are the prime numbers in the range 749 and 759?

Exercise 2.39 Find the prime factor decomposition of 117.

Exercise 2.40 Simplify the expression

$$\sqrt[3]{2125}$$

Exercise 2.41 Simplify the expression

$$\sqrt[3]{3000}$$

Exercise 2.42 Find the prime factor decomposition of 183.

Exercise 2.43 Simplify the expression

$$\sqrt{1600}$$

Exercise 2.44 What are the highest common factor (HCF) of 117 and 522?

Exercise 2.45 What are the prime numbers in the range 137 and 149?

Exercise 2.46 What are the prime numbers in the range 63 and 76?

Exercise 2.47 What are the prime numbers in the range 48 and 66?

Exercise 2.48 Simplify the expression

$$\sqrt[3]{48}$$

Exercise 2.49 Simplify the expression

$$\sqrt[3]{19000}$$

Exercise 2.50 Find the prime factor decomposition of 43.

Exercise 2.51 What are the prime numbers in the range 156 and 175?

Exercise 2.52 What are the prime numbers in the range 164 and 177?

Exercise 2.53 What are the highest common factor (HCF) of 616 and 91?

Exercise 2.54 Simplify the expression

$$\sqrt[3]{112}$$

Exercise 2.55 Find the prime factor decomposition of 150.

Exercise 2.56 What are the highest common factor (HCF) of 790 and 844?

Exercise 2.57 Find the prime factor decomposition of 18.

Exercise 2.58 Find the prime factor decomposition of 134.

Exercise 2.59 What are the prime numbers in the range 171 and 187?

Exercise 2.60 What are the lowest common multiple (LCM) of 711 and 59?

Exercise 2.61 Simplify the expression

$$\sqrt[3]{56}$$

Exercise 2.62 What are the prime numbers in the range 235 and 255?

Exercise 2.63 What are the prime numbers in the range 470 and 487?

Exercise 2.64 What are the prime numbers in the range 32 and 48?

Exercise 2.65 Simplify the expression

$$\sqrt{56}$$

- Exercise 2.66** Find the prime factor decomposition of 65.
Exercise 2.67 What are the highest common factor (HCF) of 237 and 880?
Exercise 2.68 Find the prime factor decomposition of 98.
Exercise 2.69 What are the lowest common multiple (LCM) of 797 and 481?
Exercise 2.70 What are the highest common factor (HCF) of 8 and 958?
Exercise 2.71 What are the lowest common multiple (LCM) of 654 and 797?
Exercise 2.72 Simplify the expression

$$\sqrt{833}$$

- Exercise 2.73** What are the lowest common multiple (LCM) of 516 and 866?
Exercise 2.74 Find the prime factor decomposition of 106.
Exercise 2.75 Simplify the expression

$$\sqrt{208}$$

- Exercise 2.76** What are the highest common factor (HCF) of 392 and 39?
Exercise 2.77 Simplify the expression

$$\sqrt[3]{625}$$

- Exercise 2.78** What are the prime numbers in the range 848 and 858?
Exercise 2.79 What are the prime numbers in the range 200 and 215?
Exercise 2.80 Find the prime factor decomposition of 64.
Exercise 2.81 Simplify the expression

$$\sqrt[3]{832}$$

- Exercise 2.82** What are the prime numbers in the range 19 and 36?
Exercise 2.83 What are the prime numbers in the range 100 and 120?
Exercise 2.84 What are the lowest common multiple (LCM) of 115 and 234?
Exercise 2.85 Find the prime factor decomposition of 40.
Exercise 2.86 What are the prime numbers in the range 992 and 1007?
Exercise 2.87 What are the lowest common multiple (LCM) of 473 and 449?
Exercise 2.88 Simplify the expression

$$\sqrt{243}$$

- Exercise 2.89** Find the prime factor decomposition of 120.
Exercise 2.90 What are the highest common factor (HCF) of 121 and 680?
Exercise 2.91 What are the highest common factor (HCF) of 700 and 297?
Exercise 2.92 Simplify the expression

$$\sqrt{384}$$

Exercise 2.93 Simplify the expression

$$\sqrt{216}$$

Exercise 2.94 Simplify the expression

$$\sqrt{686}$$

Exercise 2.95 Simplify the expression

$$\sqrt[3]{4608}$$

Exercise 2.96 Simplify the expression

$$\sqrt{288}$$

Exercise 2.97 What are the lowest common multiple (LCM) of 10 and 833?

Exercise 2.98 What are the lowest common multiple (LCM) of 374 and 476?

Exercise 2.99 What are the highest common factor (HCF) of 5 and 248?

Exercise 2.100 What are the prime numbers in the range 171 and 183?

2.2 Solutions

1.

$$109^1$$

2. 133210

3. 79, 83, 89

4.

$$6 \cdot \sqrt[3]{14}$$

5.

$$13^1 \cdot 2^1 \cdot 5^1$$

6. 291928

7. 509

8. 1

9.

$$5 \cdot \sqrt[3]{10}$$

10.

$$10 \cdot \sqrt[3]{20}$$

11.

$$67^1$$

12.

15

13. 157, 163, 167

14. 77539

15. 146674

16. 167, 173, 179, 181

17.

$$7 \cdot \sqrt[3]{5}$$

18. 14

19. 353, 359

20. 59, 61, 67, 71, 73

21. 1

22. 163, 167

23.

$$9 \cdot \sqrt[3]{18}$$

24. 139, 149, 151, 157

25.

$$13^1 \cdot 2^1 \cdot 5^1$$

26.

$$2\sqrt{6}$$

27.

$$8\sqrt{17}$$

28. 121863

29.

$$103^1$$

30. 631, 641

31. 1

32.

$$3\sqrt{17}$$

33. 521, 523

34. 2175

35. 41202

36. 11, 13, 17, 19, 23

37.

15

38. 751, 757

39.

$$13^1 \cdot 3^2$$

40.

$$5 \cdot \sqrt[3]{17}$$

41.

$$10 \cdot \sqrt[3]{3}$$

42.

$$3^1 \cdot 61^1$$

43.

$$40$$

44. 9

45. 137, 139

46. 67, 71, 73

47. 53, 59, 61

48.

$$2 \cdot \sqrt[3]{6}$$

49.

$$10 \cdot \sqrt[3]{19}$$

50.

$$43^1$$

51. 157, 163, 167, 173

52. 167, 173

53. 7

54.

$$2 \cdot \sqrt[3]{14}$$

55.

$$2^1 \cdot 3^1 \cdot 5^2$$

56. 2

57.

$$2^1 \cdot 3^2$$

58.

$$2^1 \cdot 67^1$$

59. 173, 179, 181

60. 41949

61.

$$2 \cdot \sqrt[3]{7}$$

62. 239, 241, 251

63. 479

64. 37, 41, 43, 47

65.

$$2\sqrt{14}$$

66.

$$13^1 \cdot 5^1$$

67. 1

68.

$$2^1 \cdot 7^2$$

69. 383357

70. 2

71. 521238

72.

$$7\sqrt{17}$$

73. 223428

74.

$$2^1 \cdot 53^1$$

75.

$$4\sqrt{13}$$

76. 1

77.

$$5 \cdot \sqrt[3]{5}$$

78. 853, 857

79. 211

80.

$$2^6$$

81.

$$4 \cdot \sqrt[3]{13}$$

82. 19, 23, 29, 31

83. 101, 103, 107, 109, 113

84. 26910

85.

$$2^3 \cdot 5^1$$

86. 997

87. 212377

88.

$$9\sqrt{3}$$

89.

$$2^3 \cdot 3^1 \cdot 5^1$$

90. 1

91. 1

92.

$$8\sqrt{6}$$

93.

$$6\sqrt{6}$$

94.

$$7\sqrt{14}$$

95.

$$8 \cdot 3^{\frac{2}{3}}$$

96.

$$12\sqrt{2}$$

97. 8330

98. 5236

99. 1

100. 173, 179, 181

3. Algebra

3.1 Problems

Exercise 3.1 Simplify

$$\frac{6ac^4 - 3c}{6a^5c^8 - 3a^4c^5 + 18a^2c^4 - 9ac}$$

Exercise 3.2 Solve the following inequality

$$-x - 3 \geq -1$$

Exercise 3.3 Simplify

$$\frac{5b^2}{10b^3 - 10b^2} + \frac{12b}{11}$$

Exercise 3.4 Solve the following inequality for w

$$8b^2 + 4w \leq -6b + w$$

Exercise 3.5 Expand

$$(-2s^3 - 4s)^2 (-5r^4s + 7r^4)$$

Exercise 3.6 Simplify

$$-\frac{u}{9} + \frac{1}{18} + \frac{3w-2}{-2w-1} + \frac{5}{14w}$$

Exercise 3.7 Expand

$$(6m^2 + 7mn)(-3m^3 + 6m^2)^2$$

Exercise 3.8 Expand

$$(a^2b^3 - 6ac^3d - 2c)^2$$

Exercise 3.9 Complete the square for the following expression and then solve the equation:

$$-28n^2 + 78n$$

Exercise 3.10 Expand and simplify

$$(9p^3 - 6q^3)^2 (-p^3q^4 - 10)$$

Exercise 3.11 Simplify

$$\frac{-25l^4m^3 + 45l^4n^2 - 35l^3m^3 + 63l^3n^2}{-5l^4 - 7l^3}$$

Exercise 3.12 Simplify

$$\frac{7r^2s}{(-1)21r^3s}$$

Exercise 3.13 If $a = -p^2 + 6p$, show that

$$(a - x)^2 + (a + x)^2 = 2p^4 - 24p^3 + 72p^2 + 2x^2$$

Exercise 3.14 Simplify

$$\frac{-5r^4s^4 + 30r^2s^4 + 7r^2s^3 - 42s^3}{r^2 - 6}$$

Exercise 3.15 Simplify

$$\frac{\frac{-2w-8}{6w+1} - \frac{15}{-6w-10}}{-\frac{4w-10}{5w-10} + \frac{10-2w}{3-9w}}$$

Exercise 3.16 Simplify

$$\frac{-10ab^3c^3 + 14ab^3 + 20ac^5 - 28ac^2 - 20b^4c^4 + 28b^4c + 40b^2c^6 - 56b^2c^3}{5c^3 - 7}$$

Exercise 3.17 Solve the following inequality

$$m - 2 < -8$$

Exercise 3.18 Simplify

$$\frac{3b}{2q+10} + \frac{6-7q}{-9b-8q} - \frac{18}{6bq-9}$$

Exercise 3.19 Simplify

$$\frac{(-1)7p^4q^2}{35p^7q^3 + 21p^5q^5 + 14p^4q^4}$$

Exercise 3.20 Simplify

$$\frac{15a^3b^3c^7 - 30a^2c^5}{5a^3b^3c^4 - 10a^2c^2}$$

Exercise 3.21 Solve the following simultaneous equations for x and y :

$$6x - 5y + 6 = 0$$

$$-7x + 10y + 8 = 0$$

Exercise 3.22 Simplify

$$10 + \frac{10p - 1}{3p + 10}$$

Exercise 3.23 Expand

$$(3m^4n^3 - 8)(-10m^4n^4 + 10m^4n^2 + 4m^4n)$$

Exercise 3.24 Solve the following inequality for s

$$-5s - 6w^2 \leq s - 3$$

Exercise 3.25 Simplify

$$-\frac{10w}{7} - \frac{2}{-9dw + w} + \frac{3 - 8d}{5d}$$

Exercise 3.26 Simplify

$$-\frac{9v^3 - 2v}{-3v^2 - 8v} - \frac{1}{2v^2}$$

Exercise 3.27 Solve the following simultaneous equations for x and y :

$$8x + 2y - 5 = 0$$

$$-2x + 6y + 1 = 0$$

Exercise 3.28 Expand

$$(p^2 + 5r^2)(-2p^4q^4 - q^2r + 9r^4)$$

Exercise 3.29 Solve the equation in x :

$$ax + x = 0$$

Exercise 3.30 Solve the following simultaneous equations for x and y :

$$-4x + 3y - 2 = 0$$

$$-x - 4y - 10 = 0$$

Exercise 3.31 Solve the following simultaneous equations for x and y :

$$-7x + y - 3 = 0$$

$$-3x + 10y + 7 = 0$$

Exercise 3.32 Simplify

$$\frac{16r^4s^7 + 36r^2s^3 - 40s^4}{4s^3}$$

Exercise 3.33 Expand

$$(6l^3 + 2m^2n^3)(-5l^2m^4n + 1)(-l^4m^3n^2 + 4m^4n^3)$$

Exercise 3.34 Simplify

$$\frac{-18m^8n^5 - 26m^5n - 13m^4n^5 - 13mn - 2n^5}{-2m^4n - n}$$

Exercise 3.35 If $a = 7pq^3 + 10q^4$, show that

$$-(a-x)^3 + (a+x)^3 = (7pq^3 + 10q^4 - x)^3 + (7pq^3 + 10q^4 + x)^3$$

Exercise 3.36 Complete the square for the following expression and then solve the equation:

$$12u^2 + 20u$$

Exercise 3.37 Solve the equation in x :

$$8ax + 16x + 5 = 0$$

Exercise 3.38 Expand

$$(l^2 + m^3n^3)(-9l^4m^3n^2 + l^2m^2n^4)^2$$

Exercise 3.39 Expand

$$(5a^4b^2c^2 + 9a^3b^3 + 7a^3c)^2$$

Exercise 3.40 Simplify

$$\frac{\frac{-6z-2}{-5z-5} - \frac{10z+7}{-10z-6}}{-\frac{8z-9}{-5z-3} + \frac{7-10z}{4z}}$$

Exercise 3.41 Simplify

$$\frac{\frac{8l-7}{-2l-2} - \frac{1}{-6l-4}}{-\frac{2l}{5} - \frac{4l}{9-3l} + \frac{6}{5}}$$

Exercise 3.42 Solve the following simultaneous equations for x and y :

$$-9x - 2y - 4 = 0$$

$$-4x + 2y - 9 = 0$$

Exercise 3.43 Solve the following simultaneous equations for x and y :

$$12ax + 56ay + 6a = 0$$

$$25ay - 36x + 5 = 0$$

Exercise 3.44 Expand and simplify

$$(8bc^3 - 5d^4)^2 \cdot (8a^2bc^2 + 1)$$

Exercise 3.45 Factorize the polynomial

$$-18p^2 - 101p - 90$$

Exercise 3.46 Simplify

$$\frac{-15p^4q^3 + 18p^3q^3}{3q^3}$$

Exercise 3.47 Expand

$$\left(-10p^3q^4 - \frac{1}{3p^3r^3}\right)^2$$

Exercise 3.48 Simplify

$$\frac{x-4}{6x-6} - \frac{1}{-5x-6}$$

Exercise 3.49 Simplify

$$\frac{9-8t^2}{t^2-9t} + \frac{8t^2+10t}{7t^2-8t}$$

Exercise 3.50 Complete the square for the following expression and then solve the equation:

$$45x^2 + 41x$$

Exercise 3.51 Simplify

$$\frac{\frac{20}{5x+2} - \frac{4x+6}{12x}}{\frac{-3x-2}{2x-5} + \frac{10x-2}{2-5x}}$$

Exercise 3.52 Solve the equation in x :

$$6a + 7x^2 + 6 = 0$$

Exercise 3.53 Expand and simplify

$$(-6a^2b^2c^4d^2 - d)^2 \cdot (6a^4b^4cd^3 + 6d^4)$$

Exercise 3.54 Solve the following simultaneous equations for x and y :

$$2x - 8y - 1 = 0$$

$$12ax - 3y - 5 = 0$$

Exercise 3.55 Solve the following inequality for p

$$-8p + 2r^2 \leq -p + 3r$$

Exercise 3.56 Simplify

$$\frac{15q}{1-7q} + \frac{4}{2q+5}$$

Exercise 3.57 Solve the following inequality for n

$$9n + 7u < -6n - 2$$

Exercise 3.58 Solve the equation in x :

$$9ax^2 - 6a + 5x^2 - 11x = 0$$

Exercise 3.59 Simplify

$$\frac{(-1)9q^3}{27pq^5r^4 - 90q^7 - 18q^3}$$

Exercise 3.60 Simplify

$$\frac{1}{14} - \frac{2sw + 2}{-2w - 6} - \frac{-2sw + 4}{8 - 10w}$$

Exercise 3.61 Simplify

$$-\frac{2w}{11} - \frac{6}{11} + \frac{10w - 3}{5w}$$

Exercise 3.62 If $a = 5 - 4t^3$, show that

$$(a - x)^2 + (a + x)^2 = 32t^6 - 80t^3 + 2x^2 + 50$$

Exercise 3.63 Expand

$$(-6c^4 + 5c^2)^2 (-6bc^3d^4 - 8c^2)$$

Exercise 3.64 If $a = -4p^4q - 7p^2$, show that

$$-(a - x)^3 + (a + x)^3 = -(4p^4q + 7p^2 - x)^3 - (4p^4q + 7p^2 + x)^3$$

Exercise 3.65 Simplify

$$\frac{(-1)15lm^3n^3}{3lm^3n^2}$$

Exercise 3.66 Expand

$$(-n^3 - 9)(-4m^2n + 7n^4)(-2m^3n^4 + 9n^2)$$

Exercise 3.67 Simplify

$$-\frac{2 - 6r}{3 - 6r} - 5$$

Exercise 3.68 Simplify

$$\frac{-50a^7b^7c^3d^6 - 10a^7b^5c^4d^3 - 100a^7b^3c^2d^6 - 20a^7bc^3d^3}{10a^4b^3d^3 + 2a^4bc}$$

Exercise 3.69 Simplify

$$\frac{3y - 9}{-10ry + 8} - \frac{1}{3r - 10} - \frac{8y + 2}{10r}$$

Exercise 3.70 Expand

$$(5uv^2 - u)(u^3w^2 - 7v^4)^2$$

Exercise 3.71 Solve the equation in x :

$$10ax + 12x - 8 = 0$$

Exercise 3.72 Expand

$$(8s^3 - 3s^2)(9r^4s^2 - 7r^3)^2$$

Exercise 3.73 If $a = b^2c^3d^2 - 7c^2$, show that

$$(a - x)^2 + (a + x)^2 = 2b^4c^6d^4 - 28b^2c^5d^2 + 98c^4 + 2x^2$$

Exercise 3.74 Simplify

$$\frac{-4q^8 - 10q^4}{4q^4 + 10}$$

Exercise 3.75 Simplify

$$\frac{-54p^4q^4r^4 - 54pq^5}{6p^4r^4 + 6pq}$$

Exercise 3.76 Simplify

$$\frac{56m^4n^2 + 49m^4 + 14mn^4 + 49mn^2}{7m}$$

Exercise 3.77 Solve the following simultaneous equations for x and y :

$$50ax + 8ay - a = 0$$

$$-3a - 40x - 50y = 0$$

Exercise 3.78 Solve the following simultaneous equations for x and y :

$$12ay + 42x + 8 = 0$$

$$-60ay + 100x - 2 = 0$$

Exercise 3.79 Simplify

$$\frac{(-1)8d^3}{(-1)72a^2b^3d^7}$$

Exercise 3.80 Expand

$$(-8n^4 + n)(6l^2m^3 + 6lm^3)(6l^3m^3n^3 + 5m^2)$$

Exercise 3.81 Solve the following inequality

$$l - 6 \leq -7$$

Exercise 3.82 Expand

$$\left(-9p - \frac{1}{7p^2r^3}\right)^2$$

Exercise 3.83 Expand

$$(7c^4 - 1)(6a^3c^4 + 2ab)(-4b^4c + 5)$$

Exercise 3.84 Simplify

$$\frac{l}{4l^2 + 8l} - \frac{l}{-6l^2 - 7}$$

Exercise 3.85 Expand

$$-3y^3(3x^2z^4 + xy^2z)^2$$

Exercise 3.86 Complete the square for the following expression and then solve the equation:

$$-30u^2 + 37u$$

Exercise 3.87 Expand

$$(l^4 + 9)(9l^4 - 3l^2 - n^4)$$

Exercise 3.88 If $a = 8l^3m^2n^2 + 2l^2m^4$, show that

$$-(a-x)^3 + (a+x)^3 = (8l^3m^2n^2 + 2l^2m^4 - x)^3 + (8l^3m^2n^2 + 2l^2m^4 + x)^3$$

Exercise 3.89 Expand and simplify

$$(-6rs^4 - 5)^2(-r^2s^4t^2 + 2s^2t^4)$$

Exercise 3.90 Expand

$$\left(-8a^4b^3c^4 + \frac{1}{6b^3}\right)^2$$

Exercise 3.91 Expand and simplify

$$(7r^3t^4 + 4r^3)(-7r^4s^2 - 3r^3)^2$$

Exercise 3.92 If $a = -4rs^2t^3 - 8s^4$, show that

$$(a-x)^2 + (a+x)^2 = 32r^2s^4t^6 + 128rs^6t^3 + 128s^8 + 2x^2$$

Exercise 3.93 Solve the equation in x :

$$-17a - 7x^2 - 6 = 0$$

Exercise 3.94 Expand

$$(2xy^2 - 2y^2 + 2yz^2)^2$$

Exercise 3.95 Solve the equation in x :

$$-8ax - 5x + 9 = 0$$

Exercise 3.96 Solve the following inequality

$$10n + 10 < -6$$

Exercise 3.97 Simplify

$$-\frac{5}{t}$$

Exercise 3.98 Simplify

$$\frac{(-1)6p^2}{(-1)18p^3q}$$

Exercise 3.99 Expand and simplify

$$(3uv + 3u)(-5u^4v^3 - 10u^3w)^2$$

Exercise 3.100 Simplify

$$\frac{(-1)4a^4c^2}{-12a^5b^2c^3 + 4a^4b^2c^5 + 32a^4b^2c^3}$$

Exercise 3.101 Complete the square for the following expression and then solve the equation:

$$21l^2 - 31l$$

Exercise 3.102 Simplify

$$\frac{-35p^8q^6 + 49p^7q^4 + 49p^4q^8}{(-1)7p^4q^4}$$

Exercise 3.103 Factorize the polynomial

$$-12n^2 - 10n + 8$$

Exercise 3.104 Solve the following simultaneous equations for x and y :

$$-10x + 3y - 7 = 0$$

$$-4x - 10y + 8 = 0$$

Exercise 3.105 Complete the square for the following expression and then solve the equation:

$$-10l^2 - 61l$$

Exercise 3.106 Solve the following inequality

$$-8a - 7 > -8$$

Exercise 3.107 Complete the square for the following expression and then solve the equation:

$$-30p^2 - 55p$$

Exercise 3.108 Solve the equation in x :

$$-4a^2 + 7ax^2 - 3a = 0$$

Exercise 3.109 Expand

$$(2p^3 - 6q)(6q + 4r^3)(4p^3q^3r^2 + 7p^2q^3r^4)$$

Exercise 3.110 Expand and simplify

$$(3m^2 - 3m)^2(-10m^3n^2 - 2m^2n^3)$$

Exercise 3.111 Expand

$$(-7rs^3 - 7s^4)(-3rs^3 - 6s^4)(-r^4s^3 + 5s^4)$$

Exercise 3.112 Solve the following simultaneous equations for x and y :

$$-10x + 3y + 9 = 0$$

$$-6x + 7y = 0$$

Exercise 3.113 Expand

$$(-3x^4y^3z^4 + 3x^2z - 8z^2)^2$$

Exercise 3.114 Simplify

$$\frac{-\frac{3p+4}{-p-1} + \frac{9p-3}{6-3p}}{2 - \frac{2}{p}}$$

Exercise 3.115 Expand

$$(9s^3 + 4t^4)(10r^2s^4t^3 - 7rt^3 + s^3)$$

Exercise 3.116 If $a = 8p^3q^4 + 2pq^4r$, show that

$$-(a-x)^3 + (a+x)^3 = (8p^3q^4 + 2pq^4r - x)^3 + (8p^3q^4 + 2pq^4r + x)^3$$

Exercise 3.117 Solve the equation in x :

$$9ax - 5a - 3x = 0$$

Exercise 3.118 Expand

$$(-2p^3 + 6q^2r^4)(-5q^2 + 3r^4)(-7p^2q + 7)$$

Exercise 3.119 Factorize the polynomial

$$15w^2 - 20w - 20$$

Exercise 3.120 Simplify

$$\frac{-10r^4s^2 + 5r^4}{-90r^8s^2 + 45r^8 + 50r^4s^2 - 25r^4}$$

Exercise 3.121 Simplify

$$-\frac{4u^2}{11} + \frac{10u}{11} + \frac{8u^3 - u^2}{-9u^2 - u}$$

Exercise 3.122 Solve the following simultaneous equations for x and y :

$$-ax + 4a - 40y = 0$$

$$-20ax - 100y + 8 = 0$$

Exercise 3.123 If $a = 10x^2z^4 - 3y$, show that

$$-(a-x)^3 + (a+x)^3 = -(-10x^2z^4 + x + 3y)^3 + (10x^2z^4 + x - 3y)^3$$

Exercise 3.124 Expand

$$(5m - n^4)(7l^3m^4n + 3m^4 - 9m^2n^2)$$

Exercise 3.125 Expand

$$64s^6(-4r^3 - 4)$$

Exercise 3.126 Simplify

$$\frac{\frac{2z-3}{7-7z} + \frac{10-3z}{16z}}{-\frac{5z}{3z-2} + \frac{1}{z}}$$

Exercise 3.127 Expand

$$(10r^2st^4 + 10t^3 + 2)^2$$

Exercise 3.128 Expand

$$(lm^4n^2 + 7m)(8l^3m^2n - 4lm^4n^2 - 10m^3)$$

Exercise 3.129 Solve the following simultaneous equations for x and y :

$$-9x + 7y - 3 = 0$$

$$-6x + 4y - 2 = 0$$

Exercise 3.130 Simplify

$$\frac{4r}{5r+4} + \frac{10-7r}{-3r-7t} + \frac{-2rt-7r}{-9r+6t}$$

Exercise 3.131 Simplify

$$\frac{12p^5q^4 - 48p^4q^7 - 14p^4q + 56p^3q^4 - 2p^2q + 8pq^4 + 14pq^3 - 56q^6}{2p - 8q^3}$$

Exercise 3.132 Simplify

$$\frac{-9a^4bc^4 - 6ab^4}{-18a^8b^2c^5 - 12a^5b^5c}$$

Exercise 3.133 Solve the following inequality for m

$$3a - 9m < -a^2 - 10m$$

Exercise 3.134 Expand

$$(4c^2d^4 - 1)(8a^3bc^3 - 10a^3d^2)(-9a^4b^4cd^4 - 6bc^2d^3)$$

Exercise 3.135 Expand

$$(8p^4 + 3)(5q^4 + 2q^2)(-7p^2q - 7p)$$

Exercise 3.136 Simplify

$$\frac{2w+4}{-6y-6} + \frac{7wy+3}{3w+3} + \frac{w-y}{7w}$$

Exercise 3.137 Solve the following simultaneous equations for x and y :

$$42x + 40y + 9 = 0$$

$$20x + 30y + 6 = 0$$

Exercise 3.138 If $a = -3s - 9t^4$, show that

$$(a-x)^2 + (a+x)^2 = 18s^2 + 108st^4 + 162t^8 + 2x^2$$

Exercise 3.139 Simplify

$$\frac{12b^5c^2 - 18b^4d^4 + 6b^3c^5 - 9b^2c^3d^4 - 4bc^6 + 6c^4d^4}{2bc^2 - 3d^4}$$

Exercise 3.140 Solve the equation in x :

$$9a^2x + ax - 12a - 7 = 0$$

Exercise 3.141 Simplify

$$\frac{5n}{12} - \frac{-2n-10}{7x-1} + \frac{-2nx-n}{10n-9}$$

Exercise 3.142 Complete the square for the following expression and then solve the equation:

$$5a^2 - 5a$$

Exercise 3.143 Simplify

$$-\frac{104q}{31 \cdot (7q-8)} - \frac{13 \cdot (5-5q)}{434q}$$

Exercise 3.144 Simplify

$$\frac{(-1)10a}{(-1)30ab^3}$$

Exercise 3.145 Solve the following inequality

$$-8q - 7 \leq -4$$

Exercise 3.146 Expand

$$\left(-3r^2 + \frac{1}{8s^4}\right)^2$$

Exercise 3.147 Complete the square for the following expression and then solve the equation:

$$4z^2 - 12z$$

Exercise 3.148 Solve the following simultaneous equations for x and y :

$$5x - 10y - 3 = 0$$

$$-5x - 4y + 4 = 0$$

Exercise 3.149 Solve the following inequality for r

$$7m + 2r < 5m + 5r$$

Exercise 3.150 If $a = -2m^4 + n$, show that

$$-(a-x)^3 + (a+x)^3 = (-2m^4 + n + x)^3 - (2m^4 - n + x)^3$$

Exercise 3.151 Simplify

$$\frac{-z^3 + 7z^2}{1 - 9z^2} + \frac{1}{z}$$

Exercise 3.152 Solve the following inequality

$$m + 2 < 5$$

Exercise 3.153 If $a = 5p^3 + 10q^3$, show that

$$-(a-x)^3 + (a+x)^3 = (5p^3 + 10q^3 - x)^3 + (5p^3 + 10q^3 + x)^3$$

Exercise 3.154 Solve the following inequality

$$-2t - 2 \leq -4$$

Exercise 3.155 If $a = -2q - 5$, show that

$$(a-x)^2 + (a+x)^2 = 8q^2 + 40q + 2x^2 + 50$$

Exercise 3.156 Simplify

$$\frac{8m^2n^3}{n^3}$$

Exercise 3.157 If $a = -3b^3c^2d^4 - 9b$, show that

$$-(a-x)^3 + (a+x)^3 = -(3b^3c^2d^4 + 9b - x)^3 - (3b^3c^2d^4 + 9b + x)^3$$

Exercise 3.158 Expand

$$\left(-2p^3r^4 - \frac{1}{4pr^3}\right)^2$$

Exercise 3.159 Simplify

$$\frac{28r^7s^5t^4 - 21r^5s^3t^2 + 70r^4s^4t^4 + 12r^3s^2t^5 - 9rt^3 + 30st^5}{-7r^4s^3t^2 - 3t^3}$$

Exercise 3.160 Expand

$$(5x^4y^4z^2 + 4x^2y^4z^2 - 1)^2$$

Exercise 3.161 Solve the following inequality for z

$$-4n^2 - 2z \geq -8z - 9$$

Exercise 3.162 Solve the following simultaneous equations for x and y :

$$7x + y = 0$$

$$-8x - 4y + 3 = 0$$

Exercise 3.163 If $a = -b^2c^4 - 5b^2c$, show that

$$(a - x)^2 + (a + x)^2 = 2b^4c^8 + 20b^4c^5 + 50b^4c^2 + 2x^2$$

Exercise 3.164 Simplify

$$\frac{3q - 6}{10q - 7} + \frac{8q - 8}{n + 8} + \frac{7n + q}{5q}$$

Exercise 3.165 Solve the following simultaneous equations for x and y :

$$-x - 2y + 4 = 0$$

$$-8x + 5y - 3 = 0$$

Exercise 3.166 Simplify

$$-\frac{5m}{8} + \frac{11m}{-9m - 5}$$

Exercise 3.167 Simplify

$$\frac{9b^3 + 5c^3d^4}{36a^2b^7c^2d^4 + 20a^2b^4c^5d^8 - 54b^3c^4 + 81b^3 - 30c^7d^4 + 45c^3d^4}$$

Exercise 3.168 Factorize the polynomial

$$-35c^2 - 94c - 63$$

Exercise 3.169 Simplify

$$-\frac{3}{5v - 6} - \frac{1}{2v}$$

Exercise 3.170 Simplify

$$\frac{4p - 10}{-3p - 4} + \frac{3}{-9p - 8}$$

Exercise 3.171 Solve the following simultaneous equations for x and y :

$$8x - 6y - 9 = 0$$

$$5x - 6y + 2 = 0$$

Exercise 3.172 Expand and simplify

$$(-5a^4 - a^2bd^4)(10cd^2 - 4d^3)^2$$

Exercise 3.173 Expand and simplify

$$320b^3c^2 + 128b$$

Exercise 3.174 Simplify

$$\frac{\frac{12}{7} + \frac{15}{7-6d}}{-\frac{6d}{6d+8} - \frac{2-6d}{2d+2}}$$

Exercise 3.175 Simplify

$$\frac{-7m+6r}{-5r-7} - \frac{4}{-5m+8r} + \frac{3r-5}{-6m-1}$$

Exercise 3.176 Simplify

$$\frac{4r}{3} + \frac{5r}{2-9r} + \frac{10}{3}$$

Exercise 3.177 Expand and simplify

$$(3ln^4 - 4m^4)(-6mn^4 + 10n^3)^2$$

Exercise 3.178 Simplify

$$-b + 2 + \frac{7b+2}{4b}$$

Exercise 3.179 Factorize the polynomial

$$-3t^2 + 3t + 6$$

Exercise 3.180 Expand

$$(-ln^4 + n^4)(-4l^4m^2n + 9m^4n^4 + 5n^2)$$

Exercise 3.181 Solve the equation in x :

$$-8ax - 4x - 3 = 0$$

Exercise 3.182 Simplify

$$\frac{9r}{7} - \frac{3}{7} - \frac{-r-3}{5-2r}$$

Exercise 3.183 Simplify

$$\frac{-60x^7yz^7 + 42x^6y^3z^5}{(-1)6x^3yz^4}$$

Exercise 3.184 Expand

$$(4p^3 + p - 10q^2)^2$$

Exercise 3.185 Solve the following simultaneous equations for x and y :

$$-3x - 9y + 1 = 0$$

$$9x + 6y - 6 = 0$$

Exercise 3.186 Solve the following inequality

$$2c - 2 > -9$$

Exercise 3.187 Complete the square for the following expression and then solve the equation:

$$-12q^2 - 49q$$

Exercise 3.188 Solve the following inequality for t

$$-8n^2 + 5t \leq 10 - 4t$$

Exercise 3.189 Expand

$$(2 - s)^2$$

Exercise 3.190 Simplify

$$\frac{\frac{16}{-9t-2} - \frac{10-9t}{14t}}{-\frac{1}{-4t-6} + \frac{3}{t}}$$

Exercise 3.191 Simplify

$$\frac{-36ab^5c^4 + 36ab^5c^2 - 99b^5c^3 + 99b^5c}{-9b^2c^3 + 9b^2c}$$

Exercise 3.192 Simplify

$$\frac{2(-9y-9)}{9 \cdot (4y-1)} + \frac{2 \cdot (3y-9)}{9 \cdot (6y+8)}$$

Exercise 3.193 Solve the following inequality for s

$$8b + 3s \leq -2b - 2s$$

Exercise 3.194 Expand

$$(7a^4 + 4a^2c^3d^2)(2a^4c^4 + 5a^3)(6a^4bc^2 + 2d^2)$$

Exercise 3.195 Complete the square for the following expression and then solve the equation:

$$6t^2 - 12t$$

Exercise 3.196 Simplify

$$\frac{-4a^3b^4 - 5a^3b^2c^3}{32a^5b^4 + 40a^5b^2c^3}$$

Exercise 3.197 Solve the equation in x :

$$5a^2 - 9x + 10 = 0$$

Exercise 3.198 If $a = 9p^4r^2 + 9q^4r^3$, show that

$$(a-x)^2 + (a+x)^2 = 162p^8r^4 + 324p^4q^4r^5 + 162q^8r^6 + 2x^2$$

Exercise 3.199 Expand

$$(-2ab^3c^3 + 8c)(-9a^3b^4c^3 - 3ab^2d^3 + 7b^4c^2d^4)$$

Exercise 3.200 Solve the following inequality for d

$$-4d + w \leq -8d + 6w^2$$

3.2 Solutions

1.

$$\frac{1}{a(a^3c^4 + 3)}$$

2.

$$x \leq -2$$

3.

$$\frac{24b^2 - 24b + 11}{22b - 22}$$

4.

$$w \leq -\frac{8b^2}{3} - 2b$$

5.

$$-20r^4s^7 + 28r^4s^6 - 80r^4s^5 + 112r^4s^4 - 80r^4s^3 + 112r^4s^2$$

6.

$$\frac{-28uw^2 - 14uw - 364w^2 + 349w + 45}{252w^2 + 126w}$$

7.

$$54m^8 + 63m^7n - 216m^7 - 252m^6n + 216m^6 + 252m^5n$$

8.

$$a^4b^6 - 12a^3b^3c^3d - 4a^2b^3c + 36a^2c^6d^2 + 24ac^4d + 4c^2$$

9.

$$\left[\frac{2}{7}, \frac{5}{2} \right]$$

10.

$$-81p^9q^4 + 108p^6q^7 - 810p^6 - 36p^3q^{10} + 1080p^3q^3 - 360q^6$$

11.

$$5m^3 - 9n^2$$

12.

$$-\frac{1}{3r}$$

13. Provided in problem statement

14.

$$s^3(-5r^2s + 7)$$

15.

$$\frac{-540w^4 + 2250w^3 - 5595w^2 + 7485w - 1950}{1656w^4 + 3324w^3 + 3508w^2 + 4700w + 700}$$

16.

$$-2ab^3 + 4ac^2 - 4b^4c + 8b^2c^3$$

17.

$$m < -6$$

18.

$$\frac{54b^3q + 48b^2q^2 - 81b^2 + 28bq^3 + 116bq^2 - 300bq - 540b - 138q^2 - 654q + 180}{36b^2q^2 + 180b^2q + 32bq^3 + 160bq^2 - 54bq - 270b - 48q^2 - 240q}$$

19.

$$-\frac{1}{q(5p^3 + 3pq^2 + 2q)}$$

20.

$$3c^3$$

21.

$$\left\{ x : -4, y : -\frac{18}{5} \right\}$$

22.

$$\frac{40p + 99}{3p + 10}$$

23.

$$-30m^8n^7 + 30m^8n^5 + 12m^8n^4 + 80m^4n^4 - 80m^4n^2 - 32m^4n$$

24.

$$s \geq \frac{1}{2} - w^2$$

25.

$$\frac{-450d^2w^2 - 504d^2w + 50dw^2 + 245dw + 70d - 21w}{315d^2w - 35dw}$$

26.

$$\frac{18v^4 - 4v^2 - 3v - 8}{6v^3 + 16v^2}$$

27.

$$\left\{ x : \frac{8}{13}, y : \frac{1}{26} \right\}$$

28.

$$-2p^6q^4 - 10p^4q^4r^2 - p^2q^2r + 9p^2r^4 - 5q^2r^3 + 45r^6$$

29.

$$\{0\}$$

30.

$$\{x : -2, y : -2\}$$

31.

$$\left\{ x : -\frac{37}{67}, y : -\frac{58}{67} \right\}$$

32.

$$4r^4s^4 + 9r^2 - 10s$$

33.

$$30l^9m^7n^3 - 6l^7m^3n^2 + 10l^6m^9n^6 - 120l^5m^8n^4 - 2l^4m^5n^5 + 24l^3m^4n^3 - 40l^2m^{10}n^7 + 8m^6n^6$$

34.

$$9m^4n^4 + 13m + 2n^4$$

35. Provided in problem statement

36.

$$\left[-2, \frac{1}{3} \right]$$

37.

$$\left\{ -\frac{5}{8(a+2)} \right\}$$

38.

$$81l^{10}m^6n^4 + 81l^8m^9n^7 - 18l^8m^5n^6 - 18l^6m^8n^9 + l^6m^4n^8 + l^4m^7n^{11}$$

39.

$$25a^8b^4c^4 + 90a^7b^5c^2 + 70a^7b^2c^3 + 81a^6b^6 + 126a^6b^3c + 49a^6c^2$$

40.

$$\frac{-220z^3 - 282z^2 - 94z}{90z^3 + 245z^2 + 50z - 105}$$

41.

$$\frac{360l^3 - 1170l^2 + 45l + 675}{36l^4 - 276l^3 - 212l^2 + 316l + 216}$$

42.

$$\left\{ x : -1, y : \frac{5}{2} \right\}$$

43.

$$\left\{ x : \frac{140 - 75a}{150a + 1008}, y : -\frac{23}{25a + 168} \right\}$$

44.

$$512a^2b^3c^8 - 640a^2b^2c^5d^4 + 200a^2bc^2d^8 + 64b^2c^6 - 80bc^3d^4 + 25d^8$$

45.

$$2p + 9$$

and

$$-9p - 10$$

46.

$$p^3 \cdot (6 - 5p)$$

47.

$$100p^6q^8 + \frac{20q^4}{3r^3} + \frac{1}{9p^6r^6}$$

48.

$$\frac{5x^2 - 8x - 30}{30x^2 + 6x - 36}$$

49.

$$\frac{-48t^3 + 2t^2 - 27t - 72}{7t^3 - 71t^2 + 72t}$$

50.

$$\left[-\frac{4}{5}, -\frac{1}{9} \right]$$

51.

$$\frac{100x^4 - 1300x^3 + 3089x^2 - 1184x + 60}{1050x^4 - 1080x^3 - 420x^2 + 72x}$$

52.

$$\left\{ -\frac{\sqrt{42}\sqrt{-a-1}}{7}, \frac{\sqrt{42}\sqrt{-a-1}}{7} \right\}$$

53.

$$6d^5 \cdot (36a^8b^8c^9d^2 + 12a^6b^6c^5d + 36a^4b^4c^8d^3 + a^4b^4c + 12a^2b^2c^4d^2 + d)$$

54.

$$\left\{ x: \frac{37}{96a-6}, y: \frac{5-6a}{48a-3} \right\}$$

55.

$$p \geq \frac{2r^2}{7} - \frac{3r}{7}$$

56.

$$\frac{-30q^2 - 47q - 4}{14q^2 + 33q - 5}$$

57.

$$n < -\frac{7u}{15} - \frac{2}{15}$$

58.

$$\left\{ -\frac{\sqrt{216a^2 + 120a + 121}}{2 \cdot (9a + 5)} + \frac{11}{2 \cdot (9a + 5)}, \frac{\sqrt{216a^2 + 120a + 121}}{2 \cdot (9a + 5)} + \frac{11}{2 \cdot (9a + 5)} \right\}$$

59.

$$\frac{1}{-3pq^2r^4 + 10q^4 + 2}$$

60.

$$\frac{56sw^2 - 98sw + 5w^2 + 109w + 16}{70w^2 + 154w - 168}$$

61.

$$\frac{-10w^2 + 80w - 33}{55w}$$

62. Provided in problem statement

63.

$$-216bc^{11}d^4 + 360bc^9d^4 - 150bc^7d^4 - 288c^{10} + 480c^8 - 200c^6$$

64. Provided in problem statement

65.

$$-5n$$

66.

$$-8m^5n^8 - 72m^5n^5 + 14m^3n^{11} + 126m^3n^8 + 36m^2n^6 + 324m^2n^3 - 63n^9 - 567n^6$$

67.

$$\frac{17 - 36r}{6r - 3}$$

68.

$$5a^3c^2d^3(-b^4c - 2)$$

69.

$$\frac{-120r^2y^2 - 125r^2y + 135r^2 + 400ry^2 + 346ry - 386r - 320y - 80}{150r^3y - 500r^2y - 120r^2 + 400r}$$

70.

$$5u^7v^2w^4 - u^7w^4 - 70u^4v^6w^2 + 14u^4v^4w^2 + 245uv^{10} - 49uv^8$$

71.

$$\left\{ \frac{4}{5a+6} \right\}$$

72.

$$648r^8s^7 - 243r^8s^6 - 1008r^7s^5 + 378r^7s^4 + 392r^6s^3 - 147r^6s^2$$

73. Provided in problem statement

74.

$$-q^4$$

75.

$$-9q^4$$

76.

$$8m^3n^2 + 7m^3 + 2n^4 + 7n^2$$

77.

$$\left\{ x : \frac{6a}{545} + \frac{5}{218}, y : -\frac{15a}{218} - \frac{2}{109} \right\}$$

78.

$$\left\{ x : -\frac{19}{155}, y : -\frac{221}{930a} \right\}$$

79.

$$\frac{1}{9a^2b^3d^4}$$

80.

$$-288l^5m^6n^7 + 36l^5m^6n^4 - 288l^4m^6n^7 + 36l^4m^6n^4 - 240l^2m^5n^4 + 30l^2m^5n - 240lm^5n^4 + 30lm^5n$$

81.

$$l \leq -1$$

82.

$$81p^2 + \frac{18}{7pr^3} + \frac{1}{49p^4r^6}$$

83.

$$-168a^3b^4c^9 + 24a^3b^4c^5 + 210a^3c^8 - 30a^3c^4 - 56ab^5c^5 + 8ab^5c + 70abc^4 - 10ab$$

84.

$$\frac{10l^2 + 8l + 7}{24l^3 + 48l^2 + 28l + 56}$$

85.

$$-27x^4y^3z^8 - 18x^3y^5z^5 - 3x^2y^7z^2$$

86.

$$\left[\frac{2}{5}, \frac{5}{6} \right]$$

87.

$$9l^8 - 3l^6 - l^4n^4 + 81l^4 - 27l^2 - 9n^4$$

88. Provided in problem statement

89.

$$s^2t^2(-36r^4s^{10} - 60r^3s^6 + 72r^2s^8t^2 - 25r^2s^2 + 120rs^4t^2 + 50t^2)$$

90.

$$64a^8b^6c^8 - \frac{8a^4c^4}{3} + \frac{1}{36b^6}$$

91.

$$r^9 \cdot (343r^2s^4t^4 + 196r^2s^4 + 294rs^2t^4 + 168rs^2 + 63t^4 + 36)$$

92. Provided in problem statement

93.

$$\left\{ -\frac{\sqrt{7}\sqrt{-17a-6}}{7}, \frac{\sqrt{7}\sqrt{-17a-6}}{7} \right\}$$

94.

$$4x^2y^4 - 8xy^4 + 8xy^3z^2 + 4y^4 - 8y^3z^2 + 4y^2z^4$$

95.

$$\left\{ \frac{9}{8a+5} \right\}$$

96.

$$n < -\frac{8}{5}$$

97.

$$-\frac{5}{t}$$

98.

$$\frac{1}{3pq}$$

99.

$$75u^7(u^2v^7 + u^2v^6 + 4uv^4w + 4uv^3w + 4vw^2 + 4w^2)$$

100.

$$\frac{1}{b^2c(3a - c^2 - 8)}$$

101.

$$\left[\frac{1}{3}, \frac{8}{7} \right]$$

102.

$$5p^4q^2 - 7p^3 - 7q^4$$

103.

$$4n - 2$$

and

$$-3n - 4$$

104.

$$\left\{ x : -\frac{23}{56}, y : \frac{27}{28} \right\}$$

105.

$$\left[-\frac{9}{2}, -\frac{8}{5} \right]$$

106.

$$a < \frac{1}{8}$$

107.

$$\left[-1, -\frac{5}{6} \right]$$

108.

$$\left\{ -\frac{\sqrt{7}\sqrt{4a+3}}{7}, \frac{\sqrt{7}\sqrt{4a+3}}{7} \right\}$$

109.

$$48p^6q^4r^2 + 32p^6q^3r^5 + 84p^5q^4r^4 + 56p^5q^3r^7 - 144p^3q^5r^2 - 96p^3q^4r^5 - 252p^2q^5r^4 - 168p^2q^4r^7$$

110.

$$18m^4n^2(-5m^3 - m^2n + 10m^2 + 2mn - 5m - n)$$

111.

$$-21r^6s^9 - 63r^5s^{10} - 42r^4s^{11} + 105r^2s^{10} + 315rs^{11} + 210s^{12}$$

112.

$$\left\{ x : \frac{63}{52}, y : \frac{27}{26} \right\}$$

113.

$$9x^8y^6z^8 - 18x^6y^3z^5 + 48x^4y^3z^6 + 9x^4z^2 - 48x^2z^3 + 64z^4$$

114.

$$\frac{-4p^2 - 7p}{2p^3 - 4p^2 - 2p + 4}$$

115.

$$90r^2s^7t^3 + 40r^2s^4t^7 - 63rs^3t^3 - 28rt^7 + 9s^6 + 4s^3t^4$$

116. Provided in problem statement

117.

$$\left\{ \frac{5a}{3 \cdot (3a - 1)} \right\}$$

118.

$$-70p^5q^3 + 42p^5qr^4 + 70p^3q^2 - 42p^3r^4 + 210p^2q^5r^4 - 126p^2q^3r^8 - 210q^4r^4 + 126q^2r^8$$

119.

$$3w + 2$$

and

$$5w - 10$$

120.

$$\frac{1}{9r^4 - 5}$$

121.

$$\frac{-36u^3 - 2u^2 + 21u}{99u + 11}$$

122.

$$\left\{ x : \frac{16 - 20a}{35a}, y : \frac{4a}{35} - \frac{2}{175} \right\}$$

123. Provided in problem statement

124.

$$35l^3m^5n - 7l^3m^4n^5 + 15m^5 - 3m^4n^4 - 45m^3n^2 + 9m^2n^6$$

125.

$$-256r^3s^6 - 256s^6$$

126.

$$\frac{159z^3 - 523z^2 + 488z - 140}{560z^3 - 896z^2 + 560z - 224}$$

127.

$$100r^4s^2t^8 + 200r^2st^7 + 40r^2st^4 + 100t^6 + 40t^3 + 4$$

128.

$$8l^4m^6n^3 + 56l^3m^3n - 4l^2m^8n^4 - 10lm^7n^2 - 28lm^5n^2 - 70m^4$$

129.

$$\left\{ x : -\frac{1}{3}, y : 0 \right\}$$

130.

$$\frac{30r^3t + 528r^3 + 70r^2t^2 + 239r^2t - 114r^2 - 112rt^2 + 328rt - 360r + 240t}{135r^3 + 225r^2t + 108r^2 - 210rt^2 + 180rt - 168t^2}$$

131.

$$q(6p^4q^3 - 7p^3 - p + 7q^2)$$

132.

$$\frac{1}{2a^4bc}$$

133.

$$m < -a^2 - 3a$$

134.

$$-288a^7b^5c^6d^8 + 72a^7b^5c^4d^4 + 360a^7b^4c^3d^{10} - 90a^7b^4cd^6 - 192a^3b^2c^7d^7 + 48a^3b^2c^5d^3 + 240a^3bc^4d^9 - 60a^3bc^2d^5$$

135.

$$-280p^6q^5 - 112p^6q^3 - 280p^5q^4 - 112p^5q^2 - 105p^2q^5 - 42p^2q^3 - 105pq^4 - 42pq^2$$

136.

$$\frac{-7w^3 + 49w^2y^2 + 52w^2y - 18w^2 - 3wy^2 + 21wy + 10w - 3y^2 - 3y}{21w^2y + 21w^2 + 21wy + 21w}$$

137.

$$\left\{ x : -\frac{3}{46}, y : -\frac{18}{115} \right\}$$

138. Provided in problem statement

139.

$$6b^4 + 3b^2c^3 - 2c^4$$

140.

$$\left\{ \frac{12a+7}{a(9a+1)} \right\}$$

141.

$$\frac{350n^2x + 190n^2 - 168nx^2 - 375nx + 1041n - 1080}{840nx - 120n - 756x + 108}$$

142.

$$[-2, 3]$$

143.

$$\frac{-1001q^2 - 975q + 520}{3038q^2 - 3472q}$$

144.

$$\frac{1}{3b^3}$$

145.

$$-\frac{3}{8} \leq q$$

146.

$$9r^4 - \frac{3r^2}{4s^4} + \frac{1}{64s^8}$$

147.

$$\left[\frac{1}{2}, \frac{5}{2} \right]$$

148.

$$\left\{ x : \frac{26}{35}, y : \frac{1}{14} \right\}$$

149.

$$r > \frac{2m}{3}$$

150. Provided in problem statement

151.

$$\frac{z^4 - 7z^3 + 9z^2 - 1}{9z^3 - z}$$

152.

$$m < 3$$

153. Provided in problem statement

154.

$$1 \leq t$$

155. Provided in problem statement

156.

$$8m^2$$

157. Provided in problem statement

158.

$$4p^6r^8 + p^2r + \frac{1}{16p^2r^6}$$

159.

$$-4r^3s^2t^2 + 3r - 10st^2$$

160.

$$25x^8y^8z^4 + 40x^6y^8z^4 + 16x^4y^8z^4 - 10x^4y^4z^2 - 8x^2y^4z^2 + 1$$

161.

$$z \geq \frac{2n^2}{3} - \frac{3}{2}$$

162.

$$\left\{x: -\frac{3}{20}, y: \frac{21}{20}\right\}$$

163. Provided in problem statement

164.

$$\frac{70n^2q - 49n^2 + 25nq^2 + 523nq - 392n + 400q^3 - 480q^2 - 16q}{50nq^2 - 35nq + 400q^2 - 280q}$$

165.

$$\left\{x: \frac{2}{3}, y: \frac{5}{3}\right\}$$

166.

$$\frac{-45m^2 - 113m}{72m + 40}$$

167.

$$\frac{1}{4a^2b^4c^2d^4 - 6c^4 + 9}$$

168.

$$5c + 7$$

and

$$-7c - 9$$

169.

$$\frac{6 - 11v}{10v^2 - 12v}$$

170.

$$\frac{-36p^2 + 49p + 68}{27p^2 + 60p + 32}$$

171.

$$\left\{x: \frac{11}{3}, y: \frac{61}{18}\right\}$$

172.

$$a^2d^4(-500a^2c^2 + 400a^2cd - 80a^2d^2 - 100bc^2d^4 + 80bcd^5 - 16bd^6)$$

173.

$$320b^3c^2 + 128b$$

174.

$$\frac{216d^3 - 63d^2 - 1035d - 756}{252d^3 - 42d^2 - 462d + 196}$$

175.

$$\frac{-210m^3 + 516m^2r - 35m^2 - 213mr^2 - 54mr - 343m - 120r^3 - 16r^2 + 260r - 28}{-150m^2r - 210m^2 + 240mr^2 + 311mr - 35m + 40r^2 + 56r}$$

176.

$$\frac{36r^2 + 67r - 20}{27r - 6}$$

177.

$$n^6 \cdot (108lm^2n^6 - 360lmn^5 + 300ln^4 - 144m^6n^2 + 480m^5n - 400m^4)$$

178.

$$\frac{-4b^2 + 15b + 2}{4b}$$

179.

$$2 - t$$

and

$$3t + 3$$

180.

$$4l^5m^2n^5 - 4l^4m^2n^5 - 9lm^4n^8 - 5ln^6 + 9m^4n^8 + 5n^6$$

181.

$$\left\{ -\frac{3}{4 \cdot (2a + 1)} \right\}$$

182.

$$\frac{18r^2 - 58r - 6}{14r - 35}$$

183.

$$x^3z(10xz^2 - 7y^2)$$

184.

$$16p^6 + 8p^4 - 80p^3q^2 + p^2 - 20pq^2 + 100q^4$$

185.

$$\left\{ x : \frac{16}{21}, y : -\frac{1}{7} \right\}$$

186.

$$-\frac{7}{2} < c$$

187.

$$\left[-\frac{10}{3}, -\frac{3}{4} \right]$$

188.

$$t \leq \frac{8n^2}{9} + \frac{10}{9}$$

189.

$$s^2 - 4s + 4$$

190.

$$\frac{162t^3 - 349t^2 - 928t - 60}{819t^2 + 1316t + 252}$$

191.

$$b^3 \cdot (4ac + 11)$$

192.

$$\frac{-14y^2 - 55y - 21}{36y^2 + 39y - 12}$$

193.

$$s \leq -2b$$

194.

$$84a^{12}bc^6 + 210a^{11}bc^2 + 48a^{10}bc^9d^2 + 120a^9bc^5d^2 + 28a^8c^4d^2 + 70a^7d^2 + 16a^6c^7d^4 + 40a^5c^3d^4$$

195.

$$[-1, 3]$$

196.

$$-\frac{1}{8a^2}$$

197.

$$\left\{ \frac{5a^2}{9} + \frac{10}{9} \right\}$$

198. Provided in problem statement

199.

$$18a^4b^7c^6 - 72a^3b^4c^4 + 6a^2b^5c^3d^3 - 14ab^7c^5d^4 - 24ab^2cd^3 + 56b^4c^3d^4$$

200.

$$d \leq \frac{3w^2}{2} - \frac{w}{4}$$

4. Functions

4.1 Problems

Exercise 4.1 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = 7 - 6x$$

and

$$g(x) = 7a - 6x$$

Exercise 4.2 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -8ax^2 + 10a - 6x$$

and

$$g(x) = -3ax + 4$$

Exercise 4.3 If $f(x)$ is defined as

$$f(x) = 3ax - 10x^2$$

and $g(x)$ is defined as

$$g(x) = 3x + 6$$

solve the equation

$$g(f(x)) = 0$$

Exercise 4.4 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -10a + 2x$$

and

$$g(x) = 2a - x$$

Exercise 4.5 If $f(x)$ is defined as

$$f(x) = \frac{x+1}{5-8x}$$

find $f^{-1}(x)$.

Exercise 4.6 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = 3ax^2 - 10a + 6x$$

and

$$g(x) = a(-5x - 4)$$

Exercise 4.7 If $f(x)$ is defined as

$$f(x) = \frac{9x-2}{-3x-1}$$

find $f^{-1}(x)$.

Exercise 4.8 If $f(x)$ is defined as

$$f(x) = 7x + 8$$

and $g(x)$ is defined as

$$g(x) = -10f(-8x - 10)x^2 - 5x$$

simplify $g(x)$ in terms of x .

Exercise 4.9 If $f(x)$ is defined as

$$f(x) = \frac{ax-a}{7a+5x}$$

find the value of a if $f^{-1}(3) = 5$.

Exercise 4.10 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = 6ax^2 - 8a - 7x$$

and

$$g(x) = a(5 - 6x)$$

Exercise 4.11 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -4a + 10x$$

and

$$g(x) = 4a - 10x$$

Exercise 4.12 If $f(x)$ is defined as

$$f(x) = -4x^2 - x - 5$$

and $g(x)$ is defined as

$$g(x) = -8f(-2x^2 - x + 1)x^2 - 8x$$

simplify $g(x)$ in terms of x .

Exercise 4.13 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = 4ax^2 + 5a - 9x$$

and

$$g(x) = 6ax - 7$$

Exercise 4.14 If $f(x)$ is defined as

$$f(x) = \frac{x-6}{6-2x}$$

what is the domain and range of $f(x)$?

Exercise 4.15 For the following two functions,

$$f(x) = 3ax^2 + ax - 7$$

and

$$g(x) = a(x^2 - 10x - 2)$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.16 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -3a - x$$

and

$$g(x) = -7a - 4x$$

Exercise 4.17 For the following two functions,

$$f(x) = a(-6x^2 + 9x - 8)$$

and

$$g(x) = -10ax - 5$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.18 If $f(x)$ is defined as

$$f(x) = x - 1$$

and $g(x)$ is defined as

$$g(x) = 7x^2 + 2x + 4$$

simplify $g(f(x))$.

Exercise 4.19 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -5a + 4x$$

and

$$g(x) = -6a + 10x$$

Exercise 4.20 For the following two functions,

$$f(x) = 7ax^2 + 4ax - 10$$

and

$$g(x) = 8a(1 - x)$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.21 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -6ax^2 + 4a + x$$

and

$$g(x) = 2a(-2x - 5)$$

Exercise 4.22 If $f(x)$ is defined as

$$f(x) = -10x^2 - x + 7$$

and $g(x)$ is defined as

$$g(x) = 5f(-7x^2 + 9x - 4)x^2 - 6x$$

simplify $g(x)$ in terms of x .

Exercise 4.23 If $f(x)$ is defined as

$$f(x) = 8x + 5$$

and $g(x)$ is defined as

$$g(x) = -f(9x - 8)x^2 - 9x$$

simplify $g(x)$ in terms of x .

Exercise 4.24 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = 9 - 7x$$

and

$$g(x) = 9a + 7x$$

Exercise 4.25 If $f(x)$ is defined as

$$f(x) = 7x + 4$$

and $g(x)$ is defined as

$$g(x) = 10x^2 + x + 8$$

simplify $g(f(x))$.

Exercise 4.26 If $f(x)$ is defined as

$$f(x) = 6x + 4$$

and $g(x)$ is defined as

$$g(x) = -7x^2 - 9x + 8$$

simplify $g(f(x))$.

Exercise 4.27 For the following two functions,

$$f(x) = 7ax^2 - 8ax + 7$$

and

$$g(x) = 4a(2 - x)$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.28 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = 5a + x$$

and

$$g(x) = -7a + 10x$$

Exercise 4.29 If $f(x)$ is defined as

$$f(x) = 9x - 3$$

and $g(x)$ is defined as

$$g(x) = -8f(-9x - 7)x^2 + 2x$$

simplify $g(x)$ in terms of x .

Exercise 4.30 If $f(x)$ is defined as

$$f(x) = \frac{6x + 7}{-8x - 7}$$

find $f^{-1}(x)$.

Exercise 4.31 If $f(x)$ is defined as

$$f(x) = \frac{1 - 4x}{2x + 1}$$

find $f^{-1}(x)$.

Exercise 4.32 For the following two functions,

$$f(x) = a(-6x^2 + 5x + 2)$$

and

$$g(x) = 2ax + 1$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.33 If $f(x)$ is defined as

$$f(x) = \frac{9 - 6x}{-3x - 3}$$

find $f^{-1}(x)$.

Exercise 4.34 For the following two functions,

$$f(x) = 6ax^2 + 8ax + 4$$

and

$$g(x) = -9ax - 8$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.35 If $f(x)$ is defined as

$$f(x) = \frac{4x - 5}{-9x - 2}$$

what is the domain and range of $f(x)$?

Exercise 4.36 If $f(x)$ is defined as

$$f(x) = -2x^2 + 2x + 5$$

and $g(x)$ is defined as

$$g(x) = -4f(3x^2 + 2x + 8)x^2 - 5x$$

simplify $g(x)$ in terms of x .

Exercise 4.37 If $f(x)$ is defined as

$$f(x) = -x^2 - 4x - 4$$

and $g(x)$ is defined as

$$g(x) = 3f(10x^2 - 10x - 10)x^2 + x$$

simplify $g(x)$ in terms of x .

Exercise 4.38 If $f(x)$ is defined as

$$f(x) = 10x - 4$$

and $g(x)$ is defined as

$$g(x) = -8x^2 + 10x - 1$$

simplify $g(f(x))$.

Exercise 4.39 For the following two functions,

$$f(x) = -6ax^2 + ax - 5$$

and

$$g(x) = 8ax^2 - 9ax + 1$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.40 If $f(x)$ is defined as

$$f(x) = \frac{7-6x}{2x+8}$$

what is the domain and range of $f(x)$?

Exercise 4.41 If $f(x)$ is defined as

$$f(x) = \frac{-9x-6}{5x-7}$$

what is the domain and range of $f(x)$?

Exercise 4.42 If $f(x)$ is defined as

$$f(x) = 3x^2 - x + 10$$

and $g(x)$ is defined as

$$g(x) = 10f(-3x^2 - 3x + 2)x^2 - 7x$$

simplify $g(x)$ in terms of x .

Exercise 4.43 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = ax^2 - 4a - 7x$$

and

$$g(x) = a(9x + 5)$$

Exercise 4.44 For the following two functions,

$$f(x) = ax^2 + ax - 3$$

and

$$g(x) = ax - 10$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.45 For the following two functions,

$$f(x) = 10ax^2 - 6ax + 7$$

and

$$g(x) = -10ax^2 - 5ax - 4$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.46 If $f(x)$ is defined as

$$f(x) = \frac{8-5x}{9x+8}$$

find $f^{-1}(x)$.

Exercise 4.47 If $f(x)$ is defined as

$$f(x) = 4ax - 10x^2$$

and $g(x)$ is defined as

$$g(x) = 7x - 2$$

solve the equation

$$g(f(x)) = 0$$

Exercise 4.48 If $f(x)$ is defined as

$$f(x) = 9x^2 - 8x + 10$$

and $g(x)$ is defined as

$$g(x) = -5f(x^2 + 9x - 8)x^2 - 2x$$

simplify $g(x)$ in terms of x .

Exercise 4.49 If $f(x)$ is defined as

$$f(x) = 2x + 10$$

and $g(x)$ is defined as

$$g(x) = 4x^2 - x - 3$$

simplify $g(f(x))$.

Exercise 4.50 If $f(x)$ is defined as

$$f(x) = 3ax - 6x^2$$

and $g(x)$ is defined as

$$g(x) = -7x - 5$$

solve the equation

$$g(f(x)) = 0$$

Exercise 4.51 If $f(x)$ is defined as

$$f(x) = 5x + 7$$

and $g(x)$ is defined as

$$g(x) = -f(9x + 7)x^2 + 10x$$

simplify $g(x)$ in terms of x .

Exercise 4.52 If $f(x)$ is defined as

$$f(x) = \frac{6x - 8}{5x - 10}$$

find $f^{-1}(x)$.

Exercise 4.53 If $f(x)$ is defined as

$$f(x) = 2 - 6x$$

and $g(x)$ is defined as

$$g(x) = -x^2 + 2x - 5$$

simplify $g(f(x))$.

Exercise 4.54 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -x - 6$$

and

$$g(x) = 3a + x$$

Exercise 4.55 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = 7ax^2 + 3a - 10x$$

and

$$g(x) = a(10x - 9)$$

Exercise 4.56 If $f(x)$ is defined as

$$f(x) = 4x^2 - 3x + 4$$

and $g(x)$ is defined as

$$g(x) = -5f(9x^2 + 9x + 10)x^2 - x$$

simplify $g(x)$ in terms of x .

Exercise 4.57 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = 8x - 9$$

and

$$g(x) = -2a - 8x$$

Exercise 4.58 If $f(x)$ is defined as

$$f(x) = \frac{-9x - 7}{10x + 6}$$

find $f^{-1}(x)$.

Exercise 4.59 For the following two functions,

$$f(x) = 2ax^2 + 10ax + 3$$

and

$$g(x) = 6ax^2 + 4ax - 6$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.60 If $f(x)$ is defined as

$$f(x) = 7x - 4$$

and $g(x)$ is defined as

$$g(x) = -2f(6x + 10)x^2 + 7x$$

simplify $g(x)$ in terms of x .

Exercise 4.61 If $f(x)$ is defined as

$$f(x) = \frac{8ax - 7a}{-7a + 2x}$$

find the value of a if $f^{-1}(6) = 10$.

Exercise 4.62 If $f(x)$ is defined as

$$f(x) = 6 - 3x$$

and $g(x)$ is defined as

$$g(x) = 4x^2 + 3x + 8$$

simplify $g(f(x))$.

Exercise 4.63 If $f(x)$ is defined as

$$f(x) = 10 - 4x$$

and $g(x)$ is defined as

$$g(x) = 6f(-7x - 4)x^2 + 9x$$

simplify $g(x)$ in terms of x .

Exercise 4.64 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -8ax^2 + 9a - 8x$$

and

$$g(x) = -7ax - 2$$

Exercise 4.65 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = 9ax^2 - 8a - 10x$$

and

$$g(x) = -ax + 8$$

Exercise 4.66 If $f(x)$ is defined as

$$f(x) = \frac{8x - 2}{5 - 6x}$$

find $f^{-1}(x)$.

Exercise 4.67 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -2ax^2 - 5a - 3x$$

and

$$g(x) = 3ax + 5$$

Exercise 4.68 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = 5ax^2 - 7a - 4x$$

and

$$g(x) = 2a(3x + 5)$$

Exercise 4.69 If $f(x)$ is defined as

$$f(x) = 7x - 9$$

and $g(x)$ is defined as

$$g(x) = -10f(6x - 7)x^2 - 4x$$

simplify $g(x)$ in terms of x .

Exercise 4.70 If $f(x)$ is defined as

$$f(x) = 9ax + 9x^2$$

and $g(x)$ is defined as

$$g(x) = -7x - 10$$

solve the equation

$$g(f(x)) = 0$$

Exercise 4.71 If $f(x)$ is defined as

$$f(x) = \frac{-x - 3}{10x - 5}$$

find $f^{-1}(x)$.

Exercise 4.72 If $f(x)$ is defined as

$$f(x) = \frac{x - 4}{10 - 8x}$$

find $f^{-1}(x)$.

Exercise 4.73 If $f(x)$ is defined as

$$f(x) = 8x^2 - 3x - 3$$

and $g(x)$ is defined as

$$g(x) = -7f(3x^2 - x - 7)x^2 + 4x$$

simplify $g(x)$ in terms of x .

Exercise 4.74 If $f(x)$ is defined as

$$f(x) = 10x - 2$$

and $g(x)$ is defined as

$$g(x) = 9f(x-3)x^2 - 5x$$

simplify $g(x)$ in terms of x .

Exercise 4.75 If $f(x)$ is defined as

$$f(x) = \frac{-9ax - 3a}{-6a + 10x}$$

find the value of a if $f^{-1}(1) = 6$.

Exercise 4.76 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -10a - 5x$$

and

$$g(x) = a - 2x$$

Exercise 4.77 For the following two functions,

$$f(x) = 6ax^2 - 3ax + 2$$

and

$$g(x) = 2a(4x^2 + 3x + 3)$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.78 For the following two functions,

$$f(x) = 8ax^2 - 5ax - 7$$

and

$$g(x) = ax^2 + 10ax + 5$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.79 For the following two functions,

$$f(x) = -4ax^2 - ax + 9$$

and

$$g(x) = -10ax^2 - 2ax - 2$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.80 If $f(x)$ is defined as

$$f(x) = \frac{-6ax + 9a}{-7a - 10x}$$

find the value of a if $f^{-1}(3) = 10$.

Exercise 4.81 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = ax^2 + 4a + 5x$$

and

$$g(x) = a(-2x - 3)$$

Exercise 4.82 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = 10ax^2 + 4a - 3x$$

and

$$g(x) = a(3x + 1)$$

Exercise 4.83 If $f(x)$ is defined as

$$f(x) = \frac{-7ax + 5a}{-3a - 6x}$$

find the value of a if $f^{-1}(5) = 3$.

Exercise 4.84 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -6a + 9x$$

and

$$g(x) = -a - 7x$$

Exercise 4.85 If $f(x)$ is defined as

$$f(x) = -6x - 3$$

and $g(x)$ is defined as

$$g(x) = 10f(4 - 2x)x^2 - x$$

simplify $g(x)$ in terms of x .

Exercise 4.86 If $f(x)$ is defined as

$$f(x) = 10 - 4x$$

and $g(x)$ is defined as

$$g(x) = -7f(2x + 3)x^2 - 6x$$

simplify $g(x)$ in terms of x .

Exercise 4.87 For the following two functions,

$$f(x) = 10ax^2 - 8ax + 5$$

and

$$g(x) = a(7 - 3x)$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.88 If $f(x)$ is defined as

$$f(x) = -4x^2 - 9x - 4$$

and $g(x)$ is defined as

$$g(x) = -6f(6x^2 - 10x - 2)x^2 + 7x$$

simplify $g(x)$ in terms of x .

Exercise 4.89 If $f(x)$ is defined as

$$f(x) = -4x - 6$$

and $g(x)$ is defined as

$$g(x) = 2x^2 - x + 7$$

simplify $g(f(x))$.

Exercise 4.90 For the following two functions,

$$f(x) = 4ax^2 - 4ax + 1$$

and

$$g(x) = -5ax^2 - 4ax + 7$$

what values of a will ensure that the graphs of these functions never intersect?

Exercise 4.91 If $f(x)$ is defined as

$$f(x) = -8x^2 - 5x + 7$$

and $g(x)$ is defined as

$$g(x) = f(-7x^2 + 9x + 3)x^2 + 8x$$

simplify $g(x)$ in terms of x .

Exercise 4.92 If $f(x)$ is defined as

$$f(x) = \frac{2x - 3}{6 - 2x}$$

find $f^{-1}(x)$.

Exercise 4.93 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -3ax^2 + 5a + 9x$$

and

$$g(x) = a(3x + 5)$$

Exercise 4.94 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = 7ax^2 - 9a + 4x$$

and

$$g(x) = 2a(1 - x)$$

Exercise 4.95 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -10a - 10x$$

and

$$g(x) = -a + 3x$$

Exercise 4.96 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = 5ax^2 - 3a - 3x$$

and

$$g(x) = 2a(4x - 3)$$

Exercise 4.97 Find the value of x at which the graphs of the following functions intersect:

$$f(x) = -9a + 2x$$

and

$$g(x) = 4a - 8x$$

Exercise 4.98 If $f(x)$ is defined as

$$f(x) = -3ax + 2x^2$$

and $g(x)$ is defined as

$$g(x) = 3 - 8x$$

solve the equation

$$g(f(x)) = 0$$

Exercise 4.99 If $f(x)$ is defined as

$$f(x) = -6ax + 7x^2$$

and $g(x)$ is defined as

$$g(x) = 5x + 5$$

solve the equation

$$g(f(x)) = 0$$

Exercise 4.100 If $f(x)$ is defined as

$$f(x) = -10x^2 + x + 9$$

and $g(x)$ is defined as

$$g(x) = 8f(-9x^2 - 8x + 3)x^2 - 5x$$

simplify $g(x)$ in terms of x .

4.2 Solutions

1.

$$\emptyset$$

2.

$$\left\{ \frac{3(a-2)}{16a} - \frac{\sqrt{329a^2 - 164a + 36}}{16a}, \frac{3(a-2)}{16a} + \frac{\sqrt{329a^2 - 164a + 36}}{16a} \right\}$$

3.

$$\left\{ \frac{3a}{20} - \frac{\sqrt{9a^2 + 80}}{20}, \frac{3a}{20} + \frac{\sqrt{9a^2 + 80}}{20} \right\}$$

4.

$$\{4a\}$$

5.

$$\left\{ \frac{5y-1}{8y+1} \right\}$$

6.

$$\left\{ -\frac{5a+6}{6a} - \frac{\sqrt{97a^2 + 60a + 36}}{6a}, -\frac{5a+6}{6a} + \frac{\sqrt{97a^2 + 60a + 36}}{6a} \right\}$$

7.

$$\left\{ -\frac{y-2}{3(y+3)} \right\}$$

8.

$$560x^3 + 620x^2 - 5x$$

9.

$$-4.412$$

10.

$$\left\{ -\frac{6a-7}{12a} - \frac{\sqrt{348a^2 - 84a + 49}}{12a}, -\frac{6a-7}{12a} + \frac{\sqrt{348a^2 - 84a + 49}}{12a} \right\}$$

11.

$$\left\{ \frac{2a}{5} \right\}$$

12.

$$128x^6 + 128x^5 - 112x^4 - 72x^3 + 80x^2 - 8x$$

13.

$$\left\{ \frac{3 \cdot (2a+3)}{8a} - \frac{\sqrt{-44a^2 - 4a + 81}}{8a}, \frac{3 \cdot (2a+3)}{8a} + \frac{\sqrt{-44a^2 - 4a + 81}}{8a} \right\}$$

14. Domain:

$$(-\infty, 3) \cup (3, \infty)$$

Range:

$$\left(-\infty, -\frac{1}{2} \right) \cup \left(-\frac{1}{2}, \infty \right)$$

15.

$$-\frac{8}{15} < a, a < 0$$

16.

$$\left\{ -\frac{4a}{3} \right\}$$

17.

$$-\frac{120}{169} < a, a < 0$$

18.

$$7x^2 - 12x + 9$$

19.

$$\left\{ \frac{a}{6} \right\}$$

20.

$$-\frac{35}{46} < a, a < 0$$

21.

$$\left\{ \frac{4a+1}{12a} - \frac{\sqrt{352a^2 + 8a + 1}}{12a}, \frac{4a+1}{12a} + \frac{\sqrt{352a^2 + 8a + 1}}{12a} \right\}$$

22.

$$-2450x^6 + 6300x^5 - 6815x^4 + 3555x^3 - 745x^2 - 6x$$

23.

$$-72x^3 + 59x^2 - 9x$$

24.

$$\left\{ \frac{9}{14} - \frac{9a}{14} \right\}$$

25.

$$490x^2 + 567x + 172$$

26.

$$-252x^2 - 390x - 140$$

27.

$$0 < a, a < \frac{49}{60}$$

28.

$$\left\{ \frac{4a}{3} \right\}$$

29.

$$648x^3 + 528x^2 + 2x$$

30.

$$\left\{ -\frac{7(y+1)}{2 \cdot (4y+3)} \right\}$$

31.

$$\left\{ -\frac{y-1}{2(y+2)} \right\}$$

32.

$$0 < a, a < \frac{8}{19}$$

33.

$$\left\{ -\frac{y+3}{y-2} \right\}$$

34.

$$0 < a, a < \frac{288}{289}$$

35. Domain:

$$\left(-\infty, -\frac{2}{9} \right) \cup \left(-\frac{2}{9}, \infty \right)$$

Range:

$$\left(-\infty, -\frac{4}{9} \right) \cup \left(-\frac{4}{9}, \infty \right)$$

36.

$$72x^6 + 96x^5 + 392x^4 + 240x^3 + 428x^2 - 5x$$

37.

$$-300x^6 + 600x^5 + 180x^4 - 480x^3 - 192x^2 + x$$

38.

$$-800x^2 + 740x - 169$$

39.

$$0 < a, a < \frac{84}{25}$$

40. Domain:

$$(-\infty, -4) \cup (-4, \infty)$$

Range:

$$(-\infty, -3) \cup (-3, \infty)$$

41. Domain:

$$\left(-\infty, \frac{7}{5}\right) \cup \left(\frac{7}{5}, \infty\right)$$

Range:

$$\left(-\infty, -\frac{9}{5}\right) \cup \left(-\frac{9}{5}, \infty\right)$$

42.

$$270x^6 + 540x^5 - 60x^4 - 330x^3 + 200x^2 - 7x$$

43.

$$\left\{ \frac{9a+7}{2a} - \frac{\sqrt{117a^2+126a+49}}{2a}, \frac{9a+7}{2a} + \frac{\sqrt{117a^2+126a+49}}{2a} \right\}$$

44.

$$0 < a$$

45.

$$0 < a, a < 880$$

46.

$$\left\{ -\frac{8(y-1)}{9y+5} \right\}$$

47.

$$\left\{ \frac{a}{5} - \frac{\sqrt{7}\sqrt{7a^2-5}}{35}, \frac{a}{5} + \frac{\sqrt{7}\sqrt{7a^2-5}}{35} \right\}$$

48.

$$-45x^6 - 810x^5 - 2885x^4 + 6840x^3 - 3250x^2 - 2x$$

49.

$$16x^2 + 158x + 387$$

50.

$$\left\{ \frac{a}{4} - \frac{\sqrt{21}\sqrt{21a^2+40}}{84}, \frac{a}{4} + \frac{\sqrt{21}\sqrt{21a^2+40}}{84} \right\}$$

51.

$$-45x^3 - 42x^2 + 10x$$

52.

$$\left\{ \frac{2 \cdot (5y-4)}{5y-6} \right\}$$

53.

$$-36x^2 + 12x - 5$$

54.

$$\left\{ -\frac{3a}{2} - 3 \right\}$$

55.

$$\left\{ \frac{5(a+1)}{7a} - \frac{\sqrt{-59a^2+50a+25}}{7a}, \frac{5(a+1)}{7a} + \frac{\sqrt{-59a^2+50a+25}}{7a} \right\}$$

56.

$$-1620x^6 - 3240x^5 - 5085x^4 - 3465x^3 - 1870x^2 - x$$

57.

$$\left\{ \frac{9}{16} - \frac{a}{8} \right\}$$

58.

$$\left\{ -\frac{6y+7}{10y+9} \right\}$$

59.

$$-4 < a, a < 0$$

60.

$$-84x^3 - 132x^2 + 7x$$

61.

$$1.043$$

62.

$$36x^2 - 153x + 170$$

63.

$$168x^3 + 156x^2 + 9x$$

64.

$$\left\{ \frac{7a-8}{16a} - \frac{\sqrt{337a^2-48a+64}}{16a}, \frac{7a-8}{16a} + \frac{\sqrt{337a^2-48a+64}}{16a} \right\}$$

65.

$$\left\{ -\frac{a-10}{18a} - \frac{\sqrt{289a^2+268a+100}}{18a}, -\frac{a-10}{18a} + \frac{\sqrt{289a^2+268a+100}}{18a} \right\}$$

66.

$$\left\{ \frac{5y+2}{2 \cdot (3y+4)} \right\}$$

67.

$$\left\{ -\frac{\sqrt{-(a+1)(31a-9)}}{4a} - \frac{3(a+1)}{4a}, \frac{\sqrt{-(a+1)(31a-9)}}{4a} - \frac{3(a+1)}{4a} \right\}$$

68.

$$\left\{ \frac{3a+2}{5a} - \frac{\sqrt{2}\sqrt{47a^2+6a+2}}{5a}, \frac{3a+2}{5a} + \frac{\sqrt{2}\sqrt{47a^2+6a+2}}{5a} \right\}$$

69.

$$-420x^3 + 580x^2 - 4x$$

70.

$$\left\{ -\frac{a}{2} - \frac{\sqrt{7}\sqrt{63a^2-40}}{42}, -\frac{a}{2} + \frac{\sqrt{7}\sqrt{63a^2-40}}{42} \right\}$$

71.

$$\left\{ \frac{5y-3}{10y+1} \right\}$$

72.

$$\left\{ \frac{2 \cdot (5y+2)}{8y+1} \right\}$$

73.

$$-504x^6 + 336x^5 + 2359x^4 - 805x^3 - 2870x^2 + 4x$$

74.

$$90x^3 - 288x^2 - 5x$$

75.

$$-1.176$$

76.

$$\left\{ -\frac{11a}{3} \right\}$$

77.

$$-\frac{16}{33} < a, a < 0$$

78.

$$-\frac{112}{75} < a, a < 0$$

79.

$$0 < a, a < 264$$

80.

$$10.0$$

81.

$$\left\{ -\frac{2a+5}{2a} - \frac{\sqrt{-24a^2+20a+25}}{2a}, -\frac{2a+5}{2a} + \frac{\sqrt{-24a^2+20a+25}}{2a} \right\}$$

82.

$$\left\{ \frac{3(a+1)}{20a} - \frac{\sqrt{3}\sqrt{-37a^2+6a+3}}{20a}, \frac{3(a+1)}{20a} + \frac{\sqrt{3}\sqrt{-37a^2+6a+3}}{20a} \right\}$$

83.

$$90.0$$

84.

$$\left\{ \frac{5a}{16} \right\}$$

85.

$$120x^3 - 270x^2 - x$$

86.

$$56x^3 + 14x^2 - 6x$$

87.

$$0 < a, a < \frac{40}{61}$$

88.

$$864x^6 - 2880x^5 + 2148x^4 + 420x^3 + 12x^2 + 7x$$

89.

$$32x^2 + 100x + 85$$

90.

$$a < 0$$

91.

$$-392x^6 + 1008x^5 - 277x^4 - 477x^3 - 80x^2 + 8x$$

92.

$$\left\{ \frac{3 \cdot (2y+1)}{2(y+1)} \right\}$$

93.

$$\left\{ 0, -\frac{a-3}{a} \right\}$$

94.

$$\left\{ -\frac{a+2}{7a} - \frac{\sqrt{2}\sqrt{39a^2+2a+2}}{7a}, -\frac{a+2}{7a} + \frac{\sqrt{2}\sqrt{39a^2+2a+2}}{7a} \right\}$$

95.

$$\left\{ -\frac{9a}{13} \right\}$$

96.

$$\left\{ \frac{8a+3}{10a} - \frac{\sqrt{4a^2+48a+9}}{10a}, \frac{8a+3}{10a} + \frac{\sqrt{4a^2+48a+9}}{10a} \right\}$$

97.

$$\left\{ \frac{13a}{10} \right\}$$

98.

$$\left\{ \frac{3a}{4} - \frac{\sqrt{3}\sqrt{3a^2+1}}{4}, \frac{3a}{4} + \frac{\sqrt{3}\sqrt{3a^2+1}}{4} \right\}$$

99.

$$\left\{ \frac{3a}{7} - \frac{\sqrt{9a^2-7}}{7}, \frac{3a}{7} + \frac{\sqrt{9a^2-7}}{7} \right\}$$

100.

$$-6480x^6 - 11520x^5 - 872x^4 + 3776x^3 - 624x^2 - 5x$$

5. Differential calculus

5.1 Problems

Exercise 5.1 Double derivative: Differentiate the following expression with respect to n twice

$$8n^5w + 8n^4 + 10w^4 + 8w - 7$$

Exercise 5.2 Differentiate the following expression with respect to d

$$2d^6 - 7dn^6 + n^6 - 4$$

Exercise 5.3 Differentiate the following expression with respect to d

$$9d^{\frac{2}{5}}n^{\frac{3}{5}} + 3d^{\frac{2}{3}}\sqrt{n} - 4n^{\frac{6}{5}} - 2n^2 + 5$$

Exercise 5.4 Differentiate the polynomial with respect to c

$$5c^3 + 7c^2 + 2c - 10$$

Exercise 5.5 Double derivative: Differentiate the following expression with respect to n twice

$$-d^3 + 10d + 2n^6 - 8n^5 + 3$$

Exercise 5.6 Differentiate the following expression with respect to n

$$7n^{\frac{5}{6}} + \sqrt[6]{n} - n^{\frac{5}{4}} - 6$$

Exercise 5.7 Double derivative: Differentiate the following expression with respect to t twice

$$n^3 \cdot (6n^3t^6 - 9n^2 - 5)$$

Exercise 5.8 Differentiate the following expression with respect to y

$$-9v^{\frac{2}{5}} + 7v^2y^3 - 9vy^6 + 9\sqrt[5]{y}$$

Exercise 5.9 Differentiate the following expression with respect to a

$$-9a^5 - 2an^4 - 12n$$

Exercise 5.10 Differentiate the following expression with respect to r

$$4\sqrt[5]{l} - 5r^{\frac{3}{5}} - 2r$$

Exercise 5.11 Differentiate the following expression with respect to s

$$9a^4s - 6a^4 + s^2 - 9$$

Exercise 5.12 Differentiate the following expression with respect to t

$$6c^{\frac{5}{6}}t^{\frac{2}{5}} + c^{\frac{3}{4}}t^{\frac{6}{5}} - 10\sqrt[3]{ct^{\frac{3}{5}}} - 5t^6 - 4$$

Exercise 5.13 Differentiate the following expression with respect to z

$$-8s^{\frac{6}{5}}z^2 - 8s^{\frac{3}{5}} - 9s^{\frac{2}{3}}z + 10\sqrt[3]{sz^{\frac{3}{2}}} + 6\sqrt[3]{sz^3}$$

Exercise 5.14 Differentiate the following expression with respect to d

$$d^{\frac{4}{3}} + 5d^{\frac{3}{2}} - 2\sqrt{s}$$

Exercise 5.15 Differentiate the following expression with respect to w

$$-6w$$

Exercise 5.16 Differentiate the polynomial with respect to z

$$-2z^4 + 6z^3 + 10z^2 - 6z - 9$$

Exercise 5.17 Differentiate the following expression with respect to q

$$-4l^{\frac{5}{6}} + 4l^{\frac{3}{2}} + 7\sqrt[3]{q}$$

Exercise 5.18 Differentiate the polynomial with respect to r

$$4r^4 + 5r^3 + 10r^2 + 2r - 4$$

Exercise 5.19 Differentiate the following expression with respect to s

$$3s^5 + s^3x^2 - s^2 - 1$$

Exercise 5.20 Double derivative: Differentiate the following expression with respect to z twice

$$-y^3 - 7z^3 + 9z^2 + 2z$$

Exercise 5.21 Double derivative: Differentiate the following expression with respect to c twice

$$c(8c^5 + 3c^3l^4 + 2c^3l + 4c - l^2)$$

Exercise 5.22 Differentiate the following expression with respect to s

$$4\sqrt[6]{ms^{\frac{4}{3}}} - 6m^{\frac{3}{4}}\sqrt[6]{s} + 5m^2\sqrt[5]{s}$$

Exercise 5.23 Double derivative: Differentiate the following expression with respect to x twice

$$-6t^6 - 6t^4 + t^3 + 6x^2 - 8x$$

Exercise 5.24 Differentiate the following expression with respect to l

$$9l^{\frac{5}{6}}m - 2l^2\sqrt[4]{m} - 7lm^{\frac{6}{5}}$$

Exercise 5.25 Double derivative: Differentiate the following expression with respect to s twice

$$l^3(-4ls^3 + 9ls^2 - 10)$$

Exercise 5.26 Differentiate the following expression with respect to l

$$3\sqrt[4]{l} - 9l^6n^5 + 8l^3n^3 + 10ln^4 + 5n^{\frac{4}{5}}$$

Exercise 5.27 Differentiate the following expression with respect to s

$$2\sqrt[4]{d}s + 2s^{\frac{2}{5}} + 9$$

Exercise 5.28 Differentiate the following expression with respect to b

$$-3b^4n^4 + 8b^3 - 9b^2n - n^2 - 3$$

Exercise 5.29 Differentiate the following expression with respect to n

$$b^2(-4b^3 - 7b^2n^6 + 8bn^4 + 8bn + 10n^5)$$

Exercise 5.30 Differentiate the following expression with respect to d

$$-2a^{\frac{5}{3}} - 4a^{\frac{2}{3}}\sqrt[3]{d} - 8\sqrt{a}\sqrt[3]{d} - 7\sqrt[6]{d}$$

Exercise 5.31 Differentiate the following expression with respect to z

$$z(8u^5z^4 + 9u^3z^4 - 8)$$

Exercise 5.32 Differentiate the polynomial with respect to z

$$2z^4 - 7z^3 - 10z^2 - 9z + 5$$

Exercise 5.33 Double derivative: Differentiate the following expression with respect to r twice

$$r(-9r^4 - 4r^2 - 4)$$

Exercise 5.34 Differentiate the polynomial with respect to y

$$-8y^3 - 7y^2 - 3y + 1$$

Exercise 5.35 Differentiate the following expression with respect to p

$$p^2v^2 \cdot (4p^2v^3 + 8p - v^4)$$

Exercise 5.36 Differentiate the polynomial with respect to s

$$-2s^3 - 5s^2 + 8s - 6$$

Exercise 5.37 Double derivative: Differentiate the following expression with respect to m twice

$$-2l - m^5 + 5m^4$$

Exercise 5.38 Differentiate the following expression with respect to b

$$-5b^6n^4 - 4b^5 - 8b^3 + 4n + 8$$

Exercise 5.39 Differentiate the following expression with respect to n

$$8n^2 \cdot (2 - n^4)$$

Exercise 5.40 Differentiate the polynomial with respect to r

$$8r^4 - 10r^3 - r^2 - 6r - 7$$

Exercise 5.41 Differentiate the polynomial with respect to p

$$6p^3 + 9p^2 - 5p - 2$$

Exercise 5.42 Differentiate the following expression with respect to x

$$9l^3x^3 - 8l^2 - 2x^2$$

Exercise 5.43 Double derivative: Differentiate the following expression with respect to v twice

$$-3r^3v^4 - 5r^3 - 7r^2v^2 - 2r + 6v^5$$

Exercise 5.44 Differentiate the following expression with respect to y

$$-3u^{\frac{5}{4}} + 3u^{\frac{2}{3}} + \sqrt{u}y^3 + 5u$$

Exercise 5.45 Differentiate the following expression with respect to d

$$3d^6v^3 + 5d^2v^6 - 3v^2 - 1$$

Exercise 5.46 Differentiate the following expression with respect to x

$$\sqrt[3]{l}x^3 - 10x^{\frac{6}{5}} - 9\sqrt{x}$$

Exercise 5.47 Differentiate the following expression with respect to m

$$-4l^5m^6 + 3m^5 + 3m^3 + 10m - 3$$

Exercise 5.48 Differentiate the following expression with respect to m

$$-10b^5m^3 - 5b^5 - 2bm^6 + 3m^2$$

Exercise 5.49 Differentiate the following expression with respect to n

$$4d^{\frac{4}{3}}n^4 - 9dn^{\frac{3}{2}} + 9n$$

Exercise 5.50 Differentiate the following expression with respect to p

$$-6b^{\frac{2}{5}} - b^{\frac{5}{3}} + 9bp - 6\sqrt[3]{p}$$

Exercise 5.51 Double derivative: Differentiate the following expression with respect to z twice

$$-6r - 9z^5 + 8z^3 + 10z^2 + 2$$

Exercise 5.52 Differentiate the polynomial with respect to b

$$5b^3 + 5b^2 + 3b + 6$$

Exercise 5.53 Differentiate the polynomial with respect to s

$$10s^4 + s^3 + 8s^2 - 9s + 5$$

Exercise 5.54 Differentiate the following expression with respect to r

$$a^3 \cdot (4a^3r + 3a - 9r^3 - 10r)$$

Exercise 5.55 Differentiate the following expression with respect to n

$$u^2 \cdot (10n^6u^2 + 4n^5u^2 + 10)$$

Exercise 5.56 Differentiate the following expression with respect to m

$$m(-10d^3m - 3d^2 - 6m)$$

Exercise 5.57 Differentiate the polynomial with respect to a

$$2a^3 - 8a^2 + 2a - 10$$

Exercise 5.58 Differentiate the following expression with respect to w

$$-5u^{\frac{3}{5}} + 9\sqrt[4]{u} - u^{\frac{5}{3}}w + 10u^{\frac{2}{3}}$$

Exercise 5.59 Differentiate the following expression with respect to m

$$-10\sqrt{m}\sqrt[6]{w} + 3w^{\frac{5}{3}} - 10w^{\frac{5}{2}} - 9w$$

Exercise 5.60 Double derivative: Differentiate the following expression with respect to d twice

$$s(-3d^6 - 4d^5s + 5d^4s^5 - 9s)$$

Exercise 5.61 Differentiate the following expression with respect to x

$$6p^{\frac{5}{2}}x^3 + p^{\frac{3}{2}}\sqrt[6]{x} - 3p^2 + 5px^4 + 10x^{\frac{5}{2}}$$

Exercise 5.62 Differentiate the following expression with respect to y

$$3a^{\frac{5}{6}}\sqrt[5]{y} + 2\sqrt[3]{ay^{\frac{3}{5}}} - 6y$$

Exercise 5.63 Double derivative: Differentiate the following expression with respect to v twice

$$v(7v^5z^2 - 4vz^6 - 9vz - 3)$$

Exercise 5.64 Double derivative: Differentiate the following expression with respect to w twice

$$-5v^6 + 10w^2 - 6w$$

Exercise 5.65 Double derivative: Differentiate the following expression with respect to z twice

$$-2n^4z^3 + 10n^4 + 10n^2 - 5z^5 + 5$$

Exercise 5.66 Differentiate the following expression with respect to n

$$-8\sqrt[4]{n}z^3 + 2n^{\frac{5}{3}}z - 8n^{\frac{3}{2}} - 4\sqrt{z} + 2$$

Exercise 5.67 Double derivative: Differentiate the following expression with respect to p twice

$$-2p^5v^4 + 4p^5 - 6v^6 - 5$$

Exercise 5.68 Double derivative: Differentiate the following expression with respect to r twice

$$b(-7b^5r + 4b^4 - 4r^4)$$

Exercise 5.69 Differentiate the polynomial with respect to s

$$-2s^3 - 4s^2 + 8s + 3$$

Exercise 5.70 Double derivative: Differentiate the following expression with respect to z twice

$$2r^5 - 10r^2 - 7r - 8z^6 + 3z^3$$

Exercise 5.71 Double derivative: Differentiate the following expression with respect to p twice

$$x(3p^5x^2 + 3p - 8x^2)$$

Exercise 5.72 Differentiate the following expression with respect to c

$$2c^5l^5 - 9c^5 + 9c^2 - 9l^2$$

Exercise 5.73 Differentiate the following expression with respect to s

$$10p^6 - 3p^5 - 7p - 3s^3$$

Exercise 5.74 Differentiate the polynomial with respect to a

$$-2a^4 + 3a^3 + 9a^2 - 7a - 8$$

Exercise 5.75 Differentiate the polynomial with respect to b

$$10b^4 + b^3 + 6b^2 + 5b - 4$$

Exercise 5.76 Double derivative: Differentiate the following expression with respect to x twice

$$-9x^5z^6 + 4x^2 - 8x - 10z^4 + 5z^2$$

Exercise 5.77 Differentiate the following expression with respect to b

$$4b^6 + 3b^2x^2 - 2b + 9x^3 + 1$$

Exercise 5.78 Differentiate the polynomial with respect to x

$$2x^2 - 5x + 5$$

Exercise 5.79 Differentiate the following expression with respect to s

$$2s + 13$$

Exercise 5.80 Double derivative: Differentiate the following expression with respect to d twice

$$d^4 - 10dv^3 + 3v^6 - 5v^3$$

Exercise 5.81 Differentiate the following expression with respect to s

$$-3s^{\frac{6}{5}} - 6s + 8w^5$$

Exercise 5.82 Double derivative: Differentiate the following expression with respect to m twice

$$9d^5m^6 + 4d^4 + 6m^4$$

Exercise 5.83 Differentiate the polynomial with respect to t

$$-7t^3 + 9t^2 + 7t + 4$$

Exercise 5.84 Differentiate the following expression with respect to q

$$3q^{\frac{3}{4}}r^{\frac{5}{3}} + 7q^{\frac{5}{3}}r^{\frac{2}{5}} + q^{\frac{5}{3}} + 4\sqrt{q} + r^{\frac{3}{4}}$$

Exercise 5.85 Differentiate the following expression with respect to z

$$w^2 \cdot (6w^4 + 9z^6 - 2z^3)$$

Exercise 5.86 Double derivative: Differentiate the following expression with respect to d twice

$$8d^6z^2 + 10d^6 + 4d^5 + 5$$

Exercise 5.87 Differentiate the following expression with respect to r

$$\sqrt{q}(10q^2 - 4q^4\sqrt{r} + 9)$$

Exercise 5.88 Differentiate the following expression with respect to c

$$-13c^5 - 10c^3 + 10n^3 + 4$$

Exercise 5.89 Double derivative: Differentiate the following expression with respect to x twice

$$-5p^5x^6 + 10x^6 + 6$$

Exercise 5.90 Differentiate the polynomial with respect to b

$$10b^3 + 9b^2 - 7b + 2$$

Exercise 5.91 Differentiate the polynomial with respect to a

$$9a^4 - 4a^3 - 9a^2 - 6a - 3$$

Exercise 5.92 Double derivative: Differentiate the following expression with respect to y twice

$$5y^3z^6 - 4y^2 - 7z^2 - 8$$

Exercise 5.93 Differentiate the following expression with respect to c

$$-9c^6w^6 - 10c^2w^3 + 9c^2 + 3$$

Exercise 5.94 Differentiate the following expression with respect to z

$$-4\sqrt[4]{sz^{\frac{2}{5}}} + 5s^{\frac{5}{2}} - 6s^{\frac{3}{2}}\sqrt[6]{z} - 9\sqrt{z}$$

Exercise 5.95 Double derivative: Differentiate the following expression with respect to y twice

$$-9y^3z^4 - 5y^3z^3 - 5y^2 + 10z^4 + z^3$$

Exercise 5.96 Double derivative: Differentiate the following expression with respect to n twice

$$-n^4 + 3n^3 + 6w^6$$

Exercise 5.97 Differentiate the polynomial with respect to s

$$-6s^4 - 6s^3 + 10s^2 - 3s - 6$$

Exercise 5.98 Differentiate the polynomial with respect to l

$$3l^3 - 2l^2 - 2l + 9$$

Exercise 5.99 Double derivative: Differentiate the following expression with respect to x twice

$$-3r^4x^4 - 3r^3x - 10r^2x + 9r^2 + 2$$

Exercise 5.100 Double derivative: Differentiate the following expression with respect to z twice

$$z^2 \cdot (5v^5z + 2z - 10)$$

5.2 Solutions

1.

$$n^2 \cdot (160nw + 96)$$

2.

$$12d^5 - 7n^6$$

3.

$$\frac{2\sqrt{n}}{\sqrt[3]{d}} + \frac{18n^{\frac{3}{5}}}{5d^{\frac{3}{5}}}$$

4.

$$15c^2 + 14c + 2$$

5.

$$n^3 \cdot (60n - 160)$$

6.

$$\frac{70n^{\frac{5}{6}} + 2\sqrt[6]{n} - 15n^{\frac{5}{4}}}{12n}$$

7.

$$180n^6t^4$$

8.

$$21v^2y^2 - 54vy^5 + \frac{9}{5y^{\frac{4}{5}}}$$

9.

$$-45a^4 - 2n^4$$

10.

$$-2 - \frac{3}{r^{\frac{2}{5}}}$$

11.

$$9a^4 + 2s$$

12.

$$\frac{6 \cdot (2c^{\frac{5}{6}}t^{\frac{2}{5}} + c^{\frac{3}{4}}t^{\frac{6}{5}} - 5\sqrt[3]{ct^{\frac{3}{5}}} - 25t^6)}{5t}$$

13.

$$-16s^{\frac{6}{5}}z - 9s^{\frac{2}{3}} + 15\sqrt[3]{s}\sqrt{z} + 18\sqrt[3]{s}z^2$$

14.

$$\frac{4\sqrt[3]{d}}{3} + \frac{15\sqrt{d}}{2}$$

15.

$$-6$$

16.

$$-8z^3 + 18z^2 + 20z - 6$$

17.

$$\frac{7}{3q^{\frac{2}{3}}}$$

18.

$$16r^3 + 15r^2 + 20r + 2$$

19.

$$s(15s^3 + 3sx^2 - 2)$$

20.

$$18 - 42z$$

21.

$$240c^4 + 36c^2l^4 + 24c^2l + 8$$

22.

$$\frac{16\sqrt[6]{m}\sqrt[3]{s}}{3} - \frac{m^{\frac{3}{4}}}{s^{\frac{5}{6}}} + \frac{m^2}{s^{\frac{4}{5}}}$$

23.

$$12$$

24.

$$-4l\sqrt[4]{m} - 7m^{\frac{6}{5}} + \frac{15m}{2\sqrt[6]{l}}$$

25.

$$l^4 \cdot (18 - 24s)$$

26.

$$-54l^5n^5 + 24l^2n^3 + 10n^4 + \frac{3}{4l^{\frac{3}{4}}}$$

27.

$$2\sqrt[4]{d} + \frac{4}{5s^{\frac{3}{5}}}$$

28.

$$6b(-2b^2n^4 + 4b - 3n)$$

29.

$$b^2(-42b^2n^5 + 32bn^3 + 8b + 50n^4)$$

30.

$$\frac{8d^{\frac{5}{6}}\left(-a^{\frac{2}{3}} - 2\sqrt{a}\right) - 7d^{\frac{2}{3}}}{6d^{\frac{3}{2}}}$$

31.

$$40u^5z^4 + 45u^3z^4 - 8$$

32.

$$8z^3 - 21z^2 - 20z - 9$$

33.

$$-180r^3 - 24r$$

34.

$$-24y^2 - 14y - 3$$

35.

$$2pv^2 \cdot (8p^2v^3 + 12p - v^4)$$

36.

$$-6s^2 - 10s + 8$$

37.

$$20m^2 \cdot (3 - m)$$

38.

$$b^2(-30b^3n^4 - 20b^2 - 24)$$

39.

$$-48n^5 + 32n$$

40.

$$32r^3 - 30r^2 - 2r - 6$$

41.

$$18p^2 + 18p - 5$$

42.

$$x(27l^3x - 4)$$

43.

$$-36r^3v^2 - 14r^2 + 120v^3$$

44.

$$3\sqrt{u}y^2$$

45.

$$18d^5v^3 + 10dv^6$$

46.

$$3\sqrt[3]{l}x^2 - 12\sqrt[5]{x} - \frac{9}{2\sqrt{x}}$$

47.

$$-24l^5m^5 + 15m^4 + 9m^2 + 10$$

48.

$$6m(-5b^5m - 2bm^4 + 1)$$

49.

$$16d^{\frac{4}{3}}n^3 - \frac{27d\sqrt{n}}{2} + 9$$

50.

$$9b - \frac{2}{p^{\frac{2}{3}}}$$

51.

$$-180z^3 + 48z + 20$$

52.

$$15b^2 + 10b + 3$$

53.

$$40s^3 + 3s^2 + 16s - 9$$

54.

$$a^3 \cdot (4a^3 - 27r^2 - 10)$$

55.

$$n^4u^4 \cdot (60n + 20)$$

56.

$$-20d^3m - 3d^2 - 12m$$

57.

$$6a^2 - 16a + 2$$

58.

$$-u^{\frac{5}{3}}$$

59.

$$-\frac{5\sqrt[6]{w}}{\sqrt{m}}$$

60.

$$10d^2s(-9d^2 - 8ds + 6s^5)$$

61.

$$18p^{\frac{5}{2}}x^2 + \frac{p^{\frac{3}{2}}}{6x^{\frac{5}{6}}} + 20px^3 + 25x^{\frac{3}{2}}$$

62.

$$\frac{3a^{\frac{5}{6}}}{5y^{\frac{4}{5}}} + \frac{6\sqrt[3]{a}}{5y^{\frac{2}{5}}} - 6$$

63.

$$2z(105v^4z - 4z^5 - 9)$$

64.

$$20$$

65.

$$-12n^4z - 100z^3$$

66.

$$\frac{2 \cdot \left(5n^{\frac{17}{12}}z - 18n^{\frac{5}{4}} - 3z^3\right)}{3n^{\frac{3}{4}}}$$

67.

$$40p^3 \cdot (2 - v^4)$$

68.

$$-48br^2$$

69.

$$-6s^2 - 8s + 8$$

70.

$$-240z^4 + 18z$$

71.

$$60p^3x^3$$

72.

$$c(10c^3l^5 - 45c^3 + 18)$$

73.

$$-9s^2$$

74.

$$-8a^3 + 9a^2 + 18a - 7$$

75.

$$40b^3 + 3b^2 + 12b + 5$$

76.

$$-180x^3z^6 + 8$$

77.

$$24b^5 + 6bx^2 - 2$$

78.

$$4x - 5$$

79.

$$2$$

80.

$$12d^2$$

81.

$$-\frac{18\sqrt[5]{s}}{5} - 6$$

82.

$$m^2 \cdot (270d^5m^2 + 72)$$

83.

$$-21t^2 + 18t + 7$$

84.

$$\frac{20q^{\frac{17}{12}} \cdot \left(7r^{\frac{2}{3}} + 1\right) + 24\sqrt[4]{q} + 27\sqrt{q}r^{\frac{5}{3}}}{12q^{\frac{3}{4}}}$$

85.

$$w^2z^2 \cdot (54z^3 - 6)$$

86.

$$d^3 \cdot (240dz^2 + 300d + 80)$$

87.

$$-\frac{q^{\frac{3}{2}}}{r^{\frac{3}{4}}}$$

88.

$$c^2(-65c^2 - 30)$$

89.

$$150x^4 \cdot (2 - p^5)$$

90.

$$30b^2 + 18b - 7$$

91.

$$36a^3 - 12a^2 - 18a - 6$$

92.

$$30yz^6 - 8$$

93.

$$2c(-27c^4w^6 - 10w^3 + 9)$$

94.

$$-\frac{8\sqrt[4]{s}}{5z^{\frac{3}{5}}} - \frac{s^{\frac{3}{2}}}{z^{\frac{5}{6}}} - \frac{9}{2\sqrt{z}}$$

95.

$$-54yz^4 - 30yz^3 - 10$$

96.

$$6n(3 - 2n)$$

97.

$$-24s^3 - 18s^2 + 20s - 3$$

98.

$$9l^2 - 4l - 2$$

99.

$$-36r^4x^2$$

100.

$$30v^5z + 12z - 20$$

6. Integral calculus

6.1 Problems

Exercise 6.1 Evaluate the definite integral for the following function, where the limits are 0 and 3.

$$f(d) = d(d^3 + 12)$$

Exercise 6.2 Integrate the polynomial with respect to a

$$-a^4 + 9a^3 + 10a^2 - 7a + 3$$

Exercise 6.3 Evaluate the following indefinite integral

$$\int -b^4 - 9b^2 + 2bv^6 - 1dv$$

Exercise 6.4 Evaluate the definite integral for the following function, where the limits are 1 and 5.

$$f(y) = -2y^{\frac{5}{4}} - 7y^3 + 9y$$

Exercise 6.5 Evaluate the following indefinite integral

$$\int q^3 \cdot (4q^2w^2 + 3q^2 - 2w^5) dw$$

Exercise 6.6 Evaluate the following indefinite integral

$$\int 4\sqrt[5]{w}z^{\frac{3}{5}} - 9w^{\frac{3}{2}} - 9w^6\sqrt[4]{z} + 2z^6 + 4dz$$

Exercise 6.7 Evaluate the following indefinite integral

$$\int -5b^{\frac{2}{3}}\sqrt{t} + b^3t + 6\sqrt{t} - 9tdb$$

Exercise 6.8 Evaluate the following indefinite integral

$$\int -9n^{\frac{3}{5}}y^{\frac{6}{5}} + 8n^{\frac{5}{4}}y - 9n^{\frac{3}{2}}y^5 + 5y^{\frac{4}{3}} dy$$

Exercise 6.9 Evaluate the definite integral for the following function, where the limits are 2 and 4.

$$f(v) = 8v^{\frac{4}{5}} + 6\sqrt[4]{v} - 7v^{\frac{4}{3}} - 8v^2 - 8$$

Exercise 6.10 Evaluate the definite integral for the following function, where the limits are 1 and 5.

$$f(a) = -6\sqrt[4]{a} + 2a^2 - 4a$$

Exercise 6.11 Evaluate the following indefinite integral

$$\int -10m - p^{\frac{5}{2}} + p^4 + 10dp$$

Exercise 6.12 Integrate the polynomial with respect to z

$$7z^3 + z^2 + 3z - 7$$

Exercise 6.13 Evaluate the following indefinite integral

$$\int -4q^5u - q^5 + 6q^2u^2 + 8u^6 dq$$

Exercise 6.14 Integrate the polynomial with respect to r

$$6r^2 + 10r - 9$$

Exercise 6.15 Evaluate the following indefinite integral

$$\int -8r^6 - 5r^3y + 2ry^5 + 9r + 3dr$$

Exercise 6.16 Evaluate the definite integral for the following function, where the limits are 3 and 7.

$$f(a) = -7a^{\frac{4}{3}} + 7a^{\frac{2}{3}} - 5\sqrt{a} + 8a^5 - 5a$$

Exercise 6.17 Integrate the polynomial with respect to w

$$-4w^4 + 2w^3 - 3w^2 + 9w + 4$$

Exercise 6.18 Integrate the polynomial with respect to u

$$-3u^2 + 3u + 8$$

Exercise 6.19 Evaluate the following indefinite integral

$$\int 7t^{\frac{3}{4}} + 4t^3u - 8u^{\frac{6}{5}} - u^{\frac{4}{5}} + 8u^3 dt$$

Exercise 6.20 Evaluate the definite integral for the following function, where the limits are 3 and 4.

$$f(z) = 8z^4 - 2z - 2$$

Exercise 6.21 Evaluate the following indefinite integral

$$\int x^3 (-10b^2x^3 - 10x + 1) db$$

Exercise 6.22 Evaluate the following indefinite integral

$$\int 5a^5 - 4a^4 + 10z^5 - 1da$$

Exercise 6.23 Evaluate the definite integral for the following function, where the limits are 2 and 3.

$$f(p) = p^{\frac{5}{6}} + 8p^{\frac{2}{3}} + 10p^{\frac{3}{2}} - 8p^4 + 3p^3$$

Exercise 6.24 Evaluate the following indefinite integral

$$\int -4b^{\frac{3}{2}}q^{\frac{3}{2}} - 8b^2q^{\frac{4}{5}} + bq^{\frac{4}{3}} + 6q^{\frac{3}{5}} - 6db$$

Exercise 6.25 Evaluate the following indefinite integral

$$\int -7a^2p^3 - 3a + 3p^{\frac{3}{4}} - 7p^3 da$$

Exercise 6.26 Evaluate the following indefinite integral

$$\int -6c^{\frac{5}{3}} - 10c^{\frac{3}{2}} + 6\sqrt{c}s^2 - 9c^2 - 10s^{\frac{3}{4}} dc$$

Exercise 6.27 Evaluate the following indefinite integral

$$\int 6q^{\frac{3}{2}}x + q^2 + 9q + 4dx$$

Exercise 6.28 Evaluate the following indefinite integral

$$\int -2\sqrt{m} - 9m^2p + 6m\sqrt[6]{p} - 3dm$$

Exercise 6.29 Evaluate the definite integral for the following function, where the limits are 3 and 7.

$$f(c) = 6c^3 + 9c$$

Exercise 6.30 Evaluate the definite integral for the following function, where the limits are 2 and 3.

$$f(s) = -4s^{\frac{4}{5}} + 4s^{\frac{3}{5}} + 7s^{\frac{2}{5}} - 3s^4 - 5s$$

Exercise 6.31 Evaluate the following indefinite integral

$$\int 6lr^2 - 7l + 2r^6 dr$$

Exercise 6.32 Evaluate the definite integral for the following function, where the limits are 2 and 5.

$$f(t) = -10\sqrt[3]{t} + 5t^5 + t^2$$

Exercise 6.33 Evaluate the following indefinite integral

$$\int -5\sqrt[9]{q}x^6 + 8q - 6dq$$

Exercise 6.34 Integrate the polynomial with respect to v

$$-7v^2 + 6v - 1$$

Exercise 6.35 Evaluate the following indefinite integral

$$\int 4l^5 + 5l^4 + 8n^6 dn$$

Exercise 6.36 Evaluate the following indefinite integral

$$\int r^{\frac{2}{5}} + r^{\frac{3}{2}}x^{\frac{5}{2}} + 7r^5x^{\frac{3}{2}} + 10rx^5 + 8xdx$$

Exercise 6.37 Evaluate the definite integral for the following function, where the limits are 0 and 4.

$$f(w) = -w^{\frac{4}{5}} - 9\sqrt[3]{w} + 3\sqrt{w}$$

Exercise 6.38 Evaluate the definite integral for the following function, where the limits are 3 and 6.

$$f(v) = 7v^{\frac{6}{5}} + 8v^{\frac{5}{4}} - 1$$

Exercise 6.39 Integrate the polynomial with respect to u

$$10u^2 - 6u - 5$$

Exercise 6.40 Integrate the polynomial with respect to x

$$-9x^4 + 5x^3 + 2x^2 + 2x + 5$$

Exercise 6.41 Evaluate the definite integral for the following function, where the limits are 3 and 6.

$$f(n) = -8n^{\frac{5}{6}} - 10n^{\frac{2}{3}} - n^6 + 6$$

Exercise 6.42 Evaluate the following indefinite integral

$$\int 2c^6 - 4c^3 + 7p^2 dp$$

Exercise 6.43 Evaluate the definite integral for the following function, where the limits are 3 and 4.

$$f(l) = 10l^{\frac{4}{3}} + 4\sqrt[5]{l} + 8l^2 + 8$$

Exercise 6.44 Evaluate the following indefinite integral

$$\int -4n^3 z^{\frac{2}{3}} - 4n^2 z + 9nz^{\frac{5}{2}} + 2dz$$

Exercise 6.45 Integrate the polynomial with respect to r

$$2r^4 - 3r^3 - 2r^2 + 5r - 5$$

Exercise 6.46 Evaluate the definite integral for the following function, where the limits are 1 and 4.

$$f(u) = u^{\frac{3}{5}} - 3\sqrt[5]{u} + 8$$

Exercise 6.47 Evaluate the following indefinite integral

$$\int -5\sqrt[6]{l}\sqrt{z} - 7\sqrt[3]{l}z^{\frac{2}{3}} + 6z^{\frac{2}{3}} dl$$

Exercise 6.48 Evaluate the following indefinite integral

$$\int 3n^4 + 6n^3 r^6 - 7n^3 - 6r^3 dn$$

Exercise 6.49 Evaluate the following indefinite integral

$$\int 4c^2 z^2 - 6c - 10dc$$

Exercise 6.50 Integrate the polynomial with respect to s

$$10s^2 - 5s - 9$$

Exercise 6.51 Evaluate the following indefinite integral

$$\int -2\sqrt{rs}^{\frac{2}{3}} + 2r^2 - 4r + s^{\frac{3}{2}} + 4sdr$$

Exercise 6.52 Evaluate the following indefinite integral

$$\int 8m^3 - 3m^2\sqrt{v} + 2v^{\frac{2}{5}} + 2vdm$$

Exercise 6.53 Evaluate the following indefinite integral

$$\int 7b^6 - 5b^5w + 8b^2w^3 - 2b - 6w^5 dw$$

Exercise 6.54 Evaluate the following indefinite integral

$$\int -6b^3 + 9r^2 + 9rdb$$

Exercise 6.55 Evaluate the definite integral for the following function, where the limits are 0 and 4.

$$f(x) = 10x^{\frac{4}{5}} + 4x^{\frac{2}{5}} + x^6 + 3x$$

Exercise 6.56 Evaluate the following indefinite integral

$$\int -8b^{\frac{5}{4}} - \sqrt[3]{b} + 8b^4 db$$

Exercise 6.57 Evaluate the following indefinite integral

$$\int -s^5w^3 + 4s^5w^2 - 5s^5w + 3ds$$

Exercise 6.58 Evaluate the following indefinite integral

$$\int 9c^{\frac{6}{5}} - 4\sqrt[5]{c} - q^{\frac{6}{5}} - 10q dq$$

Exercise 6.59 Integrate the polynomial with respect to y

$$8y^3 + 9y^2 - 4y + 1$$

Exercise 6.60 Evaluate the following indefinite integral

$$\int b^3 \cdot (4bz^2 - 7b - z^2) db$$

Exercise 6.61 Integrate the polynomial with respect to y

$$-10y^4 - 10y^3 + 10y^2 - 2y + 1$$

Exercise 6.62 Evaluate the following indefinite integral

$$\int 13l + 8udl$$

Exercise 6.63 Evaluate the following indefinite integral

$$\int 2w^{\frac{5}{6}}y^{\frac{3}{2}} + w^{\frac{4}{5}} + 9w^{\frac{3}{4}}\sqrt[3]{y} - 10w^{\frac{5}{3}}\sqrt[3]{y} - 5\sqrt{y} dy$$

Exercise 6.64 Evaluate the following indefinite integral

$$\int s(10s^5 - 2s^2 + 9t^5) dt$$

Exercise 6.65 Integrate the polynomial with respect to c

$$4c^4 + 9c^3 - 2c^2 - 2c + 2$$

Exercise 6.66 Integrate the polynomial with respect to c

$$9c^3 + c^2 + 5c + 2$$

Exercise 6.67 Integrate the polynomial with respect to u

$$-2u^4 + 7u^3 - 4u^2 + 2u - 10$$

Exercise 6.68 Evaluate the following indefinite integral

$$\int -2c^4 l^3 - 4c^3 l^4 + 8l^6 - 5dl$$

Exercise 6.69 Evaluate the following indefinite integral

$$\int -v^2 + 4\sqrt[3]{w} + 2\sqrt{w}dw$$

Exercise 6.70 Integrate the polynomial with respect to v

$$-7v^2 + 4v + 1$$

Exercise 6.71 Evaluate the following indefinite integral

$$\int 6w^{\frac{3}{4}} + 10w^3 - 2w^2 + 9z^{\frac{3}{5}} + z^{\frac{2}{5}}dw$$

Exercise 6.72 Evaluate the following indefinite integral

$$\int r(-3a^5 r^5 - 3ar^3 + 4r^4 - 8) dr$$

Exercise 6.73 Evaluate the following indefinite integral

$$\int 7a^5 u^4 + 10a^5 u^3 - a^2 - 8au - 10udu$$

Exercise 6.74 Evaluate the following indefinite integral

$$\int m^2(-2m^3 t - 3m - 8t^5) dm$$

Exercise 6.75 Evaluate the following indefinite integral

$$\int 10b^{\frac{6}{5}}y^{\frac{3}{2}} - 3y^{\frac{5}{4}} - 3y^3 + 3y db$$

Exercise 6.76 Integrate the polynomial with respect to w

$$-7w^2 + w + 6$$

Exercise 6.77 Evaluate the definite integral for the following function, where the limits are 1 and 3.

$$f(p) = 8p^{\frac{6}{5}} - 8\sqrt[5]{p} - 7p^4 + 3p - 1$$

Exercise 6.78 Evaluate the following indefinite integral

$$\int 2\sqrt[4]{c} - 7m^{\frac{3}{5}} - 9\sqrt{m} dc$$

Exercise 6.79 Evaluate the definite integral for the following function, where the limits are 0 and 2.

$$f(d) = -8d^{\frac{5}{6}} + 3d^{\frac{4}{3}}$$

Exercise 6.80 Evaluate the definite integral for the following function, where the limits are 0 and 1.

$$f(q) = 6\sqrt[6]{q} + 10q^{\frac{5}{4}} - 5q^{\frac{3}{2}} + 9\sqrt{q} - 5q$$

Exercise 6.81 Evaluate the definite integral for the following function, where the limits are 0 and 3.

$$f(r) = 7r^{\frac{5}{6}} + 2r^{\frac{3}{5}} - 7r^{\frac{2}{3}} - 3r^2 + 2$$

Exercise 6.82 Evaluate the following indefinite integral

$$\int b^2 (-9b^4x^2 + 7b^4 + 6b^3x - 9b^2 - 4x^6) db$$

Exercise 6.83 Evaluate the definite integral for the following function, where the limits are 0 and 4.

$$f(v) = v^{\frac{2}{5}} - 4\sqrt[4]{v} + 10v^{\frac{3}{2}} - 9v$$

Exercise 6.84 Evaluate the following indefinite integral

$$\int 2\sqrt[6]{q} - q^5\sqrt{y} + 4q^3y^2 + 3y^{\frac{5}{4}} dq$$

Exercise 6.85 Evaluate the definite integral for the following function, where the limits are 2 and 3.

$$f(c) = -10c^{\frac{6}{5}} - 4c^2 + 5c$$

Exercise 6.86 Evaluate the definite integral for the following function, where the limits are 1 and 3.

$$f(l) = l^{\frac{5}{4}} + \sqrt[3]{l} - 5l^6 + 3$$

Exercise 6.87 Evaluate the following indefinite integral

$$\int -6b^{\frac{4}{3}} - 3b^3 - 2w^{\frac{2}{3}} + 3w^3 dw$$

Exercise 6.88 Evaluate the following indefinite integral

$$\int -4q^6 t^3 - 2q^4 + 2qt^3 + 4qt^2 - 10dq$$

Exercise 6.89 Integrate the polynomial with respect to p

$$-6p^3 + p^2 - 8p - 1$$

Exercise 6.90 Evaluate the following indefinite integral

$$\int a^3 (-9a^3 - 2y^5 - 9y) da$$

Exercise 6.91 Integrate the polynomial with respect to b

$$-7b^4 - 8b^3 - b^2 - 4b + 4$$

Exercise 6.92 Evaluate the following indefinite integral

$$\int 2n^6 p^4 + 7n^5 p^4 - 7n^5 - 7n^3 - 2dp$$

Exercise 6.93 Evaluate the following indefinite integral

$$\int -10pq - 4q^{\frac{5}{6}} - 8q^{\frac{5}{3}} + 3dp$$

Exercise 6.94 Evaluate the following indefinite integral

$$\int z (-7xz^5 + 4xz^3 + 7z - 5) dz$$

Exercise 6.95 Evaluate the following indefinite integral

$$\int 8c^4 r^6 - 10c^4 + 4cr^2 + 5r^2 dc$$

Exercise 6.96 Evaluate the definite integral for the following function, where the limits are 1 and 4.

$$f(a) = 4a^{\frac{4}{5}} + 4a^{\frac{5}{2}} - 6a^4 + 4a$$

Exercise 6.97 Evaluate the following indefinite integral

$$\int a(5a^4u^3 + 2a^2 - 10u) du$$

Exercise 6.98 Evaluate the following indefinite integral

$$\int -b^6s^3 - 9b^3 - 10db$$

Exercise 6.99 Integrate the polynomial with respect to z

$$z^2 - 4z - 9$$

Exercise 6.100 Integrate the polynomial with respect to r

$$2r^3 - 8r^2 - 4r + 2$$

6.2 Solutions

1. The indefinite integral is

$$\frac{d^2(d^3 + 30)}{5}$$

The value of the definite integral is 102.600000000000.

- 2.

$$\frac{a(-12a^4 + 135a^3 + 200a^2 - 210a + 180)}{60}$$

- 3.

$$v\left(-b^4 - 9b^2 + \frac{2bv^6}{7} - 1\right)$$

4. The indefinite integral is

$$-\frac{8y^{\frac{9}{4}}}{9} - \frac{7y^4}{4} + \frac{9y^2}{2}$$

The value of the definite integral is -1016.34108402714.

- 5.

$$\frac{q^3w(4q^2w^2 + 9q^2 - w^5)}{3}$$

- 6.

$$\frac{5\sqrt[5]{wz^{\frac{8}{5}}}}{2} - 9w^{\frac{3}{2}}z - \frac{36w^6z^{\frac{5}{4}}}{5} + \frac{2z^7}{7} + 4z$$

- 7.

$$-3b^{\frac{5}{3}}\sqrt{t} + \frac{b^4t}{4} + 6b\sqrt{t} - 9bt$$

8.

$$-\frac{45n^{\frac{3}{5}}y^{\frac{11}{5}}}{11} + 4n^{\frac{5}{4}}y^2 - \frac{3n^{\frac{3}{2}}y^6}{2} + \frac{15y^{\frac{7}{3}}}{7}$$

9. The indefinite integral is

$$\frac{40v^{\frac{9}{5}}}{9} + \frac{24v^{\frac{5}{4}}}{5} - 3v^{\frac{7}{3}} - \frac{8v^3}{3} - 8v$$

The value of the definite integral is -172.257329007093.

10. The indefinite integral is

$$-\frac{24a^{\frac{5}{4}}}{5} + \frac{2a^3}{3} - 2a^2$$

The value of the definite integral is 3.57829591735737.

11.

$$-10mp - \frac{2p^{\frac{7}{2}}}{7} + \frac{p^5}{5} + 10p$$

12.

$$\frac{z(21z^3 + 4z^2 + 18z - 84)}{12}$$

13.

$$\frac{q(-q^5 \cdot (4u + 1) + 12q^2u^2 + 48u^6)}{6}$$

14.

$$r(2r^2 + 5r - 9)$$

15.

$$\frac{r(-32r^6 - 35r^3y + 14r(2y^5 + 9) + 84)}{28}$$

16. The indefinite integral is

$$-3a^{\frac{7}{3}} + \frac{21a^{\frac{5}{3}}}{5} - \frac{10a^{\frac{3}{2}}}{3} + \frac{4a^6}{3} - \frac{5a^2}{2}$$

The value of the definite integral is 155588.034030386.

17.

$$\frac{w(-8w^4 + 5w^3 - 10w^2 + 45w + 40)}{10}$$

18.

$$\frac{u(-2u^2 + 3u + 16)}{2}$$

19.

$$4t^{\frac{7}{4}} + t^4u - 8tu^{\frac{6}{5}} - tu^{\frac{4}{5}} + 8tu^3$$

20. The indefinite integral is

$$\frac{z(8z^4 - 5z - 10)}{5}$$

The value of the definite integral is 1240.600000000000.

21.

$$\frac{bx^3(-10b^2x^3 - 30x + 3)}{3}$$

22.

$$\frac{a(25a^5 - 24a^4 + 300z^5 - 30)}{30}$$

23. The indefinite integral is

$$\frac{6p^{\frac{11}{6}}}{11} + \frac{40p^{\frac{7}{5}}}{7} + 4p^{\frac{5}{2}} - \frac{8p^5}{5} + \frac{3p^4}{4}$$

The value of the definite integral is -235.456672956732.

24.

$$-\frac{8b^{\frac{5}{2}}q^{\frac{3}{2}}}{5} - \frac{8b^3q^{\frac{4}{5}}}{3} + \frac{b^2q^{\frac{4}{3}}}{2} + 6bq^{\frac{3}{5}} - 6b$$

25.

$$\frac{a(-14a^2p^3 - 9a + 18p^{\frac{3}{4}} - 42p^3)}{6}$$

26.

$$-\frac{9c^{\frac{8}{3}}}{4} - 4c^{\frac{5}{2}} + 4c^{\frac{3}{2}}s^2 - 3c^3 - 10cs^{\frac{3}{4}}$$

27.

$$x(3q^{\frac{3}{2}}x + q^2 + 9q + 4)$$

28.

$$-\frac{4m^{\frac{3}{2}}}{3} - 3m^3p + 3m^2\sqrt[6]{p} - 3m$$

29. The indefinite integral is

$$\frac{3c^2(c^2 + 3)}{2}$$

The value of the definite integral is 3660.000000000000.

30. The indefinite integral is

$$-\frac{20s^{\frac{9}{5}}}{9} + \frac{5s^{\frac{8}{5}}}{2} + 5s^{\frac{7}{5}} - \frac{3s^5}{5} - \frac{5s^2}{2}$$

The value of the definite integral is -130.413717000468.

31.

$$\frac{r(14lr^2 - 49l + 2r^6)}{7}$$

32. The indefinite integral is

$$-\frac{15t^{\frac{4}{3}}}{2} + \frac{5t^6}{6} + \frac{t^3}{3}$$

The value of the definite integral is 12961.2747177480.

33.

$$-\frac{30q^{\frac{7}{6}}x^6}{7} + 4q^2 - 6q$$

34.

$$\frac{v(-7v^2 + 9v - 3)}{3}$$

35.

$$\frac{n(l^4 \cdot (28l + 35) + 8n^6)}{7}$$

36.

$$r^{\frac{2}{5}}x + \frac{2r^{\frac{3}{2}}x^{\frac{7}{2}}}{7} + \frac{14r^5x^{\frac{5}{2}}}{5} + \frac{5rx^6}{3} + 4x^2$$

37. The indefinite integral is

$$-\frac{5w^{\frac{9}{5}}}{9} - \frac{27w^{\frac{4}{3}}}{4} + 2w^{\frac{3}{2}}$$

The value of the definite integral is -33.5963464765209.

38. The indefinite integral is

$$\frac{35v^{\frac{11}{5}}}{11} + \frac{32v^{\frac{9}{4}}}{9} - v$$

The value of the definite integral is 283.454257490695.

39.

$$\frac{u(10u^2 - 9u - 15)}{3}$$

40.

$$\frac{x(-108x^4 + 75x^3 + 40x^2 + 60x + 300)}{60}$$

41. The indefinite integral is

$$-\frac{48n^{\frac{11}{6}}}{11} - \frac{50n^{\frac{7}{5}}}{7} - \frac{n^7}{7} + 6n$$

The value of the definite integral is -39798.7661423313.

42.

$$\frac{p(6c^3(c^3 - 2) + 7p^2)}{3}$$

43. The indefinite integral is

$$\frac{50l^{\frac{9}{5}}}{9} + \frac{10l^{\frac{6}{5}}}{3} + \frac{8l^3}{3} + 8l$$

The value of the definite integral is 139.030898726601.

44.

$$-\frac{12n^3z^{\frac{5}{3}}}{5} - 2n^2z^2 + \frac{18nz^{\frac{7}{2}}}{7} + 2z$$

45.

$$\frac{r(24r^4 - 45r^3 - 40r^2 + 150r - 300)}{60}$$

46. The indefinite integral is

$$\frac{5u^{\frac{8}{5}}}{8} - \frac{5u^{\frac{6}{5}}}{2} + 8u$$

The value of the definite integral is 18.4234126672562.

47.

$$-\frac{30l^{\frac{7}{6}}\sqrt{z}}{7} - \frac{21l^{\frac{4}{3}}z^{\frac{2}{3}}}{4} + 6lz^{\frac{2}{3}}$$

48.

$$\frac{n(12n^4 + n^3 \cdot (30r^6 - 35) - 120r^3)}{20}$$

49.

$$\frac{c(4c^2z^2 - 9c - 30)}{3}$$

50.

$$\frac{s(20s^2 - 15s - 54)}{6}$$

51.

$$-\frac{4r^{\frac{3}{2}}s^{\frac{2}{3}}}{3} + \frac{2r^3}{3} - 2r^2 + rs^{\frac{3}{2}} + 4rs$$

52.

$$m(2m^3 - m^2\sqrt{v} + 2v^{\frac{2}{5}} + 2v)$$

53.

$$\frac{w(14b^6 - 5b^5w + 4b^2w^3 - 4b - 2w^5)}{2}$$

54.

$$\frac{3b(-b^3 + 6r(r+1))}{2}$$

55. The indefinite integral is

$$\frac{50x^{\frac{9}{5}}}{9} + \frac{20x^{\frac{7}{5}}}{7} + \frac{x^7}{7} + \frac{3x^2}{2}$$

The value of the definite integral is 2451.83490789485.

56.

$$-\frac{32b^{\frac{9}{4}}}{9} - \frac{3b^{\frac{4}{3}}}{4} + \frac{8b^5}{5}$$

57.

$$\frac{s(s^5w(-w^2 + 4w - 5) + 18)}{6}$$

58.

$$9c^{\frac{6}{5}}q - 4\sqrt[5]{c}q - \frac{5q^{\frac{11}{5}}}{11} - 5q^2$$

59.

$$y(2y^3 + 3y^2 - 2y + 1)$$

60.

$$b^4 \left(\frac{b(4z^2 - 7)}{5} - \frac{z^2}{4} \right)$$

61.

$$\frac{y(-12y^4 - 15y^3 + 20y^2 - 6y + 6)}{6}$$

62.

$$\frac{l(13l + 16u)}{2}$$

63.

$$\frac{4w^{\frac{5}{6}}y^{\frac{5}{2}}}{5} + w^{\frac{4}{5}}y + \frac{27w^{\frac{3}{4}}y^{\frac{4}{3}}}{4} - \frac{15w^{\frac{5}{3}}y^{\frac{4}{3}}}{2} - \frac{10y^{\frac{3}{2}}}{3}$$

64.

$$\frac{st(20s^5 - 4s^2 + 3t^5)}{2}$$

65.

$$\frac{c(48c^4 + 135c^3 - 40c^2 - 60c + 120)}{60}$$

66.

$$\frac{c(27c^3 + 4c^2 + 30c + 24)}{12}$$

67.

$$\frac{u(-24u^4 + 105u^3 - 80u^2 + 60u - 600)}{60}$$

68.

$$\frac{l(-35c^4l^3 - 56c^3l^4 + 80l^6 - 350)}{70}$$

69.

$$-v^2w + 3w^{\frac{4}{3}} + \frac{4w^{\frac{3}{2}}}{3}$$

70.

$$\frac{v(-7v^2 + 6v + 3)}{3}$$

71.

$$\frac{24w^{\frac{7}{4}}}{7} + \frac{5w^4}{2} - \frac{2w^3}{3} + 9wz^{\frac{3}{5}} + wz^{\frac{2}{5}}$$

72.

$$\frac{r^2(-45a^5r^5 - 63ar^3 + 70r^4 - 420)}{105}$$

73.

$$\frac{u(14a^5u^4 + 25a^5u^3 - 10a^2 - 40au - 50u)}{10}$$

74.

$$\frac{m^3(-4m^3t - 9m - 32t^5)}{12}$$

75.

$$\frac{50b^{\frac{11}{5}}y^{\frac{3}{2}}}{11} - 3by^{\frac{5}{4}} - 3by^3 + 3by$$

76.

$$\frac{w(-14w^2 + 3w + 36)}{6}$$

77. The indefinite integral is

$$\frac{40p^{\frac{11}{5}}}{11} - \frac{20p^{\frac{6}{5}}}{3} - \frac{7p^5}{5} + \frac{3p^2}{2} - p$$

The value of the definite integral is -309.914939556409.

78.

$$\frac{8c^{\frac{5}{4}}}{5} - 7cm^{\frac{3}{5}} - 9c\sqrt{m}$$

79. The indefinite integral is

$$-\frac{48d^{\frac{11}{6}}}{11} + \frac{9d^{\frac{7}{3}}}{7}$$

The value of the definite integral is -9.07063820028891.

80. The indefinite integral is

$$\frac{36q^{\frac{7}{6}}}{7} + \frac{40q^{\frac{9}{4}}}{9} - 2q^{\frac{5}{2}} + 6q^{\frac{3}{2}} - \frac{5q^2}{2}$$

The value of the definite integral is 11.0873015873016.

81. The indefinite integral is

$$\frac{42r^{\frac{11}{6}}}{11} + \frac{5r^{\frac{8}{3}}}{4} - \frac{21r^{\frac{5}{3}}}{5} - r^3 + 2r$$

The value of the definite integral is -11.3456015788855.

82.

$$\frac{b^3 \cdot (15b^4 \cdot (7 - 9x^2) + 105b^3x - 189b^2 - 140x^6)}{105}$$

83. The indefinite integral is

$$\frac{5v^{\frac{7}{5}}}{7} - \frac{16v^{\frac{5}{4}}}{5} + 4v^{\frac{5}{2}} - \frac{9v^2}{2}$$

The value of the definite integral is 42.8726410490308.

84.

$$\frac{12q^{\frac{7}{6}}}{7} - \frac{q^6\sqrt{y}}{6} + q^4y^2 + 3qy^{\frac{5}{4}}$$

85. The indefinite integral is

$$-\frac{50c^{\frac{11}{5}}}{11} - \frac{4c^3}{3} + \frac{5c^2}{2}$$

The value of the definite integral is -42.9096289540220.

86. The indefinite integral is

$$\frac{4l^{\frac{9}{4}}}{9} + \frac{3l^{\frac{4}{3}}}{4} - \frac{5l^7}{7} + 3l$$

The value of the definite integral is -1548.11365828801.

87.

$$-6b^{\frac{4}{3}}w - 3b^3w - \frac{6w^{\frac{5}{3}}}{5} + \frac{3w^4}{4}$$

88.

$$\frac{q(-20q^6t^3 - 14q^4 + 35qt^2(t+2) - 350)}{35}$$

89.

$$\frac{p(-9p^3 + 2p^2 - 24p - 6)}{6}$$

90.

$$\frac{a^4(-36a^3 - 14y^5 - 63y)}{28}$$

91.

$$\frac{b(-21b^4 - 30b^3 - 5b^2 - 30b + 60)}{15}$$

92.

$$\frac{p(n^5p^4 \cdot (2n+7) - 35n^5 - 35n^3 - 10)}{5}$$

93.

$$p\left(-5pq - 4q^{\frac{5}{6}} - 8q^{\frac{5}{3}} + 3\right)$$

94.

$$\frac{z^2(-30xz^5 + 24xz^3 + 70z - 75)}{30}$$

95.

$$\frac{c(c^4 \cdot (8r^6 - 10) + 10cr^2 + 25r^2)}{5}$$

96. The indefinite integral is

$$\frac{20a^{\frac{9}{5}}}{9} + \frac{8a^{\frac{7}{2}}}{7} - \frac{6a^5}{5} + 2a^2$$

The value of the definite integral is -1027.73329278585.

97.

$$\frac{au(5a^4u^3 + 8a^2 - 20u)}{4}$$

98.

$$\frac{b(-4b^6s^3 - 63b^3 - 280)}{28}$$

99.

$$\frac{z(z^2 - 6z - 27)}{3}$$

100.

$$\frac{r(3r^3 - 16r^2 - 12r + 12)}{6}$$