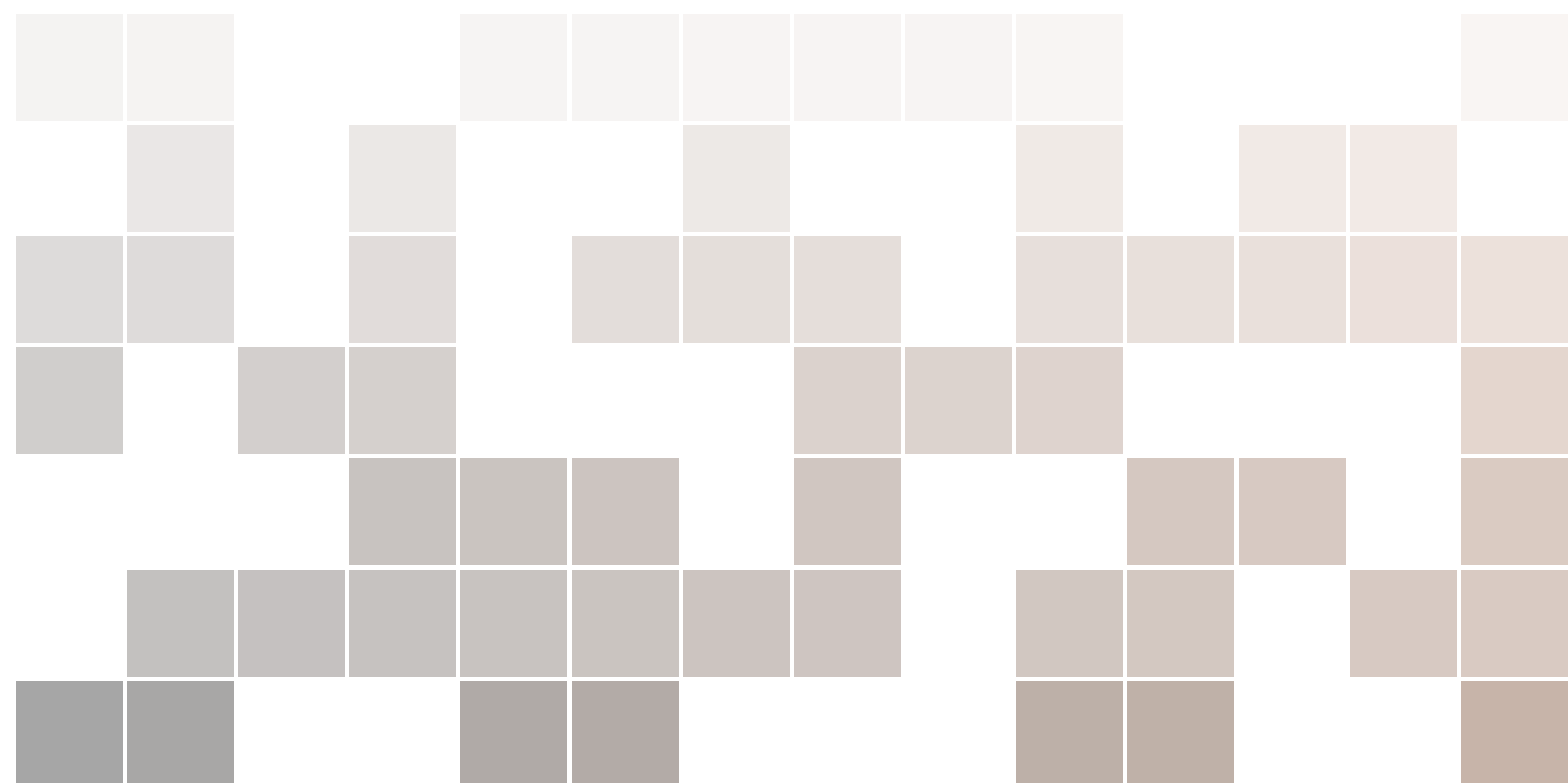


# VCE Mathematics Methods: Practice problems and solutions: Functions and Calculus



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# 1. Introduction

## 1.1 Motivation

My year 9 daughter kept asking me to write her new math problems, apart from those in the text book, so she can do more practice. I'd take a piece of paper and write problems, but it's a tedious process. So I thought: how about I **generate tons** of them?

## 1.2 Problem generation method

The problems and solutions have all been generated using the python programming language, and relied on a number of open source python packages, especially the SymPy package. Therefore, you can trust the accuracy of the solutions.

## 1.3 Structure

This document includes hundreds of mathematics problems that have been designed for year 11 students enrolled in VCE Mathematics Methods.

Most of the problems require the student to perform a single task, such as calculating the derivative of a function or finding the prime numbers within a range of integers. In addition, a number of questions have been designed based on the Mathematics Methods exam questions, which require the student to perform two or more tasks.

I hope you find them useful.



## 2. Functions

### 2.1 Problems

**Exercise 2.1** For the following two functions,

$$f(x) = -6ax^2 - 2ax - 8$$

and

$$g(x) = 3ax + 2$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.2** If  $f(x)$  is defined as

$$f(x) = x - 4$$

and  $g(x)$  is defined as

$$g(x) = 2x^2 - 5x + 7$$

simplify  $g(f(x))$ .

**Exercise 2.3** For the following two functions,

$$f(x) = a(-8x^2 + 4x + 9)$$

and

$$g(x) = 7ax^2 - 8ax - 1$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.4** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -8a - 10x$$

and

$$g(x) = 6a + 8x$$

**Exercise 2.5** For the following two functions,

$$f(x) = ax^2 - 9ax + 5$$

and

$$g(x) = -9ax^2 + 9ax + 9$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.6** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -4x - 3$$

and

$$g(x) = 3a - 7x$$

**Exercise 2.7** For the following two functions,

$$f(x) = ax^2 - 9ax + 9$$

and

$$g(x) = -3ax + 1$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.8** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -2ax^2 - 10a + 5x$$

and

$$g(x) = 6ax - 9$$

**Exercise 2.9** If  $f(x)$  is defined as

$$f(x) = -x - 1$$

and  $g(x)$  is defined as

$$g(x) = 5x^2 + 8x + 3$$

simplify  $g(f(x))$ .

**Exercise 2.10** If  $f(x)$  is defined as

$$f(x) = \frac{-3x - 8}{3x + 6}$$

find  $f^{-1}(x)$ .

**Exercise 2.11** For the following two functions,

$$f(x) = -10ax^2 - 3ax + 7$$

and

$$g(x) = a(4x^2 + 7x - 10)$$

what values of  $a$  will ensure that the graphs of these functions never intersect?



**Exercise 2.12** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 2x + 1$$

and

$$g(x) = -9a - 10x$$

**Exercise 2.13** For the following two functions,

$$f(x) = a(5x^2 + 2x - 3)$$

and

$$g(x) = ax^2 - 4ax - 9$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.14** If  $f(x)$  is defined as

$$f(x) = -6x - 8$$

and  $g(x)$  is defined as

$$g(x) = -6x^2 + 5x + 4$$

simplify  $g(f(x))$ .

**Exercise 2.15** If  $f(x)$  is defined as

$$f(x) = \frac{-7ax - 7a}{3a - 10x}$$

find the value of  $a$  if  $f^{-1}(1) = 10$ .

**Exercise 2.16** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 3ax^2 - 3a - x$$

and

$$g(x) = -9ax - 1$$

**Exercise 2.17** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -x - 6$$

and

$$g(x) = 3a - 10x$$

**Exercise 2.18** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 2 - 2x$$

and

$$g(x) = 6a - 7x$$

**Exercise 2.19** If  $f(x)$  is defined as

$$f(x) = \frac{2ax + 5a}{-4a + 10x}$$

find the value of  $a$  if  $f^{-1}(3) = 10$ .

**Exercise 2.20** For the following two functions,

$$f(x) = -3ax^2 - 2ax - 6$$

and

$$g(x) = 5ax^2 + 9ax + 9$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.21** If  $f(x)$  is defined as

$$f(x) = \frac{-3ax - 2a}{6a + 8x}$$

find the value of  $a$  if  $f^{-1}(10) = 1$ .

**Exercise 2.22** For the following two functions,

$$f(x) = 7ax^2 - 6ax - 8$$

and

$$g(x) = 10ax^2 + 6ax + 6$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.23** For the following two functions,

$$f(x) = -4ax^2 + 5ax - 6$$

and

$$g(x) = -2ax - 8$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.24** If  $f(x)$  is defined as

$$f(x) = 2ax + 5x^2$$

and  $g(x)$  is defined as

$$g(x) = -9x - 2$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.25** If  $f(x)$  is defined as

$$f(x) = ax - 7x^2$$

and  $g(x)$  is defined as

$$g(x) = -8x - 6$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.26** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 10ax^2 - 9a + x$$

and

$$g(x) = -2ax - 6$$

**Exercise 2.27** If  $f(x)$  is defined as

$$f(x) = \frac{-6x - 6}{9x + 5}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.28** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -9ax^2 - 9a - 9x$$

and

$$g(x) = -8ax - 4$$

**Exercise 2.29** If  $f(x)$  is defined as

$$f(x) = \frac{-9ax - a}{-2a - 7x}$$

find the value of  $a$  if  $f^{-1}(3) = 3$ .

**Exercise 2.30** If  $f(x)$  is defined as

$$f(x) = \frac{-7ax + a}{-9a - 7x}$$

find the value of  $a$  if  $f^{-1}(7) = 7$ .

**Exercise 2.31** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 3ax^2 + 6a + 4x$$

and

$$g(x) = a(9 - x)$$

**Exercise 2.32** If  $f(x)$  is defined as

$$f(x) = \frac{-x - 7}{9x - 6}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.33** If  $f(x)$  is defined as

$$f(x) = \frac{-3ax + 2a}{-9a - 3x}$$

find the value of  $a$  if  $f^{-1}(10) = 6$ .

**Exercise 2.34** If  $f(x)$  is defined as

$$f(x) = \frac{2-4x}{7x+3}$$

find  $f^{-1}(x)$ .

**Exercise 2.35** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -8a - 3x$$

and

$$g(x) = -6a + 6x$$

**Exercise 2.36** If  $f(x)$  is defined as

$$f(x) = \frac{-3x-5}{9-10x}$$

find  $f^{-1}(x)$ .

**Exercise 2.37** If  $f(x)$  is defined as

$$f(x) = -4ax - 5x^2$$

and  $g(x)$  is defined as

$$g(x) = x - 5$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.38** If  $f(x)$  is defined as

$$f(x) = \frac{-3x-1}{3-4x}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.39** If  $f(x)$  is defined as

$$f(x) = \frac{7x-8}{-10x-10}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.40** If  $f(x)$  is defined as

$$f(x) = \frac{8x+1}{10-9x}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.41** If  $f(x)$  is defined as

$$f(x) = \frac{-2ax+6a}{6a+6x}$$

find the value of  $a$  if  $f^{-1}(4) = 8$ .

**Exercise 2.42** For the following two functions,

$$f(x) = a(-6x^2 - 2x + 5)$$

and

$$g(x) = 2ax^2 - 6ax - 2$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.43** If  $f(x)$  is defined as

$$f(x) = \frac{10ax + 8a}{-10a - 6x}$$

find the value of  $a$  if  $f^{-1}(3) = 10$ .

**Exercise 2.44** If  $f(x)$  is defined as

$$f(x) = \frac{x + 8}{-8x - 8}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.45** For the following two functions,

$$f(x) = 10ax^2 + 7ax - 9$$

and

$$g(x) = a(3x^2 + 9x + 5)$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.46** If  $f(x)$  is defined as

$$f(x) = 2x + 7$$

and  $g(x)$  is defined as

$$g(x) = -5x^2 - 7x + 3$$

simplify  $g(f(x))$ .

**Exercise 2.47** If  $f(x)$  is defined as

$$f(x) = \frac{1 - 10x}{1 - 4x}$$

find  $f^{-1}(x)$ .

**Exercise 2.48** If  $f(x)$  is defined as

$$f(x) = 9 - x$$

and  $g(x)$  is defined as

$$g(x) = 4x^2 - 7x - 4$$

simplify  $g(f(x))$ .

**Exercise 2.49** For the following two functions,

$$f(x) = a(-5x^2 - 10x - 7)$$

and

$$g(x) = 8ax + 9$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.50** If  $f(x)$  is defined as

$$f(x) = \frac{-10x - 8}{3x - 8}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.51** If  $f(x)$  is defined as

$$f(x) = \frac{-ax - 9a}{7a - 5x}$$

find the value of  $a$  if  $f^{-1}(6) = 5$ .

**Exercise 2.52** If  $f(x)$  is defined as

$$f(x) = -x - 9$$

and  $g(x)$  is defined as

$$g(x) = 6x^2 - 3x + 2$$

simplify  $g(f(x))$ .

**Exercise 2.53** If  $f(x)$  is defined as

$$f(x) = -6ax - x^2$$

and  $g(x)$  is defined as

$$g(x) = 9 - 9x$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.54** If  $f(x)$  is defined as

$$f(x) = \frac{6x + 4}{-10x - 7}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.55** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 5a - 8x$$

and

$$g(x) = -4a - 10x$$

**Exercise 2.56** For the following two functions,

$$f(x) = -9ax^2 - ax - 5$$

and

$$g(x) = a(-6x - 7)$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.57** For the following two functions,

$$f(x) = 4ax^2 + 9ax - 5$$

and

$$g(x) = 3ax^2 + 6ax + 5$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.58** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 2x + 7$$

and

$$g(x) = -8a - 3x$$

**Exercise 2.59** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 6 - 6x$$

and

$$g(x) = 4a + 7x$$

**Exercise 2.60** If  $f(x)$  is defined as

$$f(x) = \frac{x+3}{3-3x}$$

find  $f^{-1}(x)$ .

**Exercise 2.61** If  $f(x)$  is defined as

$$f(x) = \frac{10-3x}{5x+4}$$

find  $f^{-1}(x)$ .

**Exercise 2.62** If  $f(x)$  is defined as

$$f(x) = \frac{3-10x}{7x+2}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.63** If  $f(x)$  is defined as

$$f(x) = \frac{6ax-8a}{-3a-6x}$$

find the value of  $a$  if  $f^{-1}(9) = 7$ .

**Exercise 2.64** If  $f(x)$  is defined as

$$f(x) = -ax + 6x^2$$

and  $g(x)$  is defined as

$$g(x) = x + 9$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.65** If  $f(x)$  is defined as

$$f(x) = 4x - 1$$

and  $g(x)$  is defined as

$$g(x) = -2x^2 + 3x - 2$$

simplify  $g(f(x))$ .

**Exercise 2.66** For the following two functions,

$$f(x) = 7ax^2 - 6ax + 10$$

and

$$g(x) = a(5x^2 + 6x + 10)$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.67** For the following two functions,

$$f(x) = 2ax^2 + 5ax + 2$$

and

$$g(x) = -4ax^2 + ax - 4$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.68** If  $f(x)$  is defined as

$$f(x) = \frac{7x - 5}{-2x - 3}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.69** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -5ax^2 - 10a - 10x$$

and

$$g(x) = 2ax - 7$$



**Exercise 2.70** If  $f(x)$  is defined as

$$f(x) = -5x - 5$$

and  $g(x)$  is defined as

$$g(x) = 10x^2 - 6x + 1$$

simplify  $g(f(x))$ .

**Exercise 2.71** If  $f(x)$  is defined as

$$f(x) = \frac{9 - 7x}{3 - 2x}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.72** If  $f(x)$  is defined as

$$f(x) = \frac{6ax - a}{-8a - 6x}$$

find the value of  $a$  if  $f^{-1}(3) = 8$ .

**Exercise 2.73** If  $f(x)$  is defined as

$$f(x) = \frac{3x - 9}{4x - 10}$$

find  $f^{-1}(x)$ .

**Exercise 2.74** For the following two functions,

$$f(x) = -3ax^2 + 6ax - 4$$

and

$$g(x) = 6ax^2 - 5ax + 6$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.75** If  $f(x)$  is defined as

$$f(x) = \frac{-8x - 6}{6 - 9x}$$

find  $f^{-1}(x)$ .

**Exercise 2.76** If  $f(x)$  is defined as

$$f(x) = \frac{9x - 9}{8x + 8}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.77** If  $f(x)$  is defined as

$$f(x) = 2x - 3$$

and  $g(x)$  is defined as

$$g(x) = 8x^2 - 3x + 4$$

simplify  $g(f(x))$ .

**Exercise 2.78** If  $f(x)$  is defined as

$$f(x) = \frac{7x-3}{8x+3}$$

find  $f^{-1}(x)$ .

**Exercise 2.79** For the following two functions,

$$f(x) = 10ax^2 - 4ax + 5$$

and

$$g(x) = -10ax^2 + 4ax - 8$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.80** For the following two functions,

$$f(x) = ax^2 + 9ax + 5$$

and

$$g(x) = -9ax^2 - 6ax + 2$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.81** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -9ax^2 - 4a + 5x$$

and

$$g(x) = a(8x + 9)$$

**Exercise 2.82** For the following two functions,

$$f(x) = a(-x^2 - 7x - 8)$$

and

$$g(x) = ax + 3$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.83** If  $f(x)$  is defined as

$$f(x) = x + 6$$

and  $g(x)$  is defined as

$$g(x) = -4x^2 + 6x + 7$$

simplify  $g(f(x))$ .

**Exercise 2.84** For the following two functions,

$$f(x) = -8ax^2 - 6ax + 2$$

and

$$g(x) = 3a(3x + 2)$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.85** If  $f(x)$  is defined as

$$f(x) = \frac{-4ax + 6a}{-8a - x}$$

find the value of  $a$  if  $f^{-1}(2) = 4$ .

**Exercise 2.86** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 3ax^2 - 10a + x$$

and

$$g(x) = 10ax - 7$$

**Exercise 2.87** If  $f(x)$  is defined as

$$f(x) = \frac{3ax - 5a}{-3a - 7x}$$

find the value of  $a$  if  $f^{-1}(6) = 3$ .

**Exercise 2.88** If  $f(x)$  is defined as

$$f(x) = \frac{10x + 6}{4 - 4x}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.89** If  $f(x)$  is defined as

$$f(x) = \frac{8ax + 9a}{-7a - 3x}$$

find the value of  $a$  if  $f^{-1}(6) = 4$ .

**Exercise 2.90** If  $f(x)$  is defined as

$$f(x) = \frac{8x + 1}{10x + 5}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.91** If  $f(x)$  is defined as

$$f(x) = \frac{-6x - 9}{-9x - 9}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.92** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 8x + 4$$

and

$$g(x) = 6a + 8x$$

**Exercise 2.93** If  $f(x)$  is defined as

$$f(x) = \frac{-ax - 10a}{7a - 5x}$$

find the value of  $a$  if  $f^{-1}(1) = 2$ .

**Exercise 2.94** If  $f(x)$  is defined as

$$f(x) = \frac{6-x}{4x+4}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.95** For the following two functions,

$$f(x) = -9ax^2 - 7ax + 2$$

and

$$g(x) = 8ax - 8$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.96** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -8a + 5x$$

and

$$g(x) = -5a + 5x$$

**Exercise 2.97** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 2x + 2$$

and

$$g(x) = 6a - 3x$$

**Exercise 2.98** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = x + 2$$

and

$$g(x) = 7a + 4x$$

**Exercise 2.99** If  $f(x)$  is defined as

$$f(x) = \frac{6ax + 5a}{-7a + 7x}$$

find the value of  $a$  if  $f^{-1}(1) = 7$ .

**Exercise 2.100** For the following two functions,

$$f(x) = -7ax^2 + 8ax - 2$$

and

$$g(x) = 5ax^2 - 4ax + 1$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.101** If  $f(x)$  is defined as

$$f(x) = 6 - 2x$$

and  $g(x)$  is defined as

$$g(x) = -x^2 - 4x + 3$$

simplify  $g(f(x))$ .

**Exercise 2.102** If  $f(x)$  is defined as

$$f(x) = 8x + 7$$

and  $g(x)$  is defined as

$$g(x) = -4x^2 - 2x + 8$$

simplify  $g(f(x))$ .

**Exercise 2.103** If  $f(x)$  is defined as

$$f(x) = -5ax - 10x^2$$

and  $g(x)$  is defined as

$$g(x) = -5x - 4$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.104** If  $f(x)$  is defined as

$$f(x) = 4ax + 3x^2$$

and  $g(x)$  is defined as

$$g(x) = -10x - 2$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.105** If  $f(x)$  is defined as

$$f(x) = -6x - 9$$

and  $g(x)$  is defined as

$$g(x) = 3x^2 - 9x + 4$$

simplify  $g(f(x))$ .

**Exercise 2.106** For the following two functions,

$$f(x) = 4ax^2 + 3ax + 1$$

and

$$g(x) = 2ax - 4$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.107** If  $f(x)$  is defined as

$$f(x) = 5x - 10$$

and  $g(x)$  is defined as

$$g(x) = 2x^2 - 9x - 7$$

simplify  $g(f(x))$ .

**Exercise 2.108** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -6x - 3$$

and

$$g(x) = 4a + 3x$$

**Exercise 2.109** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 2a + 2x$$

and

$$g(x) = 5a - 5x$$

**Exercise 2.110** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -5ax^2 - 6a - 4x$$

and

$$g(x) = -8ax - 7$$

**Exercise 2.111** If  $f(x)$  is defined as

$$f(x) = \frac{2x+7}{9-10x}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.112** If  $f(x)$  is defined as

$$f(x) = 4 - 9x$$

and  $g(x)$  is defined as

$$g(x) = 2x^2 + 3x + 8$$

simplify  $g(f(x))$ .

**Exercise 2.113** If  $f(x)$  is defined as

$$f(x) = \frac{4-10x}{3x+4}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.114** For the following two functions,

$$f(x) = a(8x^2 + 5x - 4)$$

and

$$g(x) = 7ax^2 - 9ax + 10$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.115** If  $f(x)$  is defined as

$$f(x) = \frac{9ax + 2a}{-2a + 9x}$$

find the value of  $a$  if  $f^{-1}(1) = 2$ .

**Exercise 2.116** If  $f(x)$  is defined as

$$f(x) = 2x + 10$$

and  $g(x)$  is defined as

$$g(x) = 6x^2 + 9x - 4$$

simplify  $g(f(x))$ .

**Exercise 2.117** If  $f(x)$  is defined as

$$f(x) = \frac{5x + 8}{-7x - 3}$$

find  $f^{-1}(x)$ .

**Exercise 2.118** For the following two functions,

$$f(x) = a(-2x^2 + 5x + 4)$$

and

$$g(x) = ax^2 + 9ax + 8$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.119** If  $f(x)$  is defined as

$$f(x) = 10 - 6x$$

and  $g(x)$  is defined as

$$g(x) = 9x^2 - 10x - 10$$

simplify  $g(f(x))$ .

**Exercise 2.120** If  $f(x)$  is defined as

$$f(x) = ax - 10x^2$$

and  $g(x)$  is defined as

$$g(x) = -8x - 1$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.121** If  $f(x)$  is defined as

$$f(x) = \frac{9x+5}{-x-5}$$

find  $f^{-1}(x)$ .

**Exercise 2.122** If  $f(x)$  is defined as

$$f(x) = \frac{7-5x}{4-x}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.123** If  $f(x)$  is defined as

$$f(x) = -5ax - 7x^2$$

and  $g(x)$  is defined as

$$g(x) = 9x - 1$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.124** If  $f(x)$  is defined as

$$f(x) = \frac{-ax+2a}{-4a+7x}$$

find the value of  $a$  if  $f^{-1}(1) = 9$ .

**Exercise 2.125** If  $f(x)$  is defined as

$$f(x) = 9 - 8x$$

and  $g(x)$  is defined as

$$g(x) = 4x^2 - 10x - 3$$

simplify  $g(f(x))$ .

**Exercise 2.126** If  $f(x)$  is defined as

$$f(x) = -4ax + 10x^2$$

and  $g(x)$  is defined as

$$g(x) = -6x - 8$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.127** If  $f(x)$  is defined as

$$f(x) = \frac{9-6x}{x-8}$$

find  $f^{-1}(x)$ .



**Exercise 2.128** If  $f(x)$  is defined as

$$f(x) = \frac{4x + 4}{-10x - 8}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.129** If  $f(x)$  is defined as

$$f(x) = 3x + 6$$

and  $g(x)$  is defined as

$$g(x) = -2x^2 + 4x + 7$$

simplify  $g(f(x))$ .

**Exercise 2.130** If  $f(x)$  is defined as

$$f(x) = 10ax + 8x^2$$

and  $g(x)$  is defined as

$$g(x) = -9x - 4$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.131** If  $f(x)$  is defined as

$$f(x) = -3x - 10$$

and  $g(x)$  is defined as

$$g(x) = 6x^2 - 9x - 9$$

simplify  $g(f(x))$ .

**Exercise 2.132** If  $f(x)$  is defined as

$$f(x) = -x - 10$$

and  $g(x)$  is defined as

$$g(x) = 4x^2 + 10x + 2$$

simplify  $g(f(x))$ .

**Exercise 2.133** If  $f(x)$  is defined as

$$f(x) = \frac{x - 5}{8 - 7x}$$

find  $f^{-1}(x)$ .

**Exercise 2.134** If  $f(x)$  is defined as

$$f(x) = 2ax - 2x^2$$

and  $g(x)$  is defined as

$$g(x) = 4x + 9$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.135** For the following two functions,

$$f(x) = 4ax^2 - 10ax - 3$$

and

$$g(x) = a(-4x^2 - x + 2)$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.136** If  $f(x)$  is defined as

$$f(x) = 8ax + x^2$$

and  $g(x)$  is defined as

$$g(x) = 9x - 4$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.137** If  $f(x)$  is defined as

$$f(x) = \frac{6ax + 6a}{5a - 2x}$$

find the value of  $a$  if  $f^{-1}(9) = 2$ .

**Exercise 2.138** If  $f(x)$  is defined as

$$f(x) = \frac{10 - 8x}{9x - 2}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.139** If  $f(x)$  is defined as

$$f(x) = \frac{7x + 4}{3 - 6x}$$

find  $f^{-1}(x)$ .

**Exercise 2.140** If  $f(x)$  is defined as

$$f(x) = -3ax + 2x^2$$

and  $g(x)$  is defined as

$$g(x) = -10x - 9$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.141** For the following two functions,

$$f(x) = -6ax^2 - 3ax + 3$$

and

$$g(x) = a(5x^2 + 3x - 1)$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.142** If  $f(x)$  is defined as

$$f(x) = -7x - 2$$

and  $g(x)$  is defined as

$$g(x) = -2x^2 - 3x + 5$$

simplify  $g(f(x))$ .

**Exercise 2.143** If  $f(x)$  is defined as

$$f(x) = \frac{5 - 4x}{-8x - 4}$$

find  $f^{-1}(x)$ .

**Exercise 2.144** If  $f(x)$  is defined as

$$f(x) = 9ax + 4x^2$$

and  $g(x)$  is defined as

$$g(x) = -8x - 10$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.145** If  $f(x)$  is defined as

$$f(x) = \frac{2x + 5}{2x + 8}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.146** For the following two functions,

$$f(x) = -6ax^2 + 9ax - 9$$

and

$$g(x) = a(-5x - 2)$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.147** If  $f(x)$  is defined as

$$f(x) = \frac{9ax + 4a}{-6a + 9x}$$

find the value of  $a$  if  $f^{-1}(5) = 9$ .

**Exercise 2.148** If  $f(x)$  is defined as

$$f(x) = \frac{5ax - a}{-9a + 6x}$$

find the value of  $a$  if  $f^{-1}(6) = 6$ .

**Exercise 2.149** If  $f(x)$  is defined as

$$f(x) = \frac{-9ax - 5a}{7a - 2x}$$

find the value of  $a$  if  $f^{-1}(1) = 1$ .

**Exercise 2.150** If  $f(x)$  is defined as

$$f(x) = 5 - 3x$$

and  $g(x)$  is defined as

$$g(x) = -7x^2 + 5x - 8$$

simplify  $g(f(x))$ .

**Exercise 2.151** For the following two functions,

$$f(x) = 9ax^2 + 8ax + 9$$

and

$$g(x) = -ax - 5$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.152** If  $f(x)$  is defined as

$$f(x) = 2x + 3$$

and  $g(x)$  is defined as

$$g(x) = -9x^2 - 10x + 6$$

simplify  $g(f(x))$ .

**Exercise 2.153** For the following two functions,

$$f(x) = a(-x^2 - 10x - 8)$$

and

$$g(x) = -2ax - 7$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.154** For the following two functions,

$$f(x) = a(-2x^2 - 9x + 6)$$

and

$$g(x) = ax + 7$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.155** If  $f(x)$  is defined as

$$f(x) = 7x + 9$$

and  $g(x)$  is defined as

$$g(x) = 6x^2 - 2x - 9$$

simplify  $g(f(x))$ .

**Exercise 2.156** If  $f(x)$  is defined as

$$f(x) = 3ax - 7x^2$$

and  $g(x)$  is defined as

$$g(x) = -5x - 4$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.157** For the following two functions,

$$f(x) = -6ax^2 - 3ax + 8$$

and

$$g(x) = 2a(5x^2 + 4x - 2)$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.158** If  $f(x)$  is defined as

$$f(x) = \frac{-10ax - 6a}{-7a - 6x}$$

find the value of  $a$  if  $f^{-1}(2) = 6$ .

**Exercise 2.159** If  $f(x)$  is defined as

$$f(x) = \frac{-8ax - 10a}{a + 4x}$$

find the value of  $a$  if  $f^{-1}(2) = 2$ .

**Exercise 2.160** If  $f(x)$  is defined as

$$f(x) = \frac{-7ax - 9a}{a - x}$$

find the value of  $a$  if  $f^{-1}(6) = 7$ .

**Exercise 2.161** For the following two functions,

$$f(x) = a(2x^2 + 10x - 9)$$

and

$$g(x) = -ax + 4$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.162** If  $f(x)$  is defined as

$$f(x) = -7x - 6$$

and  $g(x)$  is defined as

$$g(x) = -7x^2 - x + 3$$

simplify  $g(f(x))$ .

**Exercise 2.163** For the following two functions,

$$f(x) = 6ax^2 - 2ax - 1$$

and

$$g(x) = -ax - 2$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.164** If  $f(x)$  is defined as

$$f(x) = \frac{4x+8}{9x+9}$$

find  $f^{-1}(x)$ .

**Exercise 2.165** For the following two functions,

$$f(x) = -4ax^2 + 10ax + 7$$

and

$$g(x) = a(7-x)$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.166** If  $f(x)$  is defined as

$$f(x) = 1 - 3x$$

and  $g(x)$  is defined as

$$g(x) = 9x^2 - 5x + 7$$

simplify  $g(f(x))$ .

**Exercise 2.167** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -8ax^2 + 7a + 5x$$

and

$$g(x) = -6ax + 9$$

**Exercise 2.168** If  $f(x)$  is defined as

$$f(x) = \frac{4-5x}{x-3}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.169** If  $f(x)$  is defined as

$$f(x) = \frac{8ax+2a}{3a-2x}$$

find the value of  $a$  if  $f^{-1}(7) = 7$ .

**Exercise 2.170** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = ax^2 - a + 8x$$

and

$$g(x) = -3ax - 10$$

**Exercise 2.171** For the following two functions,

$$f(x) = a(4x^2 - 10x - 5)$$

and

$$g(x) = 2ax^2 - 7ax + 8$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.172** For the following two functions,

$$f(x) = a(-7x^2 - 6x + 10)$$

and

$$g(x) = -2ax + 1$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.173** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = 5x + 5$$

and

$$g(x) = 5a + 10x$$

**Exercise 2.174** If  $f(x)$  is defined as

$$f(x) = -ax + 3x^2$$

and  $g(x)$  is defined as

$$g(x) = 7x + 2$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.175** If  $f(x)$  is defined as

$$f(x) = \frac{7x + 5}{2 - 3x}$$

find  $f^{-1}(x)$ .

**Exercise 2.176** If  $f(x)$  is defined as

$$f(x) = \frac{9 - 9x}{9x - 3}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.177** For the following two functions,

$$f(x) = -ax^2 - 2ax + 2$$

and

$$g(x) = 7ax^2 + 8ax + 9$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.178** If  $f(x)$  is defined as

$$f(x) = 3x + 1$$

and  $g(x)$  is defined as

$$g(x) = 6x^2 + x + 9$$

simplify  $g(f(x))$ .

**Exercise 2.179** If  $f(x)$  is defined as

$$f(x) = 3ax - 6x^2$$

and  $g(x)$  is defined as

$$g(x) = 7x + 5$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.180** If  $f(x)$  is defined as

$$f(x) = -7ax + 9x^2$$

and  $g(x)$  is defined as

$$g(x) = 6 - 2x$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.181** For the following two functions,

$$f(x) = a(-10x^2 - 2x + 7)$$

and

$$g(x) = 5ax - 4$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.182** If  $f(x)$  is defined as

$$f(x) = \frac{5x + 9}{4 - 5x}$$

what is the domain and range of  $f(x)$ ?



**Exercise 2.183** For the following two functions,

$$f(x) = 6ax^2 - 3ax + 1$$

and

$$g(x) = 3a(x + 3)$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.184** If  $f(x)$  is defined as

$$f(x) = \frac{8 - 6x}{3x - 3}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.185** For the following two functions,

$$f(x) = a(-9x^2 + 5x + 2)$$

and

$$g(x) = -5ax - 5$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.186** If  $f(x)$  is defined as

$$f(x) = 6x - 7$$

and  $g(x)$  is defined as

$$g(x) = -10x^2 + 9x - 10$$

simplify  $g(f(x))$ .

**Exercise 2.187** If  $f(x)$  is defined as

$$f(x) = \frac{ax + 8a}{-9a + x}$$

find the value of  $a$  if  $f^{-1}(6) = 10$ .

**Exercise 2.188** For the following two functions,

$$f(x) = -8ax^2 - 3ax + 2$$

and

$$g(x) = -2ax^2 - 8ax - 9$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.189** If  $f(x)$  is defined as

$$f(x) = \frac{-ax - a}{-2a + 9x}$$

find the value of  $a$  if  $f^{-1}(5) = 1$ .

**Exercise 2.190** If  $f(x)$  is defined as

$$f(x) = 2ax - 9x^2$$

and  $g(x)$  is defined as

$$g(x) = 3x + 1$$

solve the equation

$$g(f(x)) = 0$$

**Exercise 2.191** If  $f(x)$  is defined as

$$f(x) = \frac{9ax + 3a}{-5a + x}$$

find the value of  $a$  if  $f^{-1}(2) = 6$ .

**Exercise 2.192** For the following two functions,

$$f(x) = 5ax^2 - 6ax - 7$$

and

$$g(x) = ax^2 + 5ax - 4$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.193** For the following two functions,

$$f(x) = 5ax^2 + 5ax - 1$$

and

$$g(x) = 10ax^2 + 3ax - 9$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.194** If  $f(x)$  is defined as

$$f(x) = \frac{-2x - 5}{7 - 3x}$$

find  $f^{-1}(x)$ .

**Exercise 2.195** For the following two functions,

$$f(x) = a(-7x^2 - x + 5)$$

and

$$g(x) = -6ax - 9$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.196** For the following two functions,

$$f(x) = 2a(-5x^2 - x + 1)$$

and

$$g(x) = 4ax + 10$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

**Exercise 2.197** If  $f(x)$  is defined as

$$f(x) = \frac{10x+6}{5x+5}$$

find  $f^{-1}(x)$ .

**Exercise 2.198** Find the value of  $x$  at which the graphs of the following functions intersect:

$$f(x) = -8ax^2 + 9a - 4x$$

and

$$g(x) = -3ax - 8$$

**Exercise 2.199** If  $f(x)$  is defined as

$$f(x) = \frac{10x+7}{7x+7}$$

what is the domain and range of  $f(x)$ ?

**Exercise 2.200** For the following two functions,

$$f(x) = -3ax^2 + 2ax - 7$$

and

$$g(x) = a(-x^2 + 7x - 5)$$

what values of  $a$  will ensure that the graphs of these functions never intersect?

## 2.2 Solutions

1.

$$0 < a, a < \frac{48}{5}$$

2.

$$2x^2 - 21x + 59$$

3.

$$-\frac{5}{57} < a, a < 0$$

4.

$$\left\{ -\frac{7a}{9} \right\}$$

5.

$$-\frac{40}{81} < a, a < 0$$

6.

$$\{a+1\}$$

7.

$$0 < a, a < \frac{8}{9}$$

8.

$$\left\{ -\frac{6a-5}{4a} - \frac{\sqrt{-44a^2+12a+25}}{4a}, -\frac{6a-5}{4a} + \frac{\sqrt{-44a^2+12a+25}}{4a} \right\}$$

9.

$$5x^2 + 2x$$

10.

$$\left\{ -\frac{2 \cdot (3y+4)}{3(y+1)} \right\}$$

11.

$$-\frac{98}{165} < a, a < 0$$

12.

$$\left\{ -\frac{3a}{4} - \frac{1}{12} \right\}$$

13.

$$0 < a, a < \frac{12}{7}$$

14.

$$-216x^2 - 606x - 420$$

15.

$$1.25$$

16.

$$\left\{ -\frac{9a-1}{6a} - \frac{\sqrt{117a^2-30a+1}}{6a}, -\frac{9a-1}{6a} + \frac{\sqrt{117a^2-30a+1}}{6a} \right\}$$

17.

$$\left\{ \frac{a}{3} + \frac{2}{3} \right\}$$

18.

$$\left\{ \frac{6a}{5} - \frac{2}{5} \right\}$$

19.

$$8.108$$

20.

$$0 < a, a < \frac{480}{121}$$

21.

$$-1.231$$

22.

$$0 < a, a < \frac{7}{6}$$

23.

$$-\frac{32}{49} < a, a < 0$$

24.

$$\left\{ -\frac{a}{5} - \frac{\sqrt{9a^2 - 10}}{15}, -\frac{a}{5} + \frac{\sqrt{9a^2 - 10}}{15} \right\}$$

25.

$$\left\{ \frac{a}{14} - \frac{\sqrt{a^2 + 21}}{14}, \frac{a}{14} + \frac{\sqrt{a^2 + 21}}{14} \right\}$$

26.

$$\left\{ -\frac{2a+1}{20a} - \frac{\sqrt{364a^2 - 236a + 1}}{20a}, -\frac{2a+1}{20a} + \frac{\sqrt{364a^2 - 236a + 1}}{20a} \right\}$$

27. Domain:

$$\left( -\infty, -\frac{5}{9} \right) \cup \left( -\frac{5}{9}, \infty \right)$$

Range:

$$\left( -\infty, -\frac{2}{3} \right) \cup \left( -\frac{2}{3}, \infty \right)$$

28.

$$\left\{ -\frac{\sqrt{81 - 260a^2}}{18a} + \frac{8a - 9}{18a}, \frac{\sqrt{81 - 260a^2}}{18a} + \frac{8a - 9}{18a} \right\}$$

29.

$$2.864$$

30.

$$-22.867$$

31.

$$\left\{ -\frac{a+4}{6a} - \frac{\sqrt{37a^2+8a+16}}{6a}, -\frac{a+4}{6a} + \frac{\sqrt{37a^2+8a+16}}{6a} \right\}$$

32. Domain:

$$\left( -\infty, \frac{2}{3} \right) \cup \left( \frac{2}{3}, \infty \right)$$

Range:

$$\left( -\infty, -\frac{1}{9} \right) \cup \left( -\frac{1}{9}, \infty \right)$$

33.

$$-2.432$$

34.

$$\left\{ -\frac{3y-2}{7y+4} \right\}$$

35.

$$\left\{ -\frac{2a}{9} \right\}$$

36.

$$\left\{ \frac{9y+5}{10y-3} \right\}$$

37.

$$\left\{ -\frac{2a}{5} - \frac{\sqrt{(2a-5)(2a+5)}}{5}, -\frac{2a}{5} + \frac{\sqrt{(2a-5)(2a+5)}}{5} \right\}$$

38. Domain:

$$\left( -\infty, \frac{3}{4} \right) \cup \left( \frac{3}{4}, \infty \right)$$

Range:

$$\left( -\infty, \frac{3}{4} \right) \cup \left( \frac{3}{4}, \infty \right)$$

39. Domain:

$$(-\infty, -1) \cup (-1, \infty)$$

Range:

$$\left( -\infty, -\frac{7}{10} \right) \cup \left( -\frac{7}{10}, \infty \right)$$

40. Domain:

$$\left(-\infty, \frac{10}{9}\right) \cup \left(\frac{10}{9}, \infty\right)$$

Range:

$$\left(-\infty, -\frac{8}{9}\right) \cup \left(-\frac{8}{9}, \infty\right)$$

41.

$$-5.647$$

42.

$$-\frac{4}{11} < a, a < 0$$

43.

$$-1.304$$

44. Domain:

$$(-\infty, -1) \cup (-1, \infty)$$

Range:

$$\left(-\infty, -\frac{1}{8}\right) \cup \left(-\frac{1}{8}, \infty\right)$$

45.

$$-\frac{7}{4} < a, a < 0$$

46.

$$-20x^2 - 154x - 291$$

47.

$$\left\{ \frac{y-1}{2 \cdot (2y-5)} \right\}$$

48.

$$4x^2 - 65x + 257$$

49.

$$0 < a, a < \frac{45}{46}$$

50. Domain:

$$\left(-\infty, \frac{8}{3}\right) \cup \left(\frac{8}{3}, \infty\right)$$

Range:

$$\left(-\infty, -\frac{10}{3}\right) \cup \left(-\frac{10}{3}, \infty\right)$$

51.

$$2.679$$

52.

$$6x^2 + 111x + 515$$

53.

$$\left\{ -3a - \sqrt{(3a-1)(3a+1)}, -3a + \sqrt{(3a-1)(3a+1)} \right\}$$

54. Domain:

$$\left( -\infty, -\frac{7}{10} \right) \cup \left( -\frac{7}{10}, \infty \right)$$

Range:

$$\left( -\infty, -\frac{3}{5} \right) \cup \left( -\frac{3}{5}, \infty \right)$$

55.

$$\left\{ -\frac{9a}{2} \right\}$$

56.

$$0 < a, a < \frac{180}{277}$$

57.

$$-\frac{40}{9} < a, a < 0$$

58.

$$\left\{ -\frac{8a}{5} - \frac{7}{5} \right\}$$

59.

$$\left\{ \frac{6}{13} - \frac{4a}{13} \right\}$$

60.

$$\left\{ \frac{3(y-1)}{3y+1} \right\}$$

61.

$$\left\{ -\frac{2 \cdot (2y-5)}{5y+3} \right\}$$



62. Domain:

$$\left(-\infty, -\frac{2}{7}\right) \cup \left(-\frac{2}{7}, \infty\right)$$

Range:

$$\left(-\infty, -\frac{10}{7}\right) \cup \left(-\frac{10}{7}, \infty\right)$$

63.

$$-6.197$$

64.

$$\left\{ \frac{a}{12} - \frac{\sqrt{a^2 - 216}}{12}, \frac{a}{12} + \frac{\sqrt{a^2 - 216}}{12} \right\}$$

65.

$$-32x^2 + 28x - 7$$

66.

$$0 < a, a < \frac{5}{14}$$

67.

$$0 < a, a < 9$$

68. Domain:

$$\left(-\infty, -\frac{3}{2}\right) \cup \left(-\frac{3}{2}, \infty\right)$$

Range:

$$\left(-\infty, -\frac{7}{2}\right) \cup \left(-\frac{7}{2}, \infty\right)$$

69.

$$\left\{ -\frac{a+5}{5a} - \frac{\sqrt{-49a^2 + 45a + 25}}{5a}, -\frac{a+5}{5a} + \frac{\sqrt{-49a^2 + 45a + 25}}{5a} \right\}$$

70.

$$250x^2 + 530x + 281$$

71. Domain:

$$\left(-\infty, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$$

Range:

$$\left(-\infty, \frac{7}{2}\right) \cup \left(\frac{7}{2}, \infty\right)$$

72.

$$-2.028$$

73.

$$\left\{ \frac{10y-9}{4y-3} \right\}$$

74.

$$0 < a, a < \frac{360}{121}$$

75.

$$\left\{ \frac{6(y+1)}{9y-8} \right\}$$

76. Domain:

$$(-\infty, -1) \cup (-1, \infty)$$

Range:

$$\left( -\infty, \frac{9}{8} \right) \cup \left( \frac{9}{8}, \infty \right)$$

77.

$$32x^2 - 102x + 85$$

78.

$$\left\{ -\frac{3(y+1)}{8y-7} \right\}$$

79.

$$0 < a, a < \frac{65}{4}$$

80.

$$0 < a, a < \frac{8}{15}$$

81.

$$\left\{ -\frac{8a-5}{18a} - \frac{\sqrt{-404a^2-80a+25}}{18a}, -\frac{8a-5}{18a} + \frac{\sqrt{-404a^2-80a+25}}{18a} \right\}$$

82.

$$0 < a, a < \frac{3}{8}$$

83.

$$-4x^2 - 42x - 101$$

84.

$$-\frac{64}{33} < a, a < 0$$

85.

$$-1.333$$

86.

$$\left\{ \frac{10a-1}{6a} - \frac{\sqrt{220a^2-104a+1}}{6a}, \frac{10a-1}{6a} + \frac{\sqrt{220a^2-104a+1}}{6a} \right\}$$

87.

$$-5.727$$

88. Domain:

$$(-\infty, 1) \cup (1, \infty)$$

Range:

$$\left(-\infty, -\frac{5}{2}\right) \cup \left(-\frac{5}{2}, \infty\right)$$

89.

$$-0.867$$

90. Domain:

$$\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$$

Range:

$$\left(-\infty, \frac{4}{5}\right) \cup \left(\frac{4}{5}, \infty\right)$$

91. Domain:

$$(-\infty, -1) \cup (-1, \infty)$$

Range:

$$\left(-\infty, \frac{2}{3}\right) \cup \left(\frac{2}{3}, \infty\right)$$

92.

$$\emptyset$$

93.

$$0.526$$

94. Domain:

$$(-\infty, -1) \cup (-1, \infty)$$

Range:

$$\left(-\infty, -\frac{1}{4}\right) \cup \left(-\frac{1}{4}, \infty\right)$$

95.

$$-\frac{8}{5} < a, a < 0$$

96.

$$\emptyset$$

97.

$$\left\{ \frac{6a}{5} - \frac{2}{5} \right\}$$

98.

$$\left\{ \frac{2}{3} - \frac{7a}{3} \right\}$$

99.

$$0.907$$

100.

$$0 < a, a < 1$$

101.

$$-4x^2 + 32x - 57$$

102.

$$-256x^2 - 464x - 202$$

103.

$$\left\{ -\frac{a}{4} - \frac{\sqrt{25a^2 + 32}}{20}, -\frac{a}{4} + \frac{\sqrt{25a^2 + 32}}{20} \right\}$$

104.

$$\left\{ -\frac{2a}{3} - \frac{\sqrt{5}\sqrt{20a^2 - 3}}{15}, -\frac{2a}{3} + \frac{\sqrt{5}\sqrt{20a^2 - 3}}{15} \right\}$$

105.

$$108x^2 + 378x + 328$$

106.

$$0 < a, a < 80$$

107.

$$50x^2 - 245x + 283$$

108.

$$\left\{ -\frac{4a}{9} - \frac{1}{3} \right\}$$

109.

$$\left\{ \frac{3a}{7} \right\}$$

110.

$$\left\{ \frac{2 \cdot (2a - 1)}{5a} - \frac{\sqrt{-14a^2 + 19a + 4}}{5a}, \frac{2 \cdot (2a - 1)}{5a} + \frac{\sqrt{-14a^2 + 19a + 4}}{5a} \right\}$$

111. Domain:

$$\left( -\infty, \frac{9}{10} \right) \cup \left( \frac{9}{10}, \infty \right)$$

Range:

$$\left( -\infty, -\frac{1}{5} \right) \cup \left( -\frac{1}{5}, \infty \right)$$

112.

$$162x^2 - 171x + 52$$

113. Domain:

$$\left( -\infty, -\frac{4}{3} \right) \cup \left( -\frac{4}{3}, \infty \right)$$

Range:

$$\left( -\infty, -\frac{10}{3} \right) \cup \left( -\frac{10}{3}, \infty \right)$$

114.

$$-\frac{10}{53} < a, a < 0$$

115.

$$0.818$$

116.

$$24x^2 + 258x + 686$$

117.

$$\left\{ -\frac{3y+8}{7y+5} \right\}$$

118.

$$0 < a, a < \frac{3}{2}$$

119.

$$324x^2 - 1020x + 790$$

120.

$$\left\{ \frac{a}{20} - \frac{\sqrt{a^2+5}}{20}, \frac{a}{20} + \frac{\sqrt{a^2+5}}{20} \right\}$$

121.

$$\left\{ -\frac{5(y+1)}{y+9} \right\}$$

122. Domain:

$$(-\infty, 4) \cup (4, \infty)$$

Range:

$$(-\infty, 5) \cup (5, \infty)$$

123.

$$\left\{ -\frac{5a}{14} - \frac{\sqrt{225a^2-28}}{42}, -\frac{5a}{14} + \frac{\sqrt{225a^2-28}}{42} \right\}$$

124.

$$-21.0$$

125.

$$256x^2 - 496x + 231$$

126.

$$\left\{ \frac{a}{5} - \frac{\sqrt{3}\sqrt{3a^2-10}}{15}, \frac{a}{5} + \frac{\sqrt{3}\sqrt{3a^2-10}}{15} \right\}$$

127.

$$\left\{ \frac{8y+9}{y+6} \right\}$$

128. Domain:

$$\left(-\infty, -\frac{4}{5}\right) \cup \left(-\frac{4}{5}, \infty\right)$$

Range:

$$\left(-\infty, -\frac{2}{5}\right) \cup \left(-\frac{2}{5}, \infty\right)$$

129.

$$-18x^2 - 60x - 41$$

130.

$$\left\{-\frac{5a}{8} - \frac{\sqrt{225a^2 - 32}}{24}, -\frac{5a}{8} + \frac{\sqrt{225a^2 - 32}}{24}\right\}$$

131.

$$54x^2 + 387x + 681$$

132.

$$4x^2 + 70x + 302$$

133.

$$\left\{\frac{8y+5}{7y+1}\right\}$$

134.

$$\left\{\frac{a}{2} - \frac{\sqrt{2}\sqrt{2a^2+9}}{4}, \frac{a}{2} + \frac{\sqrt{2}\sqrt{2a^2+9}}{4}\right\}$$

135.

$$-\frac{96}{145} < a, a < 0$$

136.

$$\left\{-4a - \frac{2\sqrt{36a^2+1}}{3}, -4a + \frac{2\sqrt{36a^2+1}}{3}\right\}$$

137.

$$1.333$$

138. Domain:

$$\left(-\infty, \frac{2}{9}\right) \cup \left(\frac{2}{9}, \infty\right)$$

Range:

$$\left(-\infty, -\frac{8}{9}\right) \cup \left(-\frac{8}{9}, \infty\right)$$

139.

$$\left\{ \frac{3y-4}{6y+7} \right\}$$

140.

$$\left\{ \frac{3a}{4} - \frac{3\sqrt{5}\sqrt{5a^2-4}}{20}, \frac{3a}{4} + \frac{3\sqrt{5}\sqrt{5a^2-4}}{20} \right\}$$

141.

$$-\frac{33}{20} < a, a < 0$$

142.

$$-98x^2 - 35x + 3$$

143.

$$\left\{ -\frac{4y+5}{4 \cdot (2y-1)} \right\}$$

144.

$$\left\{ -\frac{9a}{8} - \frac{\sqrt{81a^2-20}}{8}, -\frac{9a}{8} + \frac{\sqrt{81a^2-20}}{8} \right\}$$

145. Domain:

$$(-\infty, -4) \cup (-4, \infty)$$

Range:

$$(-\infty, 1) \cup (1, \infty)$$

146.

$$0 < a, a < \frac{54}{61}$$

147.

$$3.522$$

148.

$$2.602$$

149.

$$0.095$$

150.

$$-63x^2 + 195x - 158$$



151.

$$0 < a, a < \frac{56}{9}$$

152.

$$-36x^2 - 128x - 105$$

153.

$$-\frac{7}{8} < a, a < 0$$

154.

$$0 < a, a < \frac{14}{37}$$

155.

$$294x^2 + 742x + 459$$

156.

$$\left\{ \frac{3a}{14} - \frac{\sqrt{5}\sqrt{45a^2 + 112}}{70}, \frac{3a}{14} + \frac{\sqrt{5}\sqrt{45a^2 + 112}}{70} \right\}$$

157.

$$-\frac{512}{377} < a, a < 0$$

158.

$$1.385$$

159.

$$-0.571$$

160.

$$0.656$$

161.

$$-\frac{32}{193} < a, a < 0$$

162.

$$-343x^2 - 581x - 243$$

163.

$$0 < a, a < 24$$

164.

$$\left\{ -\frac{9y-8}{9y-4} \right\}$$

165.

$$-\frac{112}{9} < a, a < 0$$

166.

$$81x^2 - 39x + 11$$

167.

$$\left\{ \frac{6a+5}{16a} - \frac{\sqrt{260a^2 - 228a + 25}}{16a}, \frac{6a+5}{16a} + \frac{\sqrt{260a^2 - 228a + 25}}{16a} \right\}$$

168. Domain:

$$(-\infty, 3) \cup (3, \infty)$$

Range:

$$(-\infty, -5) \cup (-5, \infty)$$

169.

$$-2.649$$

170.

$$\left\{ -\frac{3a+8}{2a} - \frac{\sqrt{13a^2 + 8a + 64}}{2a}, -\frac{3a+8}{2a} + \frac{\sqrt{13a^2 + 8a + 64}}{2a} \right\}$$

171.

$$-\frac{64}{49} < a, a < 0$$

172.

$$0 < a, a < \frac{7}{74}$$

173.

$$\{1-a\}$$

174.

$$\left\{ \frac{a}{6} - \frac{\sqrt{7}\sqrt{7a^2 - 24}}{42}, \frac{a}{6} + \frac{\sqrt{7}\sqrt{7a^2 - 24}}{42} \right\}$$

175.

$$\left\{ \frac{2y-5}{3y+7} \right\}$$

176. Domain:

$$\left(-\infty, \frac{1}{3}\right) \cup \left(\frac{1}{3}, \infty\right)$$

Range:

$$(-\infty, -1) \cup (-1, \infty)$$

177.

$$0 < a, a < \frac{56}{25}$$

178.

$$54x^2 + 39x + 16$$

179.

$$\left\{ \frac{a}{4} - \frac{\sqrt{21}\sqrt{21a^2 + 40}}{84}, \frac{a}{4} + \frac{\sqrt{21}\sqrt{21a^2 + 40}}{84} \right\}$$

180.

$$\left\{ \frac{7a}{18} - \frac{\sqrt{49a^2 + 108}}{18}, \frac{7a}{18} + \frac{\sqrt{49a^2 + 108}}{18} \right\}$$

181.

$$-\frac{160}{329} < a, a < 0$$

182. Domain:

$$\left(-\infty, \frac{4}{5}\right) \cup \left(\frac{4}{5}, \infty\right)$$

Range:

$$(-\infty, -1) \cup (-1, \infty)$$

183.

$$0 < a, a < \frac{2}{21}$$

184. Domain:

$$(-\infty, 1) \cup (1, \infty)$$

Range:

$$(-\infty, -2) \cup (-2, \infty)$$

185.

$$-\frac{45}{43} < a, a < 0$$

186.

$$-360x^2 + 894x - 563$$

187.

$$0.833$$

188.

$$-\frac{264}{25} < a, a < 0$$

189.

$$5.625$$

190.

$$\left\{ \frac{a}{9} - \frac{\sqrt{a^2+3}}{9}, \frac{a}{9} + \frac{\sqrt{a^2+3}}{9} \right\}$$

191.

$$0.179$$

192.

$$-\frac{48}{121} < a, a < 0$$

193.

$$-40 < a, a < 0$$

194.

$$\left\{ \frac{7y+5}{3y-2} \right\}$$

195.

$$-\frac{84}{55} < a, a < 0$$

196.

$$0 < a, a < \frac{100}{29}$$

197.

$$\left\{ -\frac{5y-6}{5(y-2)} \right\}$$

198.

$$\left\{ \frac{3a-4}{16a} - \frac{\sqrt{297a^2+232a+16}}{16a}, \frac{3a-4}{16a} + \frac{\sqrt{297a^2+232a+16}}{16a} \right\}$$

199. Domain:

$$(-\infty, -1) \cup (-1, \infty)$$

Range:

$$\left(-\infty, \frac{10}{7}\right) \cup \left(\frac{10}{7}, \infty\right)$$

200.

$$0 < a, a < \frac{56}{65}$$



## 3. Calculus

### 3.1 Problems

**Exercise 3.1** Evaluate the following indefinite integral

$$\int 10t^{\frac{4}{5}}v^{\frac{5}{2}} + 10t^{\frac{2}{3}} + 9\sqrt{t} - 9t^5\sqrt{v} dt$$

**Exercise 3.2** Double derivative: Differentiate the following expression with respect to  $p$  twice

$$b^2(b^4p^4 + 10b^2 + 14b + 8p^5)$$

**Exercise 3.3** Differentiate the following expression with respect to  $s$

$$-70d^2s \sec(8ds^2) + 5\cot^3(5ds)$$

**Exercise 3.4** Integrate the expression with respect to  $w$

$$\frac{\sqrt{2} \cdot (5w + \frac{5}{2})}{\sqrt{5w^2 + 5w + 4}}$$

**Exercise 3.5** Differentiate the following expression with respect to  $t$

$$-6\tan(3s) + 10\sec(s^2)$$

**Exercise 3.6** Integrate the polynomial with respect to  $d$

$$-5d^2 + 5d + 10$$

**Exercise 3.7** Integrate the expression with respect to  $y$

$$\frac{5}{2\sqrt{5y+8}}$$

**Exercise 3.8** Integrate the expression with respect to  $s$

$$\frac{2s+1}{\sqrt{2s^2+2s-9}}$$

**Exercise 3.9** Differentiate the polynomial with respect to  $v$

$$7v^3 - 5v^2 - 3v - 10$$

**Exercise 3.10** Double derivative: Differentiate the following expression with respect to  $d$  twice

$$l^2 \cdot (5d^6l + 7d^5 + 10d^3l + 10d^2l - 3)$$

**Exercise 3.11** Evaluate the definite integral for the following function, where the limits are 1 and 2.

$$f(q) = 5\sqrt[6]{q} - 4q^{\frac{3}{2}} + 8\sqrt{q}$$

**Exercise 3.12** Differentiate the following expression with respect to  $z$

$$-6e^{10z} \cos(5q) + 4 \tan^2(3q^2z)$$

**Exercise 3.13** Double derivative: Differentiate the following expression with respect to  $q$  twice

$$q^2t^2 \cdot (8q^4t + q^2 + 8t^2)$$

**Exercise 3.14** Evaluate the following indefinite integral

$$\int 3t^5 - 2t^4v^5 + 5vdt$$

**Exercise 3.15** Differentiate the polynomial with respect to  $w$

$$w^2 - w + 7$$

**Exercise 3.16** Differentiate the following expression with respect to  $c$

$$-\frac{35e^{9c^2}}{\sin^2(7cz^2)} - \frac{9}{\tan(10c)}$$

**Exercise 3.17** Integrate the expression with respect to  $n$

$$-\frac{9}{2\sqrt{-9n-5}}$$

**Exercise 3.18** Evaluate the following indefinite integral

$$\int 9n^5w^4 - 2n^4w + 7n^2 + 4w^3 + 10w^2dw$$

**Exercise 3.19** Differentiate the following expression with respect to  $q$

$$70e^{8d^2q} \sec(3d^2q) - 6\cos^2(7dq^2)$$

**Exercise 3.20** Differentiate the following expression with respect to  $b$

$$\frac{-3 \tan(7b^2q^3) + 5 \sec(b^3)}{4 \sin^3(6b^3q)}$$



**Exercise 3.21** Double derivative: Differentiate the following expression with respect to  $n$  twice

$$n(-17n^4 - 4n^2q^2 - nq^4 - 2)$$

**Exercise 3.22** Integrate the polynomial with respect to  $a$

$$-2a^4 + 10a^3 + 4a^2 - 3a - 5$$

**Exercise 3.23** Evaluate the following indefinite integral

$$\int 9w^{\frac{5}{6}}y - 8w\sqrt{y} + 8wy^2 - 7y^4 - 5dw$$

**Exercise 3.24** Differentiate the following expression with respect to  $n$

$$-6\sec^2(10n^2s^2)$$

**Exercise 3.25** Differentiate the following expression with respect to  $m$

$$-27e^{10m^2}\csc(5m^2) + 2\csc^2(m^2)$$

**Exercise 3.26** Double derivative: Differentiate the following expression with respect to  $s$  twice

$$10d^4s + 5d^4 - 3s^5$$

**Exercise 3.27** Differentiate the following expression with respect to  $z$

$$30z^4\sec^2(7s^3z^2) + 5\sin^3(2z)$$

**Exercise 3.28** Differentiate the polynomial with respect to  $x$

$$-2x^3 + 9x^2 + 9x + 7$$

**Exercise 3.29** Differentiate the following expression with respect to  $b$

$$\frac{10\tan^3(9b) - 3\sec^3(7b^2w)}{4\tan^2(3b)}$$

**Exercise 3.30** Integrate the expression with respect to  $l$

$$\frac{3(-4l - 3)}{2\sqrt{-6l^2 - 9l - 4}}$$

**Exercise 3.31** Integrate the expression with respect to  $r$

$$\frac{1008r^3}{\sin^2(8r^3)} + 210r^2\cos(7r^3) - 42\cot(8r^3)$$

**Exercise 3.32** Differentiate the following expression with respect to  $n$

$$7\tan^2(5n) + 6\tan^2(10n^2y)$$

**Exercise 3.33** Evaluate the following indefinite integral

$$\int 6c^{\frac{5}{6}}v^{\frac{5}{6}} - 3c^{\frac{3}{5}} + 5c^{\frac{5}{4}}v^6 - 8\sqrt[3]{c}\sqrt[5]{v} - 5c\sqrt[6]{v}dv$$

**Exercise 3.34** Integrate the expression with respect to  $l$

$$\frac{-4l - 9}{3(-2l^2 - 9l + 7)^{\frac{2}{3}}}$$

**Exercise 3.35** Differentiate the following expression with respect to  $m$

$$12e^{4u^2} \cot(m^2u)$$

**Exercise 3.36** Differentiate the polynomial with respect to  $y$

$$-8y^3 - 7y^2 - 5y - 6$$

**Exercise 3.37** Integrate the expression with respect to  $n$

$$-40 \sin(4n) \cos(4n) - 4 \tan^2(4n) - 4$$

**Exercise 3.38** Differentiate the following expression with respect to  $l$

$$-5l^{\frac{6}{5}}t^{\frac{4}{5}} - 7l^{\frac{2}{3}}\sqrt[3]{t} + 3l^{\frac{2}{3}} + 10l^3$$

**Exercise 3.39** Differentiate the following expression with respect to  $m$

$$8m^2 - 6mq^2 - 3q^2 - 16$$

**Exercise 3.40** Differentiate the following expression with respect to  $r$

$$8 \csc(3r^2)$$

**Exercise 3.41** Integrate the expression with respect to  $c$

$$-\frac{\sqrt{2}}{\sqrt{-2c-5}}$$

**Exercise 3.42** Differentiate the following expression with respect to  $n$

$$24a^3n^2 \cos^3(3an^2) - 7 \sec^3(8an^3)$$

**Exercise 3.43** Integrate the expression with respect to  $m$

$$\frac{54 \cos(9m)}{\sin^3(9m)} - \frac{70 \cos(7m)}{\sin^3(7m)}$$

**Exercise 3.44** Integrate the polynomial with respect to  $y$

$$9y^2 - 8y - 4$$

**Exercise 3.45** Differentiate the following expression with respect to  $l$

$$8 \cot^2(10l)$$

**Exercise 3.46** Double derivative: Differentiate the following expression with respect to  $m$  twice

$$8m^3v + v^6 - 7v^4 + 7$$

**Exercise 3.47** Differentiate the following expression with respect to  $d$

$$-3d^{\frac{2}{3}}\sqrt[3]{m} + 4\sqrt{d}\sqrt[4]{m} + 4d^4$$

**Exercise 3.48** Integrate the expression with respect to  $c$

$$40c \sin(2c^2) - 24 \tan(2c) \sec^2(2c)$$

**Exercise 3.49** Differentiate the following expression with respect to  $m$

$$-\frac{4}{\tan(5p)} - \frac{5}{\sin(2mp^2)}$$

**Exercise 3.50** Integrate the expression with respect to  $c$

$$\frac{2 \cdot (2 - c)}{3(-c^2 + 4c - 9)^{\frac{2}{3}}}$$

**Exercise 3.51** Integrate the expression with respect to  $v$

$$\frac{7}{2\sqrt{7v+10}}$$

**Exercise 3.52** Differentiate the following expression with respect to  $z$

$$-3 \cos^2(3z) - \frac{10}{\tan(7l^2z)}$$

**Exercise 3.53** Evaluate the definite integral for the following function, where the limits are 2 and 5.

$$f(t) = 4t^{\frac{5}{6}} + 2t^{\frac{3}{5}} + 4\sqrt{t} + 9t - 2$$

**Exercise 3.54** Differentiate the following expression with respect to  $q$

$$32e^{4n^2q^2}n^2q^3 \sin^3(2n^2q^3) + 5 \cot^3(6q)$$

**Exercise 3.55** Differentiate the following expression with respect to  $t$

$$80e^{4t^2} \sec^2(5bt^2) + 7 \sin^2(6b)$$

**Exercise 3.56** Integrate the polynomial with respect to  $n$

$$-8n^3 + 7n^2 - 5n + 2$$

**Exercise 3.57** Differentiate the following expression with respect to  $l$

$$-15e^{5d} \cos(9dl)$$

**Exercise 3.58** Integrate the expression with respect to  $t$

$$-\frac{216t^5 \tan(3t^2)}{\cos^2(3t^2)} - 72t^3 \tan^2(3t^2) + 96 \cot(8t) \csc^2(8t)$$

**Exercise 3.59** Differentiate the following expression with respect to  $v$

$$-27dv^4 \sin^2(d^2v^3) - 6 \sin^2(v^2)$$

**Exercise 3.60** Differentiate the following expression with respect to  $t$

$$16t^3 \cos^2(4n^2t) + 8 \cos^3(7nt^2)$$

**Exercise 3.61** Differentiate the following expression with respect to  $q$

$$-18e^{2q} \cot^2(8b^2q^2) + 8 \tan(7q)$$

**Exercise 3.62** Differentiate the following expression with respect to  $w$

$$2 \cos^2(5w^2) - 2 \sec(7n^2)$$

**Exercise 3.63** Differentiate the polynomial with respect to  $s$

$$6s^2 + 5s - 4$$

**Exercise 3.64** Integrate the expression with respect to  $l$

$$l^2 \cdot (4800l^3 \tan(10l^2) \sec^3(10l^2) + 320l \sec^3(10l^2) - 900 \sin^2(10l^3) \cos(10l^3))$$

**Exercise 3.65** Integrate the polynomial with respect to  $u$

$$-6u^2 - 8u - 3$$

**Exercise 3.66** Differentiate the following expression with respect to  $r$

$$48nr \cot^3(2nr) - 5 \csc^3(8n)$$

**Exercise 3.67** Integrate the expression with respect to  $l$

$$\frac{10}{3(10l-1)^{\frac{2}{3}}}$$

**Exercise 3.68** Differentiate the following expression with respect to  $b$

$$4\sqrt{b}c^3 - 4b^6c^{\frac{5}{4}} - 3b^3c$$

**Exercise 3.69** Integrate the expression with respect to  $b$

$$-30 \sin(6b) - 90 \cot(9b) \csc^2(9b)$$

**Exercise 3.70** Differentiate the following expression with respect to  $b$

$$(3 \cos(6v^2) - 2 \sec^2(7b^3v)) \tan^3(6v^2)$$

**Exercise 3.71** Differentiate the following expression with respect to  $t$

$$63t \sin^2(6p^3t) + 3 \cot^3(9p^2t)$$

**Exercise 3.72** Integrate the expression with respect to  $q$

$$100 \sin(20q) - \frac{24}{\cos^2(8q)}$$

**Exercise 3.73** Differentiate the following expression with respect to  $w$

$$48e^{10u^2w} \sec(5u^2w^2)$$

**Exercise 3.74** Double derivative: Differentiate the following expression with respect to  $n$  twice

$$n^4 \cdot (3nq^6 - 4n + 5q^4)$$

**Exercise 3.75** Differentiate the following expression with respect to  $w$

$$-35u^4 \cos^3(5w^2) + 10 \cot(5u^2w^2)$$

**Exercise 3.76** Evaluate the following indefinite integral

$$\int 7\sqrt[3]{p}v^4 - p^{\frac{5}{2}} + 3p^{\frac{3}{2}} + 2\sqrt{p} + 5dv$$

**Exercise 3.77** Evaluate the following indefinite integral

$$\int -10a^{\frac{5}{2}}\sqrt{l} - 2dl$$

**Exercise 3.78** Differentiate the following expression with respect to  $m$

$$-10 \tan^2(3mv^2)$$

**Exercise 3.79** Differentiate the following expression with respect to  $a$

$$-100e^{7a^2} \cot^2(7t^2)$$

**Exercise 3.80** Evaluate the following indefinite integral

$$\int -10\sqrt[5]{n}x^{\frac{3}{5}} + 6n^{\frac{2}{3}} - 9x^{\frac{5}{2}} + 10\sqrt{x}dx$$

**Exercise 3.81** Integrate the expression with respect to  $u$

$$6u \left( -\frac{324u^2 \cot^2(4u^2)}{\sin^2(4u^2)} + 10 \tan(5u^2) \sec^3(5u^2) + 27 \cot^3(4u^2) \right)$$

**Exercise 3.82** Integrate the expression with respect to  $d$

$$\frac{d+1}{\sqrt{d^2+2d-5}}$$

**Exercise 3.83** Differentiate the following expression with respect to  $c$

$$10c^3 \cot^3(2l) - 10 \cos^3(9l^3)$$

**Exercise 3.84** Evaluate the definite integral for the following function, where the limits are 3 and 4.

$$f(s) = -6s^{\frac{6}{5}} - 9s^2 + 10$$

**Exercise 3.85** Differentiate the polynomial with respect to  $a$

$$-2a^2 + 6a + 6$$

**Exercise 3.86** Differentiate the polynomial with respect to  $q$

$$-3q^3 + 4q^2 - 5q + 3$$

**Exercise 3.87** Differentiate the following expression with respect to  $d$

$$d^{\frac{5}{6}} + 4d^5 - 4dv^{\frac{3}{2}} - v^{\frac{5}{6}} - 7v^2$$

**Exercise 3.88** Differentiate the following expression with respect to  $r$

$$-3 \cot(7x^2)$$

**Exercise 3.89** Double derivative: Differentiate the following expression with respect to  $x$  twice

$$5t^6 + 7t^4x - 5t^2x - 8x^2$$

**Exercise 3.90** Integrate the expression with respect to  $t$

$$12t \left( \frac{3}{\cos^2(9t^2)} - \frac{1}{\cos^2(2t^2)} \right)$$

**Exercise 3.91** Integrate the expression with respect to  $u$

$$\frac{5u - 2}{\sqrt{5u^2 - 4u - 1}}$$

**Exercise 3.92** Differentiate the polynomial with respect to  $b$

$$-4b^4 + 9b^3 + 10b^2 + 2b - 5$$

**Exercise 3.93** Differentiate the following expression with respect to  $b$

$$\frac{(8 \csc^3(4b^2m) - 9 \sec(6m^2)) \tan^2(8bm^2)}{2}$$

**Exercise 3.94** Differentiate the following expression with respect to  $n$

$$-9a^{\frac{3}{4}} - 5a^{\frac{2}{3}}n^{\frac{6}{5}} + 9\sqrt{a} - 5a^3$$

**Exercise 3.95** Evaluate the following indefinite integral

$$\int -9b^6 - 3bn^{\frac{3}{4}} + 9n^{\frac{4}{5}} dn$$

**Exercise 3.96** Differentiate the following expression with respect to  $d$

$$2 \sin^2(6z) - 8 \sec^2(10z)$$

**Exercise 3.97** Differentiate the following expression with respect to  $n$

$$3n(-2n^4x - 5n^2 + x^4)$$

**Exercise 3.98** Integrate the expression with respect to  $c$

$$18c^2 \tan(c^3) \sec^2(c^3) - 720c^2 \cot(4c^2) \csc^2(4c^2) + 45 \csc^2(4c^2)$$

**Exercise 3.99** Integrate the expression with respect to  $u$

$$-3360u^2 \sin^2(10u^2) \cos(10u^2) - 18u^2 \cot(3u^3) \csc(3u^3) - 56 \sin^3(10u^2)$$

**Exercise 3.100** Integrate the expression with respect to  $w$

$$-\frac{3}{2\sqrt{-w-1}}$$

**Exercise 3.101** Differentiate the following expression with respect to  $c$

$$7b^{\frac{5}{6}} + 6c^{\frac{3}{5}} + 6c^{\frac{2}{3}} + 8$$

**Exercise 3.102** Evaluate the following indefinite integral

$$\int 4a^6 s^6 - 2a^4 + 7a^2 + 6s^6 + 8da$$

**Exercise 3.103** Evaluate the following indefinite integral

$$\int 9r^{\frac{5}{4}} y^{\frac{3}{2}} - 3r^2 - 10\sqrt[5]{y} - 3y^{\frac{3}{2}} - 4dr$$

**Exercise 3.104** Differentiate the following expression with respect to  $t$

$$2 \tan(4bt) + 10 \csc^2(4b^2 t^2)$$

**Exercise 3.105** Evaluate the definite integral for the following function, where the limits are 0 and 1.

$$f(u) = 3u^{\frac{4}{3}} - 4u^{\frac{3}{2}} + \sqrt{u} - 1$$

**Exercise 3.106** Evaluate the following indefinite integral

$$\int -\sqrt[3]{c} \sqrt[3]{w} + 3wdw$$

**Exercise 3.107** Differentiate the following expression with respect to  $q$

$$-5b^5 + 7b^3 q^5 - 6b^2 q^2 - 9b + 8q^2$$

**Exercise 3.108** Differentiate the following expression with respect to  $u$

$$\frac{\frac{3 \sin^3(10s^3)}{4} + \cos(10s^2)}{\tan(2s^2)}$$

**Exercise 3.109** Differentiate the following expression with respect to  $v$

$$\frac{-3 \cos^2(4v) + 2 \csc^2(8z^3)}{8 \cos(8v^2)}$$

**Exercise 3.110** Differentiate the polynomial with respect to  $q$

$$-5q^3 - 4q^2 - 4q - 10$$

**Exercise 3.111** Double derivative: Differentiate the following expression with respect to  $w$  twice

$$-b^4w^4 + 3b^2w - 3b^2 - w^3$$

**Exercise 3.112** Evaluate the following indefinite integral

$$\int s(-9s^3 + 7s^2t^6 + 10t^2) ds$$

**Exercise 3.113** Differentiate the following expression with respect to  $y$

$$-48e^{6yz^2} \cos^2(5yz)$$

**Exercise 3.114** Integrate the expression with respect to  $l$

$$-\frac{\sqrt[3]{3}}{(-3l-2)^{\frac{2}{3}}}$$

**Exercise 3.115** Double derivative: Differentiate the following expression with respect to  $u$  twice

$$-15q^4 + qu^4 + 2u^3$$

**Exercise 3.116** Differentiate the following expression with respect to  $w$

$$-54w^3 \cot(10l^3) - 3 \sec^3(6l^2)$$

**Exercise 3.117** Integrate the expression with respect to  $q$

$$12q \left( \frac{\sin(2q^2)}{2} - 9 \tan(9q^2) \sec(9q^2) \right)$$

**Exercise 3.118** Double derivative: Differentiate the following expression with respect to  $m$  twice

$$m^3 \cdot (9m^2p^2 - 8m^2 + 8p^5)$$

**Exercise 3.119** Differentiate the following expression with respect to  $u$

$$-\frac{20du}{\cos(2d^2u^2)} - \frac{7}{\tan^2(du)}$$

**Exercise 3.120** Integrate the expression with respect to  $v$

$$\frac{3-2v}{3(-v^2+3v+9)^{\frac{2}{3}}}$$

**Exercise 3.121** Integrate the polynomial with respect to  $c$

$$c^2 + 2c - 10$$

**Exercise 3.122** Differentiate the following expression with respect to  $y$

$$-2e^{9y^2} \tan^2(4y)$$



**Exercise 3.123** Differentiate the following expression with respect to  $z$

$$-7r^{\frac{2}{3}}\sqrt[4]{z} - 8r^{\frac{3}{2}} + r^2$$

**Exercise 3.124** Evaluate the definite integral for the following function, where the limits are 2 and 4.

$$f(v) = 3$$

**Exercise 3.125** Evaluate the definite integral for the following function, where the limits are 3 and 4.

$$f(b) = -2b^{\frac{3}{5}} + 10b^{\frac{2}{3}} + 7\sqrt{b} - 10b^2 - 3b$$

**Exercise 3.126** Differentiate the following expression with respect to  $l$

$$\frac{-\cos(5w) - 5\sec^3(2lw^3)}{2\cos(4w^3)}$$

**Exercise 3.127** Integrate the expression with respect to  $v$

$$-\frac{1}{3(3-v)^{\frac{2}{3}}}$$

**Exercise 3.128** Integrate the expression with respect to  $x$

$$-\frac{168x^4}{\sin^2(3x^2)} + 84x^2 \cot(3x^2) + 105 \cot(5x) \csc^3(5x)$$

**Exercise 3.129** Differentiate the following expression with respect to  $m$

$$-8e^{10m} \cos^2(2m^2r^2)$$

**Exercise 3.130** Differentiate the following expression with respect to  $p$

$$-5\sqrt[6]{p} - 7p^6q + 7p$$

**Exercise 3.131** Differentiate the following expression with respect to  $b$

$$4\cos(2bz^2)$$

**Exercise 3.132** Evaluate the following indefinite integral

$$\int -6s^5z^2 - 9s^3z^4 - 2s^3 - 3z^2 dz$$

**Exercise 3.133** Differentiate the following expression with respect to  $w$

$$21w^2y^4 \sec^3(2wy^3) - 4 \tan^2(2wy^3)$$

**Exercise 3.134** Integrate the expression with respect to  $l$

$$\frac{\sqrt{2}}{2\sqrt{l-4}}$$

**Exercise 3.135** Integrate the expression with respect to  $n$

$$40n(18n^2 \tan(2n^2) \sec^3(2n^2) - 5 \cot(10n^2) \csc^2(10n^2) + 3 \sec^3(2n^2))$$

**Exercise 3.136** Differentiate the following expression with respect to  $t$

$$\frac{(2 \tan^3(n) + 5 \cot(nt^3)) \sin(10n^3)}{5}$$

**Exercise 3.137** Differentiate the following expression with respect to  $l$

$$18e^{6lx^2} \cot^2(7l^2x^2)$$

**Exercise 3.138** Evaluate the following indefinite integral

$$\int 8r^{\frac{6}{5}}v^{\frac{3}{2}} - 6r^{\frac{5}{3}} - 10r^{\frac{3}{2}}v + 3dv$$

**Exercise 3.139** Integrate the expression with respect to  $v$

$$1600v^2 \cot(5v^2) \csc^2(5v^2) + 14v \sin(2v^2) - 80 \csc^2(5v^2)$$

**Exercise 3.140** Evaluate the following indefinite integral

$$\int 7n^4t - 6n - 5t^6 + 10t^4 + 10tdn$$

**Exercise 3.141** Differentiate the following expression with respect to  $s$

$$8b^3s^4 + 8b^2s^5 - 4b + 8s^5 + 6$$

**Exercise 3.142** Differentiate the following expression with respect to  $s$

$$-7 \sec^2(y)$$

**Exercise 3.143** Differentiate the following expression with respect to  $w$

$$-6 \tan(3lw)$$

**Exercise 3.144** Differentiate the following expression with respect to  $c$

$$c^{\frac{3}{5}}d^6 - 7c^{\frac{3}{2}}d - 3d - 8$$

**Exercise 3.145** Differentiate the following expression with respect to  $n$

$$-18e^{10y^2} \sin(2n^2y^2) - 6 \sec^2(2ny)$$

**Exercise 3.146** Differentiate the polynomial with respect to  $b$

$$-2b^4 + 4b^3 + 8b^2 - 9b + 6$$

**Exercise 3.147** Evaluate the definite integral for the following function, where the limits are 1 and 2.

$$f(s) = -4s^{\frac{6}{5}} - 6s^{\frac{2}{3}} + 8s^4$$

**Exercise 3.148** Differentiate the following expression with respect to  $l$

$$5d^{\frac{3}{2}}l^{\frac{3}{5}} - 10d^2\sqrt[3]{l} + 5dl + 9d + 8l^{\frac{2}{3}}$$

**Exercise 3.149** Differentiate the following expression with respect to  $a$

$$9\cot(9z^2)$$

**Exercise 3.150** Differentiate the following expression with respect to  $v$

$$18e^{4v}\cos^2(10vx^2) + 6\sin(x^2)$$

**Exercise 3.151** Differentiate the polynomial with respect to  $y$

$$-3y^4 + 2y^3 - 6y^2 + 7y - 5$$

**Exercise 3.152** Integrate the expression with respect to  $z$

$$216z^4\tan(3z^3)\sec^2(3z^3) + 24z\sec^2(3z^3) + 28\cot(2z)\csc^2(2z)$$

**Exercise 3.153** Differentiate the following expression with respect to  $t$

$$-60e^{4rt}\sin^2(7r)$$

**Exercise 3.154** Differentiate the following expression with respect to  $a$

$$2a^3 - 6a + 4w^4$$

**Exercise 3.155** Differentiate the following expression with respect to  $n$

$$-6\cos(10n) + 7\tan(9n^2z^2)$$

**Exercise 3.156** Differentiate the following expression with respect to  $d$

$$-6\sin(8d)$$

**Exercise 3.157** Differentiate the polynomial with respect to  $t$

$$-9t^4 + t^3 + 5t^2 + 3t + 3$$

**Exercise 3.158** Integrate the expression with respect to  $z$

$$\frac{3}{2\sqrt{3z+5}}$$

**Exercise 3.159** Differentiate the following expression with respect to  $m$

$$8\sqrt[4]{m} + 3m^{\frac{3}{2}} - 7m^6 - 8m^2$$

**Exercise 3.160** Integrate the expression with respect to  $q$

$$\frac{7 \cdot (1 - 2q)}{3(-7q^2 + 7q - 9)^{\frac{2}{3}}}$$

**Exercise 3.161** Differentiate the following expression with respect to  $u$

$$d^3(-4d^3 + d^2 - 6u + 8)$$

**Exercise 3.162** Differentiate the following expression with respect to  $u$

$$12n^4u^5 - 2n^3 - 8$$

**Exercise 3.163** Differentiate the following expression with respect to  $a$

$$-24ae^{6a^2y} \cos^3(a^2) + 9 \tan^2(8a^2y^2)$$

**Exercise 3.164** Differentiate the polynomial with respect to  $q$

$$-3q^2 - 10q + 9$$

**Exercise 3.165** Evaluate the following indefinite integral

$$\int r(8q^5 + 5q^4r^2 - 6r^5 - 2r) dq$$

**Exercise 3.166** Differentiate the polynomial with respect to  $u$

$$-10u^4 + 7u^3 - 10u^2 + 8u - 7$$

**Exercise 3.167** Differentiate the following expression with respect to  $z$

$$\frac{2(-4 \cot^3(7m^3) - \csc^2(8z^3))}{7 \tan(3m^3z)}$$

**Exercise 3.168** Integrate the expression with respect to  $z$

$$-400z \tan(10z^2) \sec^2(10z^2) - 72 \sin(4z) \cos(4z)$$

**Exercise 3.169** Double derivative: Differentiate the following expression with respect to  $c$  twice

$$z(-6c^4 - c^3z^2 + 10z^3 + 9z)$$

**Exercise 3.170** Differentiate the following expression with respect to  $c$

$$c(-c^2 + 3cy - 6)$$

**Exercise 3.171** Evaluate the following indefinite integral

$$\int 10\sqrt[5]{t}y^{\frac{6}{5}} + 3\sqrt{t}y^{\frac{5}{6}} + 7\sqrt{t} + 2t\sqrt[3]{y} + 4t\sqrt{y} dy$$

**Exercise 3.172** Differentiate the following expression with respect to  $p$

$$10l^3 \cos^2(7p^2) + 8 \sec^3(l^3p^3)$$

**Exercise 3.173** Differentiate the following expression with respect to  $m$

$$14n^4 \tan^3(8m^2) + 3 \cos^2(m)$$

**Exercise 3.174** Evaluate the following indefinite integral

$$\int 2n^{\frac{4}{5}} + 8n^3\sqrt{s} + 10s^{\frac{5}{2}} - 9s^2 dn$$

**Exercise 3.175** Differentiate the following expression with respect to  $r$

$$\frac{-8\cos^3(2r^2) + 7\cot^3(7ar)}{4\cos^2(3a^2r^2)}$$

**Exercise 3.176** Differentiate the polynomial with respect to  $n$

$$2n^3 - 8n^2 + 3n - 8$$

**Exercise 3.177** Differentiate the polynomial with respect to  $w$

$$-3w^3 - 10w^2 + 6w + 2$$

**Exercise 3.178** Differentiate the following expression with respect to  $t$

$$20e^{4t^2} \sec(d^2t)$$

**Exercise 3.179** Integrate the expression with respect to  $x$

$$\frac{18x - 1}{3(9x^2 - x + 9)^{\frac{2}{3}}}$$

**Exercise 3.180** Integrate the expression with respect to  $q$

$$3q \left( -1176q^4 \sin^2(8q^3) \cos(8q^3) - 49q \sin^3(8q^3) - \frac{200 \tan^2(10q^2)}{\cos^2(10q^2)} \right)$$

**Exercise 3.181** Differentiate the following expression with respect to  $m$

$$-9m^4v^3 + 18m^4 - 6m^2v^5 - 9$$

**Exercise 3.182** Evaluate the definite integral for the following function, where the limits are 3 and 7.

$$f(d) = 1 - d$$

**Exercise 3.183** Integrate the expression with respect to  $x$

$$\frac{-3x - 5}{\sqrt{-3x^2 - 10x - 5}}$$

**Exercise 3.184** Differentiate the following expression with respect to  $r$

$$-224e^{5n}n^4r^2 \sin(r) - 10\cos^2(2n^2r)$$

**Exercise 3.185** Integrate the expression with respect to  $x$

$$\frac{7 \cdot (2x - 1)}{3(7x^2 - 7x - 9)^{\frac{2}{3}}}$$

**Exercise 3.186** Integrate the expression with respect to  $p$

$$p^2 \cdot \left( \frac{108p^3 \cos(p^3)}{\sin^3(p^3)} + \frac{30}{\sin^2(2p^3)} - \frac{54}{\sin^2(p^3)} \right)$$

**Exercise 3.187** Integrate the expression with respect to  $w$

$$-\frac{180 \cos(9w)}{\sin^3(9w)} - \frac{140 \cos(7w)}{\sin^3(7w)}$$

**Exercise 3.188** Differentiate the following expression with respect to  $x$

$$4 \tan(6x^2) - 3 \cot(lx)$$

**Exercise 3.189** Differentiate the polynomial with respect to  $s$

$$-10s^2 - 5s + 10$$

**Exercise 3.190** Differentiate the following expression with respect to  $s$

$$-9 \cos(9s)$$

**Exercise 3.191** Integrate the expression with respect to  $m$

$$\frac{8}{\sin^2(2m)} + \frac{5}{\sin^2(m)}$$

**Exercise 3.192** Differentiate the polynomial with respect to  $p$

$$8p^4 - 5p^3 + 7p^2 - p - 4$$

**Exercise 3.193** Differentiate the following expression with respect to  $b$

$$-60e^{9r^2} \sin(3b) + 10 \tan(7b^2 r^2)$$

**Exercise 3.194** Differentiate the following expression with respect to  $n$

$$\frac{(-4 \cos^2(10ns^2) + 3 \cot^3(5n^2 s^3)) \tan(5n^2 s)}{9}$$

**Exercise 3.195** Evaluate the following indefinite integral

$$\int t^3 w + 8w^5 - w^2 - 6w + 4dw$$

**Exercise 3.196** Differentiate the following expression with respect to  $b$

$$-7 \cos^2(7b^2 v^2) - \frac{9}{\tan^2(6b^2 v^2)}$$

**Exercise 3.197** Differentiate the following expression with respect to  $r$

$$3r^6 - 10r^5 t^4 + 4r^3 t^3 + r^3 t - 3$$

**Exercise 3.198** Differentiate the following expression with respect to  $x$

$$x(-9u^2 x^4 + 8ux^5 - 2)$$

**Exercise 3.199** Differentiate the following expression with respect to  $z$

$$-10e^{6z} \csc(7r^2 z) + 8 \sec(2r)$$

**Exercise 3.200** Integrate the expression with respect to  $n$

$$-\frac{2}{3(9-2n)^{\frac{2}{3}}}$$

### 3.2 Solutions

1.

$$\frac{50t^{\frac{9}{5}}v^{\frac{5}{2}}}{9} + 6t^{\frac{5}{3}} + 6t^{\frac{3}{2}} - \frac{9t^2\sqrt[5]{v}}{2}$$

2.

$$b^2p^2 \cdot (12b^4 + 160p)$$

3.

$$5d \left( -224d^2s^2 \tan(8ds^2) \sec(8ds^2) - 14d \sec(8ds^2) - \frac{15 \cot^2(5ds)}{\sin^2(5ds)} \right)$$

4.

$$C + \sqrt{10w^2 + 10w + 8}$$

5.

$$0$$

6.

$$\frac{5d(-2d^2 + 3d + 12)}{6}$$

7.

$$C + \sqrt{5y + 8}$$

8.

$$C + \sqrt{2s^2 + 2s - 9}$$

9.

$$21v^2 - 10v - 3$$

10.

$$l^2 \cdot (150d^4l + 140d^3 + 60dl + 20l)$$

11. The indefinite integral is

$$\frac{30q^{\frac{7}{6}}}{7} - \frac{8q^{\frac{5}{2}}}{5} + \frac{16q^{\frac{3}{2}}}{3}$$

The value of the definite integral is 7.63603351830078.

12.

$$-60e^{10z} \log(e) \cos(5q) + \frac{24q^2 \tan(3q^2z)}{\cos^2(3q^2z)}$$

13.

$$t^2 \cdot (240q^4t + 12q^2 + 16t^2)$$

14.

$$\frac{t(5t^5 - 4t^4v^5 + 50v)}{10}$$

15.

$$2w - 1$$

16.

$$-\frac{630ce^{9c^2}\log(e)}{\sin^2(7cz^2)} + \frac{490e^{9c^2}z^2\cos(7cz^2)}{\sin^3(7cz^2)} + \frac{90}{\sin^2(10c)}$$

17.

$$C + \sqrt{-9n - 5}$$

18.

$$\frac{w(27n^5w^4 - 15n^4w + 105n^2 + 15w^3 + 50w^2)}{15}$$

19.

$$14d\left(40de^{8d^2q}\log(e)\sec(3d^2q) + 15de^{8d^2q}\tan(3d^2q)\sec(3d^2q) + 6q\sin(14dq^2)\right)$$

20.

$$\frac{3b\left(18bq(3\tan(7b^2q^3) - 5\sec(b^3))\cos(6b^3q) + \left(5b\tan(b^3)\sec(b^3) - \frac{14q^3}{\cos^2(7b^2q^3)}\right)\sin(6b^3q)\right)}{4\sin^4(6b^3q)}$$

21.

$$-340n^3 - 24nq^2 - 2q^4$$

22.

$$\frac{a(-12a^4 + 75a^3 + 40a^2 - 45a - 150)}{30}$$

23.

$$\frac{54w^{\frac{11}{6}}y}{11} - 4w^2\sqrt{y} + 4w^2y^2 - 7wy^4 - 5w$$

24.

$$-240ns^2\tan(10n^2s^2)\sec^2(10n^2s^2)$$

25.

$$2m\left(-270e^{10m^2}\log(e)\csc(5m^2) + 135e^{10m^2}\cot(5m^2)\csc(5m^2) - 4\cot(m^2)\csc^2(m^2)\right)$$

26.

$$-60s^3$$



27.

$$840s^3z^5 \tan(7s^3z^2) \sec^2(7s^3z^2) + 120z^3 \sec^2(7s^3z^2) + 30 \sin^2(2z) \cos(2z)$$

28.

$$-6x^2 + 18x + 9$$

29.

$$\frac{3 \cdot \left( 3 \left( -7bw \tan(7b^2w) \sec^3(7b^2w) + \frac{15 \tan^2(9b)}{\cos^2(9b)} \right) \tan(3b) - \frac{10 \tan^3(9b) - 3 \sec^3(7b^2w)}{\cos^2(3b)} \right)}{2 \tan^3(3b)}$$

30.

$$C + \sqrt{-6l^2 - 9l - 4}$$

31.

$$C - 42r \cot(8r^3) + 10 \sin(7r^3)$$

32.

$$\frac{240ny \tan(10n^2y)}{\cos^2(10n^2y)} + \frac{70 \tan(5n)}{\cos^2(5n)}$$

33.

$$\frac{36c^{\frac{5}{6}}v^{\frac{11}{6}}}{11} - 3c^{\frac{3}{5}}v + \frac{5c^{\frac{5}{4}}v^7}{7} - \frac{20\sqrt[3]{c}v^{\frac{6}{5}}}{3} - \frac{30cv^{\frac{7}{6}}}{7}$$

34.

$$C + \sqrt[3]{-2l^2 - 9l + 7}$$

35.

$$-\frac{24e^{4u^2}mu}{\sin^2(m^2u)}$$

36.

$$-24y^2 - 14y - 5$$

37.

$$C + 5 \cos^2(4n) - \tan(4n)$$

38.

$$\frac{2 \left( -9l^{\frac{8}{15}}t^{\frac{4}{5}} + 45l^{\frac{7}{3}} - 7\sqrt[3]{t} + 3 \right)}{3\sqrt[3]{l}}$$

39.

$$16m - 6q^2$$

40.

$$-48r \cot(3r^2) \csc(3r^2)$$

41.

$$C + \sqrt{-4c - 10}$$

42.

$$24an(-18a^3n^2 \sin(3an^2) \cos^2(3an^2) + 2a^2 \cos^3(3an^2) - 21n \tan(8an^3) \sec^3(8an^3))$$

43.

$$C - 3 \cot^2(9m) + 5 \csc^2(7m)$$

44.

$$y(3y^2 - 4y - 4)$$

45.

$$-\frac{160 \cot(10l)}{\sin^2(10l)}$$

46.

$$48mv$$

47.

$$16d^3 + \frac{2\sqrt[4]{m}}{\sqrt{d}} - \frac{2\sqrt[3]{m}}{\sqrt[3]{d}}$$

48.

$$C - 10 \cos(2c^2) - 6 \sec^2(2c)$$

49.

$$10p^2 \cot(2mp^2) \csc(2mp^2)$$

50.

$$C + \sqrt[3]{-c^2 + 4c - 9}$$

51.

$$C + \sqrt{7v + 10}$$

52.

$$\frac{70l^2}{\sin^2(7l^2z)} + 9 \sin(6z)$$

53. The indefinite integral is

$$\frac{24t^{\frac{11}{6}}}{11} + \frac{5t^{\frac{8}{5}}}{4} + \frac{8t^{\frac{3}{2}}}{3} + \frac{9t^2}{2} - 2t$$

The value of the definite integral is 157.335403624731.

54.

$$576e^{4n^2q^2}n^4q^5\sin^2(2n^2q^3)\cos(2n^2q^3)+256e^{4n^2q^2}n^4q^4\log(e)\sin^3(2n^2q^3)+96e^{4n^2q^2}n^2q^2\sin^3(2n^2q^3)-90\cot^4(6q)$$

55.

$$320e^{4t^2}t(5b\tan(5bt^2)+2\log(e))\sec^2(5bt^2)$$

56.

$$\frac{n(-12n^3+14n^2-15n+12)}{6}$$

57.

$$135de^{5d}\sin(9dl)$$

58.

$$C-18t^4\tan^2(3t^2)-6\csc^2(8t)$$

59.

$$-81d^3v^6\sin(2d^2v^3)-108dv^3\sin^2(d^2v^3)-12v\sin(2v^2)$$

60.

$$16t(-8n^2t^2\sin(4n^2t)\cos(4n^2t)-21n\sin(7nt^2)\cos^2(7nt^2)+3t\cos^2(4n^2t))$$

61.

$$\frac{576b^2e^{2q}q\cot(8b^2q^2)}{\sin^2(8b^2q^2)}-36e^{2q}\log(e)\cot^2(8b^2q^2)+56\tan^2(7q)+56$$

62.

$$-20w\sin(10w^2)$$

63.

$$12s+5$$

64.

$$C+80l^4\sec^3(10l^2)-10\sin^3(10l^3)$$

65.

$$u(-2u^2-4u-3)$$

66.

$$48n \left( -\frac{6nr}{\sin^2(2nr)} + \cot(2nr) \right) \cot^2(2nr)$$

67.

$$C + \sqrt[3]{10l-1}$$

68.

$$-24b^5c^{\frac{5}{4}} - 9b^2c + \frac{2c^3}{\sqrt{b}}$$

69.

$$C - 10\sin^2(3b) + 5\csc^2(9b)$$

70.

$$-84b^2v\tan^3(6v^2)\tan(7b^3v)\sec^2(7b^3v)$$

71.

$$378p^3t\sin(12p^3t) - \frac{81p^2\cot^2(9p^2t)}{\sin^2(9p^2t)} + 63\sin^2(6p^3t)$$

72.

$$C - 10\cos^2(10q) - 3\tan(8q)$$

73.

$$480e^{10u^2w}u^2(w\tan(5u^2w^2) + \log(e))\sec(5u^2w^2)$$

74.

$$n^2 \cdot (60nq^6 - 80n + 60q^4)$$

75.

$$50u^2w \left( 21u^2\sin(5w^2)\cos^2(5w^2) - \frac{2}{\sin^2(5u^2w^2)} \right)$$

76.

$$v \left( \frac{7\sqrt[3]{p}v^4}{5} - p^{\frac{5}{2}} + 3p^{\frac{3}{2}} + 2\sqrt{p} + 5 \right)$$

77.

$$-\frac{20a^{\frac{5}{2}}l^{\frac{3}{2}}}{3} - 2l$$

78.

$$-\frac{60v^2\tan(3mv^2)}{\cos^2(3mv^2)}$$

79.

$$-1400ae^{7a^2} \log(e) \cot^2(7t^2)$$

80.

$$-\frac{25\sqrt[5]{n}x^{\frac{8}{5}}}{4} + 6n^{\frac{2}{3}}x - \frac{18x^{\frac{7}{2}}}{7} + \frac{20x^{\frac{3}{2}}}{3}$$

81.

$$C + 81u^2 \cot^3(4u^2) + 2 \sec^3(5u^2)$$

82.

$$C + \sqrt{d^2 + 2d - 5}$$

83.

$$30c^2 \cot^3(2l)$$

84. The indefinite integral is

$$-\frac{30s^{\frac{11}{5}}}{11} - 3s^3 + 10s$$

The value of the definite integral is -128.001494861345.

85.

$$6 - 4a$$

86.

$$-9q^2 + 8q - 5$$

87.

$$20d^4 - 4v^{\frac{3}{2}} + \frac{5}{6\sqrt[6]{d}}$$

88.

$$0$$

89.

$$-16$$

90.

$$C - 3 \tan(2t^2) + 2 \tan(9t^2)$$

91.

$$C + \sqrt{5u^2 - 4u - 1}$$

92.

$$-16b^3 + 27b^2 + 20b + 2$$

93.

$$8m \left( -12b \cot(8bm^2) \cot(4b^2m) \csc^3(4b^2m) + \frac{m(8 \csc^3(4b^2m) - 9 \sec(6m^2))}{\sin^2(8bm^2)} \right) \tan^3(8bm^2)$$

94.

$$-6a^{\frac{2}{3}} \sqrt[5]{n}$$

95.

$$-9b^6n - \frac{12bn^{\frac{7}{4}}}{7} + 5n^{\frac{9}{5}}$$

96.

$$0$$

97.

$$-30n^4x - 45n^2 + 3x^4$$

98.

$$C + 45c \csc^2(4c^2) + 3 \sec^2(c^3)$$

99.

$$C - 56u \sin^3(10u^2) + 2 \csc(3u^3)$$

100.

$$C + 3\sqrt{-w-1}$$

101.

$$\frac{4}{\sqrt[3]{c}} + \frac{18}{5c^{\frac{2}{5}}}$$

102.

$$\frac{a(60a^6s^6 - 42a^4 + 245a^2 + 630s^6 + 840)}{105}$$

103.

$$4r^{\frac{9}{4}}y^{\frac{3}{2}} - r^3 - 10r\sqrt[5]{y} - 3ry^{\frac{3}{2}} - 4r$$

104.

$$8b(-20bt \cot(4b^2t^2) \csc^2(4b^2t^2) + \tan^2(4bt) + 1)$$

105. The indefinite integral is

$$\frac{9u^{\frac{7}{3}}}{7} - \frac{8u^{\frac{5}{2}}}{5} + \frac{2u^{\frac{3}{2}}}{3} - u$$

The value of the definite integral is -0.647619047619048.

106.

$$-\frac{3\sqrt[3]{cw^{\frac{4}{3}}}}{4} + \frac{3w^2}{2}$$

107.

$$q(35b^3q^3 - 12b^2 + 16)$$

108.

$$0$$

109.

$$\frac{-2v(3\cos^2(4v) - 2\csc^2(8z^3))\sin(8v^2) + 3\sin(4v)\cos(4v)\cos(8v^2)}{\cos^2(8v^2)}$$

110.

$$-15q^2 - 8q - 4$$

111.

$$6w(-2b^4w - 1)$$

112.

$$\frac{s^2(-36s^3 + 35s^2t^6 + 100t^2)}{20}$$

113.

$$48e^{6yz^2}z(-3z\log(e)\cos(10yz) - 3z\log(e) + 5\sin(10yz))$$

114.

$$C + \sqrt[3]{-9l - 6}$$

115.

$$12u(qu + 1)$$

116.

$$-162w^2\cot(10l^3)$$

117.

$$C - 3\cos^2(q^2) - 6\sec(9q^2)$$

118.

$$180m^3p^2 - 160m^3 + 48mp^5$$

119.

$$2d \left( -40d^2u^2 \tan(2d^2u^2) \sec(2d^2u^2) - 10 \sec(2d^2u^2) + \frac{7 \cot(du)}{\sin^2(du)} \right)$$

120.

$$C + \sqrt[3]{-v^2 + 3v + 9}$$

121.

$$\frac{c(c^2 + 3c - 30)}{3}$$

122.

$$-4e^{9y^2} \cdot \left( 9y \log(e) \tan(4y) + \frac{4}{\cos^2(4y)} \right) \tan(4y)$$

123.

$$-\frac{7r^{\frac{2}{3}}}{4z^{\frac{3}{4}}}$$

124. The indefinite integral is

$$3v$$

The value of the definite integral is 6.000000000000000.

125. The indefinite integral is

$$-\frac{5b^{\frac{8}{5}}}{4} + 6b^{\frac{5}{3}} + \frac{14b^{\frac{3}{2}}}{3} - \frac{10b^3}{3} - \frac{3b^2}{2}$$

The value of the definite integral is -101.951560607421.

126.

$$-\frac{15w^3 \tan(2lw^3) \sec^3(2lw^3)}{\cos(4w^3)}$$

127.

$$C + \sqrt[3]{3-v}$$

128.

$$C + 28x^3 \cot(3x^2) - 7 \csc^3(5x)$$

129.

$$e^{10m} (32mr^2 \sin(4m^2r^2) - 40 \log(e) \cos(4m^2r^2) - 40 \log(e))$$



130.

$$-42p^5q + 7 - \frac{5}{6p^{\frac{5}{6}}}$$

131.

$$-8z^2 \sin(2bz^2)$$

132.

$$z \left( -\frac{9s^3z^4}{5} - 2s^3 + z^2(-2s^5 - 1) \right)$$

133.

$$\frac{y^3(-126w^2y^4 \tan(2wy^3) - 42wy + 32 \sin(wy^3) \cos(wy^3))}{(2 \sin^2(wy^3) - 1)^3}$$

134.

$$C + \sqrt{2l - 8}$$

135.

$$C + 60n^2 \sec^3(2n^2) + 5 \csc^2(10n^2)$$

136.

$$-\frac{3nt^2 \sin(10n^3)}{\sin^2(nt^3)}$$

137.

$$36e^{6lx^2}x^2 \left( -\frac{14l}{\sin^2(7l^2x^2)} + 3 \log(e) \cot(7l^2x^2) \right) \cot(7l^2x^2)$$

138.

$$\frac{16r^{\frac{6}{3}}v^{\frac{5}{2}}}{5} - 6r^{\frac{5}{3}}v - 5r^{\frac{3}{2}}v^2 + 3v$$

139.

$$C - 80v \csc^2(5v^2) - 7 \cos^2(v^2)$$

140.

$$\frac{n(7n^4t - 15n + 25t(-t^5 + 2t^3 + 2))}{5}$$

141.

$$s^3 \cdot (32b^3 + 40b^2s + 40s)$$

142.

$$0$$

143.

$$-\frac{18l}{\cos^2(3lw)}$$

144.

$$\frac{-105c^{\frac{9}{10}}d + 6d^6}{10c^{\frac{2}{5}}}$$

145.

$$24y \left( -3e^{10y^2} n y \cos(2n^2 y^2) - \tan(2ny) \sec^2(2ny) \right)$$

146.

$$-8b^3 + 12b^2 + 16b - 9$$

147. The indefinite integral is

$$-\frac{20s^{\frac{11}{5}}}{11} - \frac{18s^{\frac{5}{3}}}{5} + \frac{8s^5}{5}$$

The value of the definite integral is 35.2347243894869.

148.

$$\frac{3d^{\frac{3}{2}}}{l^{\frac{2}{5}}} - \frac{10d^2}{3l^{\frac{2}{3}}} + 5d + \frac{16}{3\sqrt[3]{l}}$$

149.

$$0$$

150.

$$36e^{4v} \left( -5x^2 \sin(20vx^2) + \log(e) \cos(20vx^2) + \log(e) \right)$$

151.

$$-12y^3 + 6y^2 - 12y + 7$$

152.

$$C + 12z^2 \sec^2(3z^3) - 7 \csc^2(2z)$$

153.

$$-240e^{4rt} r \log(e) \sin^2(7r)$$

154.

$$6a^2 - 6$$

155.

$$\frac{126nz^2}{\cos^2(9n^2z^2)} + 60 \sin(10n)$$

156.

$$-48 \cos(8d)$$

157.

$$-36t^3 + 3t^2 + 10t + 3$$

158.

$$C + \sqrt{3z + 5}$$

159.

$$\frac{9\sqrt{m}}{2} - 42m^5 - 16m + \frac{2}{m^{\frac{3}{4}}}$$

160.

$$C + \sqrt[3]{-7q^2 + 7q - 9}$$

161.

$$-6d^3$$

162.

$$60n^4u^4$$

163.

$$-288a^2e^{6a^2y}y\log(e)\cos^3(a^2) + 144a^2e^{6a^2y}\sin(a^2)\cos^2(a^2) + \frac{288ay^2\tan(8a^2y^2)}{\cos^2(8a^2y^2)} - 24e^{6a^2y}\cos^3(a^2)$$

164.

$$-6q - 10$$

165.

$$\frac{qr(4q^5 + 3q^4r^2 - 18r^5 - 6r)}{3}$$

166.

$$-40u^3 + 21u^2 - 20u + 8$$

167.

$$\frac{6\left(\frac{m^3 \cdot (4\cot^3(7m^3) + \csc^2(8z^3))}{\cos^2(3m^3z)} + 16z^2 \tan(3m^3z) \cot(8z^3) \csc^2(8z^3)\right)}{7 \tan^2(3m^3z)}$$

168.

$$C - 9\sin^2(4z) - 10\sec^2(10z^2)$$

169.

$$6cz(-12c - z^2)$$

170.

$$-3c^2 + 6cy - 6$$

171.

$$\frac{50\sqrt[5]{ty^{\frac{11}{5}}}}{11} + \frac{18\sqrt{ty^{\frac{11}{6}}}}{11} + 7\sqrt{ty} + \frac{3ty^{\frac{4}{3}}}{2} + \frac{8ty^{\frac{3}{2}}}{3}$$

172.

$$8l^3p \left( 9p \tan(l^3p^3) \sec^3(l^3p^3) - \frac{35 \sin(14p^2)}{2} \right)$$

173.

$$\frac{672mn^4 \tan^2(8m^2)}{\cos^2(8m^2)} - 3 \sin(2m)$$

174.

$$\frac{10n^{\frac{9}{5}}}{9} + 2n^4\sqrt{s} + 10ns^{\frac{5}{2}} - 9ns^2$$

175.

$$\frac{3 \left( -4a^2r(8\cos^3(2r^2) - 7\cot^3(7ar)) \sin(3a^2r^2) + \left( -\frac{49a\cot^2(7ar)}{\sin^2(7ar)} + 32r \sin(2r^2) \cos^2(2r^2) \right) \cos(3a^2r^2) \right)}{4\cos^3(3a^2r^2)}$$

176.

$$6n^2 - 16n + 3$$

177.

$$-9w^2 - 20w + 6$$

178.

$$20e^{4t^2} (d^2 \tan(d^2t) + 8t \log(e)) \sec(d^2t)$$

179.

$$C + \sqrt[3]{9x^2 - x + 9}$$

180.

$$C - 49q^3 \sin^3(8q^3) - 10 \tan^3(10q^2)$$

181.

$$12m(-3m^2v^3 + 6m^2 - v^5)$$

182. The indefinite integral is

$$\frac{d(2-d)}{2}$$

The value of the definite integral is -16.00000000000000.

183.

$$C + \sqrt{-3x^2 - 10x - 5}$$

184.

$$n^2(-224e^{5n}n^2r^2\cos(r) - 448e^{5n}n^2r\sin(r) + 20\sin(4n^2r))$$

185.

$$C + \sqrt[3]{7x^2 - 7x - 9}$$

186.

$$C - 18p^3 \csc^2(p^3) - 5 \cot(2p^3)$$

187.

$$C + 10 \cot^2(7w) + 10 \csc^2(9w)$$

188.

$$\frac{3l}{\sin^2(lx)} + \frac{48x}{\cos^2(6x^2)}$$

189.

$$-20s - 5$$

190.

$$81 \sin(9s)$$

191.

$$C - 5 \cot(m) - 4 \cot(2m)$$

192.

$$32p^3 - 15p^2 + 14p - 1$$

193.

$$\frac{140br^2}{\cos^2(7b^2r^2)} - 180e^{9r^2} \cos(3b)$$

194.

$$10s \left( -\frac{n(4\cos^2(10ns^2) - 3\cot^3(5n^2s^3))}{\sin^2(5n^2s)} + s \left( -\frac{9ns\cot^2(5n^2s^3)}{\sin^2(5n^2s^3)} + 4\sin(20ns^2) \right) \cot(5n^2s) \right) \tan^2(5n^2s)$$

195.

$$\frac{w(8w^5 - 2w^2 + 3w(t^3 - 6) + 24)}{6}$$

196.

$$4bv^2 \cdot \left( \frac{49 \sin(14b^2v^2)}{2} + \frac{54 \cot(6b^2v^2)}{\sin^2(6b^2v^2)} \right)$$

197.

$$r^2 \cdot (18r^3 - 50r^2t^4 + 12t^3 + 3t)$$

198.

$$-45u^2x^4 + 48ux^5 - 2$$

199.

$$10e^{6z} (7r^2 \cot(7r^2z) - 6 \log(e)) \csc(7r^2z)$$

200.

$$C + \sqrt[3]{9 - 2n}$$